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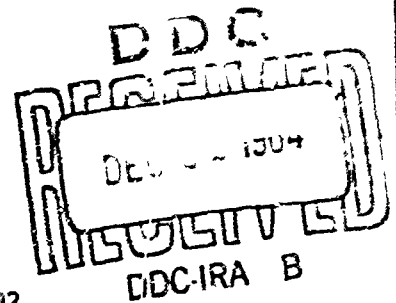


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Air Force Systems Command
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FOREWORD

The research reported herein was performed by The Moore School of Electrical Engineering, University of Pennsylvania under contract number AF 30(602)-2888 between the University of Pennsylvania and Rome Air Development Center, Air Force Systems Command, United States Air Force, Griffiss Air Force Base, New York. Project direction at RADC was obtained from Mr. R. Powers.

This report (contractor's number 64-25) was written by H. Kritikos, R. Dresp and K-C Lang of The Moore School of Electrical Engineering.

ABSTRACT

This work is divided into two parts: A theoretical study and a discussion of actual applications. In the first part, a brief introduction of the common antenna synthesis procedures is presented. Two new novel approaches were also suggested. First, the use of the signal-to-noise ratio as a criterion of design of arrays. Second, the use of the prolate spheroidal functions for aperture synthesis.

In the second part, a number of high performance antennas as well as experimental work performed in the University of Pennsylvania is discussed. Practical techniques, such as tunneling application of microwave absorbing material, are described and evaluated.

PUBLICATION REVIEW

This report has been reviewed and is approved. For further technical information on this project, contact Mr. Wayne Woodward, RADC (EMCVR), GAFB, NY, X-6148.

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I. Introduction

The ever increasing technical requirements placed on present day communication systems have created the need of improving the effectiveness of the presently available types of antennas. In particular, with the congestion of radiating systems and the high levels of interference, side-lobe reduction is one of the most important problems which one faces. The purpose of this study is to examine some of the presently available methods of side-lobe reductions and to offer new contributions.

This study is basically divided into two parts: the first part examines side-lobe reductions from an idealized theoretical point of view and the second considers practical applications.

The theoretical studies are of a very general nature and are intended to provide broad conceptual outlines. General problems of antenna synthesis will be examined for arrays as well as apertures. Other types of antennas will be examined in future studies. In the area of linear arrays, a new method of design is developed which maximizes the signal to noise ratio rather than simply maximizing the gain in order to avoid the rigidity of only the side-lobe reduction point of view. The work of Chu (1948) and Woodward (1947) is also extended to cover supergain phenomena in aperture antennas.

In the second part, practical applications are examined. In order to illustrate some of the theoretical points and to demonstrate the present state of the art, a number of interesting cases will be examined. Practical arrays having sidelobe levels below - 30 db, the

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horn parabola antenna having far sidelobes of - 50 db and other interesting antennas will be examined.

PART A

THEORETICAL ASPECTS OF ANTENNA DESIGN

2.0 Arrays

The simplest and most fundamental form of radiators are arrays of isotropic sources, and have been examined by S. A. Schelkunoff (1943), J. D. Kraus (1950), S. Ramo and Whinnery (1953), H. Friis and S. A. Schelkunoff (1952) and by H. Jasik (1961). These are idealizations because in practice isotropic sources do not exist and other problems such as inter-element interaction and losses can introduce serious design problems. This idealized discussion however is important for introducing the conceptual framework for further work.

Perhaps one of the most important contributions to array synthesis was made by Schelkunoff (1943), who was able to show that the number of elements of an array are its degrees of freedom. If one allows the number to go to infinity and is free to choose amplitude and phase, any pattern can be approximated with a vanishing error. The number of elements can increase either by keeping the inter-element distance constant or by keeping the total dimensions of the array constant.

In the first case one has physically large cumbersome structures and in the second the problems of supergain set in. In supergain some of the problems are the following: large inter-element interaction, the presence of high reactive fields and consequently losses and narrow bandwidth. Practice dictates the placing of an upper limit on the number of elements and the minimum inter-element distance.

In addition, extremely high values of currents should always be avoided.

Present day array synthesis therefore are attempts to find the best compromise for a given number of elements, minimum inter-element distance and practical excitation values. A brief discussion of the commonly used techniques which are readily available in the literature will be given below as an introduction to our contribution which is in the next section.

2.1 Uniformly Spaced Arrays

The simplest forms of arrays are the uniformly spaced arrays which have a uniform amplitude distribution. S. A. Schelkunoff (1943) and later S. Silver (1949) were among the first to examine these arrays. The main properties for a large number of elements are listed in Table I.

Table I

	<u>End Fire</u>	<u>Broadside</u>
Gain	$\frac{4l}{\lambda}$	$\frac{4l}{\lambda}$
First side-lobe	- 13.5 db	- 13.5 db
Beam Width	$2\sqrt{\frac{2\lambda}{nl}}$	$\sqrt{\frac{2\lambda}{nl}}$

where λ is the wavelength, l is the length of the array, and n is the number of elements.

Notice that in the first two parameters, the number of the elements does not appear. Also it is important to realize that neither the aperture length or the length of the array affect the side-lobe

level. This is similar to the radiation pattern of a uniform aperture where the first sidelobe level is -17 db regardless of the aperture size.

2.2 Progressive Phasing

Hansen and Woodward (1938) have shown that progressive phasing (amplitude constant and phase difference between pairs of elements not constant) has the effect of increasing the gain and narrowing the beamwidth at the expense of increasing the sidelobes. Obviously, this is an undesirable method but it is instructive because it shows that a narrow beamwidth is always obtained with large sidelobes.

2.3 Binomial Distribution

This is another classical case which illustrates the opposite of the above situation. E. C. Jordan (1950), J. D. Kraus (1950), and S. Ramo and Whinnery (1953) give a good discussion of this method. In this method the amplitudes of the elements have a binomial distribution but the phase difference between elements is constant, with the result that the sidelobes are reduced at the expense of a thicker beamwidth.

