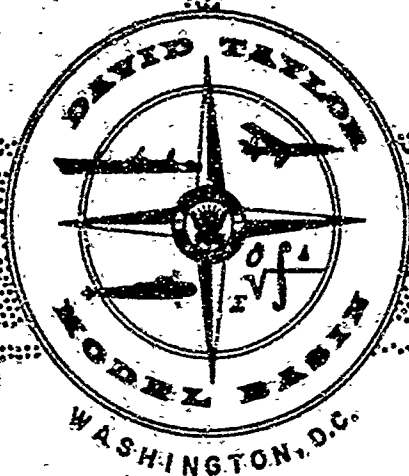


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DEPARTMENT OF THE NAVY

HYDROMECHANICS

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PROPELLER DESIGN BASED ON LIFTING-SURFACE THEORY
PART I

UNIFORM CHORDWISE LOAD DISTRIBUTION

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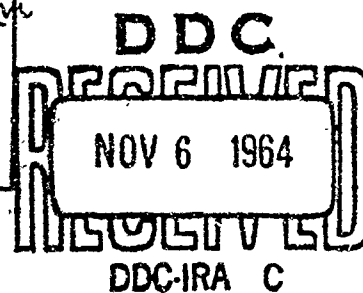
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NOTATION

A_1	Area of a lifting surface
A_2	Area of the helical surface behind the trailing edge of a lifting surface extended to infinity
A_3	Area between a superimposed lifting line and the trailing edge
a_n	Coefficients as defined in Equation [17]
b_n	Coefficients as defined in Equation [21]
C_m	Ordinate of mean line
D	Diameter of propeller
J_s	Advance coefficient based on ship velocity
L	Chordwise coordinate of any point with respect to reference line
L_l	Chordwise coordinate of leading edge
L_t	Chordwise coordinate of trailing edge
L_T	Chord length
n	Revolutions per unit time
P	Point on lifting surface at which mean line is sought
p	Pitch-diameter ratio
$\overline{R}$	Distance vector between two points
r	Radial coordinate
r_h	Radial coordinate of hub
r_0	Radial coordinate of point P
$\overline{dr}$	Length vector of an elementary bound vortex line on a lifting surface; $\overline{dr}/ dr $ is a unit vector tangent to the bound vortex line
$\overline{ds}$	Length vector of an elementary free vortex line, $\overline{ds}/ rd\phi $ is a unit vector tangent to the free vortex line

u_a, u_t	Nondimensional axial and tangential components, respectively, of induced velocity from lifting-line calculation based on ship velocity
V	Nondimensional inflow velocity based on ship speed
$\overline{V}_B$	Induced velocity vector at point P due to radial bound circulation
V_{ba}, V_{bt}	Axial and tangential components of $\overline{V}_B$
$\overline{V}_F$	Induced velocity vector at point P due to free circulation
V_{fa}, V_{ft}	Axial and tangential components of $\overline{V}_F$
$\overline{V}_L$	Velocity vector at a lifting line induced by trailing free vortex sheets
$\overline{V}_P$	Total induced velocity vector at point P
$\overline{V}_{PL}$	Total induced velocity vector at point P relative to $\overline{V}_L$
V_s	Ship velocity
V_x	Nondimensional local longitudinal velocity based on ship velocity
$\overline{v}$	Nondimensional induced velocity
v_a, v_t	Nondimensional axial and tangential components, respectively, of induced velocity v based on ship velocity
v_n	Nondimensional induced downwash at point P normal to helical chord line based on ship velocity
v_{ba}, v_{bt}	Axial and tangential components of $\overline{V}_B$ within a strip between r_1 and r_2
v_{fa}, v_{ft}	Axial and tangential components of $\overline{V}_F$ within a strip between r_1 and r_2
x, y, z	Cartesian coordinates
x	Chordwise coordinate
w_x	Local wake fraction
y	Radial coordinate as defined in Equation [17]
Z	Number of propeller blades

β	Advance angle
β_i	Hydrodynamic pitch angle
Γ	Nondimensional circulation = circulation πDV_s
$\Gamma(r)$	Nondimensional bound circulation distribution along a lifting line
$\Gamma_c(r)$	Nondimensional free circulation distribution on free vortex sheet trailing a lifting line or a lifting surface
$\Gamma_f(r, \theta)$	Nondimensional free circulation distribution
$\Gamma_r(r, \theta)$	Nondimensional bound circulation distribution on a lifting surface
$\Gamma_\theta(r, \theta)$	Nondimensional free circulation distribution on a lifting surface
γ	Circulation per unit angle = $\frac{\Gamma(r)}{\theta_T}$
δ_0	Half width of a strip between r_1 and r_2
ξ	$x - x_0$
θ	Angular coordinate = $L \cos \beta_i / r$
θ_l	Angular coordinate of leading edge
θ_0	Angular coordinate of point P
θ_t	Angular coordinate of trailing edge
θ_T	Total chord length in terms of angular coordinate
ϕ	$\theta - \theta_0$
ϕ_l	$\theta_l - \theta_0$
ϕ_m	$\phi + 2\pi(m-1)/Z$

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ABSTRACT

This report presents a method of the propeller camber calculations for the case of uniform chordwise load distribution employing the lifting-surface theory. This is essentially a refinement of Pien's published work on propeller lifting-surface theory. Mathematical development pertinent to the numerical computation is reviewed. Details of the computational procedure and method are outlined, and results of sample calculations are included. A detailed instruction for preparation of input data for the computer, samples of the computer input data and output, and the FORTRAN listing of the computer program are also given.

INTRODUCTION

In Reference 1, Pien proposed an improved method for the calculations of marine propellers by applying the lifting-surface theory.¹ Included in Reference 1 was an outline for the numerical computation by means of a high-speed computer, IBM 709. The computation was programmed for the case of uniform chordwise load distribution. After his paper was published, a model propeller (Model 3916) was designed by using this improved method and was tested; the results showed a thrust deficiency of approximately 7 percent.

In the fall of 1962 an active propeller research project was planned and initiated by Pien. The overall program included the formulations and the numerical evaluations for the following cases:

1. Propeller camber calculation method for uniform chordwise load distribution,
2. Propeller camber calculation method for arbitrary (nonuniform) chordwise load distribution,
3. Prediction of performance for a given propeller in steady inflow, and
4. Prediction of performance for a given propeller in unsteady inflow.

This report deals with the first case, the camber calculation method for uniform chordwise load distribution. With the overall project in view, the numerical computation procedure and the programming, reported in Reference 1, were revised. This new program incorporates features that will provide a greater degree of flexibility to facilitate handling of a larger number of parameters and to serve as a basic program for the subsequent works.

A brief discussion of the formulation of the problem and a detailed description of the numerical computation procedure are given in this report. Results of a number of examples with different design parameters such as number of blades and area ratios are also included.

¹References are listed on page 61.

A detailed instruction for preparation of input data, and the FORTRAN listing of the computer program, as well as samples of computer input and output data are given in the appendixes.

FORMULATION

The detailed formulation of the problem is given in Reference 1. For the sake of completeness of presentation, a brief discussion of the mathematical development of the problem and the derivation of the pertinent equations, on which the numerical computations are based, are given.

COORDINATE SYSTEM

A Cartesian (x, y, z) coordinate system, shown in Figure 1, is fixed on the propeller. The x -axis is the axis of the propeller shaft with its positive pointing upstream, the y -axis is an axis on the first blade arbitrarily selected so as to pass through the midchord at $r = 0.7R$ with its positive outward, and the z -axis is the third axis and has its positive in accordance with the right-hand rule. Another coordinate system is also shown in Figure 1. It is a cylindrical coordinate system in which a point in space can be identified by (r, θ, x) where r is the radial coordinate, θ is the angular coordinate (positive clockwise looking forward), and x is the same as defined above. Along a helical line the axial coordinate can be expressed in terms of pitch-diameter ratio $p(r)$, i.e., $x = p(r)\theta/\pi$ as shown in Figure 1. This cylindrical coordinate system is used in the formulation of the basic equations.

For convenience in the numerical computations a "moving" cylindrical coordinate is devised. This system has r, ϕ , and ξ as its coordinates, where r is the radial coordinate, ϕ is the new angular coordinate and is defined as $\phi = \theta - \theta_0$, and ξ is a new axial coordinate and is defined as $\xi = x - x_0$, where (r, θ, x) and (r_0, θ_0, x_0) identify a field point $Q(r, \theta, x)$ and a control point $P(r_0, \theta_0, x_0)$, respectively, in the "stationary" cylindrical coordinate system. Also defined are the unit vectors, $\bar{i}, \bar{j}, \bar{k}$, in the directions of r_0, ϕ , and ξ , respectively.

In this new "moving" cylindrical coordinate system the control point $P(r_0, \theta_0, x_0)$ is identified by the vector $\bar{i} r_0$, and the distance vector $\bar{R}$ between a field point $Q(r, \theta, x)$ and P is expressed as follows:

$$\bar{R} = \bar{i}(r_0 - r \cos \phi) - \bar{j} r \sin \phi - \bar{k} \phi p(r)/\pi$$

Similarly, the vectorial expressions for the element of length of the bound vortex line on the lifting surface, $\bar{dr}$, and for the element of length on the free vortex line, $\bar{ds}$, are as follows:

$$\begin{aligned} \bar{dr} &= dr (\bar{i} \cos \phi + \bar{j} \sin \phi + \bar{k} \phi p'(r)/\pi) \\ \bar{ds} &= d\phi (-\bar{i} r \sin \phi + \bar{j} r \cos \phi + \bar{k} p(r)/\pi) \end{aligned}$$

These two expressions will be used in the evaluation of the induced velocities.

BASIC EQUATIONS

It is assumed that the propeller is operating in incompressible, inviscid fluid. The thickness of the propeller blade is assumed to be zero. The theory postulates that the propeller blade may be represented by a lifting surface of the same outline. On this surface, vorticity distribution is so placed and with such intensity that for a given design condition, a certain force distribution is developed.

Let r , θ , and x be the coordinates of a field point Q (see Figures 1 and 2) where the intensity of bound vorticity is denoted as $\Gamma_r(r, \theta)$. Then the following relationship holds:

$$\int_{\theta_t(r)}^{\theta_l(r)} \Gamma_r(r, \theta) d\theta = \Gamma(r) \quad [1]$$

where chordwise coordinates $\theta_l(r)$ and $\theta_t(r)$ define, respectively, the leading and trailing edges of the lifting surface, and $\Gamma(r)$ is the radial bound circulation distribution which can be computed from lifting line theory to satisfy certain design thrust or torque conditions.

Because of the variation of $\Gamma(r)$ along the radial direction, a free vortex sheet is shed and trails behind. It is assumed that this trailing vortex sheet is a general helical surface formed by helical lines, neglecting the radial contraction of the slipstream. As an approximation the helical surface is extended to the leading edge of the lifting surface and the lifting surface is assumed to be a part of this helical surface. In the lifting-surface representation there exist two vortex systems: the radial bound system on the surface with an intensity of $\Gamma_r(r, \theta)$, and the free system with an intensity of $\Gamma_f(r, \theta)$. The latter may be considered as consisting of two parts: the part that is on the lifting surface with an intensity designated as $\Gamma_\theta(r, \theta)$, and the part that trails behind the trailing edge in the slipstream which has an intensity of $\Gamma_c(r)$. In the slipstream, the circulation $\Gamma_c(r)$ is

$$\Gamma_c(r) = - \frac{d}{dr} \Gamma(r) dr \quad [2]$$

Combining Equations [1] and [2], we have

$$\Gamma_c(r) = - \frac{d}{dr} \left[\int_{\theta_t(r)}^{\theta_l(r)} \Gamma_r(r, \theta) d\theta \right] dr \quad [3]$$

The free circulation on the lifting surface may be expressed as

$$\Gamma_{\theta}(r, \theta) = - \frac{d}{dr} \left[\int_{\theta}^{\theta_l} \Gamma_r(r, \theta) d\theta \right] dr$$

The induced velocity at any point P on the lifting surface due to these two vortex systems is derived by the use of the law of Biot-Savart. The resulting general expression is as follows:

$$\overline{V}_P = \frac{1}{4\pi} \iint_{A_1} \Gamma_r(r, \theta) d\theta \frac{\overline{dr} \times \overline{R}}{R^3} + \frac{1}{4\pi} \iint_{A_1 + A_2} \Gamma_f(r, \theta) \frac{\overline{ds} \times \overline{R}}{R^3} \quad [4]$$

where $\overline{V}_P$ = total induced velocity vector at point P on the lifting surface,

$\overline{R}$ = distance vector between a point P and a point where elementary vortex is located,

$\overline{ds}$ = length vector of an elementary vortex line in the free system,

$\overline{dr}$ = length vector of an elementary bound vortex line on the lifting surface,

A_1 = area of the lifting surface, and

A_2 = area of the helical surface behind trailing edge of the lifting surface extended to infinity downstream.

The second term of Equation [4] may be expressed as follows:

$$\frac{1}{4\pi} \iint_{A_1 + A_2} \Gamma_f(r, \theta) \frac{\overline{ds} \times \overline{R}}{R^3} = \frac{1}{4\pi} \iint_{A_1} \Gamma_{\theta}(r, \theta) \frac{\overline{ds} \times \overline{R}}{R^3} + \frac{1}{4\pi} \iint_{A_2} \Gamma_c(r) \frac{\overline{ds} \times \overline{R}}{R^3} \quad [5]$$

Substituting Equation [5] into Equation [4] and adding and subtracting a quantity of

$$\frac{1}{4\pi} \iint_{A_3} - \frac{d}{dr} \Gamma(r) dr \frac{\overline{ds} \times \overline{R}}{R^3}, \text{ where } A_3 \text{ is the enclosed area between the line drawn on}$$

the lifting surface through the centerline of shaft and the point P and the trailing edge, we obtain the following expression for the induced velocity:

$$\begin{aligned} \overline{V}_P = \frac{1}{4\pi} & \left[\iint_{A_1} \Gamma_r(r, \theta) d\theta \frac{\overline{dr} \times \overline{R}}{R^3} + \iint_{A_1} \Gamma_{\theta}(r, \theta) \frac{\overline{ds} \times \overline{R}}{R^3} + \iint_{A_3} \frac{d}{dr} \Gamma(r) dr \frac{\overline{ds} \times \overline{R}}{R^3} \right] \\ & + \frac{1}{4\pi} \left[\iint_{A_2} - \frac{d}{dr} \Gamma(r) dr \frac{\overline{ds} \times \overline{R}}{R^3} + \iint_{A_3} - \frac{d}{dr} \Gamma(r) dr \frac{\overline{ds} \times \overline{R}}{R^3} \right] \quad [6] \end{aligned}$$

since $\Gamma_c(r) = - \frac{d}{dr} \Gamma(r) dr$ by Equation [2].

The second square bracketed term is a function of r only and is denoted as $\bar{V}_L$. It expresses the induced velocity at a lifting line with a bound circulation $\Gamma(r)$. This term plays a very important role in our problem of hydrodynamic aspect of propeller design using lifting surface theory. Consider the case in which the pitch of the lifting surface is constant. If a lifting line with bound circulation $\Gamma(r)$ is superimposed on the surface through the control point P , then it will shed a vortex system with a circulation of $-\frac{d}{dr} \Gamma(r) dr$ and result in an induced velocity at P which is identical to $\bar{V}_L$. The values of this induced velocity $\bar{V}_L$ at various radii may be computed using lifting line theory based on a chosen radial load distribution. From these values the helical surface near the propeller disk is then determined, upon which we place the lifting surface with a chosen blade outline. Since $\bar{V}_L$ is independent of θ , it is convenient to use it as the reference velocity for the induced velocity due to a lifting surface. Let us define the total induced velocity at point P relative to $\bar{V}_L$, $\bar{V}_{PL} = \bar{V}_P - \bar{V}_L$.

If $\bar{V}_{PL}$ is zero everywhere, then $\bar{V}_P = \bar{V}_L$ and the streamline on the lifting surface will coincide with the helical chord at each radius. If $\bar{V}_{PL}$ is not zero everywhere, then it will induce a mean line with respect to the helical chord at each radius. Since we are only interested in the mean line with respect to the helical chord, $\bar{V}_{PL}$ is the only item that needs to be computed and we can disregard $\bar{V}_P$. Carrying this idea through, we obtain from Equation [6] the following expression for $\bar{V}_{PL}$:

$$\begin{aligned} \bar{V}_{PL} = \frac{1}{4\pi} \left[\iint_{A_1} \Gamma_r(r, \theta) d\theta \frac{\bar{dr} \times \bar{R}}{R^3} + \iint_{A_1} \Gamma_\theta(r, \theta) \frac{\bar{ds} \times \bar{R}}{R^3} \right. \\ \left. + \iint_{A_3} \frac{d}{dr} \Gamma(r) dr \frac{\bar{ds} \times \bar{R}}{R^3} \right] = \bar{V}_B + \bar{V}_F \end{aligned} \quad [7]$$

where

$$\bar{V}_B = \frac{1}{4\pi} \iint_{A_1} \Gamma_r(r, \theta) d\theta \frac{\bar{dr} \times \bar{R}}{R^3} \quad [8]$$

$$\overline{V}_F = \frac{1}{4\pi} \left[\iint_{A_1} \Gamma_\theta(r, \theta) \frac{\overline{ds} \times \overline{R}}{R^3} + \iint_{A_3} \frac{d}{dr} \Gamma(r) dr \frac{\overline{ds} \times \overline{R}}{R^3} \right] \quad [9]$$

If we compare Equation [7] with Equation [6], it is obvious that the amount of numerical computation is reduced quite appreciably. In other words, our problem has been greatly simplified by taking advantage of these shortcuts. This is the unique feature of Pien's approach to the problem.

It should be mentioned here that the foregoing procedure is true only for the case where the pitch of the lifting surface is constant. If the pitch changes with radius, the free vortex sheet of a lifting line may not exactly coincide with that of the lifting surface. However, the influence of this on the induced downwash is rather small, and this procedure, therefore, may also be applied to the cases where the pitch is not constant.

The integrals on the right-hand side of Equation [7] now have bounded limits of integration and can be handled conveniently.

From the axial and tangential components of the relative induced velocity $\overline{V}_{PL}$, the downwash distribution v_n , normal to the chord, is obtained. By integrating this downwash distribution curve, the required mean line relative to the chord is obtained.

$$C_m(r_0, \theta) = \frac{r_0 D}{2V \cos \beta_i} \int_{\theta}^{\theta_l} v_n(r_0, \theta_0) d\theta_0 \quad [10]$$

EQUATIONS FOR NUMERICAL COMPUTATION

The major portion of the work is in the evaluation of the induced velocities. By using Equation [7] as a basis, the following working equations for numerical computations are developed.

Induced Velocity at Point P Due to Bound Vortex System, $\overline{V}_B$

The induced velocity at any control point P due to the bound vortex system is expressed by Equation [8]. To reduce it into a working form, we use the "moving" cylindrical coordinate system for convenience. Making use of the expressions for $\overline{dr}$ and $\overline{R}$ developed previously, we obtain the following equation for induced velocity due to the bound vortex system:

$$\begin{aligned} \overline{V}_B = \frac{1}{4\pi} \iint_{A_1} \frac{\Gamma_r(r, \phi)}{R^3} \{ \bar{i} [-\phi \sin \phi p(r)/\pi + r \phi \sin \phi p'(r)/\pi] \\ + \bar{j} [\phi \cos \phi p(r)/\pi + \phi (r_0 - r \cos \phi) p'(r)/\pi] \\ + \bar{k} [-r_0 \sin \phi] \} d\phi dr \end{aligned} \quad [11]$$

where $R = [r^2 - 2r_0 r \cos \phi + r_0^2 + (\phi p(r)/\pi)^2]^{1/2}$.

Induced Velocity at Point P Due to Free Vortex System, $\overline{V}_F$

In a similar manner we put Equation [9] into a working form by substituting the expressions for $\overline{R}$ and $\overline{ds}$ developed previously. It follows that

$$\begin{aligned} \overline{V}_F = \frac{1}{4\pi} \iint_{A_1} \frac{1}{R^3} [\bar{i} (-\phi \cos \phi + \sin \phi) p(r)/\pi \\ + \bar{j} (r_0 - r \cos \phi - r \phi \sin \phi) p(r)/\pi \\ + \bar{k} r (r - r_0 \cos \phi)] [\Gamma_\theta(r, \phi) + \Gamma'(r) dr] d\phi \end{aligned} \quad [12]$$

where $\Gamma'(r) = \frac{d}{dr} \Gamma(r)$ equals zero when $\phi > 0$.

The total induced velocity at point P relative to $\overline{V}_L, \overline{V}_{PL}$ is therefore the sum of Equations [11] and [12]. For the present problem the components that are to be evaluated are the axial ($\bar{k}$) and the tangential ($\bar{j}$) components.

To reduce Equation [12] it is necessary to obtain an expression for the intensity of the free vortex system that is on the lifting surface, $\Gamma_\theta(r, \phi)$. This may be obtained by performing the differentiation in Equation [3].

$$\Gamma_c(r) = -dr \int_{\theta_l(r)}^{\theta_l(r)} \frac{\partial}{\partial r} \Gamma_r(r, \theta) d\theta - \Gamma_r(r, \theta_l) \frac{d}{dr} \theta_l(r) dr + \Gamma_r(r, \theta_l) \frac{d}{dr} \theta_l(r) dr$$

Then, within the lifting surface, the free circulation $\Gamma_\theta(r, \phi)$ can be expressed as follows:

$$\Gamma_\theta(r, \phi) = -dr \int_{\phi}^{\phi_l(r)} \frac{\partial}{\partial r} \Gamma_r(r, \phi) d\phi - \Gamma_r(r, \phi) \frac{d}{dr} \phi_l(r) dr \quad [13]$$

For the case of uniform chordwise load distribution, the bound circulation is uniformly distributed along the chord. Its intensity per unit angle is obtained from Equation [1]:

$$\Gamma_r(r, \phi) = \frac{\Gamma(r)}{\theta_l(r) - \theta_t(r)} = \frac{\Gamma(r)}{\theta_T(r)} = \gamma(r) \quad [14]$$

Then, Equation [13] can be further simplified and becomes

$$\Gamma_\theta(r, \phi) = -dr \left[(\phi_l(r) - \phi) \frac{d}{dr} \gamma(r) + \gamma(r) \frac{d}{dr} \theta_l(r) \right] \quad [15]$$

With Equation [13] substituted into Equation [11], and Equation [15] into Equation [12], we obtain the following expressions for the axial and tangential components of the induced velocities:

$$V_{ba} = \frac{1}{4\pi} \iint_{A_1} -r_0 \gamma(r) \frac{\sin \phi}{R^3} d\phi dr \quad [16a]$$

$$V_{bt} = \frac{1}{4\pi} \iint_{A_1} \frac{1}{\pi} \gamma(r) \frac{\phi}{R^3} [p(r) \cos \phi + (r_0 - r \cos \phi) p'(r)] d\phi dr \quad [16b]$$

$$V_{fa} = \frac{1}{4\pi} \iint_{A_1} - \frac{r(r - r_0 \cos \phi)}{R^3} \{[(\phi_l(r) - \phi) \gamma'(r) + \gamma(r) \theta_l'(r)] - \Gamma'(r)\} d\phi dr \quad [16c]$$

$$V_{ft} = \frac{1}{4\pi} \iint_{A_1} - \frac{p(r)}{rR^3} (r_0 - r \cos \phi - r \phi \sin \phi) \{[(\phi_l(r) - \phi) \gamma'(r) + \gamma(r) \theta_l'(r)] - \Gamma'(r)\} d\phi dr \quad [16d]$$

for $\phi > 0$ the $\Gamma'(r)$ term in Equations [16c] and [16d] is equal to zero.

The total axial component of the induced velocity is, therefore, the sum of Equations [16a] and [16c], and the total tangential component is the sum of Equations [16b] and [16d]. These equations are the basic working equations that are to be evaluated numerically.

The Equations [16a] through [16d] are developed for the induced velocity at a point P on the first blade due to a single blade, the first blade itself. Since all the blades may be represented by lifting surfaces, there will be contributions due to each of these surfaces to this induced velocity. The coordinates of a field point on m^{th} blade corresponding to a field

point (r, ϕ, ξ) on the first blade at the same radius is (r, ϕ_m, ξ) where $\phi_m = \phi + 2\pi(m-1)/Z$, $m = 1, 2, 3, \dots, Z$, assuming the blades are arranged symmetrically with equal angular spacings. To derive an expression for the induced velocity due to the m^{th} blade, it is only necessary then to replace $\sin \phi$ by $\sin \phi_m$ and $\cos \phi$ by $\cos \phi_m$ in Equation [16] since the blades are assumed to be identical to each other. Thus, for example, the axial component of the induced velocity due to the bound vortex system of the m^{th} blade is derived from Equation [16a]:

$$\frac{1}{4\pi} \iint_{A_1} -r_0 \gamma(r) \frac{\sin \phi_m}{[r_0^2 - 2r_0 r \cos \phi_m + r^2 + (\phi p(r)/\pi)^2]^{3/2}} d\phi dr$$

It follows that if we sum up, we would obtain an expression for this induced velocity due to all the blades:

$$V_{ba} = \frac{1}{4\pi} \sum_{m=1}^Z \iint_{A_1} -r_0 \gamma(r) \frac{\sin \phi_m}{[r_0^2 - 2r_0 r \cos \phi_m + r^2 + (\phi p(r)/\pi)^2]^{3/2}} d\phi dr$$

Similarly, the various other induced velocity components due to all the blades may be derived for the three remaining equations.

NUMERICAL COMPUTATION

As stated previously, the radial bound circulation distribution and the hydrodynamic pitch ratio distribution are obtained from the lifting-line theory based on certain design conditions. (At the David Taylor Model Basin the program entitled "AML Problem No. 106 - Moderately Loaded Propeller Using Induction Factors" may be used for this purpose.) These quantities along with the blade outline and the chord-length ratios L_l/D and L_T/D , which are chosen based on the considerations of cavitation, blade element efficiency, and strength, are used as input at given radii for the numerical computations.

For the purpose of interpolating these quantities at radii other than those specified in the input, trigonometric series representations in finite number of terms of these quantities are made by using the expression

$$X' = \sum_{n=0}^N a_n \cos n y \quad [17]$$

where X denotes any of the preceding quantities

y = radial coordinate defined:

$$\cos y = (1 + r_h - 2r)/(1 - r_h)$$

a_n = coefficients determined from input data

N = number of terms.

From this expression, the quantity X can be calculated at any desired radial position. The derivatives of these quantities with respect to r can be easily expressed as

$$\frac{d}{dr} X = - \frac{2}{(1 - r_h) \sin y} \sum_{n=0}^N n a_n \sin n y \quad [18]$$

and they can also be computed at any desired radius.

For each radius r_0 at which the blade element mean line relative to the chord line is sought, various values of θ_0 are chosen so that the distributions of components of the induced velocity can be obtained. At any point $P(r_0, \theta_0)$ the induced velocities are computed according to Equations [16a], [16b], [16c], and [16d].

It should be noted that in these equations the integrand has a singular point at $r = r_0$ and $\theta = \theta_0$ ($\phi = 0$). The integrations are carried out by isolating the singular point. The integrals are evaluated numerically in the regions of $|r - r_0| \geq \delta_0$, where δ_0 is a quantity that can be made as small as practicable. Within the strip $|r - r_0| < \delta_0$, the region of integration is divided into two parts. The first part is over the portion $|\phi| \geq \epsilon_0$, and the second part is over the portion $|\phi| < \epsilon_0$. The first portion again can be carried out numerically. The second portion, $|r - r_0| < \delta_0$ and $|\phi| < \epsilon_0$, is not evaluated, for when δ_0 and ϵ_0 are very small, the contribution to the camber due to this portion is negligible even though the induced velocity may be infinite at some of the singular points. (See Appendix A.)

