

AD 607748

University of Oklahoma Research Institute

COPY	2	OF	2	10/1
HARD COPY	\$ 1.00			
MICROFICHE	\$ 0.50			

A Closed System Model for Target Acquisition

by

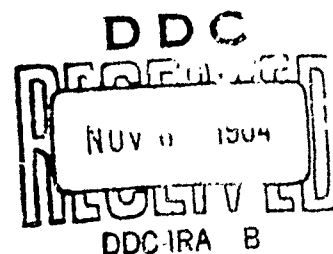
James A. Nickel and James D. Palmer

Report No. 1107-3-4

Contract No. DA 34-031-AIV-679

June 19, 1964

Technical Report



A CLOSED SYSTEM MODEL FOR TARGET ACQUISITION

The fundamental problem of Target Acquisition is to locate the desired target providing the necessary parameters or coordinates to enable the weapon to fire upon the target with a maximum probability of a hit. It is well known that the weapon itself is not absolutely accurate and that its rounds will fall within some probability distribution. Furthermore it is known that any sensor device, in locating the target will also locate the target to within some probability distribution.

It is the purpose of this report to consider the nature of the probability distribution in locating the target through three basic methods for detection, and to consider a proposed closed system employing weapon, detector and target for estimating this cumulative probability distribution.

TARGET LOCATION BY DIRECTION ONLY

The simplest means of locating a target is by means of determining its direction from two fixed points relative to a reference coordinate system. The intersection of the lines of sight then gives the target location. In practice this procedure is complicated by a number of factors which introduce errors in the final target location. Obvious error producing factors are:

- 1) Inaccurate knowledge of location of detection devices
 - a) distances between detectors only approximate
 - b) reference coordinate directions not necessarily invariant or easy to use due to curvature of earth, magnetic declination, etc.
- 2) Error distribution in reading direction
 - a) instrumentation errors built into the system
 - b) operator errors

Further errors will be introduced in the target location if

- 3) The two detectors are nearly on a line with the target.
- 4) The base line between detectors is "small" compared to the distance to the target.

A modeling of this type is illustrated in Figure I. The angles α and β are determined from observation and the length of the base line B between the target location devices $TL1$ and $TL2$ is presumed known. The distance D from $TL1$, for example, can be obtained from an application of the Law of Sines

giving

$$\frac{D}{\sin \beta} = \frac{B}{\sin (180 - \alpha - \beta)} = \frac{B}{\sin (\alpha + \beta)}$$

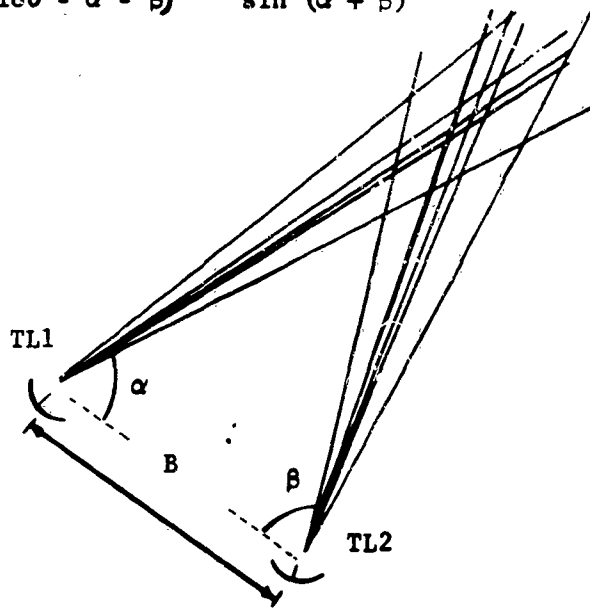


Figure I

If we introduce errors in the length of the base line and errors in the reference angles α and β , the distance D can only be approximated. Let ΔB be the error in the length of the base line and $\Delta\alpha$, $\Delta\beta$ be the errors in measuring the length of the base line. These errors may be positive or negative. To a first order approximation, the distance from TL1 to the target is given by

$$D = \frac{B \sin \beta}{\sin (\alpha + \beta)} \left[1 + \frac{\Delta B}{B} - \frac{\cos (\alpha + \beta)}{\sin (\alpha + \beta)} \Delta\alpha + \frac{\sin \alpha}{\sin \beta \sin (\alpha + \beta)} \Delta\beta \right]$$

This is the distance to the target from TL1 and it still remains to locate the target relative to the weapon.

The shape of the probability distribution locating the target in general will be elliptical. The orientation of the error ellipse will depend on the distances to the target from both target locator devices

and the relative directions. In a subsequent section the orientation of probability ellipses will be considered.