2.4 Tchebytcheff Distribution

This is perhaps the most useful and most widely used method of synthesis because it offers the best compromise between sidelobe levels and beamwidth. Specifically, it provides for a given sidelobe the minimum beamwidth and vice-versa. Again, E. C. Jordan (1950), J. D. Kraus (1950) and S. Ramo and Whinnery (1953) give a good discussion of this method.

2.5 Associated Polynomial Methods

These are general methods which have been developed by S. A. Schelkunoff (1943) and S. Silver (1949) to approximate patterns of a more complex nature. The previously mentioned techniques can be considered as special cases of the associated polynomial method. Specifically, for the sidelobe reduction problem this method does not offer any specific advantages except the additional flexibility of including other constraints in the design.

2.6 Non-Uniformly Spaced Arrays

Harrington (1963) has shown that in some cases non-uniformly spaced arrays compared favorably with the well known design techniques. Harrington's work has initiated the interest of a number of workers in this field. The advantage which aroused this interest is the attractive prospect that no phasing or amplitude control is required but only the positioning of the elements.

A summary of these design techniques can be found in Table II.

TABLE II
DESIGN METHODS OF ARRAYS

Methods	Advantages	Disadvantages
Uniform Distribution	Simplicity	High Sidelobes - 13 db
Progressive Phasing	High Gain, Narrow Beam	High Sidelobes
Binomial Distribution	Thick Beam	Low Sidelobes
Tchebytcheff Distribution	Optimum Compromise	
Associate Polynomial	Flexibility	Mathematical Complexity
Non-Uniform Positioning	Flexibility	Mathematical Complexity
	Possibility of keeping excitations the same	

3.0 The Signal to Noise Ratio as a Design Criterion of Linear Arrays

In the past few years the trend in antenna design has been to consider and evaluate the antenna in terms of the environmental conditions. In particular, the acceptance of the criterion of maximization of the signal to noise ratio of a receiving antenna introduces a new way of thinking, which if carried to its logical conclusion is capable of dictating design procedures which optimize the antenna performance. This has been recognized by a number of workers in the field and several contributors have appeared in the literature. Bracewell (1962) has considered a number of problems involved with the design of high gain pencil beam antennas with particular interest in radio astronomy applications. Dawirs et al. (1961) have examined the problem of the optimum illumination of a parabolic reflector using the signal to noise ratio.

In this paper the design problem of the simple but yet important linear one-dimensional radiating array of isotropic sources will be examined. A design procedure is developed which maximizes the signal to noise ratio for the case where the array is used as a receiving antenna in a noisy environment.

3.1 Formulation of the Problem

One of the main difficulties in parameter optimization problems is the development of a procedure which gives the optimum value of the parameters and at the same time is manageable by common computational techniques. A particular array whose optimization is possible without excessive computational complications is examined below.

Consider $2N + 1$ finite isotropic sources arranged along a straight line (see Fig. 1). The spacing s and phase difference between adjacent elements is specified by δ (where $\delta = \frac{2\pi s}{\lambda} = \frac{2\pi}{\rho}$, ρ is a constant, λ is the wavelength). The excitation of the n^{th} elements is of the form $\frac{1}{2} (x_{n+1} \pm jx_n)$ where x_n, x_{n+1} are the parameters to be optimized. The far field amplitude pattern $g(\theta)$ of such an array is

$$g(\theta) = \frac{e^{jkR_0}}{R_0} \frac{1}{2} \left[(x_N - jx_{N-1}) e^{-jN\delta(\cos \theta - 1)} \dots (x_2 - jx_1) e^{-j\delta(\cos \theta - 1)} + x_0 + (x_2 + jx_1) e^{j\delta(\cos \theta - 1)} \dots (x_N + jx_{N-1}) e^{jN\delta(\cos \theta - 1)} \right] \quad (1)$$

Letting $\psi = \delta(\cos \theta - 1)$ the gain of such an array is

$$G(\psi) = C \left[\sum_{n=0}^N x_{2n} \cos n\psi + \sum_{n=1}^{N-1} x_{2n+1} \sin n\psi \right]^2 \quad (2)$$

where C is the normalization constant determined from the condition

$$\int G d\Omega = 4\pi \quad (3)$$

where $d\Omega$ is an element of solid angle.

The gain then for $\theta = 0$ is

$$G(\psi = 0) = C \left[\sum_{n=0}^N x_{2n} \right]^2 \quad (4)$$

The noise temperature T of an antenna is given by the expression

$$T = \frac{1}{4\pi} \int G(\psi, \phi) t_1(\psi, \phi) d\Omega \quad (5)$$

where $G(\psi, \phi)$ is the gain of antenna and $t_1(\psi, \phi)$ is the temperature of a ray in the direction θ, ϕ

$$d\Omega = d(\cos \theta) d\phi$$

but

$$\delta d(\cos \theta) = d\psi \quad (6)$$

Therefore

$$T = \frac{1}{4\pi\delta} \int_{-2\delta}^0 \int_0^{2\pi} G(\psi, \phi) t_1(\psi, \phi) d\psi d\phi \quad (7)$$

Since G is function only of ψ , then one has

$$T = \int_{-2\delta}^0 G(\psi) t(\psi) d\psi \quad (8)$$

where

$$t(\psi) = \frac{1}{4\pi\delta} \int_0^{2\pi} t_1(\psi, \phi) d\phi \quad (9)$$

The signal to noise ratio from equations (2), (4), (8) is

$$\frac{S}{N} = \frac{G(\psi=0)}{T} = \frac{\left[\sum_{n=0}^N x_{2n} \right]^2}{\int_{-2\delta}^0 \left[\sum_{n=0}^N x_{2n} \cos n\psi + \sum_{n=1}^{N-1} x_{2n+1} \sin n\psi \right]^2 t(\psi) d\psi} \quad (10)$$

The problem now can be stated in terms of the array parameters.