The numerical computation for the narrow strip of the lifting surface within which the control point P is located is handled differently from the remaining portions of the lifting surface. Thus, to facilitate the numerical calculation the lifting surface A is divided geometrically into three zones (Figure 2): the "mid-zone," which is the narrow strip with a width of $2\delta_0$ from $r - \delta_0$ to $r_0 + \delta_0$; the "tip-zone," which is the area between $r_0 + \delta_0$ and the tip; and the "hub-zone," which is the area between the hub and $r - \delta_0$. The division of zones varies with the radial position r_0 of the point P . The computation procedures in the tip-zone and the hub-zone are identical. For numerical integration the Simpson method is used. In the tip- and hub-zones, the chordwise integration is performed first, and in the narrow mid-zone, which has a width of $2\delta_0$, the radial direction integration is done first. Within this mid-zone the pitch of the chord $p(r)$, $\frac{d}{dr} p(r)$ and the circulation functions $\Gamma_r(r, \phi)$, $\dot{\Gamma}_\theta(r, \phi)$, and $\frac{d}{dr} \Gamma(r)$ are considered to be independent of the radial coordinate, and the mean values

evaluated at the midwidth, r_0 , of the strip are used. Then the integrations in Equations [16a], [16b], [16c], and [16d] can be carried out for the interval between $r_0 - \delta_0$ and $r_0 + \delta_0$:

$$V_{ba} = \frac{1}{4\pi} \int_{\phi_i(r_0)}^{\phi_l(r_0)} -r_0 \gamma(r_0) \frac{(r - r_0 \cos \phi) \sin \phi}{[r_0^2 \sin^2 \phi + (\phi p(r_0)/\pi)^2] R} \Big|_{r_0 - \delta_0}^{r_0 + \delta_0} d\phi \quad [19a]$$

$$V_{bt} = \frac{1}{4\pi} \int_{\phi_i(r)}^{\phi_l(r)} \frac{\phi}{\pi} \gamma(r_0) \frac{1}{r_0^2 \sin^2 \phi + (\phi p(r_0)/\pi)^2} \left[\begin{aligned} & (p(r_0) \cos \phi + r_0 p'(r_0) \frac{r - r_0 \cos \phi}{R} \\ & + p' \cos \phi \frac{r_0^2 + (\phi p(r_0)/\pi)^2 - r r_0 \cos \phi}{R} \end{aligned} \right] \Big|_{r_0 - \delta_0}^{r_0 + \delta_0} d\phi \quad [19b]$$

$$V_{fa} = \frac{1}{4\pi} \int_{\phi_i(r_0)}^{\phi_l(r_0)} - (SF) \left[\ln (r - r_0 \cos \phi + R) - \frac{r}{R} \right] \Big|_{r_0 - \delta_0}^{r_0 + \delta_0} d\phi \quad [19c]$$

$$V_{ft} = \frac{1}{4\pi} \int_{\phi_i(r_0)}^{\phi_l(r_0)} - (SF) \frac{p(r_0)}{\pi} \left\{ \frac{r r_0 (\sin^2 \phi - \phi \sin \phi \cos \phi) + r_0^2 \phi \sin \phi + (\phi p(r_0)/\pi)^2 (\cos \phi + \phi \sin \phi)}{[r_0^2 \sin^2 \phi + (\phi p(r_0)/\pi)^2] R} \right\} \Big|_{r_0 - \delta_0}^{r_0 + \delta_0} d\phi \quad [19d]$$

where $SF = (\phi_l(r_0) - \phi) \gamma'(r_0) + \gamma(r_0) \theta_l'(r_0) - \Gamma'(r_0)$

Γ' term equals zero for $\phi > 0$. These chordwise integrations are then computed numerically by the Simpson method.

The contribution of other blades is computed in the same manner with the blade represented by a lifting surface. In the computation for the m^{th} blade the $\sin \phi$ and $\cos \phi$ terms are replaced by $\sin (\phi + 2\pi(m-1)/Z)$ and $\cos (\phi + 2\pi(m-1)/Z)$ respectively, where Z is the number of blades, $m = 2, 3, \dots, Z$.

The total axial and tangential induced velocity components v_a and v_t are then expressed as

$$v_a = v_{ba} + v_{fa} + v_{ba}' + v_{fa}'$$

$$v_t = v_{bt} + v_{ft} + v_{bt}' + v_{ft}'$$

where the primes denote the velocity components due to other blades.

From these induced velocity components the downwash distributions relative to the chord is calculated by using

$$v_n(r_0, \theta_0) = v_a(r_0, \theta_0) \cos \beta_i - v_t(r_0, \theta_0) \sin \beta_i \quad [20]$$

For calculating the blade mean line, the downwash is expressed in a power series in terms of a new nondimensional angular coordinate $\frac{\theta_0}{\theta_T}$ as follows:

$$v_n(r_0, \theta_0) = \sum_{n=0}^M b_n(r_0) \left(\frac{\theta_0}{\theta_T} \right)^n \quad [21]$$

where the integer M equals the number of chordwise coordinates minus one, and the coefficients $b_n(r_0)$ are evaluated from $(M+1)$ simultaneous equations. With this scheme the downwash at any point can be obtained by either interpolation or extrapolation, and the integration of Equation [10] can easily be performed to obtain the blade mean line ordinates:

$$C_m \left(r_0, \frac{x}{L_T} \right) = \frac{L_T(r_0)}{V(r_0)} \sum_{n=0}^M \frac{b_n}{n+1} \left(\frac{x}{L_T} \right)^{n+1} \quad [22]$$

where $V(r_0)$ is the mean resultant velocity. The mean resultant velocity is computed based on the following geometric relationship; see Figure 3.

$$V(r_0) = [(V_x(r_0) + u_a(r_0))^2 + (\pi r_0/J_s - u_t(r_0))^2]^{1/2}$$

where V_x = nondimensional local longitudinal velocity based on ship velocity;
 u_a, u_t = nondimensional axial and tangential components, respectively, of induced velocity from lifting-line calculation based on ship velocity; and
 J_s = advance coefficient based on ship velocity.

THE COMPUTER PROGRAM

The numerical computation has been programed into the IBM 7090 machine. The computer program is designated as Applied Mathematics Laboratory Problem XPLU. First, the program calculates the angular positions, ϕ , of field points relative to any point P at which the blade element mean line is sought according to the following relationships:

$$\phi_{k,n} = \phi_{1,n-k+1} - \phi_{1,k}$$

with

$$\phi_{1,n} = \phi_{1,n-1} + (n-1) (\Delta\phi)_{\min}$$

and

$$\phi_{k,1} = 0$$

where $k = 1, 2, 3, \dots$ and $n = 2, 3, 4, \dots$ are the indices for the radial and chordwise stations, respectively.

$(\Delta\phi)_{\min}$ is the minimum spacing specified, and $(n-1) (\Delta\phi)_{\min}$ is spacing between the n^{th} and the $(n-1)^{\text{th}}$ stations. When it exceeds a prescribed maximum value $(\Delta\phi)_{\max}$ in the input, it will take on the value of $(\Delta\phi)_{\max}$, i.e., the spacing $\Delta\phi$ is limited to $(\Delta\phi)_{\min} \leq \Delta\phi \leq (\Delta\phi)_{\max}$. This shows that the field points are so located that they have smaller spacings in the close neighborhood of point P , and the spacing increases as the distance between P and the field point increases. The fineness of the network varies depending upon the $(\Delta\phi)_{\min}$ and $(\Delta\phi)_{\max}$ specified. For each of these angular positions the sine and cosine of the angles are computed, compiled into tables, and stored. These tables are used for providing ready information of ϕ , $\sin \phi$, and $\cos \phi$, required in the later stage of the computation. Since only one set of angles is computed, a common pattern of network of field points for any point P is thus established. The specified maximum range of angles for which the tables are made is sufficiently large so that they may be used for any propeller of the conventional configuration. Next, the program calculates the coefficients a_n in Equation [17] with the input data supplied. The number of terms, N , depends on the number of intervals of radii r in the input data which can be chosen arbitrarily. A 9-interval of r is recommended, and the present program is programed for 9 intervals.

The computer then selects, according to the input, the radial coordinate (starting from the innermost radius) r_0 of point P at which the blade element mean line is sought. The radial stations where the elemental vortex is located in each of the tip- and hub-zones are calculated according to the spacings specified in the input data.

At each of these radial stations r , quantities that are functions of r and the derivatives required in the computation of the induced velocities are then calculated and stored.

For a given radial station r the factors, which are functions of r and ϕ , appearing in the expressions of the induced velocities are first calculated for various angles at r and then are stored.

The computer proceeds to select a chordwise coordinate (starting from the leading edge) θ_0 of point P according to the specified intervals. The chordwise stations ϕ of the field points are then computed with the aid of the table of ϕ compiled at the beginning of the program at the corresponding radial station. The number of chordwise stations depends on the difference between r and r_0 and the blade outline of the particular propeller involved. This may be better explained by a graphic illustration. Figure 4 shows a pattern of a network of field points for point P , where both the radial and the chordwise stations are symmetrical about the point P . In Figure 5, a propeller blade outline is superimposed on Figure 4 with the point P coinciding. The stations between the line passing P and the leading edge, and between P and the trailing edge at various radial stations, including the stations at the leading and the trailing edges, are the field points at which the vortex elements are assumed to be distributed.

Then the computation of the induced velocity components at point $P (r_0, \theta_0)$ follows. This is done by computing the integrands at the selected chordwise stations and integrating them along the chord. The values of the integrations, which represent the contribution of the vortex systems at r to the induced velocities, are stored. It might be instructive to note here that in the calculation of Equations [16c] and [16d] the strength of the free vortex system, which are the factors enclosed in braces in these equations, may be represented graphically as shown in Figure 6. They may be considered to consist of two parts, one is the contribution due to lifting surface and the other due to the superimposed lifting line.

The procedure is repeated for the next θ_0 as point P moves along the chord toward the trailing edge at r_0 , and for another r , and so on. Then, the radial integration for various θ_0 is done and the values are stored.

When the calculations for the tip- and hub-zones of the "first blade," and for the other blades are completed, the calculation for the mid-zone of the first blade at various θ_0 follows.

The total induced velocity components at $P(r_0, \theta_0)$ are obtained by adding the values computed for the mid-zone to the radial integrated values previously computed. The entire procedure is then repeated for the next r_0 and so on.

The downwash $v_n(r_0, \theta_0)$ at each point P specified is then computed from the total induced velocity components according to Equation [20]. The downwash at points where mean line ordinates are desired is extrapolated or interpolated by using Equation [21]. Finally, the ordinates of the mean line at any point $\left(r_0, \frac{z}{L_T}\right)$ are calculated based on Equation [22].

Basically, the new computer program is similar to the original program in the method of approach. But there are two areas in which improvements have been made in this new program that are worth mentioning. One is in the method of setting up the network of field points. The other is in the integration of the induced velocity in the tip- and hub-zones.

As noted above, the network of field points is mapped so that it is common to all of the control points, and the field points are located so that they are closely spaced in the

neighborhood of the control point and are symmetrical about it. This feature of symmetry improves the accuracy of numerical computation, especially in the neighborhood of the singular point.

In the original program the lifting surface is divided into strips by cylinders of various radii. Within each of the strips the pitch distribution and the circulation functions are considered to be independent of the radial coordinate. Thus, the integration with respect to the radial coordinate can be carried out first by using the mean values of these quantities and then the chordwise integration is done numerically, whereas in the new program this simplification is made only for the narrow midstrip. The double integrations for the remaining part of the lifting surface are carried out numerically. This again is considered to be an improvement for better accuracy in the evaluation of the integrals.

RESULTS OF SAMPLE CALCULATIONS

The computer program discussed in the previous section has been completed. A number of cases have been worked out by using this new program and the results are reported herein.

NUMBER OF BLADES

The first case is a 5-bladed propeller similar to Propeller 3916, reported in Reference 1, except that the calculation is done by the new computer program. This propeller is designated as Propeller 3916A. The calculated induced velocities and the camber distributions are shown in Table 1 and in Figure 15 (Appendix C), and the maximum mean line ordinates due to all five blades (Curve 1) and due to the one blade (Curve 2) are plotted in Figure 7. For comparison, the results for five blades reported in Reference 1, calculated from the original computing program, is also shown (Curve 3). The difference between Curves 1 and 3 is due to the refinement made in the present computing program.

BLADE OUTLINE

It is worth noting that the camber distribution is affected by the shape of the blade outline selected. When the blade outline of Propeller 3916A is slightly modified, keeping the same expanded area ratio, as shown in the following table, a somewhat different camber distribution is obtained. The maximum mean line ordinates are shown also in Figure 7 (Curve 4). This propeller with the modified blade outline is designated as Propeller 3916B.

L_{τ}/D		
r/R	Original	Modified
0.2	0.308	0.294
0.3	0.372	0.358
0.4	0.431	0.430
0.5	0.462	0.488
0.6	0.464	0.488
0.7	0.435	0.458
0.8	0.370	0.364
0.9	0.268	0.228
1.0	0.040	0.000

COMPARISON WITH TWO-DIMENSIONAL CASE

The second case is for a propeller having a rectangular expanded outline with an aspect ratio of 10. It has zero hydrodynamic pitch ratio and a uniform bound-circulation distribution along the radius. This fictitious propeller resembles a two-dimensional wing, although the motion involved here is a rotational one rather than a linear translational. The purpose of this calculation is to determine the mean lines at various radii induced by the uniform bound-circulation distribution only and to compare them with a two-dimensional airfoil. The calculated mean line distributions at radii of 0.3, 0.6, and 0.9 are plotted in Figure 8 in which the NACA $a = 1$ mean line distribution is also shown.

In this fictitious case, the mean lines at various radii are quite close to the two-dimensional case. However, for the actual propeller, the mean lines at various radii obtained for uniform chordwise loading distribution are somewhat different from the $a = 1$ mean line as shown in Figure 9 for Propeller 3916A. Figure 10 shows the camber distributions of Propeller 3916A at various radii calculated by using the new method compared with those calculated by using the method outlined in Reference 2 for $a = 1$ mean line with camber correction factors k_1 and k_2 :

$$\text{Camber ratio, corrected} = 0.05515 k_1 k_2 C_L$$

where C_L is lift coefficient,

SKEWED BLADES

To further demonstrate the effect of blade outline on camber distribution, a third case was worked out. This case is also a 5-bladed propeller (Propeller 3917A). The loading, pitch, and blade chord length distributions are the same as those for Propeller 3916A, except that the blades are skewed. The amount of the skew is the same as that for Propeller 3917.¹ The results are shown in Table 2, and the comparative camber distributions calculated on one blade at various radii are plotted in Figure 11. From these curves, we notice that blade skew not only distorted the mean line but also changed the effective angle of attack. The range of angle of attack for this particular sample problem is in the order of -0.4 degree near the hub and -0.6 degree at the tip.

EXPANDED AREA RATIO

To show the dependence of camber distribution on expanded area ratio (EAR), calculations were made for propellers having the same basic blade outline as Propeller 3916A but with different EAR ranging from 0.5 to 1.2. The results based on one blade are shown in Table 3 and in Figure 12. In this figure, the maximum mean line ordinates due to the bound circulation only are also shown. The contribution due to the other four blades as percentage of the contribution due to one blade is shown in Figure 13 for the various EAR.

LOAD DISTRIBUTION

Calculations were made for a propeller having the same blade outline as Propeller 3916A but with a different loading, i.e., the radial bound circulation distribution is different. The radial load distribution is altered by arbitrarily changing the hydrodynamic pitch ratio to a constant. This constant hydrodynamic pitch propeller is designated as Propeller 3916C. The load and pitch distributions for these two propellers are shown in Figure 14. The camber distributions are shown in Table 4. The comparative camber distributions at various radii are shown in Figure 15.

DISCUSSION

The preceding sample calculations were presented to provide a clearer picture of the information that can be obtained from the program and of the various parameters that affect the mean line distribution.

From the results of these limited sample calculations, we may make the following remarks:

For a uniform chordwise load distribution, as in the case of Propeller 3916A, the three-dimensional mean line distribution differs somewhat from that obtained from the two-dimensional method with correction factors.

The mean line distribution of blades generally depends on the pitch ratio, radial load distribution, blade chord length distribution, and blade outline. Consequently, the expanded area ratio and the skew are very important parameters in the design of mean line, especially the latter, for the skew not only distorts the mean but also changes the effective angle of attack.

The presence of other blades has some effect upon the mean line of a blade. It has more effect in the hub region than it has near the tip. The contribution to the mean line due to other blades also increases as the expanded area ratio increases for propellers having the same blade outline.

It may be of interest to mention that some of these findings disagree with the conclusions drawn in Reference 3 in which van Manen and Bakker reported the numerical results of a number of propeller design problems using lifting-surface theory. They reported, in part,

that for a uniform pressure distribution along the chord, the three-dimensional camber line of a propeller blade section is identical to the camber line of the two-dimensional profiles and that the effect of radial load distribution is not very pronounced. They also reported that the effect of skew back is negligible. Based on the evidence shown in this report, their conclusions appear to be doubtful.

FUTURE PLANS

To obtain some experimental verification of the theory, tests of model propellers designed by using this method are contemplated. The tests are to be done in conjunction with other research projects that are currently going on at the David Taylor Model Basin. It is also felt that the test of the theory could be easily accomplished by comparing the results from the inverse problem of performance prediction for the steady inflow case with reliable published open-water experimental data. If close agreement between the theoretical and experimental results is obtained, of course, the validity of the theory is firmly established. If, however, some discrepancies are found, we hope to be able to derive some consistent parametric relationships. When this is accomplished, propeller open-water design charts can be constructed with data obtained from computations for a large range of parameters rather than from costly experiments.

With these considerations, we shall proceed with the work on the remaining part of the overall program. The work on the case of an arbitrary chordwise load distribution is already underway and will be reported in the near future. It is anticipated that the work on the inverse problem of predicting performance for a given propeller design will commence shortly.

Also anticipated is the inclusion of the thickness effect in the investigation. Work in this area has been done recently by Kerwin and Leopold; their results are reported in References 4 and 5. Their work may be incorporated in the present computation to take account of the thickness effect.

Finally, from the investigation of the unsteady inflow case, it is hoped that one can estimate the time dependent force fluctuations produced by a propeller operating in a nonuniform wake field behind a ship. Thus, the phenomena of cavitation due to unsteadiness and propeller-induced vibration and noise could be better understood.

ACKNOWLEDGMENTS

The overall project was initiated by Dr. P.C. Pien, who has taken an active interest in the development of this work. The author is grateful for having the opportunity to work with him and wishes to express his sincere thanks to Dr. Pien for his continuing guidance and help. The author also wishes to thank Mr. L. Mueller of the Applied Mathematics Laboratory for his help in the computer work.