In passing, it should be observed that the angle α and β as indicated in the figure can be relative, that is if TL1 can see the target and TL2, the angle α is the difference in directions.

TARGET LOCATION BY RANGE ONLY

If the target location device is capable of measuring range but not of determining direction (to any sensible degree of accuracy), two devices will again be needed. In this modeling structure, the three sides of a triangle are measured or known. It is possible that the base line will actually be measured by the devices but could be an actual survey distance. This situation is illustrated in Figure II, where again the measure of ranges will have a probability distribution which in general will be a function of the range of operation. Again the target will be located to within some probability ellipse. The orientation of the probability ellipse will again depend upon the relative locations of the target locating devices.

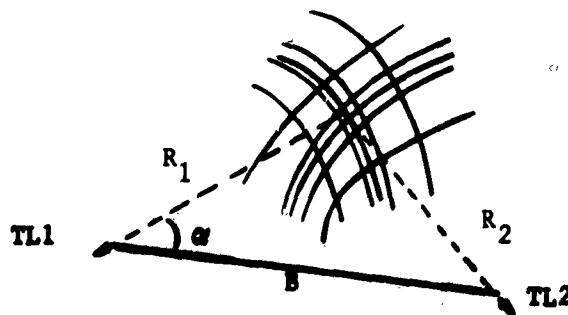


Figure II

For a complete description of the target location, an angle α is needed. Let R_1 and R_2 be the ranges as determined by the target locators TL1 and TL2, then the angle α can be determined from the Law of Cosines to get

$$\cos \alpha = \frac{B^2 + R_1^2 - R_2^2}{2BR_1}$$

If B , R_1 and R_2 are determined to within errors of ΔB , ΔR_1 and ΔR_2 , this value for the cosine will be determined to a first order approximation by

$$\cos \alpha = \frac{B^2 + R_1^2 - R_2^2}{2BR_1} \left[1 + \frac{\Delta R_1}{R} - \frac{\Delta B}{B} \right] + \left[\frac{\Delta B}{R_1} + \frac{\Delta R_1}{B} - \frac{R_2 \Delta R_2}{BR_1} \right]$$

Again it should be observed that the errors may be positive or negative.

TARGET LOCATION BY RANGE AND DIRECTION

In the event that the target location device can sense range and direction as most electromagnetic systems can, a single unit is all that is required for locating the target. This modeling structure is indicated by Figure III. It is to be anticipated that range errors and azimuth errors will in general be different, giving rise to a probability ellipse with one of the axes in the θ' radial direction from the target location device. For the previous models suggested, this orientation of the probability ellipse would not be known without considerable computation. Not only is the orientation of the probability ellipse known, but also the lengths of the semi-major axes can be assumed to be proportional to the range and azimuth errors.

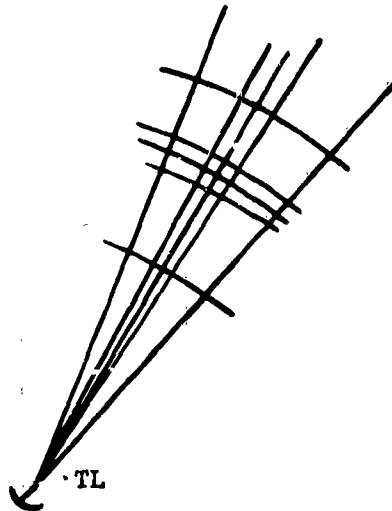


Figure III

TARGET ACQUISITION SYSTEM MODEL

It was initially stated in this paper, that the problem of Target Acquisition was that of locating the target in such a way as to provide adequate information for the firing of the weapon yielding the maximum probability of striking the target. In designing the following model, the premise has been made that the problem of target acquisition does not involve the accuracy of the sensing device alone, but also the capability of the weapon using the information (a communication problem in part) and lastly the weapons accuracy capability. The distribution of error resulting from the target acquisition device has been briefly discussed in the preceding sections, and a familiarity of some weapon accuracy problems is assumed. Let us now briefly consider the problem of weapon capability of using the sensor information.

Fundamentally the target acquisition device locates the target relative to the device with reference to some coordinate system. In order to get usable information to the weapons, its location must be known relative to the target locator. This means that both the target locator and weapon

must be using the same coordinate system (or at least know exactly the rules of transformation of one to the other). This presents a source of error to the acquisition system which on occasion has been referred to as a bias. This error involves distance and direction, which on computing the target location relative to the weapon, influence the resulting accuracy.