For a signal of unit strength in the direction $\theta = 0$ and a noise picture given by $t(\psi)$, find the parameters x_n such that the signal to noise ratio is a maximum.

3.2 Determination of the Parameters

Consider the denominator of Eq. (10). Since $t(\psi)$ is positive ($t(\psi) > 0$) the integral is always a positive quantity. Furthermore, it is of a quadratic form with respect to the variable x_n .

In a matrix form the denominator is

$$\frac{T}{C} = X'AX \quad (11)$$

where X' is a row matrix and X is a column matrix; x' is the transpose of X and

$$A = \begin{vmatrix} a_{11} & a_{12} \cdots a_{1n} \\ a_{21} & a_{22} \cdots \cdot \\ \vdots & \vdots \quad \vdots \\ a_{n1} & a_{n2} \cdots a_{nn} \end{vmatrix} \quad (12)$$

The quadratic form $X'AX$ is positive definite because $X'AX > 0$. It is therefore always possible through some transformation to put it in the form of sum of squares.

$$X'AX = Y'KY \quad (13)$$

where K is a diagonal matrix. The transformation does not have to be orthogonal and the transformation which seems to be appropriate here is that which is obtained from the completion of the squares.

It can be shown that for a positive definite quadratic form [see G. Hadley (1961)]

$$X'AX = \sum_n k_n y_n^2 \quad (14)$$

where $Y = SX$ and $S = S_{n-1} \dots S_2 S_1$ (15)

$$S_1 = \begin{vmatrix} 1 & a_{12} & a_{13} & \dots & a_{1n} \\ & a_{11} & & & \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{vmatrix} \quad S_2 = \begin{vmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & a_{23} & \dots & a_{2n} \\ & & a_{22} & & \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{vmatrix} \text{ etc.} \quad (16)$$

also

$$k_1 = a_{11}$$

$$k_2 = \frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{a_{11}}$$

$$k_3 = \frac{\begin{vmatrix} a_{12} & a_{11} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & & \\ & a_{12} & \\ & & a_{22} \end{vmatrix}} \text{ etc.} \quad (17)$$

Inverting $Y = SX$, one has $X = S^{-1} Y$. Therefore, in general, equation (4) becomes

$$G(\psi = 0) = C \left[\sum_{n=0}^N x_{2n} \right]^2 = C \left[\sum_n b_n y_n \right]^2 \quad (18)$$

Equation (10) finally becomes

$$\frac{S}{N} = \frac{\left(\sum_n b_n y_n\right)^2}{\left(\sum_n k_n y_n\right)^2} \quad (19)$$

Making the substitution

$$z_n = \sqrt{k_n} y_n \quad (20)$$

one has

$$\frac{S}{N} = \frac{\left(\sum_n \frac{b_n}{\sqrt{k_n}} z_n\right)^2}{\sum_n z_n^2} \quad (21)$$

From Schwartz's inequality one has

$$\left(\sum_n \frac{b_n}{\sqrt{k_n}} z_n\right)^2 \leq \left[\sum_n \frac{b_n}{\sqrt{k_n}}\right]^2 \left[\sum_n z_n^2\right] \quad (22)$$

or

$$\frac{\left(\sum_n \frac{b_n}{\sqrt{k_n}} z_n\right)^2}{\sum_n z_n^2} \leq \sum_n \left(\frac{b_n}{\sqrt{k_n}}\right)^2 \quad (23)$$

The greatest upper bound of the $\frac{S}{N}$ which is $\sum_n \left(\frac{b_n}{\sqrt{k_n}}\right)^2$ is obtained by letting

$$z_n = \frac{b_n}{\sqrt{k_n}} \quad (24)$$

The unknown parameters X therefore can be finally obtained and they are

$$\mathbf{X} = \mathbf{S}^{-1} \mathbf{Y}, \quad y_n = \frac{z_n}{\sqrt{k_n}} = \frac{b_n}{k_n} \quad (25)$$

The Schwartz inequality therefore is a powerful tool by which one can find the greatest upper bound of the $\frac{S}{N}$ ratio without having to go into the lengthy procedure of finding the partial derivatives with respect to each x and then solve a complex system of equations for the optimum values of the parameters \mathbf{X} .

3.3 An Application to a Typical Array

Consider an array of three elements having a $\delta = \frac{\pi}{2}$ and an environment with a noise temperature function of the following form

$$t(\psi) = -\sin \psi - \pi < \psi < 0 \quad (26)$$

The gain of such an array is

$$G = C \left[x_0 + x_1 \sin \psi + x_2 \cos \psi \right]^2 \quad (27)$$

The noise temperature is

$$\begin{aligned} T = -C \int_{-\pi}^0 & \left[x_0^2 + x_1^2 \sin^2 \psi + x_2^2 \cos^2 \psi + 2x_0 x_1 \sin \psi + \right. \\ & \left. + 2x_0 x_2 \cos \psi + 2x_1 x_2 \sin \psi \cos \psi \right] \sin \psi \, d\psi = \quad (28) \\ & = \left[2x_0^2 + \frac{4}{3} x_1^2 + \frac{2}{3} x_2^2 - \pi x_0 x_1 \right] \end{aligned}$$

The A matrix is of the following form

$$A = \begin{pmatrix} 2 & -\frac{\pi}{2} & 0 \\ -\frac{\pi}{2} & \frac{4}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{pmatrix} \quad (29)$$

$$S_1 = \begin{pmatrix} 1 & \frac{\pi}{4} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad S_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (30)$$

$$S = S_2 S_1 = \begin{pmatrix} 1 & \frac{\pi}{4} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (31)$$

$$k_1 = 2 \quad k_2 = \frac{\begin{vmatrix} 2 & -\frac{\pi}{2} \\ \frac{\pi}{2} & \frac{4}{3} \end{vmatrix}}{2} = .105$$

$$k_3 = \frac{\begin{vmatrix} 2 & -\frac{\pi}{2} & 0 \\ -\frac{\pi}{2} & \frac{4}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{vmatrix}}{\begin{vmatrix} 2 & -\frac{\pi}{2} \\ -\frac{\pi}{2} & \frac{4}{3} \end{vmatrix}} = .665$$