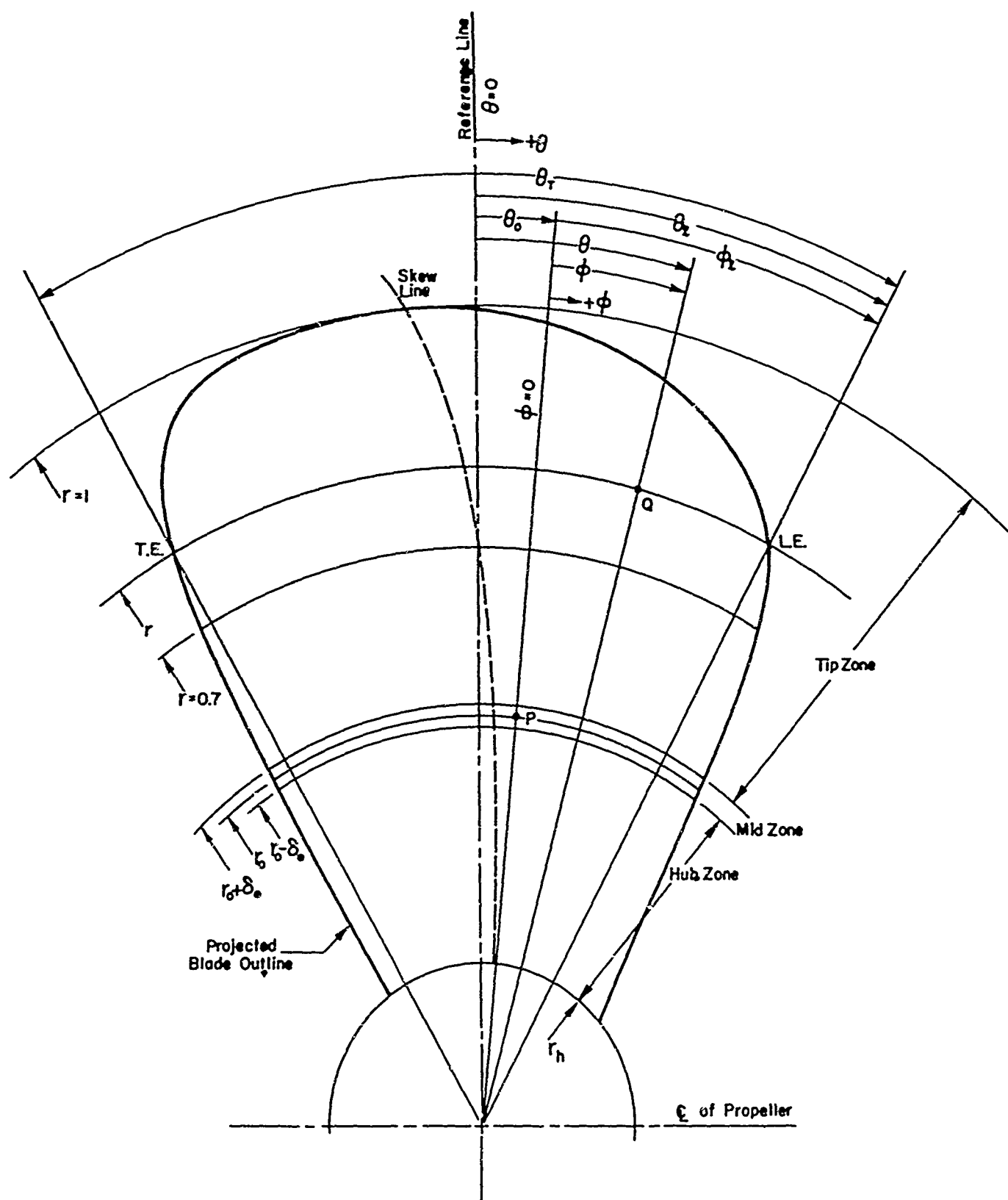


Figure 2 – Coordinate System in Projected Plane and Regions of Integration

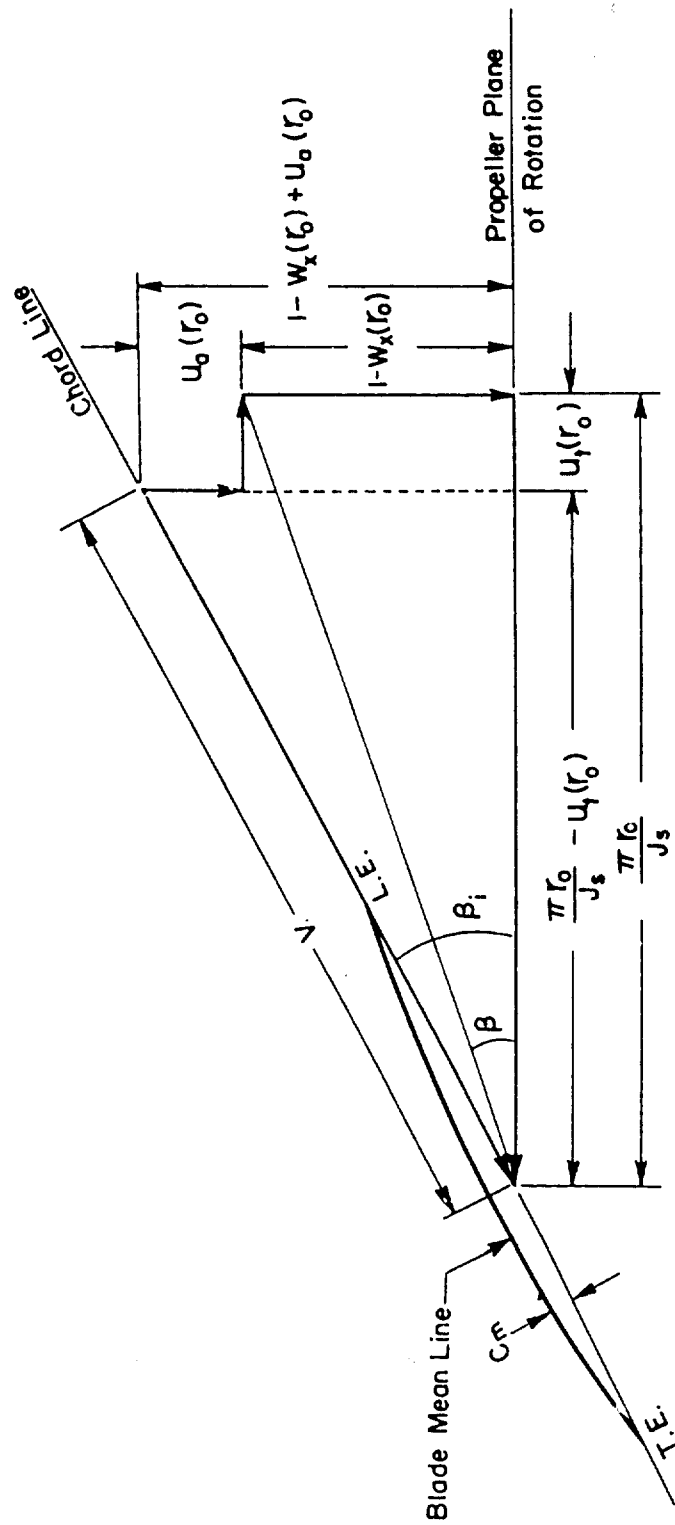


Figure 3 - Velocity Diagram at Radius r_0

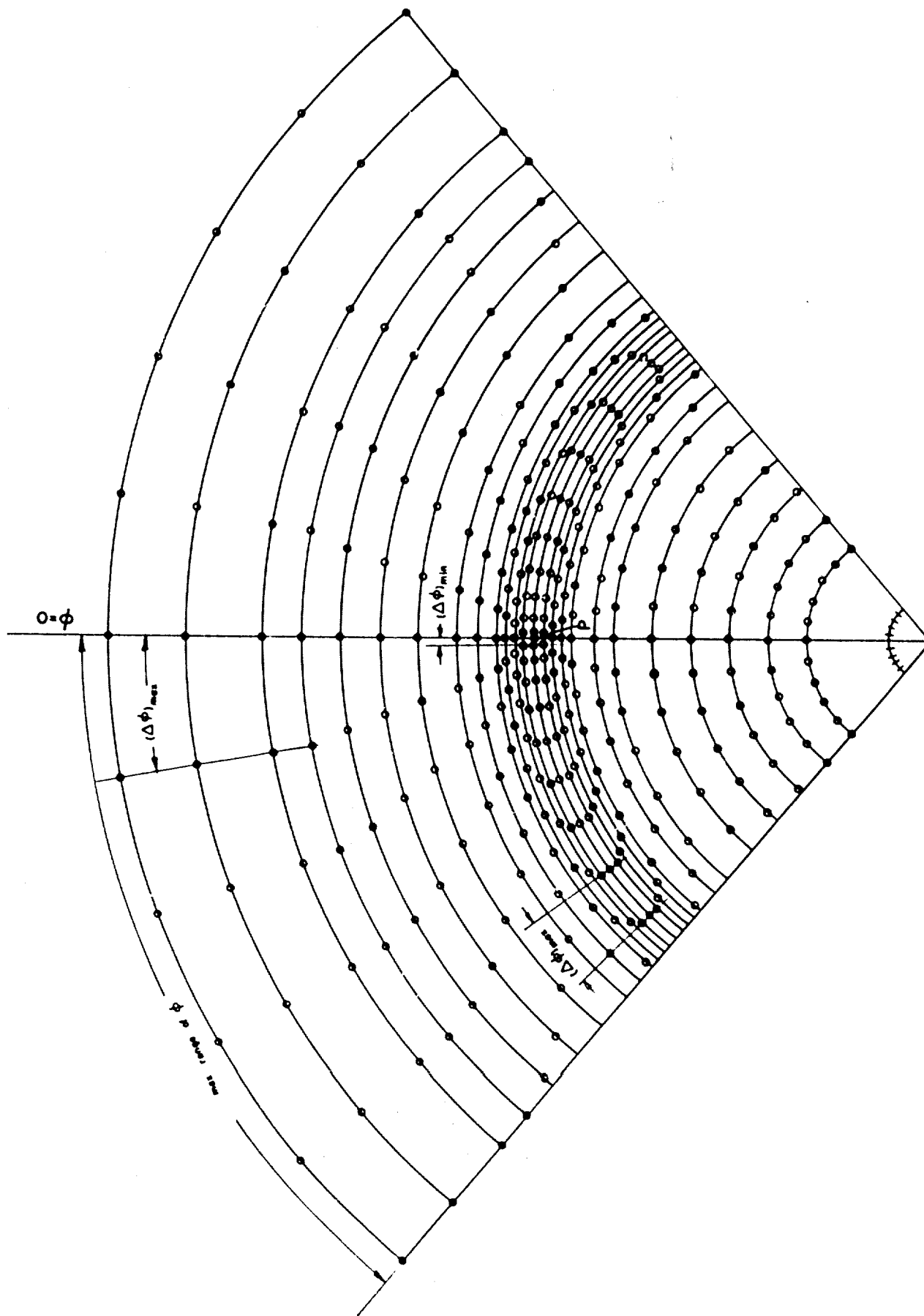


Figure 4 -- General Pattern of Network of Field Points for Point P

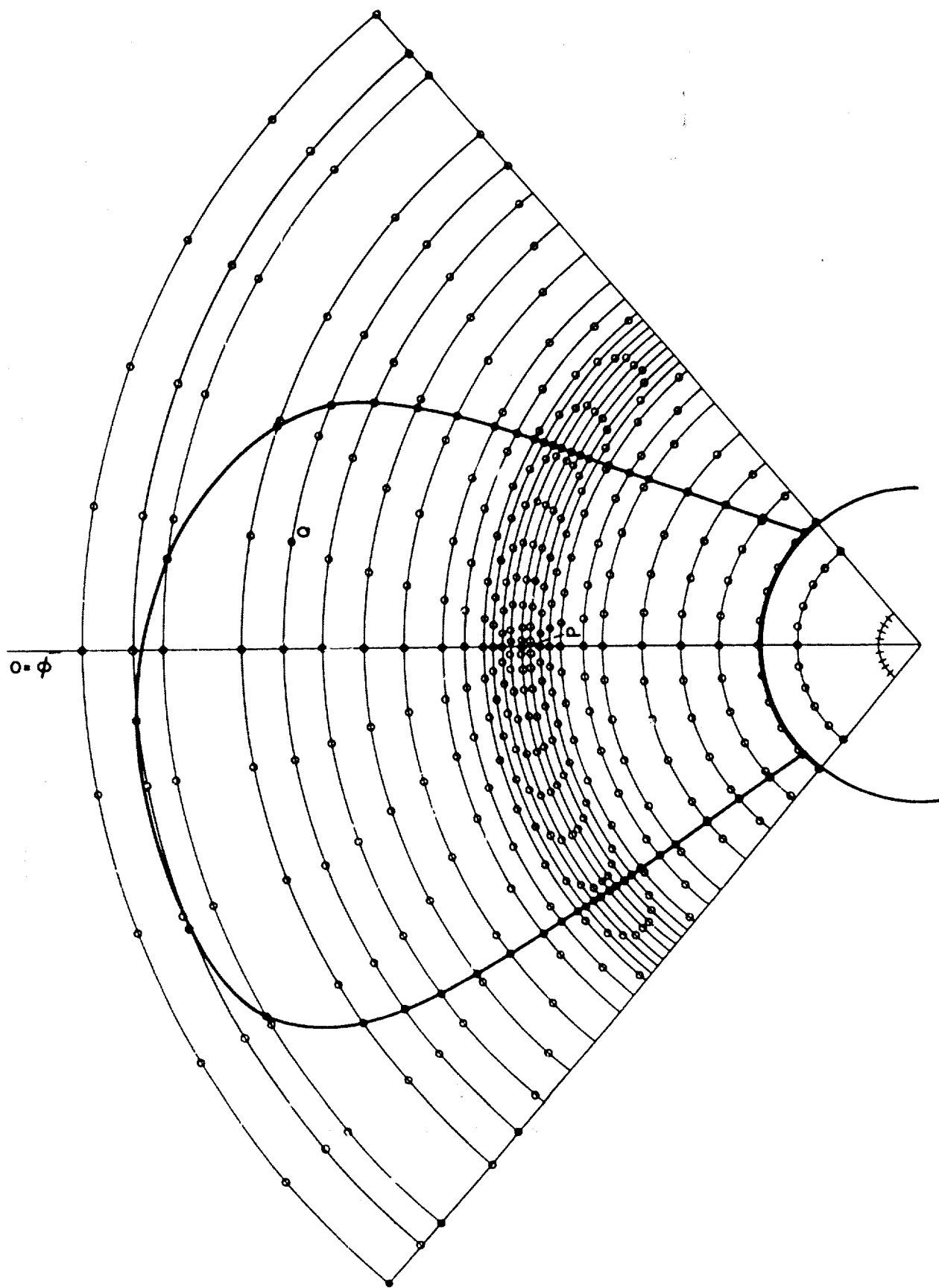


Figure 5 — Field Points for Point *P* on Blade

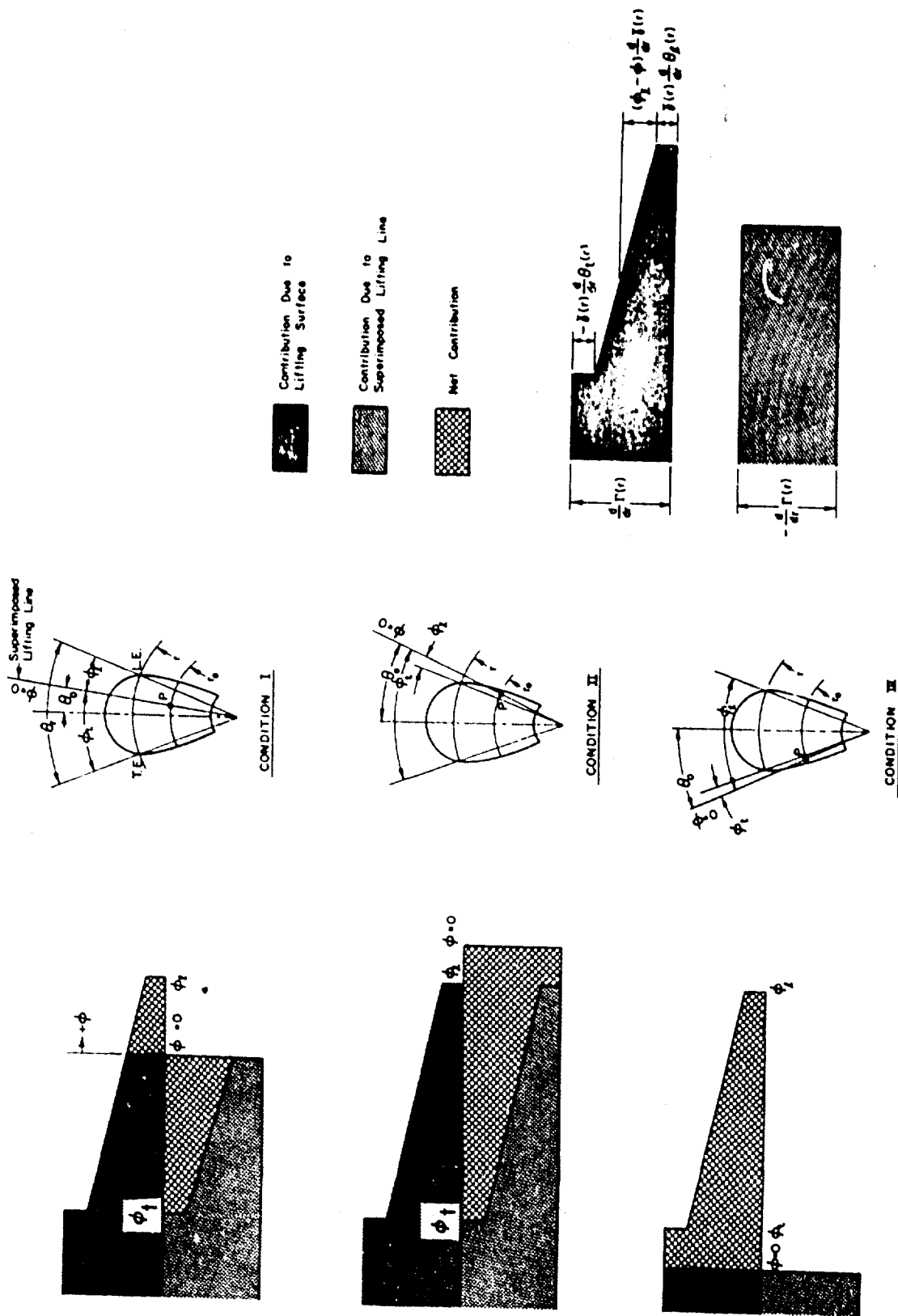


Figure 6 — Graphic Representation of Free Vortex Strength Distribution Along a Helical Line at r Due to a Lifting Surface and a Superimposed Lifting Line at $\phi = 0$

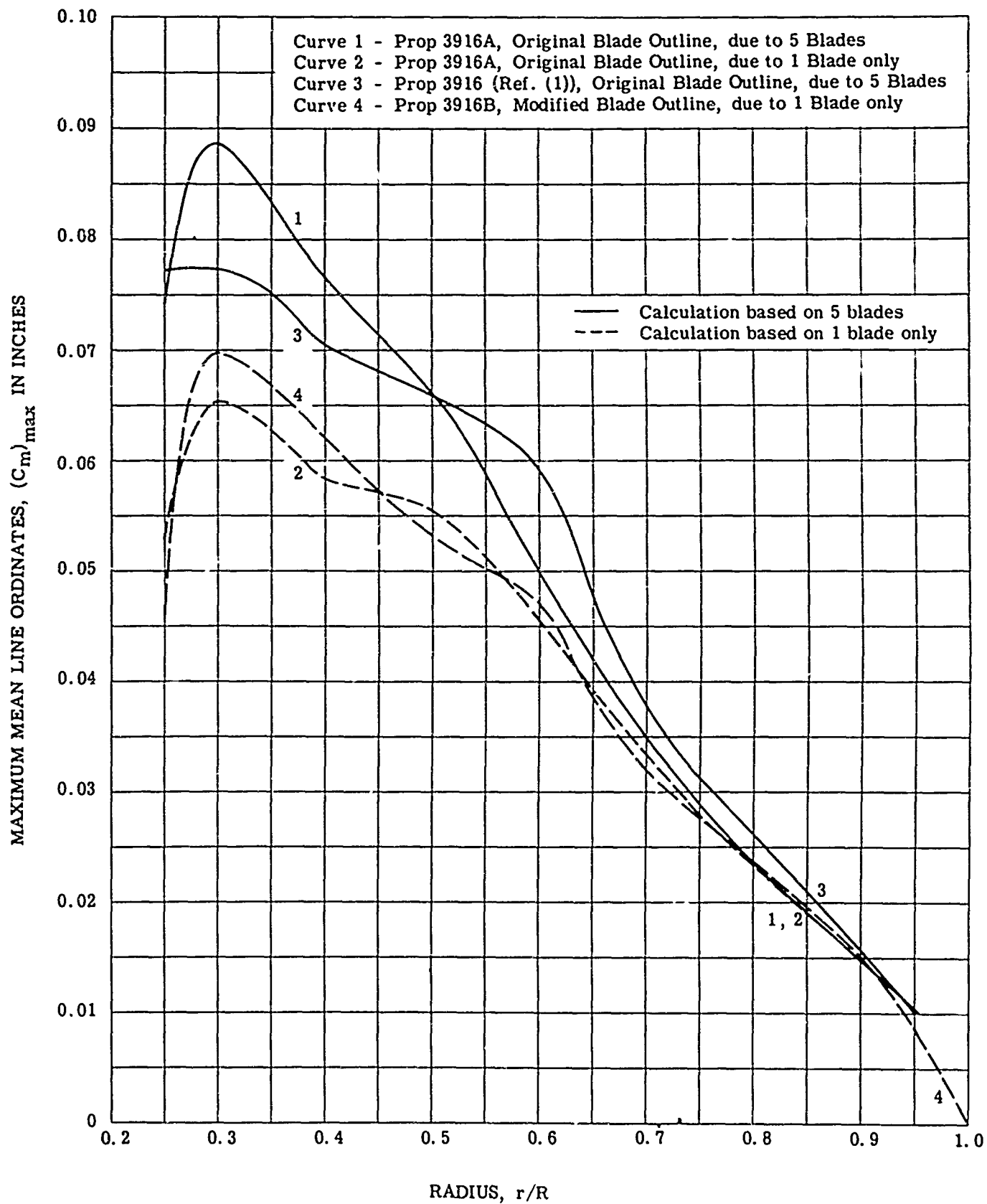


Figure 7 -- Maximum Mean Line Ordinates for Propellers 3916, 3916A, and 3916B

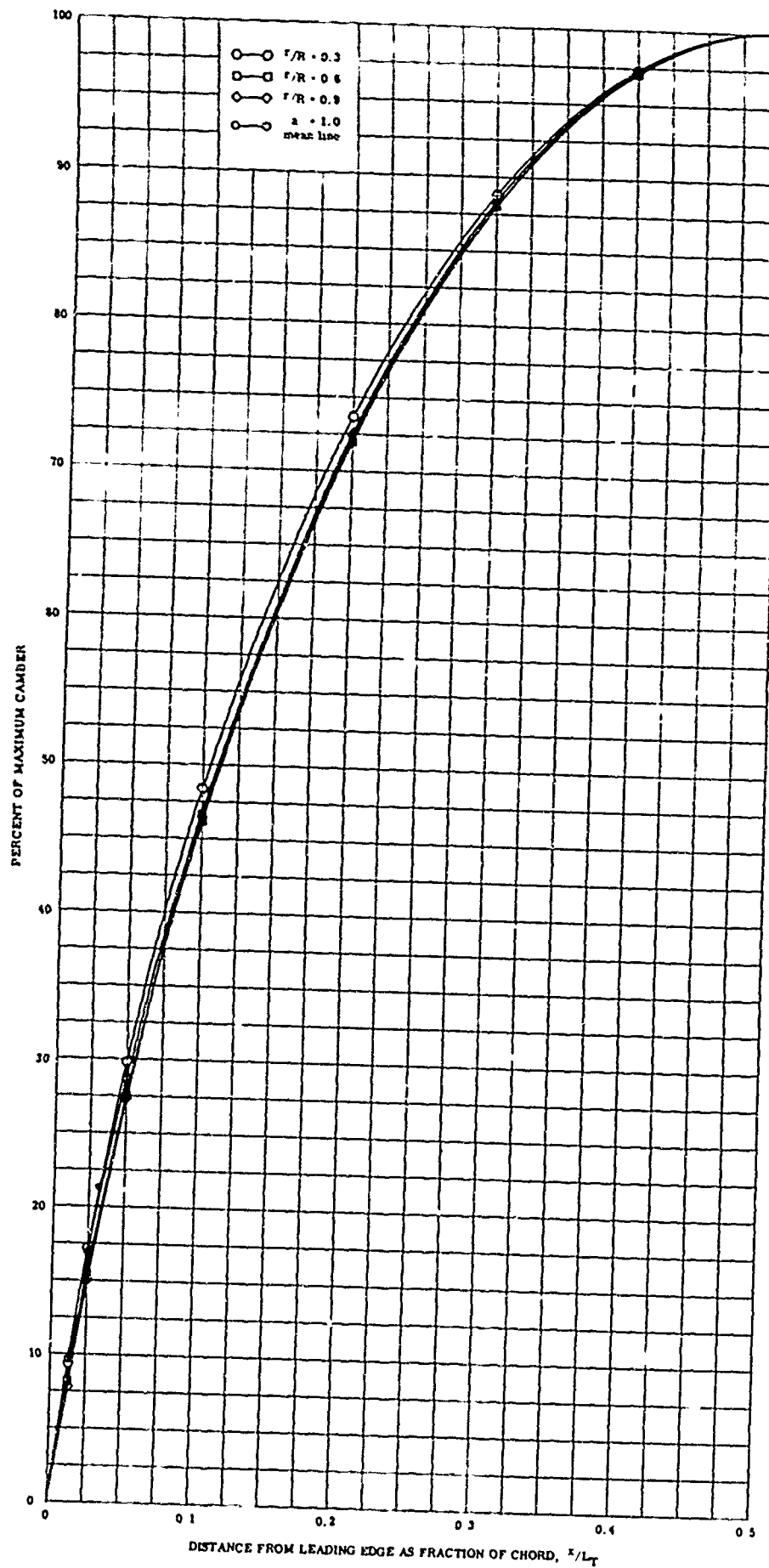


Figure 8 – Camber Line Distribution of a Two-Dimensional Propeller Blade

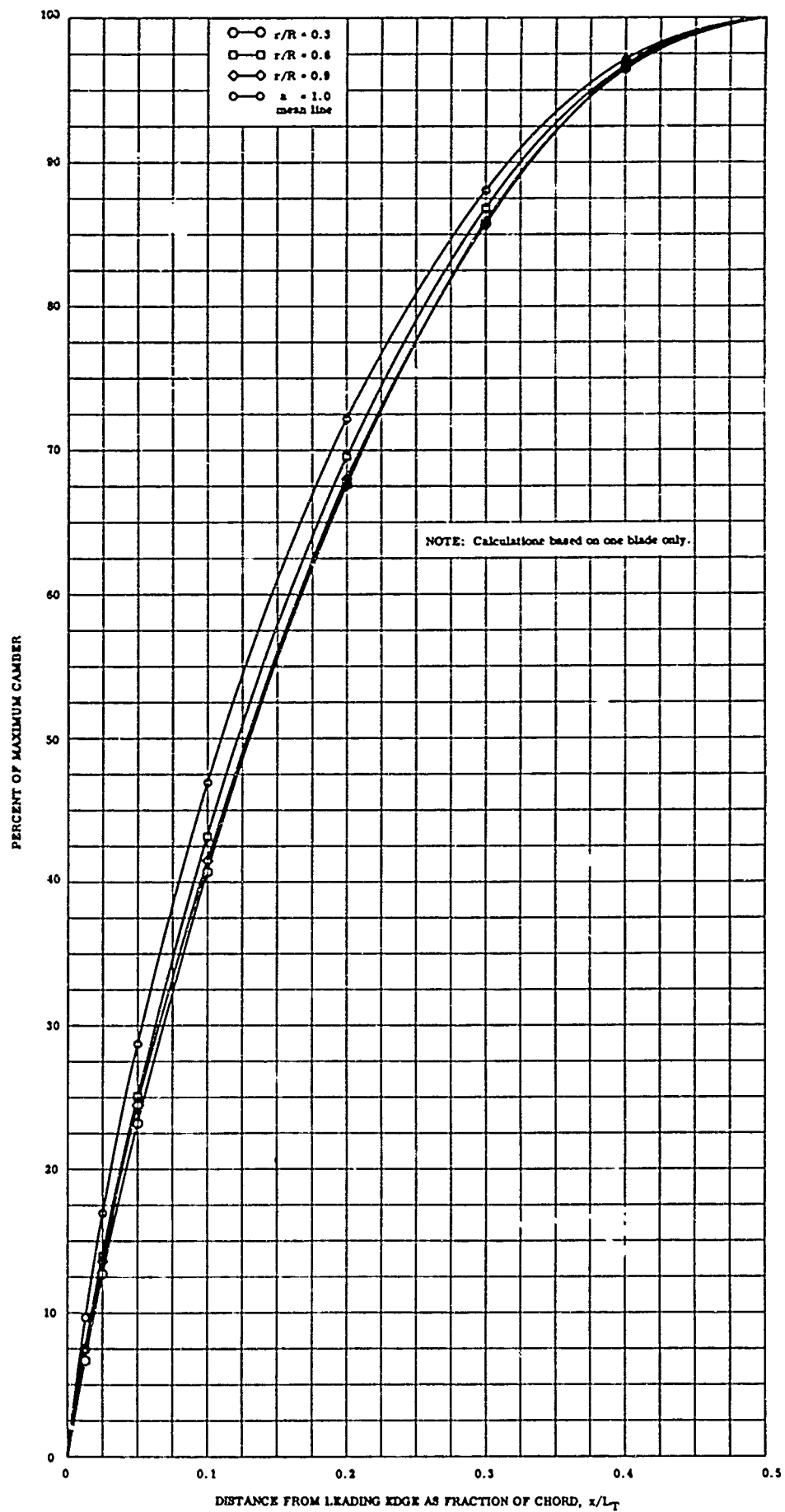


Figure 9 - Camber Line Distribution of Propeller 3916A

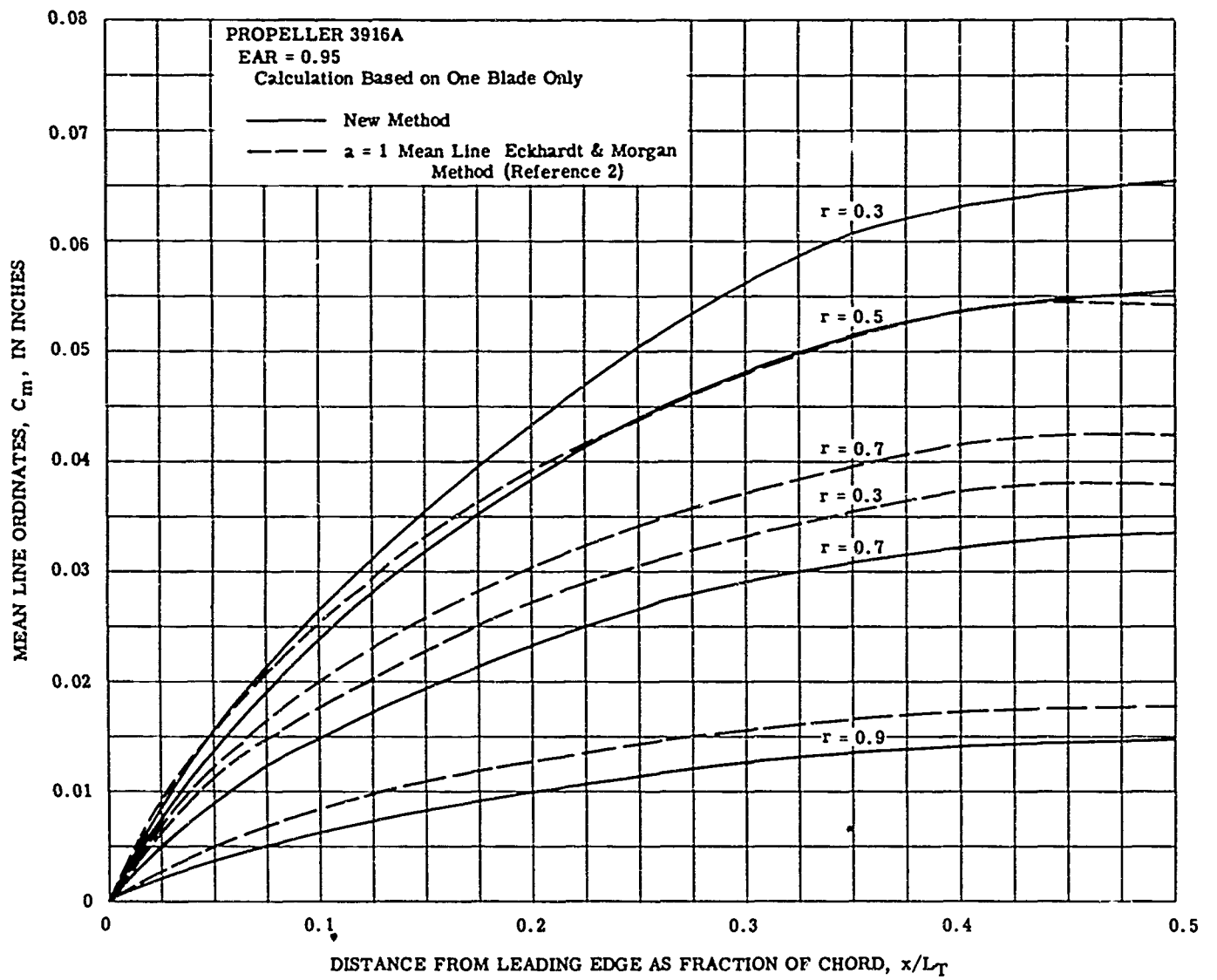


Figure 10 – Comparison of Camber Distribution for Propeller 3916A

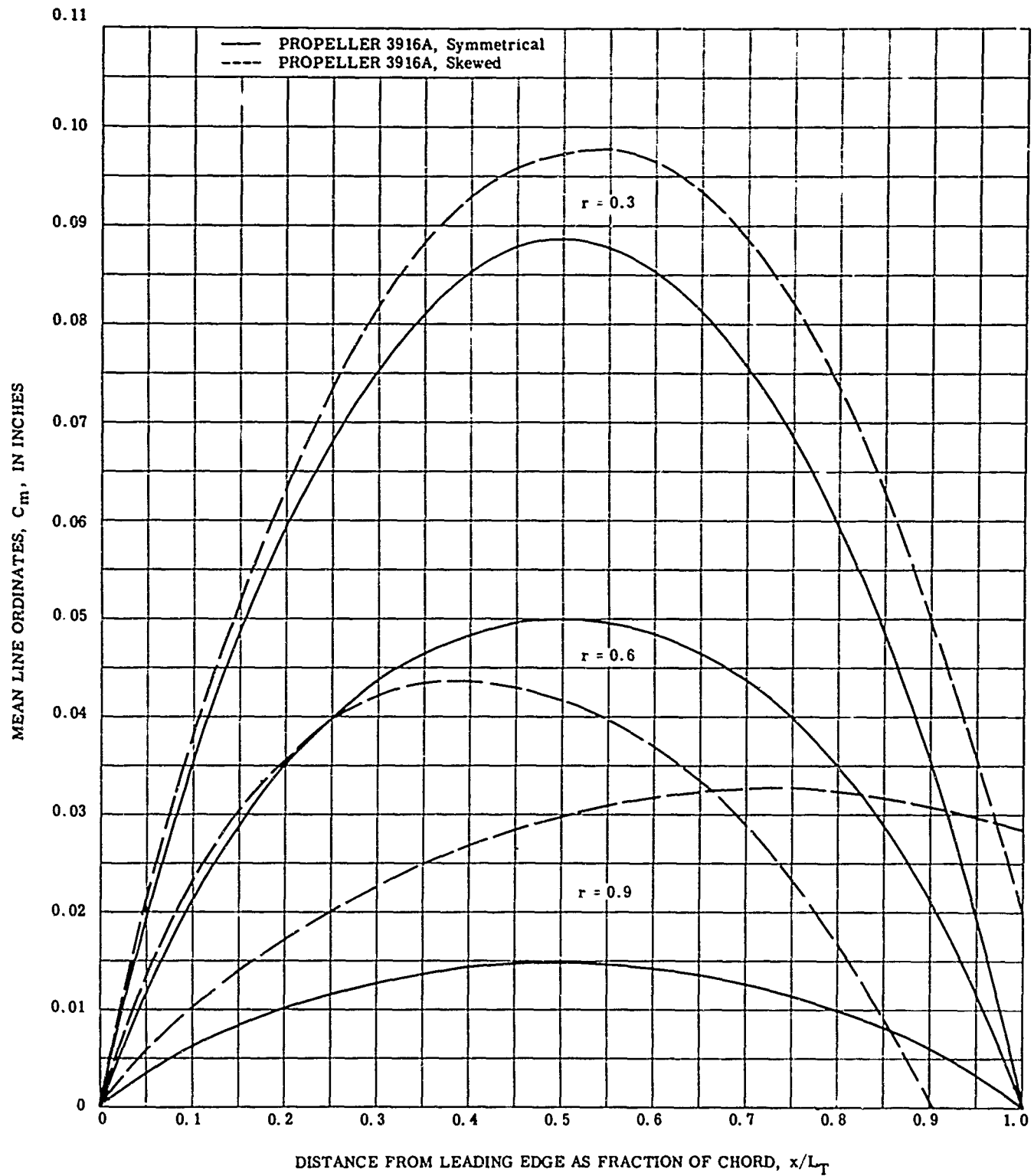


Figure 11 – Comparison of Camber Distributions, Symmetrical versus Skewed Blade Propellers

