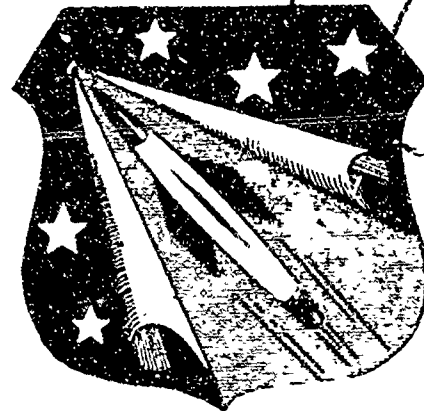


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Magnetohydrodynamics and Aerodynamic Heating

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Los Angeles, CaliforniaAbstract

The basic equations and some fundamental concepts of magnetoaerodynamics are discussed. As an application, the problem of the flow near the stagnation point of a body of revolution is reviewed and an exact solution is given. Data are presented for the heat transfer coefficient at the stagnation point, and for its gradient in the stagnation point region. The magnetic field strength required to accomplish an appreciable reduction of aerodynamic heating in hypersonic flight is discussed, for the case in which ionization is due to thermal motion. Alternatively, methods which involve electrical breakdown of the air are considered.

Introduction

At hypersonic speeds, exceeding a flight Mach number of approximately 15, the air behind a normal shock reaches a temperature, which is sufficient to ionize the air to an appreciable extent. It is then possible to alter the characteristics of the flow, by means of externally applied magnetic fields, (1)^{**} to (8). Due to the motion of the ionized gas (plasma) through the magnetic lines of force, currents are induced

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^{**) Number in parentheses indicate References at end of paper.}

in the gas, which in turn give rise to a force $\vec{j} \times \vec{B}$, per unit volume, acting on the gas. In this expression, $\vec{j}$ is the current density and $\vec{B}$ the magnetic induction. The plasma currents in turn give rise to a magnetic field, which adds to the field produced by external currents. $\vec{B}$ represents the total field. The theoretical analysis of the ensuing flow therefore requires in general a solution of Maxwell's equations for the electromagnetic field, coupled with the equations of fluid-mechanics containing the additional force term. The equation of energy is also modified, in a manner which will be discussed below.

Confining the present considerations to the stationary flow of a fluid of constant properties, the usual equations of magnetohydrodynamics take the following form:

$$\text{curl } \vec{H} = \vec{j} \quad (1)$$

$$\text{curl } \vec{E} = 0 \quad (2)$$

$$\text{div } \vec{B} = 0 \quad (3)$$

where $\vec{H}$ and $\vec{E}$ are the magnetic and electric field strengths, respectively. Maxwell's equations are supplemented by the constitutive equation

$$\vec{B} = \mu \vec{H} \quad (4)$$

and Ohm's law for moving, isotropic media,

$$\vec{j} = \sigma (\vec{E} + \vec{v} \times \vec{B}) \quad (5)$$

where μ is the magnetic permeability, σ the conductivity and $\vec{v}$ the velocity. Furthermore,

$$\text{div } \vec{v} = 0 \quad (6)$$

since a fluid of constant density is assumed. Supplementing the Navier-Stokes equation by the magnetic force term,

$$\rho \frac{D\vec{v}}{Dt} = - \text{grad } p + \eta \nabla^2 \vec{v} + \vec{j} \times \vec{B} \quad (7)$$

(ρ = density, p = pressure, η = viscosity).

Eqs. (1) to (7) suffice to determine for instance the velocity and the current distribution in the fluid. For the purpose of computing the distribution of temperature and rates of heat transfer, consideration must be given to the equation of energy^{*)},

$$\rho c \frac{DT}{Dt} - \nabla^2 T = \frac{1}{\sigma} j^2 + \Phi \quad (8)$$

(c = specific heat per unit mass, T = temperature, κ = thermal conductivity), where

$$\Phi = \tau_{ij} \frac{\partial v_i}{\partial x_j}; \quad \tau_{ij} = \eta \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (9)$$

The terms on the left of Eq. (8) represent the rate of heat removed per unit volume by convection and by heat conduction, respectively. The terms on the right are the rates of heat added through Joule heating and through viscous dissipation.

Stagnation Point Flow

As an illustration, the rotationally symmetric stagnation point flow is considered. The corresponding two-dimensional case was treated by Neuringer and McIlroy (10). In the present analysis, a rotationally symmetric magnetic field is assumed, which vanishes at a large distance forward of the stagnation point and has a normal component on the surface which is constant. It follows from simple symmetry arguments that a solution exists, for which

*) A full discussion of the equation of energy in magnetohydrodynamics, applicable to more general conditions than needed here, is due to Goldstein and Chu (9).

$$\left. \begin{aligned}
 j_r &= j_z = 0 \\
 B_\varphi &= 0 \\
 v_\varphi &= 0 \\
 E_r &= E_z = E_\varphi = 0
 \end{aligned} \right\} (10)$$

where r , z and φ indicate the corresponding components in cylindrical coordinates. It is seen that the current flows in circular loops about the z -axis. The magnetic force $\vec{j} \times \vec{B}$ is in the meridional plane, and its direction is such as to impede the motion of the fluid.

The trial solution

$$\left. \begin{aligned}
 v_r &= r f'(z) \\
 v_z &= -2 f(z) \\
 B_r &= r g'(z) \\
 B_z &= -2 g(z) \\
 \bar{p} - p &= h(z) + r^2 k(z) \\
 \bar{T} - T &= l(z) + r^2 m(z)
 \end{aligned} \right\} (11)$$

where f, g, h, k, l and m are functions of the coordinate z alone, and where $\bar{p}$ and $\bar{T}$ are the pressure and temperature at the stagnation point, satisfies Eqs. (1) to (9) identically, resulting in a set of differential equations for these functions. Full details are given elsewhere (11).

The following boundary conditions are imposed on the surface, i.e. at $z = 0$,

- (a) the normal component of the total magnetic field is constant, i.e. $B_z(0) = \bar{B}$ where $\bar{B}$ is a given constant.
- (b) the velocity vanishes, i.e. $v_r(0) = v_z(0) = 0$.
- (c) the temperature of the surface is constant, i.e. $T(0) = \bar{T}$.

At a large distance from the surface, the following conditions are imposed,

- (d) the magnetic field vanishes. In particular, it is required that $B_z(\infty) = 0$.
- (e) the velocity and pressure distributions approach the corresponding distributions for an inviscous, non-magnetic stagnation-point flow. In particular, $v_r(\infty) = ar$, where the constant "a" is determined by the characteristics of the flow outside the stagnation point region.
- (f) the temperature is independent of the radius. In particular, $T(\infty) = \bar{T}$ where $\bar{T}$ is a given constant.