One therefore has

$$\begin{aligned}y_0 &= x_0 - \frac{\pi}{4} x_1 \\y_1 &= x_1 \\y_2 &= x_2\end{aligned}\tag{32}$$

or

$$\begin{aligned}x_0 &= y_0 + \frac{\pi}{4} y_1 \\x_1 &= y_1 \\x_2 &= y_2\end{aligned}\tag{33}$$

Also

$$\begin{aligned}x_0 + x_2 &= y_0 b_0 + y_1 b_1 + y_2 b_2 = y_0 + \frac{\pi}{4} y_1 + y_2 \\b_0 &= 1 \quad b_1 = \frac{\pi}{4} \quad b_2 = 1\end{aligned}\tag{34}$$

The signal to noise ratio is then

$$\frac{S}{N} = \frac{[y_0 + .785 y_1 + y_2]^2}{2 y_0^2 + .105 y_1^2 + .666 y_2^2}\tag{35}$$

The maximum value of this ratio takes place for

$$y_n = \frac{b_n}{k_n}$$

or

$$y_0 = .500 \quad y_1 = 7.48 \quad y_2 = 1.50$$

The maximum value of $\frac{S}{N}$ is

$$\frac{S}{N} \max = \sum \frac{b_n^2}{k_n} = .500 + 5.87 + 1.50 = 7.87$$

and

$$x_0 = 6.37 \quad x_1 = 7.48 \quad x_2 = 1.50$$

From Eq. (3) one finds finally that $C = .110$.

It is instructive to compare this with a normal array having constant excitation. One has then

$$x_0' = 1 \quad x_1' = 0 \quad x_2' = 1$$

The gain is

$$G' = C' \left[1 + \cos \psi \right]^2 \quad (36)$$

From the normalization condition $C' = .666$.

The noise temperature is

$$\frac{T'}{C'} = - \int_{-\pi}^0 \left[1 + \cos^2 \psi + 2 \cos \psi \right] \sin \psi \, d\psi = \frac{8}{3}$$

The signal to noise ratio is

$$\frac{S'}{N'} = \frac{4}{\frac{8}{3}} = 1.5$$

An improvement therefore by a factor of $\frac{S}{N} / \frac{S'}{N'} = 5.3$ has resulted by optimizing the excitation of the elements. Figure II shows the patterns of the optimized and the common array. The common array has a pattern which has most of the gain concentrated in the front hemisphere and very little in the back. This is a common conservative pattern which one should choose for cases where the interfering noise is not known and it is desirable to keep the average sidelobes down. Also in Fig. II one can examine the optimized pattern which shows a high forward gain at the expense of a large backlobe. The optimized pattern is stretched

in the forward and backward direction in order to avoid the highly noisy area which occurs in the equatorial region ($\theta \simeq \frac{\pi}{2}$).

3.4 Discussion of Results

The particular technique which was developed here touches only a small part of the many optimization problems present in antenna synthesis and design. It deals, however, with one of the simplest forms of radiating systems, the linear array constrained with a finite number of isotropic sources in a noisy environment receiving a signal from a point source in space. For this case a technique has been developed which provides the amplitude and the phase of each element of the array such that the signal to noise ratio is maximum. The great advantage of the suggested method is that it is simple from a computational point of view. The greatest difficulty encountered in the development is the inversion of a matrix [Eq. (15)] whose elements below the diagonal are zero.

The technique followed in this paper can be applied to a variety of other situations. For example, instead of isotropic sources one can consider dipoles and other three dimensional spatial configurations of the elements. Also another degree of complexity can be introduced by considering the signal not only as a point source but as having a certain distribution in space. The common goal of all the problems should be to reduce the signal to noise ratio into some form of the Schwartz inequality and thus take advantage of its powerful results.

4.0 Aperture Antennas

For aperture antennas the pattern is partitioned in two regions: first the main lobe and near sidelobe region, and second the far sidelobe and shadow region. The mathematical methods for the determination of the pattern depend on the type of antenna.

For parabolic reflectors the complete antenna pattern can be found by the induced current technique, as described by S. Silver (1949) and H. Jasik (1961). The shortcoming of the induced current technique is its complicated form and the additional burden of numerical calculation. Simpler and easier to use approximations are the aperture methods. These methods are described by S. A. Schelkunoff (1943), S. Silver (1949), E. C. Jordan (1950) and by J. D. Kraus (1950). The normal aperture method provides the pattern in the main and near sidelobe region and the extended aperture methods provide the far sidelobe and shadow region. A detailed discussion of these was given by Kritikos (1963).

The great advantage of these aperture methods is their flexibility, no numerical complication, and the possibility of creating useful intuitive models for the engineer. Another method which is used for the far sidelobes and shadow region is the diffracted ray techniques, which were recently presented by L. Peters, Jr. and P. C. Ruddock (1963). These techniques are very easy to use and are widely used for the shadow radiation of horns and parabolic reflectors. Their shortcoming is the fact that in the region of caustics the field has singularities and has to be supplemented by other techniques. A compilation of these methods appears in Table III.

Most of the aperture antenna synthesis problems are referred only to the main near-sidelobe region. Much work has been done in this area but it is mostly of cut and try nature. The general trends and the effect of tapering the distribution in the edges are well known and are extensively reported in the literature. In this report an effort was made to establish the synthesis on a more rigorous basis by using the newly recognized prolate spheroidal function. A discussion of this synthesis procedure appears in the next section.

TABLE III
THEORETICAL METHODS OF PREDICTING
PATTERNS OF APERTURE ANTENNAS

Method	Application	Advantage	Disadvantage
Exact Solution	Does not exist		
Induced Current	Parabolic Reflector	Provides whole pattern	Numerically cumbersome
Aperture	Parabolic Reflectors Horns	Simplicity	Applicable only to near sidelobes
Extended Aperture	Parabolic Reflector	Simplicity	Applicable only to shadow region
Diffracted Ray Techniques	Parabolic Reflector Horns	Simplicity	Applicable only to far side-lobe regions. Have infinities at caustics.

