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INLET EFFICIENCY PARAMETERS FOR SUPERSONIC COMBUSTION RAMJET ENGINES

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FOREWORD

This report was prepared by the Ramjet Analysis Branch of the AF Aero Propulsion Laboratory, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio. The work described was accomplished under Task 301201, "Special Ramjets," of Project 3012, "Ramjet Technology," and represents work accomplished from November 1962 to April 1963.

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ABSTRACT

In this report, the various definitions of inlet efficiency, which are currently in use for supersonic combustion ramjet engines, are surveyed. Derivations of each of these parameters and the relationships between them are presented. Charts for the conversion from one parameter of efficiency to another are included. These charts are based on real gas properties. Attention is drawn to the application of available energy concepts to the supersonic combustion ramjet engine, and a definition of inlet effectiveness is introduced.

This report has been reviewed and is approved.

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LIST OF SYMBOLS

b	availability parameter	
C_p	specific heat at constant pressure	$\frac{Btu}{lb\ °R}$
C_v	specific heat at constant pressure	$\frac{Btu}{lb\ °R}$
g	gravitation constant (g = 32.174 ft/sec)	
h	static enthalpy	$\frac{Btu}{lb}$
h_o	isentropic expansion enthalpy (See Figures 1 and 3)	$\frac{Btu}{lb}$
h_t	total enthalpy	$\frac{Btu}{lb}$
J	mechanical equivalent of heat ($J = 778 \frac{ft\ lb}{Btu}$)	
K_D	process efficiency	
m	molecular weight	$\frac{lb}{mole}$
M	mach number	
P	static pressure	atm
P_r	relative pressure function (See Reference 11)	
P_t	total pressure	atm
P_{2i}	ideal static pressure at station 2	atm
R	gas constant for air ($R = \frac{\bar{R}}{m}$)	$\frac{ft\ lb}{lb\ °R}$
$\bar{R}$	universal gas constant ($\bar{R} = 1545 \frac{ft\ lb}{mole\ °R}$)	

LIST OF SYMBOLS (CONT'D)

s	entropy	$\frac{Btu}{lb\ ^\circ R}$
T	static temperature	°R
T_o'	isentropic expansion temperature ($T_o' = f(h_o')$)	°R
T_t	total temperature	°R
u	internal energy	$\frac{Btu}{lb}$
V	velocity	ft/sec
V_o'	isentropic expansion velocity (See Figures 1 and 3)	ft/sec
γ	ratio of specific heats	
η_1	hypothetical efficiency parameter (See Equation 5)	
η_2	hypothetical efficiency parameter (See Equation 7)	
η_{EL}	polytropic efficiency	
η_I	total pressure recovery	
η_{KE}	kinetic energy efficiency	
$(\eta_{KE})_{MIN}$	minimum value of kinetic energy efficiency possible for a compression process	
η_R	static pressure recovery	
η_{TH}	effectiveness or thermodynamic efficiency	
ρ	density	lb/ft^3
$\Phi(T)$	entropy function (See Reference 11)	
$\frac{\Delta s}{R}$	dimensionless entropy change	

Subscripts:

- o Condition in Air Stream Preceding Inlet
- 2 Condition at Exit Plane of Inlet
- i Ideal Condition (Arrived at Isentropically)

LIST OF SYMBOLS (CONT'D)

is Isentropic Process

P Constant Pressure Process

r Reference Condition

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INTRODUCTION

In various analyses of the performance of the supersonic combustion* ramjet engine, some confusion has been caused by the presentation of inlet performance in terms of different definitions of efficiency. Although this confusion has been associated with the definitions of inlet efficiency, it is symptomatic of the difficulty of defining meaningful efficiencies for each major component of the engine -- the inlet, the combustor, and the nozzle. These difficulties can become acute when non-adiabatic, non-uniform flow is encountered.

The authors are convinced that the only rational manner of formulating meaningful efficiencies is to base such efficiencies on the available energy concepts expounded by Keenan (References 1 and 2), Stepanoff (Reference 3), and Bruges (Reference 4). Application of this approach to the inlet leads to the definition of the "effectiveness" of the inlet.

The aims of this report are to draw attention to available-energy methods and to present a preliminary survey of the existing parameters that may be used to define the performance of inlets operating with supersonic discharge velocities.

In this report, only adiabatic uniform flow is considered. However, the extension of the available-energy approach to non-adiabatic cases can be accomplished quite easily.

PRELIMINARY DISCUSSION

In the case of subsonic burning ramjets, the primary losses in the inlet were associated with the shock configuration. For a given configuration, the subsonic exit velocity desired could be obtained by correct choice of the area ratio of the subsonic diffuser. Normally, this subsonic velocity was kept fairly low to maintain the loading of the combustor within acceptable limits and to reduce the losses of internal total pressure. The losses associated with the subsonic flow were usually small in comparison with those suffered through the shock system and, in project groups, a common assumption was that, for a given shock configuration, the inlet efficiency was primarily dependent on the free stream Mach number. For such ramjet engines, the inlet efficiencies most commonly used were the total pressure recovery (References 5 and 6), η_I, and the kinetic energy efficiency (Reference 6), η_{KE}. A hypothetical process of compression for such an inlet is shown in Figure 1 where it is seen that for the low subsonic exit velocities the flow is diffused almost to stagnation conditions. The performance of subsonic-burning engines decreases markedly with increasing hypersonic speeds. One of the principal causes of this degradation in performance was the decrease in efficiency of a given inlet with increasing flight Mach number.

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*The term "supersonic combustion ramjet engine" will be used in this report to mean a ramjet engine in which the flow is supersonic at the combustion chamber entrance, and combustion occurs under supersonic flow conditions.

If the amount of diffusion accomplished by the inlet were reduced, resulting in supersonic exit velocities from the diffuser, then the losses associated with the inlet could be decreased. Thus, if a disproportionate increase in combustor and nozzle losses could be avoided, a performance superior to that of the subsonic combustion engine should be obtained (Reference 7). Thus, the superior performance of the supersonic combustion engine is seen to depend on the relative trade-off between the increased efficiency of the inlet and the increased losses associated with the combustion-expansion system as the amount of slow diffusion is reduced. Therefore, it is apparent that, in the case of the supersonic combustion engine operating at a given flight condition, the overall performance is linked closely to the variation of inlet performance with the amount of diffusion performed.

In early performance studies of the supersonic combustion engine system it was necessary, in the absence of experimental data, to use empirical relationships relating the inlet performance to the discharge velocity. Dugger (Reference 8) utilized the relationship

$$\eta_{KE} = 0.94 + 0.06 \frac{M_2}{M_0} . \tag{1}$$

The boundary conditions for this relationship were appropriate: as M_2 approached low subsonic values, η_{KE} tended to an acceptable value (0.94), and, with zero diffusion, $M_2 = M_0$, and η_{KE} became equal to unity. Another analysis performed by C. Lindley (Reference 9) of The Marquardt Corporation utilized the empirical relationship

$$\eta_{KE} = K_D + (1 - K_D) \left(\frac{V_2}{V_0} \right)^2 . \tag{2}$$

It was realized later that the term K_D in Equation (2) possessed thermodynamic significance and was of interest as a parameter of inlet efficiency in its own right. However, the reader should note that in both of the preceding cases (Equations 1 and 2), the inlet efficiency was expressed in terms of the familiar kinetic energy efficiency. More recently, various definitions have appeared in the literature and two additional definitions will be considered in this report.

THE CONCEPT OF AVAILABLE ENERGY

In many thermodynamic processes, efficiencies are defined that compare the amount of work obtained from the process with a theoretical output of work based on some ideal process that takes place within the same boundary conditions as those for the real process. Thus, in the case of a simple expansion of the nozzle, the actual output of thrust may be compared with the thrust that would be obtained by an expansion on an isentropic path through the same pressure ratio as that of the actual process. An essential factor in the definition of efficiency is that a common boundary condition is used for both real and hypothetical processes. For example, in this case, the same initial and final pressures, but not the same initial and final states are used. The authors suggest that a more meaningful way of defining efficiencies is to compare the output of a given real process with the maximum available output that could be produced in a hypothetical process with the same change of state as the real process. Obviously, the available output must be available with respect to some reference state. For an air-breathing engine system, the obvious reference state corresponds to ambient conditions. In order to assess what the change in the available work is between any two states, we first quote an expression for the work available from our working fluid at state (P_2, T_2) relative to ambient conditions (P_0, T_0). The maximum work will be obtained if the system changes reversibly between these states.

Following Keenan (Reference 1) and Bruges (Reference 2), it can be shown that the available energy, with respect to the given reference state, is generally given by

$$\left[u - T_0 s + \frac{P}{\rho}\right]_0^2 .$$

and, for the case of steady flow with negligible kinetic energy, is expressed as

$$\left[h - T_0 s\right]_0^2 .$$

Keenan has termed the quantity $[h - T_0 s]_0^2$ the availability, and it is commonly denoted by the symbol "b". The change in availability between any two states 0 and 2 is correspondingly

$$\left[h - T_0 s\right]_0^2 = b_2 - b_0 .$$

Of course, for components of air-breathing engines, the kinetic energy of the flow is appreciable and must be included in the above expressions, in which case one obtains as the available energy:

$$\left[h + \frac{V^2}{2gJ} - T_0 s\right]_0^2 = \left[b + \frac{V^2}{2gJ}\right]_0^2 .$$

It is interesting to note that the availability, b, can be represented on the h - s diagram, i.e., since

$$b = h - T_0 s$$

we have

$$h = b + T_0 s$$

and, for a given reference state, we have a straight-line representation as shown in Figure 2.

For a more rigorous and complete treatment of the available-energy concept, the reader is directed to References 1 through 4. Reference 12 is also challenging, but informative, reading.

The preceding brief comments have been made to stimulate thought regarding the concepts of efficiency and to introduce the concept of available energy. We will now examine the various parameters used to define the efficiency of the inlet of a supersonic combustion ramjet engine.

SURVEY OF DEFINITIONS OF EFFICIENCY

GENERAL

The parameters of efficiency that will be considered here are:

- η_{TH} effectiveness or thermodynamic efficiency;
- η_{KE} kinetic energy efficiency
- K_D process efficiency;
- η_I total pressure recovery;
- η_R static pressure recovery;
- η_{EL} polytropic efficiency;
- $\frac{\Delta s}{R}$ increase in dimensionless entropy.

In the following paragraphs it will be assumed that the gas is thermally perfect and that the flow is adiabatic, uniform, and one-dimensional. For some of the derivations, the analysis must be restricted to gases that are calorically perfect. We will also assume that the inlet considered will operate with a specified amount of diffusion, an amount corresponding to a specified $\frac{V_2}{V_0}$. Thus, the process will be an increase in enthalpy from h_0 to h_2.

In relation to heat engines, Keenan (Reference 2) has proposed the use of an efficiency termed the effectiveness. In this definition, the actual work done is compared to an ideal work that is taken to be the change in available energy that corresponds to the actual change in the state of the working fluid. This definition is now applied to the process of compression. By referring to Figure 2, we see that

$$\text{IDEAL WORK} = (h_2 - T_0 s_2) - (h_0 - T_0 s_0)$$
$$= (h_2 - h_0) - T_0 (s_2 - s_0)$$

$$\text{ACTUAL WORK} = h_2 - h_0 .$$

So the effectiveness is

$$\eta_{TH} = 1 - \frac{T_0 (s_2 - s_0)}{h_2 - h_0} .$$

At first sight, this appears to be a meaningful and easily evaluated definition since calculations are performed directly in terms of entropy and enthalpy and are confined to the lower areas of the Mollier Chart. Additionally, the extension to non-adiabatic flows is not expected to be difficult since absorption of heat through the inlet walls can be accounted for on an available-energy basis. The appearance of the reference temperature T_0 in the expression may seem surprising. However, T_0 is to be expected on thermodynamic grounds. It will be interesting to see whether the effectiveness is a more characteristic quality of a given inlet than alternative definitions of efficiency. Some theoretical values of η_{TH} will be presented in a later section of this report.

KINETIC ENERGY EFFICIENCY

The kinetic energy efficiency is defined as the ratio of the kinetic energy available by isentropic expansion from the final condition (P_2, T_2) to the initial pressure P_0, to the initial kinetic energy in the free stream. The efficiency is given by

$$\eta_{KE} = \frac{h_t - h'_0}{h_t - h_0} = \frac{V_0'^2}{V_0^2} . \quad (3)$$

This process is illustrated in Figure 3. One of the most frequent criticisms of this parameter is that it has a value very close to unity and is, therefore, a very insensitive parameter. This objection can be appreciated most easily if the lower level of η_{KE} is estimated. The minimum value of η_{KE} that is of interest corresponds to a process following a constant pressure line. For this condition,

$$(\eta_{KE})_{MIN} = \left(\frac{V_2}{V_0}\right)^2 , \quad (4)$$

and the minimum value of η_{KE} is directly related to the amount of diffusion.

Thus, for processes of interest we have

$$1 > \eta_{KE} > \left(\frac{V_2}{V_0}\right)^2 .$$

In some cases the supersonic combustion engine may operate at a diffusion value of 0.975 in which case η_{KE} can only vary between 0.95 and 1.00. Of course for the subsonic combustion engine operating at very low values of V_2/V_0, the kinetic energy efficiency could vary through much wider limits. It is of interest to consider an efficiency parameter, say η_1, which takes the value zero at $(\eta_{KE})_{MIN}$ and increases to unity when $\eta_{KE} = 1.00$. If a linear variation of η_1 in the interval $\eta_{KE} = (\eta_{KE})_{MIN}$ to $\eta_{KE} = 1.00$ is assumed, then

$$\eta_1 = \frac{\eta_{KE} - \left(\frac{V_2}{V_0}\right)^2}{1 - \left(\frac{V_2}{V_0}\right)^2} ,$$

or alternatively,

$$\eta_{KE} = \eta_1 + (1 - \eta_1)\left(\frac{V_2}{V_0}\right)^2 . \quad (5)$$

The preceding derivation leads to the definition of the process efficiency "K_D" (compare with Equation 2). Despite the narrowing range of η_{KE} at high values of the velocity ratio, this efficiency definition is easily utilized in calculations of ramjet performance because the necessary calculations can be confined to the lower area of the Mollier diagram. Thus, the entropy increase associated with the process AB shown on Figure 3 can be evaluated along the isobar AC. Thus, given h_0, V_0, $\frac{V_2}{V_0}$, and η_{KE}, one can calculate

$$h'_0 = (1 - \eta_{KE})(h_t - h_0) + h_0$$

and obtain s_2 directly from a Mollier chart, or one can utilize Reference 11, where appropriate, to calculate s_2 from the relationship

$$s_2(P_2, T_2) - s_0(P_0, T_0) = s_2(P_0, T'_0) - s_0(P_0, T_0)$$

$$= \phi(T'_0) - \phi(T_0)$$

where

$$\phi(T) = \int_{T_r}^{T} \frac{C_p\, dT}{T} \quad .$$

Thus, for given initial conditions, the entropy rise associated with the process is implicity a function of η_{KE}. For a polytropic gas, one may write

$$s_2(P_0, T'_0) - s_0(P_0, T_0) = C_p \ln \frac{T'_0}{T_0}$$

$$= C_p \ln \frac{h'_0}{h_0}$$

so that

$$\eta_{KE} = \frac{\frac{h_t}{h_0} - e^{\frac{s_2 - s_0}{C_p}}}{\frac{h_t}{h_0} - 1} \quad . \qquad (6)$$

One can say in summarizing that, for given initial conditions, η_{KE} determines the entropy rise associated with the process. The evaluation of engine performance is simplified because calculations generally are confined to the lower regions of the Mollier diagram. However, the limited range of η_{KE}, particularly at low amounts of diffusion, does not make this efficiency a very sensitive measure of inlet performance.

PROCESS EFFICIENCY

As mentioned in the preliminary discussion, the parameter of process efficiency was introduced by The Marquardt Corporation. The process efficiency was defined in the relationship

$$\eta_{KE} = K_D + (1 - K_D)\left(\frac{V_2}{V_0}\right)^2 .$$

The process efficiency can, however, be derived by considering an efficiency similar to η_{KE}, but based on the change of kinetic energy in the diffusion process, rather than on the absolute values of the kinetic energy. Thus, referring to Figure 3, and writing

$$\eta_2 = \frac{V_0'^2 - V_2^2}{V_0^2 - V_2^2} = \frac{h_2 - h'_0}{h_2 - h_0} \quad , \qquad (7)$$

we have

$$(1-\eta_2)\left(\frac{V_2}{V_0}\right)^2 = \left(\frac{V_0^2 - V_0'^2}{V_0^2 - V_2^2}\right)\left(\frac{V_2}{V_0}\right)^2 . \tag{8}$$

By summing Equations (8) and (7) we have

$$\eta_2 + (1-\eta_2)\left(\frac{V_2}{V_0}\right)^2 = \left(\frac{V_0'}{V_0}\right)^2 = \eta_{KE} .$$

Thus,

$$K_D = \eta_2 = \frac{h_2 - h_0'}{h_2 - h_0} .$$

Obviously, for subsonic burning engines where V_2/V_0 is small, the value of K_D approaches that of η_{KE}. If inlet performance is given as a function of K_D and V_2/V_0, then obviously η_{KE} can be calculated. It follows that calculations once again will be confined to the lower regions of the Mollier diagram.

TOTAL PRESSURE RECOVERY (η_I)

The total pressure recovery of an intake system is defined as

$$\eta_I = \frac{P_{t2}}{P_{t0}} . \tag{9}$$

For adiabatic flow, it is evident that once P_{to} and η_I are specified, the entropy rise associated with the process, in theory, can be determined. In practice, however, we find that existing Mollier charts do not cover the range required. Since the only state conditions of real interest are those corresponding to points A and B (Figure 3), the extension of existing Mollier charts to embrace this definition of efficiency is not warranted.

STATIC PRESSURE RECOVERY

The static pressure recovery is defined as

$$\eta_R = \frac{P_2}{P_{2i}} \tag{10}$$

where

P_2 = static pressure at exit from diffuser

P_{2i} = ideal static pressure corresponding to isentropic compression from T_0 to T_2.