Numerical results were obtained by means of a digital computer. Introducing the dimensionless quantities

$$\zeta = \sqrt{\mu \sigma a} \ z \quad (12)$$

$$F(\zeta) = \sqrt{\frac{\mu \sigma}{a}} \ f(z) \quad (13)$$

$$G(\zeta) = - \frac{2}{\bar{B}} \ g(z) \quad (14)$$

typical distributions of velocity and magnetic induction are plotted in Fig.1. These results depend upon two dimensionless parameters, viz.

$$\alpha = \eta \ a \ (B^2/2\mu)^{-1} \quad (15)$$

$$\beta = \mu \sigma \ \eta / \rho \quad (16)$$

In aerodynamic applications, α and β are invariably very small, typically of the order of 10^{-6} . The ratio

$$\gamma = \beta/2\alpha = \frac{\sigma B^2}{4\rho a} \quad (17)$$

which is a measure for the ratio of the magnetic forces to the inertial forces, however is generally of order one. Letting α and β tend to zero gives rise to a boundary layer problem (11). The velocity and the magnetic field distributions then depend only upon the single parameter γ . Figs. 2 and 3 represent the two velocity components outside this boundary layer ($\zeta_1 \equiv \zeta$, $F_1 \equiv F$). Similarly, Figs. 4 and 5 give the velocity distribution in the boundary layer, where $\zeta_2 \equiv \beta^{-1/2}\zeta$ and $F_2 \equiv \beta^{-1/2}F$.

As one recognizes from the velocity distributions, an important effect of the magnetic field consists in displacing the flow in the direction of the positive z -axis. The thickness of this magnetic cushion, defined by

$$\bar{\zeta} = \gamma \overline{\mu \sigma a} \bar{z}; \quad \bar{z} = \lim_{z \rightarrow \infty} (z - f(z)/a) \quad (18)$$

is plotted in Fig. 7.

Defining the Nusselt number

$$Nu(r) = h(r) \frac{1}{K} \left(\frac{\eta}{\rho a} \right)^{1/2} \quad (19)$$

where $h(r)$ is the heat transfer coefficient, one obtains

$$Nu(r) = Nu^{(0)} + Nu^{(1)} \frac{a^2 r^2}{c(\bar{T} - \bar{T})} \quad (20)$$

The coefficients $Nu^{(0)}$ and $Nu^{(1)}$ have been computed for a Prandtl number of 0.70 and are given in Fig. 8. With increasing magnetic field strength, the rate of heat transfer at the stagnation point is seen to decrease, a result, which is in qualitative agreement with the computations by Neuringer and McIlroy (10) for the two-dimensional case. In the non-magnetic case, which corresponds to $\gamma = 0$, the result agrees with data computed by Cohen and Reshotko (12).

$Nu^{(1)}$ increases with increasing field strength, indicating an increased slope of the heat transfer rate considered as a function of the radius. In applications to aerodynamic heating, this is a detrimental effect. The cross-over point, at which the advantage of a reduced heat transfer at the stagnation point is cancelled by an increased heat transfer at some distance from this point, depends upon the details of the application.

Required Magnetic Field Strength

It is seen that γ must be large (of the order of 100), if an appreciable reduction in heat transfer at the stagnation point is to be achieved. The same does not need to be true, however, for configurations of the magnetic field differing from the one presently considered. The contribution of the plasma currents to the total magnetic field is substantial, as seen from the curves in Fig. 6 where the magnetic field outside the boundary layer is plotted in terms of $G_1(\zeta_1) \approx G(\zeta)$.

The required field strength depends critically upon the conductivity of the medium. The conductivity of air in thermodynamic equilibrium has been estimated for temperatures up to 24,000°K (Fig. 9). The quotient ρ/ρ_0 in this figure is the ratio of the density to the density at n.t.p. These data have been computed from the electron collision

cross sections (13) and the calculated equilibrium composition of air (14). Details are given in an Appendix to this paper. It must be emphasized that in view of the uncertainty in the presently available data on the appropriate cross sections, the degree of accuracy is small.

In order to permit a quick orientation, the conductivity of air behind a normal shock is plotted in Fig. 10 directly as a function of the flight velocity V and of the altitude H above sea level. In Fig. 11, the magnitude of the magnetic induction B is plotted, which is required to make the ratio of the magnetic forces to the inertial forces, i.e. $\sigma B^2 l / \rho u$ equal to unity. Here, σ , ρ and u are the conductivity, density, and velocity behind a normal shock in air (taking into account dissociation and ionization). l is a characteristic length of the aerodynamic body, and was taken as 1m. With these assumptions, a definite field strength is associated with any given velocity and altitude. The curve labeled by the equation $\omega_e = \gamma_e$ is discussed in the Appendix.

It appears from these data, that large field strengths are required, if the heat transfer at the stagnation point is to be reduced substantially, at altitudes for which aerodynamic heating is critical. Different configurations of the magnetic field may be more advantageous.

In some cases, the temperature of the electron gas may be many times larger than the temperature of the gas, corresponding to an effective increase of the conductivity. This is the case for instance with microwave breakdown and glow discharges. The device shown in Fig. 12 contains a large number of electrodes of alternating polarity, which are imbedded in an aerodynamic surface. It has been found possible to maintain a glow discharge in a supersonic airstream, ionizing thereby the air in the boundary layer. A typical current voltage characteristic is given in Fig. 13.

Acknowledgment

The author is indebted to Dr. Lee O. Heflinger, who supervised the digital computer programming, and to Dr. Allan B. Schaffer, who obtained the data presented in Fig. 13.

Appendix

The electrical conductivity of air in thermodynamic equilibrium at temperatures up to about $7,000^{\circ}\text{K}$ is known with considerable reliability from shock tube experiments reported by Lamb and Lin (13). At these temperatures, the contribution of the positive ions to the total electron collision probability is still relatively small, but becomes increasingly important at higher temperatures.

It appears that no detailed experimental data are available at present for temperatures in excess of $7,000^{\circ}\text{K}$. On the other hand, many of the contemplated applications of magnetoaerodynamics involve considerably higher temperatures. In this paper, estimates of the conductivity up to $24,000^{\circ}\text{K}$ are computed. This upper limit was decided upon because it coincides with the limit of Gilmore's computations of the equilibrium composition of air (14).

The usual practice in computing the conductivity of partially ionized gases is to add the Maxwell-averaged total electron collision cross sections (weighted with the number density) of the neutral molecules to a corresponding equivalent cross section of the ions, the latter being computed on the basis of a theory which applies to fully ionized gases. The degree of approximation involved in this assumption is not known. In the case of air, a further element of uncertainty is introduced by the fact that the cross sections of some of the neutral species, notably N, are not known with certainty.

The conductivity σ is expressed by

$$\sigma = \frac{n_e e^2}{m_e \bar{C}_e \sum_j n_j \bar{Q}_j} \quad (\text{A-1})$$

where n_e is the number density of free electrons, e the electronic charge, m_e the electronic mass, n_j the number density of the species j , $\bar{Q}_j$ its Maxwell-averaged total electron collision cross section, and

$$\bar{C}_e = \sqrt{\frac{8kT}{\pi m_e}}$$

(k = Boltzmann's constant, T = temperature) the mean speed of the electrons. Thermodynamic equilibrium with an electron temperature equal to the gas temperature is assumed.