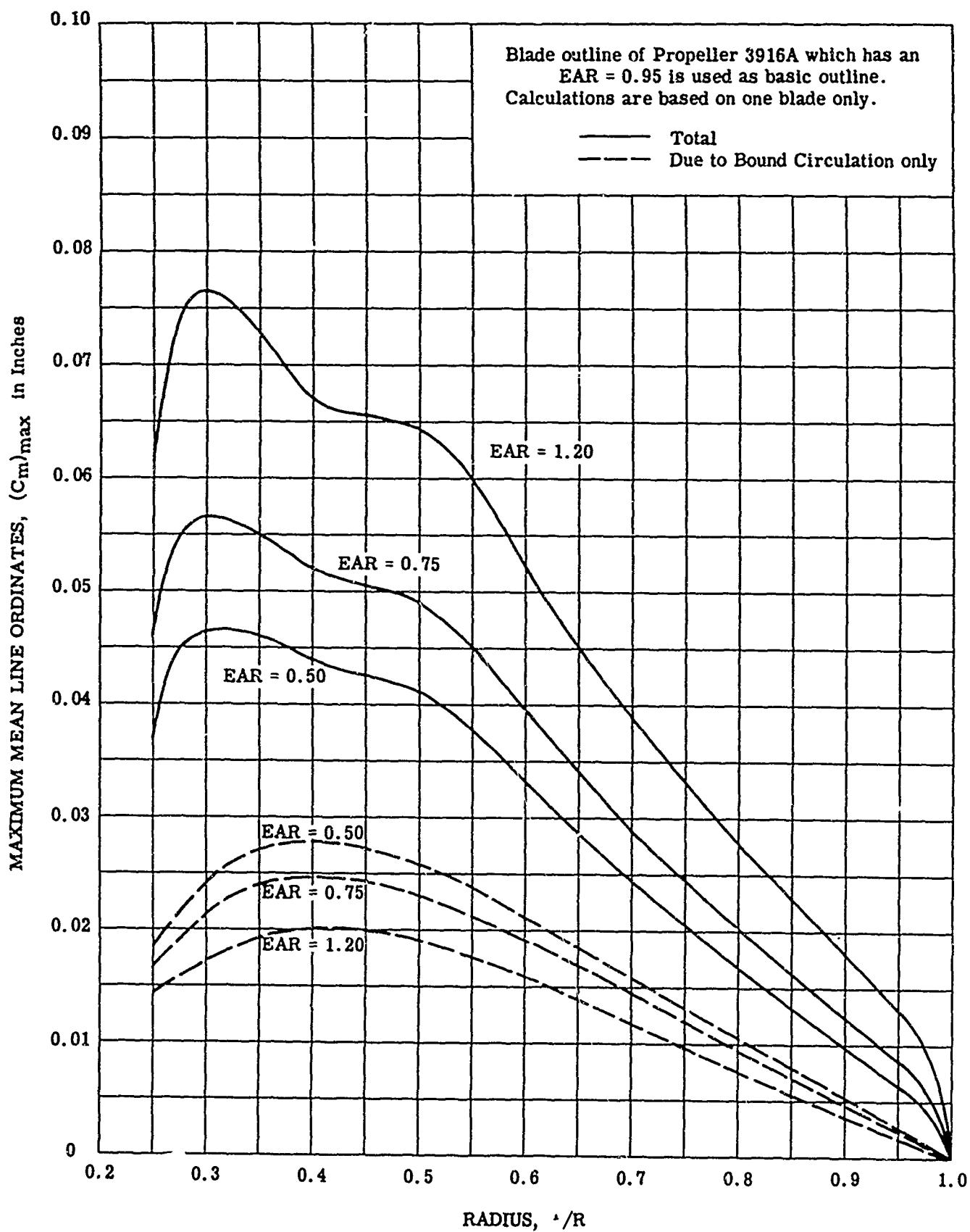


Figure 12 - Maximum Mean Line Ordinates for Propellers with Different EAR

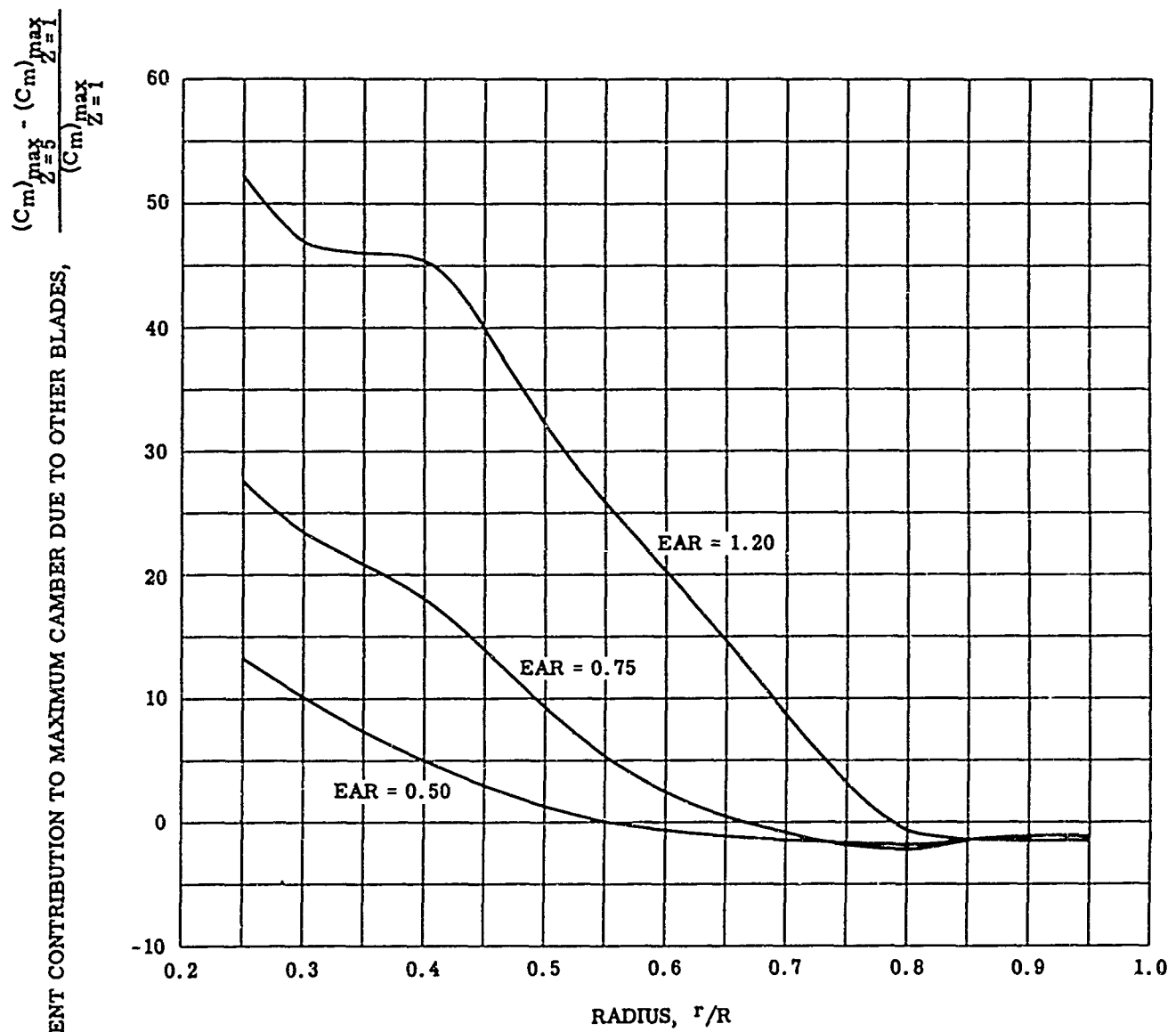


Figure 13 – Percentage Contribution to Maximum Mean Line Ordinates Due to Other Blades for Propellers with Different EAR

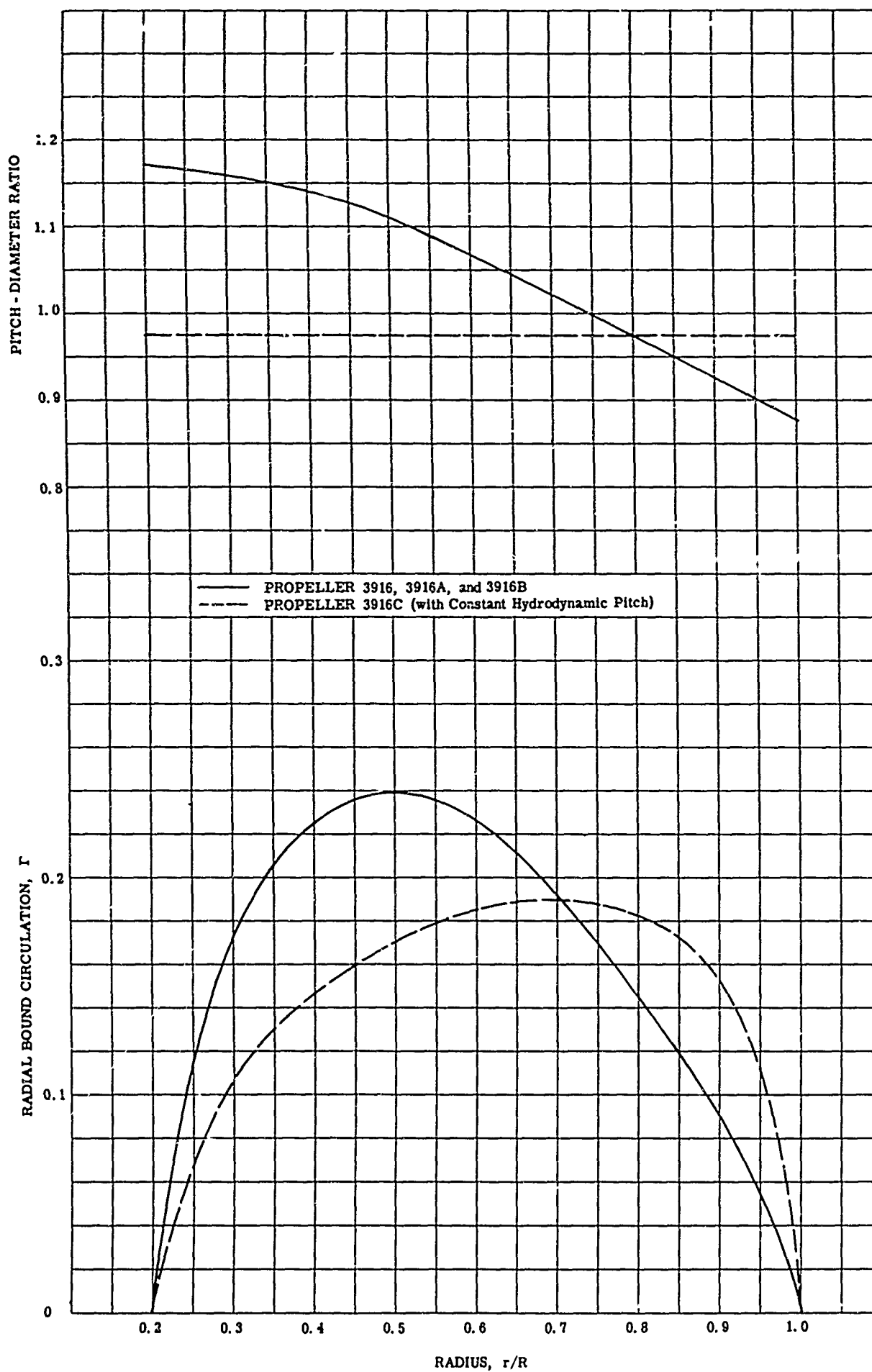


Figure 14 – Pitch Ratio and Bound Circulation Distribution for Propellers 3916 and 3916C

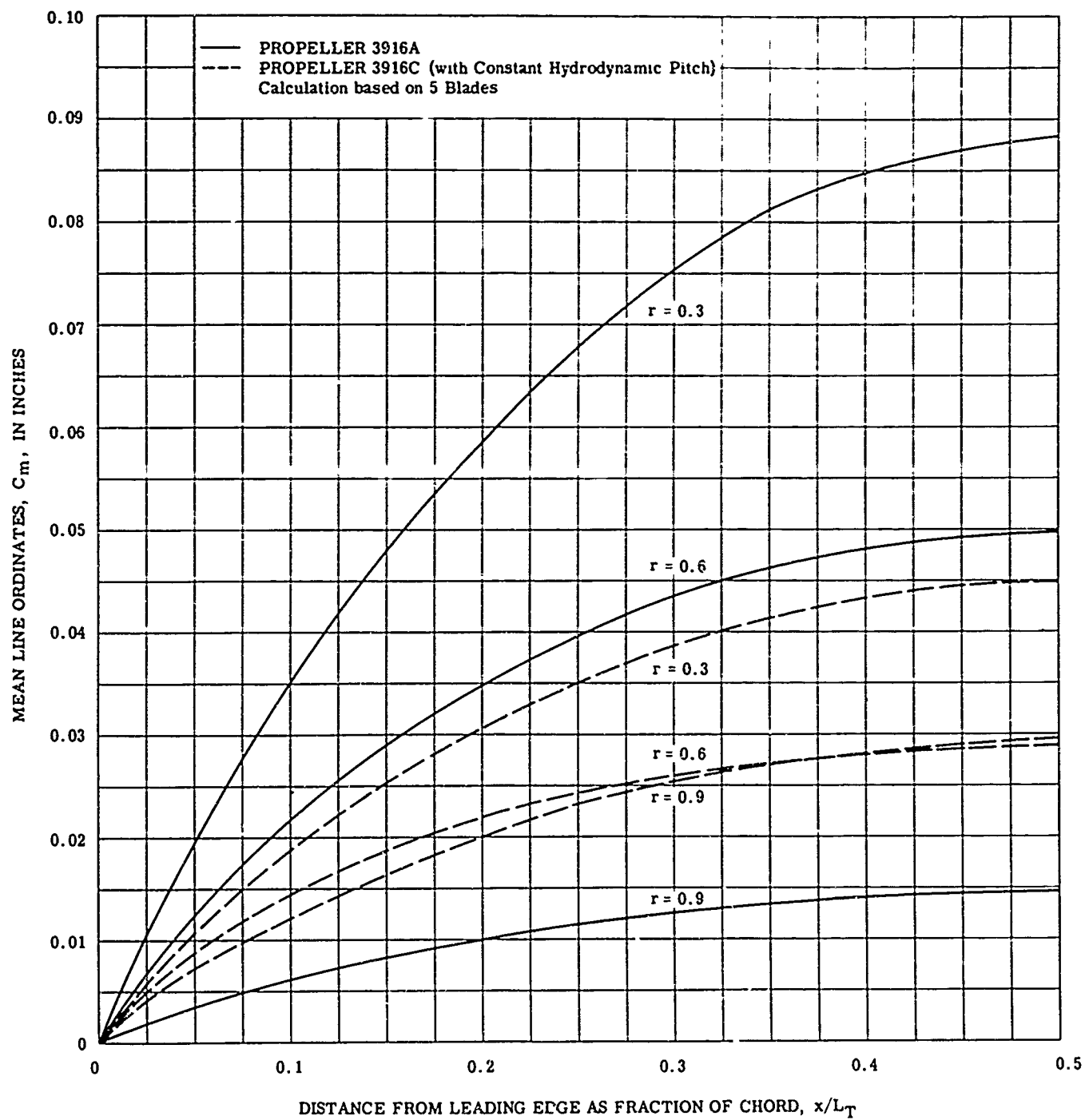


Figure 15 – Comparison of Camber Distribution for Propellers 3916A and 3916C

TABLE 1
Calculated Induced Velocities and Camber Distribution for Propeller 3916A

INPUT DATA									
18	0.00500	0.10000	2.00000	150.00000					
0.00500	0.01000	0.01600	0.02400	0.03500	0.05000	0.07000	0.09600	0.12900	
0.17000	0.22000	0.28000	0.35200	0.44000	0.55200	0.70000	0.94500	1.00000	
1	8	9	0	9	-0	-0			
0.00993	0.05003	0.11862	0.20414	0.29586	0.38138	0.44917	0.49007		
0.00993	0.01250	0.02500	0.05000	0.10000	0.20000	0.30000	0.40000	0.50000	
0.20000	0.30000	0.40000	0.50000	0.60000	0.70000	0.80000	0.90000	1.00000	
0.15400	0.18600	0.21500	0.23100	0.23200	0.21100	0.18500	0.13400	0.02000	
0.30800	0.37200	0.43000	0.46200	0.46400	0.43500	0.37000	0.26800	0.04000	
1.86210	1.22635	0.90766	0.70429	0.56622	0.46392	0.38698	0.32722	0.27950	
0.	0.01730	0.02266	0.02428	0.02283	0.01936	0.01463	0.00898	0.	
0.25000	0.30000	0.40000	0.50000	0.60000	0.70000	0.80000	0.90000	0.95000	
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	
0.14500	0.16402	0.20861	0.22173	0.20765	0.17661	0.13584	0.08937	0.06500	
0.16010	0.16678	0.16039	0.13704	0.10517	0.07463	0.04889	0.02767	0.01770	
1.00000	0.82600	0.60000	1.00000	1.00000	1.00000				
OUTPUT									
AXIAL INDUCED VELOCITY COMPONENT									
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
X/LT									
0.0099	0.0673	0.1046	0.1276	0.1496	0.1513	0.1487	0.1425	0.1288	0.1184
0.0508	0.0505	0.0775	0.0889	0.1053	0.1086	0.1026	0.0992	0.0913	0.0936
0.1186	0.0349	0.0591	0.0661	0.0758	0.0789	0.0701	0.0688	0.0687	0.0738
0.2041	0.0287	0.0432	0.0465	0.0552	0.0551	0.0521	0.0494	0.0498	0.0581
0.2959	0.0175	0.0289	0.0304	0.0362	0.0363	0.0339	0.0323	0.0336	0.0380
0.3814	0.0116	0.0165	0.0169	0.0206	0.0205	0.0192	0.0184	0.0193	0.0249
0.4492	0.0056	0.0071	0.0068	0.0089	0.0087	0.0081	0.0079	0.0082	0.0111
0.4901	0.0021	0.0015	0.0008	0.0020	0.0017	0.0016	0.0016	0.0014	0.0026
TANGENTIAL INDUCED VELOCITY COMPONENT									
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
X/LT									
0.0099	-0.1153	-0.1377	-0.1198	-0.1059	-0.0847	-0.0675	-0.0538	-0.0414	-0.0343
0.0508	-0.0844	-0.1040	-0.0856	-0.0758	-0.0609	-0.0466	-0.0373	-0.0293	-0.0275
0.1186	-0.0703	-0.0805	-0.0641	-0.0553	-0.0446	-0.0323	-0.0261	-0.0220	-0.0217
0.2041	-0.0526	-0.0594	-0.0460	-0.0402	-0.0314	-0.0239	-0.0186	-0.0159	-0.0170
0.2959	-0.0359	-0.0349	-0.0302	-0.0266	-0.0207	-0.0156	-0.0121	-0.0107	-0.0111
0.3814	-0.0217	-0.0229	-0.0169	-0.0152	-0.0118	-0.0088	-0.0069	-0.0061	-0.0073
0.4492	-0.0100	-0.0099	-0.0069	-0.0066	-0.0050	-0.0037	-0.0030	-0.0026	-0.0033
0.4901	-0.0033	-0.0020	-0.0009	-0.0015	-0.0010	-0.0007	-0.0006	-0.0004	-0.0008
INDUCED DOWNWASH DUE TO FIRST BLADE ONLY									
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
X/LT									
0.0099	0.1333	0.1748	0.1750	0.1833	0.1734	0.1634	0.1523	0.1353	0.1213
0.0508	0.1023	0.1296	0.1234	0.1305	0.1245	0.1127	0.1060	0.0959	0.0975
0.1186	0.0801	0.0998	0.0970	0.0938	0.0907	0.0772	0.0736	0.0721	0.0769
0.2041	0.0597	0.0734	0.0653	0.0683	0.0634	0.0573	0.0528	0.0523	0.0605
0.2959	0.0407	0.0492	0.0428	0.0449	0.0418	0.0373	0.0345	0.0353	0.0396
0.3814	0.0241	0.0282	0.0239	0.0256	0.0237	0.0211	0.0197	0.0203	0.0260
0.4492	0.0114	0.0121	0.0096	0.0111	0.0101	0.0070	0.0084	0.0086	0.0116
0.4901	0.0039	0.0025	0.0012	0.0025	0.0020	0.0018	0.0017	0.0015	0.0027
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
V/D	1.1560	1.1577	1.1406	1.1063	1.0673	1.0202	0.9726	0.9252	0.8976
L.C.	1.3513	1.4880	1.7240	1.8480	1.8560	1.7400	1.4800	1.0720	0.7457
CHORD	2.7213	2.7760	3.4400	3.6960	3.7120	3.4800	2.9600	2.1440	1.4943
CAMBER DISTRIBUTION									
X/LT									
0.0099	0.0027	0.0035	0.0034	0.0033	0.0027	0.0021	0.0015	0.0009	0.0005
0.0125	0.0034	0.0044	0.0043	0.0041	0.0034	0.0026	0.0018	0.0011	0.0006
0.0250	0.0064	0.0083	0.0081	0.0077	0.0063	0.0047	0.0034	0.0020	0.0012
0.0500	0.0118	0.0152	0.0145	0.0139	0.0114	0.0088	0.0062	0.0036	0.0022
0.1000	0.0209	0.0266	0.0242	0.0238	0.0176	0.0147	0.0104	0.0061	0.0040
0.2000	0.0351	0.0442	0.0404	0.0383	0.0316	0.0232	0.0165	0.0100	0.0068
0.3000	0.0448	0.0561	0.0505	0.0480	0.0394	0.0290	0.0205	0.0126	0.0088
0.4000	0.0507	0.0631	0.0563	0.0536	0.0439	0.0322	0.0228	0.0142	0.0100
0.5000	0.0529	0.0654	0.0581	0.0555	0.0454	0.0333	0.0236	0.0147	0.0105
CAMBER RATIO									
X/LT									
0.0099	0.0010	0.0012	0.0010	0.0009	0.0007	0.0006	0.0005	0.0004	0.0003
0.0125	0.0012	0.0015	0.0013	0.0011	0.0009	0.0007	0.0006	0.0005	0.0004
0.0250	0.0024	0.0028	0.0024	0.0021	0.0017	0.0014	0.0012	0.0009	0.0008
0.0500	0.0043	0.0051	0.0042	0.0038	0.0031	0.0025	0.0021	0.0017	0.0015
0.1000	0.0077	0.0089	0.0072	0.0064	0.0053	0.0042	0.0035	0.0029	0.0027
0.2000	0.0129	0.0148	0.0117	0.0104	0.0085	0.0067	0.0056	0.0047	0.0046
0.3000	0.0165	0.0188	0.0147	0.0130	0.0106	0.0083	0.0069	0.0059	0.0059
0.4000	0.0186	0.0212	0.0164	0.0145	0.0118	0.0093	0.0077	0.0066	0.0067
0.5000	0.0197	0.0220	0.0169	0.0150	0.0122	0.0096	0.0080	0.0069	0.0070

NOTES: 1. Calculation based on one blade only.
2. Calculated camber in inches.

TABLE 2

Calculated Induced Velocities and Camber Distribution for Propeller 3917A (Skewed)

INPUT DATA

18
0.00500 0.10000 2.00000 150.00000
0.00500 0.01000 0.01000 0.02000 0.03500 0.05000 0.07000 0.09000 0.12000
0.17000 0.22000 0.28000 0.35200 0.44000 0.55200 0.70000 0.94500 1.00000
5 8 9 0 9 -0 -0
0.01250 0.10166 0.23724 0.40828 0.59172 0.76276 0.89834 0.98014
0.01250 0.02500 0.05000 0.10000 0.20000 0.40000 0.60000 0.80000 0.95000
0.20000 0.30000 0.40000 0.50000 0.60000 0.70000 0.80000 0.90000 1.00000
0.12500 0.20900 0.26400 0.28600 0.27300 0.22700 0.14900 0.03300 0.11300
0.30800 0.37200 0.43000 0.46200 0.46400 0.43500 0.37000 0.26800 0.04000
1.08210 1.22835 0.90766 0.70429 0.56622 0.46392 0.38696 0.32722 0.27960

0. 0.01730 0.02266 0.02428 0.02283 0.01936 0.01463 0.00898 0.
0.25000 0.30000 0.40000 0.50000 0.60000 0.70000 0.80000 0.90000 0.95000
1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000
0.14500 0.16902 0.20861 0.22173 0.20765 0.17661 0.13584 0.08937 0.06500
0.16010 0.16678 0.16039 0.13704 0.10917 0.07483 0.04889 0.02767 0.01770
1.00000 0.82600 0.00000 1.00000 1.00000

Card 1 (see Appendix B for explanation)

Card 2

Card 3

Card 5

Card 6

Card 7

Card 8

r
 $L_f(r)/D$
 $L_t(r)/D$
 $\tan \beta_1(r)$

 $\Gamma(r)$ r_0 $V_x(r_0)$ $u_x(r_0)$ $u_t(r_0)$

Card 9

OUTPUT

AXIAL INDUCED VELOCITY COMPONENT

RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
X/LT									
0.0125	0.0809	0.1115	0.1158	0.1427	0.1642	0.1770	0.1818	0.1922	0.1680
0.1017	0.0573	0.0669	0.0604	0.0713	0.0800	0.0872	0.0984	0.1257	0.1343
0.2372	0.0450	0.0400	0.0232	0.0200	0.0320	0.0430	0.0480	0.0844	0.1034
0.4083	0.0341	0.0163	-0.0067	-0.0122	-0.0049	0.0090	0.0213	0.0358	0.0720
0.5917	0.0228	-0.0065	-0.0322	-0.0425	-0.0355	-0.0201	-0.0067	0.0253	0.0335
0.7628	0.0126	-0.0310	-0.0575	-0.0718	-0.0650	-0.0480	-0.0339	-0.0038	-0.0005
0.8983	-0.0133	-0.0589	-0.0876	-0.1049	-0.0975	-0.0783	-0.0620	-0.0333	-0.0323
0.9801	-0.0429	-0.1002	-0.1296	-0.1512	-0.1446	-0.1231	-0.1031	-0.0729	-0.0927

TANGENTIAL INDUCED VELOCITY COMPONENT

RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
X/LT									
0.0125	-0.1953	-0.2042	-0.1637	-0.1423	-0.1226	-0.8994	-0.0758	-0.0662	-0.0479
0.1017	-0.1479	-0.1418	-0.1073	-0.0890	-0.0718	-0.0547	-0.0418	-0.0436	-0.0375
0.2372	-0.1066	-0.0934	-0.0632	-0.0460	-0.0374	-0.0285	-0.0201	-0.0289	-0.0279
0.4083	-0.0562	-0.0372	-0.0153	-0.0049	-0.0052	-0.0063	-0.0064	-0.0179	-0.0179
0.5917	-0.0022	0.0231	0.0346	0.0335	0.0243	0.0139	0.0075	-0.0064	-0.0060
0.7628	0.0465	0.0794	0.0806	0.0703	0.0511	0.0325	0.0206	0.0046	0.0045
0.8983	0.0952	0.1291	0.1217	0.1038	0.0763	0.0506	0.0334	0.0153	0.0144
0.9801	0.1404	0.1795	0.1653	0.1404	0.1052	0.0732	0.0501	0.0289	0.0326

INDUCED DOWNWASH DUE TO ALL BLADES

RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
X/LT									
0.0125	0.2073	0.2228	0.1958	0.1986	0.2033	0.2024	0.1969	0.2033	0.1747
0.1017	0.1348	0.1822	0.1168	0.1095	0.1050	0.1021	0.1049	0.1330	0.1395
0.2372	0.1135	0.0977	0.0597	0.0449	0.0462	0.0511	0.0520	0.0892	0.1072
0.4083	0.0657	0.0391	0.0053	-0.0060	-0.0017	0.0108	0.0221	0.0585	0.0741
0.5917	0.0145	-0.0220	-0.0471	-0.0540	-0.0429	-0.0241	-0.0089	0.0260	0.0338
0.7628	-0.0332	-0.0812	-0.0968	-0.0997	-0.0818	-0.0572	-0.0390	-0.0050	-0.0017
0.8983	-0.0803	-0.1373	-0.1466	-0.1455	-0.1224	-0.0923	-0.0699	-0.0364	-0.0351
0.9801	-0.1405	-0.2025	-0.2071	-0.2044	-0.1777	-0.1425	-0.1143	-0.0783	-0.0981
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
P/D	1.1560	1.1577	1.1406	1.1063	1.0673	1.0202	0.9726	0.9252	0.8976
L.E.	1.3624	1.6720	2.1120	2.2880	2.1840	1.8160	1.1920	0.2640	-0.3133
CHORD	2.7213	2.9760	3.4400	3.6960	3.7120	3.4800	2.9600	2.1440	1.4943

CAMBER DISTRIBUTION

X/LT	0.0125	0.0250	0.0500	0.1000	0.2000	0.4000	0.6000	0.8000	0.9500
0.0125	0.0052	0.0058	0.0048	0.0045	0.0040	0.0033	0.0024	0.0016	0.0009
0.0250	0.0101	0.0112	0.0093	0.0086	0.0076	0.0062	0.0046	0.0031	0.0017
0.0500	0.0194	0.0210	0.0171	0.0158	0.0138	0.0112	0.0084	0.0057	0.0034
0.1000	0.0357	0.0375	0.0299	0.0270	0.0232	0.0187	0.0143	0.0102	0.0063
0.2000	0.0626	0.0630	0.0476	0.0411	0.0351	0.0284	0.0218	0.0170	0.0114
0.4000	0.1004	0.0928	0.0624	0.0497	0.0435	0.0372	0.0293	0.0264	0.0189
0.6000	0.1160	0.0962	0.0544	0.0394	0.0367	0.0355	0.0306	0.0315	0.0233
0.8000	0.1100	0.0730	0.0248	0.0109	0.0166	0.0245	0.0254	0.0323	0.0241
0.9500	0.0873	0.0354	-0.0144	-0.0246	-0.0093	0.0066	0.0164	0.0294	0.0223

CAMBER RATIO

X/LT	0.0125	0.0250	0.0500	0.1000	0.2000	0.4000	0.6000	0.8000	0.9500
0.0125	0.0019	0.0020	0.0014	0.0012	0.0011	0.0009	0.0008	0.0007	0.0006
0.0250	0.0037	0.0038	0.0027	0.0023	0.0020	0.0018	0.0015	0.0014	0.0012
0.0500	0.0071	0.0071	0.0050	0.0043	0.0037	0.0032	0.0025	0.0027	0.0023
0.1000	0.0131	0.0126	0.0087	0.0073	0.0063	0.0054	0.0048	0.0048	0.0042
0.2000	0.0231	0.0212	0.0138	0.0111	0.0095	0.0082	0.0074	0.0079	0.0076
0.4000	0.0369	0.0312	0.0181	0.0134	0.0117	0.0107	0.0099	0.0123	0.0127
0.6000	0.0426	0.0323	0.0158	0.0107	0.0099	0.0102	0.0103	0.0147	0.0156
0.8000	0.0404	0.0245	0.0072	0.0030	0.0045	0.0071	0.0086	0.0150	0.0162
0.9500	0.0321	0.0119	-0.0042	-0.0067	-0.0025	0.0025	0.0055	0.0137	0.0188

NOTES: 1. Calculation based on five blades.