For a given amount of diffusion, P_{2i} is easily identified from the Mollier diagram, or, where appropriate, from Gas Tables (Reference 11) using the relative pressure function P_r. For a thermally perfect gas of fixed composition, the entropy increase is given by:

$$s_2 - s_0 = R \ln \frac{P_{2i}}{P_2} .$$

Obviously, if both static and total pressures are within the non-dissociating region of the Mollier chart, then

$$s_2 - s_0 = R \ln \frac{P_{2i}}{P_2} = R \ln \frac{P_{t0}}{P_{t2}}$$

and

$$\eta_I = \eta_R . \qquad (11)$$

However, for high hypersonic speeds, the equalities expressed in Equation (11) do not hold, and

$$\eta_I \neq \eta_R .$$

POLYTROPIC EFFICIENCY (η_{EL})

The polytropic efficiency parameter has not yet appeared in the literature concerning supersonic combustion engines. For the path shown in Figure 4, the polytropic, or small-stage, efficiency is defined (Reference 6) for an adiabatic process by

$$\eta_{EL} = \frac{dh_{is}}{dh} . \qquad (12)$$

The entropy change associated with the process can be derived by writing

$$Tds = dh - \frac{1}{\rho} dp . \qquad (13)$$

Thus, we have

$$dh_{is} = \frac{1}{\rho} dp .$$

so that

$$Tds = dh - dh_{is}$$

$$= \frac{dP}{\rho}\left(\frac{1}{\eta_{EL}} - 1\right)$$

or

$$Tds - \frac{dP}{\rho}\left(\frac{1}{\eta_{EL}} - 1\right) = 0 .$$

Hence,

$$s_2 - s_0 = \left(\frac{1}{\eta_{EL}} - 1\right) R \ln \frac{P_2}{P_0} .$$

This expression, together with an assigned value of h_2, allows location of the point (P_2, T_2) on the Mollier chart. Where appropriate, the entropy change can be calculated from Gas Tables.

Thus, with

$$s_2 - s_0 = \phi_2 - \phi_0 - R \ln \frac{P_2}{P_0}$$

and

$$s_2 - s_0 = \left(\frac{1}{\eta_{EL}} - 1\right) R \ln \frac{P_2}{P_0}$$

one obtains

$$s_2 - s_0 = (1 - \eta_{EL})(\phi_2 - \phi_0). \qquad (14)$$

The reader can readily see that polytropic efficiency of zero corresponds to an isobaric process.

DIMENSIONLESS ENTROPY INCREASE $\left(\frac{\Delta s}{R}\right)$

Molder, in Reference 10, has recently drawn attention to the dimensionless entropy increase parameter and recommends its use for the following reasons:

a. It does not require the determination of total conditions;

b. It is uniformly sensitive in the Mach number range of 12 through 25;

c. Calculations may be made easily with a Mollier Diagram.

However, the parameter gives no indication of relative merit. It is better to say that the relative loss is:

$$\frac{\left(\frac{\Delta s}{R}\right)}{\left(\frac{\Delta s}{R}\right)_{\text{CONSTANT PRESSURE}}}$$

so that

$$\eta_s = 1 - \frac{(\Delta s / R)}{(\Delta s / R)_p} .$$

With this definition,

$$\eta_s = \eta_{EL} .$$

RELATIONSHIPS BETWEEN THE VARIOUS PARAMETERS OF EFFICIENCY

From the previous examination of the various definitions, we will realize that the parameters of efficiency can be divided into two classes.

Class 1. Class 1 includes those parameters that implicitly define the entropy rise associated with the process, regardless of the amount of diffusion. Parameters in this class are η_{KE}, η_L, η_R, and $\frac{\Delta s}{R}$.

Class 2. Class 2 includes those parameters that relate the entropy increase to the amount of diffusion obtained from given initial conditions. Parameters in this class include η_{TH}, K_D, and η_{EL}. Class 2 parameters are more meaningful thermodynamically than are the parameters in Class 1, as shown by the derivation of η_{TH}.

In the case of the supersonic combustion engine it is always necessary, for a given flight condition, to specify efficiency as a function of the amount of diffusion. Since this specification is always required it would seem that either of the above classes of efficiency parameters could be used. Obviously, one would expect simpler calculation with Class 1 parameters in the regions where they can be utilized. For high hypersonic Mach numbers, η_I is discarded for reasons previously outlined. However, one advantage of using Class 2 parameters is that, for a given class of inlet configurations (e.g., two shock systems), there is always the possibility that the exchange rate between entropy rise and enthalpy increase is such that one parameter will suffice to describe inlet performance. In this event, a quantitative description of the inlet is simple and cycle-type calculations can be simplified. Thus, referring to Figure 5, in which the performance of a two-shock inlet at a Mach number of 10 is illustrated, we can see that K_D and η_{TH} are relatively constant over a fairly large range of velocity ratio. A similar result is shown in Figure 6, in which inlet performance at a Mach number of 25 is shown.

Figures 5 and 6 show that η_{KE} and η_R are poor parameters for use in specifying the performance of supersonic combustion ramjets. As expected, η_{KE} is near unity for the entire range and, therefore, is a poor indicator of performance. At hypersonic velocities, η_R approaches a very low value rapidly and for this reason it, too, is unsuitable.

η_{EL} is a useful parameter since it has a reasonable value over a large range of velocity ratio. However, it varys quite widely throughout the velocity ratio range and is, therefore, inferior to η_{TH} and K_D for most purposes.

At the present time, none of the parameters discussed has been universally accepted for use. For this reason it is often necessary to convert from one parameter to another. Figures 8 through 14 are conversion charts that permit conversion from one parameter to another on a real gas basis. These charts are plotted with velocity ratio $\left(\frac{V_2}{V_0}\right)$ as an independent variable.

Often the enthalpy ratio is specified instead of the velocity ratio. For this reason, Figure 7 has been included to permit conversion from enthalpy ratio to velocity ratio.

Analytical equations for conversion between the various parameters cannot be derived on a real gas basis. However, interesting characteristics are illustrated by the analytical expressions for the case of an ideal gas. The analytical expressions for an ideal gas, which relate the various parameters, are given in Table I in the Appendix. The equations presented in the table are derived in the appendix.

CONCLUSIONS

Process efficiency (K_D) has been shown to be a most useful parameter for use with supersonic combustion ramjet engines. Kinetic energy efficiency (η_{KE}), total pressure ratio (η_I), and static pressure recovery (η_R), which currently enjoy wide usage, have been shown to be relatively poor parameters for use in this application.

Two new parameters, thermodynamic efficiency (η_{TH}) and polytropic efficiency (η_{EL}), have been discussed and have been shown to be of interest when applied to supersonic combustion ramjet engines.

A number of charts have been included which permit rapid conversion from one parameter to another.

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APPENDIX

DERIVATION OF IDEAL GAS EQUATIONS RELATING VARIOUS PARAMETERS OF EFFICIENCY

In this Appendix, the equations relating the various parameters of efficiency will be derived. As pointed out previously, these derivations apply only for operation at low Mach number. Conversion between parameters at high Mach numbers can be accomplished only by use of appropriate charts that take into account real gas effects. The method employed in the derivation will be to first express the dimensionless entropy change ($\frac{\Delta s}{C_p} = \frac{S_2 - S_0}{C_p}$) in terms of each of the various parameters.

We start with the definition of η_{TH}:

$$\eta_{TH} = 1 - \frac{T_0 \, (s_2 - s_0)}{h_2 - h_0} \, .$$

This is easily tranformed to:

$$\frac{\Delta s}{C_p} = (1 - \eta_{TH}) \left(\frac{h_2}{h_0} - 1 \right) \, . \qquad (15)$$

Next, the expression for η_{EL} will be used; and we obtain

$$\eta_{EL} = \frac{1}{1 + \dfrac{s_2 - s_0}{R \ln \dfrac{P_2}{P_0}}} \, .$$

By rearranging, we find that

$$s_2 - s_0 = \left(\frac{1}{\eta_{EL}} - 1 \right) R \ln \frac{P_2}{P_0} \, . \qquad (16)$$

The entropy change can be expressed as

$$s - s_r = C_p \ln \frac{T}{T_r} - R \ln \frac{P}{P_r} \, . \qquad (17)$$

For this case,

$$s_2 - s_0 = C_p \ln \frac{T_2}{T_0} - R \ln \frac{P_2}{P_0} \, .$$

For an ideal gas,

$$\frac{h_2}{h_0} = \frac{T_2}{T_0}$$

Then,

$$R \ln \frac{P_2}{P_0} = C_p \ln \frac{h_2}{h_0} - (s_2 - s_0) \, . \qquad (18)$$

The substitution of Equation (18) into Equation (16) gives

$$s_2 - s_o = \left(\frac{1}{\eta_{EL}} - 1\right)\left[C_p \ln\frac{h_2}{h_o} - (s_2 - s_o)\right]$$

$$(s_2 - s_o)\left(\frac{1}{\eta_{EL}}\right) = \left(\frac{1-\eta_{EL}}{\eta_{EL}}\right) C_p \ln\frac{h_2}{h_o}$$

$$\frac{\Delta s}{C_p} = (1-\eta_{EL}) \ln\frac{h_2}{h_o} \quad . \tag{19}$$

Now, if we use the expression for η_R,

$$\eta_R = \frac{P_2}{P_{2i}} \quad .$$

Equation (17) for the actual compression process is, again

$$s_2 - s_o = C_p \ln\frac{T_2}{T_o} - R \ln\frac{P_2}{P_o} \quad . \tag{20}$$

For the ideal compression process Equation (17) becomes

$$s_{2i} - s_o = C_p \ln\frac{T_{2i}}{T_o} - R \ln\frac{P_{2i}}{P_o} \quad .$$

An inspection of Figure 3 shows that

$$T_{2i} = T_2$$

$$s_{2i} = s_o \quad .$$

Therefore,

$$C_p \ln\frac{T_2}{T_o} = R \ln\frac{P_{2i}}{P_o}$$

or

$$C_p \ln\frac{T_2}{T_o} = R \ln\frac{P_{2i}}{P_2}\left(\frac{P_2}{P_o}\right) = R \ln\frac{P_{2i}}{P_2} + R \ln\frac{P_2}{P_o}$$

$$C_p \ln\frac{T_2}{T_o} = R \ln\frac{1}{\eta_R} + R \ln\frac{P_2}{P_o} \quad . \tag{21}$$

By substituting Equation (21) in Equation (20), we have:

$$s_2 - s_o = R \ln\frac{1}{\eta_R} + R \ln\frac{P_2}{P_o} - R \ln\frac{P_2}{P_o}$$

or

$$\frac{\Delta s}{C_p} = \left(\frac{\gamma - 1}{\gamma}\right) \ln\frac{1}{\eta_R} \quad . \tag{22}$$

Now, when we use the expression for η_I,

$$\eta_I = \frac{P_{t2}}{P_{to}}$$

or

$$\eta_I = \frac{P_{t2}}{P_2}\left(\frac{P_o}{P_{to}}\right)\left(\frac{P_2}{P_o}\right)$$

and

$$\frac{P_{t2}}{P_2} = \left(\frac{T_{t2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_o}{P_{to}} = \left(\frac{T_o}{T_{to}}\right)^{\frac{\gamma}{\gamma-1}} .$$

Since

$$T_{t2} = T_{to}$$

$$\eta_I = \left(\frac{T_{to}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{T_o}{T_{to}}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{P_2}{P_o}\right)$$

or

$$\eta_I = \left(\frac{h_o}{h_2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{P_2}{P_o}\right) .$$

If we rearrange these expressions, we have

$$\frac{P_2}{P_o} = \frac{\eta_I}{\left(\frac{h_o}{h_2}\right)^{\frac{\gamma}{\gamma-1}}} \tag{23}$$

By using Equation (17) again and substituting Equation (23), we find that

$$s_2 - s_o = C_p \ln \frac{h_2}{h_o} - C_p\left(\frac{\gamma-1}{\gamma}\right) \ln\left[\frac{\eta_I}{\left(\frac{h_o}{h_2}\right)^{\frac{\gamma}{\gamma-1}}}\right]$$

or

$$\frac{\Delta s}{C_p} = \ln \frac{h_2}{h_o} - \ln \frac{h_2}{h_o} - \left(\frac{\gamma-1}{\gamma}\right) \ln \eta_I$$

and

$$\frac{\Delta s}{C_p} = \left(\frac{\gamma-1}{\gamma}\right) \ln \frac{1}{\eta_I} . \tag{24}$$

An inspection of Equations (22) and (24) shows that for an ideal gas η_R and η_I are identical. The expression for η_{KE} will now be used. Equation (17) for the actual process is, again

$$s_2 - s_0 = C_p \ln \frac{h_2}{h_0} - R \ln \frac{P_2}{P_0} .$$

For the ideal process Equation (17) becomes

$$s_2 - s'_0 = C_p \ln \frac{h_2}{h'_0} - R \ln \frac{P_2}{P_0} .$$

Inspection of Figure 3 shows that

$$s_2 = s'_0 .$$

Therefore,

$$R \ln \frac{P_2}{P_0} = C_p \ln \frac{h_2}{h'_0}$$

and

$$s_2 - s_0 = C_p \ln \frac{h_2}{h_0} - C_p \ln \frac{h_2}{h'_0} = C_p \ln \frac{h'_0}{h_0} . \qquad (25)$$

If we expand the expression for η_{KE},

$$\eta_{KE} = \frac{\left(h_0 + \frac{V_0^2}{2gJ} - h'_0\right) 2gJ}{V_0^2} = \frac{h_0 \left(1 + \frac{V_0^2}{2gJh_0} - \frac{h'_0}{h_0}\right) 2gJ}{V_0^2}$$

$$\eta_{KE} = \left(\frac{V_0^2}{2gJh_0}\right) = 1 + \frac{V_0^2}{2gJh_0} - \frac{h'_0}{h_0}$$

or

$$\frac{h'_0}{h_0} = 1 + (1 - \eta_{KE}) \frac{V_0^2}{2gJh_0} . \qquad (26)$$

We find by substituting Equation (26) in (25) that

$$\frac{\Delta s}{C_p} = \ln \left[1 + (1 - \eta_{KE}) \frac{V_0^2}{2gJh_0} \right] . \qquad (27)$$

Finally, using the expression for K_D,

$$K_D = \frac{\eta_{KE} - \left(\frac{V_2}{V_0}\right)^2}{1 - \left(\frac{V_2}{V_0}\right)^2} .$$

Solving Equation (27) for η_{KE} gives

$$1 + (1 - \eta_{KE}) \frac{V_0^2}{2gJh_0} = e^{\frac{\Delta s}{C_p}}$$

$$\eta_{KE} = 1 - \frac{2gJh_0}{V_0^2} \left[e^{\frac{\Delta s}{C_p}} - 1 \right] . \qquad (28)$$

When we use Equation (28) in the expression for K_D, we find that

$$K_D = \frac{1 - \frac{2gJh_0}{V_0^2} \left[e^{\frac{\Delta s}{C_p}} - 1 \right] - \left(\frac{V_2}{V_0} \right)^2}{1 - \left(\frac{V_2}{V_0} \right)^2} . \qquad (29)$$

Now, solving Equation (29) for $\frac{\Delta s}{C_p}$ gives

$$K_D \left[1 - \left(\frac{V_2}{V_0} \right)^2 \right] = 1 - \frac{2gJh_0}{V_0^2} \left[e^{\frac{\Delta s}{C_p}} - 1 \right] - \left(\frac{V_2}{V_0} \right)^2$$

$$(1 - K_D) \left[1 - \left(\frac{V_2}{V_0} \right)^2 \right] = \frac{2gJh_0}{V_0^2} \left[e^{\frac{\Delta s}{C_p}} - 1 \right]$$

$$e^{\frac{\Delta s}{C_p}} = 1 + \frac{V_0^2}{2gJh_0} (1 - K_D) \left[1 - \left(\frac{V_2}{V_0} \right)^2 \right]$$

$$\frac{\Delta s}{C_p} = \ln \left\{ 1 + \frac{V_0^2}{2gJh_0} (1 - K_D) \left[1 - \left(\frac{V_2}{V_0} \right)^2 \right] \right\} . \qquad (30)$$

It is now simply a matter of algebra to equate Equations (15) through (30) and solve for the various efficiency parameters.

When we equate Equations (15) and (19)

$$(1 - \eta_{TH}) \left(\frac{h_2}{h_0} - 1 \right) = (1 - \eta_{EL}) \ln \frac{h_2}{h_0} .$$

Solving for η_{TH}, we find

$$\boxed{\eta_{TH} = 1 - \frac{(1 - \eta_{EL}) \ln \frac{h_2}{h_0}}{\left(\frac{h_2}{h_0} - 1 \right)}} .$$

When we solve for η_{EL}, we find

$$\eta_{EL} = 1 - \frac{(1-\eta_{TH})\left(\frac{h_2}{h_0}-1\right)}{\ln\frac{h_2}{h_0}} .$$

Next we equate Equations (15) and (22) and obtain

$$(1-\eta_{TH})\left(\frac{h_2}{h_0}-1\right) = \left(\frac{\gamma-1}{\gamma}\right)\ln\frac{1}{\eta_R} .$$

We find, when solving for η_{TH}, that

$$\eta_{TH} = 1 - \frac{\left(\frac{\gamma-1}{\gamma}\right)\ln\frac{1}{\eta_R}}{\left(\frac{h_2}{h_0}-1\right)} .$$

Solving for η_R, we find

$$\frac{1}{\eta_R} = e^{\left(\frac{\gamma}{\gamma-1}\right)(1-\eta_{TH})\left(\frac{h_2}{h_0}-1\right)}$$

$$\eta_R = e^{\left(\frac{\gamma}{\gamma-1}\right)(\eta_{TH}-1)\left(\frac{h_2}{h_0}-1\right)} .$$

It was shown previously that for an ideal gas η_R and η_I are identical.