The summation in Eq. (A-1) is extended over those species, neutrals as well as ions, which contribute appreciably to the total cross section. An inspection of the equilibrium composition of air in the range from $6,000^\circ\text{K}$ to $24,000^\circ\text{K}$ and a density ratio ρ/ρ_0 (ρ_0 = standard density) varying from 10^{-3} to 10 indicates that the neutrals of major importance are N_2 , N and O (14). In view of the approximate nature of the computation, the inclusion of O_2 , NO and other, even less frequent species, would not be warranted. The cross sections for N_2 , N and O above an electron temperature of $6,000^\circ\text{K}$ are believed to be quite similar and nearly independent of the energy in the range considered. (At much higher temperatures, the effect of the neutrals becomes negligible). The computations can be based therefore on a common cross section of the neutrals, $\bar{Q}_n$, which was taken as $0.80 \times 10^{-15} \text{ cm}^2$. In view of the uncertainty in the cross sectional data of N and O , a more detailed computation would not be

justified.

The sum over all cross sections, in Eq. (A-1), can therefore be expressed as

$$\begin{aligned}\sum_j n_j \bar{Q}_j &= n_n \bar{Q}_n + n_i \bar{Q}_i \\ &= n_{A,0} \frac{\rho}{\rho_0} \left[(N_2 + N + O) \bar{Q}_n + e^- \bar{Q}_i \right]\end{aligned}\quad (A-2)$$

where n_n and n_i are the number densities of neutrals and ions, respectively.

$n_{A,0}$ is the number density of "air atoms" at standard condition; i.e.

$n_{A,0} = 5.38 \times 10^{19} \text{ cm}^{-3}$. N_2 , N , O and e^- are the number of N-molecules, N-atoms, O-atoms, and free electrons, respectively, per "air atom". The values computed by Gilmore have been used for these fractions.

Within the limits of temperature and density considered here, the effect of doubly ionized ions on the conductivity amounts to less than 8% and is neglected. Similarly, the effect of O^- is negligible. Consequently, for a neutral plasma,

$$n_i = n_e = n_{A,0} \frac{\rho}{\rho_0} e^- \quad (A-3)$$

a result, which was already utilized in the derivation of Eq. (A-2).

Spitzer and Härm's (15) results for the conductivity of a fully ionized gas, if written in terms of an equivalent cross section $\bar{Q}_i$ for the positive ions, give

$$\bar{Q}_i = \left| \frac{\pi e^2}{4kT} \right|^2 \frac{\ln(q C_e^2)}{\gamma_E} \quad (A-4)$$

for singly ionized ions, where

$$q = \frac{m_e}{2e^3} \left[\frac{kT}{2\pi n_e} \right]^{1/2}$$

and $\gamma_E = 0.582$. C_e is the rms electron velocity,

$$C_e = \left(\frac{3kT}{m_e} \right)^{1/2}$$

The results of a computation of σ , based on Eqs. (A-1), to (A-4) are given in Fig. 9. It is noted that the dependence upon density is relatively weak, and that its gradient reverses within the considered interval of temperature. Above $24,000^\circ\text{K}$ the effect of the second ionization becomes increasingly important.

At the lower end of the temperature interval, the computed conductivity agrees with the experimental data of Lamb and Lin (13). The computed values fall inside the experimental scatter. However, this agreement is to be expected, since cross sectional areas were used, which are derived from these data.

Gilmore's results for the thermodynamic variables of state of the air behind a normal shock resulting from hypersonic velocities, have been used to calculate the conductivity for different flight velocities V and altitudes H above sea level (Fig. 10). For comparison, the velocity of a satellite in a circular orbit of 7×10^6 m radius, and the escape velocity, have also been indicated.

In many applications of magnetoaerodynamics, the parameter $\sigma B^2 / \rho u$ plays an important role, since it is a measure for the ratio of the magnetic forces to the inertial forces. σ , ρ and u are defined as the conductivity, density and velocity behind a normal shock. B is the magnetic

induction, and l a characteristic dimension associated with the body producing the shock wave. In general, this parameter must be at least of order one, if the magnetic field is to produce an appreciable effect.

In order to permit a quick estimate of the typical field strength required in magnetoaerodynamics (assuming thermodynamic equilibrium), Fig. 11 has been drawn. In this figure, the required field strength is given in the case where the value of this parameter is 1, and for a length $l = 1$ m. Since from conservation of mass

$$\rho u = \rho_{\infty} V$$

where $\rho_{\infty} = \rho_{\infty}(H)$ is the free-stream density, the computation of B as a function of H and V is straight-forward.

Eq. (A-1) gives σ correctly only in the case of weak fields. Otherwise, the conductivity is no longer described by a scalar quantity. This occurs if the cyclotron frequency ω_e of the electrons is comparable or larger than their collision frequency ν_e . From

$$\omega_e = \frac{Be}{m_e}$$

and

$$\sigma = \frac{n_e e^2}{m_e \nu_e}$$

it follows from equating ω_e and ν_e , that

$$B' = \frac{n_e e}{\sigma} \quad (A-5)$$

where B' can be considered as a rough estimate of the maximum field strength B for which Eq. (A-1) is still valid. The locus of points in the