2. Calculated camber in inches.

TABLE 3
Camber Distribution for Propellers with Different Expanded Area Ratios

EAR = 0.50									
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
P/D	1.1560	1.1577	1.1406	1.1063	1.0673	1.0202	0.9726	0.9252	0.8976
L.E.	0.7119	0.7827	0.9068	0.9720	0.9763	0.9152	0.7785	0.5639	0.3920
CHORD	1.4314	1.5654	1.8074	1.9441	1.9525	1.8305	1.5570	1.1277	0.7860
CAMBER DISTRIBUTION									
X/LT									
0.0099	0.0022	0.0028	0.0028	0.0026	0.0021	0.0015	0.0010	0.0006	0.0003
0.0125	0.0027	0.0035	0.0035	0.0033	0.0026	0.0019	0.0013	0.0007	0.0004
0.0250	0.0050	0.0065	0.0065	0.0061	0.0049	0.0036	0.0024	0.0013	0.0007
0.0500	0.0071	0.0116	0.0116	0.0108	0.0088	0.0065	0.0043	0.0024	0.0014
0.1000	0.0157	0.0198	0.0175	0.0181	0.0148	0.0109	0.0074	0.0047	0.0026
0.2000	0.0257	0.0321	0.0309	0.0288	0.0215	0.0172	0.0117	0.0067	0.0043
0.3000	0.0325	0.0402	0.0382	0.0357	0.0289	0.0213	0.0146	0.0084	0.0054
0.4000	0.0365	0.0449	0.0425	0.0397	0.0322	0.0236	0.0162	0.0094	0.0061
0.5000	0.0379	0.0463	0.0438	0.0410	0.0332	0.0244	0.0157	0.0097	0.0064
EAR = 0.75									
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
P/D	1.1560	1.1577	1.1406	1.1063	1.0673	1.0202	0.9726	0.9252	0.8976
L.E.	1.0664	1.1725	1.3585	1.4562	1.4625	1.3711	1.1667	0.8447	0.5872
CHORD	2.1444	2.3451	2.7107	2.9124	2.9251	2.7422	2.3325	1.5675	1.1775
CAMBER DISTRIBUTION									
X/LT									
0.0099	0.0025	0.0032	0.0032	0.0030	0.0025	0.0018	0.0013	0.0007	0.0004
0.0125	0.0030	0.0039	0.0039	0.0037	0.0030	0.0022	0.0016	0.0009	0.0005
0.0250	0.0058	0.0074	0.0074	0.0069	0.0057	0.0042	0.0029	0.0017	0.0010
0.0500	0.0106	0.0125	0.0132	0.0124	0.0102	0.0076	0.0052	0.0031	0.0019
0.1000	0.0185	0.0235	0.0224	0.0212	0.0174	0.0128	0.0088	0.0053	0.0033
0.2000	0.0307	0.0385	0.0381	0.0341	0.0277	0.0200	0.0141	0.0085	0.0057
0.3000	0.0371	0.0485	0.0450	0.0424	0.0344	0.0251	0.0176	0.0107	0.0073
0.4000	0.0442	0.0546	0.0501	0.0472	0.0383	0.0279	0.0195	0.0120	0.0083
0.5000	0.0460	0.0565	0.0516	0.0489	0.0395	0.0288	0.0202	0.0124	0.0086
EAR = 1.20									
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
P/D	1.1560	1.1577	1.1406	1.1063	1.0673	1.0202	0.9726	0.9252	0.8976
L.E.	1.7052	1.8749	2.1722	2.3285	2.3386	2.1924	1.8648	1.3507	0.9389
CHORD	3.4289	3.7498	4.3344	4.6570	4.6771	4.3848	3.7296	2.7014	1.8829
CAMBER DISTRIBUTION									
X/LT									
0.0099	0.0030	0.0040	0.0038	0.0038	0.0032	0.0024	0.0017	0.0010	0.0006
0.0125	0.0038	0.0049	0.0043	0.0047	0.0039	0.0030	0.0022	0.0013	0.0007
0.0250	0.0072	0.0094	0.0090	0.0088	0.0073	0.0057	0.0040	0.0024	0.0014
0.0500	0.0133	0.0172	0.0163	0.0157	0.0128	0.0102	0.0072	0.0044	0.0027
0.1000	0.0238	0.0305	0.0283	0.0271	0.0219	0.0170	0.0122	0.0075	0.0050
0.2000	0.0403	0.0512	0.0462	0.0442	0.0359	0.0269	0.0194	0.0122	0.0085
0.3000	0.0517	0.0653	0.0579	0.0555	0.0450	0.0337	0.0241	0.0153	0.0110
0.4000	0.0586	0.0737	0.0647	0.0621	0.0503	0.0375	0.0267	0.0172	0.0125
0.5000	0.0613	0.0764	0.0667	0.0643	0.0520	0.0388	0.0277	0.0179	0.0131

NOTES: 1. Blade outline of Propeller 3916A which has an EAR = 0.95 is used as basic blade outline.
2. Calculations are based on one blade only.
3. Calculated camber distributions are in inches.

TABLE 4
Calculated Induced Velocities and Camber Distribution for Propeller 3916C

INPUT DATA										
18										
0.00500	0.10000	2.00000	0.15000	0.00000						Card 1 (see Appendix B for explanation)
0.00500	0.01000	0.01000	0.02400	0.03500	0.05000	0.07000	0.09600	0.12900		Card 2
0.17000	0.22000	0.28000	0.35200	0.44000	0.55200	0.70900	0.94500	1.00000		Card 3
0.00993	0.05083	0.11862	0.20414	0.29586	0.38138	0.44917	0.49007			Card 5
0.00993	0.01250	0.02500	0.05000	0.10000	0.20000	0.30000	0.40000	0.50000		Card 6
0.20000	0.30000	0.40000	0.50000	0.60000	0.70000	0.80000	0.90000	1.00000		Card 7
0.15400	0.18600	0.21550	0.23100	0.23200	0.21750	0.18500	0.13400	0.02000		Card 8
0.30800	0.37200	0.43000	0.46200	0.46400	0.43500	0.37000	0.26800	0.04000		r
1.57165	1.01777	0.78583	0.62866	0.52388	0.44904	0.39291	0.34926	0.31433		$L_d(r)/b$
0.	0.01067	0.01451	0.01698	0.01841	0.01897	0.01833	0.01537	0.		$L_t(r)/b$
0.25000	0.30000	0.40000	0.50000	0.60000	0.70000	0.80000	0.90000	0.95000		$\tan \beta_1(r)$
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000		$\Gamma(r)$
0.07500	0.09239	0.12034	0.13957	0.15287	0.16215	0.16890	0.17973	0.17900		r_0
0.09550	0.09793	0.09510	0.08817	0.08041	0.07308	0.06661	0.06302	0.06000		$V_x(r_0)$
1.09500	0.82600	0.60000	1.00000	1.00000						$u_a(r_0)$
										$u_t(r_0)$
										Card 9
OUTPUT										
AXIAL INDUCED VELOCITY COMPONENT										
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500	
X/LT										
0.0099	0.0214	0.0559	0.0649	0.0861	0.1089	0.1346	0.1777	0.2487	0.2667	
0.0508	0.0104	0.0370	0.0389	0.0556	0.0687	0.0885	0.1209	0.1802	0.2215	
0.1186	0.0050	0.0246	0.0232	0.0346	0.0410	0.0573	0.0808	0.1367	0.1732	
0.2041	0.0007	0.0133	0.0128	0.0203	0.0267	0.0361	0.0553	0.0993	0.1370	
0.2959	0.0003	0.0071	0.0064	0.0109	0.0111	0.0225	0.0352	0.0672	0.0931	
0.3814	0.0002	0.0052	0.0022	0.0058	0.0081	0.0121	0.0199	0.0387	0.0584	
0.4492	0.0007	0.0022	0.0007	0.0024	0.0033	0.0051	0.0085	0.0164	0.0258	
0.4901	0.0010	0.0005	-0.0002	0.0006	0.0006	0.0010	0.0017	0.0027	0.0057	
TANGENTIAL INDUCED VELOCITY COMPONENT										
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500	
X/LT										
0.0099	-0.1014	-0.1112	-0.0998	-0.0926	-0.0839	-0.0777	-0.0786	-0.0914	-0.0900	
0.0508	-0.0828	-0.0893	-0.0772	-0.0700	-0.0613	-0.0556	-0.0556	-0.0670	-0.0749	
0.1186	-0.0674	-0.0718	-0.0602	-0.0533	-0.0441	-0.0394	-0.0389	-0.0512	-0.0587	
0.2041	-0.0517	-0.0545	-0.0446	-0.0384	-0.0314	-0.0269	-0.0271	-0.0373	-0.0463	
0.2959	-0.0359	-0.0374	-0.0301	-0.0253	-0.0194	-0.0172	-0.0174	-0.0252	-0.0315	
0.3814	-0.0213	-0.0220	-0.0172	-0.0144	-0.0112	-0.0095	-0.0099	-0.0145	-0.0197	
0.4492	-0.0096	-0.0095	-0.0072	-0.0062	-0.0047	-0.0040	-0.0042	-0.0061	-0.0087	
0.4901	-0.0025	-0.0019	-0.0012	-0.0013	-0.0009	-0.0008	-0.0009	-0.0010	-0.0020	
INDUCED DOWNWASH DUE TO ALL BLADES										
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500	
X/LT										
0.0099	0.0927	0.1190	0.1127	0.1222	0.1254	0.1546	0.1941	0.2649	0.2814	
0.0508	0.0712	0.0901	0.0783	0.0844	0.0893	0.1035	0.1329	0.1922	0.2338	
0.1186	0.0559	0.0689	0.0555	0.0576	0.0567	0.0684	0.0845	0.1459	0.1829	
0.2041	0.0409	0.0486	0.0376	0.0376	0.0382	0.0439	0.0614	0.1060	0.1446	
0.2959	0.0283	0.0320	0.0236	0.0227	0.0189	0.0275	0.0392	0.0718	0.0983	
0.3814	0.0168	0.0195	0.0123	0.0125	0.0123	0.0150	0.0222	0.0413	0.0617	
0.4492	0.0079	0.0084	0.0050	0.0053	0.0051	0.0063	0.0095	0.0175	0.0272	
0.4901	0.0026	0.0017	0.0005	0.0012	0.0010	0.0012	0.0019	0.0027	0.0060	
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500	
P/D	0.9875	0.9875	0.9875	0.9875	0.9875	0.9875	0.9875	0.9875	0.9875	
L.E.	1.3533	1.4880	1.7240	1.8480	1.8560	1.7400	1.4800	1.0720	0.7452	
CNORD	2.7213	2.9760	3.4400	3.6960	3.7120	3.4800	2.9600	2.1440	1.4943	
CAMBER DISTRIBUTION										
X/LT										
0.0099	0.0019	0.0024	0.0022	0.0022	0.0021	0.0020	0.0019	0.0017	0.0011	
0.0125	0.0024	0.0030	0.0028	0.0028	0.0026	0.0025	0.0023	0.0021	0.0014	
0.0250	0.0045	0.0058	0.0052	0.0052	0.0049	0.0046	0.0044	0.0039	0.0028	
0.0500	0.0083	0.0106	0.0093	0.0092	0.0087	0.0082	0.0078	0.0071	0.0053	
0.1000	0.0147	0.0186	0.0158	0.0155	0.0143	0.0136	0.0131	0.0122	0.0095	
0.2000	0.0247	0.0306	0.0251	0.0241	0.0217	0.0208	0.0204	0.0201	0.0162	
0.3000	0.0315	0.0385	0.0308	0.0292	0.0259	0.0251	0.0250	0.0255	0.0211	
0.4000	0.0357	0.0432	0.0339	0.0320	0.0280	0.0275	0.0276	0.0286	0.0241	
0.5000	0.0372	0.0449	0.0348	0.0329	0.0288	0.0283	0.0285	0.0297	0.0251	
CAMBER RATIO										
X/LT										
0.0099	0.0007	0.0008	0.0007	0.0006	0.0006	0.0006	0.0006	0.0008	0.0008	
0.0125	0.0009	0.0010	0.0008	0.0007	0.0007	0.0007	0.0008	0.0010	0.0010	
0.0250	0.0017	0.0019	0.0015	0.0014	0.0013	0.0013	0.0015	0.0018	0.0019	
0.0500	0.0031	0.0035	0.0027	0.0025	0.0024	0.0024	0.0026	0.0033	0.0035	
0.1000	0.0054	0.0062	0.0046	0.0042	0.0038	0.0039	0.0044	0.0057	0.0064	
0.2000	0.0091	0.0103	0.0073	0.0065	0.0059	0.0060	0.0069	0.0094	0.0109	
0.3000	0.0116	0.0129	0.0090	0.0079	0.0070	0.0072	0.0084	0.0114	0.0141	
0.4000	0.0131	0.0145	0.0099	0.0087	0.0075	0.0079	0.0093	0.0134	0.0161	
0.5000	0.0137	0.0151	0.0101	0.0089	0.0078	0.0081	0.0096	0.0138	0.0168	

NOTES: 1. Calculation based on five blades.
2. Calculated camber in inches.

APPENDIX A

EVALUATION OF INTEGRALS

It has been stated in the text that the integrals of Equation [16] have a singularity at $r = r_0$ and $\theta = \theta_0$ (or $\phi = 0$). The integrals are integrated numerically with this singular point isolated. The justification for this procedure is that the contribution to the final mean line due to the small region that includes the singular point is negligible. To show that this is indeed the case, we shall first take a look at the induced velocities and then their contributions to the final mean line.

INDUCED VELOCITIES

The integral to be investigated which represents the contribution of the small area to the induced velocity components is in the following general form:

$$I_n(r_0, \theta_0) = \int_{r_0 - \delta_0}^{r_0 + \delta_0} \int_{\theta_0 - \epsilon_0}^{\theta_0 + \epsilon_0} L_n(r, \theta) K_n(r_0, \theta_0, r, \theta) d\theta dr$$

where $\delta_0 \ll 1$, $\epsilon_0 \ll 1$, $L_n(r, \theta)$ represents the loading function, and $K_n(r_0, \theta_0, r, \theta)$ is the kernel function which has the form

$$K_n(r_0, \theta_0, r, \theta) = \frac{F_n(r, \theta)}{[r_0^2 + r^2 - 2r_0 r \cos(\theta - \theta_0) + (p/\pi)^2 (\theta - \theta_0)^2]^{3/2}}$$

To facilitate the evaluation, we introduce a limiting process of $\lim_{X \rightarrow 0}$ such that

$$I_n(r_0, \theta_0) = \lim_{X \rightarrow 0} \left\{ \int_{r_0 - \delta_0}^{r_0 + \delta_0} \int_{\theta_0 - \epsilon_0}^{\theta_0 + \epsilon_0} \frac{L_n(r, \theta) F_n(r, \theta) d\theta dr}{[X^2 + r_0^2 + r^2 - 2r_0 r \cos(\theta - \theta_0) + (p/\pi)^2 (\theta - \theta_0)^2]^{3/2}} \right\}$$

Physically, the introduction of this limiting process means that mathematical evaluations are performed over an area on a surface which is X distance away from the control point on the lifting surface, and that in the limit the fictitious area is brought infinitesimally close to the control point. This means that the velocity induced on the surface $X=0$ is a continuous function of X including $X=0$.

With this procedure in mind, we shall attempt to deal with each of the four integrals involved.

Equation [16a]

$$V_{ba}(r_0, \theta_0) = I_1(r_0, \theta_0)$$

$$= \lim_{X \rightarrow 0} \left\{ -\frac{r_0}{4\pi} \int_{r_0 - \delta_0}^{r_0 + \delta_0} \int_{\theta_0 - \epsilon_0}^{\theta_0 + \epsilon_0} \frac{\gamma(r) \sin(\theta - \theta_0) d\theta dr}{[X^2 + r_0^2 + r^2 - 2r_0 r \cos(\theta - \theta_0) + (p/\pi)^2 (\theta - \theta_0)^2]^{3/2}} \right\}$$

For a small ϵ_0 , we may substitute $\theta - \theta_0$ for $\sin(\theta - \theta_0)$ and $1 - \frac{1}{2}(\theta - \theta_0)^2$ for $\cos(\theta - \theta_0)$, and for a linear approximation we can express

$$\gamma(r) = \gamma(r_0) + \gamma'(r_0)(r - r_0)$$

Let

$$\epsilon = \theta - \theta_0, \quad |\epsilon| \ll 1$$

$$\delta = r - r_0, \quad |\delta| \ll 1$$

we have

$$I_1(r_0, \theta_0) = \lim_{X \rightarrow 0} \left\{ -\frac{r_0}{4\pi} \int_{-\delta_0}^{\delta_0} \int_{-\epsilon_0}^{\epsilon_0} \frac{[\gamma(r_0) + \gamma'(r_0)\delta]}{[X^2 + \delta^2 + a^2 \epsilon^2]^{3/2}} d\epsilon d\delta \right\}$$

where

$$a^2 = r_0^2 + (p/\pi)^2$$

The integration of the second term with respect to δ is zero, which justifies the omission of the second term in the calculation for the mid-zone, and the integration of the first term also vanishes for $\theta_l - \epsilon_0 \geq \theta_0 \geq \theta_l + \epsilon_0$. Therefore, we only have to consider the cases of $\theta_0 > \theta_l - \epsilon_0$ and $\theta_0 < \theta_l + \epsilon_0$, i.e., the end points. Integrating the first integral with respect to ϵ and assuming $\theta_l \leq \theta_0 \leq \theta_l - \epsilon_0$, we obtain

$$I_1(r_0, \theta_0) = \lim_{X \rightarrow 0} \left\{ -\frac{r_0 \gamma(r_0)}{4\pi} \int_{-\delta_0}^{\delta_0} \frac{1}{a^2 (X^2 + \delta^2 + a^2 \epsilon^2)^{1/2}} \left| \frac{\theta_l - \theta_0}{\theta_l - 2\epsilon_0 - \theta_0} \right| d\delta \right\}$$

$$= \lim_{X \rightarrow 0} \left\{ - \frac{r_0 \gamma(r_0)}{4\pi a^2} \int_{-\delta_0}^{\delta_0} \frac{d\delta}{[X^2 + \delta^2 + a^2 (\theta_l - \theta_0)^2]^{1/2}} - \int_{-\delta_0}^{\delta_0} \frac{d\delta}{[X^2 + \delta^2 + a^2 (\theta_l - \theta_0 - 2\epsilon_0)^2]^{1/2}} \right\}$$

$$I_1(r_0, \theta_0) = 0 \quad \text{when} \quad \theta_0 = \theta_l - \epsilon_0$$

Integrating and taking the limit, we finally obtain

$$I_1(r_0, \theta_0) = - \frac{r_0 \gamma(r_0)}{4a^2 \pi} \left\{ \ln \left[\delta_0 + \sqrt{\delta_0^2 + a^2 (\theta_l - \theta_0)^2} \right] - \ln \left[-\delta_0 + \sqrt{\delta_0^2 + a^2 (\theta_l - \theta_0)^2} \right] - \ln \left[\delta_0 + \sqrt{\delta_0^2 + a^2 (\theta_l - \theta_0 - 2\epsilon_0)^2} \right] + \ln \left[-\delta_0 + \sqrt{\delta_0^2 + a^2 (\theta_l - \theta_0 - 2\epsilon_0)^2} \right] \right\}$$

or

$$I_1(r_0, \theta_0) = \frac{r_0 \gamma(r_0)}{4a^2 \pi} \ln \left[-\delta_0 + \sqrt{\delta_0^2 + a^2 (\theta_l - \theta_0)^2} \right]$$

since the second term is certainly the most important term as $\theta_0 \rightarrow \theta_l$. As expected, the induced velocity components have logarithmic singularity at the leading edge.

Equation [16b]

$$V_{bt}(r_0, \theta_0) = I_2(r_0, \theta_0)$$

$$= \lim_{X \rightarrow 0} \left\{ \frac{1}{4\pi^2} \int_{r_0 - \delta_0}^{r_0 + \delta_0} \int_{\theta_0 - \epsilon_0}^{\theta_0 + \epsilon_0} \frac{\gamma(r)(\theta - \theta_0)[p \cos(\theta - \theta_0) + p'(r_0 - r \cos(\theta - \theta_0))]}{[X^2 + r_0^2 + r^2 - 2r_0 r \cos(\theta - \theta_0) + (p/\pi)^2 (\theta - \theta_0)^2]^{3/2}} d\theta dr \right\}$$

Following the same procedure and neglecting the p' term, we obtain

$$I_2(r_0, \theta_0) = - \frac{p \gamma(r_0)}{4a^2 \pi^2} \ln \left[-\delta_0 + \sqrt{\delta_0^2 + a^2 (\theta_l - \theta_0)^2} \right]$$

Equation [16c]

$$V_{fa}(r_0, \theta_0) = I_3(r_0, \theta_0)$$

$$= \lim_{X \rightarrow 0} \left\{ -\frac{1}{4\pi} \int_{r_0 - \delta_0}^{r_0 + \delta_0} \int_{\theta_0 - \epsilon_0}^{\theta_0 + \epsilon_0} \frac{\Gamma_\theta(r, \theta) r [r - r_0 \cos(\theta - \theta_0)]}{[X^2 + r_0^2 + r^2 - 2r_0 r \cos(\theta - \theta_0) + (p/\pi)^2 (\theta - \theta_0)^2]^{3/2}} d\theta dr \right\}$$

Let

$$\Gamma_\theta(r, \theta) = \Gamma_\theta(r_0, \theta) + \Gamma'_\theta(r_0, \theta) (r - r_0)$$

we have

$$I_3(r_0, \theta_0) = \lim_{X \rightarrow 0} \left\{ -\frac{r_0}{4\pi} \int_{-\epsilon_0}^{\epsilon_0} \int_{-\delta_0}^{\delta_0} \frac{\Gamma_\theta(r_0, \epsilon) + \Gamma'_\theta(r_0, \epsilon) \delta}{(X^2 + \delta^2 + a^2 \epsilon^2)^{3/2}} \delta d\delta d\epsilon \right\}$$

$$= \lim_{X \rightarrow 0} \left\{ -\frac{r_0}{4\pi} \int_{-\epsilon_0}^{\epsilon_0} \int_{-\delta_0}^{\delta_0} \frac{\Gamma_\theta(r_0, \epsilon) + C(r_0) \delta - \gamma'(r_0) \epsilon \delta}{(X^2 + \delta^2 + a^2 \epsilon^2)^{3/2}} \delta d\delta d\epsilon \right\}$$

since $\Gamma'_\theta(r_0, \epsilon)$ can be expressed as the sum of $C(r_0)$ and $-\gamma'(r_0)\epsilon$ where $C(r_0)$ represents quantities evaluated at r_0 , a constant.

The integration of the first term with respect to δ and that of the third term with respect to ϵ vanish, and the only remaining term is the second term. Therefore

$$I_3(r_0, \theta_0) = \lim_{X \rightarrow 0} \left\{ -\frac{r_0 C(r_0)}{4\pi} \int_{-\epsilon_0}^{\epsilon_0} \int_{-\delta_0}^{\delta_0} \frac{\delta^2}{(X^2 + \delta^2 + a^2 \epsilon^2)^{3/2}} d\delta d\epsilon \right\}$$

$$= \lim_{X \rightarrow 0} \left\{ -\frac{r_0 C(r_0)}{4\pi} \int_{-\epsilon_0}^{\epsilon_0} \left[\frac{-2\delta_0}{(X^2 + \delta_0^2 + a^2 \epsilon^2)^{1/2}} + \ln \left(\delta_0 + \sqrt{X^2 + \delta_0^2 + a^2 \epsilon^2} \right) \right. \right.$$

$$\left. \left. - \ln \left(-\delta_0 + \sqrt{X^2 + \delta_0^2 + a^2 \epsilon^2} \right) \right] d\epsilon \right\}$$

Here again we observe that the most important term is the last term. Carrying out the integration, taking the limit, and using the approximation of

$$-\delta_0 + (\delta_0^2 + a^2 \epsilon^2)^{1/2} \rightarrow \delta_0 \frac{a^2}{2} \left(\frac{\epsilon}{\delta_0} \right)^2$$

we finally have

$$I_3(r_0, \theta_0) = \frac{r_0 C(r_0)}{4\pi} \left[\epsilon \ln \frac{a^2}{2\delta_0} + 2\epsilon (\ln \epsilon - 1) \right]_{-\epsilon_0}^{\epsilon_0}$$

which vanishes as $\epsilon_0 \rightarrow 0$.

Equation [16d]

$$V_{ft}(r_0, \theta_0) = I_4(r_0, \theta_0)$$

$$\begin{aligned} & \lim_{X \rightarrow 0} \left\{ \frac{1}{4\pi^2} \int_{r_0 - \delta_0}^{r_0 + \delta_0} \int_{\theta_0 - \epsilon_0}^{\theta_0 + \epsilon_0} \frac{p \Gamma_\theta(r_0, \theta) [r_0 - r \cos(\theta - \theta_0) - r(\theta - \theta_0) \sin(\theta - \theta_0)]}{[X^2 + r_0^2 + r^2 - 2r_0 r \cos(\theta - \theta_0) + (p/\pi)^2 (\theta - \theta_0)^2]^{3/2}} d\theta dr \right\} \\ &= \lim_{X \rightarrow 0} \left\{ \frac{1}{4\pi^2} \int_{-\epsilon_0}^{\epsilon_0} \int_{-\delta_0}^{\delta_0} \frac{-p \Gamma_\theta(r_0, \epsilon) \delta}{[X^2 + \delta^2 + a^2 \epsilon^2]^{3/2}} d\delta d\epsilon \right\} \end{aligned}$$

which vanishes by following the same produced as for I_3 .

This shows that the only contribution is from I_1 and I_2 .

CONTRIBUTION TO MEAN LINE

The contribution to the mean line due to the small area in the close neighborhood of the singular point when $\theta_l \geq \theta_0 \geq \theta_l - \epsilon_0$ is

$$\Delta C_m(r_0, \epsilon_0) = \frac{D r_0}{2V \cos \beta_i} \int_{\theta_l - \epsilon_0}^{\theta_l} v_n(r_c, \theta_0) d\theta_0$$

where v_n is the induced downwash, which can be expressed as

$$v_n(r_0, \theta_0) = I_1(r_0, \theta_0) \cos \beta_i(r_0) - I_2(r_0, \theta_0) \sin \beta_i(r_0)$$

Substituting the results of I_1 and I_2 and making the following approximation

$$-\delta_0 + \sqrt{\delta_0^2 + a^2 (\theta_l - \theta_0)^2} \rightarrow \delta_0 \frac{a^2 (\theta_l - \theta_0)^2}{2 \delta_0}$$

we get

$$v_n(r_0, \theta_0) = f(r_0) \ln \left[\frac{\delta_0 a^2 (\theta_l - \theta_0)^2}{2 \delta_0} \right]$$

where

$$f(r_0) = \frac{\gamma(r_0)}{4 a^2 \pi} [r_0 \cos \beta_i(r_0) + (p(r_0)/\pi)^2 \sin \beta_i(r_0)]$$

Performing the integration, we obtain

$$\Delta C_m(r_0, \epsilon_0) = \frac{D r_0}{2 V \cos \beta_i} f(r_0) \epsilon_0 \left[\ln \frac{\delta_0 a^2}{2} + 2 \left(\ln \frac{\epsilon_0}{\delta_0} - 1 \right) \right]$$

Since $\epsilon_0 \ll 1$, this contribution to the mean line is indeed negligible. An estimate for a case where $\epsilon_0 = 0.001$ and $\delta_0 = 0.005$ shows that the contribution of this small area as compared to that of the remaining area is in the order of tenths of 1 percent. These values of ϵ_0 and δ_0 are actually used in the computing program.

APPENDIX B

PREPARATION OF INPUT DATA

The input data consists mainly of two parts: (1) program controls and (2) propeller data. The following discussion is outlined in the order of the input data format; see Figure 16.

Card 1 Format 9I8

NL Number of radial stations in the common network of field points (maximum number of radial stations in the tip or hub zones)
Specify NL = 18

Card 2 Format 8F9.5

DY δ_0 , half-width of mid-zone
Specify DY = 0.005

DFD Minimum chordwise spacing, $(\Delta\phi)_{\min}$ in the common network of field points in degrees
Specify DFD = 0.1*

FDFD Maximum chordwise spacing, $(\Delta\phi)_{\max}$ in the common network of field points in degrees
Specify FDFD = 2.0*

DFFD Maximum range of ϕ in the common network of field points in degrees
Specify DFFD = 150.0*

Card 3 (more than one card) Format 9F8.5

SPR Spacings of radial stations in the common network of field points. Total number of entries should equal NL. The following set of numbers may be specified:
0.005, 0.010, 0.016, 0.024, 0.035, 0.050, 0.070, 0.096, 0.129, 0.170, 0.220, 0.280, 0.352, 0.440, 0.552, 0.709, 0.945, 1.000

If Cards 1, 2, and 3 as specified are used, no entry for these cards is necessary.

Card 4 Format 12A6

Propeller identification not more than 72 digits.
(This is the first line in the print out.)

Card 5 Format 9I8

NN Number of blades

NKN Number of chordwise coordinates θ_0 at which induced velocities are calculated.

*Total number of chordwise stations on either side of control point should not exceed 90.

- NCL Number of chordwise coordinates x/L_T at which camber distribution is calculated.
- NCTR Program control.
 When calculation for a given set of input data is completed, if:
 NCTR = 1, the computer reads another completed set of input data (cards 4 through 9) and proceeds to compute.
 NCTR = -1, the computer reads another set of data for NN, NCTR, NPT, KSP, VM, DY, SM, SW and proceeds to compute.
 NCTR = 0, program terminates.
- KRN Number of radial coordinates r_0 at which induced velocities and camber distribution are calculated.
- NPT Specify NPT = 0. Program control for print out. (See Appendix C.)
- KSP Specify KSP = 0. Program control for print out. (See Appendix C.)
- Card 6 Format 9F8.5
- DK Used to calculate chordwise coordinates at which induced velocities are calculated. The j^{th} chordwise coordinate θ_0^j is calculated by using the formula:

$$\theta_0^j = \theta_l(r_0) - \text{DK}(j) \theta_T(r_0).$$
 Total number of entries should equal NKN.
- Card 7 Format 9F8.5
- XL Chordwise coordinates at which camber distribution is calculated.
 Total number of entries should equal NCL.
- Card 8 (9 cards) Format 9F8.5
 Propeller data:
- 1st card – Radii r at which input data is given in the following four cards.
- 2nd card – Dimensionless leading edge coordinate from reference line at r , L_T/D .
- 3rd card – Dimensionless chord length at r , L_T/D .
- 4th card – Tangent of hydrodynamic pitch angle at r , $\tan \beta_i$, taken from the lifting line calculation output.*
- 5th card – Dimensionless bound circulation distribution $\Gamma(r)$ based on ship speed taken from the lifting-line calculation output.*
- 6th card – Radii r_0 at which induced velocities and camber distribution are calculated.
 Total number of entries should equal KRN.
- 7th card – Advance velocity at r_0 , V_x .
- 8th card – Axial induced velocity u_a at r_0 taken from the lifting-line calculation output.*
- 9th card – Tangential induced velocity u_t at r_0 taken from the lifting-line calculation output.*

*AML Problem No. 106, "Moderately Loaded Propellers Using Induction Factors."