Therefore,

$$\eta_{TH} = 1 - \frac{\left(\frac{\gamma-1}{\gamma}\right)\ln\frac{1}{\eta_I}}{\left(\frac{h_2}{h_0}-1\right)}$$

and

$$\eta_I = e^{\left(\frac{\gamma}{\gamma-1}\right)(\eta_{TH}-1)\left(\frac{h_2}{h_0}-1\right)} .$$

Equating Equations (15) and (27) gives

$$(1-\eta_{TH})\left(\frac{h_2}{h_0}-1\right) = \ln\left[1+(1-\eta_{KE})\frac{V_0^2}{2gJh_0}\right] .$$

Then when we solve for η_{TH}, we find

$$\boxed{\eta_{TH} = 1 - \frac{\ln\left[1 + (1-\eta_{KE})\frac{V_o^2}{2gJh_o}\right]}{\left(\frac{h_2}{h_o} - 1\right)}} \; .$$

Solving for η_{KE}, we find

$$1 + (1-\eta_{KE})\frac{V_o^2}{2gJh_o} = e^{(1-\eta_{TH})\left(\frac{h_2}{h_o} - 1\right)}$$

$$\boxed{\eta_{KE} = 1 - \frac{2gJh_o}{V_o^2}\left[e^{(1-\eta_{TH})\left(\frac{h_2}{h_o} - 1\right)} - 1\right]} \; .$$

When Equations (15) and (30) are equated,

$$(1-\eta_{TH})\left(\frac{h_2}{h_o} - 1\right) = \ln\left\{1 + \frac{V_o^2}{2gJh_o}(1-K_D)\left[1 - \left(\frac{V_2}{V_o}\right)^2\right]\right\} .$$

We find, when solving for η_{TH}, that

$$\eta_{TH} = 1 - \frac{\ln\left\{1 + \frac{V_o^2}{2gJh_o}(1-K_D)\left[1 - \left(\frac{V_2}{V_o}\right)^2\right]\right\}}{\left(\frac{h_2}{h_o} - 1\right)} \; .$$

Use the relation

$$\frac{V_o^2}{2gJh_o}\left[1 - \left(\frac{V_2}{V_o}\right)^2\right] = \frac{V_o^2 - V_2^2}{2gJh_o} = \frac{h_2}{h_o} - 1 \; . \qquad (31)$$

Then,

$$\boxed{\eta_{TH} = 1 - \frac{\ln\left[1 + (1-K_D)\left(\frac{h_2}{h_o} - 1\right)\right]}{\left(\frac{h_2}{h_o} - 1\right)}} \; .$$

We solve for K_D and find

$$1 + \frac{V_o^2}{2gJh_o}(1 - K_D)\left[1 - \left(\frac{V_2}{V_o}\right)^2\right] = e^{(1-\eta_{TH})\left(\frac{h_2}{h_o}-1\right)}$$

$$K_D = 1 - \frac{2gJh_o}{V_o^2}\left[\frac{1 - e^{(1-\eta_{TH})\left(\frac{h_2}{h_o}-1\right)}}{1-\left(\frac{V_2}{V_o}\right)^2}\right] .$$

By use of Equation (31)

$$\boxed{K_D = 1 - \frac{1 - e^{(1-\eta_{TH})\left(\frac{h_2}{h_o}-1\right)}}{\frac{h_2}{h_o} - 1}} .$$

By equating Equations (19) and (22)

$$(1 - \eta_{EL})\, \ln \frac{h_2}{h_o} = \left(\frac{\gamma - 1}{\gamma}\right) \ln \frac{1}{\eta_R} .$$

We solve for η_{EL} and find

$$\boxed{\eta_{EL} = 1 - \frac{\left(\frac{\gamma-1}{\gamma}\right)\ln \frac{1}{\eta_R}}{\ln \frac{h_2}{h_o}}} .$$

We solve for η_R and find find that

$$\left(\frac{h_2}{h_o}\right)^{1-\eta_{EL}} = \left(\frac{1}{\eta_R}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\eta_R = \left(\frac{h_2}{h_o}\right)^{\frac{\gamma}{\gamma-1}(\eta_{EL}-1)}} .$$

Also,

$$\eta_{EL} = 1 - \frac{\left(\frac{\gamma - 1}{\gamma}\right) \ln \frac{1}{\eta_I}}{\ln \frac{h_2}{h_o}}$$

$$\eta_I = \left(\frac{h_2}{h_o}\right)^{\frac{\gamma}{\gamma - 1}(\eta_{EL} - 1)} .$$

When we equate Equations (19) and (27),

$$(1 - \eta_{EL}) \ln \frac{h_2}{h_o} = \ln \left[1 + (1 - \eta_{KE}) \frac{V_o^2}{2gJh_o} \right] .$$

By solving for η_{EL}, we find

$$\eta_{EL} = 1 - \frac{\ln \left[1 + (1 - \eta_{KE}) \frac{V_o^2}{2gJh_o} \right]}{\ln \frac{h_2}{h_o}} .$$

Solving for η_{KE}, we find

$$1 + (1 - \eta_{KE}) \frac{V_o^2}{2gJh_o} = \left(\frac{h_2}{h_o}\right)^{1 - \eta_{EL}}$$

$$\eta_{KE} = 1 - \frac{2gJh_o}{V_o^2} \left[\left(\frac{h_2}{h_o}\right)^{1 - \eta_{EL}} - 1 \right] .$$

Equations (19) and (30), when equated give

$$(1 - \eta_{EL}) \ln \frac{h_2}{h_o} = \ln \left\{ 1 + \frac{V_o^2}{2gJh_o} (1 - K_D) \left[1 - \left(\frac{V_2}{V_o}\right)^2 \right] \right\} .$$

When we solve for η_{EL}, we obtain

$$\eta_{EL} = 1 - \frac{\ln \left\{ 1 + \frac{V_o^2}{2gJh_o} (1 - K_D) \left[1 \quad \left(\frac{V_2}{V_o}\right)^2 \right] \right\}}{\ln \frac{h_2}{h_o}} .$$

Using Equation (31) and rearranging gives

$$\eta_{EL} = 1 - \frac{\ln\left[K_D + \frac{h_2}{h_o}(1 - K_D)\right]}{\ln \frac{h_2}{h_o}} .$$

When we solve for K_D, we find

$$1 + \frac{V_o^2}{2gJh_o}(1 - K_D)\left[1 - \left(\frac{V_2}{V_o}\right)^2\right] = \left(\frac{h_2}{h_o}\right)^{1-\eta_{EL}}$$

$$K_D = 1 - \frac{2gJh_o}{V_o^2}\left[\frac{\left(\frac{h_2}{h_o}\right)^{1-\eta_{EL}} - 1}{1 - \left(\frac{V_2}{V_o}\right)^2}\right] .$$

By using Equation (31),

$$K_D = 1 - \frac{\left(\frac{h_2}{h_o}\right)^{1-\eta_{EL}} - 1}{\frac{h_2}{h_o} - 1} .$$

Equate Equations (22) and (27) and obtain

$$\left(\frac{\gamma - 1}{\gamma}\right) \ln \frac{1}{\eta_R} = \ln\left[1 + (1 - \eta_{KE})\frac{V_o^2}{2gJh_o}\right] .$$

We find, when solving for η_R, that

$$\left(\frac{1}{\eta_R}\right)^{\frac{\gamma-1}{\gamma}} = 1 + (1 - \eta_{KE})\frac{V_o^2}{2gJh_o}$$

$$\eta_R = \left[1 + (1 - \eta_{KE})\frac{V_o^2}{2gJh_o}\right]^{\frac{\gamma}{1-\gamma}} .$$

Solving for η_{KE}, we find

$$\eta_{KE} = 1 - \frac{2gJh_o}{V_o^2}\left[\left(\frac{1}{\eta_R}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] .$$

Also,

$$\eta_I = \left[1 + (1-\eta_{KE})\frac{V_o^2}{2gJh_o}\right]^{\frac{\gamma}{1-\gamma}}$$

$$\eta_{KE} = 1 - \frac{2gJh_o}{V_o^2}\left[\left(\frac{1}{\eta_I}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] .$$

Equating Equations (22) and (30) gives

$$\left(\frac{\gamma-1}{\gamma}\right)\ln\frac{1}{\eta_R} = \ln\left\{1 + \frac{V_o^2}{2gJh_o}(1-K_D)\left[1-\left(\frac{V_2}{V_o}\right)^2\right]\right\} .$$

If we solve for η_o,

$$\frac{1}{\eta_R} = \left\{1 + \frac{V_o^2}{2gJh_o}(1-K_D)\left[1-\left(\frac{V_2}{V_o}\right)^2\right]\right\}^{\frac{\gamma}{\gamma-1}}$$

$$\eta_R = \left\{1 + \frac{V_o^2}{2gJh_o}(1-K_D)\left[1-\left(\frac{V_2}{V_o}\right)^2\right]\right\}^{\frac{\gamma}{1-\gamma}} .$$

The use of Equation (31) gives

$$\eta_R = \left[1 + (1-K_D)\left(\frac{h_2}{h_o} - 1\right)\right]^{\frac{\gamma}{1-\gamma}} .$$

Solve for K_D and obtain

$$K_D = 1 - \frac{2gJh_o}{V_o^2}\left[\frac{\left(\frac{1}{\eta_R}\right)^{\frac{\gamma-1}{\gamma}} - 1}{1-\left(\frac{V_2}{V_o}\right)^2}\right] .$$

Then, using Equation (31) gives

$$\boxed{K_D = 1 - \frac{\left(\frac{1}{\eta_R}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{h_2}{h_o} - 1}} .$$

Also,

$$\eta_I = \left[1 + (1 - K_D)\left(\frac{h_2}{h_o} - 1\right)\right]^{\frac{\gamma}{1-\gamma}}$$

$$\boxed{K_D = 1 - \frac{\left(\frac{1}{\eta_I}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{h_2}{h_o} - 1}} .$$

When we equate Equations (27) and (30),

$$\ln\left[1 + (1 - \eta_{KE})\frac{V_o^2}{2gJh_o}\right] = \ln\left\{1 + \frac{V_o^2}{2gJh_o}(1 - K_D)\left[1 - \left(\frac{V_2}{V_o}\right)^2\right]\right\}$$

Solving for η_{KE} gives

$$1 + (1 - \eta_{KE})\frac{V_o^2}{2gJh_o} = 1 + \frac{V_o^2}{2gJh_o}(1 - K_D)\left[1 - \left(\frac{V_2}{V_o}\right)^2\right]$$

$$\boxed{\eta_{KE} = K_D + (1 - K_D)\left(\frac{V_2}{V_o}\right)^2} .$$

Now we solve for K_D and derive

$$\boxed{K_D = \frac{\eta_{KE} - \left(\frac{V_2}{V_o}\right)^2}{1 - \left(\frac{V_2}{V_o}\right)}} .$$

The results of the preceding derivations have been tabulated in Table I.

TABLE 1

TABULATION OF EQUATIONS RELATING VARIOUS EFFICIENCY PARAMETERS
(For Ideal Gas)

PARAMETER DESIRED	KNOWN PARAMETER					
	η_{TH}	η_{KE}	K_D	η_I	η_R	η_{EL}
η_{TH} =		$1-\frac{\ln\left[1+(1-\eta_{KE})\frac{V_0^2}{2gJh_0}\right]}{\left(\frac{h_2}{h_0}-1\right)}$	$1-\frac{\ln\left[1+(1-K_D)\left(\frac{h_2}{h_0}-1\right)\right]}{\frac{h_2}{h_0}-1}$	$1-\frac{\left(\frac{\gamma-1}{\gamma}\right)\ln\frac{1}{\eta_I}}{\left(\frac{h_2}{h_0}-1\right)}$	$1-\frac{\left(\frac{\gamma-1}{\gamma}\right)\ln\frac{1}{\eta_R}}{\left(\frac{h_2}{h_0}-1\right)}$	$1-\frac{(1-\eta_{EL})\ln\frac{h_2}{h_0}}{\left(\frac{h_2}{h_0}-1\right)}$
η_{KE} =	$1-\frac{2gJh_0}{V_0^2}\left[e^{(1-\eta_{TH})\left(\frac{h_2}{h_0}-1\right)}-1\right]$		$K_D+(1-K_D)\left(\frac{V_2}{V_0}\right)^2$	$1-\frac{2gJh_0}{V_0^2}\left[\left(\frac{1}{\eta_I}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$	$1-\frac{2gJh_0}{V_0^2}\left[\left(\frac{1}{\eta_R}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$	$1-\frac{2gJh_0}{V_0^2}\left[\left(\frac{h_2}{h_0}\right)^{1-\eta_{EL}}-1\right]$
K_D =	$1-\frac{1-e^{(1-\eta_{TH})\left(\frac{h_2}{h_0}-1\right)}}{\frac{h_2}{h_0}-1}$	$\frac{\eta_{KE}-\left(\frac{V_2}{V_0}\right)^2}{1-\left(\frac{V_2}{V_0}\right)^2}$		$1-\frac{\left(\frac{1}{\eta_I}\right)^{\frac{\gamma-1}{\gamma}}-1}{\frac{h_2}{h_0}-1}$	$1-\frac{\left(\frac{1}{\eta_R}\right)^{\frac{\gamma-1}{\gamma}}-1}{\frac{h_2}{h_0}-1}$	$1-\frac{\left(\frac{h_2}{h_0}\right)^{1-\eta_{EL}}-1}{\frac{h_2}{h_0}-1}$
η_I =	$e^{\left(\frac{\gamma}{\gamma-1}\right)(\eta_{TH}-1)\left(\frac{h_2}{h_0}-1\right)}$	$\left[1+(1-\eta_{KE})\frac{V_0^2}{2gJh_0}\right]^{\frac{\gamma}{1-\gamma}}$	$\left[1+(1-K_D)\left(\frac{h_2}{h_0}-1\right)\right]^{\frac{\gamma}{1-\gamma}}$		η_R	$\left(\frac{h_2}{h_0}\right)^{\frac{\gamma}{\gamma-1}(\eta_{EL}-1)}$
η_R =	$e^{\left(\frac{\gamma}{\gamma-1}\right)(\eta_{TH}-1)\left(\frac{h_2}{h_0}-1\right)}$	$\left[1+(1-\eta_{KE})\frac{V_0^2}{2gJh_0}\right]^{\frac{\gamma}{1-\gamma}}$	$\left[1+(1-K_D)\left(\frac{h_2}{h_0}-1\right)\right]^{\frac{\gamma}{1-\gamma}}$	η_I		$\left(\frac{h_2}{h_0}\right)^{\frac{\gamma}{\gamma-1}(\eta_{EL}-1)}$
η_{EL} =	$1-\frac{(1-\eta_{TH})\left(\frac{h_2}{h_0}-1\right)}{\ln\frac{h_2}{h_0}}$	$1-\frac{\ln\left[1+(1-\eta_{KE})\frac{V_0^2}{2gJh_0}\right]}{\ln\frac{h_2}{h_0}}$	$1-\frac{\ln\left[K_D+\frac{h_2}{h_0}(1-K_D)\right]}{\ln\frac{h_2}{h_0}}$	$1-\frac{\left(\frac{\gamma-1}{\gamma}\right)\ln\frac{1}{\eta_I}}{\ln\frac{h_2}{h_0}}$	$1-\frac{\left(\frac{\gamma-1}{\gamma}\right)\ln\frac{1}{\eta_R}}{\ln\frac{h_2}{h_0}}$	