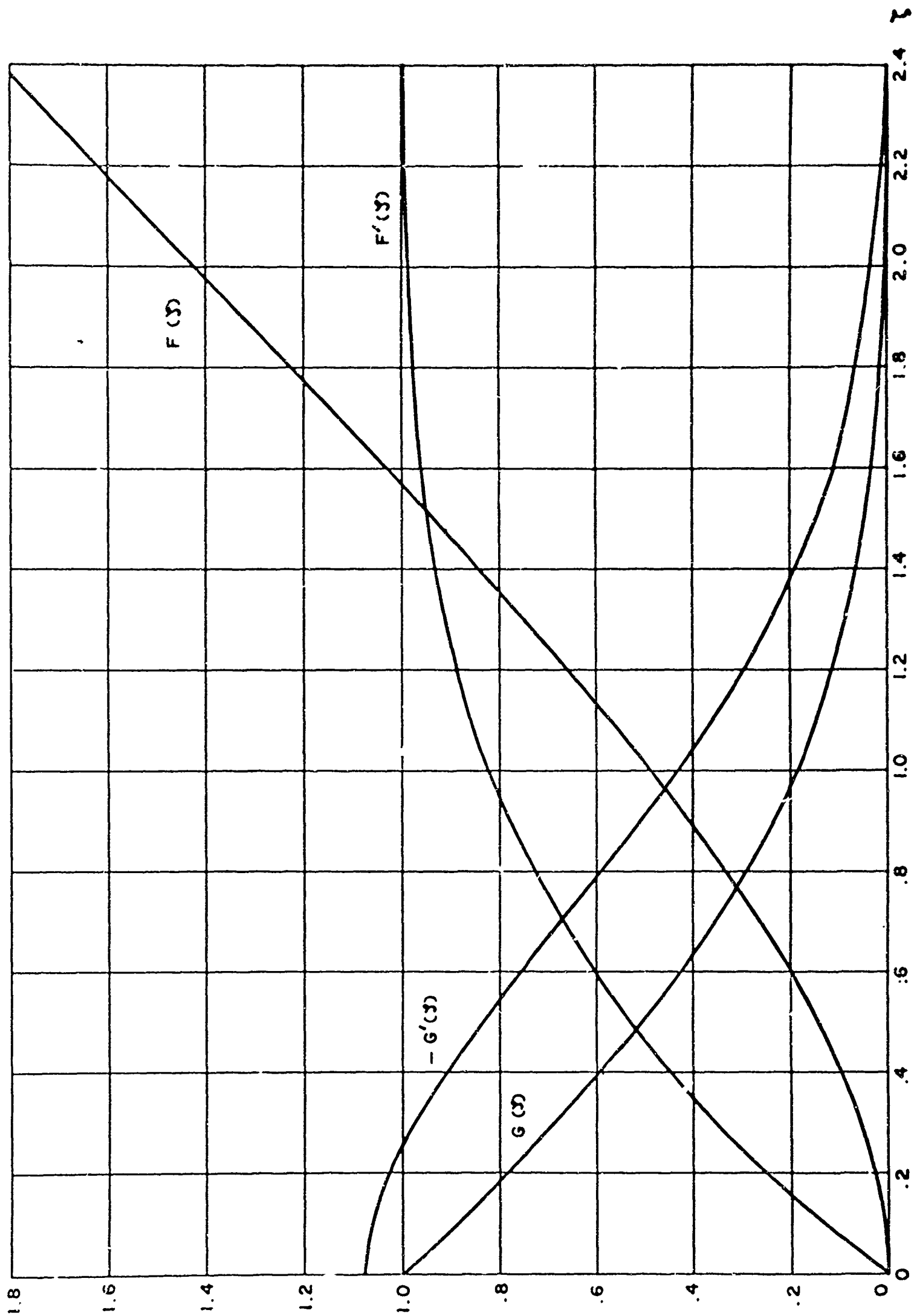
H - V diagram for which this limiting condition exists, is indicated in the figure. To the left of this curve, $\omega_e > \gamma_e$.

