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Extension of Propeller Blade Stress Program

GASL TECHNICAL REPORT 466

By

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I INTRODUCTION

The existing IBM 7090 computer program to calculate static stress distributions of propeller blades (ref.2) has been revised to execute the full potential of the program, i.e. to generate 108 degrees of freedom. As noted in the conclusions of ref. 2, the round-off error accumulated in the process of solving a system of more than 90 equations had precluded the full exploitation of the existing program. The aforementioned revision consisted of developing a technique which would result in a solution for a system as high as 108 equations.

Two techniques were investigated. One was to utilize a "brute force" approach and invert the coefficient matrix in double precision, using a routine which ignored the fact that the matrix was positive-definite. The other approach was to partition the matrix into 9 sub-matrices (6 independent due to symmetry) and generate the solution vector; the inverse is not calculated. Either technique would then be followed by appropriate iteration schemes which would refine the solution vector to the required precision.

The direct inversion method enjoyed the advantage of always being able to generate a solution vector; past experience, however, indicated that direct inversion of such a large system invariably produced large round-off errors and a corresponding relatively inaccurate solution vector. It was necessary to determine whether the subsequent iteration schemes would filter out the "noise" in this solution within a reasonable amount of machine time.

The Gauss-Seidel iteration is unconditionally convergent for a positive-definite matrix; however, convergence can be extremely slow - perhaps prohibitively slow - if the trial solution vector is a very poor approximation of the true solution.

The partitioning method operates on a family of sub-matrices of size 36 x 36 at most, rather than a single 108 x 108 matrix. Three inversions are executed on these smaller-sized matrices; the resulting round-off error using double-precision arithmetic is virtually nill. However, as shown in detail later, the subsequent operations include subtractions of matrices of like terms; it is here that the introduction of noise is likely to occur. Thus, while a solution vector is always generated, its accuracy is subject to question.

This report should be viewed as a modification to ref.2. All symbols are defined in ref. 2 and descriptive figures are contained therein.

II ANALYSIS

A. Direct Inversion Method

Given the system $Ax = f$, the inverse of the coefficient matrix, A^{-1} is calculated leading to the solution vector, $x = A^{-1} f$. The inverse actually generated is in error due to round-off error; call it $\tilde{A}^{-1}$. Thus, the resulting solution $\tilde{x} = \tilde{A}^{-1} f$ is also in error. If we write $x = \tilde{x} + \Delta x$ and define $\tilde{f} = A\tilde{x}$ and $f = \tilde{f} + \Delta f$, then we may write the system as

$$A\,(\tilde{x} + \Delta x) = (\tilde{f} + \Delta f)$$

which leads to $A\,\Delta x = \Delta f$, where $\Delta f = f - \tilde{f}$ may be calculated. If the inverse, $\tilde{A}^{-1}$ is "sufficiently" accurate (ref.3), then we may set up an iterative scheme as follows:

$$\Delta x^{(n)} = \tilde{A}^{-1}\,\Delta f^{(n)}$$

$$x^{(n+1)} = x^{(n)} + \Delta x^{(n)},\quad f^{(n+1)} = Ax^{(n+1)},\quad \Delta f^{(n+1)} = f - f^{(n+1)}$$

Thus, if the iteration is convergent, Δx approaches zero and the desired precision of the solution vector is attained.

Subsequently, a Sidelian iteration is performed to further refine the solution. During the course of each sweep thru the system, the elements of the solution vector are calculated in terms of the "most recent" values:

$$x_i = \frac{1}{a_{ii}} \left[f_i - \sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij}\, x_j \right] \quad ; \; i = 1,2,3, \ldots, N$$

In the above discussion,

A	is the matrix of coefficients (strain energy matrix, ref. 1)
x	is the (unknown) solution vector
f	is the forcing vector which is calculated previously by the program, utilizing the pressure loading input data.
N	is the size of the system ≤ 108
Δx, Δf	are error vectors

B. Partitioning Method

The system $Ax = f$ may be partitioned into sub-matrices, a_{ij}, and vectors, x_i and f_i as follows:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