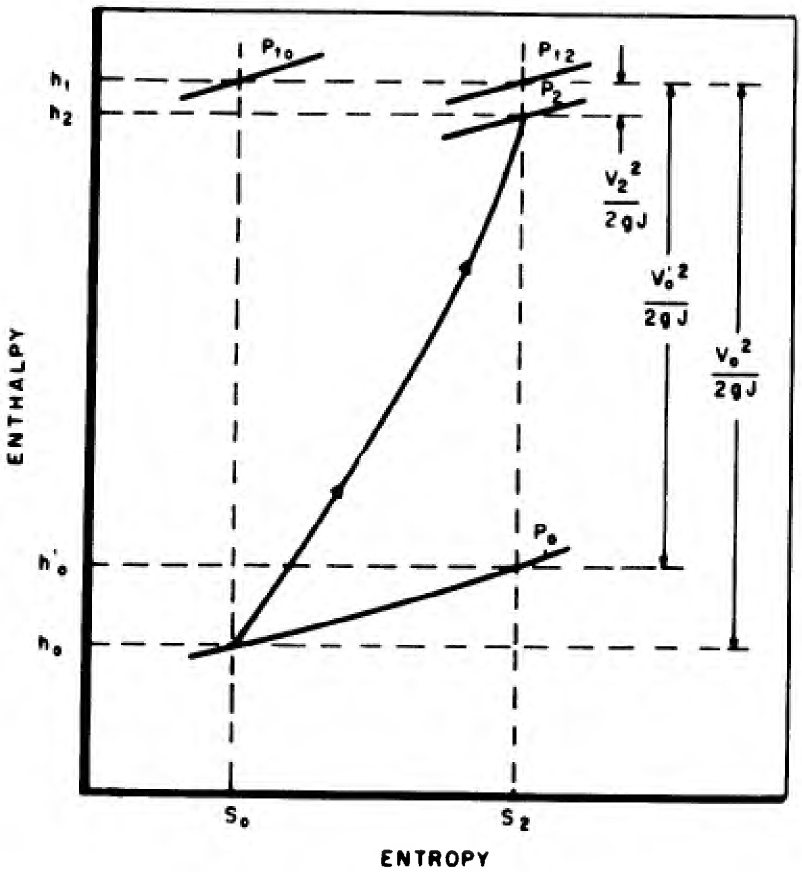

Figure 1. Subsonic Combustion Ramjet Inlet Process

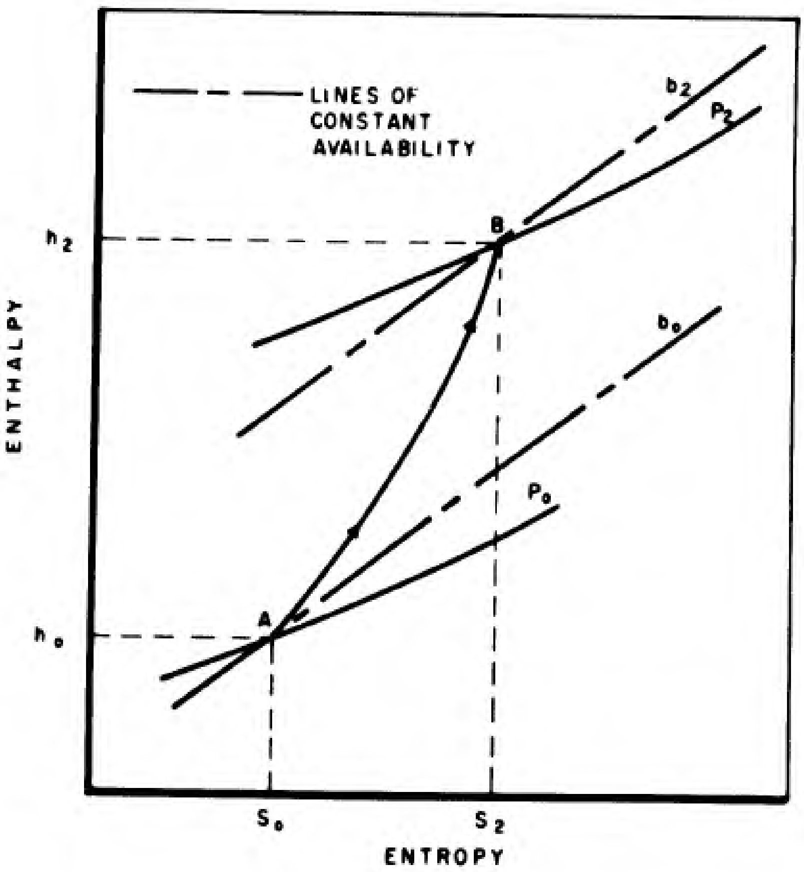

Figure 2. Inlet Process With Available Energy Concept Illustrated

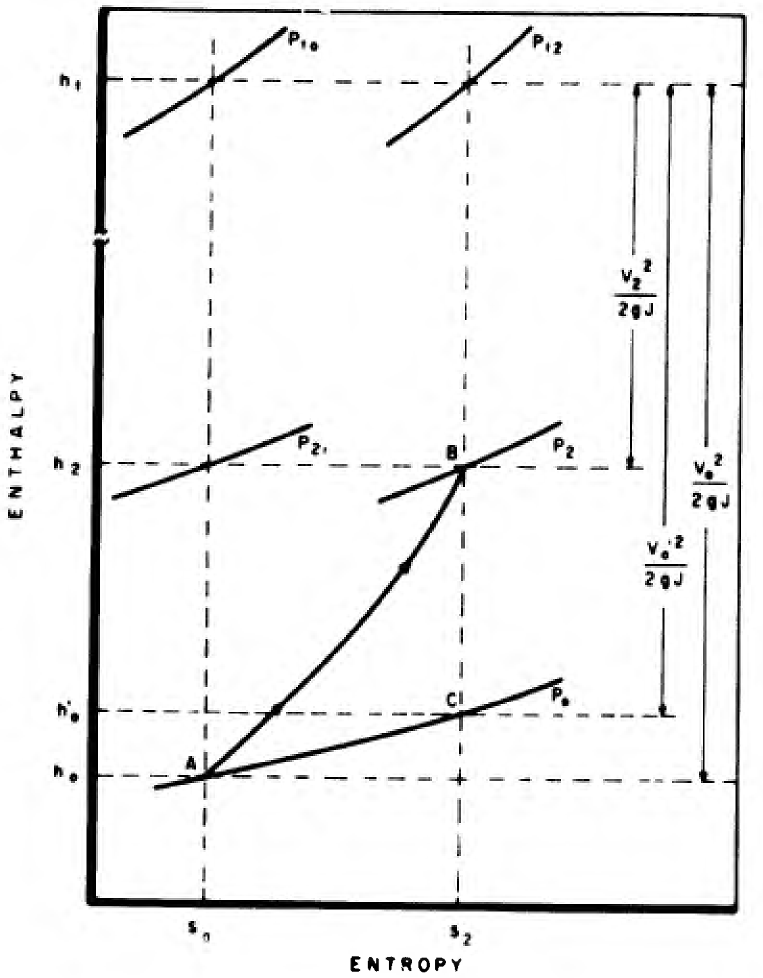

Figure 3. Supersonic Combustion Ramjet Inlet Process

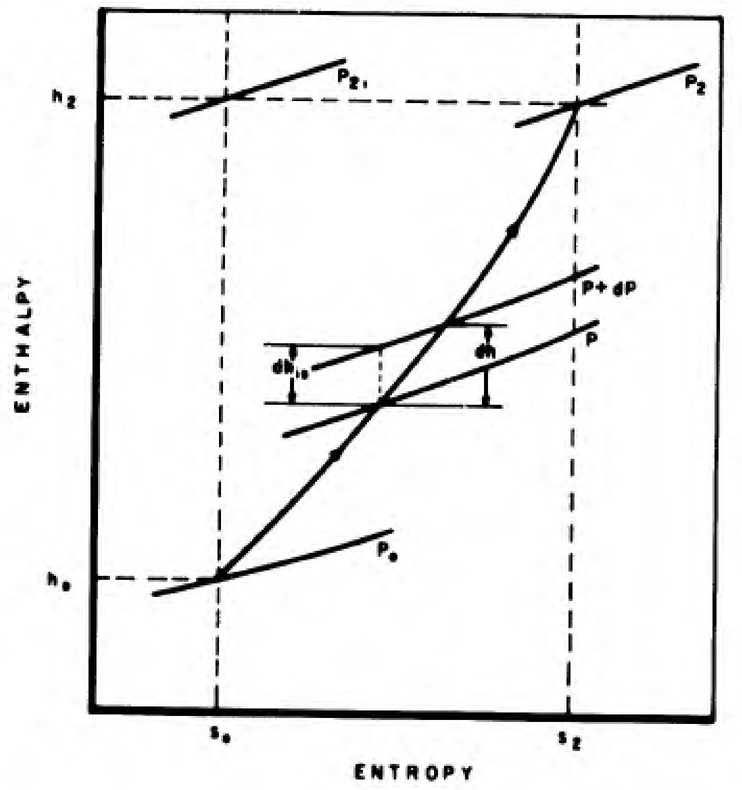

Figure 4. Illustration of Polytropic Efficiency Definition

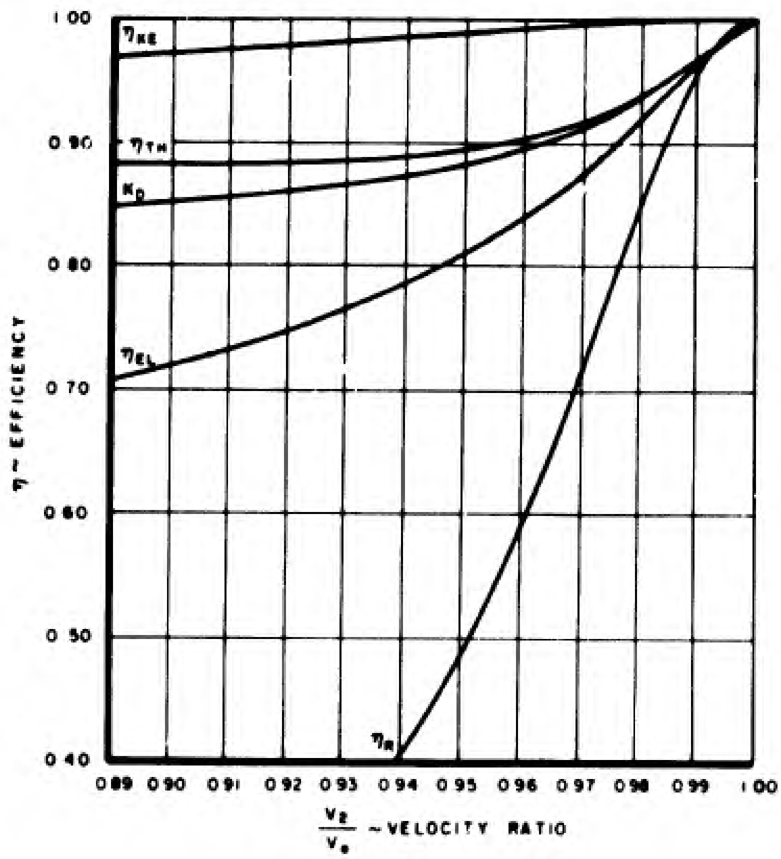

Figure 5. Efficiency Variation for a Two-Shock Inlet Operating at Mach 10

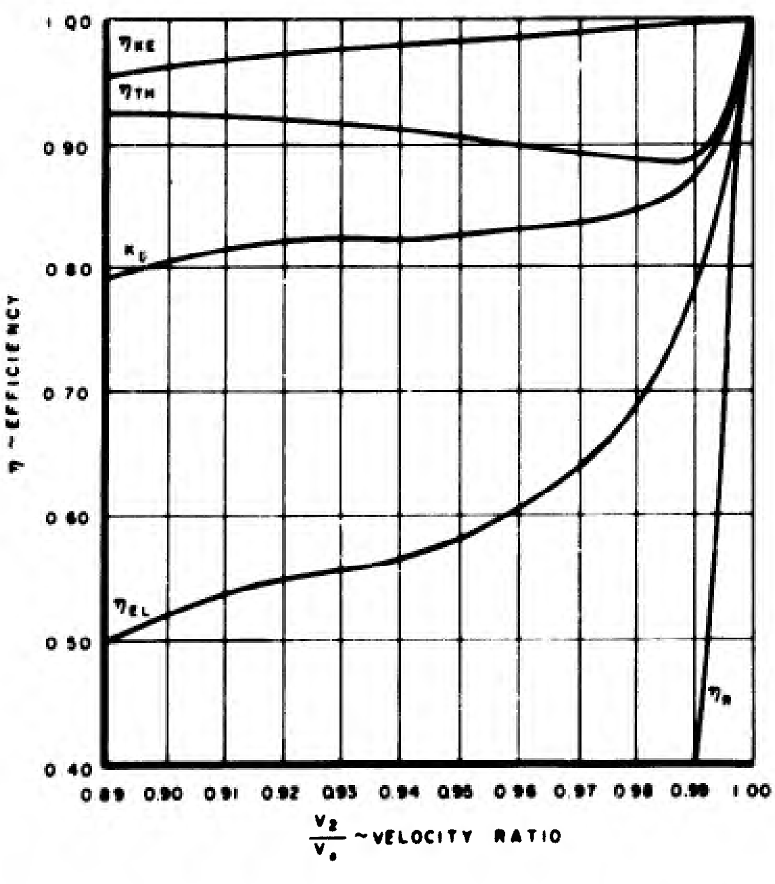

Figure 6. Efficiency Variation for a Two-Shock Inlet Operating at Mach 25

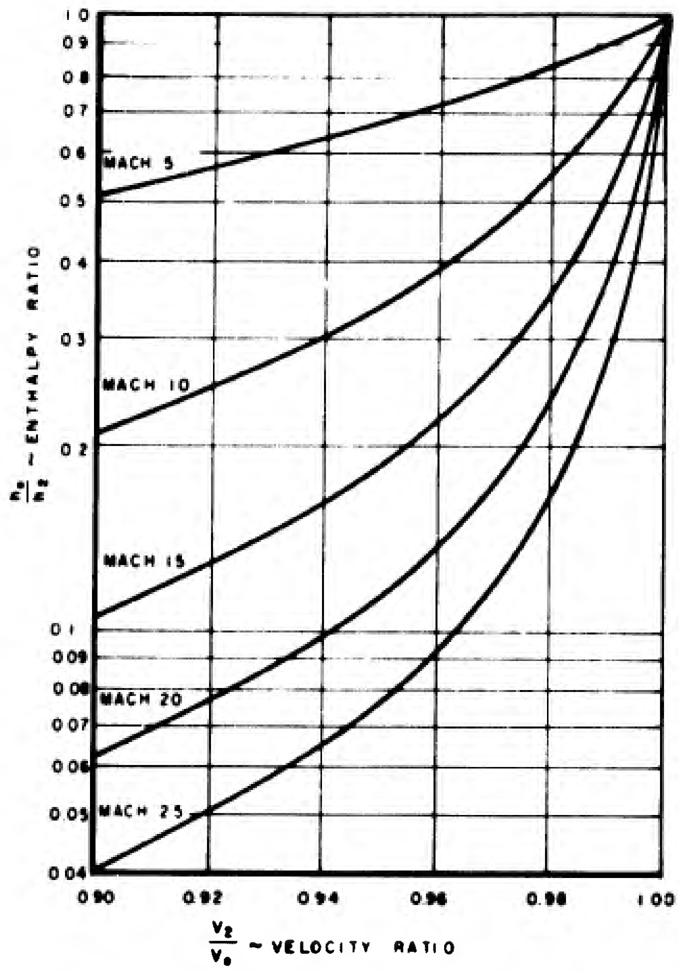

Figure 7. Chart for Conversion Between Enthalpy Ratio and Velocity Ratio

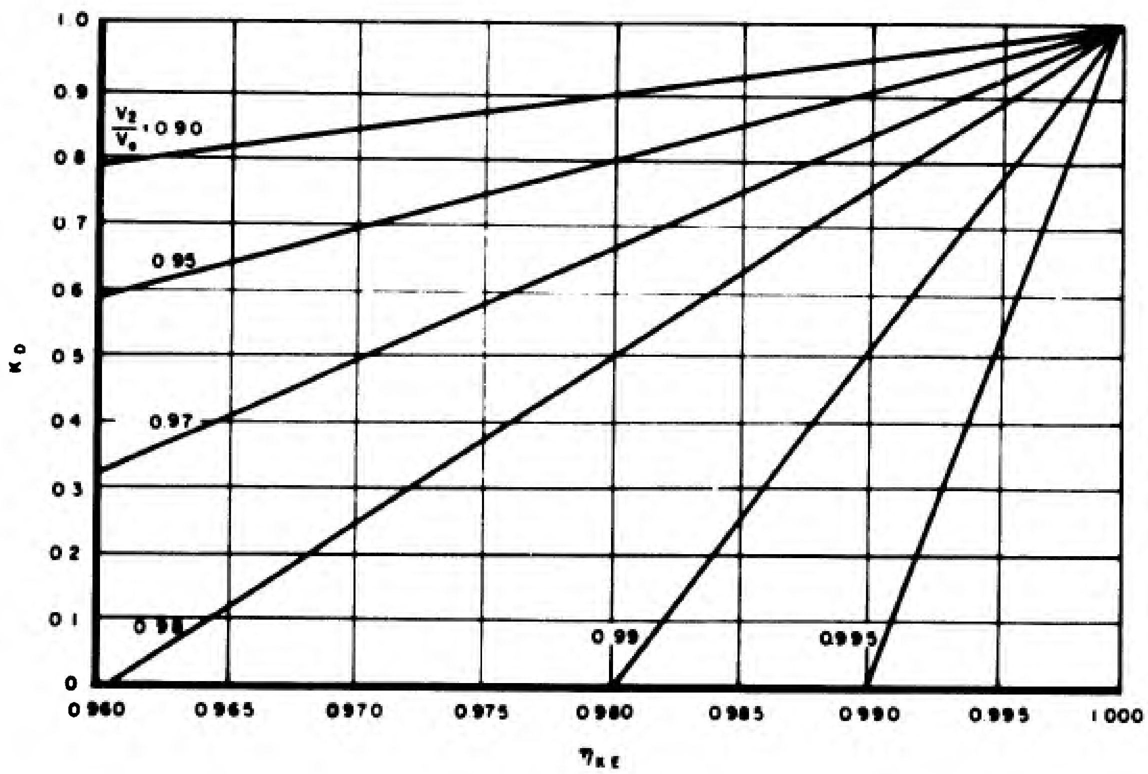

Figure 8. Chart for Conversion Between Process Efficiency and Kinetic Energy Efficiency

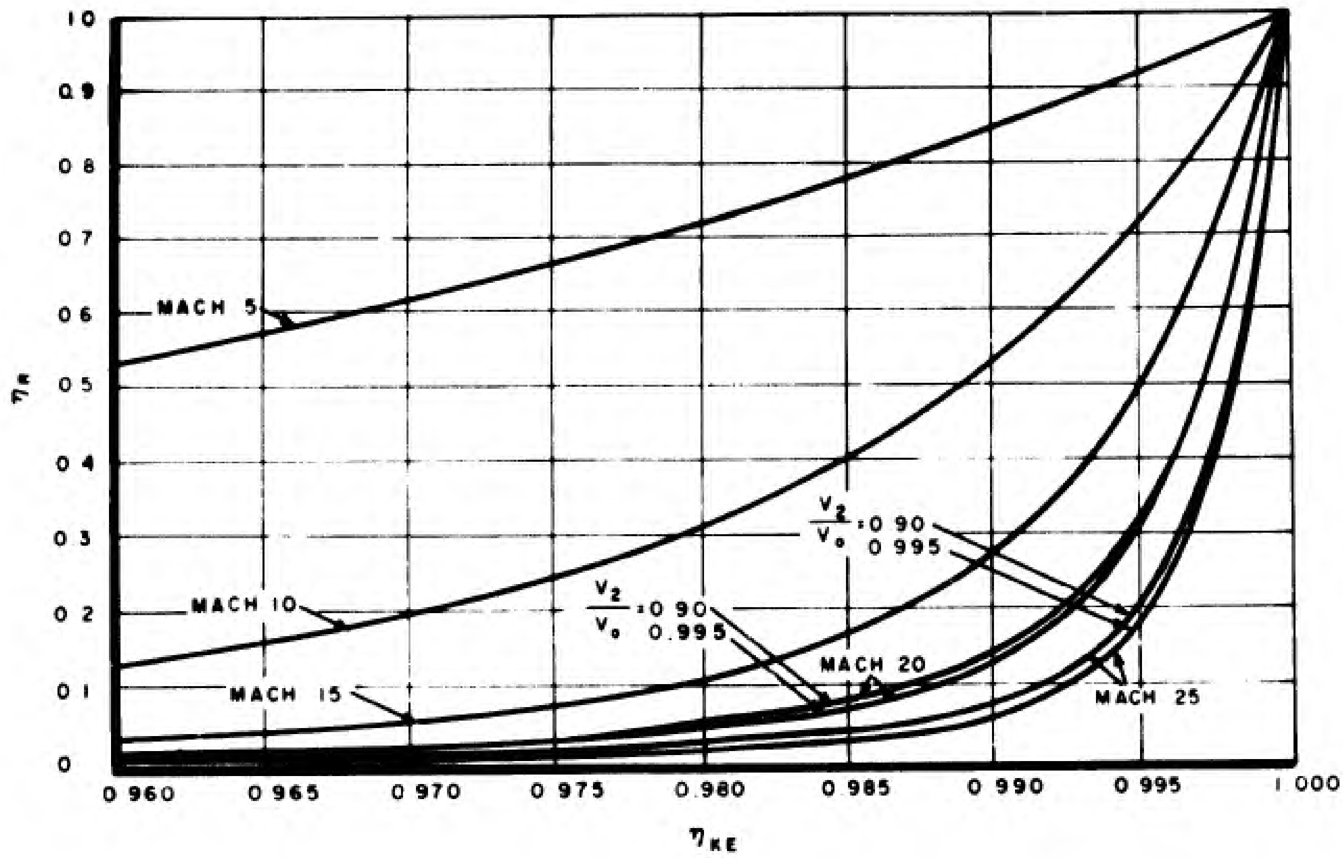

Figure 9. Chart for Conversion Between Pressure Recovery and Kinetic Energy Efficiency