Card 9 Format 9F8.5

VM Scale factor for chord length used for calculating the camber for different expanded area ratios with respect to a reference EAR.
Specify VM = 1.0 if the input data L_T/D (third card of Card 8) give the required EAR. For other values of EAR it is only necessary to change the value of VM, e.g., VM = 1.25 represents an EAR which is 25 percent larger.

AJ Coefficient of advance, based on ship velocity.

DIA Propeller diameter.

SM Specify SM = 1.0.
If only the camber due to bound circulation is to appear in the print out, specify SM = 0.

SW Specify SW = 1.0.
If only the camber due to free circulation is to appear in the print out, specify SW = 0.

Note 1. The program is written so that more than one problem can be handled at one time. The computer performs complete sets of calculations, one for each problem, successively. The problem could be

- a. for entirely different propellers, in which case one set of the input data Cards 4 through 9 for each propeller will be required,
- b. for a propeller the same as the one specified in the immediate preceding set except that the expanded area ratio (EAR) is different,
- c. for the same propeller as specified in the immediate preceding set except calculation is to be performed with different NN, NPT, KSP or SM, or
- d. for a combination of b and c.

For either b, c, or d, a set of two input cards will be required. They specify the quantities involved in the following order:

Card i NN, NCTR, NPT, KSP (Format 4I8)

Card ii VM, DY, SM, SW (Format 4F8.5).

In each case, appropriate values of NCTR should be specified. The sets of input cards are to be arranged consecutively in an appropriate order.

Note 2. The computing time depends mainly on the number of blades, the shape of blade outline, and the fineness of network of field points, i.e., the number of stations. For a conventional propeller as the one shown in Figure 16, the computing time required is tabulated for various values for Z, NL, $(\Delta\phi)_{\min}$, $(\Delta\phi)_{\max}$, and maximum range of ϕ :

Z	NL	$(\Delta\phi)_{\min}$	$(\Delta\phi)_{\max}$	Range of ϕ	Time
1	20	0.10°	2.0°	150°	9.4 min
1	11	0.10°	2.0°	150°	6.0 min
1	11	0.10°	3.0°	220°	5.5 min
5	20	0.10°	2.0°	150°	11.4 min

APPLIED MATHEMATICS LABORATORY PROBLEM XPLU
PROPELLER DESIGN BASED ON LIFTING SURFACE THEORY
UNIFORM CHORDWISE LOAD DISTRIBUTION

*PROPELLER 3916A

XPLU

Card 4 (see Appendix B for explanation)

INPUT DATA

18
0.00500 0.10000 2.00000 150.00000
0.00500 0.01000 0.01000 0.02400 0.03500 0.05000 0.07000 0.09000 0.12000
0.17000 0.22000 0.28000 0.35200 0.44000 0.55200 0.70000 0.94500 1.00000
5 8 9 0 9 6 3
0.00993 0.05083 0.11862 0.20414 0.29586 0.38138 0.44917 0.49007
0.00993 0.01250 0.02500 0.05000 0.10000 0.20000 0.30000 0.40000 0.50000
0.20000 0.30000 0.40000 0.50000 0.60000 0.70000 0.80000 0.90000 1.00000
0.15400 0.18600 0.21550 0.23100 0.23200 0.21750 0.18500 0.13400 0.02000
0.30800 0.37200 0.43000 0.46200 0.46400 0.43500 0.37000 0.26800 0.04000
1.04210 1.22835 0.90766 0.70429 0.56622 0.46392 0.38698 0.32722 0.27960

0. 0.01730 0.02266 0.02428 0.02283 0.01936 0.01463 0.00898-0.
0.25000 0.38900 0.48000 0.50000 0.60000 0.70000 0.80000 0.90000 0.95000
1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000
0.14500 0.16902 0.20861 0.22173 0.20765 0.17661 0.13594 0.08937 0.06500
0.16810 0.16678 0.16039 0.13704 0.10517 0.07483 0.04889 0.02767 0.01770
1.00000 0.82600 0.00000 1.00000 1.00000

Card 1
Card 2
Card 3

Card 5
Card 6
Card 7
Card 8

r
 $L_f(r)/D$
 $L_g(r)/D$
 $\tan \beta_1(r)$

$\Gamma(r)$
 r_0
 $V_a(r_0)$
 $u_a(r_0)$
 $u_t(r_0)$

Card 9

OUTPUT

AXIAL INDUCED VELOCITY COMPONENT

RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
X/LT									
0.0099	0.0590	0.1047	0.1314	0.1541	0.1522	0.1460	0.1385	0.1259	0.1146
0.0508	0.0421	0.0765	0.0903	0.1091	0.1084	0.0995	0.0954	0.0887	0.0919
0.1186	0.0305	0.0568	0.0662	0.0759	0.0774	0.0662	0.0651	0.0665	0.0724
0.2041	0.0214	0.0403	0.0448	0.0540	0.0524	0.0488	0.0466	0.0481	0.0570
0.2959	0.0141	0.0264	0.0285	0.0343	0.0337	0.0313	0.0303	0.0324	0.0372
0.3814	0.0083	0.0149	0.0154	0.0191	0.0188	0.0175	0.0172	0.0186	0.0245
0.4492	0.0043	0.0064	0.0061	0.0082	0.0079	0.0074	0.0074	0.0079	0.0109
0.4901	0.0019	0.0013	0.0006	0.0018	0.0016	0.0015	0.0015	0.0013	0.0026

TANGENTIAL INDUCED VELOCITY COMPONENT

RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
X/LT									
0.0099	-0.1699	-0.1953	-0.1730	-0.1480	-0.1142	-0.0849	-0.0620	-0.0450	-0.0360
0.0508	-0.1404	-0.1578	-0.1353	-0.1147	-0.0877	-0.0623	-0.0447	-0.0333	-0.0290
0.1186	-0.1150	-0.1277	-0.1073	-0.0809	-0.0671	-0.0454	-0.0323	-0.0253	-0.0230
0.2041	-0.0885	-0.0974	-0.0805	-0.0664	-0.0486	-0.0336	-0.0232	-0.0184	-0.0180
0.2959	-0.0614	-0.0571	-0.0547	-0.0447	-0.0323	-0.0220	-0.0153	-0.0124	-0.0119
0.3814	-0.0363	-0.0391	-0.0315	-0.0257	-0.0184	-0.0125	-0.0087	-0.0071	-0.0078
0.4492	-0.0163	-0.0168	-0.0132	-0.0111	-0.0078	-0.0053	-0.0038	-0.0030	-0.0035
0.4901	-0.0043	-0.0034	-0.0022	-0.0024	-0.0015	-0.0010	-0.0008	-0.0005	-0.0009

INDUCED DOWNWASH DUE TO ALL BLADES

RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
X/LT									
0.0099	0.1740	0.2175	0.2136	0.2112	0.1887	0.1682	0.1514	0.1339	0.1201
0.0508	0.1400	0.1707	0.1578	0.1553	0.1376	0.1164	0.1051	0.0947	0.0964
0.1186	0.1125	0.1348	0.1212	0.1132	0.1004	0.0791	0.0724	0.0710	0.0759
0.2041	0.0854	0.1010	0.0873	0.0824	0.0696	0.0584	0.0519	0.0514	0.0598
0.2959	0.0508	0.0687	0.0579	0.0538	0.0453	0.0376	0.0337	0.0347	0.0391
0.3814	0.0348	0.0397	0.0325	0.0304	0.0254	0.0211	0.0192	0.0199	0.0257
0.4492	0.0159	0.0111	0.0134	0.0131	0.0107	0.0090	0.0082	0.0084	0.0115
0.4901	0.0046	0.0035	0.0020	0.0029	0.0021	0.0018	0.0017	0.0014	0.0027
RX/RO	0.2500	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
P/D	1.1560	1.1577	1.1406	1.1063	1.0673	1.0202	0.9726	0.9252	0.8976
L/D	1.3533	1.4880	1.7240	1.8480	1.8560	1.7400	1.4800	1.0720	0.7452
CHORD	2.7213	2.9760	3.4400	3.6960	3.7120	3.4800	2.9600	2.1440	1.4943

CAMBER DISTRIBUTION

X/LT									
0.0099	0.0035	0.0044	0.0042	0.0038	0.0029	0.0022	0.0015	0.0008	0.0005
0.0125	0.0044	0.0055	0.0052	0.0047	0.0037	0.0027	0.0018	0.0011	0.0006
0.0250	0.0084	0.0105	0.0099	0.0089	0.0069	0.0050	0.0034	0.0020	0.0012
0.0500	0.0157	0.0194	0.0180	0.0162	0.0125	0.0090	0.0061	0.0035	0.0022
0.1000	0.0283	0.0346	0.0314	0.0280	0.0216	0.0151	0.0103	0.0061	0.0039
0.2000	0.0483	0.0585	0.0519	0.0455	0.0348	0.0238	0.0163	0.0099	0.0067
0.3000	0.0624	0.0751	0.0656	0.0572	0.0433	0.0297	0.0202	0.0125	0.0087
0.4000	0.0709	0.0849	0.0735	0.0639	0.0482	0.0330	0.0225	0.0140	0.0099
0.5000	0.0740	0.0882	0.0760	0.0661	0.0497	0.0341	0.0232	0.0145	0.0104

CAMBER RATIO

X/LT									
0.0099	0.0013	0.0015	0.0012	0.0010	0.0008	0.0006	0.0005	0.0004	0.0003
0.0125	0.0016	0.0018	0.0015	0.0013	0.0010	0.0008	0.0006	0.0005	0.0004
0.0250	0.0031	0.0035	0.0029	0.0024	0.0019	0.0014	0.0012	0.0009	0.0006
0.0500	0.0058	0.0065	0.0052	0.0044	0.0034	0.0026	0.0021	0.0016	0.0013
0.1000	0.0104	0.0116	0.0091	0.0076	0.0058	0.0043	0.0035	0.0026	0.0020
0.2000	0.0177	0.0197	0.0151	0.0123	0.0094	0.0068	0.0055	0.0046	0.0045
0.3000	0.0229	0.0252	0.0191	0.0155	0.0117	0.0085	0.0068	0.0058	0.0058
0.4000	0.0261	0.0285	0.0214	0.0173	0.0130	0.0095	0.0076	0.0065	0.0066
0.5000	0.0272	0.0296	0.0221	0.0179	0.0134	0.0098	0.0078	0.0068	0.0069

Figure 16 - Sample Computer Output (for Propeller 3916A)

APPENDIX C

SAMPLES OF OUTPUT

The program prints out the following information in tabular form; see Figure 16.

Axial induced velocity component.
Tangential induced velocity component.
Induced downwash.
Camber distribution (in unit same as propeller diameter).
Camber ratio.

In addition, the program has a feature of printing out certain intermediate information when desired; see Figure 17. It will print out the chordwise integrated values of the axial and tangential induced velocity components in the following general order:

At $r_0 = r_0^1$

Tip-zone

$r = r_1$

chordwise integrated values of v_a at various θ_0

chordwise integrated values of v_t at various θ_0

$r = r_2$

(same print out as for $r = r_1$)

...

etc.

Hub-zone

$r = r_1$

(same print out as in the tip-zone)

etc.

Integrated values of v_a in mid-zone at various θ_0

Integrated values of v_t in mid-zone at various θ_0

At $r = r_0^2$

(same print out for $r_0 = r_0^1$)

etc.

Also printed out are the values of integrands in the equations expressing the induced velocity components (Equation [16]) for any specified control point $P(r_0^i, \theta_0^j)$ in the following order:

At $r = r_0^i$ and $\theta_0 = \theta_0^j$

Tip- and Hub-Zones

At $r = r_1$

Chordwise stations ϕ , PHI

Axial components due to superimposed lifting line (to be exact, the second term of Equation [16c]) corresponding to ϕ , UHH

Tangential components due to superimposed lifting line (the second term of Equation [16d]) corresponding to ϕ , VHH

Axial components due to free system (the first term of Equation [16c]) corresponding to ϕ - leading edge portion (from P to leading edge), UHL

Axial component due to free system - trailing edge portion (from P to trailing edge), UHT

Tangential component due to free system (the first term of Equation [16d]) corresponding to ϕ - leading edge portion, VHL

Tangential component due to free system - trailing edge portion, VHT

Axial component due to bound system (Equation [16a]) corresponding to ϕ - leading edge portion, UBL

Axial component due to bound system - trailing edge portion, UBT

Tangential component due to bound system (Equation [16b]) corresponding to ϕ & leading edge portion, VBL

Tangential component due to bound system - trailing edge portion, VBT

Value of r_1

Chordwise integrated values of v_a at various θ_0

Chordwise integrated values of v_t at various θ_0

At $r = r_2$

(similar print out as for $r = r_1$)

Mid-Zone

(similar print out for the axial and tangential components as in the tip- and hub-zones.)

Integrated values of v_a at various θ_0

Integrated values of v_t at various θ_0

Figure 16 shows the output for a typical problem.

Figure 17 shows the partial output of the same problem as in Figure 16, except intermediate information for point P at $r_0 = r_0^6 = 0.7$ and $\theta_0 = \theta_0^3$, i.e., NPT = 6, KSP = 3, is also printed.

CHORDWISE INTEGRATED VALUES OF VA AND VT AT RO=0.2500

TIP-ZONE

R=.2450

-0.92330E 02 -0.88920E 02 -0.76231E 02 -0.59221E 02 -0.40714E 02 -0.23373E 02 -0.96042E 01 -0.12934E 01
0.12463E 03 0.12039E 03 0.10310E 03 0.79949E 02 0.54884E 02 0.31479E 02 0.12926E 02 0.17336E 01

R=.2600

-0.43474E 02 -0.42415E 02 -0.36732E 02 -0.28635E 02 -0.19690E 02 -0.11264E 02 -0.45621E 01 -0.51539E 00
0.53491E 02 0.52674E 02 0.45597E 02 0.35442E 02 0.24303E 02 0.13377E 02 0.56111E 01 0.62383E 00

R=.2650

-0.27450E 02 -0.26974E 02 -0.23570E 02 -0.18440E 02 -0.12681E 02 -0.72284E 01 -0.28839E 01 -0.25932E-00
0.31435E 02 0.30613E 02 0.26603E 02 0.20897E 02 0.14317E 02 0.81378E 01 0.32361E 01 0.27870E-00

R=.2710

-0.18773E 02 -0.18244E 02 -0.16081E 02 -0.12634E 02 -0.86913E 01 -0.49327E 01 -0.19321E 01 -0.11851E-00
0.18613E 02 0.18477E 02 0.16404E 02 0.12848E 02 0.87964E 01 0.49722E 01 0.19356E 01 0.10324E-00

R=.2790

-0.12769E 02 -0.12348E 02 -0.10976E 02 -0.86697E 01 -0.59673E 01 -0.37687E 01 -0.12885E 01 -0.30404E-01
0.10670E 02 0.10628E 02 0.96099E 01 0.75844E 01 0.51929E 01 0.29138E 01 0.11008E 01 0.64386E-02

R=.2900

-0.86030E 01 -0.82547E 01 -0.73858E 01 -0.58724E 01 -0.40470E 01 -0.22715E 01 -0.34486E 00 0.18607E-01
0.55540E 01 0.55438E 01 0.51422E 01 0.41175E 01 0.28226E 01 0.15689E 01 0.56807E 00 0.36896E-01

R=.3050

-0.57757E 01 -0.55001E 01 -0.49364E 01 -0.35543E 01 -0.27333E 01 -0.15289E 01 -0.55646E 00 0.32799E-01
0.34654E 01 0.24804E 01 0.23945E 01 0.19710E 01 0.13599E 01 0.74806E 00 0.25614E-00 0.41402E-01

R=.3250

-0.33345E 01 -0.37260E 01 -0.33443E 01 -0.27002E 01 -0.18794E 01 -0.10567E 01 -0.38838E-00 0.17191E-01
0.85655E 00 0.81850E 00 0.86667E 00 0.76249E 00 0.53796E 00 0.29408E-00 0.94912E-01 0.26149E-01

R=.3510

-0.27583E 01 -0.26069E 01 -0.23398E 01 -0.19063E 01 -0.13445E 01 -0.77153E 00 -0.30233E-00 -0.16820E-01
-0.21427E-01 0.19507E-01 0.10948E-00 0.14947E-00 0.12007E-00 0.67050E-01 0.20969E-01 0.70912E-02

R=.3840

-0.19466E 01 -0.18439E 01 -0.16612E 01 -0.13699E 01 -0.98560E 00 -0.58511E 00 -0.25316E-00 -0.50310E-01
-0.36189E-00 -0.32050E-00 -0.22770E-00 -0.13562E-00 -0.77593E-01 -0.40323E-01 -0.13105E-01 0.30270E-02

R=.4250

-0.12396E 01 -0.11825E 01 -0.10776E 01 -0.90568E 00 -0.66841E 00 -0.41163E-00 -0.19434E-00 -0.60573E-01
-0.48537E-00 -0.44828E-00 -0.36607E-00 -0.26283E-00 -0.17013E-00 -0.93214E-01 -0.33322E-01 0.29735E-02

R=.4750

-0.48479E-00 -0.47437E-00 -0.45143E-00 -0.39982E-00 -0.30965E-00 -0.19871E-00 -0.99283E-01 -0.36932E-01
-0.56934E 00 -0.47852E-00 -0.41042E-00 -0.31307E-00 -0.21085E-00 -0.11915E-00 -0.45986E-01 -0.33592E-02

R=.5350

0.26920E-00 0.24622E-00 0.18772E-00 0.12158E-00 0.67230E-01 0.32020E-01 0.11463E-01 0.62007E-03
-0.47020E-00 -0.44832E-00 -0.39421E-00 -0.31059E-00 -0.21381E-00 -0.12291E-00 -0.51212E-01 -0.80420E-02

R=.6070

0.87715E 00 0.60378E 00 0.69260E 00 0.53207E 00 0.36193E-00 0.20795E-00 0.88375E-01 0.17508E-01
-0.38850E-00 -0.36828E-00 -0.32879E-00 -0.26298E-00 -0.18292E-00 -0.10551E-00 -0.44063E-01 -0.73862E-02

R=.6950

0.12581E 01 0.11588E 01 0.97708E 00 0.78258E 00 0.53912E 00 0.30983E-00 0.12777E-00 0.17030E-01
-0.27872E-00 -0.26327E-00 -0.23118E-00 -0.18790E-00 -0.13159E-00 -0.76142E-01 -0.31659E-01 -0.48009E-02

R=.8070

0.14985E 01 0.13791E 01 0.11787E 01 0.91945E 00 0.64975E 00 0.37749E-00 0.15960E-00 0.28209E-01
-0.16825E-00 -0.15780E-00 -0.13844E-00 -0.11082E-00 -0.79373E-01 -0.46670E-01 -0.19740E-01 -0.33492E-02

R=.9640

0.22769E 01 0.21293E 01 0.18186E 01 0.14191E 01 0.97915E 00 0.56103E 00 0.24208E-00 0.40819E-01
-0.96521E-01 -0.90102E-01 -0.78728E-01 -0.63117E-01 -0.44669E-01 -0.26020E-01 -0.11284E-01 -0.19084E-02

R=.9999

0.35523E 01 0.32648E 01 0.27879E 01 0.21730E 01 0.15050E 01 0.88792E 00 0.38283E-00 0.57501E-01
-0.12182E-00 -0.11358E-00 -0.99279E-01 -0.79526E-01 -0.56407E-01 -0.33789E-01 -0.14664E-01 -0.22053E-02

HUB-ZONE

R=.2450

0.11191E 03 0.96770E 02 0.81272E 02 0.62740E 02 0.43119E 02 0.24910E 02 0.70499E 02 0.18078E 01
-0.17782E 03 -0.15463E 03 -0.13021E 03 -0.10075E 03 -0.69354E 02 -0.40103E 02 -0.16910E 02 -0.29149E 01

R=.2400

0.58851E 02 0.50487E 02 0.42074E 02 0.32376E 02 0.22261E 02 0.12893E 02 0.54902E 01 0.10278E 01
-0.10081E 03 -0.87487E 02 -0.73373E 02 -0.56769E 02 -0.39141E 02 -0.22712E 02 -0.96795E 01 -0.18136E 01

R=.2350

0.39929E 02 0.35000E 02 0.29040E 02 0.22317E 02 0.15331E 02 0.88974E 01 0.38204E 01 0.76114E 00
-0.74085E 02 -0.65768E 02 -0.55128E 02 -0.42684E 02 -0.29478E 02 -0.17157E 02 -0.73750E 01 -0.14697E 01

R=.2290

0.28988E 02 0.26033E 02 0.21597E 02 0.16574E 02 0.11383E 02 0.66166E 01 0.28607E 01 0.59852E 00
-0.59481E 02 -0.53998E 02 -0.45422E 02 -0.35231E 02 -0.24377E 02 -0.14228E 02 -0.61607E 01 -0.12885E 01

R=.2210

0.21292E 02 0.19671E 02 0.16440E 02 0.12618E 02 0.86615E 01 0.50365E 01 0.21841E 01 0.46701E-00
-0.50405E 02 -0.46791E 02 -0.39726E 02 -0.30921E 02 -0.21443E 02 -0.12540E 02 -0.54499E 01 -0.11645E 01

R=.2100

0.15763E 02 0.14654E 02 0.12640E 02 0.97364E 01 0.66762E 01 0.38680E 01 0.16602E 01 0.33166E-00
-0.45977E 02 -0.43212E 02 -0.37220E 02 -0.29140E 02 -0.20247E 02 -0.11823E 02 -0.50903E 01 -0.10169E 01

R=.2001

0.12794E 02 0.12185E 02 0.10558E 02 0.81820E 01 0.56004E 01 0.32168E 01 0.13416E 01 0.21329E-00
-0.48627E 02 -0.43075E 02 -0.37490E 02 -0.29495E 02 -0.20487E 02 -0.11878E 02 -0.49729E 01 -0.79124E 00

INTEGRATED VA,VT IN MID-ZONE

0.43029E-02 -0.78018E-03 -0.12828E-02 -0.11450E-02 -0.82964E-03 -0.48787E-03 -0.20401E-03 -0.30532E-04
-0.82402E-02 -0.59352E-03 0.41177E-03 0.54312E-03 0.43664E-03 0.26657E-03 0.11250E-03 0.16235E-04

(Similar print out for Ro = 0.3, 0.4, ..., 0.95)

Figure 17 - Sample Computer Output with Intermediate Information

CHORDS/SE INTEGRATED VALUES OF VA AND VT AT R0=7000

YIP-ZONE

THETASUB0 3

INTERMEDIATE INFORMATION OF PHI,UMH,VHM,UM,UMT,VHL,VMT,UBL,UBT,VBL,VBT AT R 1

0.	0.17454E-02	0.52361E-02	0.10472E-01	0.17454E-01	0.26180E-01	0.36652E-01	0.48870E-01	0.62833E-01
0.76541E-01	0.95994E-01	0.11519E-00	0.12762E-00	0.13883E-00	0.18326E-00	0.26944E-00	0.23737E-00	0.26704E-00
0.29846E-00	0.33162E-00	0.36652E-00	0.40143E-00	0.43634E-00	0.47125E-00	0.50615E-00	0.54106E-00	0.57597E-00
0.61087E-00	0.64578E-00	0.68069E-00	0.71559E-00	0.75050E-00	0.78541E-00	0.82032E-00	0.85522E-00	0.89013E-00
0.92504E-00	0.95994E-00	0.98762E-00						
0.74907E-04	0.67493E-04	0.35233E-04	0.10951E-04	0.32120E-03	0.10872E-03	0.43490E-02	0.20254E-02	0.10782E-02
0.64513E-01	0.42682E-01	0.30716E-01	0.26057E-01	0.19244E-01	0.16315E-01	0.14263E-01	0.12818E-01	0.11725E-01
0.10889E-01	0.10233E-01	0.97094E-00	0.93025E-00	0.89774E-00	0.87109E-00	0.84867E-00	0.82926E-00	0.81186E-00
0.79571E-00	0.78005E-00	0.76424E-00	0.74769E-00	0.72989E-00	0.71039E-00	0.68892E-00	0.66536E-00	0.63985E-00
0.61277E-00	0.58460E-00	0.56255E-00						
-0.34444E-04	-0.31009E-04	-0.16186E-04	-0.50275E-03	-0.14714E-03	-0.49492E-02	-0.19515E-02	-0.88341E-01	-0.44786E-01
-0.24857E-01	-0.14796E-01	-0.92665E-00	-0.71041E-00	-0.39141E-00	-0.25161E-00	-0.15202E-00	-0.77208E-01	-0.18116E-01
0.30867E-01	0.73399E-01	0.11197E-00	0.14671E-00	0.17919E-00	0.21044E-00	0.24110E-00	0.27156E-00	0.30200E-00
0.33240E-00	0.36255E-00	0.39202E-00	0.42015E-00	0.44605E-00	0.46861E-00	0.48651E-00	0.49838E-00	0.50249E-00
0.49897E-00	0.48605E-00	0.46942E-00						
-0.37433E-04	-0.33701E-04	-0.17592E-04	-0.54675E-03	-0.16036E-03	-0.54273E-02	-0.21709E-02	-0.10109E-02	-0.53808E-01
-0.32190E-01	-0.21294E-01	-0.15322E-01	-0.12996E-01					
-0.37433E-04	-0.33702E-04	-0.17594E-04	-0.54685E-03	-0.16041E-03	-0.54298E-02	-0.21723E-02	-0.10118E-02	-0.53869E-01
-0.32236E-01	-0.21330E-01	-0.15353E-01	-0.13026E-01	-0.96225E-00	-0.81598E-00	-0.71453E-00	-0.64138E-00	-0.56687E-00
-0.54514E-00	-0.51247E-00	-0.48641E-00	-0.46617E-00	-0.45002E-00	-0.43680E-00	-0.42569E-00	-0.41608E-00	-0.40749E-00
-0.39950E-00	-0.39176E-00	-0.38194E-00	-0.37574E-00	-0.36691E-00	-0.35722E-00	-0.34653E-00	-0.33478E-00	-0.32205E-00
-0.30852E-00	-0.29452E-00	-0.28339E-00						
0.17199E-04	0.15484E-04	0.80815E-03	0.25101E-03	0.73457E-02	0.24707E-02	0.97410E-01	0.44092E-01	0.22350E-01
0.12403E-01	0.73819E-00	0.46222E-00	0.35432E-00					
0.17199E-04	0.15484E-04	0.80822E-03	0.25106E-03	0.73480E-02	0.24719E-02	0.97475E-01	0.44130E-01	0.22375E-01
0.12420E-01	0.73946E-00	0.46318E-00	0.35513E-00	0.19572E-00	0.12584E-00	0.76048E-01	0.38634E-01	0.90685E-02
-0.15454E-01	-0.36759E-01	-0.56095E-01	-0.73518E-01	-0.89824E-01	-0.10552E-00	-0.12093E-00	-0.13626E-00	-0.15158E-00
-0.16689E-00	-0.18208E-00	-0.19694E-00	-0.21114E-00	-0.22423E-00	-0.23564E-00	-0.24472E-00	-0.25076E-00	-0.25311E-00
-0.25122E-00	-0.24479E-00	-0.23647E-00						
0.37451E-05	-0.94873E-03	-0.14831E-04	-0.91637E-03	-0.44145E-03	-0.21771E-03	-0.11592E-03	-0.66498E-02	-0.40620E-02
-0.24122E-02	-0.17519E-02	-0.12163E-02	-0.98599E-01					
-0.37451E-05	-0.94873E-03	-0.14831E-04	-0.91637E-03	-0.44145E-03	-0.21771E-03	-0.11592E-03	-0.66498E-02	-0.40620E-02
0.24122E-02	0.17519E-02	0.12163E-02	0.98599E-01	0.63559E-01	0.47404E-01	0.35913E-01	0.27541E-01	0.21304E-01
0.14560E-01	0.12878E-01	0.99629E-00	0.77097E-00	0.59095E-00	0.44331E-00	0.31921E-00	0.21270E-00	0.11973E-00
0.37578E-01	-0.35539E-01	-0.10062E-00	-0.15807E-00	-0.20779E-00	-0.24926E-00	-0.28171E-00	-0.30421E-00	-0.31592E-00
-0.31631E-00	-0.30535E-00	-0.28966E-00						
0.	0.44014E-03	0.68805E-03	0.42510E-03	0.20477E-03	0.10097E-03	0.53748E-02	0.30823E-02	0.18526E-02
0.12097E-02	0.81099E-01	0.56293E-01	0.45817E-01					
-0.	-0.44014E-03	-0.68805E-03	-0.42510E-03	-0.20477E-03	-0.10097E-03	-0.53748E-02	-0.30823E-02	-0.18526E-02
-0.12097E-02	-0.81099E-01	-0.56293E-01	-0.45817E-01	-0.29454E-01	-0.22021E-01	-0.16761E-01	-0.12957E-01	-0.10157E-01
-0.80637E-00	-0.64422E-00	-0.52796E-00	-0.44030E-00	-0.37600E-00	-0.32926E-00	-0.29636E-00	-0.27492E-00	-0.26339E-00
-0.26083E-00	-0.26665E-00	-0.28047E-00	-0.30200E-00	-0.33087E-00	-0.36652E-00	-0.40605E-00	-0.45409E-00	-0.50276E-00
-0.53163E-00	-0.59790E-00	-0.63078E-00						