A closed system consisting of a) the target acquisition device, (radar, laser, IR, etc.) b) the weapon and communication link and c) the target, is proposed as a model for a target acquisition system. The target acquisition device locates both the target and the weapon relative to itself. The line joining the weapon and target location device then serves as the reference line for locating the direction to the target. This modeling structure is analyzed in the next section to give an analytical appraisal of the error ellipse of the distribution rounds fired at the target.

Within the context of the material considered in this paper, it may be desirable to reformulate the question of target acquisition to include the more general problem of system analysis to determine whether the probability of striking the target is sufficiently high to warrant firing at it. The question can be answered in part from a knowledge of the probabilities of "seeing" and isolating the target location yielding sufficient information to the weapon so that the probability of striking within the radius of destruction can be determined. The implicit errors of the weapon are also brought into this analysis.

THE SURFACE DISTRIBUTION OF TARGET ACQUISITION ERRORS

Within the framework of the foregoing target acquisition model, the line connecting the Target Locator and the Weapon will be taken as the base or reference line. It will also be assumed that the errors of the Target Locator Device, TLD, and the weapon will be proportional to the distance. The basic configuration of the system is shown in Figure IV.

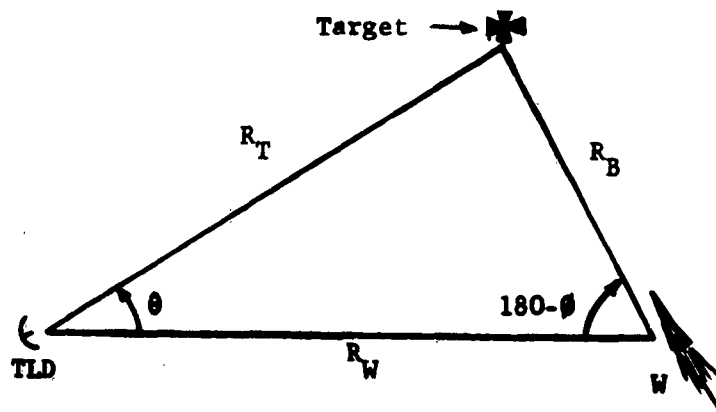


Figure IV

The notation to be employed is as follows:

R_T = Range to Target from Target Locator

R_W = Range to Weapon from Target Locator

R_E = Range of Ballistic (calculated range, weapon to target)

a = Standard deviation of target locator range error divided by range

b = Standard deviation of target locator azimuth error divided by range

A = Standard deviation of weapon range error divided by range

B = Standard deviation of weapon azimuth error divided by range

(x,y) = The coordinates of a point about the center of the distribution.

Utilizing this notation, we see that the location of the weapon would be within some probability ellipse, K_T , given by

$$K_W^2 R_W^2 = x^2/a^2 + y^2/b^2$$

In this particular case the equation of the ellipse is in standard form, since the reference or base line is the radial line from the Target Locator to the weapon.

The probability ellipse governing the target location is complicated by a problem of rotation since it is assumed that the Locator determines range and direction, hence one axis of the ellipse must lie in the radial direction. In this case a probability ellipse will have the shape determined by

$$K_T^2 R_T^2 = \frac{x^2 \cos^2 \theta + 2xy \cos \theta \sin \theta + y^2 \sin^2 \theta}{a^2} + \frac{x^2 \sin^2 \theta - 2xy \sin \theta \cos \theta + y^2 \cos^2 \theta}{b^2}$$

A third probability ellipse is introduced when one considers the errors implicit to the weapon itself. As a first approximation in determining the orientation of this ellipse it can be assumed that R_B and the angle θ are correct. The equation is then given by

$$K_B^2 R_B^2 = \frac{x^2 \cos^2 \theta + 2xy \cos \theta \sin \theta + y^2 \sin^2 \theta}{A^2} + \frac{x^2 \sin^2 \theta - 2xy \sin \theta \cos \theta + y^2 \cos^2 \theta}{B^2}$$

an equation functionally the same as the ellipse equation for the target location. Furthermore, the probability ellipse for the weapon location is also of the same form, however, $\theta = 0$.

From basic theoretical statistical considerations, it follows that the probability density of a target being "acquired" by the system is

$$f = \left[(2\pi)^3 ab R_W R_T R_B AB \right]^{-1} \exp \left[- (K_W^2 + K_T^2 + K_B^2) \right]$$

where the K's are found from the preceeding equations. By stating that the target is "acquired," it is meant that the target has been detected and the weapon has fired at it striking within the radius of lethality of the particular warhead being used.