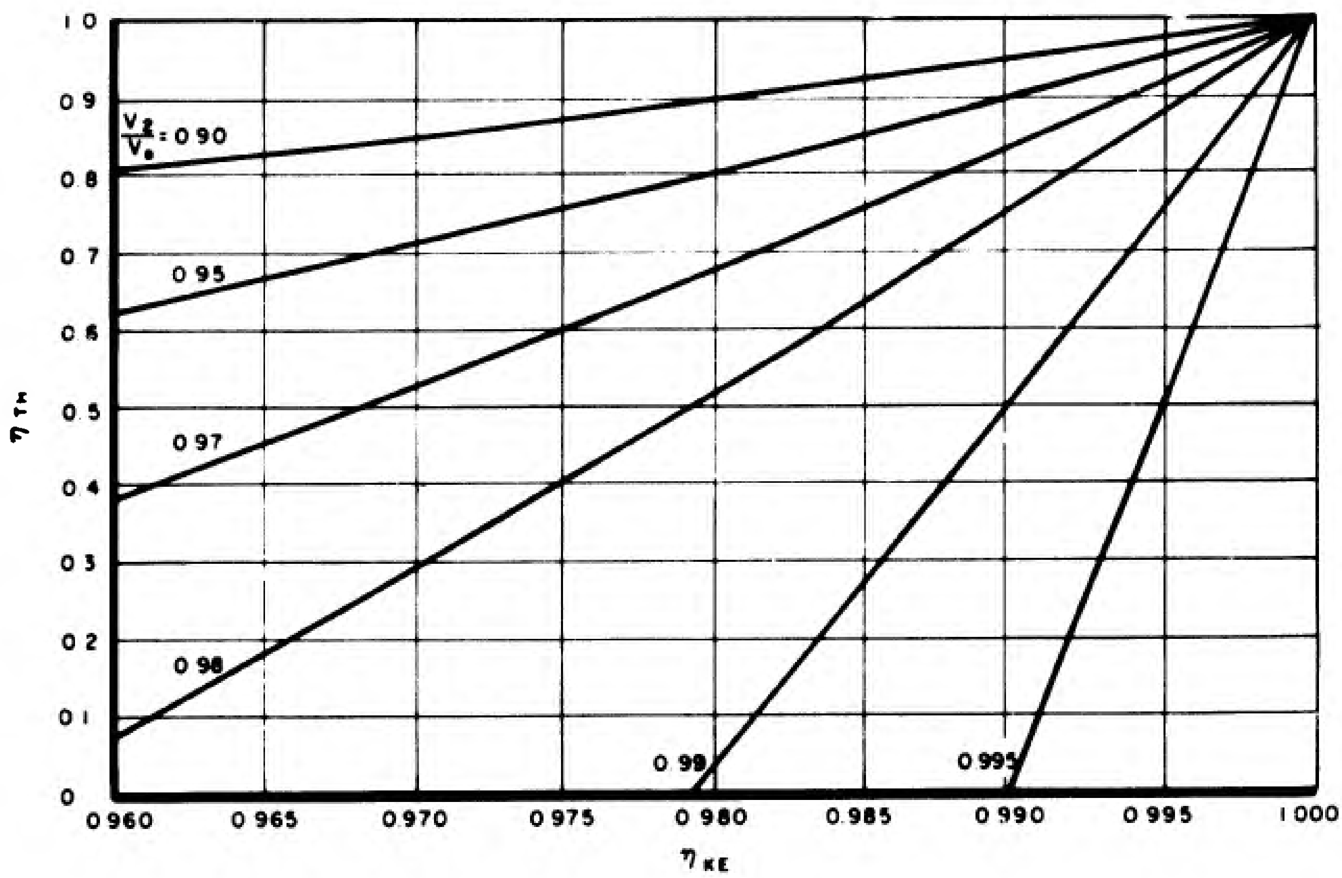

Figure 10a Chart for Conversion Between Thermodynamic Efficiency and Kinetic Energy Efficiency at Mach 5

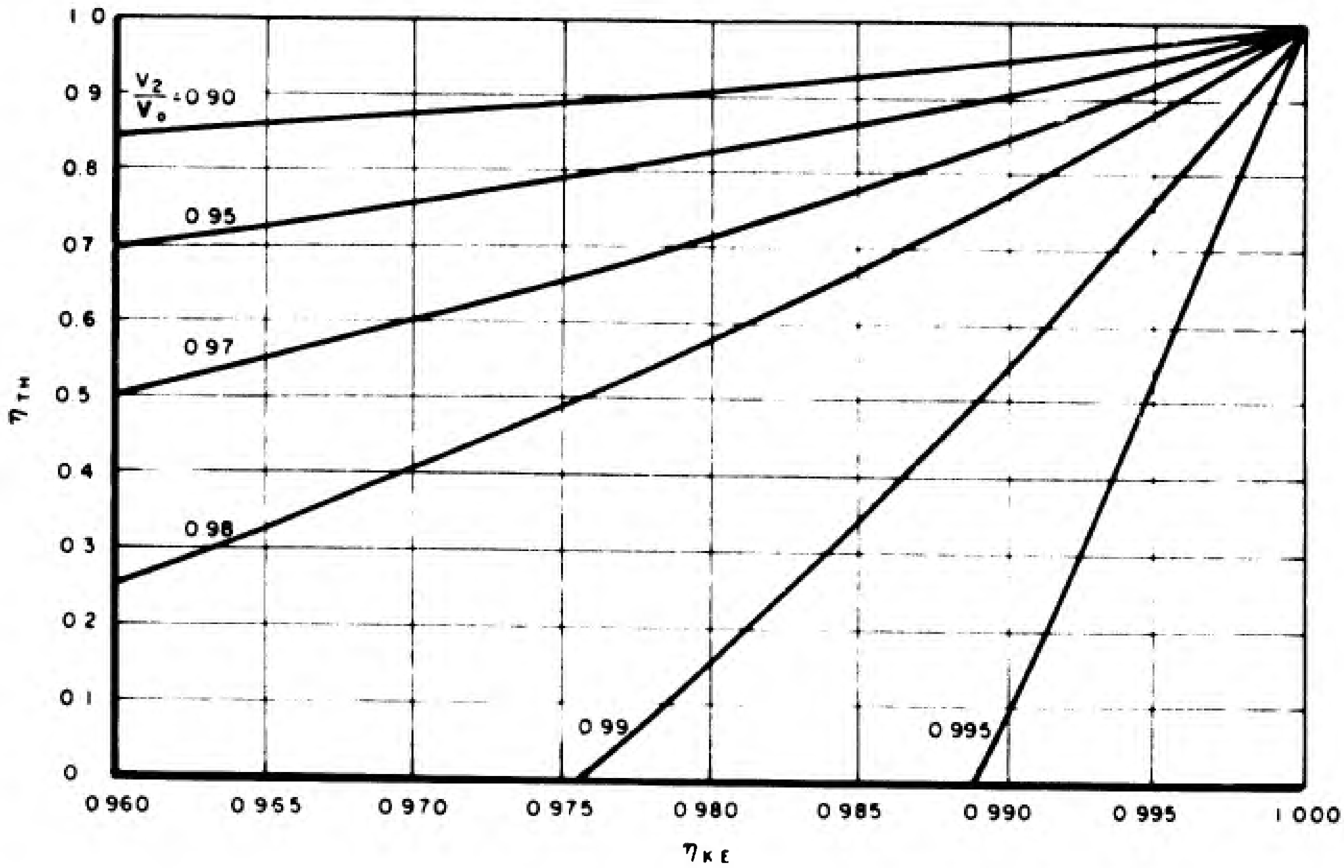

Figure 10b Chart for Conversion Between Thermodynamic Efficiency and Kinetic Energy Efficiency at Mach 10

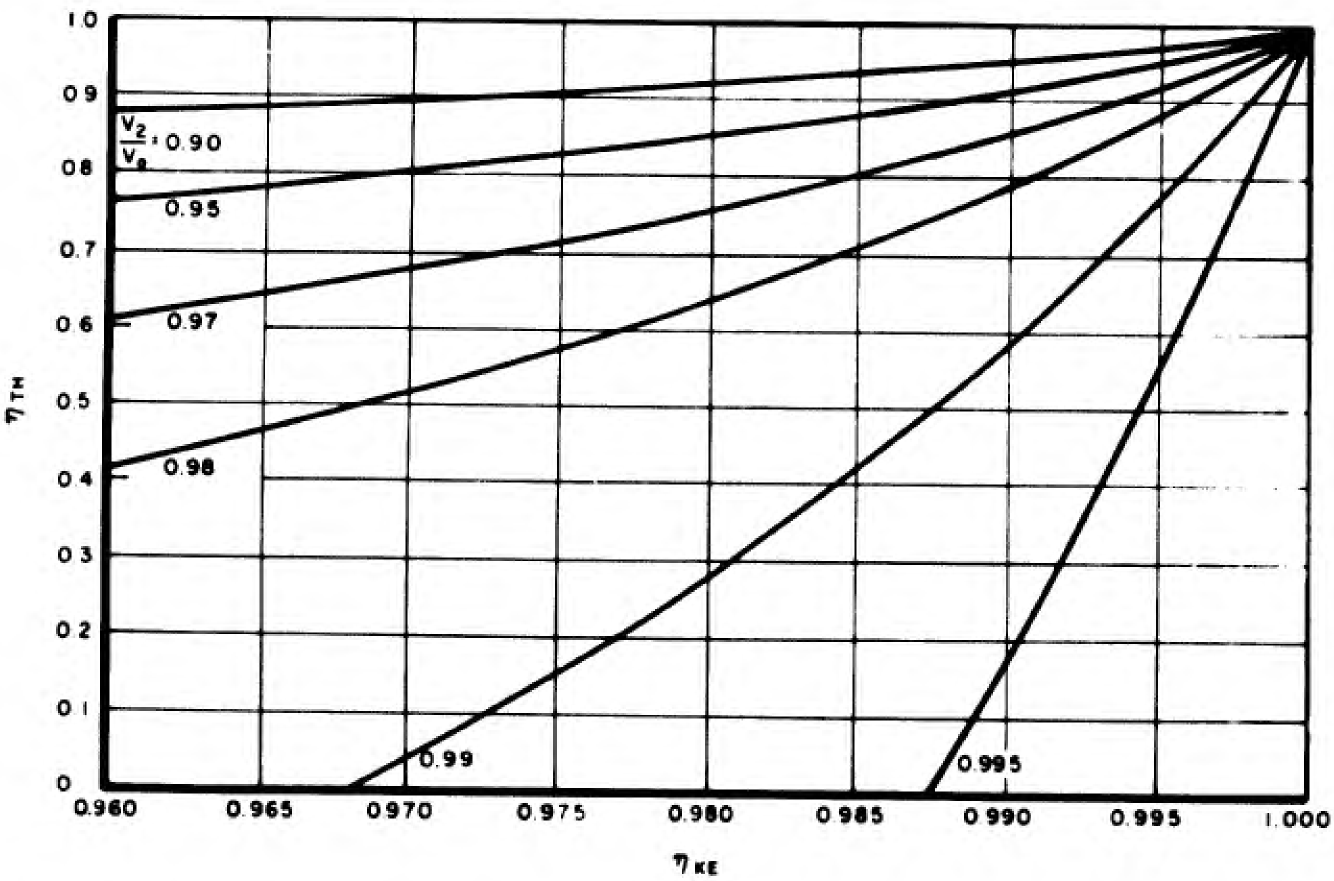

Figure 10c Chart for Conversion Between Thermodynamic Efficiency and Kinetic Energy Efficiency at Mach 15

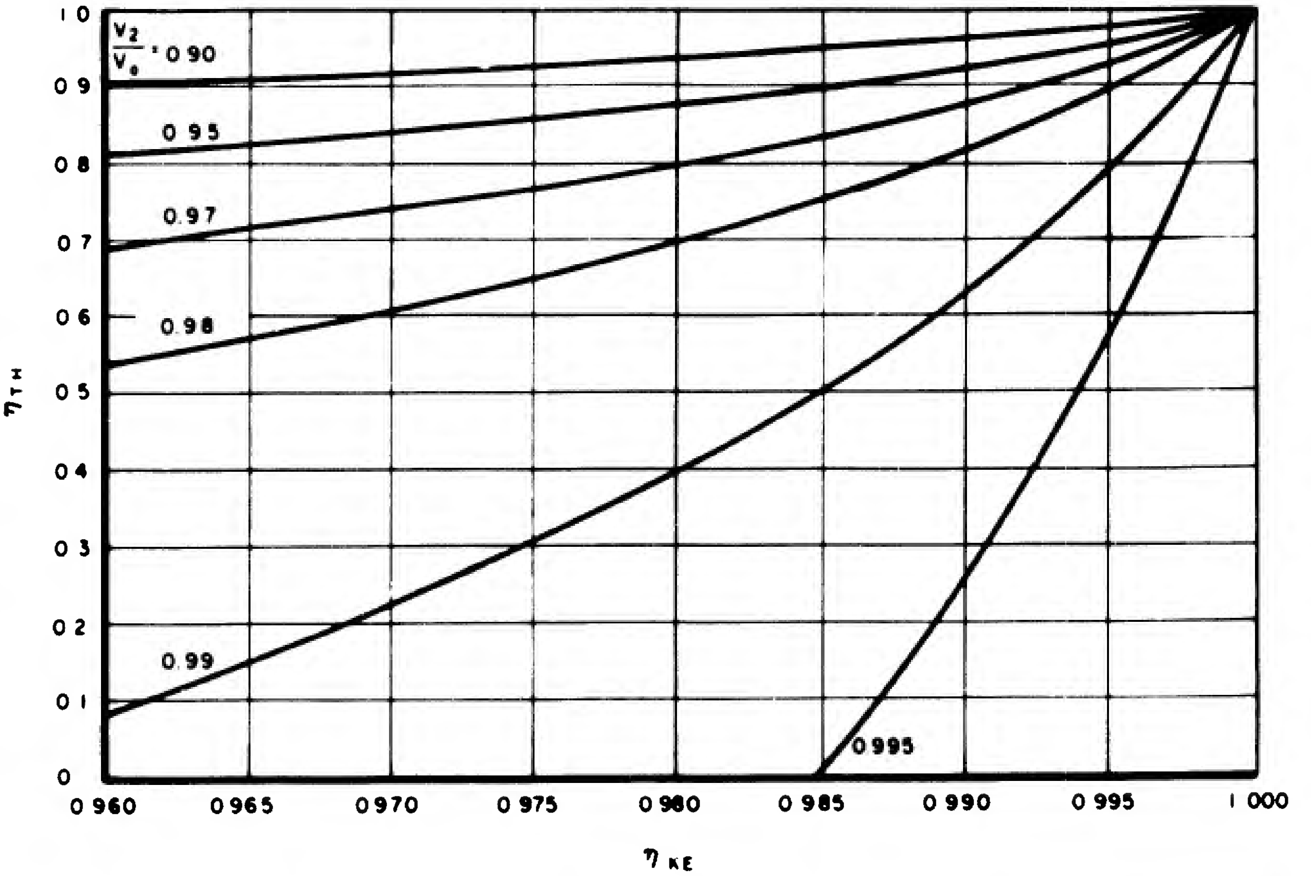

Figure 10d Chart for Conversion Between Thermodynamic Efficiency and Kinetic Energy Efficiency at Mach 20

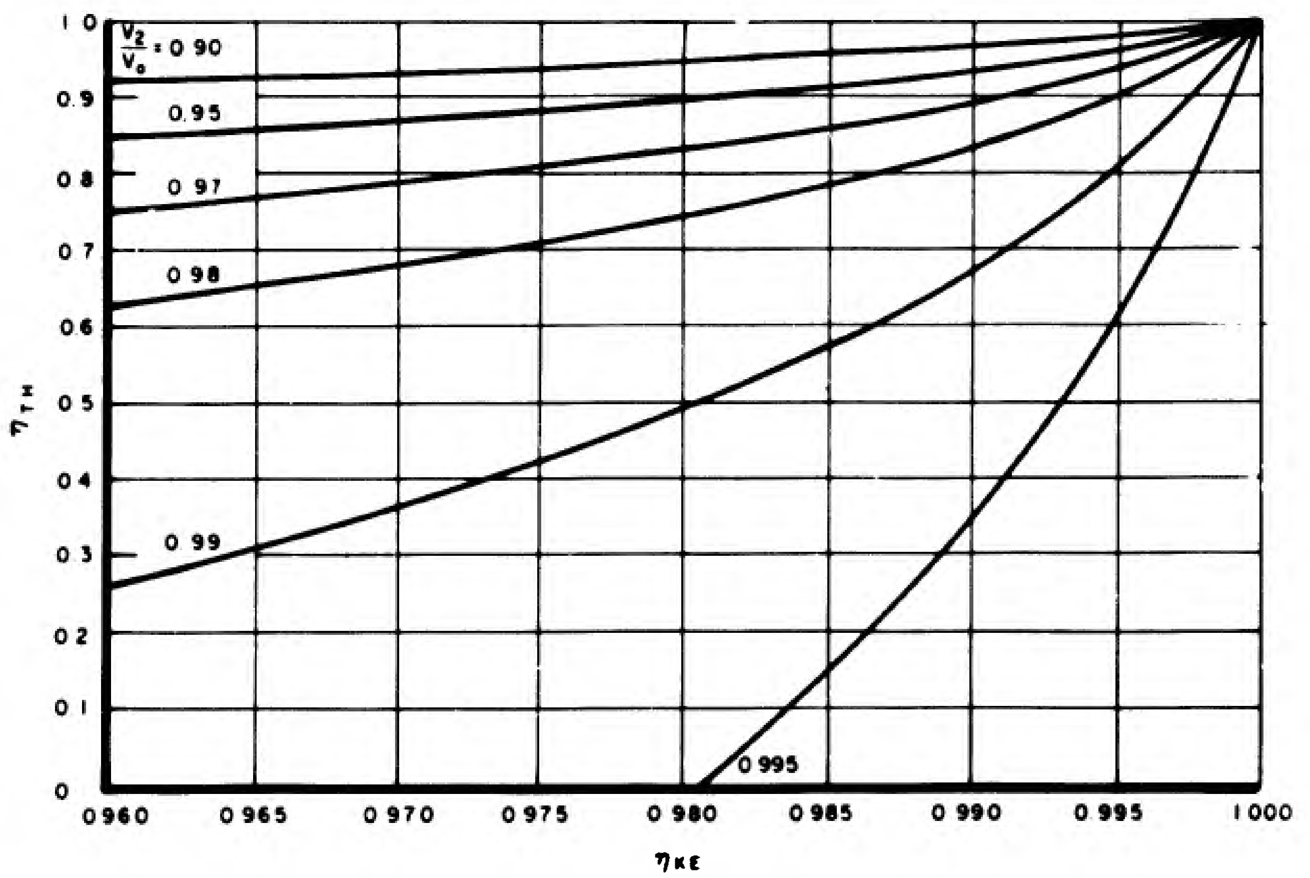

Figure 10e Chart for Conversion Between Thermodynamic Efficiency and Kinetic Energy Efficiency at Mach 25