R=7050

0.29173E-02 0.34360E-01 0.12850E-01 0.61892E-00 0.29588E-00 0.98491E-01 -0.31545E-01 -0.10538E-00

-0.13466E-02 -0.15538E-01 -0.54315E-00 -0.22687E-00 -0.84029E-01 -0.10912E-01 0.29587E-01 0.30868E-01

THETASUB0 3

INTERMEDIATE INFORMATION OF PHI,UMH,VHM,UM,UMT,VHL,VMT,UBL,UBT,VBL,VBT AT R 2

0.	0.34907E-02	0.87260E-02	0.15708E-01	0.24435E-01	0.34907E-01	0.47125E-01	0.61067E-01	0.76796E-01
0.94249E-01	0.11345E-00	0.12144E-00	0.15708E-00	0.18152E-00	0.20770E-00	0.23562E-00	0.26529E-00	0.29671E-00
0.32987E-00	0.35478E-00	0.39969E-00	0.43459E-00	0.46950E-00	0.50441E-00	0.53931E-00	0.57422E-00	0.60913E-00
0.64404E-00	0.67894E-00	0.71385E-00	0.74876E-00	0.78366E-00	0.81857E-00	0.85348E-00	0.88838E-00	0.92329E-00
0.93820E-00	0.98147E-00							
0.19153E-04	0.17231E-04	0.10909E-04	0.49377E-03	0.19924E-03	0.83594E-02	0.38475E-02	0.19611E-02	0.11028E-02
0.67913E-01	0.45405E-01	0.39576E-01	0.24954E-01	0.20092E-01	0.16870E-01	0.14640E-01	0.13061E-01	0.11893E-01
0.11011E-01	0.10330E-01	0.95166E-00	0.94177E-00	0.90993E-00	0.88384E-00	0.86187E-00	0.84275E-00	0.82548E-00
0.80922E-00	0.79321E-00	0.77679E-00	0.75936E-00	0.74042E-00	0.71959E-00	0.69669E-00	0.67174E-00	0.64507E-00
0.61723E-00	0.59846E-00							
-0.87132E-03	-0.78383E-03	-0.49619E-03	-0.22429E-03	-0.90231E-02	-0.37588E-02	-0.17046E-02	-0.84555E-01	-0.45447E-01
-0.26125E-01	-0.15839E-01	-0.13170E-01	-0.64398E-00	-0.41707E-00	-0.26386E-00	-0.15497E-00	-0.73590E-01	-0.96606E-02
0.43115E-01	0.88849E-01	0.12847E-00	0.16441E-00	0.19816E-00	0.23069E-00	0.26258E-00	0.29413E-00	0.32544E-00
0.35634E-00	0.38648E-00	0.41523E-00	0.44174E-00	0.46489E-00	0.48341E-00	0.49589E-00	0.50099E-00	0.49760E-00
0.48507E-00	0.47162E-00							
-0.95283E-03	-0.85715E-03	-0.54263E-03	-0.24557E-03	-0.99073E-02	-0.41560E-02	-0.19125E-02	-0.97457E-01	-0.54789E-01
-0.33731E-01	-0.22543E-01	-0.19646E-01						
-0.95283E-03	-0.85725E-03	-0.54279E-03	-0.24570E-03	-0.99158E-02	-0.41611E-02	-0.19156E-02	-0.97665E-01	-0.54936E-01
-0.33842E-01	-0.22632E-01	-0.19730E-01	-0.12448E-01	-0.10027E-01	-0.84231E-00	-0.73172E-00	-0.65277E-00	-0.59472E-00
-0.55092E-00	-0.51717E-00	-0.49176E-00	-0.47207E-00	-0.45638E-00	-0.44357E-00	-0.43280E-00	-0.42345E-00	-0.41603E-00
-0.40710E-00	-0.39929E-00	-0.39126E-00	-0.38271E-00	-0.37338E-00	-0.36310E-00	-0.35175E-00	-0.33936E-00	-0.32608E-00
-0.31221E-00	-0.30282E-00							
0.43306E-03	0.38991E-03	0.24676E-03	0.11155E-03	0.44868E-02	0.18688E-02	0.84730E-01	0.42019E-01	0.22579E-01
0.12975E-01	0.78641E-00	0.65379E-00						
0.43346E-03	0.38996E-03	0.24683E-03	0.11161E-03	0.44906E-02	0.18710E-02	0.84870E-01	0.42109E-01	0.22639E-01
0.13018E-01	0.78953E-00	0.65657E-00	0.32124E-00	0.20814E-00	0.13174E-00	0.77409E-01	0.36779E-01	0.48308E-02
-0.21572E-01	-0.44482E-01	-0.64359E-01	-0.82411E-01	-0.99390E-01	-0.11577E-00	-0.13186E-00	-0.14779E-00	-0.16362E-00
-0.17927E-00	-0.19455E-00	-0.20914E-00	-0.22263E-00	-0.23444E-00	-0.24392E-00	-0.25037E-00	-0.25310E-00	-0.25154E-00
-0.24535E-00	-0.23864E-00							
0.37022E-05	-0.23696E-03	-0.37414E-03	-0.30291E-03	-0.18747E-03	-0.10959E-03	-0.65356E-02	-0.40579E-02	-0.26269E-02
-0.17660E-02	-0.12265E-02	-0.10717E-02						
-0.37022E-05	-0.23696E-03	-0.37414E-03	-0.30281E-03	-0.18747E-03	-0.10959E-03	-0.65356E-02	-0.40579E-02	-0.26269E-02
-0.17660E-02	-0.12265E-02	-0.10717E-02	-0.64013E-01	-0.47700E-01	-0.36108E-01	-0.27670E-01	-0.21392E-01	-0.16622E-01
0.12925E-01	0.10001E-01	0.77429E-00	0.59422E-00	0.44658E-00	0.32255E-00	0.21616E-00	0.12330E-00	0.41211E-01
-0.31902E-01	-0.97001E-01	-0.15475E-00	-0.20484E-00	-0.24684E-00	-0.27998E-00	-0.30330E-00	-0.31594E-00	-0.31728E-00
-0.30724E-00	-0.29451E-00							
0.	0.10993E-03	0.17357E-03	0.14047E-03	0.86949E-02	0.50817E-02	0.30296E-02	0.18803E-02	0.12166E-02
0.81756E-01	0.56770E-01	0.49604E-01						
-0.	-0.10993E-03	-0.17357E-03	-0.14047E-03	-0.86949E-02	-0.50817E-02	-0.30296E-02	-0.18803E-02	-0.12166E-02
-0.81756E-01	-0.56770E-01	-0.49604E-01	-0.29663E-01	-0.22155E-01	-0.16846E-01	-0.13010E-01	-0.10189E-01	-0.80823E-01
-0.84916E-00	-0.52826E-00	-0.44014E-00	-0.37548E-00	-0.32842E-00	-0.29522E-00	-0.27347E-00	-0.26162E-00	-0.25871E-00
-0.26417E-00	-0.27762E-00	-0.29881E-00	-0.32738E-00	-0.36283E-00	-0.40430E-00	-0.45048E-00	-0.49952E-00	-0.54904E-00
-0.59824E-00	-0.62493E-00							

R=7100

0.21869E-02 0.41084E-01 0.14477E-01 0.71692E-00 0.37526E-00 0.16912E-00 0.33854E-01 -0.42853E-01

-0.10027E-02 -0.18523E-01 -0.60918E-00 -0.26517E-00 -0.11612E-00 -0.41025E-01 0.38338E-03 0.22122E-01

(Similar print out for R = 0.715, 0.721, ..., 0.9999)

Figure 17 (Continued)

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HUB-ICME
THETASUBO 3
INTERMEDIATE INFORMATION OF PHI,UPH,VPH,UHL,UNT,VHL,VHT,UOL,UBT,VBL,VBT AT R15
0. 0.17454E-02 0.52361E-02 0.10472E-01 0.17454E-01 0.26180E-01 0.36652E-01 0.48870E-01 0.62833E-01
0.78541E-01 0.95994E-01 0.11519E-00 0.13614E-00 0.13979E-00 0.18326E-00 0.20944E-00 0.23737E-00 0.26704E-00
0.29846E-00 0.33162E-00 0.36652E-00 0.40143E-00 0.43634E-00 0.47125E-00 0.50615E-00 0.54106E-00 0.57597E-00
0.61087E-00 0.64578E-00 0.68069E-00 0.71559E-00 0.75050E-00 0.78541E-00 0.82032E-00 0.85522E-00 0.89013E-00
0.92504E-00 0.95994E-00 0.99485E-00 0.99973E-00
-0.71780E-04 -0.64636E-04 -0.33798E-04 -0.10474E-04 -0.29756E-03 -0.94919E-02 -0.34052E-02 -0.13171E-02 -0.51449E-01
-0.17798E-01 -0.27934E-00 0.41438E-00 0.73744E-00 0.77066E-00 0.93952E-00 0.95256E-00 0.94385E-00 0.92494E-00
0.90186E-00 0.87779E-00 0.85430E-00 0.83312E-00 0.81413E-00 0.79698E-00 0.78127E-00 0.76656E-00 0.75238E-00
0.73827E-00 0.72378E-00 0.70843E-00 0.69180E-00 0.67351E-00 0.65327E-00 0.63092E-00 0.60650E-00 0.58029E-00
0.55282E-00 0.52486E-00 0.49737E-00 0.49362E-00
0.33626E-04 0.30280E-04 0.15835E-04 0.48875E-03 0.13987E-03 0.44751E-02 0.16439E-02 0.66557E-01 0.28953E-01
0.13199E-01 0.61700E-00 0.29362E-00 0.14470E-00 0.12962E-00 0.57704E-01 0.57088E-01 0.67901E-01 0.85090E-01
0.10583E-00 0.12892E-00 0.15383E-00 0.17916E-00 0.20474E-00 0.23127E-00 0.25821E-00 0.28580E-00 0.31346E-00
0.34251E-00 0.37111E-00 0.39926E-00 0.42673E-00 0.45110E-00 0.47275E-00 0.48987E-00 0.50110E-00 0.50517E-00
0.50107E-00 0.48826E-00 0.46685E-00 0.46321E-00
0.36111E-04 0.32518E-04 0.17004E-04 0.52446E-03 0.14972E-03 0.47762E-02 0.17136E-02 0.66288E-01 0.25897E-01
0.89595E-00 0.14064E-00 -0.20867E-00 -0.37142E-00 -0.38816E-00
0.36111E-04 0.32517E-04 0.17002E-04 0.52437E-03 0.14967E-03 0.47741E-02 0.17126E-02 0.66233E-01 0.25869E-01
0.89477E-00 0.14042E-00 -0.20827E-00 -0.37057E-00 -0.38725E-00 -0.47193E-00 -0.47837E-00 -0.47389E-00 -0.46427E-00
-0.45257E-00 -0.44036E-00 -0.42846E-00 -0.41771E-00 -0.40807E-00 -0.39936E-00 -0.39137E-00 -0.38389E-00 -0.37668E-00
-0.36950E-00 -0.36214E-00 -0.35436E-00 -0.34594E-00 -0.33669E-00 -0.32648E-00 -0.31521E-00 -0.30292E-00 -0.28975E-00
-0.27595E-00 -0.26192E-00 -0.24812E-00 -0.24624E-00
-0.16916E-04 -0.15233E-04 -0.79668E-03 -0.24590E-03 -0.70376E-02 -0.22619E-02 -0.82724E-01 -0.33497E-01 -0.14574E-01
-0.66402E-00 -0.31065E-00 -0.14786E-00 -0.72878E-01 -0.65285E-01
-0.16916E-04 -0.15233E-04 -0.79661E-03 -0.24586E-03 -0.70355E-02 -0.22609E-02 -0.82674E-01 -0.33470E-01 -0.14558E-01
-0.66314E-00 -0.31015E-00 -0.14757E-00 -0.72712E-01 -0.65132E-01 -0.28985E-01 -0.28664E-01 -0.34121E-01 -0.42711E-01
-0.53110E-01 -0.64678E-01 -0.77149E-01 -0.89827E-01 -0.10272E-00 -0.11589E-00 -0.12935E-00 -0.14313E-00 -0.15716E-00
-0.17143E-00 -0.18569E-00 -0.19971E-00 -0.21314E-00 -0.22551E-00 -0.23626E-00 -0.24474E-00 -0.25028E-00 -0.25224E-00
-0.25012E-00 -0.24365E-00 -0.23290E-00 -0.23107E-00
0.38287E-05 -0.95016E-03 -0.14931E-04 -0.92667E-03 -0.44753E-03 -0.22091E-03 -0.11767E-03 -0.67519E-02 -0.41247E-02
-0.26526E-02 -0.17791E-02 -0.12352E-02 -0.38239E-01 -0.83638E-01
-0.38287E-05 0.95016E-03 0.14931E-04 0.92667E-03 0.44753E-03 0.22091E-03 0.11767E-03 0.67519E-02 0.41247E-02
0.26526E-02 0.17791E-02 0.12352E-02 0.38239E-01 0.83638E-01 0.48136E-01 0.36466E-01 0.27963E-01 0.21627E-01
0.16808E-01 0.13069E-01 0.10107E-01 0.70177E-00 0.59005E-00 0.44916E-00 0.32322E-00 0.21521E-00 0.12105E-00
0.37952E-01 -0.35846E-01 -0.10136E-00 -0.15899E-00 -0.20864E-00 -0.24981E-00 -0.28173E-00 -0.30354E-00 -0.31466E-00
-0.31406E-00 -0.30242E-00 -0.28037E-00 -0.27654E-00
0. 0.44080E-03 0.69268E-03 0.42988E-03 0.20759E-03 0.10246E-03 0.54562E-02 0.31296E-02 0.19111E-02
0.12285E-02 0.57168E-01 0.40850E-01 0.38725E-01
0. 0.44080E-03 0.69268E-03 0.42988E-03 0.20759E-03 0.10246E-03 0.54562E-02 0.31296E-02 0.19111E-02
-0.12285E-02 -0.82350E-01 -0.57168E-01 -0.40850E-01 -0.38725E-01 -0.22364E-01 -0.17022E-01 -0.13159E-01 -0.10315E-01
-0.81091E-00 -0.65830E-00 -0.53619E-00 -0.44718E-00 -0.38189E-00 -0.33443E-00 -0.30102E-00 -0.27922E-00 -0.26746E-00
-0.26475E-00 -0.27047E-00 -0.28421E-00 -0.30562E-00 -0.33430E-00 -0.36961E-00 -0.41061E-00 -0.45587E-00 -0.50347E-00
-0.55100E-00 -0.59570E-00 -0.63470E-00 -0.63955E-00
R=0.6930
0.75944E-01 0.27328E-01 0.13224E-01 0.77237E-00 0.48859E-00 0.31300E-00 0.19714E-00 0.13113E-00
-0.35272E-01 -0.12513E-01 -0.57860E-00 -0.31229E-00 -0.18301E-00 -0.11581E-00 -0.78770E-01 -0.59363E-01
THETASUBO 3
INTERMEDIATE INFORMATION OF PHI,UPH,VPH,UHL,UNT,VHL,VHT,UOL,UBT,VBL,VBT AT R16
0. 0.34907E-02 0.87268E-02 0.15708E-01 0.24435E-01 0.34907E-01 0.47125E-01 0.61087E-01 0.76796E-01
0.94249E-01 0.11345E-00 0.13439E-00 0.14579E-00 0.18152E-00 0.20770E-00 0.23562E-00 0.26529E-00 0.29671E-00
0.32987E-00 0.36478E-00 0.39969E-00 0.43459E-00 0.46950E-00 0.50441E-00 0.53931E-00 0.57422E-00 0.60913E-00
0.64404E-00 0.67894E-00 0.71385E-00 0.74876E-00 0.78366E-00 0.81857E-00 0.85348E-00 0.88838E-00 0.92329E-00
0.95820E-00 0.99311E-00 0.10057E-01
-0.17538E-04 -0.15796E-04 -0.10026E-04 -0.45169E-03 -0.17825E-03 -0.71252E-02 -0.30096E-02 -0.13337E-02 -0.60167E-01
-0.26082E-01 -0.93665E-00 -0.84473E-01 -0.17869E-00 -0.59551E-00 0.71752E-00 0.77782E-00 0.80383E-00 0.81071E-00
0.80703E-00 0.79766E-00 0.78599E-00 0.77355E-00 0.76101E-00 0.74856E-00 0.73614E-00 0.72355E-00 0.71049E-00
0.69664E-00 0.68162E-00 0.66509E-00 0.64673E-00 0.62630E-00 0.60373E-00 0.57910E-00 0.55274E-00 0.52525E-00
0.49741E-00 0.47022E-00 0.46077E-00
0.82988E-03 0.74749E-03 0.47462E-03 0.21411E-03 0.84799E-02 0.34192E-02 0.14725E-02 0.67969E-01 0.33335E-01
0.17213E-01 0.93149E-00 0.53022E-00 0.40701E-00 0.21526E-00 0.16262E-00 0.14050E-00 0.13609E-00 0.14240E-00
0.13557E-00 0.17352E-00 0.19405E-00 0.21639E-00 0.24018E-00 0.26519E-00 0.29127E-00 0.31822E-00 0.34578E-00
0.37356E-00 0.40103E-00 0.42746E-00 0.45170E-00 0.47323E-00 0.49016E-00 0.50136E-00 0.50557E-00 0.50178E-00
0.48945E-00 0.46866E-00 0.45925E-00
0.88568E-03 0.79775E-03 0.50638E-03 0.22817E-03 0.90057E-02 0.36005E-02 0.15211E-02 0.67426E-01 0.30426E-01
0.13193E-01 0.47395E-00 0.42759E-01 -0.90470E-01
0.88568E-03 0.79765E-03 0.50623E-03 0.22805E-03 0.89981E-02 0.35961E-02 0.15186E-02 0.67283E-01 0.30344E-01
0.13150E-01 0.47208E-00 0.42560E-01 -0.90012E-01 -0.29978E-00 -0.36104E-00 -0.39119E-00 -0.40406E-00 -0.40730E-00
-0.40522E-00 -0.40027E-00 -0.39417E-00 -0.38769E-00 -0.38118E-00 -0.37471E-00 -0.36827E-00 -0.36174E-00 -0.35500E-00
-0.34786E-00 -0.34016E-00 -0.33170E-00 -0.32234E-00 -0.31197E-00 -0.30054E-00 -0.28810E-00 -0.27482E-00 -0.26099E-00
-0.24701E-00 -0.23336E-00 -0.22862E-00
-0.41910E-03 -0.37751E-03 -0.23973E-03 -0.10816E-03 -0.42842E-02 -0.17278E-02 -0.74423E-01 -0.34361E-01 -0.16857E-01
-0.87088E-00 -0.47134E-00 -0.26839E-00 -0.20608E-00
-0.41910E-03 -0.37747E-03 -0.23965E-03 -0.10810E-03 -0.42806E-02 -0.17257E-02 -0.74301E-01 -0.34288E-01 -0.16812E-01
-0.86783E-00 -0.46948E-00 -0.26714E-00 -0.20503E-00 -0.10836E-00 -0.81829E-01 -0.70664E-01 -0.68411E-01 -0.71542E-01
-0.78115E-01 -0.87070E-01 -0.97313E-01 -0.10845E-00 -0.12030E-00 -0.13275E-00 -0.14571E-00 -0.15909E-00 -0.17277E-00
-0.18654E-00 -0.20013E-00 -0.21319E-00 -0.22524E-00 -0.23572E-00 -0.24401E-00 -0.24943E-00 -0.25137E-00 -0.24933E-00
-0.24305E-00 -0.23259E-00 -0.22786E-00
0.38725E-05 -0.23764E-03 -0.37799E-03 -0.10855E-03 -0.19209E-03 -0.11263E-03 -0.67269E-02 -0.41800E-02 -0.27072E-02
-0.18204E-02 -0.12645E-02 -0.90301E-01 -0.76714E-01
-0.38725E-05 0.23764E-03 0.37799E-03 0.30855E-03 0.19209E-03 0.11263E-03 0.67269E-02 0.41800E-02 0.27072E-02
0.18204E-02 0.12645E-02 0.90301E-01 0.76714E-01 0.49179E-01 0.37224E-01 0.28521E-01 0.22045E-01 0.17124E-01
0.13310E-01 0.10293E-01 0.79636E-00 0.61063E-00 0.45845E-00 0.33072E-00 0.22131E-00 0.12602E-00 0.42038E-01
-0.32467E-01 -0.98536E-01 -0.15860E-00 -0.20656E-00 -0.24797E-00 -0.28007E-00 -0.30203E-00 -0.31311E-00 -0.31288E-00
-0.30146E-00 -0.27968E-00 -0.26957E-00
0. 0.11025E-03 0.17535E-03 0.14313E-03 0.89093E-02 0.52225E-02 0.31183E-02 0.19369E-02 0.12539E-02
0.84279E-01 0.58532E-01 0.41808E-01 0.35531E-01
0. 0.11025E-03 0.17535E-03 0.14313E-03 0.89093E-02 0.52225E-02 0.31183E-02 0.19369E-02 0.12539E-02
-0.84279E-01 -0.58532E-01 -0.41808E-01 -0.35531E-01 -0.22847E-01 -0.17372E-01 -0.13417E-01 -0.10508E-01 -0.83353E-02
-0.66950E-00 -0.54484E-00 -0.45399E-00 -0.38733E-00 -0.33881E-00 -0.30457E-00 -0.28209E-00 -0.26976E-00 -0.26655E-00
-0.27181E-00 -0.28510E-00 -0.30606E-00 -0.33425E-00 -0.36905E-00 -0.40948E-00 -0.45412E-00 -0.50107E-00 -0.54775E-00
-0.59202E-00 -0.63048E-00 -0.64246E-00
R=0.6900
0.50337E-01 0.23473E-01 0.11840E-01 0.68056E-00 0.41196E-00 0.24396E-00 0.13277E-00 0.69373E-01
-0.23239E-01 -0.10780E-01 -0.52308E-00 -0.27679E-00 -0.15231E-00 -0.86512E-01 -0.50042E-01 -0.30903E-01

```

(Similar print out for R = 0.685, 0.680, ..., 0.2001)

Figure 17 (Continued)

```

TASUBO 3
ERREDIATE INFORMATION OF PH1,UPH,VHM,UML,UMT,VML,VMT,URL,UBT,VBL,VBT IN MID ZONE
17454E-02 0.52361E-02 0.10472E-01 0.17454E-01 0.26180E-01 0.36652E-01 0.48870E-01 0.62833E-01 0.78571E-01
95994E-01 0.11519E-00 0.13374E-00 0.15883E-00 0.18326E-00 0.20944E-00 0.23737E-00 0.26704E-00 0.29846E-00
33162E-00 0.36652E-00 0.40143E-00 0.43634E-00 0.47125E-00 0.50615E-00 0.54106E-00 0.57597E-00 0.61087E-00
54578E-00 0.68069E-00 0.71559E-00 0.75050E-00 0.78541E-00 0.82032E-00 0.85522E-00 0.89013E-00 0.92504E-00
95994E-00 0.99370E-00
56907E-00 0.20416E-00 0.11276E-00 0.73859E-01 0.51331E-01 0.37340E-01 0.28250E-01 0.22079E-01 0.17711E-01
14513E-01 0.12106E-01 0.10435E-01 0.87917E-02 0.76229E-02 0.66729E-02 0.58899E-02 0.52377E-02 0.46183E-02
42214E-02 0.38214E-02 0.34910E-02 0.32137E-02 0.29775E-02 0.27740E-02 0.25969E-02 0.24414E-02 0.23037E-02
21810E-02 0.20710E-02 0.19718E-02 0.18819E-02 0.18001E-02 0.17253E-02 0.16567E-02 0.15935E-02 0.15352E-02
14812E-02 0.14326E-02
68503E-02 -0.31023E-01 -0.38242E-01 -0.30575E-01 -0.22646E-01 -0.16883E-01 -0.12915E-01 -0.10148E-01 -0.81626E-02
66975E-02 -0.55895E-02 -0.48172E-02 -0.40560E-02 -0.35130E-02 -0.30706E-02 -0.27051E-02 -0.23998E-02 -0.21418E-02
19218E-02 -0.17324E-02 -0.15756E-02 -0.14432E-02 -0.13298E-02 -0.12316E-02 -0.11455E-02 -0.10695E-02 -0.10017E-02
94091E-03 -0.88593E-03 -0.83595E-03 -0.79027E-03 -0.74830E-03 -0.70958E-03 -0.67369E-03 -0.64030E-03 -0.60913E-03
57993E-03 -0.55336E-03
28521E-00 -0.10232E-00 -0.56514E-01 -0.37017E-01 -0.25726E-01 -0.18714E-01 -0.14158E-01 -0.11065E-01 -0.88762E-02
72734E-02 -0.60672E-02 -0.52296E-02
28521E-00 -0.10232E-00 -0.56515E-01 -0.37018E-01 -0.25727E-01 -0.18714E-01 -0.14159E-01 -0.11066E-01 -0.88767E-02
72739E-02 -0.60678E-02 -0.52301E-02 -0.44066E-02 -0.38208E-02 -0.33446E-02 -0.29522E-02 -0.26253E-02 -0.23500E-02
21160E-02 -0.19155E-02 -0.17499E-02 -0.16109E-02 -0.14925E-02 -0.13906E-02 -0.13018E-02 -0.12238E-02 -0.11549E-02
10934E-02 -0.10382E-02 -0.98851E-03 -0.94346E-03 -0.90246E-03 -0.86498E-03 -0.83059E-03 -0.79893E-03 -0.76970E-03
74262E-03 -0.71827E-03
34333E-02 0.15548E-01 0.15324E-01 0.11350E-01 0.84615E-02 0.64726E-02 0.50859E-02 0.40909E-02
33566E-02 0.28013E-02 0.24142E-02
34333E-02 0.15548E-01 0.19070E-01 0.15324E-01 0.11350E-01 0.84618E-02 0.64728E-02 0.50862E-02 0.40911E-02
33569E-02 0.28015E-02 0.24145E-02 0.20329E-02 0.17608E-02 0.15391E-02 0.13559E-02 0.12029E-02 0.10736E-02
96332E-03 0.86847E-03 0.78979E-03 0.72341E-03 0.66659E-03 0.61736E-03 0.57425E-03 0.53614E-03 0.50218E-03
47169E-03 0.44413E-03 0.41908E-03 0.39618E-03 0.37515E-03 0.35574E-03 0.33775E-03 0.32102E-03 0.30540E-03
29076E-03 0.27744E-03
14034E-03 -0.37680E-02 -0.12745E-02 -0.50582E-01 -0.23277E-01 -0.12047E-01 -0.68216E-00 -0.41404E-00 -0.26545E-00
17786E-00 -0.12357E-00 -0.91688E-01
14034E-03 0.37680E-02 0.12745E-02 0.50582E-01 0.23277E-01 0.12047E-01 0.68216E-00 0.41404E-00 0.26545E-00
17786E-00 0.12357E-00 0.91688E-01 0.64999E-01 0.48806E-01 0.37348E-01 0.29057E-01 0.22938E-01 0.18343E-01
14838E-01 0.12128E-01 0.10093E-01 0.85264E-02 0.72948E-02 0.63091E-02 0.55078E-02 0.48478E-02 0.42975E-02
38340E-02 0.34399E-02 0.31020E-02 0.28100E-02 0.25561E-02 0.23339E-02 0.21382E-02 0.19651E-02 0.18111E-02
16726E-02 0.15540E-02
65108E-02 0.17480E-02 0.59124E-01 0.23462E-01 0.10795E-01 0.55851E-00 0.31608E-00 0.19170E-00 0.12276E-00
82131E-01 0.56943E-01 0.42153E-01
65108E-02 -0.17480E-02 -0.59124E-01 -0.23462E-01 -0.10795E-01 -0.55851E-00 -0.31608E-00 -0.19170E-00 -0.12276E-00
82131E-01 -0.56943E-01 -0.42153E-01 -0.29771E-01 -0.22259E-01 -0.16943E-01 -0.13097E-01 -0.10258E-01 -0.81261E-02
65002E-02 -0.52426E-02 -0.42984E-02 -0.35716E-02 -0.30002E-02 -0.25428E-02 -0.21709E-02 -0.18646E-02 -0.16092E-02
13940E-02 -0.12111E-02 -0.10542E-02 -0.91859E-03 -0.80064E-03 -0.69738E-03 -0.60645E-03 -0.52597E-03 -0.45437E-03
39040E-03 -0.33479E-03
EGRATED VA,VT IN MID-ZONE
10796E-01 0.23058E-02 0.94946E-03 0.49127E-03 0.27033E-03 0.13979E-03 0.56447E-04 0.95776E-05
49694E-02 -0.10363E-02 -0.41224E-03 -0.20608E-03 -0.11042E-03 -0.56245E-04 -0.22633E-04 -0.39378E-05

```

Figure 17 (Continued)

APPENDIX D

FORTRAN LISTING OF COMPUTER PROGRAM

The FORTRAN listing of the computer program is included in the following pages. The computer also uses the subroutine MATINV and the function SIMPUN.