These partitioned elements may be operated upon by a Gauss-Jordan reduction scheme to yield an upper triangular system:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & b_{22} & b_{23} \\ 0 & 0 & c_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ g_2 \\ g_3 \end{bmatrix}$$

where

$$b_{22} = a_{22} - a_{21}\, a_{11}^{-1}\, a_{12}$$

$$b_{23} = a_{23} - a_{21}\, a_{11}^{-1}\, a_{13}\ ;\ b_{32} = b_{23}^{T}$$

$$b_{33} = a_{33} - a_{31}\, a_{11}^{-1}\, a_{13}$$

$$c_{33} = b_{33} - b_{32}\, b_{22}^{-1}\, b_{23}$$

$$g_2 = f_2 - a_{21}\, a_{11}^{-1}\, f_1$$

$$g_3 = f_3 - a_{31}\, a_{11}^{-1}\, f_1 - b_{32}\, b_{22}^{-1} \cdot g_2$$

Then utilizing "back substitution",

$$x_3 = c_{33}^{-1}\, g_3$$

$$x_2 = b_{22}^{-1} \left[g_2 - b_{23}\, x_3 \right]$$

$$x_1 = a_{11}^{-1} \left[-a_{12}\, x_2 - a_{13} x_3 + f_1 \right]$$

Note that since the coefficient matrix, A, is symmetric, a_{ii} is symmetric and $a_{ji} = a_{ij}^{T}$. There are 3 inversions performed - submatrices, a_{11}, b_{22}, c_{33} - but the inverse A^{-1} is not generated.

After the solution vector is calculated the program utilizes the Sidelian iteration scheme to refine the results, as described previously.