The Composite Probability Ellipse

Unless the range and azimuth errors are equal, the composite probability ellipse about the target cannot be expected to be circular or to have any particular orientation. A basic problem of acquisition then becomes the determination of the orientation and shape of the error ellipse about the target location. This information is obtained from an analysis of the exponents given in the foregoing probability density.

Each of the probability or error ellipses can be described by means of a function of the form

$$K^2(x, y; \alpha, \beta, \psi, R) = \frac{x^2}{R^2} \left[\frac{\cos^2 \psi}{\alpha^2} + \frac{\sin^2 \psi}{\beta^2} \right] + \frac{2xy}{R^2} \left[\frac{1}{\alpha^2} - \frac{1}{\beta^2} \right] \sin \psi \cos \psi + \frac{y^2}{R^2} \left[\frac{\sin^2 \psi}{\alpha^2} + \frac{\cos^2 \psi}{\beta^2} \right]$$

where

$$K_W^2 = K^2(x, y; a, b, 0, R_W)$$

$$K_T^2 = K^2(x, y; a, b, \theta, R_W)$$

$$K_B^2 = K^2(x, y; A, B, \phi, R_B)$$

The exponents of the probability density can now be written

$$+ G(x, y; a, b, A, B, \theta, \phi, R_W, R_T, R_B) = f(K_W^2 + K_T^2 + K_B^2)$$

$$\begin{aligned}
&= x^2 \left[\frac{1}{a^2 R_W^2} + \frac{1}{R_T^2} \left(\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \right) + \frac{1}{R_B^2} \left(\frac{\cos^2 \theta}{A^2} + \frac{\sin^2 \theta}{B^2} \right) \right] \\
&+ 2xy \left[\frac{\cos \theta \sin \theta}{R_T^2} \left(\frac{1}{a^2} - \frac{1}{b^2} \right) + \frac{\cos \theta \sin \theta}{R_B^2} \left(\frac{1}{A^2} - \frac{1}{B^2} \right) \right] \\
&+ y^2 \left[\frac{1}{b^2 R_W^2} + \frac{1}{R_T^2} \left(\frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} \right) + \frac{1}{R_B^2} \left(\frac{\sin^2 \theta}{A^2} + \frac{\cos^2 \theta}{B^2} \right) \right]
\end{aligned}$$

The angle θ is a variable derived from the initial data supplied as in the range R_B . It would be preferable to remove either or both of these variables from the equation. From the Law of Cosines it follows that

$$R_B^2 = R_W^2 + R_T^2 - 2 R_W R_T \cos \theta.$$

However, the functional form of the above probability ellipse, G , will be easier to consider if this substitution is not made. From the Law of Sines

$$\sin \theta = R_T \sin \theta / R_B$$

and hence

$$\cos \theta = (R_W - R_T \cos \theta) / R_B.$$

Under this notational change we now have

$$G(x, y; a, b, A, B, \theta, R_W, R_T, R_B)$$

$$\begin{aligned}
&= x^2 \left[\frac{1}{a^2 R_W^2} + \frac{1}{R_T^2} \left(\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \right) + \frac{1}{R_B^4} \left(\frac{(R_W - R_T \cos \theta)^2}{A^2} + \frac{R_T^2 \sin^2 \theta}{B^2} \right) \right] \\
&+ 2xys \sin \theta \left[\frac{\cos \theta}{R_T^2} \left(\frac{1}{a^2} - \frac{1}{b^2} \right) + \frac{R_T (R_W - R_T \cos \theta)}{R_B^4} \left(\frac{1}{A^2} - \frac{1}{B^2} \right) \right] \\
&+ y^2 \left[\frac{1}{b^2 R_W^2} + \frac{1}{R_T^2} \left(\frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} \right) + \frac{1}{R_B^4} \left(\frac{R_T^2 \sin^2 \theta}{A^2} + \frac{(R_W - R_T \cos \theta)^2}{B^2} \right) \right]
\end{aligned}$$

The General Probability Ellipse

The general probability ellipse was given as

$$G(x,y) = x^2 \left[\frac{1}{a^2 R_W^2} + \frac{1}{R_T^2} \left(\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \right) + \frac{1}{R_B^4} \left(\frac{(R_W - R_T \cos \theta)^2}{A^2} + \frac{R_T^2 \sin^2 \theta}{B^2} \right) \right] \\ + 2xy \sin \theta \left[\frac{\cos \theta}{R_T^2} \left(\frac{1}{a^2} - \frac{1}{b^2} \right) + \frac{R_T (R_W - R_T \cos \theta)}{R_B^4} \left(\frac{1}{A^2} - \frac{1}{B^2} \right) \right] \\ + y^2 \left[\frac{1}{b^2 R_W^2} + \frac{1}{R_T^2} \left(\frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} \right) + \frac{1}{R_B^4} \left(\frac{R_T^2 \sin^2 \theta}{A^2} + \frac{(R_W - R_T \cos \theta)^2}{B^2} \right) \right]$$