5.0 Aperture Synthesis

In this section, the problem of aperture antenna synthesis is treated in its most general form by the use of the prolate spheroidal wave functions.* By the synthesis problem is meant that one is interested in finding an aperture distribution of current or field that gives a radiation pattern with prescribed properties such as, for example, low sidelobes or optimum gain. Previously, C. J. Bouwkamp and N. G. de Bruijn (1946), P. M. Woodward (1947), L. J. Chu (1948), P. M. Woodward and J. D. Lawson (1948), T. T. Taylor (1955) and R. C. Hansen (1960) investigated the aperture antenna synthesis problem. However, their solutions were seriously limited by the lack of direct mathematical formalism. It was L. J. Chu (1948) who first pointed out some of the fundamental concepts and limitations of the synthesis problems. He determined the optimum performance of a general antenna. His major contribution, as far as this work is concerned, is the fact that associated with the optimum performance of an antenna there exists a high reactive field. Since Chu concerned himself only with the field distribution on a spherical surface surrounding the antenna, his results are only of qualitative importance to aperture synthesis problems. In 1948, Woodward and Lawson (1948) were the first to specify the aperture field distribution for a desired

* At the time this problem was being solved, a similar approach was suggested by Donald R. Rhodes in "The Optimum Line Source for the Best Mean Square Approximation to a Given Radiation Pattern" in the IEEE Trans. on Antennas and Propagation, vol. AP-11, No. 4, July 1963. His final results, however, are of different nature.

pattern. They arrived at their results by rather cumbersome numerical analyses and by intuitive reasoning. From 1948 on, T. T. Taylor (1955) and R. C. Hansen (1960) have offered approximate methods for the solution of the problem. The limitations of these methods is that they are essentially semi-numerical methods and as such they lack the elegance and precision of more direct and rigorous solutions. In this report it is shown that the aperture antenna synthesis problem can be treated exactly in terms of the expansions of prolate spheroidal orthogonal functions. In this way the results of this report parallel those of L. J. Chu who used Legendre functions but limited his treatment only to spherical surfaces.

Closely connected with aperture antenna synthesis is the concept of supergain. It is shown here that if no restrictions are placed on the aperture illumination function, infinite gain or supergain is possible. The arbitrarily large gain is produced by an interference process in which large phase changes and large currents are used to produce a low value of effective radiating current. This well-known phenomenon is called supergain. It is generally avoided in practice because it involves, as is shown here, large energy storage and, hence, high losses. An indication of these losses is given by the supergain ratio as defined by T. T. Taylor. (1955).

The following discussion has been divided into four parts. In the first part the synthesis problem and its solution are presented. In the second we establish the criteria for maximum gain or supergain.

In the third a discussion of the significance of the results is obtained. And lastly, as an appendix, the basic properties of the prolate spheroidal wave functions are given.

5.1 Formulation of the Problem

The prolate spheroidal functions, as given by D. Slepian and H. D. Pollak (1961), will be applied to solve exactly the synthesis problem of a line source. It is well known that the relationship between the aperture and radiation pattern is given by a Fourier transform pair.

$$F(u) = \frac{1}{2\pi a} \int_{-a}^a f(t) e^{itu} dt \quad (37)$$

$$f(t) = \int_{-\infty}^{\infty} F(u) e^{-itu} du \quad (38)$$

where (see figure below)

$$u = \frac{2\pi a}{\lambda} \sin \theta$$

$$t = \frac{x}{a}$$

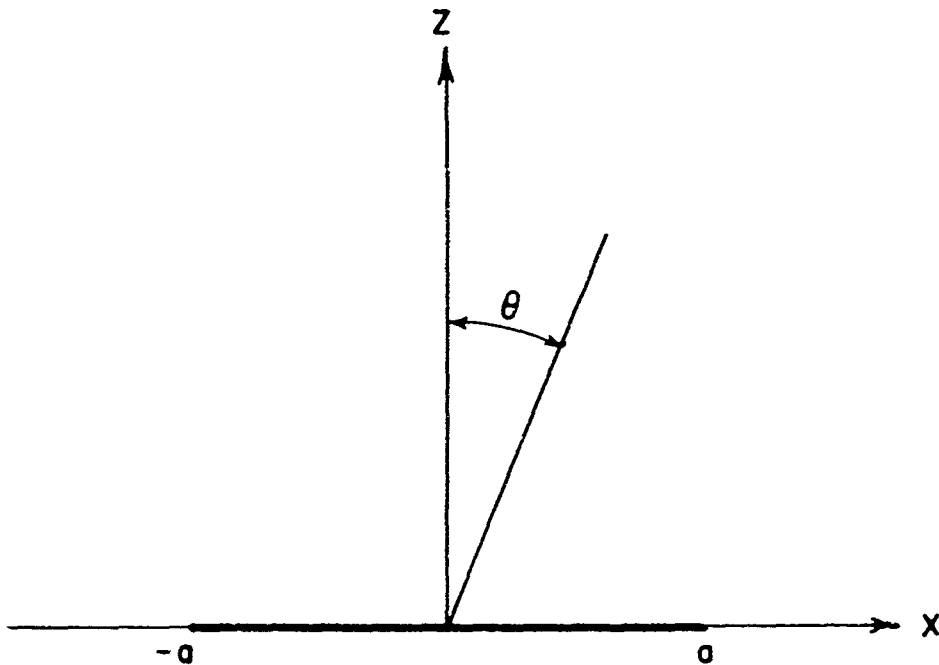
$$c = \frac{2\pi a}{\lambda}$$

$2a$ = size of the aperture

θ = the angle measured from the vertical through the center of the aperture

$F(u)$ = radiation pattern

$f(t)$ = aperture distribution function.