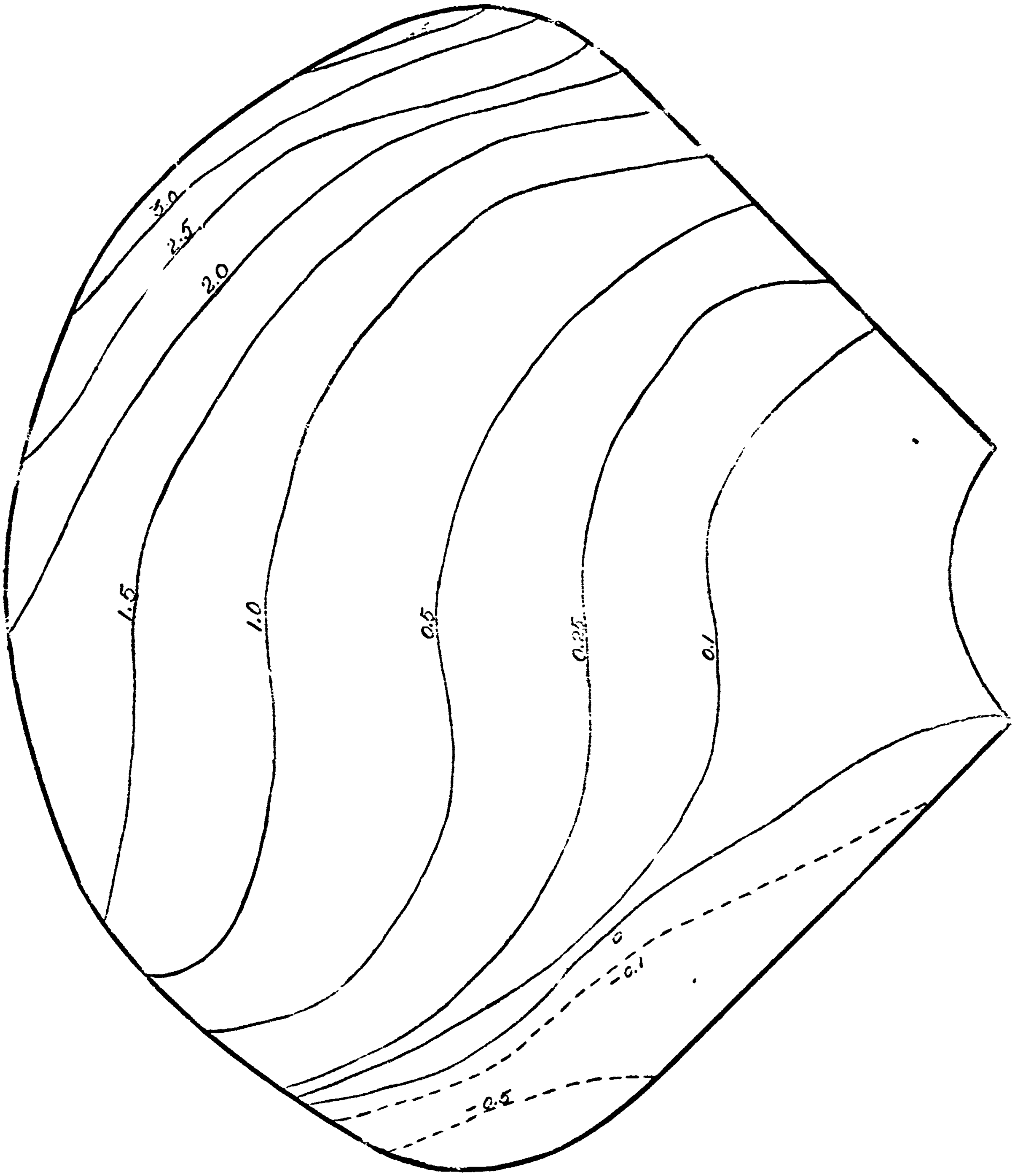
III RESULTS OF TEST RUNS

Two program decks were formed and were tested in parallel. Using the DD828 propeller blade configuration as the test vehicle, these programs were run on the DTMB IBM 7090 computer. To study the influence of error growth as the size of the system was increased, the programs utilized 30, 45, 63 then 84 degrees of freedom, in that sequence. The results of the two versions were compared, and these in turn were compared with the results obtained with the previous version of the program.

All results were in excellent agreement with the one exception of the partitioning version at 84 degrees of freedom. The two new programs were then run at 75 degrees of freedom, and it was found that several elements of the solution vectors of the two new versions, differed significantly. Further examination indicated conclusively that the partitioning version was not reliable beyond 63 degrees of freedom.