The angle of rotation necessary to put this into standard form with an axis parallel to the base line is

$$\tan 2\alpha = \frac{2 \sin \theta \left[\frac{\cos \theta}{R_T^2} \left(\frac{1}{a^2} - \frac{1}{b^2} \right) + \frac{R_T (R_W - R_T \cos \theta)}{R_B^4} \left(\frac{1}{A^2} - \frac{1}{B^2} \right) \right]}{\left(\frac{1}{a^2} - \frac{1}{b^2} \right) \left(\frac{1}{R_W^2} + \frac{\cos^2 \theta}{R_T^2} \right) + \left(\frac{1}{A^2} - \frac{1}{B^2} \right) \left[\frac{(R_W - R_T \cos \theta)^2}{R_B^4} - R_T^2 \sin^2 \theta \right]}$$

Since a , b , A and B are fixed parameters within the system, this expression can be simplified by setting

$$m = \frac{1}{a^2} - \frac{1}{b^2}, \quad M = \frac{1}{A^2} - \frac{1}{B^2}.$$

It should be observed that m and M are fixed parameters of the system, i.e. for a given system are constant. The foregoing equation can now be written

$$\tan 2\alpha = \frac{2 \sin \theta \left[\frac{m \cos \theta}{R_T^2} + \frac{M R_T (R_W - R_T \cos \theta)}{R_B^4} \right]}{m \left[\frac{1}{R_W^2} + \frac{\cos^2 \theta}{R_T^2} \right] + M \left[\frac{(R_W - R_T \cos \theta)^2}{R_B^4} - R_T^2 \sin^2 \theta \right]}$$

Lengths proportional to the semi-minor axis r_1 , and semi-major axis r_2 of the general probability ellipse are given by

$$\frac{G}{r_i^2} = \left(\frac{1}{R_W^2} + \frac{1}{R_T^2} \right) \left(\frac{1}{a^2} + \frac{1}{b^2} \right) + \frac{1}{R_B^2} \left(\frac{1}{A^2} + \frac{1}{B^2} \right) + \epsilon_{11} D$$

where $\epsilon_{11} = +1$, $\epsilon_{12} = -1$ and

$$D^2 = m^2 \left[\left(\frac{1}{R_W^2} - \frac{1}{R_T^2} \right)^2 + \frac{4 \cos^2 \theta}{R_W^2 R_T^2} \right] + \frac{M}{R_B^2} \\ + \frac{2mM}{R_W^2 R_T^2 R_B^2} \left[(R_T^2 + R_W^2 \cos 2\theta) (R_W - R_T \cos \theta)^2 + R_W^2 \sin 2\theta (R_T \sin \theta) (R_W - R_T \cos \theta) \right. \\ \left. - (R_T^2 + R_W^2 \cos 2\theta) (R_T \sin \theta)^2 \right]$$

According to some working papers of Patricia L. Milic an elliptical error distribution can be approximated by an "equivalent" circular distribution

The equivalent standard deviation for the composite distribution is given by

$$\sigma^2 = \frac{1}{2 \left[\left(\frac{1}{a^2} + \frac{1}{b^2} \right) \left(\frac{1}{R_W^2} + \frac{1}{R_T^2} \right) + \frac{1}{R_B^2} \left(\frac{1}{A^2} + \frac{1}{B^2} \right) \right]}$$

The circular probable error is obtained by multiplying σ by 1.1774.

To simplify the discussion it can be assumed that the range and azimuth errors are proportional, i.e.

$$a = kb$$

$$A = KB.$$

In general it can be anticipated that the range errors will be the larger and hence k, K are greater than 1. Under this notation the variance σ^2

becomes

$$\sigma^2 = \frac{1}{2} \left[\frac{1}{b^2} \left(1 + \frac{1}{k^2} \right) \left(\frac{1}{R_W^2} + \frac{1}{R_T^2} \right) + \frac{1}{B^2 R_B^2} \left(1 + \frac{1}{K^2} \right) \right]^{-1}.$$

For ease in estimating the parameters m, M and σ^2 the following table is supplied

k	$\frac{1}{k^2} - 1$	$\frac{1}{k^2} + 1$
1	0	2
2	-0.7500	1.2500
3	-0.8889	1.1111
4	-0.9375	1.0625
5	-0.9600	1.0400

For the distribution to be truly circular it is necessary that D vanish. This can happen if and only if

- 1) $m=M=0$, The special case of circular distribution errors within the original components of the system.
- 2) $M=0$, $R_T = R_W$, $\theta = \pi/2$. This is a very specialized case and will be briefly considered in a subsequent paragraph.
- 3) The cross product term has the same magnitude but opposite sign as the terms in m^2 and M^2 .