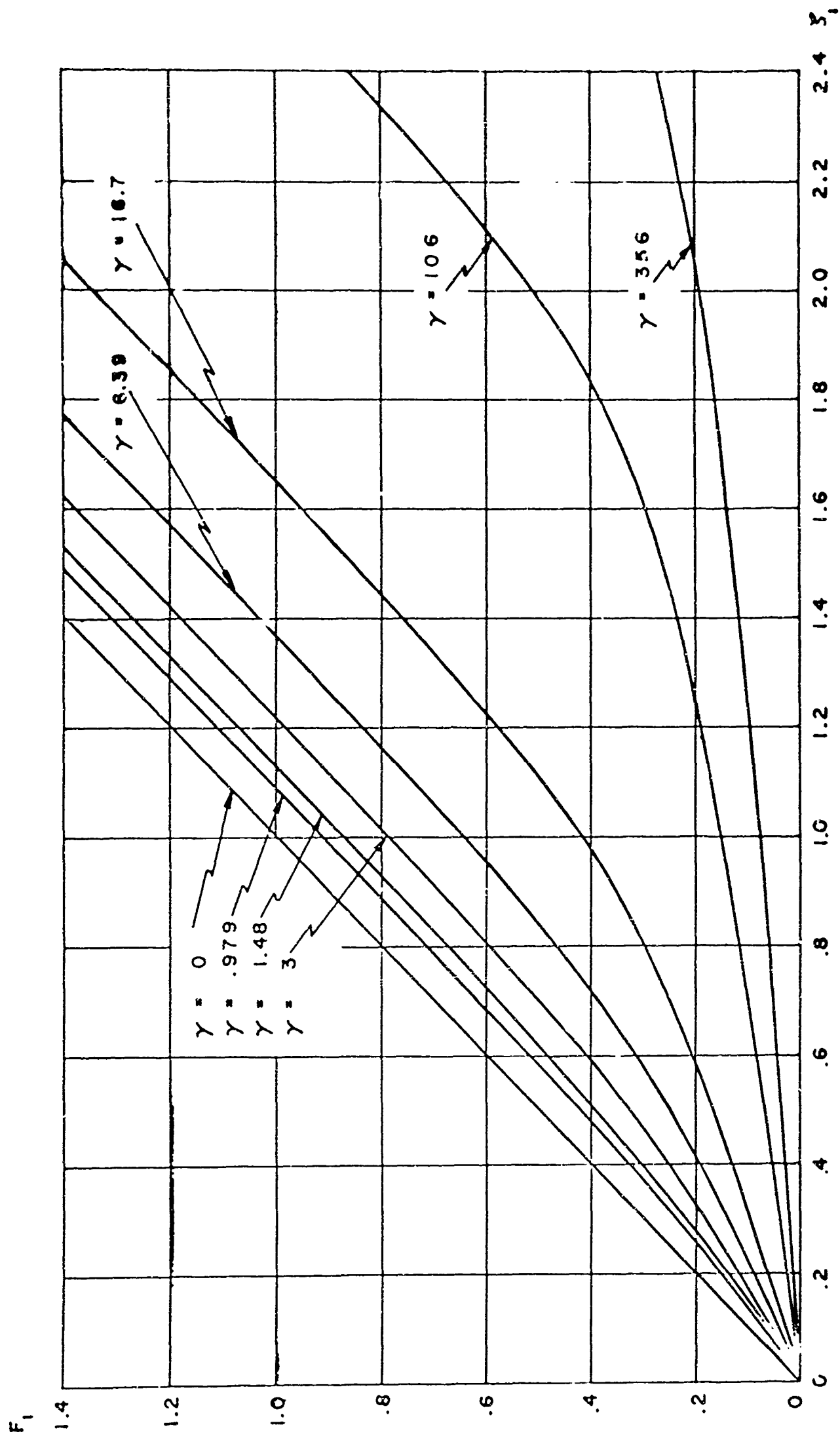
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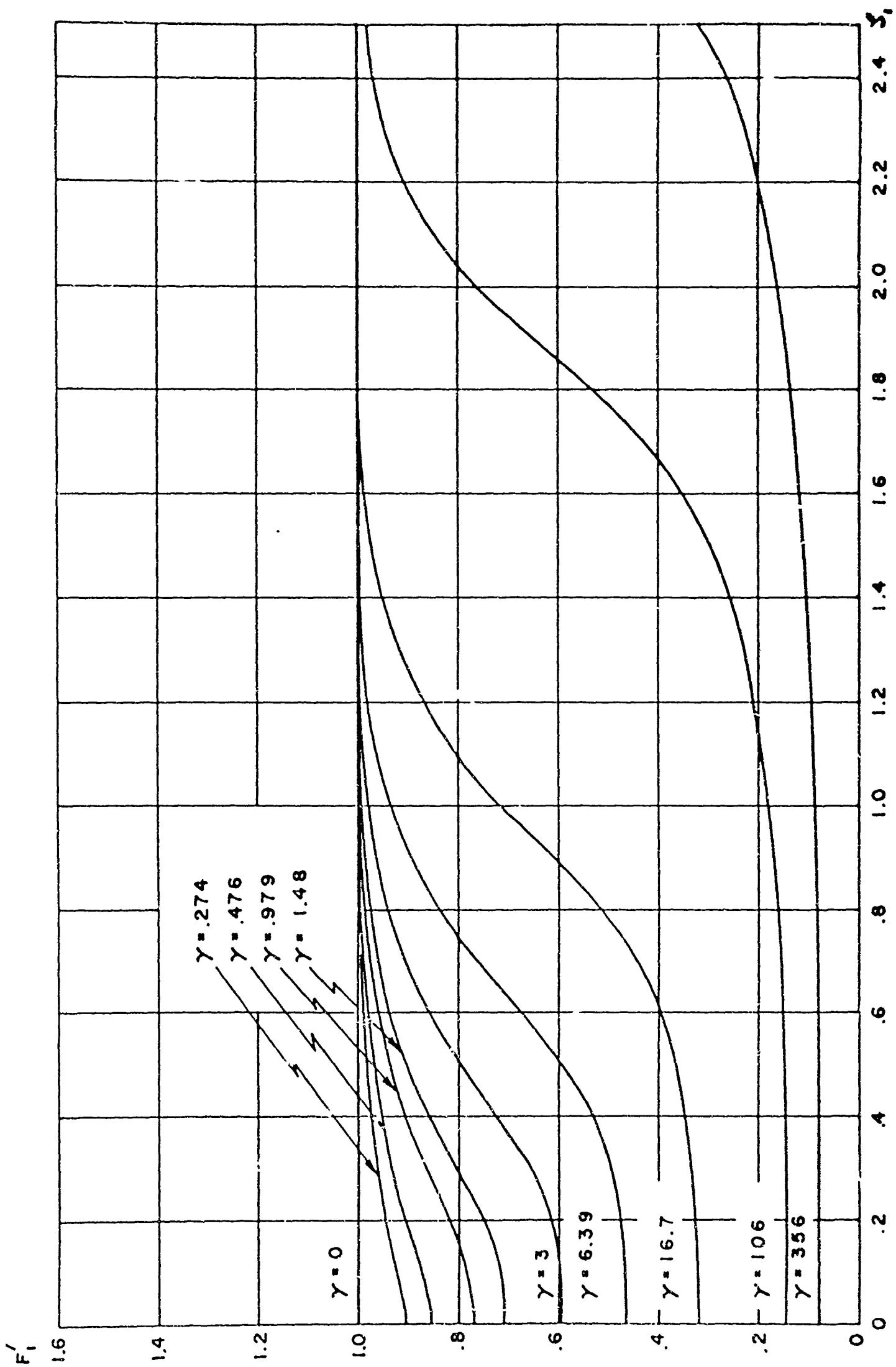
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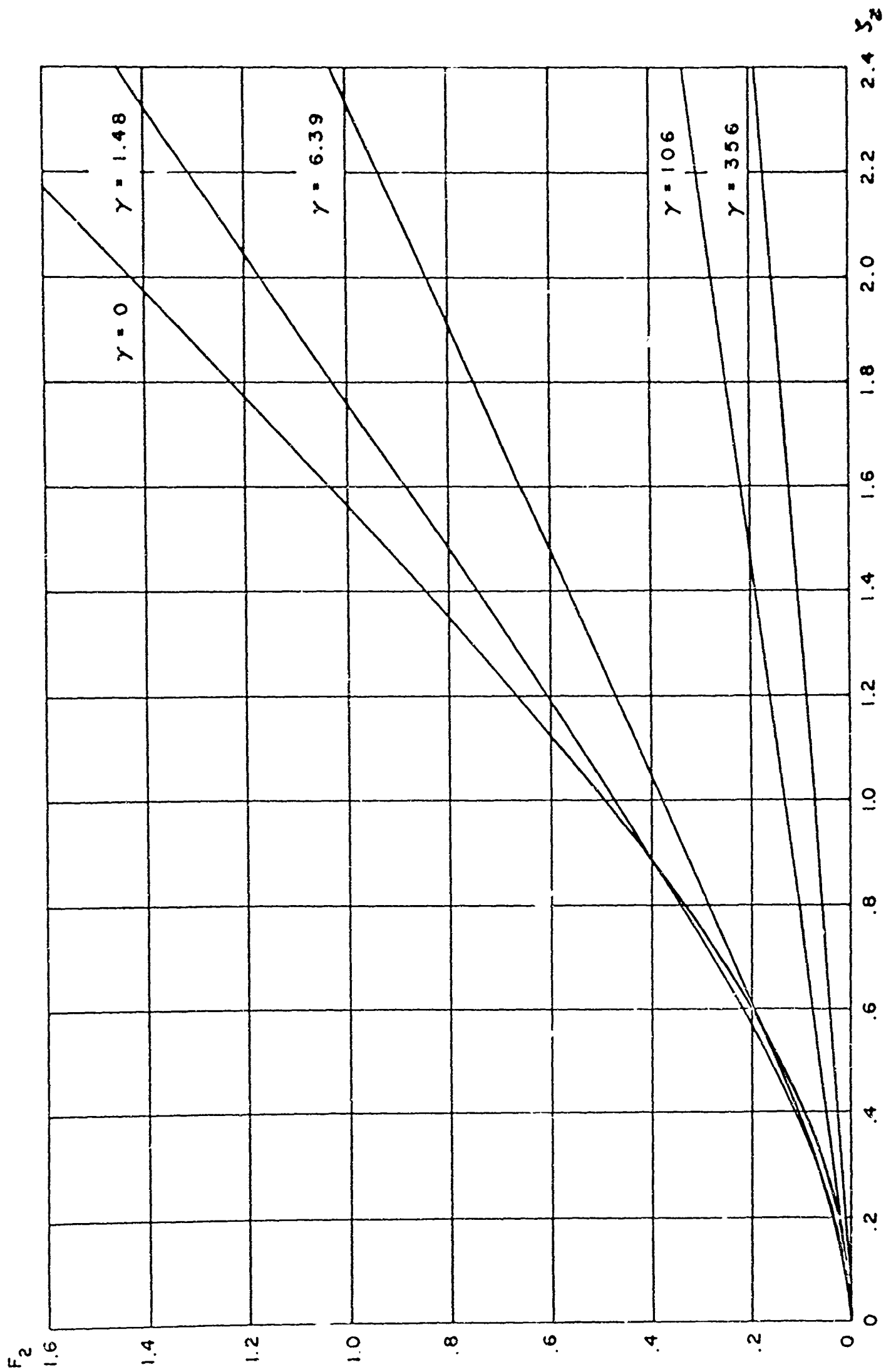
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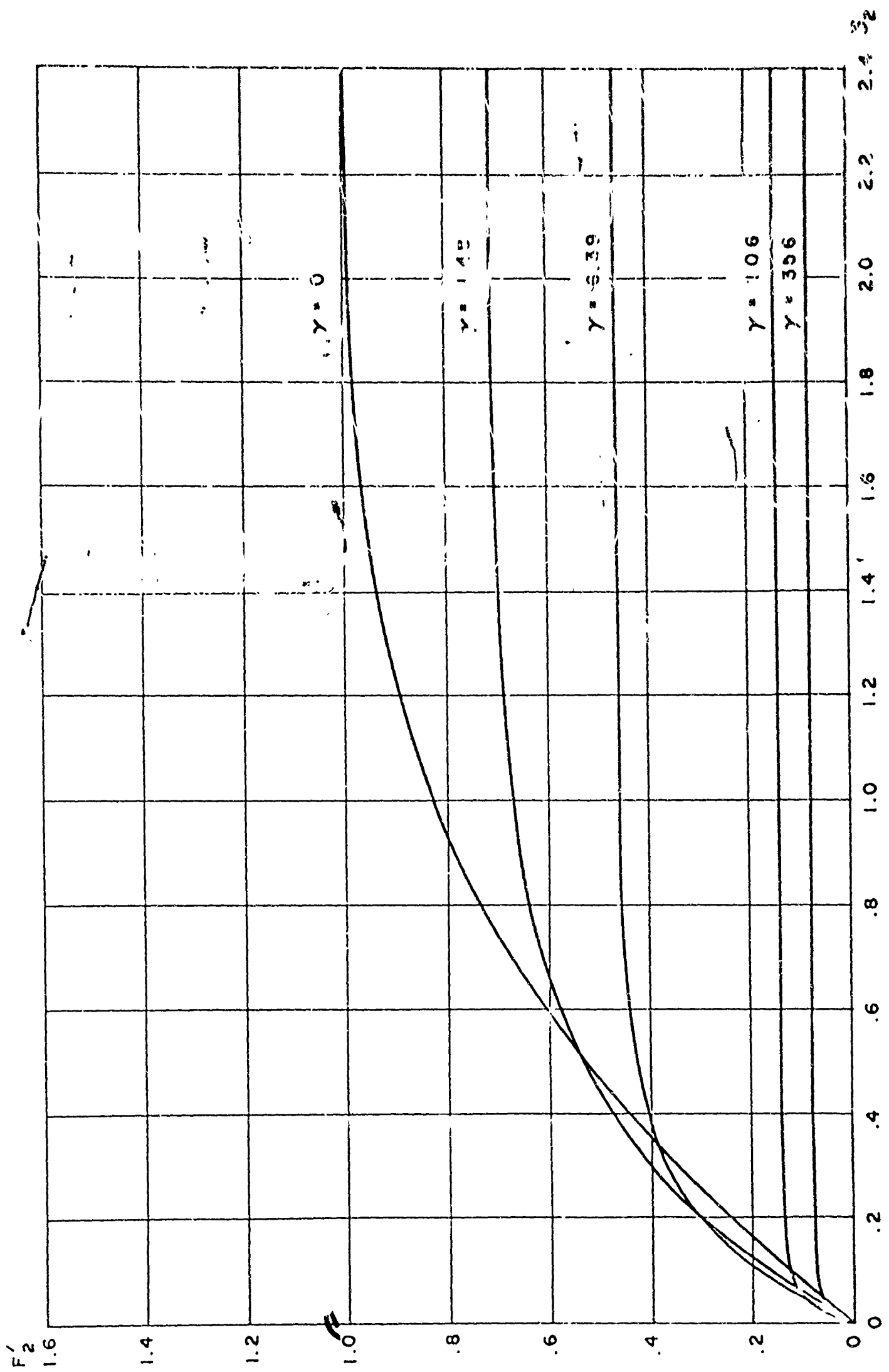
- Fig. 1 : The functions $F(\zeta)$, $G(\zeta)$ and their derivatives, for $\alpha = \beta = 1$
- Fig. 2 : Function $F_1(\zeta_1)$
- Fig. 3 : Function $F_1'(\zeta_1)$
- Fig. 4 : Function $F_2(\zeta_2)$
- Fig. 5 : Function $F_2'(\zeta_2)$
- Fig. 6 : Function $G_1(\zeta_1)$
- Fig. 7 : Magnetic cushion thickness $\bar{\zeta}$
- Fig. 8 : Nusselt numbers $Nu^{(0)}$ and $Nu^{(1)}$, for $Pr = 0.70$
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- Fig. 10 : Estimated electrical conductivity of air behind a normal shock
- Fig. 11 : Required magnitude of magnetic field for $\sigma B^2 l / \rho u = 1$ and $l = 1 \text{ m}$, behind a normal shock in air.
- Fig. 12 : Model used in glow-discharge experiments in a supersonic stream
- Fig. 13 : Typical current-voltage characteristic for glow-discharge in supersonic air stream. Copper electrodes, parallel to free-stream velocity. Cathode area = 3cm^2 .