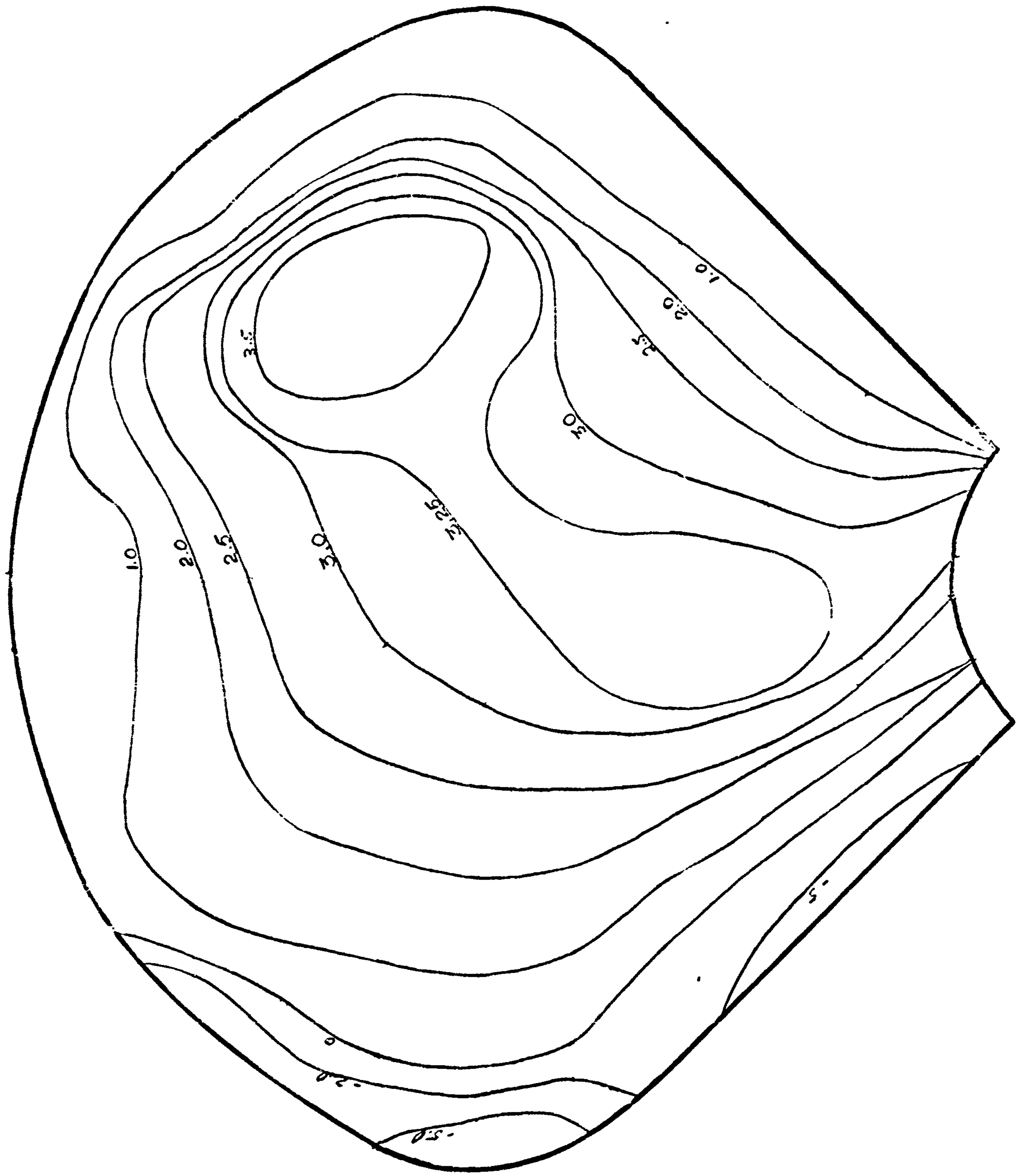
The direct inversion program was then run at full capacity - 108 degrees of freedom - and the results compared favorably with those of 84 degrees of freedom. To test the accuracy of solution, the number of iterations utilized by the Sidelian technique was doubled. This final run indicated that doubling the number of iterations did not alter the solution.

These results have been plotted and appear on the following pages.