The third condition seems to be the most promising. In order to investigate the behavior of the cross product term in D factor $(R_T^2 + R_W^2 \cos 2\theta)$ out of the equation leaving a quadratic form in $(R_W - R_T \cos \theta)$ and $R_T \sin \theta$, namely,

$$(R_W - R_T \cos \theta)^2 + \frac{2R_W^2 \sin 2\theta}{R_T^2 + R_W^2 \cos 2\theta} (R_W - R_T \cos \theta)(R_T \sin \theta) - (R_T \sin \theta)^2.$$

This quadratic form has as its discriminant

$$\frac{(R_T^2 + R_W^2)^2 - (2R_T R_W \sin \theta)^2}{(R_T^2 + R_W^2 \cos 2\theta)^2},$$

which will be positive excepting the special case of $\theta = \pi/2$ and $R_T = R_W$ for which it vanishes. Since the discriminant is positive, it has real and unequal zeros, hence the quadratic form takes on both positive and negative values.

The third condition as listed for an error distribution to be circular cannot be expected to be satisfied except in very special situations. One thus concludes that the error distribution will be elliptical in shape with the orientation depending upon

- a) range and azimuth error characteristics of the TLD
- b) range and azimuth error characteristics of the weapon
- c) angle with vertex at TLD measured from weapon to target
- d) range from TLD to target and to weapon

Instead of investigating the general case, some special configurations will be considered with their resulting simplifications.

Case I. Weapon and Target are located on the base line generated by the TLD. In this configuration $\sin \theta = 0$ and the rotational term vanishes.

If the target and the weapon are on the same side of the target locator $R_B = \frac{1}{2} (R_T + R_W)$ and $\cos \theta = 1$. The function $G(x,y)$ representing the probability ellipse now becomes

$$G(x,y) = x^2 \left[\frac{1}{a^2} \left(\frac{1}{R_W^2} + \frac{1}{R_T^2} \right) + \frac{1}{(R_W - R_T)^2 A^2} \right] + y^2 \left[\frac{1}{b^2} \left(\frac{1}{R_W^2} + \frac{1}{R_T^2} \right) + \frac{1}{(R_W - R_T)^2 B^2} \right]$$

Obviously $R_W = R_T$ must be excluded, but this means that the target cannot be at the weapon. The condition for the error pattern to be circular becomes

$$\frac{(R_T^2 + R_W^2)^2 m^2}{R_W^4 R_T^4} + \frac{2(R_T^2 - R_W^2)(R_W - R_T)^2 mM}{R_W^2 R_T^2 R_B^2} + \frac{M^2}{R_B^2} = 0$$

This can be satisfied only if $m=M=0$, i.e. the error distributions are initially circular.

If the target and the weapon are on opposite sides of the TLD, $\cos \theta = -1$ and now $R_B = R_W + R_T$. The function representing the probability ellipse is

$$G(x,y) = x^2 \left[\frac{1}{a^2} \left(\frac{1}{R_W^2} + \frac{1}{R_T^2} \right) + \frac{1}{(R_W + R_T)^2 A^2} \right] + y^2 \left[\frac{1}{b^2} \left(\frac{1}{R_W^2} + \frac{1}{R_T^2} \right) + \frac{1}{(R_W + R_T)^2 B^2} \right]$$

Another special case corresponds to placing the TLD at the weapon to give an error ellipse of

$$G(x,y) = \frac{1}{R_T^2} \left[x^2 \left(\frac{1}{a^2} + \frac{1}{A^2} \right) + y^2 \left(\frac{1}{b^2} + \frac{1}{B^2} \right) \right]$$

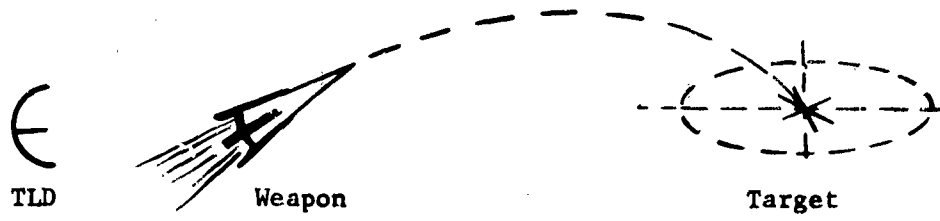
This error ellipse is of minimal dimensions of all possible configurations.