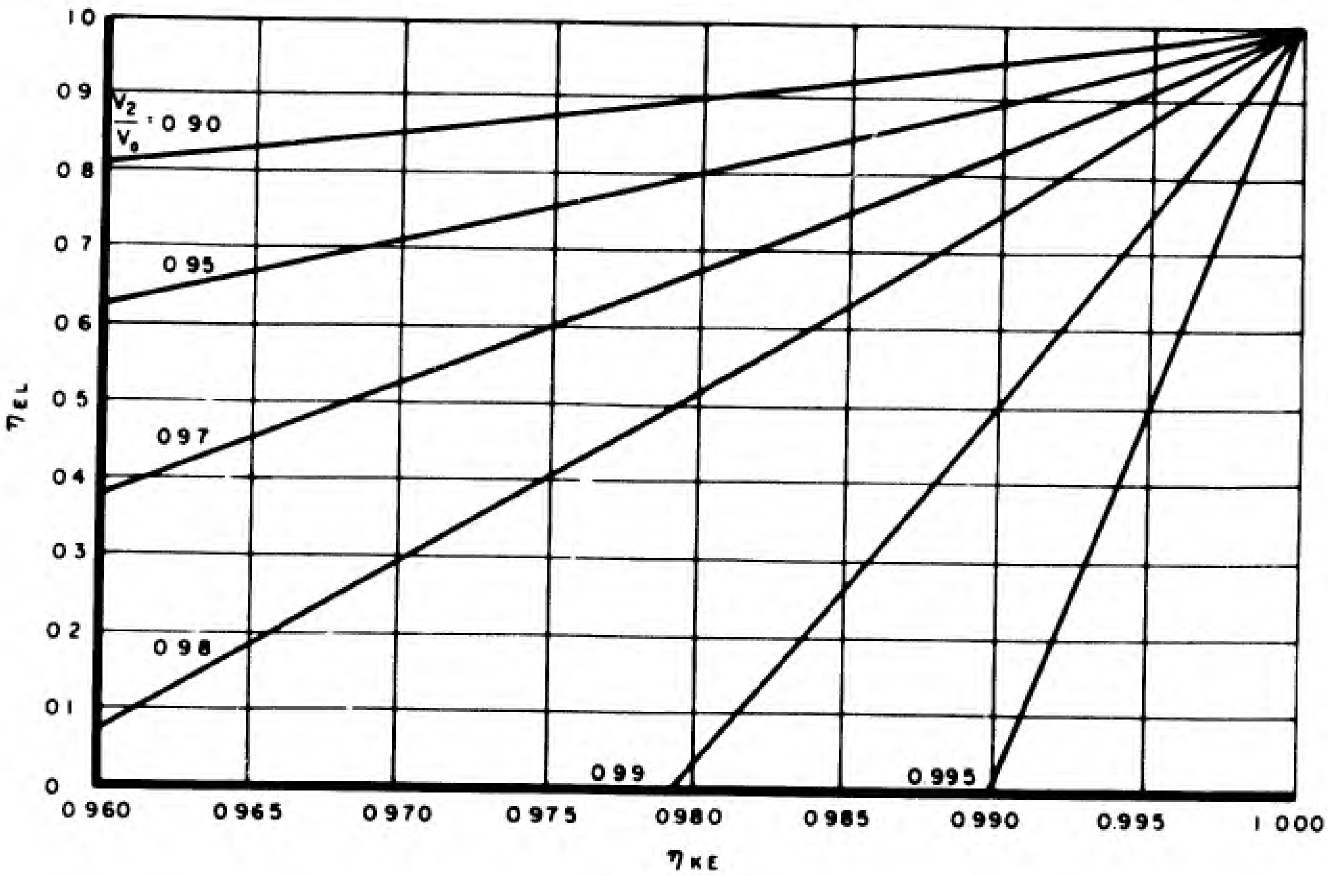

Figure 11a Chart for Conversion Between Polytropic Efficiency and Kinetic Energy Efficiency at Mach 5

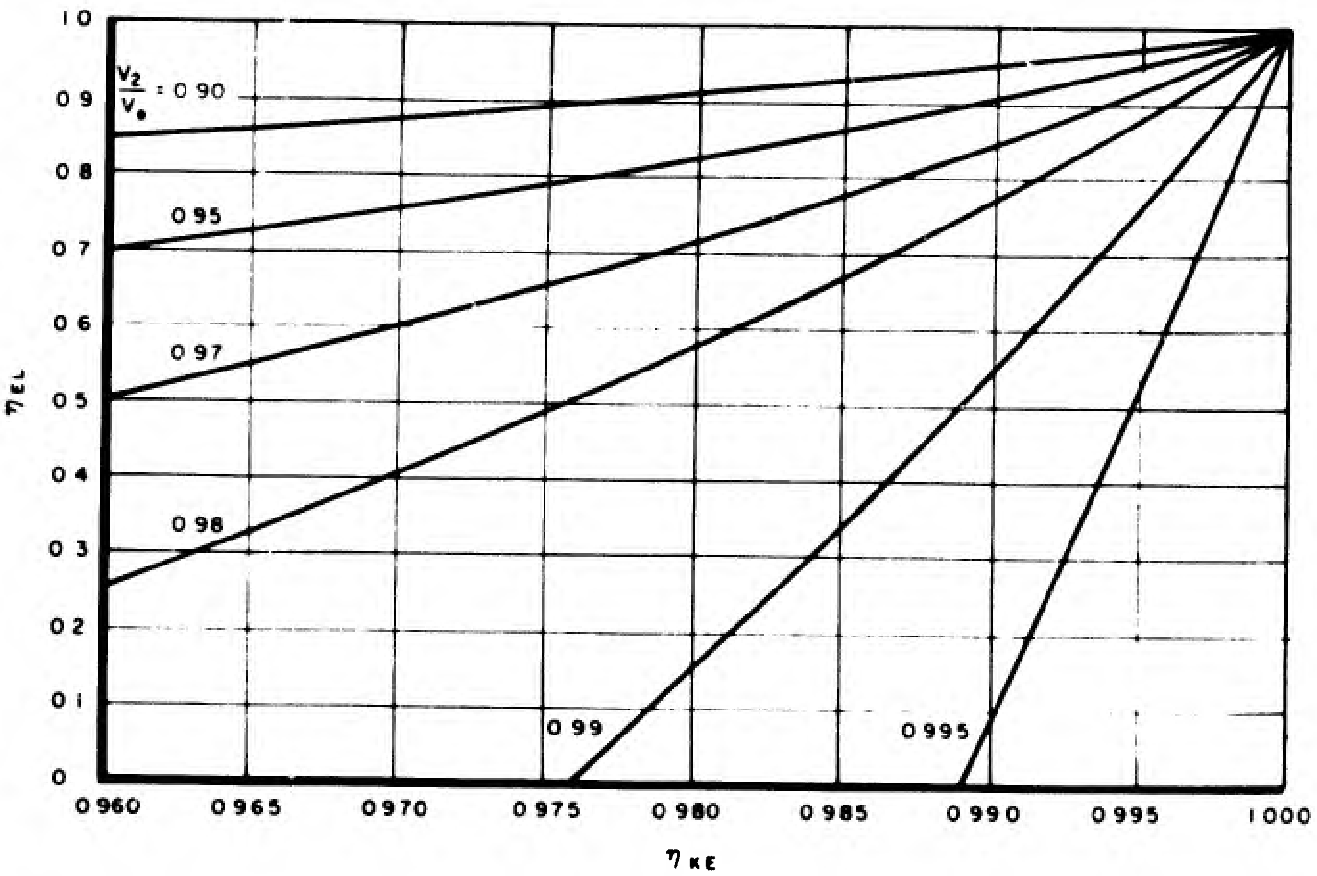

Figure 11b Chart for Conversion Between Polytropic Efficiency and Kinetic Energy Efficiency at Mach 10

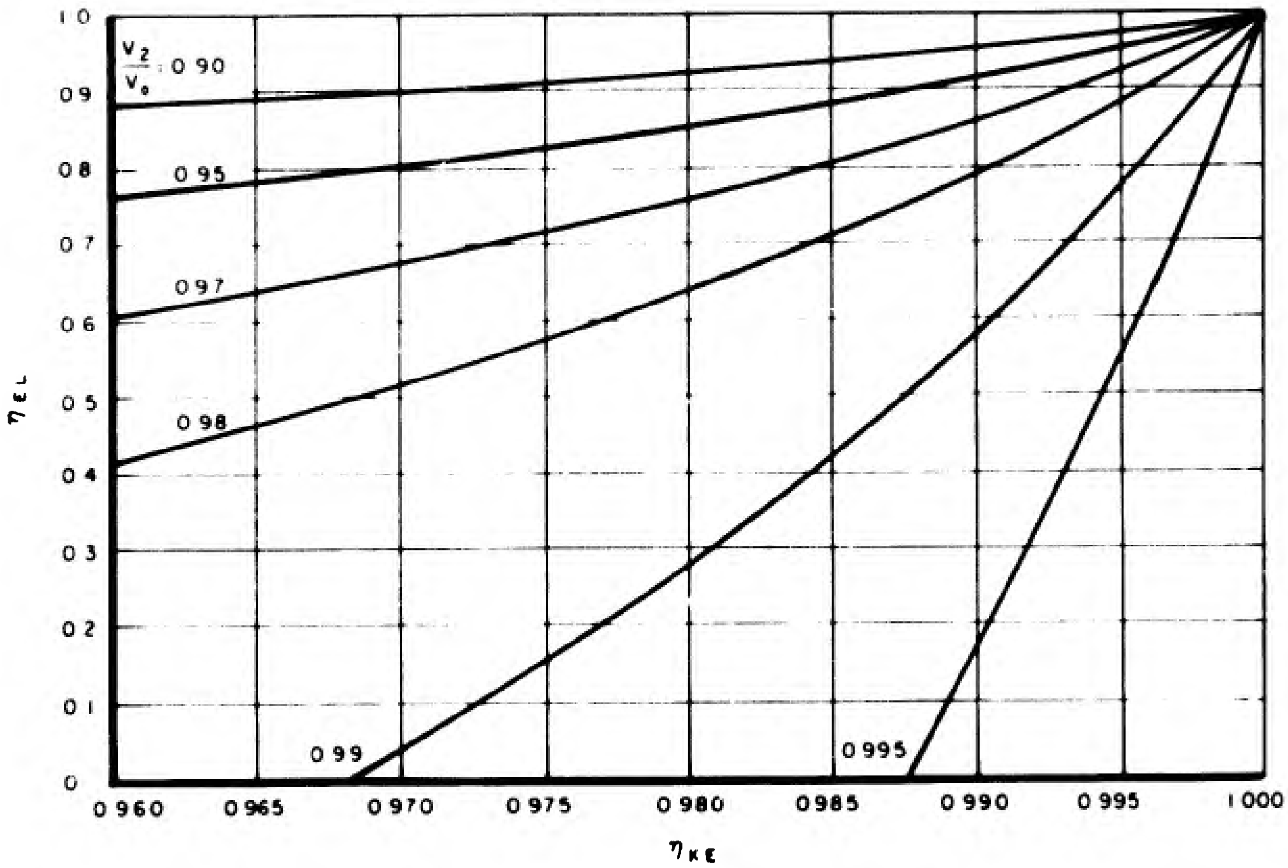

Figure 11c Chart for Conversion Between Polytropic Efficiency and Kinetic Energy Efficiency at Mach 15

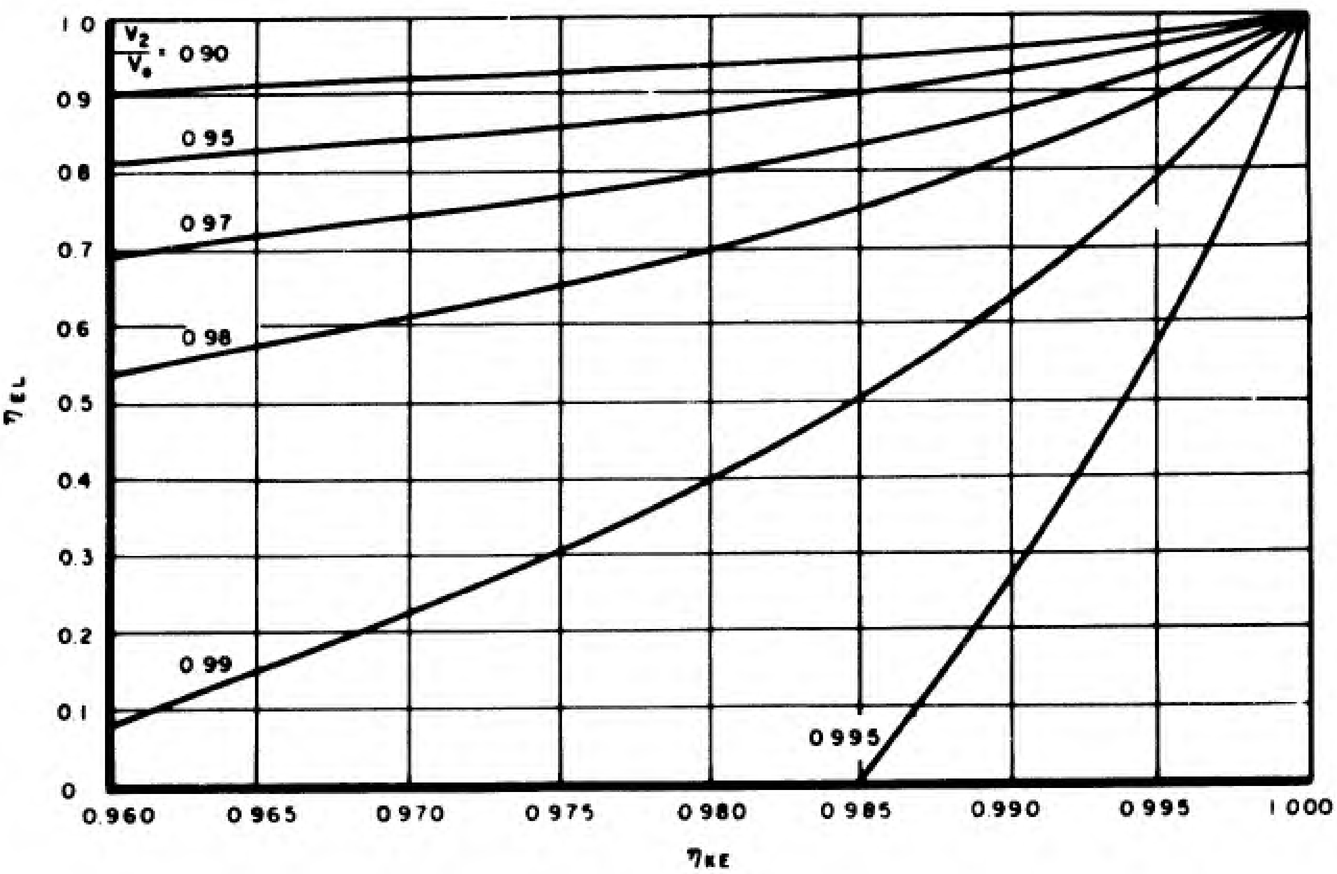

Figure 11d Chart for Conversion Between Polytropic Efficiency and Kinetic Energy Efficiency at Mach 20

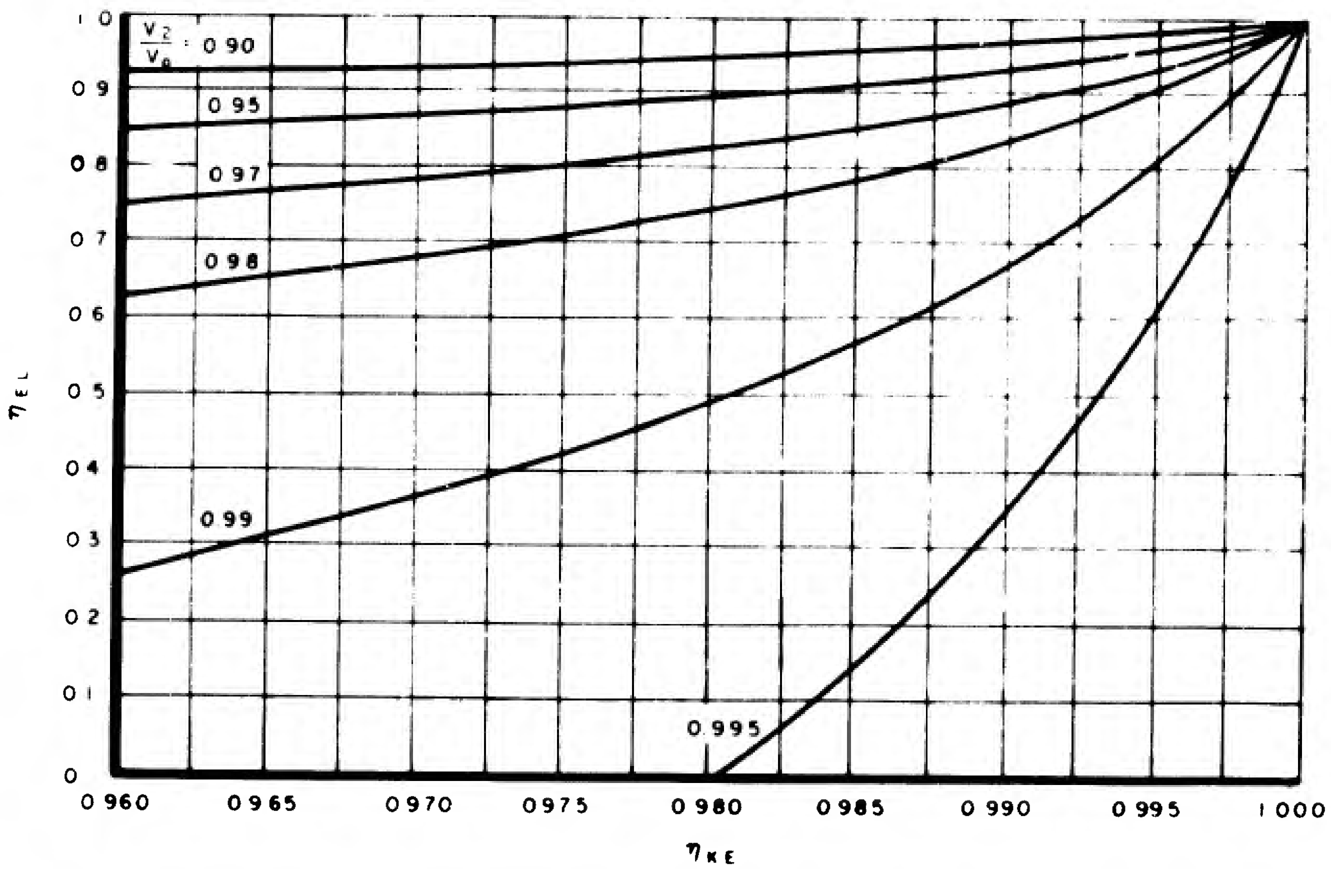

Figure 11e Chart for Conversion Between Polytropic Efficiency and Kinetic Energy Efficiency at Mach 25