The antenna aperture

Now, using the results of Appendix A, the far field pattern can be expanded in terms of the orthogonal angular prolate spheroidal wave functions.

$$F_N(u) = \sum_{n=0}^N a_n S_{0n}(c, \frac{u}{c}) \quad (39)$$

where $\frac{u}{c} = \eta = \sin \theta$ and $c = \frac{2\pi a}{\lambda}$ (40)

and a_n are constant coefficients to be determined. In equation (39) the summation to only N values has been taken. This can be done since $F(u)$ converges in the mean square sense and the mean square error can be made as small as desired by making N sufficiently large.

The coefficients a_n can be determined from the orthogonality property (eq. A15) within the band limited (aperture limited) region.

Upon multiplying both sides of eq. (39) by $S_{on}(c, \frac{u}{c})$ and integrating from -1 to +1.

$$\int_{-1}^{+1} F_N(u) S_{on}(c, \frac{u}{c}) du = \int_{-1}^{+1} \sum_{n=0}^N a_n S_{on}(c, \frac{u}{c}) S_{on}(c, \frac{u}{c}) du \quad (41)$$

so that

$$a_n = \frac{1}{N_{on}} \int_{-1}^{+1} F_N(u) S_{on}(c, \frac{u}{c}) du \quad (42)$$

And as long as

$$\sum_{n=0}^N a_n^2$$

remains finite, any set of a_n 's will describe an aperture limited function.

In order to obtain the aperture field distribution, substitute eq. (39) into eq. (37). So that

$$\sum_{n=0}^N a_n S_{on}(c, \frac{u}{c}) = \frac{1}{2\pi} \int_{-1}^{+1} f(t) e^{itu} du \quad (43)$$

and with the aid of eq. (A.18)

$$\sum_{n=0}^N a_n S_{on}(c, \frac{u}{c}) = \frac{i^n}{\pi} R_{on}(c, 1) f(t) \quad (44)$$

or

$$f(t) = \sum_{n=0}^N \frac{\pi i^{-n}}{R_{on}(c, 1)} a_n S_{on}(c, t) \quad (45)$$

and zero outside the interval.

Now the basic equations that allow one to solve the synthesis problem exactly have been derived. Once a radiation pattern $F_N(u)$ has been decided upon, the constant a_n in eq. (45) can be determined by use of eq. (42).

The usefulness of these results are illustrated by an example. Let the far field pattern be of the form of a delta function. Such a radiation pattern is closely allied with supergain. And upon solving this problem, the supergain criteria for the current distribution function on the aperture has essentially been established. The unit impulse is defined as follows.

$$\delta(u) = \begin{cases} 0 & u \neq 0 \\ \infty & u = 0 \end{cases} \quad (46)$$

and

$$\int_a^b f(u) \delta(u - u_0) du = f(u_0) \quad (47)$$

if u_0 is within the interval (a, b) . From eqs. (39) and (42)

$$F(u) = \sum_{n=0}^{\infty} a_n S_{on}(c, \frac{u}{c}) \quad (48)$$

$$a_n = \frac{1}{N_{on}} \int_{-1}^{+1} F(u) S_{on}(c, \frac{u}{c}) du \quad (49)$$

and upon substituting $\delta(u)$ for $F(u)$

$$a_n = \frac{1}{N_{on}} \int_{-1}^{+1} S_{on}(c, \frac{u}{c}) \delta(u) du \quad (50)$$

we have

$$a_n = \frac{S_{on}(c, 0)}{N_{on}} \quad (51)$$

So that

$$F_N(u) = \sum_{n=0}^N \frac{S_{on}(c, 0)}{N_{on}} S_{on}(c, \frac{u}{c}) \quad (52)$$

This is the spectrum of the delta function. Now in order to obtain the aperture distribution $f(t)$, the Fourier transform property is used. From eq. (45) the aperture distribution is obtained.

$$f(t) = \sum_{n=0}^N \frac{\pi i^{-n}}{R_{on}(c, 1)} \frac{S_{on}(c, 0)}{N_{on}} S_{on}(c, t) \quad (53)$$

Equation (53) can also be written in the form

$$f(t) = \sum_{n=0}^N \frac{\pi i^{-n}}{\sqrt{\frac{\pi \lambda_{on}}{2c}}} \frac{S_{on}(c, 0)}{N_{on}} S_{on}(c, t) \quad (54)$$

or

$$f(t) = \sqrt{2\pi c} \sum_{n=0}^N \frac{i^{-n} S_{on}(c, 0)}{\sqrt{\lambda_{on}} N_{on}} S_{on}(c, t) \quad (55)$$

The spectrum of the delta function has been plotted in Fig. 3 for $N = 8$ terms. As can be seen from the graph the figure is a very rough approximation to the delta function. Due to the lack of more extensive tables for the prolate spheroidal functions this was the best that could be calculated at the present. Equation (55), the aperture

distribution, is plotted in Fig. 4. It is seen that the variation of field amplitude is sinusoidal and becomes extremely large at the end points. If more extensive tables had been available, it would have been possible to get a better approximation of the delta function. The variation of the aperture function would then have been expected to be at an even higher frequency and extremely large currents on the edges.