Case II. The second special case to be considered has the target at right angles to the base line from the weapon. This imposes the analytical conditions that

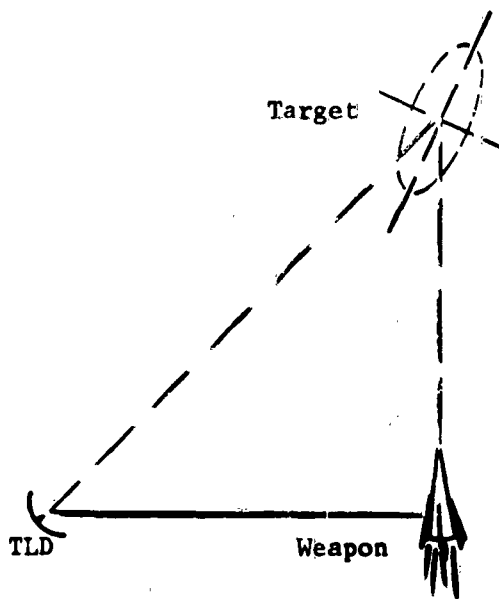
$$\cos \theta = R_W/R_T, \quad \sin \theta = \sqrt{(R_T^2 - R_W^2)}/R_T \quad R_T > R_W$$

which converts the quadratic $G(x,y)$ to

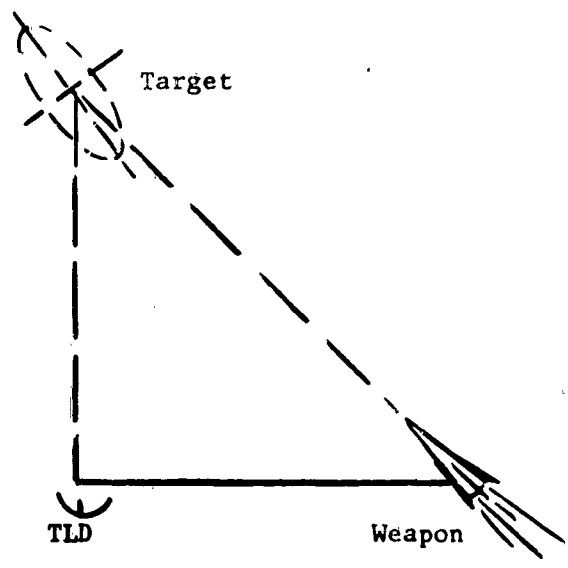
$$G(x,y) = x^2 \left[\frac{R_T^4 + R_W^4}{a^2 R_W^2 R_T^4} + \frac{B^2 (R_T^2 - R_W^2)^2 + b^2 R_T^4}{b^2 B^2 R_T^4 (R_T^2 - R_W^2)} \right]$$



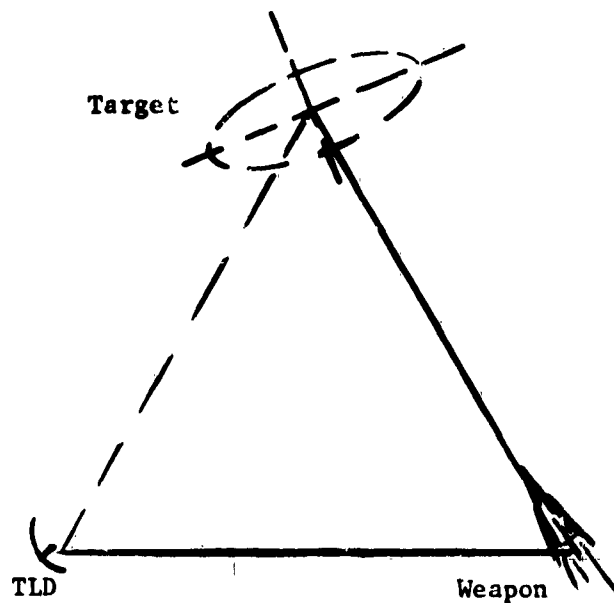
CASE I



CASE II



CASE III



CASE IV

$$+ 2xy \frac{R_W \sqrt{(R_T^2 - R_W^2)}}{R_T^2} \left(\frac{1}{a^2} - \frac{1}{b^2} \right)$$

$$+ y^2 \left[\frac{R_T^4 + R_W^4}{b^2 R_W^2 R_T^4} + \frac{A^2 (R_T^2 - R_W^2)^2 + a^2 R_T^4}{a^2 A^2 R_T^4 (R_T^2 - R_W^2)} \right]$$

since $R_B^2 = R_T^2 - R_W^2$. A more detailed investigation of this special configuration will not be considered.