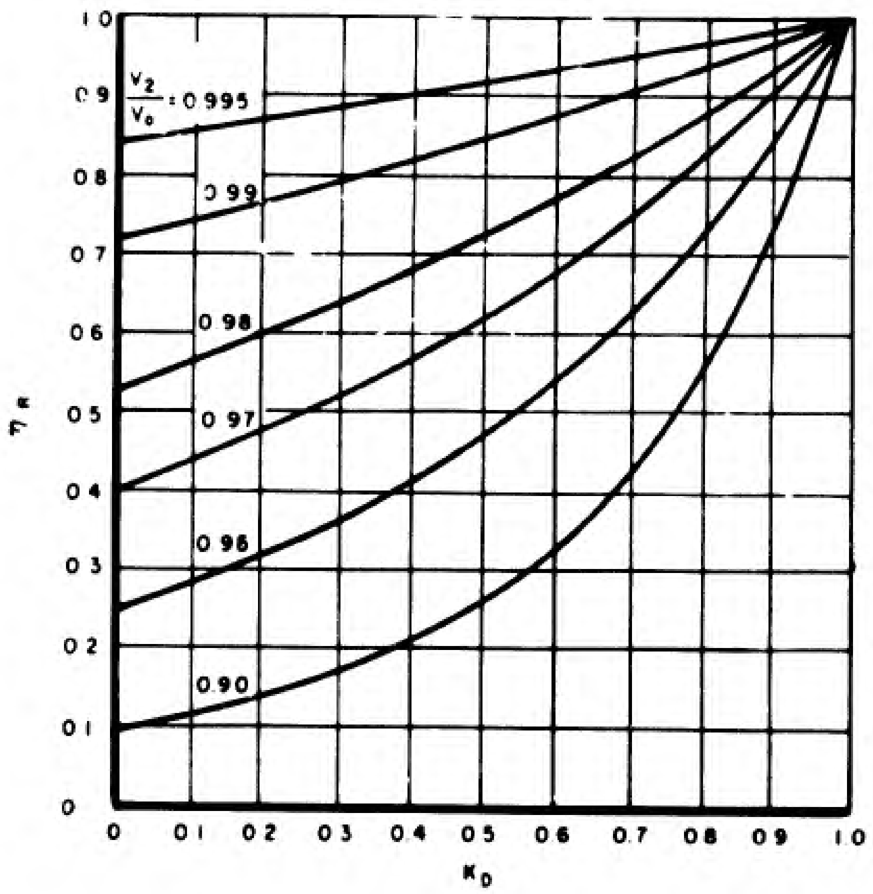

Figure 12a Chart for Conversion Between Pressure Recovery and Process Efficiency at Mach 5

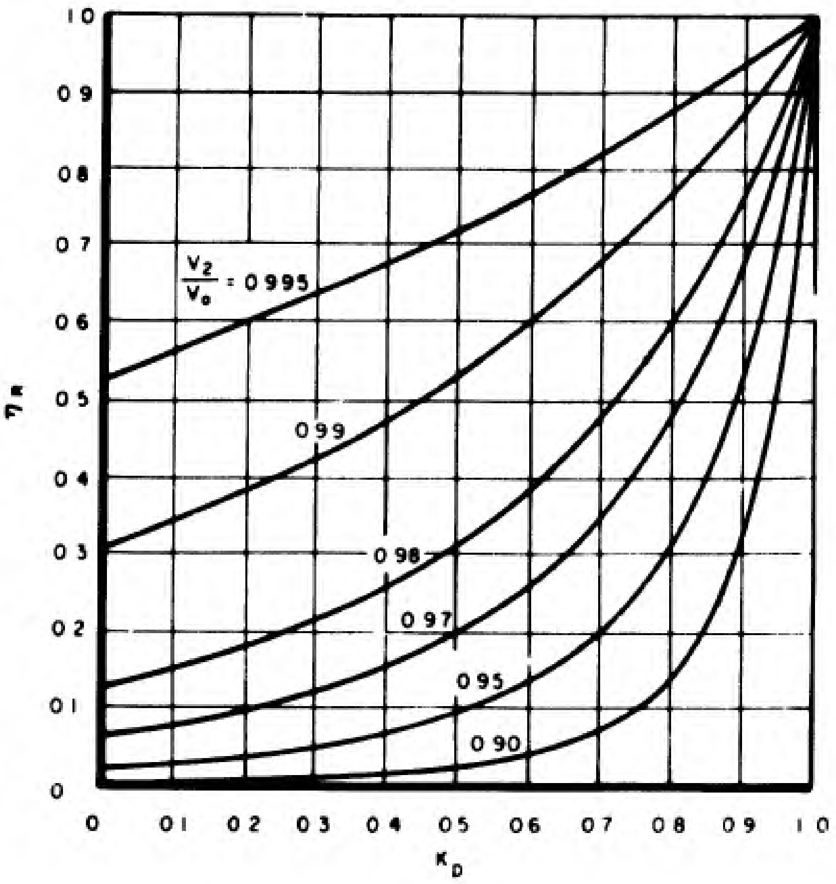

Figure 12b Chart for Conversion Between Pressure Recovery and Process Efficiency at Mach 10

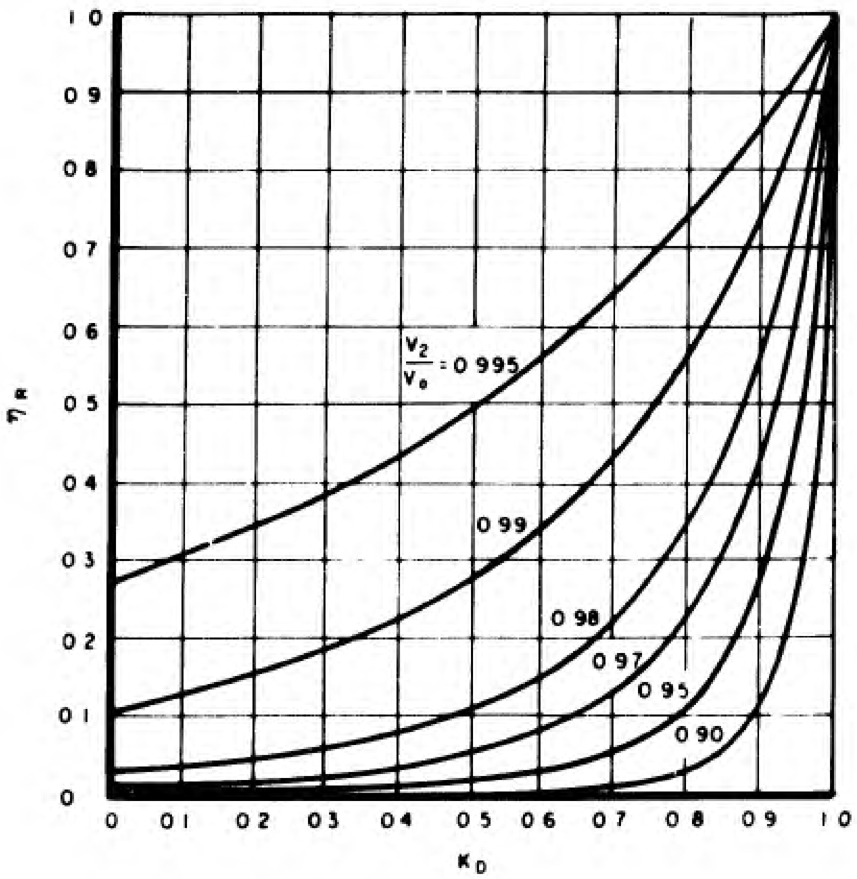

Figure 12c Chart for Conversion Between Pressure Recovery and Process Efficiency at Mach 15

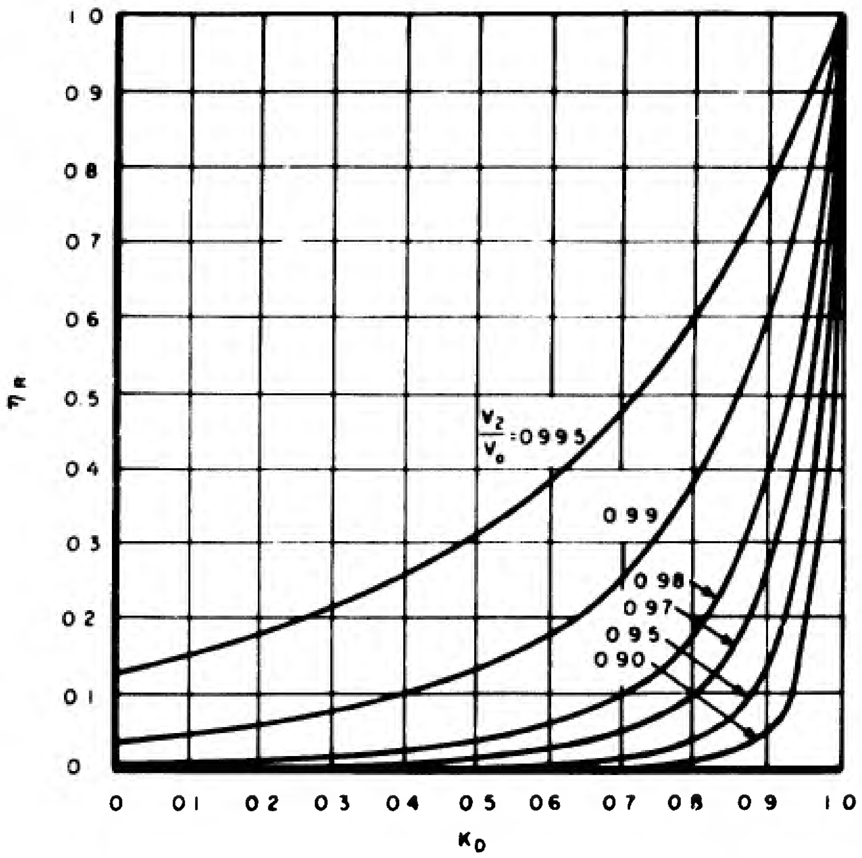

Figure 12d Chart for Conversion Between Pressure Recovery and Process Efficiency at Mach 20

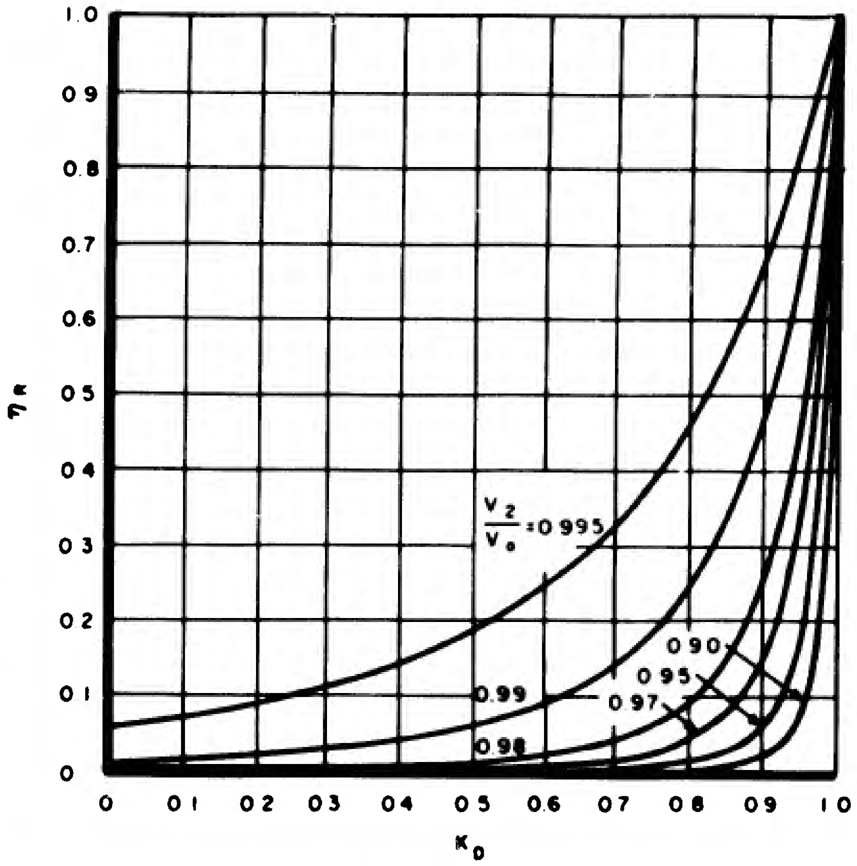

Figure 12e Chart for Conversion Between Pressure Recovery and Process Efficiency at M..ch 25

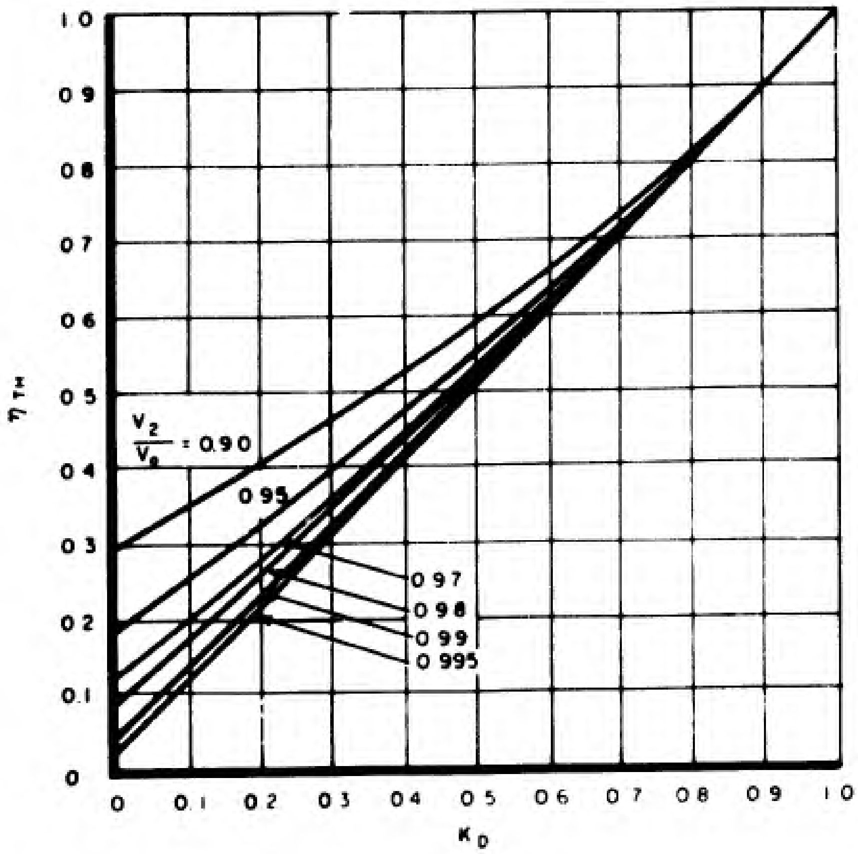

Figure 13a Chart for Conversion Between Thermodynamic Efficiency and Process Efficiency at Mach 5

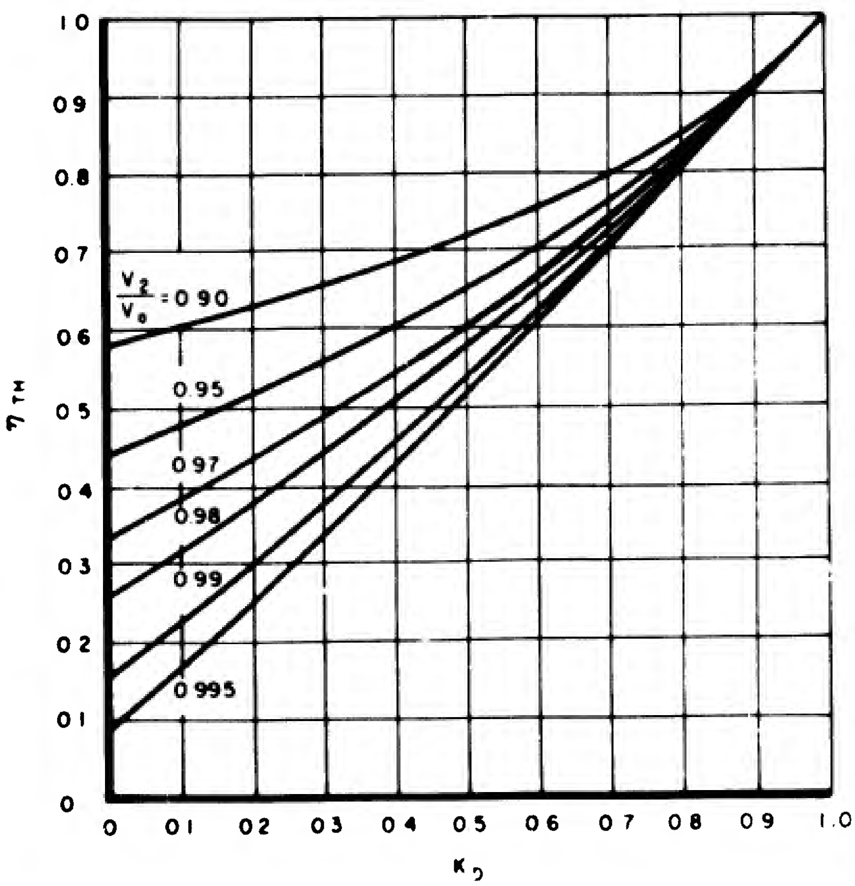

Figure 13b Chart for Conversion Between Thermodynamic Efficiency and Process Efficiency at Mach 10

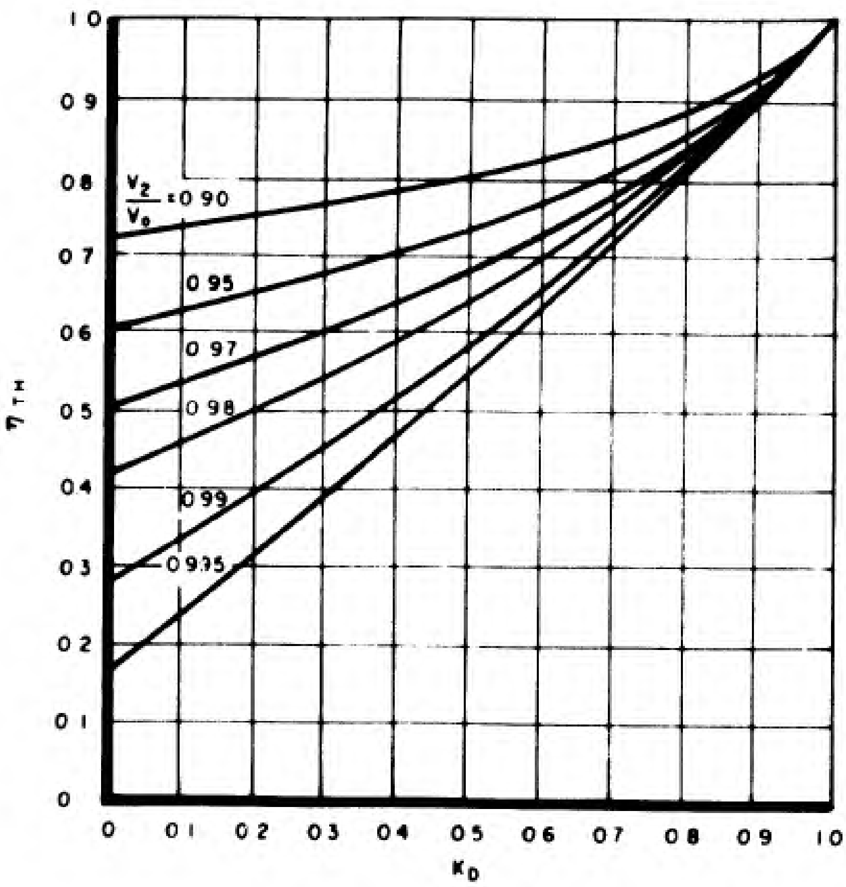

Figure 13c Chart for Conversion Between Thermodynamic Efficiency and Process Efficiency at Mach 15