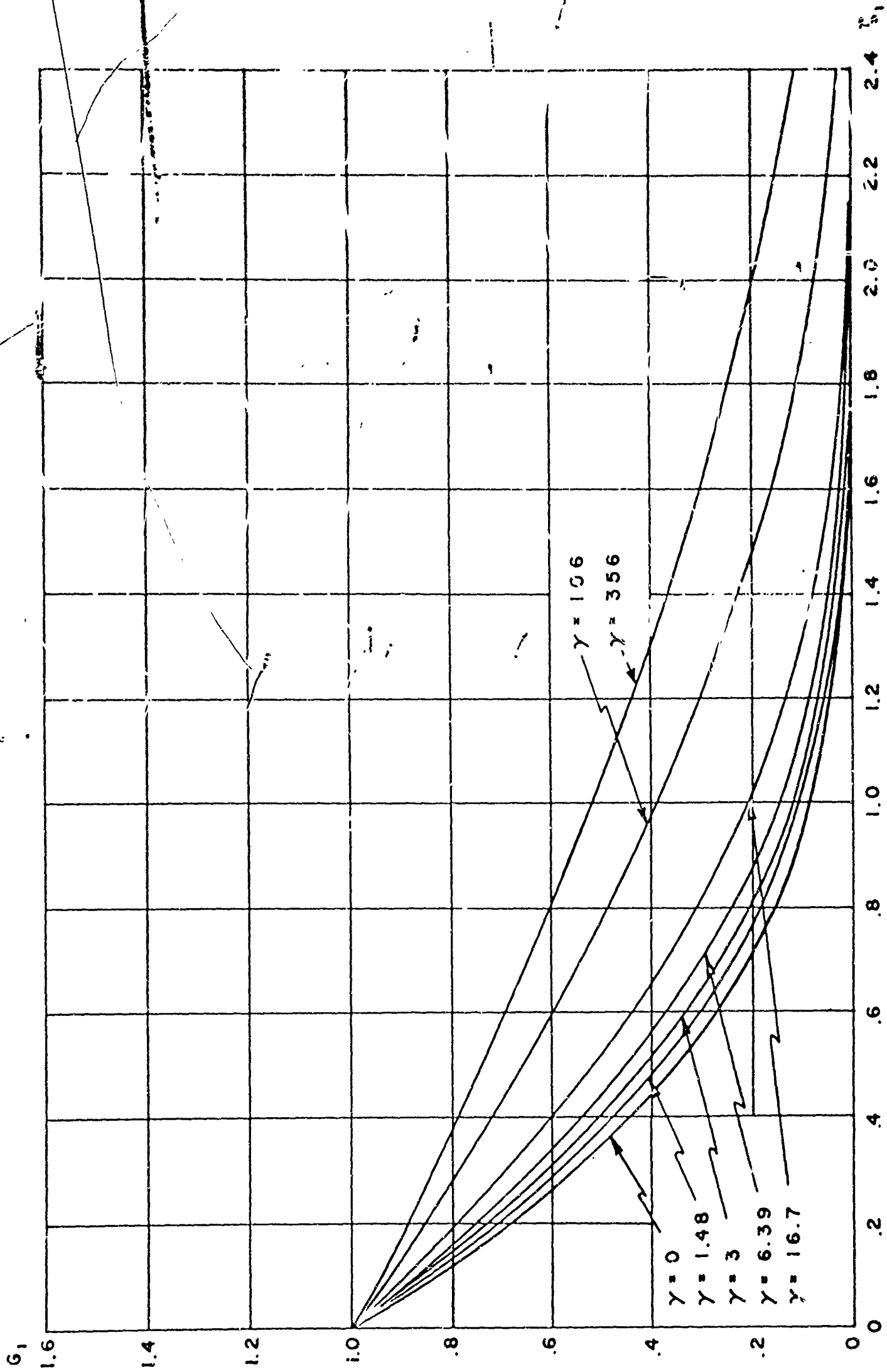


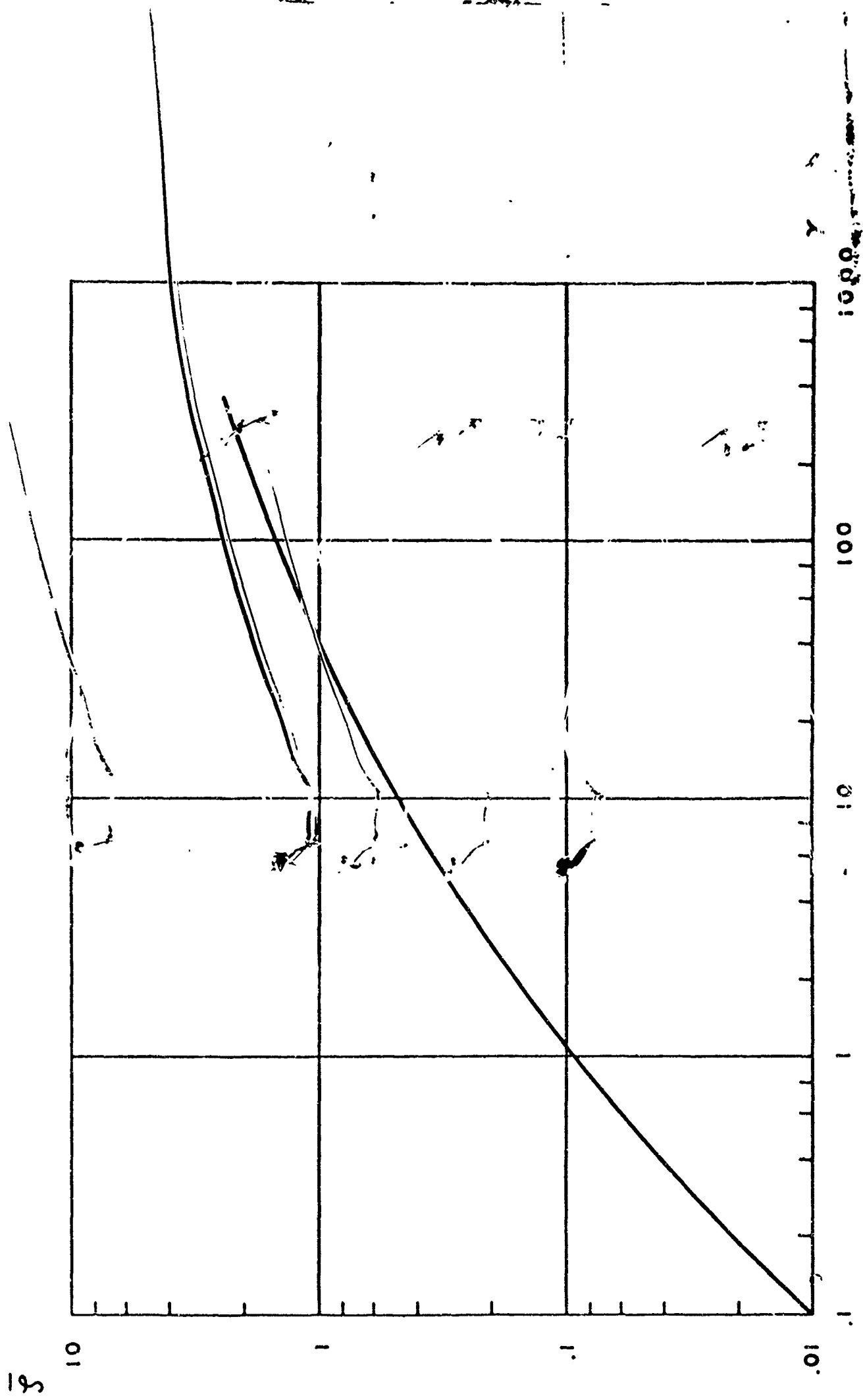


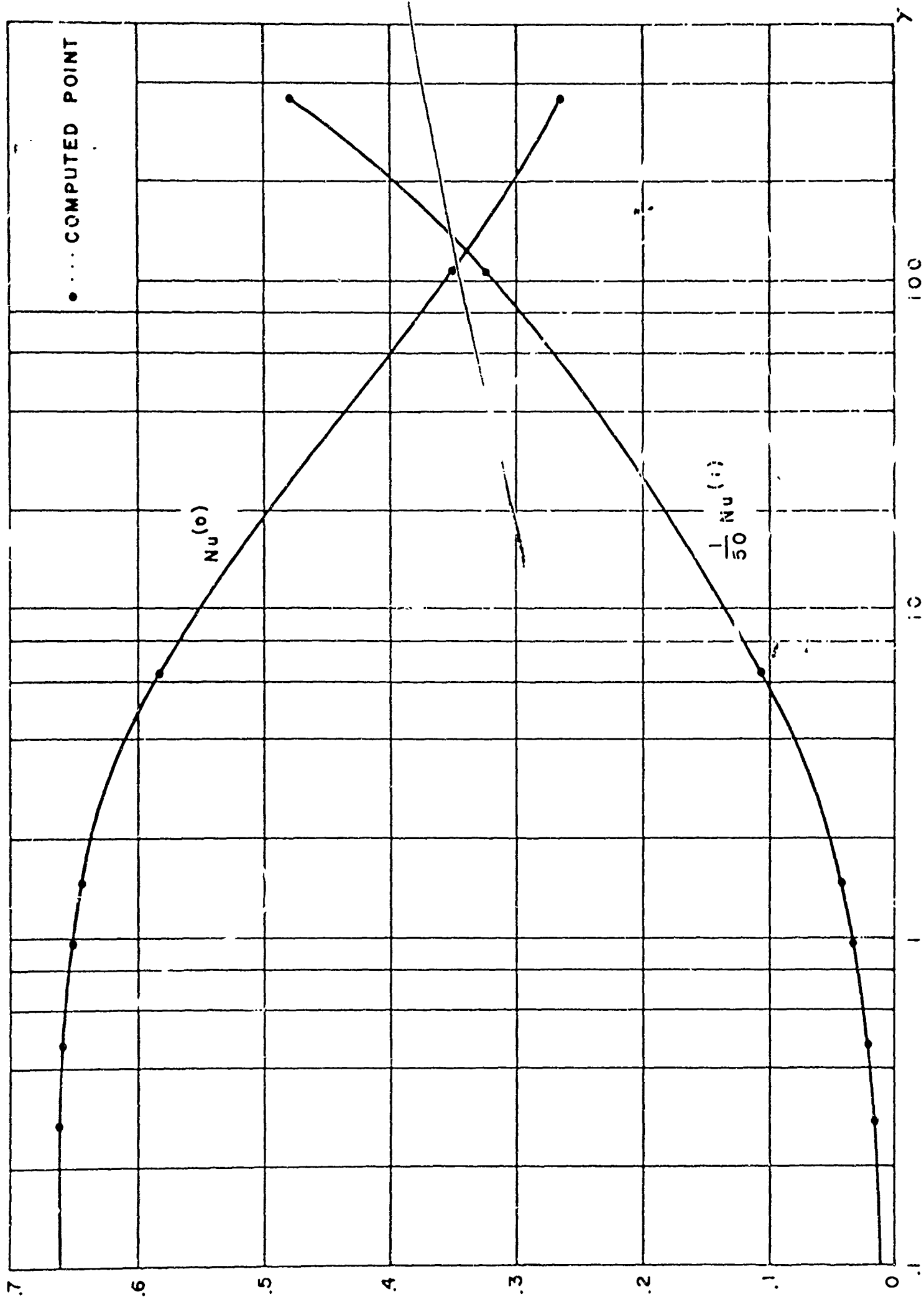












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