5.2 Criterion for Maximum Gain

Now it is desirable to investigate what the coefficient a_n should be in order to give maximum gain. The far field is given by

$$F_N(u) = \sum_{n=0}^N a_n S_{on}(c, \frac{u}{c}) \quad (56)$$

and the aperture field is given by

$$f(t) = \sum_{n=0}^N \frac{\pi i^{-n}}{R_{on}(c, l)} a_n S_{on}(c, t) \quad (57)$$

The gain is defined by

$$G = \frac{|F(o)|^2}{\int_{-1}^{+1} |F(u)|^2 du} \quad (58)$$

or

$$G = \frac{\left| \sum_{n=0}^N a_n S_{on}(c,0) \right|^2}{\int_{-1}^{+1} \left| \sum_{n=0}^N a_n S_{on}\left(c, \frac{u}{c}\right) \right|^2 du} \quad (59)$$

Upon integrating

$$G = \frac{\left| \sum_{n=0}^N a_n S_{on}(c,0) \right|^2}{\sum_{n=0}^N a_n^2 N_{on}} \quad (60)$$

where use of the following orthogonality property has been made

$$\int_{-1}^{+1} S_{on}^2(c, \eta) d\eta = N_{on} \quad (61)$$

In order for G to be a maximum (see Appendix B)

$$a_n = \frac{S_{on}(c,0)}{N_{on}} \quad (62)$$

Consequently,

$$G = \frac{\left| \sum_{n=0}^N \frac{S_{on}^2(c,0)}{N_{on}} \right|^2}{\sum_{n=0}^N \frac{S_{on}^2(c,0)}{N_{on}}} = \sum_{n=0}^N \frac{S_{on}^2(c,0)}{N_{on}} \quad (63)$$

With this value of the coefficient of a_n , the radiation pattern is

as follows

$$F_N(u) = \sum_{n=0}^N \frac{S_{on}(c,0)}{N_{on}} S_{on}\left(c, \frac{u}{c}\right) \quad (64)$$

and the aperture distribution function is given by

$$f(t) = \sum_{n=0}^N \frac{\pi i^{-n}}{R_{on}(c,1)} \frac{S_{on}(c,0)}{N_{on}} S_{on}(c,t) \quad (65)$$

By comparing eqs. (52), (53) and eqs. (64), (65), it is seen that the aperture and field distribution under the criterion of maximum gain are the same as those for the impulse function. This is, of course, as expected since theoretically there is no limit for the gain that can be obtained from a line source.

5.3 Gain and Super-Gain Ratio

The term gain of a line source is defined as the ratio of power flux radiated in the direction of maximum intensity to the value of this flux averaged over all directions. According to T. T. Taylor (1955), the gain G_o which the entire antenna would have if the element factor were isotropic and under the assumption of a broadside beam is given by

$$G_o = \frac{4a}{\lambda} \frac{|F(o)|^2}{\int_{-2a/\lambda}^{2a/\lambda} |F(u)|^2 du} \quad (66)$$

The quantity $\lambda G_o / 4a$ will be called the specific gain, or

$$\frac{\lambda G_o}{4a} = \frac{|F(o)|^2}{\int_{-2a/\lambda}^{2a/\lambda} |F(u)|^2 du} \quad (67)$$

Now consider a line source with a distribution $f(t)$ of a fixed functional form and let the ratio of physical length of the aperture to wavelength be increased. This produces no change in $F(u)$, which is related to $f(t)$ through (37). However, it obviously changes the specific gain via the limits of the integral in (67). Evidently, then, the limiting value of the specific gain is an important line source parameter, especially since it depends only upon the functional form of $f(t)$. This is shown by the following expression, in which the Parseval formula

$$\int_{-\infty}^{\infty} f(t) \overline{g(t)} dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \overline{G(\omega)} d\omega \quad (68)$$

for Fourier integrals has been employed. The bar signifies the complex conjugate. Also

$$\left(\frac{\lambda G_0}{4a}\right)_{\infty} = \frac{|F(0)|^2}{\int_{-\infty}^{\infty} |F(u)|^2 du} \quad (69)$$

$$\left(\frac{\lambda G_0}{4a}\right)_{\infty} = \frac{\left| \int_{-1}^{+1} f(t) dt \right|^2}{2\pi \int_{-1}^{+1} |f(t)|^2 dt} \quad (70)$$

If $f(t)$ is now varied, it can be shown that $(\lambda G_0/4a)_{\infty}$ takes on its greatest value, namely unity, when $f(t)$ is uniform. It is only in

this sense, that a uniform distribution has a specific gain greater than that of any other type of line source distribution. It should be pointed out that S. Silver (1949) does not make this meaning of maximum gain clear. For in this case $2a/\lambda$ is finite and arbitrarily high specific gains can be synthesized. More definite, Taylor states that super-gaining must increase the value of the ratio of $\lambda G_o/4a$ to $(\lambda G_o/4a)_\infty$ with respect to the value it would have if the distribution were uniform. Taylor has called this ratio the super-gain ratio, given by

$$\gamma = \frac{\lambda G_o/4a}{(\lambda G_c/4a)_\infty} = \frac{\int_{-\infty}^{\infty} |F(u)|^2 du}{\int_{-\frac{2a}{\lambda}}^{\frac{2a}{\lambda}} |F(u)|^2 du} \quad (71)$$

This super-gain ratio for the case of the delta function radiation pattern is now evaluated. Then

$$\gamma_N = \frac{\int_{-a}^a \left| \sum_{n=0}^N \frac{S_{on}(c,0)}{N_{on}} S_{on}(c, \frac{u}{c}) \right|^2 du}{\int_{-\infty}^{\infty} \left| \sum_{n=0}^N \frac{S_{on}(c,0)}{N_{on}} S_{on}(c, \frac{u}{c}) \right|^2 du} \quad (72)$$

$$\gamma_N = \frac{\sum_{n=0}^N \frac{|S_{on}(c,0)|^2}{\lambda_{on}}}{\sum_{n=0}^N |S_{on}(c,0)|^2} \quad (73)$$

This ratio reduces to $\gamma_0 = \frac{1}{\gamma_0}$ when $N = 0$. Thus it is seen that γ_N has a lower bound whose value is dependent upon the value of c . Equation (73) is tabulated in Table IV for various values of c . It can be seen from Table IV that the super-gain ratio increases extremely rapidly as $\frac{2\pi a}{\lambda}$ becomes smaller. This indicates that the desired radiation pattern can be obtained as closely as desired, but only at the expense of extremely high reactive power due to the large value of currents on the aperture. Unfortunately, because of the lack of complete tables for the spheroidal functions, the γ was not calculated for higher values of c .