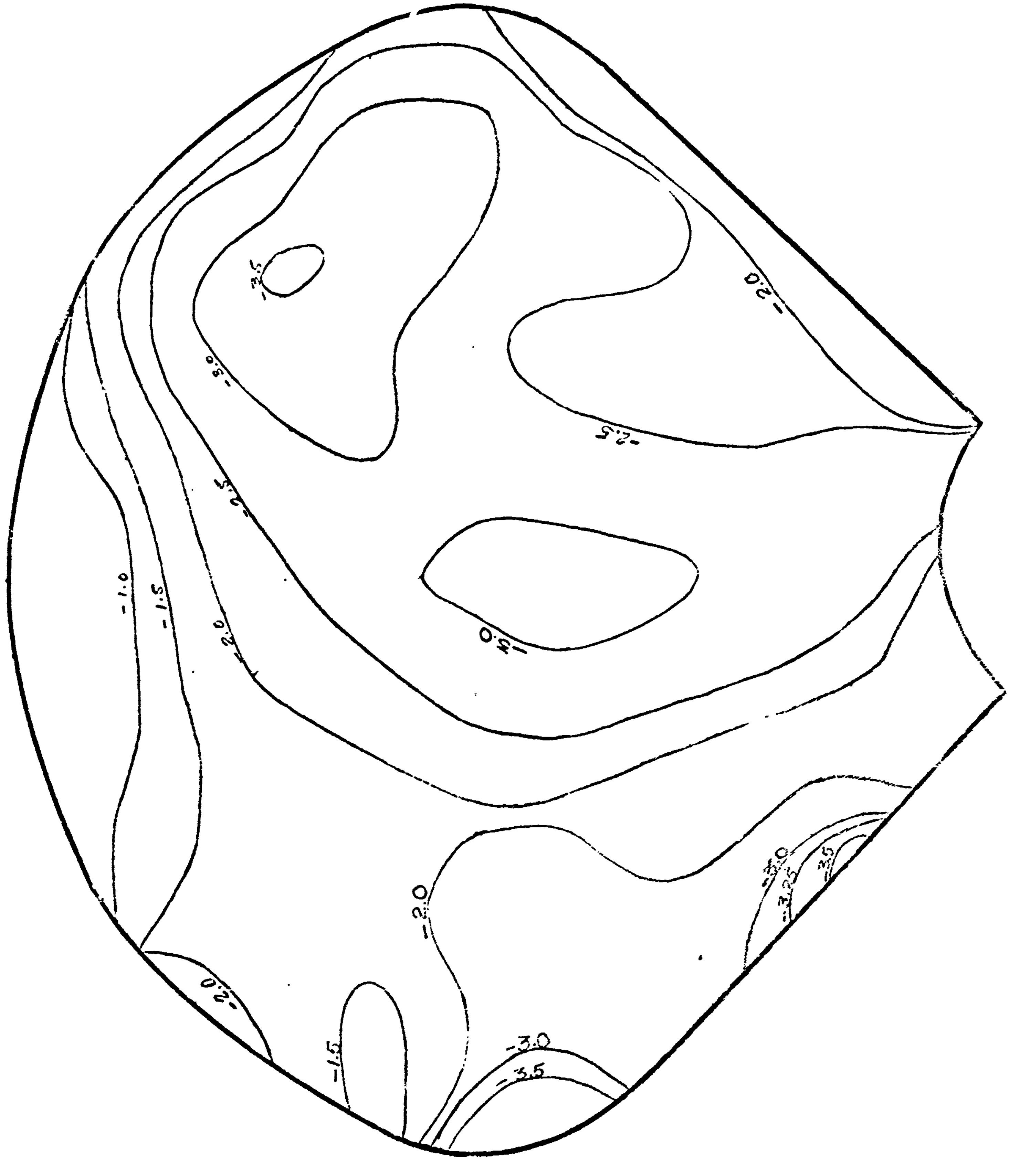


U (3) x 10^{-7}

PROPELLER BLADE

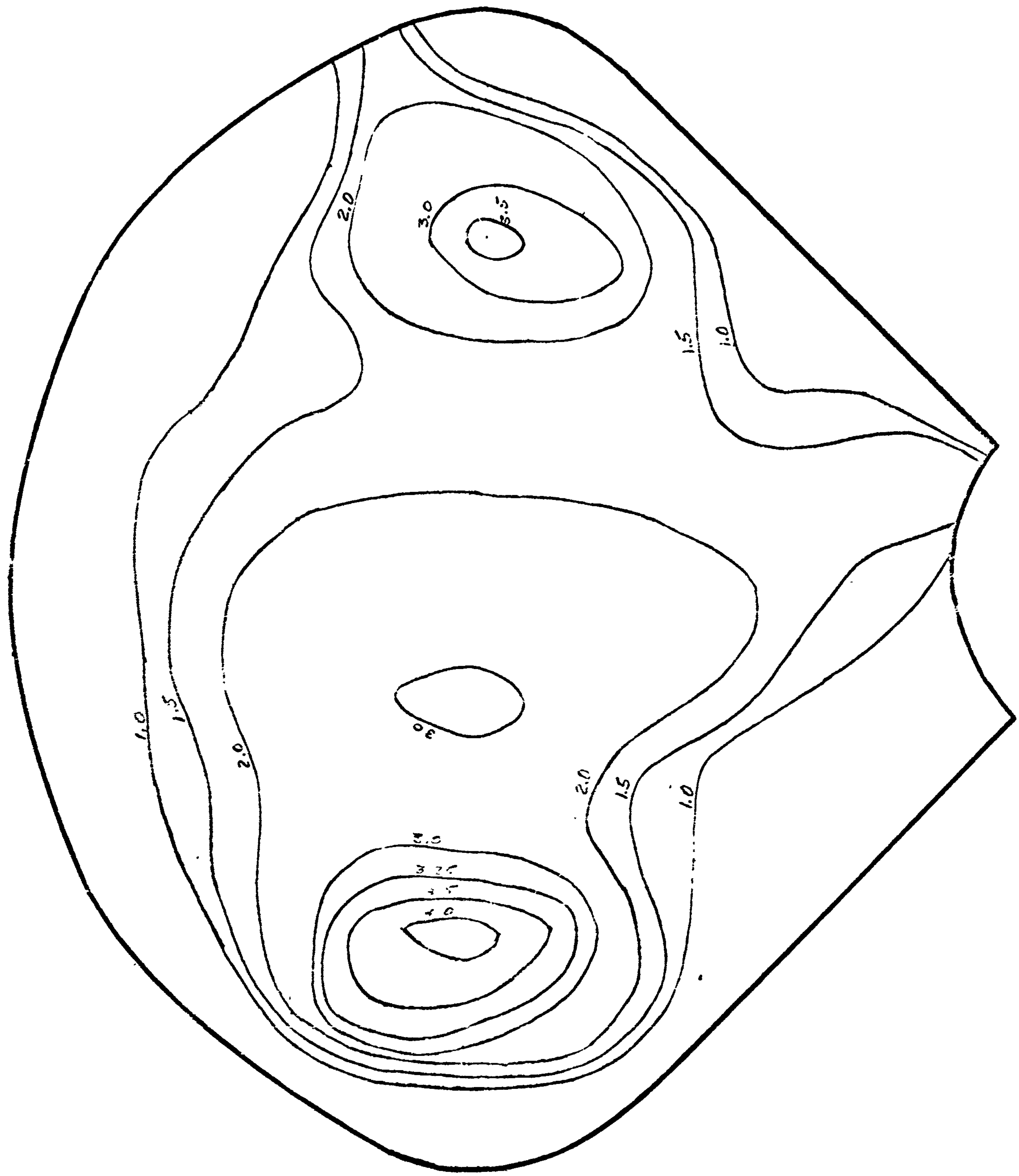


σ(1,1) Front Face x 10^{-4}

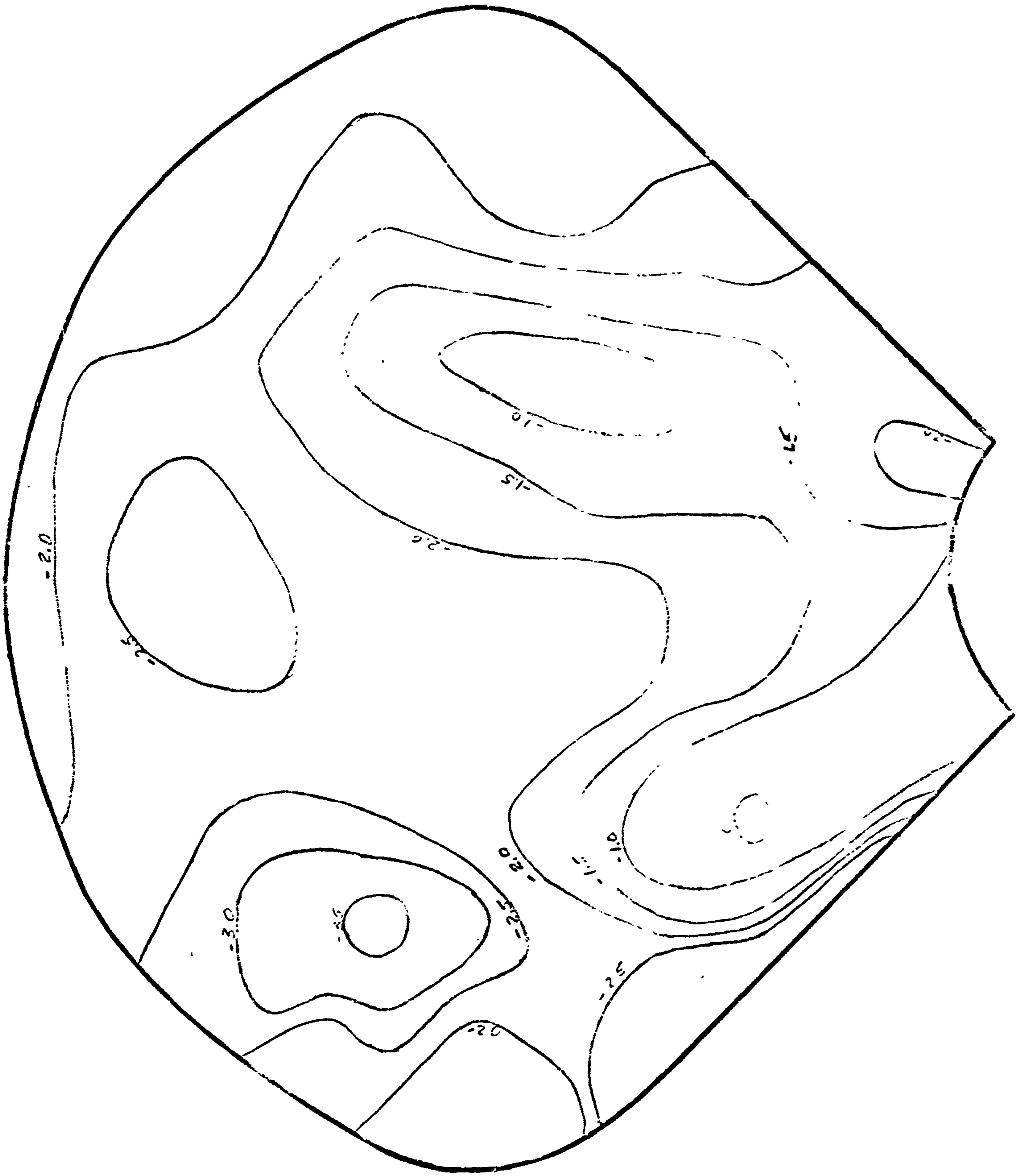


σ (1,1) Back Face x 10^{-4}

PROPELLER BLADE



σ (2,2) Front Face x 10^{-4}



σ (2,2) Back Face x 10^{-4}

IV CONCLUSIONS

The computer program now has the capability of utilizing 108 degrees of freedom in the process of generating stress distributions for propeller blades. The results of such a test run have been plotted and may be compared with similar plots of results which were generated using 84 degrees of freedom (see ref. 2)

These comparisons indicate that no radical change in stress contours occurs as a result of increasing the number of degrees of freedom from 84 to 108; indeed, there is a marked similarity between the two sets of plots. This similarity