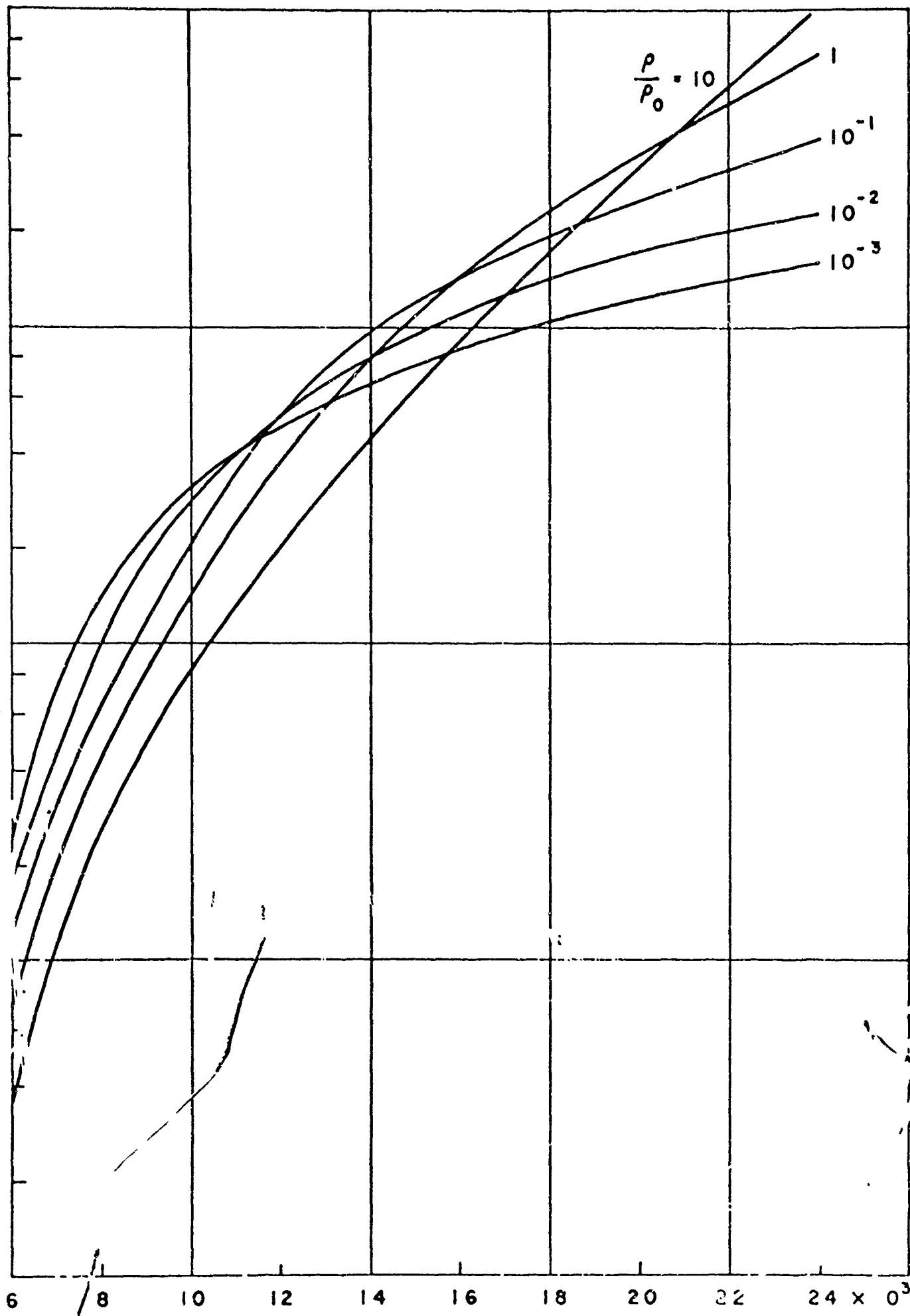
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10^4

10^3

10^2

10^1



$T (^{\circ}\text{K})$