```

PROPELLER DESIGN BASED ON LIFTING SURFACE THEORY WITH UNIFORM
CHORDWISE LOAD DISTRIBUTION
XPLU(16-22-64)
DIMENSION DK(20),XL(20),R(9,9),CO(9),AH(10,9),CD(10,9),FAH(20,20)
1,EAH(20,18),
XDB(9,16),DC(9),HA(9),BP( 3,9),DE(9),BV(16),DWSH(9,20),
X X(20,20),ZZ(20,18),TIT(12),CM(20,18),CHR(20,18)
X
DIMENSION XY(50),YX(50),BB(16,50),SY(90),
SAG(50),
JAGS(50),AB(9,9),
DWSHT(9,20),
XAGST(50),BO(16 ),
VRE(20), BPP(20)
DIMENSION H(25),T(25),SPR(20),A(25,90),NF(25),AS(25,90),AC(25,90),
XSYS(90),SYC(90),UHH(90),VHH(90),UHL(90),UHT(90),VHL(90),VHT(90),
XUHL(90),UBT(90),VBL(90),VBT(90)
DIMENSION WT(20),OT(20),Z(5,90),P(9),VP(90)
DIMENSION OSA(10,35),DSAT(10,35),R(5,90),YS(90)
READ 3,NL
READ 7,DY,DFD,DFDO,DFFO
READ 4,(SPR(J),J=1,NL)
CALCULATE ANGLE FE, SINE(FE),AND COSINE(FE) TABLES
DF=DFD/57.295
DFD=DFDO/57.295
DFF=DFFO/57.295
K=1
A(K,1)=0.
AS(K,1)=0.
AC(K,1)=1.
1=2
173 FD=FLOATF(1-1)*DF
IF(FD=DFD)171,172,172
172 FD=DFD
171 A(K,1)=A(K,1-1)+FD
SS=SINF(FD)
CC=COSF(FD)
AC(K,1)=CC*AC(K,1-1)-SS*AS(K,1-1)
AS(K,1)=CC*AS(K,1-1)+SS*AC(K,1-1)
1=1+1
IF(A(K,1-1)-DFF)173,174,174
174 NF(1)=1
NF=N*NF(1)
DO 170 K=2,NL
DO 176 J=K,NF*N
N=J-K+1
A(K,N)=A(1,J)-A(1,K)
AS(K,N)=AS(1,J)+AC(1,K)-AC(1,J)*AS(1,K)
AC(K,N)=AC(1,J)*AC(1,K)+AS(1,J)*AS(1,K)
176 CONTINUE
NF(K)=N
170 CONTINUE
101 READ 2,TIT
READ 3, NN, NKN,NCL,NCTR,KRN, NPT,KSP
READ 4,(DK(J),J=1,NKN)
READ 4,(XL(J),J=1,NCL)
READ 4,(B(K,J),J=1,9),K=1,9)
READ 4,VM,AJ,DIA,SM,SW
1 FORMAT (1H012A6)
2 FORMAT (12A6)
3 FORMAT (91B)
4 FORMAT (9F8.5)
7 FORMAT(8F9.5)

```

```

2000 FORMAT(9E13.5)
PRINT 2999
2999 FORMAT (1H1.33X.44H APPLIED MATHEMATICS LABORATORY PROBLEM XPLU)
PRINT 3000
3000 FORMAT(30X.50H PROPELLER DESIGN BASED ON LIFTING SURFACE THEORY )
PRINT 3002
3002 FORMAT(36X.38H UNIFORM CHORDWISE LOAD DISTRIBUTION )
PRINT 1,TIT
PRINT 3001
3001 FORMAT(11H0INPUT DATA)
PRINT 3,NL
PRINT 7,DY,DFD,DFDO,DFFO
PRINT 4,(SPR(J),J=1,NL)
PRINT 3,NN, NKN,NCL,NCTR,KRN, NPT,KSP
PRINT 4,(DK(J),J=1,NKN)
PRINT 4,(XL(J),J=1,NCL)
PRINT 4,(B(K,J),J=1,9),K=1,9)
PRINT 4,VM,AJ,DIA,SM,SW
DO 2800 J=1,9
2800 B(4,J)=B(4,J)*3.1416*B(1,J)
PRINT 2998
2998 FORMAT(7H0OUTPUT)
C CALCULATE COEFFICIENTS OF DATA INTERPOLATING MATRIX
1061 DO 5J=1,9
AB(2,J)=B(2,J)*VM
AB(3,J)=B(3,J)*VM
AB(5,J)=B(5,J)*6.2832
CO(J)=3.1416*B(1,J)/SORTF((3.1416*B(1,J))*2+B(4,J)*2)
AH(2,J)=(1+B(1,1)-2*B(1,J))/(1-B(1,1))
CD(2,J)=SORTF(1-AH(2,J)*2)
AH(1,J)=1.
5 CD(1,J)=0.
AH(2,1)=1.
AH(2,9)=-1.
CD(2,1)=0.
CD(2,9)=0.
DO 6 J=3,10
L=J-1
DO 6K=1,9
AH(J,K)=AH(L,K)*AH(2,K)-CD(L,K)*CD(2,K)
6 CD(J,K)=CD(L,K)*AH(2,K)+AH(L,K)*CD(2,K)
106 DO 9J=1,9
DO 11K=1,9
11 FAH(J,K)=AH(K,J)
CAH(J,1)=AB(2,J)
EAH(J,2)=AB(3,J)
EAH(J,3)=B(4,J)
FT=CO(J)*2./B(1,J)
EAH(J,4)=AB(5,J)
EAH(J,5)=CO(J)
EAH(J,6)=AB(2,J)*FT
EAH(J,7)=AB(3,J)*FT
EAH(J,8)=AB(5,J)/EAH(J,7)
9 CONTINUE
ERASE ZZ
XH=B(1,1)
CALL MATINV(FAH,9,EAH,8,V,10)
GOTO (1000,184),10

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184 PRINT 1002
GOTO 84
1000 DO 15J=1,9
DO 15K=1,8
15 DB(J,K)=EAH(J,K)
ERASE DWSH,DWSHT
KR=1
20 RP=8(6,KR)
IF(NPT)200,200,2997
2997 PRINT 2996,RP
2996 FORMAT(48HCHORDWISE INTEGRATED VALUES OF VA AND VT AT RO=FS.4)
PRINT 2993
2993 FORMAT(9H-TIP-ZONE)
C SET UP RADIAL STATIONS OF FIELD POINTS FOR POINT P
200 J=1
T(1)=RP+DY
201 J=J+1
T(J)=T(1)+SPR(J-1)
IF(T(J)-.9999)201,202,202
202 T(J)=.9999
LT=J
J=1
H(1)=RP-DY
203 J=J+1
H(J)=H(1)-SPR(J-1)
IF(H(J)-XH-.0001)204,204,203
204 H(J)=XH+.0001
LH=J
LTH=LT+LH
LTH1=L+LTH
DO 205 J=1,LT
205 XY(J)=T(J)
DO 206 J=1,LH
JJ=J+LT
XY(JJ)=H(J)
206 XY(LTH1)=RP
C INTERPOLATE INPUT DATA AT RADIAL STATIONS OF FIELD POINTS
427 ERASE IU
DO 405 L=1,LTH1
X=XY(L)
21 CY=(1.+XH-2.*X)/(1.-XH)
SZ=SORTF(1.-CY*CY)
HA(1)=1.
HA(2)=CY
DC(1)=0.
DC(2)=52
DO 17J=3,9
17J=1
HA(J)=HA(1)+HA(2)-DC(1)*DC(2)
17 DC(J)=DC(1)+HA(2)+HA(1)*DC(2)
DO 19 J=1,9
19 DE(J)=FLOATF(J-1)*DC(J)
DO 22J=1,9
DO 22K=1,8
22 DB(J,K+8)=DB(J,K)+DE(J)
23 DO 25 K=1,8
DO 25J=1,9
88(K+8,L)=88(K+8,L)+DB(J,K+8)*2./((1.-XH)*DC(2))

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25 88(K,L)=88(K,L)+DB(J,K)*HA(J)
405 CONTINUE
DO 1405 K=1,3
1405 BP(K,KR)=88(K,LTH1)
ERASE OSA,DSAT
C SET UP CHORDWISE STATIONS OF FIELD POINTS
NM=1
N=1
7601 L=N
760 Y=XY(N)
ALT=ABSF(88(6,LTH1)-88(6,N)+88(7,N))
ATL=ABSF(88(6,N)-88(6,LTH1)+88(7,LTH1))
ACT=ALT
IF(ALT-ATL) 700,701,701
700 ACT=ATL
701 DO 411 J=1,16
411 BV(J)=88(J,N)
ERASE Z
M7=1
M=1
231 FMP=6.2832*FLOATF(M-1)/FLOATF(NN)
FMPS=SINF(FMP)
FMPC=COSF(FMP)
J=1
702 SX=AS(L,J)*FMPC+AC(L,J)*FMPS
CX=AC(L,J)*FMPC-AS(L,J)*FMPS
S=AS(L,J)
YRPS=ABSF(Y+Y+RP-2.*Y+RP+CX*(BV(3)+S/3.1416))*2)
VP(J)=YRPS*.1,5
Z(1,J)=Z(1,J)-SX*RP/VP(J)
Z(5,J)=Z(5,J)+S*CX/VP(J)
Z(2,J)=Z(2,J)+S*(RP-Y*CX)/VP(J)
Z(3,J)=Z(3,J)+(RP-Y*(CX+S*CX))/VP(J)
Z(4,J)=Z(4,J)+Y*(Y-RP*CX)/VP(J)
GO TO (704,705,706).M7
704 J=J+1
710 IF(A(L,J)-ACT)702,702,710
710 M=M+1
IF(M-NN) 231,231,730
730 DO 719 K=1,5
719 P(K)=Z(K,1)
P(6)=AS(L,1)
P(7)=AC(L,1)
P(8)=A(L,1)
P(9)=VP(1)
KS=1
750 FP=88(6,LTH1)-DK(KS)*88(7,LTH1)
YL=88(6,N)-FP
YT=FP+88(7,N)-88(6,N)
SL=ABSF(YL)
ST=ABSF(YT)
SL1=SL
SL2=ST
IF(SL-ST) 520,520,521
521 SL1=ST
SL2=SL
520 J=1
523 DO 703 K=1,5

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703 R(K,J)=Z(K,J)
SY(J)=A(L,J)
J=J+1
IF(A(L,J)-SL1) 523,523,524
524 JS=J-1
SL1=SL1-A(L,JS1)
IF(SL1A-.0005)1714,524,524
1714 SL1=SL1+.0005
524 M7=2
J1=J
DO 708 K=1,5
708 Z(K,1)=0.
AS(L,1)=SINF(SL1)
AC(L,1)=COSF(SL1)
A(L,1)=SL1
SY(J1)=SL1
M=1
GO TO 231
705 M=M+1
IF(M>NN)231,231,712
712 DO 713 K=1,5
713 R(K,J1)=Z(K,1)
JK=J1
IF(SL-S1)525,526,525
525 J=J1+1
5251 IF(A(L,J)-SL2)527,527,5281
5281 JS2=J-1
SL2=SL2-A(L,JS2)
IF(SL2A-.0005)1716,528,528
1716 SL2=SL2+.0005
GO TO 528
527 DO 714 K=1,5
714 R(K,J2)=Z(K,J)
SY(J2)=A(L,J)
J=J2+1
GO TO 5251
528 M7=3
J2=J
DO 715 K=1,5
715 Z(K,1)=0.
SY(J2)=SL2
AS(L,1)=SINF(SL2)
AC(L,1)=COSF(SL2)
A(L,1)=SL2
M=1
JK=J2
GO TO 231
706 M=M+1
IF(M>NN)231,231,720
720 DO 721 K=1,5
721 R(K,J2)=Z(K,1)
526 J2=JK
DO 1721 K=1,5
1721 Z(K,1)=P(K)
VP(1)=P(9)
AS(L,1)=P(6)
AC(L,1)=P(7)
A(L,1)=P(8)

C CALCULATE INDUCED VELOCITY COMPONENTS IN TIP AND HUB ZONES
ERASE UHL,VHL,UMT,VHT,UBL,VBL,UBT,VBT,UHH,VHH
JS=1
SLM=1.
STM=1.
IF(YL)722,724,723
722 DO 7221 J=1,J1
7221 UHH(J)=BV(12)*R(4,J)
7221 VHH(J)=BV(12)*R(3,J)*BV(3)/3.1416
JS=J1
724 SLM=0.
723 TT=1.
IF(YT)725,790,727
725 DO 7251 J=1,J1
7251 UHH(J)=BV(12)*R(4,J)
7251 VHH(J)=BV(12)*R(3,J)*BV(3)/3.1416
JS=J1
TT=-1.
790 STM=0.
727 CONTINUE
KL=J2
KT=J1
IF(SL-ST)533,534,534
533 KT=J2
KL=J1
534 CONTINUE
DO 535 J=JS,J2
IF(J-KT)536,536,537
536 UHH(J)=BV(12)*R(4,J)
VHH(J)=BV(12)*R(3,J)*BV(3)/3.1416
537 S=SY(J)
SFL=(BV(4)*BV(14)+(YL-S)*(BV(12)-6V(4)/BV(15)))/BV(7)
SFT=(BV(4)*BV(14)+(YL+S)*(BV(12)-8V(4)/BV(15)))/BV(7)
UHR(4,J)
UHL(J)=SLM*SFL*UH
UMT(J)=STM*SFT*UH
VHR(3,J)=R(3,J)/3.1416
VHL(J)=SLM*SFL*VH
VHT(J)=STM*SFT*VH
VB=(BV(3)*R(5,J)+R(2,J)*BV(11))*BV(8)/(3.1416)
VBL(J)=SLM*VB
VBT(J)=STM*VB
UB=BV(8)*R(1,J)
UBL(J)=SLM*UB
UBT(J)=STM*UB
535 CONTINUE
4103 IF(KR-NPT)711,2005,711
2005 IF(KS-KSP)711,2004,711
2004 PRINT 4104,KS
4104 FORMAT(11H THETASUBO ,12)
PRINT 4102,N
4102 FORMAT(77H INTERMEDIATE INFORMATION OF PHI,UHH,VHH,UMT,VHL,VHT
X,UBL,UBT,VBL,VBT AT R,12)
PRINT 2000,(SY(JP),JP=1,J2)
PRINT 2000,(UHH(JP),JP=1,KT)
PRINT 2000,(VHH(JP),JP=1,KT)
PRINT 2000,(UHL(JP),JP=1,KL)
PRINT 2000,(VHT(JP),JP=1,KL)

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PRINT 2000.(VHL(JP).JP=1,KL)
PRINT 2000.(VHT(JP).JP=1,KT)
PRINT 2000.(UBL(JP).JP=1,KL)
PRINT 2000.(UBT(JP).JP=1,KT)
PRINT 2000.(VDL(JP).JP=1,KL)
PRINT 2000.(VBT(JP).JP=1,KT)
711 CONTINUE
C CHORDWISE INTEGRATION AT R
IF(SLM)712,711,7112
711 DO 7113 J=1,J2
UHL(J)=UHL(J)+UHT(J)
7113 VHL(J)=VHL(J)+VHT(J)
GO TO 7115
7112 IF(SLM)7115,7114,7115
7114 JJ=J1-1
DO 7116 J=1,JJ
UHL(J)=UHL(J)+UHT(J)
7116 VHL(J)=VHL(J)+VHT(J)
7115 DSA(KS,N)=SM*(SIMPUN(SY,UHL,KT)+STM+SIMPUN(SY,UHL,KL)
X+SIMPUN(SY,UHT,KT)+SLM)+SM*(SIMPUN(SY,UBL,KT)+SIMPUN(SY,UBT,KT))
DSAT(KS,N)=SM*(SIMPUN(SY,VHL,KT)+STM+SIMPUN(SY,VHL,KL)
X+SIMPUN(SY,VHT,KT)+SLM)+SM*(SIMPUN(SY,VBL,KT)+SIMPUN(SY,VBT,KT))
KS=KS+1
IF(KS=NKN)758,758,7591
7591 CONTINUE
IF(NPT)759,759,2994
2994 PRINT 2999,Y
2999 FORMAT(3H R=F5.4)
PRINT 2000.(DSA(KS,N),KS=1,NKN)
PRINT 2000.(DSAT(KS,N),KS=1,NKN)
759 MN=M+1
GO TO (770,771),NM
770 IF(N-LT)7601,7601,773
771 IF(N-LTH)772,772,761
773 NM=M+2
IF(NPT)772,772,2990
2990 PRINT 2991
2991 FORMAT(9H HUB-ZONE)
772 L=N-LY
GO TO 760
761 CONTINUE
DO 5392 KS=1,NKN
DO 539 J=1,LH
K=LT+J
YX(J)=XY(K)
AGS(J)=DSA(KS,K)
AGST(J)=DSAT(KS,K)
539 CONTINUE
DO 5391 J=1,LT
SAG(J)=DSA(KS,J)
SAGT(J)=DSAT(KS,J)
5391 DWSH(K,KS)=SIMPUN(XY,SAG,LT)-SIMPUN(XY,AGS,LH)
DWSHT(KR,KS)=SIMPUN(XY,SAGT,LT)-SIMPUN(XY,AGST,LH)
DWSH(KR,KS)=DWSH(KR,KS)/12.5664
DWSHT(KR,KS)=DWSHT(KR,KS)/12.5664
5392 CONTINUE
C CALCULATE INDUCED VELOCITY COMPONENTS IN MID ZONE OF THE FIRST
C BLADE

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ERASE Z,R,SYC,SY,S,Y,S
ERASE UHL,VHL,UHT,VHT,UBL,VBL,UBT,VBT,UHL,VHL,VHT,VHT
WT=0.
QT=0.
DO 811 K=1,16
811 BQ(K)=UB(K,LYH1)
KS=1
4501 FP=BQ(6)-DK(KS)*BQ(7)
YL=BQ(6)-FP
YT=FP+BQ(7)-BQ(6)
R1=RP-DY
R2=RP+DY
M8=1
J=2
731 S=A(1,J)
SY(J)=S
CX=AC(1,J)
SX=AS(1,J)
SYS(J)=SX
SYC(J)=CX
733 PP=(BQ(3)*S/3.1416)**2
ZP=(RP+SX)*2+PP
SOL=SORTF(ABSF(RP*RP+R1*R1-2.*RP*R1*CX+PP))
S02=SORTF(ABSF(RP*RP+R2*R2-2.*RP*R2*CX+PP))
Z(4,J)=((R2-RP*CX)/S02-(R1-RP*CX)/S01)/ZP
Z(1,J)=LOGF(ABSF((R2-RP*CX+S02)/(R1-RP*CX+S01)))-R2/S02+R1/S01
Z(2,J)=((RP*(SX+SX-SX)*S01)/ZP
X*PP)*((1/S02-1/S01)/ZP
Z(3,J)=BQ(3)*CX*Z(4,J)+
X)*((1/S02-1/S01)-RP*CX*(R2/S02-R1/S01)/ZP))
GO TO (735,736,750),M8
735 J=J+1
IF(A(1,J)-BQ(7)) 731,732,732
732 SY(J)=BQ(7)
SL1=YL
SL2=YT
IF(YL-YT)741,742,742
742 SL1=YT
SL2=YL
741 J=2
1741 IF(SY(J)-SL1)744,745,745
744 DO 743 K=1,4
743 R(K,J)=Z(K,J)
Z(5,J)=SY(J)
J=J+1
GO TO 1741
745 J1=J
DO 1745 K=1,4
1745 P(K)=Z(K,J1)
M8=2
Z(5,J1)=SL1
S=SL1
CX=COSF(S)
SX=SINF(S)
SYC(J1)=SX
SYC(J1)=CX
GO TO 733
736 DO 746 K=1,4

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R(K,J)=Z(K,J)
746 Z(K,J)=P(K)
IF (YL-YT) 1748,1750,1748
1748 J=J+1
748 IF (SY(J)-SL2) 748,749,749
748 DO 747 K=1,4
747 R(K,J)=Z(K,J)
Z(S,J)=SY(J)
J=J+1
GO TO 7481
749 J2=J
DO 1749 K=1,4
1749 P(K)=Z(K,J2)
M8=3
Z(S,J2)=SL2
S=SL2
CX=COSF(S)
SX=SINF(S)
SYS(J2)=SX
SYC(J2)=CX
GO TO 733
750 DO 751 K=1,4
R(K,J2)=Z(K,J2)
751 Z(K,J2)=P(K)
1750 J2=J
KL=MJ1
KT=MJ2
IF (YL-YT) 752,752,753
753 KL=MJ2
KT=MJ1
752 CONTINUE
DO 1112 J=2,J2
1112 YS(J)=Z(S,J)
IF (YL) 754,754,793
793 DO 755 J=2,KL
X1=R(1,J)
X2=R(2,J)
X3=R(3,J)
X4=R(4,J)
S=Z(S,J)
SX=SYS(J)
CX=SYC(J)
SFL=
BO(8)*BO(14)+(YL-S)*(BO(12)-BO(8)*BO(15))/BO(17)
UHL(J)=X1*SFL
VHL(J)=X2*BO(3)*SFL/3.1416
755 UHL(J)=X3*BO(8)*S/3.1416
754 IF (YT) 765,765,756
756 DO 757 J=2,KT
X1=R(1,J)
X2=R(2,J)
X3=R(3,J)
X4=R(4,J)
S=Z(S,J)
SX=SYS(J)
CX=SYC(J)
BO(8)*BO(14)+(YL+S)*(BO(12)-BO(8)*BO(15))/BO(17)
SFT=
UHL(J)=X1*BO(12)

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VHL(J)=X2*BO(12) *BO(3)/3.1416
UHT(J)=X1*SFT
VHT(J)=X2*SFT*BO(3)/3.1416
VBT(J)=X3*S*BO(8)/3.1416
757 UBT(J)=X4*RP*S*BO(8)
765 CONTINUE
4684 IF (KR-NPT) 4682,4683,4682
4683 IF (KS-XSP) 4682,4681,4682
4681 PRINT 4104,KS
PRINT 25000
25000 FORMAT(9H INTERMEDIATE INFORMATION OF PH1,UHM,VHM,UHL,UHT,VHL,VHT
X,UHL,UHT,VBL,VBT IN MID ZONE
PRINT 2000,{YS(JP),JP=2,J2)
PRINT 2000,(UHM(JP),JP=2,KT)
PRINT 2000,(VHM(JP),JP=2,KT)
PRINT 2000,(UHL(JP),JP=2,KL)
PRINT 2000,(UHT(JP),JP=2,KT)
PRINT 2000,(VHL(JP),JP=2,KL)
PRINT 2000,(VHT(JP),JP=2,KT)
PRINT 2000,(UBL(JP),JP=2,KL)
PRINT 2000,(UBT(JP),JP=2,KT)
PRINT 2000,(VBL(JP),JP=2,KL)
PRINT 2000,(VBT(JP),JP=2,KT)
4682 CONTINUE
WT(KS)=(SIMPUN(YS,UHM,KT) +SIMPUN(YS,UHL,KL)+SIMPUN(YS,UHT,KT))*
XSM +(SIMPUN(YS,UBL,KL)+SIMPUN(YS,UBT,KT))*SW
QT(KS) =(SIMPUN(YS,VHM,KT) +SIMPUN(YS,VHL,KL)+SIMPUN(YS,VHT,KT))
XSM +(SIMPUN(YS,VBL,KL)+SIMPUN(YS,VBT,KT))*SW
WT(KS)=WT(KS)/12.5664
QT(KS)=QT(KS)/12.5664
4502 KS=KS+1
IF (KS-NKN) 4501,4501,450
450 CONTINUE
IF (NPT) 467,467,4671
4671 PRINT 4693
4693 FORMAT(29H INTEGRATED VA,VT IN MID-ZONE)
PRINT 2000,(WT(KS),KS=1,NKN)
PRINT 2000,(QT(KS),KS=1,NKN)
467 CONTINUE
DO 5393 KS=1,NKN
DWSH (KR,KS)= DWSH (KR,KS)+WT(KS)
DWSHT(KR,KS)= DWSHT(KR,KS)+QT(KS)
5393 CONTINUE
KR=KR+1
IF (KR-NRN) 20,20,57
57 DO 60 J=1,NKN
XX(J,1)=1
XX(J,2)=DK(J)
DO 60 K=3,NKN
60 XX(J,K)=XX(J,K-1)*XX(J,2)
IF (SM) 2017,2018,2017
2018 PRINT 2019
2019 FORMAT(30HODUE TO BOUND CIRCULATION ONLY)
GO TO 12017
2017 IF (SW) 12017,12018,12017
12018 PRINT 12019
12019 FORMAT(29HODUE TO FREE CIRCULATION ONLY )
12017 PRINT 2020

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200 FORMAT(33HAXIAL INDUCED VELOCITY COMPONENT)
PRINT 262,(B(6,J),J=1,KRN)
PRINT 263
DO 2021 J=1,NKN
2021 PRINT 264,DK(J),(DWSH(K,J),K=1,KRN)
2022 PRINT 265,DK(J),(DWSH(K,J),K=1,KRN)
2022 FORMAT(36HROTATIONAL INDUCED VELOCITY COMPONENT)
PRINT 262,(B(6,J),J=1,KRN)
PRINT 263
DO 2023 J=1,NKN
2023 PRINT 264,DK(J),(DWSH(K,J),K=1,KRN)
82 JS=KRN
83 DO 63J=1,KRN
CCAA=J.1416*B(6,J)/SURT((J.1416*U(6,J))**2+B(4,J)**2)
SUB=SQRT(ABS(1.-CCAA*CCAA))
JJKRN=J+KRN
DO 63K=1,NKN
63 ZZ(K,J)=DWSH(J,K)*CCAA-DWSH(J,K)*SUB
IF(NN-1)12025,12024,12025
12024 PRINT 2024
2024 FORMAT(41H0 INDUCED DOWNWASH DUE TO FIRST BLADE ONLY)
GO TO 12026
12025 PRINT 2026
2026 FORMAT(35H0 INDUCED DOWNWASH DUE TO ALL BLADES)
12026 PRINT 262,(B(6,J),J=1,KRN)
PRINT 263
DO 2027 J=1,NKN
2027 PRINT 264,DK(J),(ZZ(J,K),K=1,KRN)
CALL MATINV(XX,MKN,ZZ,JS,V,10)
GOTO (1001,183),10
183 PRINT 1002
GOTO 84
1001 ERASE CM
DO 65 J=1,NKN
65 XX(J,19)=FLOATH(J)
DO 67J=1,JS
DO 67K=1,MKN
67 XX(K,J)=ZZ(K,J)/XX(K,19)
DO 70J=1,MCL
XX(1,20)=XL(J)
DO 73K=2,MKN
73 XX(K,20)=XX(K-1,20)*XL(J)
DO 75JJ=1,JS
DO 75KK=1,NKN
75 CH(J,JJ)=XX(KK,JJ)*XX(KK,20)+CH(KK,JJ)
70 CONTINUE
DO 78J=1,KRN
BPP(J)=BP(2,J)
VRE(J)=
X SORTF((J.1416*B(6,J)/AJ-B(9,J))**2+B(7,J)*B(8,J))**2)
78 CONTINUE
DO 77 J=1,JS
DO 77K=1,NCL
CH(K,J)=CM(K,J)/VRE(J)
77 CH(K,J)=CM(K,J)*BPP(J)*DIA
PRINT 262,(B(6,J),J=1,KRN)
262 FORMAT( /12H RX/RO .9F12.4)
PRINT 266,(BP(3,J),J=1,KRN)

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266 FORMAT( 12H P/D .9F12.4)
DO 2010 J=1,KRN
BP(1,J)=DIA*BP(1,J)
2010 BP(2,J)=DIA*BP(2,J)
90 PRINT 267,(BP(1,J),J=1,KRN)
267 FORMAT( 12H L.C. .9F12.4)
PRINT 268,(BP(2,J),J=1,KRN)
268 FORMAT( 12H CHORD .9F12.4)
PRINT 271
2711 FORMAT(20H0CAMBER DISTRIBUTION)
PRINT 263
263 FORMAT(5H X/LT)
DO 261J=1,NCL
261 PRINT 264,XL(J),(C/(J,K),K=1,KRN)
PRINT 270
270 FORMAT(13H0CAMBER RATIO)
PRINT 263
DO 265 J=1,NCL
265 PRINT 264,XL(J),(CMR(J,K),K=1,KRN)
264 FORMAT(F8.4,F16.4,DP12.4)
1002 FORMAT(19H MATQIX IS SINGULAR)
84 IF(NCTR)102,100,101
102 READ 104,MN, NCTR, NPT,KSP
104 FORMAT(418)
READ 4,VH,DY,SM,SW
PRINT 106
106 FORMAT(53H1CAMBER LINES FOR THE SAME INPUT DATA AS ABOVE EXCEPT)
PRINT 107,NCTR,MN, VH,DY,SM, NPT,KSP,SW
107 FORMAT(6HONCTR=12.2X,3HNN=12.2X, 3HVM=FB.5.2X,3HXY=FB.5.2X,3HSM=FB.5.2X,4HNPT=12.2X,4HKSP=12.2X,3HSW=FB.5.2X)
GO TO 1061
100 CALL END JOB
STOP
END

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