indicates that the solution has "settled down" and that 108 degrees of freedom are adequate for the purpose of depicting the variation of displacement components over the planform of propeller blades. Thus, increased confidence is warranted in the capability of the subject computer program to generate accurate stress distributions for static, distributed loadings on propeller blades.

V INPUT FORMAT

Card	Input Information
1	Identification
2	Blank
3	Poisson's Ratio, Blade Root Geometry
4, 7, 10,...	Section Properties
5, 8, 11,...	Offsets to Back Face Along Section
6, 9, 12,...	Offsets to Front Face Along Section
T - 1	Properties of Blade Tip
T	Code Card
T+1, T+3, T+5,...	Pressure-jumps at points on sections corresponding respectively to those sections of input cards 3, 6, 9, ... These points are from leading edge to 25% of chord.
T+2, T+4, T+6,...	Pressure-jumps at points on sections corresponding respectively to those sections of input cards 3, 6, 9, ... These points are from 30% of chord to trailing edge.
C	Control Card

Next set of inputs, if any.

Card No. 1:

Any identification statement of 71 characters or less will be accepted. The statement must be punched in columns 2-72 of a card. The digit, "1" should be punched in column 1 for convenience of output.

Card No. 2:

Blank

Card No. 3

Data	Columns	Format
ν	1 - 5	X.XXX
W_R	7 - 12	XX.XXX
W_L	14 - 19	XX.XXX
R_R	21 - 26	XX.XXX
R_L	28 - 33	XX.XXX
N	37 - 38	XX

The first word, ν, is Poisson's Ratio. The next 4 words, W_R, W_L, R_R, R_L, are indicated in Fig. 2 (ref. 2). The number of degrees of freedom for each displacement component is N. Thus for 63 degrees of freedom, N is 21. If a full-scale problem is being run (108 degrees of freedom), N may be left blank.