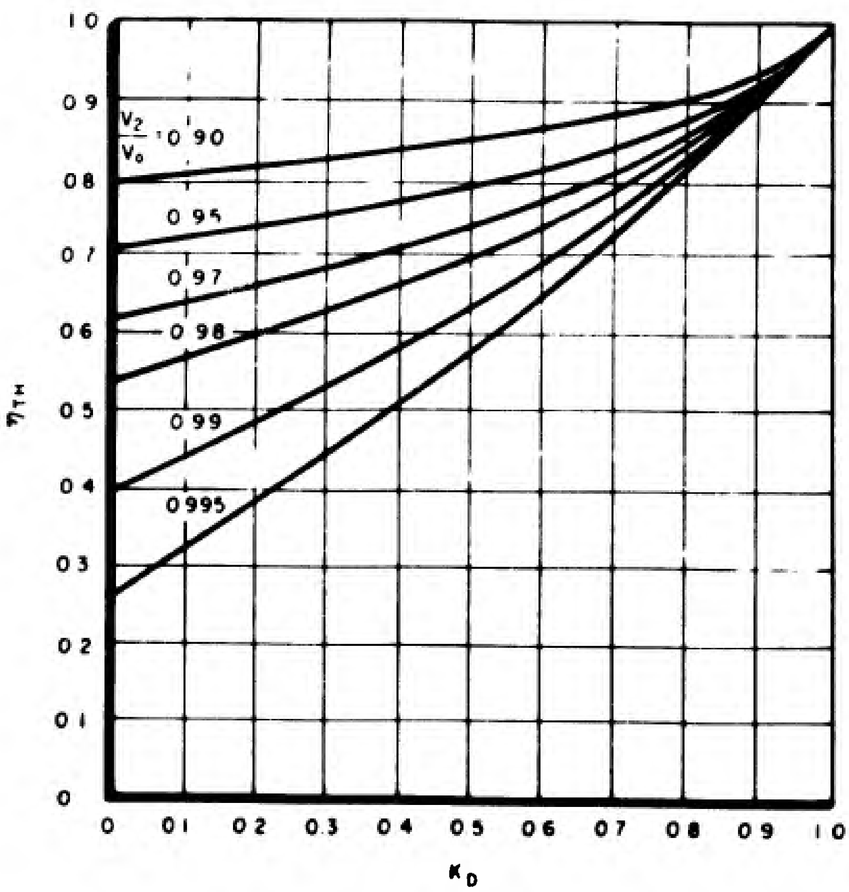

Figure 13d Chart for Conversion Between Thermodynamic Efficiency and Process Efficiency at Mach 20

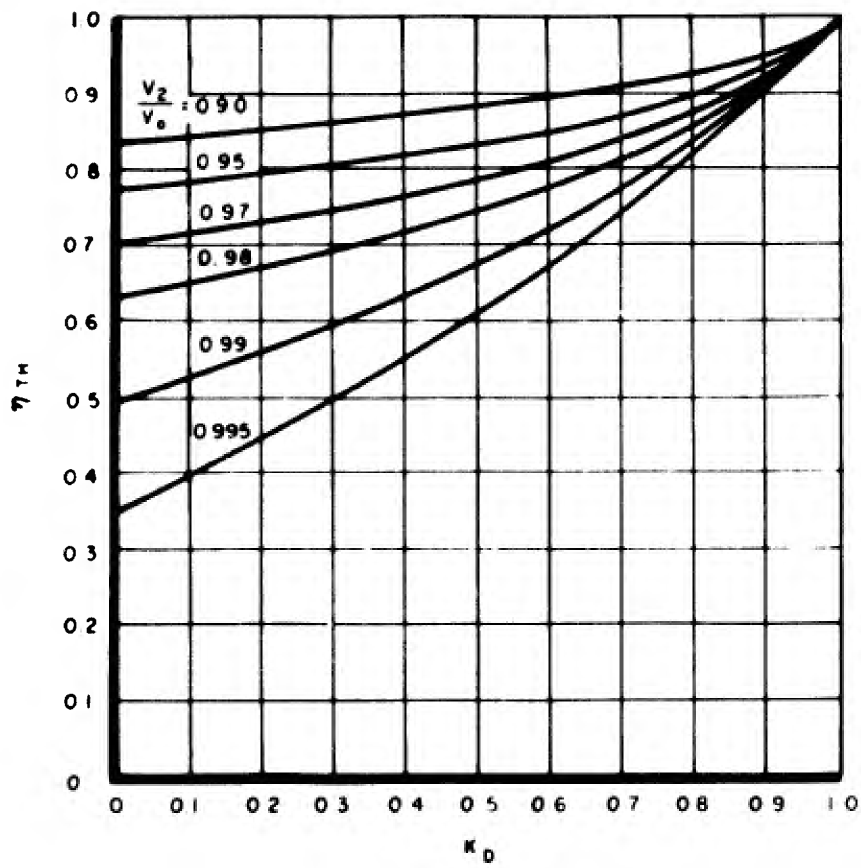

Figure 13e Chart for Conversion Between Thermodynamic Efficiency and Process Efficiency at Mach 25

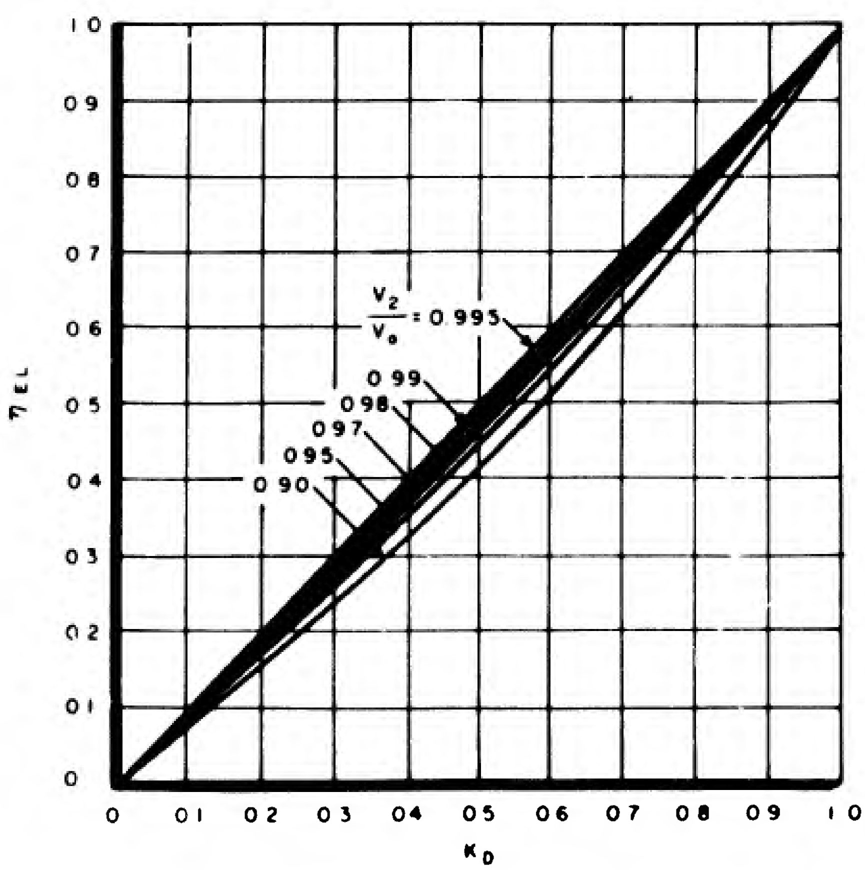

Figure 14a Chart for Conversion Between Polytropic Efficiency and Process Efficiency at Mach 5

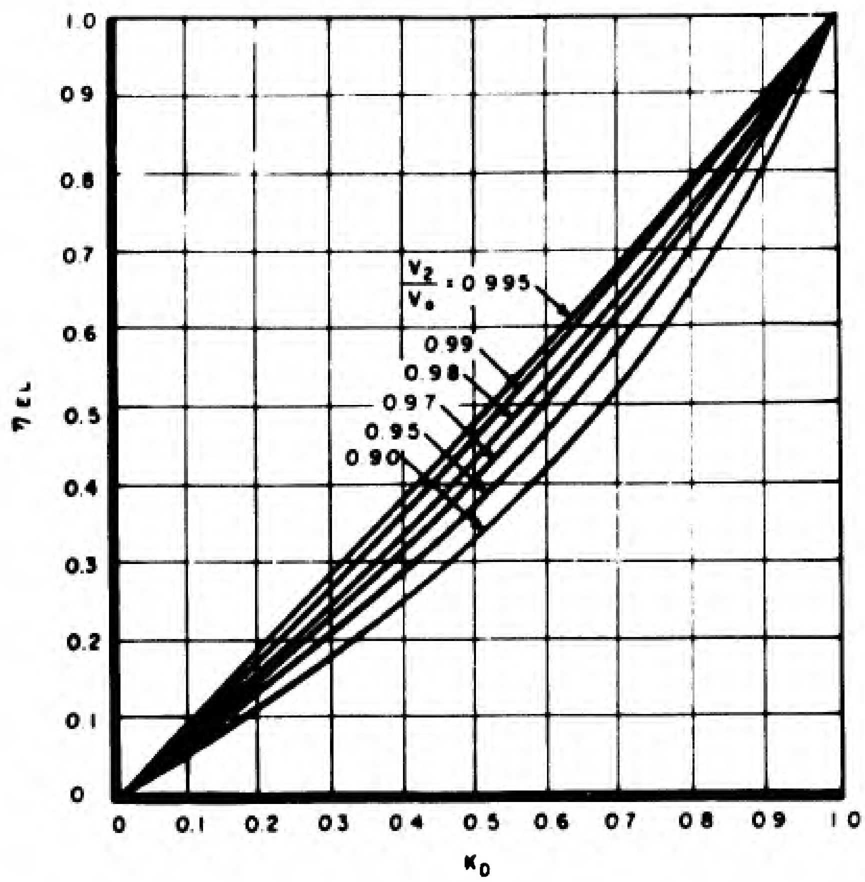

Figure 14b Chart for Conversion Between Polytropic Efficiency and Process Efficiency at Mach 10

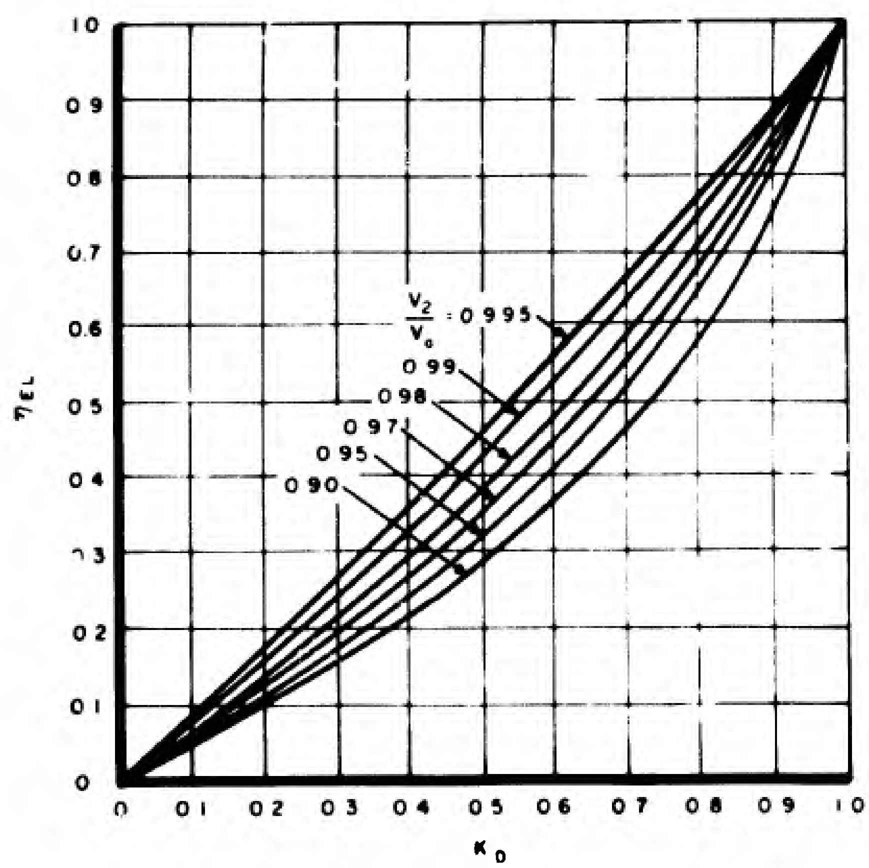

Figure 14c Chart for Conversion Between Polytropic Efficiency and Process Efficiency at Mach 15

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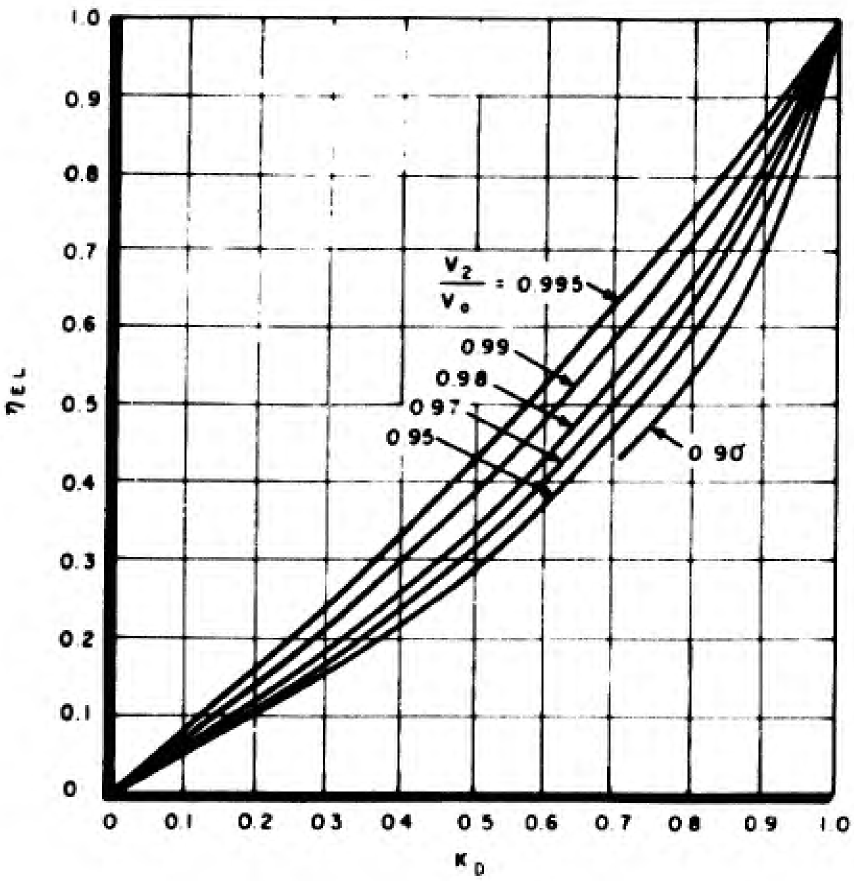

Figure 14d Chart for Conversion Between Polytropic Efficiency and Process Efficiency at Mach 20

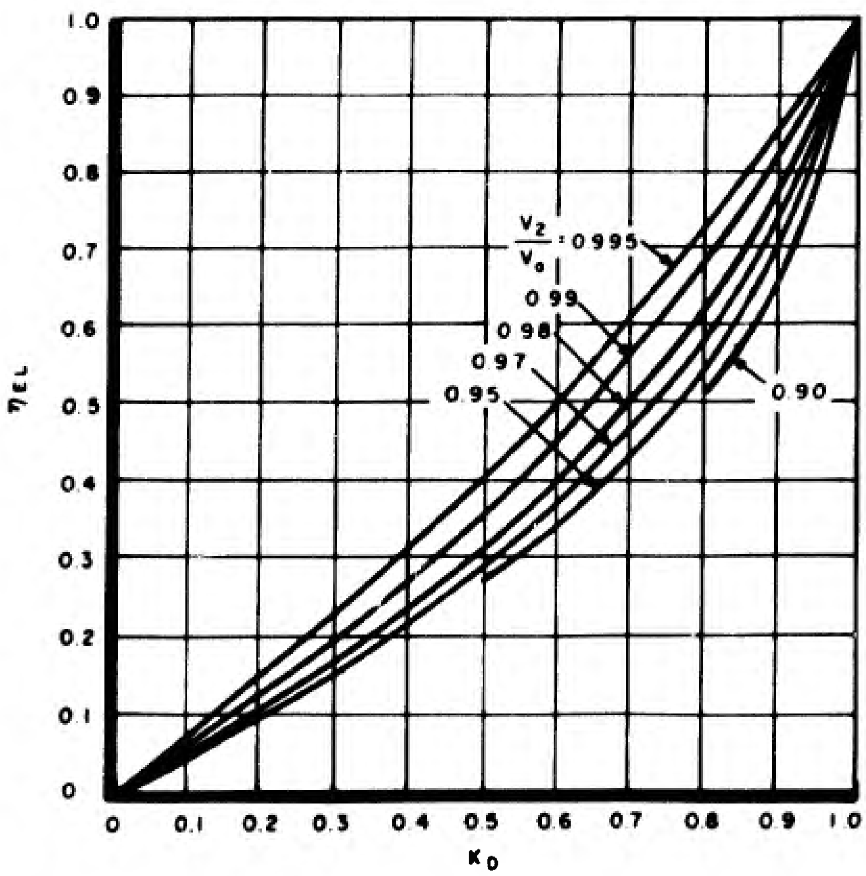

Figure 14e Chart for Conversion Between Polytropic Efficiency and Process Efficiency at Mach 25

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