Case III. This case is similar to the Case II, now however, the weapon and Target are located at right angles relative to the Target Location devise. Now however, $\cos \theta = 0$ and $R_B^2 = R_T^2 + R_W^2$. Now

$$G(x,y) = x^2 \left[\frac{1}{a^2 R_W^2} + \frac{1}{b^2 R_T^2} + \frac{B^2 R_W^2 + A^2 R_T^2}{A^2 B^2 (R_T^2 + R_W^2)^2} \right]$$

$$+ 2xy \frac{R_W R_T}{(R_T^2 + R_W^2)^2} \left(\frac{1}{A^2} - \frac{1}{B^2} \right)$$

$$+ y^2 \left[\frac{1}{b^2 R_W^2} + \frac{1}{a^2 R_T^2} + \frac{B^2 R_T^2 + A^2 R_W^2}{A^2 B^2 (R_T^2 + R_W^2)^2} \right]$$

The condition for a circular error distribution, reduces for this particular case the vanishing of

$$\frac{(R_T^2 - R_W^2)^2}{R_T^2 R_W^2} \left[\frac{m^2}{R_T^2 R_W^2} - \frac{2mM}{R_B^2} \right] + \frac{M^2}{R_B^2} = 0$$

This is possible, for example, it vanishes if the weapon has a circularly distributed error, $M = 0$, and $R_T = R_W$. There are other situations for which this expression may vanish. However, all of them form some rather special situation which in general cannot be expected to be satisfied.

Case IV. As a fourth and last special case to be considered let

$\theta = \pi/3$ and $R_W = R_T$. It follows that $R_B = R_W = R_T = R$ and hence

$$\begin{aligned} R^2 G(x,y) = & \frac{x^2}{4} \left[\frac{5}{a^2} + \frac{3}{b^2} + \frac{1}{A^2} + \frac{3}{B^2} \right] \\ & + \frac{\sqrt{3} xy}{2} \left[\left(\frac{1}{a^2} - \frac{1}{b^2} \right) + \left(\frac{1}{A^2} - \frac{1}{B^2} \right) \right] \\ & + \frac{y^2}{4} \left[\frac{3}{a^2} + \frac{5}{b^2} + \frac{3}{A^2} + \frac{1}{B^2} \right]. \end{aligned}$$

This ellipse will have an axis parallel to the base line if

$$a^{-2} + A^{-2} = b^{-2} + B^{-2}.$$

In general however it will be necessary to rotate through an angle α

given by

$$\tan 2\alpha = \sqrt{3} \frac{\left(\frac{1}{a^2} - \frac{1}{b^2} \right) + \left(\frac{1}{A^2} - \frac{1}{B^2} \right)}{\left(\frac{1}{a^2} - \frac{1}{b^2} \right) - \left(\frac{1}{A^2} - \frac{1}{B^2} \right)}.$$

Should the further condition be satisfied that $a = A$ and $b = B$, this rotation amounts to a 45° angle.

Summary

The general problem of target acquisition involves the problem of determining the location of the target, transmitting this data to the weapon to be employed and ultimately "acquiring" the target by "hitting" it. In the foregoing pages of this report, the attempt has been made to analyse the cumulative errors in accomplishing the above mission. As a model for field operation, it has been proposed that not only the target, but also the weapon employed be located relative to the Target locating device. Such a model inherently gives a reference line from which all computations can be made, removing any bias introduced by survey errors in attempting to plot the various components of the system on a map.

Thus, a methodology of target location has been developed which if utilized will greatly simplify the location problem and define accuracy restrictions in locator devices. It has been shown that the circular probable error for any such system can be approximated by

$$\frac{1.1774}{\sqrt{2}} \left[\frac{1}{b^2} \left(1 + \frac{1}{k^2} \right) \left(\frac{1}{R_W^2} + \frac{1}{R_T^2} \right) + \frac{1}{B^2 R_B^2} \left(1 + \frac{1}{K^2} \right) \right]^{-\frac{1}{2}}$$

in which b and B are the controlling locator and weapon error parameters respectively.

BIBLIOGRAPHY

Milic, Patricia L. Approximation to Circular Probable Error. (No date), Contract #DA 34-031 AIV-679.

Nickel, J. A. and Palmer, J. D., Methodology Utilized in the Determination of Weapons Systems Accuracy Requirements. Proceedings of the National Winter Convention on Military Electronics, Vol. III, I.E.E.E., Los Angeles, California, February, 1964.