Card No. 4

Data	Columns	Format
% Radius	1 - 2	XX
Radius	4 - 9	XX.XXX or XXX.XX
Span	11 - 16	XX.XXX or XXX.XX
Pitch	18 - 24	±XX.XXX or ±XXX.XX
Rake	26 - 32	±X.XXXX
Skew	34 - 40	±X.XXXX

Card No. 5

Data	Columns	Format
A	1 - 6	±X.XXX or ±XX.XX
E	7 - 12	
F	13 - 18	
G	19 - 24	
H	25 - 30	
J	31 - 36	
K	37 - 42	
L	43 - 48	
M	49 - 54	
N	55 - 60	
P	61 - 66	

Card No. 6

Identical in format to Card No. 5.

Card T-1

Data	Columns	Format
-	1 - 2	Must be blank
Radius	4 - 9	XX.XXX or XXX.XX
Thickness	11 - 16	XX.XXX
Pitch	18 - 24	XXX.XXX
Rake	26 - 32	±X.XXXX
Skew	34 - 40	±X.XXXX

Card T

Data	Column	Format
Code	1	X

Card T + 1

Data	Columns	Format
P_o	1 - 7	XXX.XXX
$P_{1.25}$	9 - 15	↓
$P_{2.5}$	17 - 23	
$P_{5.0}$	25 - 31	
$P_{7.5}$	33 - 39	
P_{10}	41 - 47	
P_{15}	49 - 55	
P_{20}	57 - 63	
P_{25}	65 - 71	

Card T + 2

Data	Columns	Format
P_{30}	1 - 7	XXX.XXX
P_{40}	9 - 15	↓
P_{50}	17 - 23	
P_{60}	25 - 31	
P_{70}	33 - 39	
P_{80}	41 - 47	
P_{90}	49 - 55	
P_{95}	57 - 63	
P_{100}	65 - 71	

Card C

Data	Column	Format
Code	1	x

The program can accept a maximum of 12 sections corresponding to a total of 66 input cards per set.

Cards 5 and 6 specify the thickness offsets along a section at equal chord increments, from leading-edge to trailing-edge. The offset of A is the offset near the leading edge, the offset of E is at one-tenth the chord distance from the leading edge, etc. The offset of P refers to trailing edge.

Card T specifies the type of peaked loading if any. The code is "1" for a "peaked" load and a fully wetted blade, and blank for any load which is not "peaked".

Cards T+1 and T+2 specify the complete pressure-jump loading acting on that section described on cards 4, 5 and 6. The subscripts of the p's are the percentages of chord, counting from leading edge to trailing edge. For example, P_O is the pressure-jump loading acting at the leading edge of the section under consideration; P_{20} acts at 20% of chord, or one-fifth the distance from the leading edge. Of course, P_{100} is the pressure-jump loading at the trailing edge (invariably, zero). Cards T+3 and T+4 specify the pressure-jump loading on that section described on cards 7, 8, and 9; cards T+5 and T+6 pertain to cards 10, 11, and 12, etc.

The control card dictates the action of the program after a "run" has been completed. If no more inputs follow, the control card may be blank. If it is desired to conduct an additional analysis on the same blade using a different loading, the control card must have a "2" punched in Column 1, and be followed immediately by cards specifying the loading, as described above (cards T through C). Should it be desired to conduct an analysis on another blade (no matter what the loading may be), a "1" must be punched in Column 1, followed by cards describing the new geometry and corresponding pressure-jump loading (cards 1, 2, 3...,C).

The formats indicated are representative. The decimal points may be shifted in either direction, so long as the input "word" is confined within the columns specified.

The program assumes that the tip of the blade is a point (span equal to zero) as is the case of a propeller blade. If the section at the blade tip has some span, such as the flat plates it will be necessary to input two sections extremely close to the tip. In general, both sections should be within the outer 1% of the total blade span.

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2. Lieberman, Edward, "Propeller Blade Vibration and Stress Analysis", GASL Technical Report No. 283, November 1963.

3. Bodewig, E., "Matrix Calculus", Interscience Publishers, 1956.

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