TABLE IV
 γ OF LINE SOURCE ANTENNA

Criterion: Maximum gain with fixed number of terms

$c = \frac{2\pi a}{\lambda}$	$\lambda_0(c)$	γ_0	γ_2	γ_4
0.5	.309	3.240	4724	$\sim \infty$
1.0	.572	1.750	120	$\approx 10^6$
2.0	.880	1.113	4.24	10^5

5.4 Conclusion

The antenna synthesis problem has been examined. The use of the prolate spheroidal wave functions allowed an exact solution of the problem. Application of the theory has been illustrated for the specific synthesis of a delta function field pattern. The graphical

results indicate that the current variation on the aperture is of a sinusoidal type with very high currents on the edges. When this graph is compared with the field pattern due to a uniform distribution, it is seen that a greater variation and higher values of current are required to obtain the delta function. An investigation of the gain and supergain ratio indicates that it is possible, theoretically at least, to obtain infinite gain. A calculation of the supergain ratio indicates that very high gain can only be achieved at the expense of extremely high reactive power. Thus, in order to reduce some of these unwanted conditions, it is necessary to place some limit on the amount of supergain allowed. The authors feel that a slight increase in reactive power will give much higher gain and a lower side-lobe level.

PART B
APPLICATIONS

6.0 General Discussion

In a report by G. Evans (1962) many interesting design descriptions of actually constructed antennas appear. Some of these antennas are examined in this report and the reasons for their high performance will be brought out and carefully discussed. Only three types of antennas will be discussed here: 1) arrays, 2) parabolic reflectors, and 3) horns.

6.1 The Litton Array

An interesting example of an array which was designed specifically to have low sidelobes (-35 db) is the one reported by the Litton Industries (1962). The important properties are listed below.

Radiating Elements: Horns with slots in the sides in order to reduce back lobes.

Number of Elements: 32

Distribution: Tchebytcheff

Excitations of Elements: By using directional couplers

Tolerances of Excitation: Amplitude 0.5% phase, 1.5 degree

Designed Sidelobe Level: 42 db

Measured: -32 db only in some parts of the L-band

The low sidelobe level of this array is obtained by the careful control of the excitation of the radiating elements. This was obtained by a very careful design of the feeding system. The feeding system

consists of a main line with individual horns fed by directional couplers. This system is superior to the one which divides the main line into a number of auxiliary channels which drive the elements, because the effects of the terminations are isolated by the directional couplers. Improvement of the performance of this array is expected to be obtained by a more precise control of the excitations and by using better component parts.

6.2 The Sylvania Array

Another high performance array which has been reported by R. M. Hergenrother and P. J. Nordquist (1962) in the literature is the one designed by Sylvania Corporation. The performance is as follows:

Radiating Elements: Open waveguides supported with horn-type tunnel.

Number of Elements: 60

Distribution: Taylor with 40 db taper

Excitation of Elements: Through a slotted main feedline

Tolerances of Excitations: Amplitude ± 0.5 db, phase $\pm 1^\circ$

Sidelobe Level: Measured better than 35 db

Frequency: 8882 kMc

This antenna has been designed primarily as a flexible array with adjustable phasing which would be capable of electronic beam steering and beam splitting. It was shown, however, that careful control of the excitations not only met the required operational flexibility but it produced low sidelobes as well.

6.3 Horn Parabola Antenna

One of the most interesting high performance antennas is the horn parabola antenna. A report on this antenna was given by A. B. Crawford, D. C. Hogg and L. E. Hunt (1961). It was developed by the Bell Telephone Laboratories and presently, despite its high cost of construction, is being used for a variety of applications ranging from satellite tracking to microwave links. A typical antenna of this type is the one used for the ECHO experiment. Its main characteristics are the following:

Gain: 43.3 db

Frequency: 2390 Mc

Aperture: 20 × 20 ft.

Aperture Illumination Efficiency: 76%

Polarization: Circular

Near Sidelobes: ~ -17 db

Far Sidelobes in Shadow Region: ~ -45 to -50 db

Spurious Sidelobes: -30 db at 342°

-35 db at 70° (Spillover lobe)

Cross-Polarization Level in Main Lobe Region: -17 db

Noise Temperature at Zenith: Approximately estimated 2.1°K

The outstanding feature of this antenna is the very low sidelobe level in the shadow region. This results in an extremely low antenna temperature of 2.1°K . This antenna temperature is, to the best of the information collected in this report, the lowest that is achieved in the present state of the art of antenna design.

The most recent horn parabola antenna is the one used for the Telstar experiment. Its performance, as reported by J. N. Hines, Trugye Li, and R. H. Turrin (1963) is as follows:

Gain: 54 db

Frequency: ~ 4170 Mc

Aperture: Circular 68 ft. in diameter

Polarization: Circular

Near Sidelobes: From -17 to -24 db

Far Sidelobes in Shadow Region: Approximately ~ -60 db

Noise Figure at Results: Approximately 2°K

Additional tests conducted at the Bell Telephone Laboratories have produced the following listed results (private communications):

TABLE V
Horn Reflector Antenna Characteristics

Frequency	4 Gc		6 Gc		11 Gc	
	Vert.	Hor.	Vert.	Hor.	Vert.	Hor.
Polarization						
Midband gain (db)	39.6	39.4	43.2	43.0	48.0	47.4
Front-to-back ratio (db)	71	77	71	71	78	71
Beamwidth (azimuth) (degrees)	2.5	1.6	1.5	1.25	1.0	0.8
Beamwidth (elevation) (degrees)	2.0	2.13	1.25	1.38	0.75	0.88
Sidelobes (db below main beam)	49	54	49	57	54	61
Cross-Polarization discrimination (db)	50	46	51	51	57	51
Side-to-side coupling (db)	81	89	120	122	94	112
Back-to-back coupling (db)	140	122	140	127	139	140

TABLE VI

Horn Reflector Antenna and Its Waveguide System

Characteristics in db at 6 Gc

	Mean	Standard Deviation
Side-to-side coupling (same polarization)	102	8.1
Side-to-side coupling (opposite polarization)	109	9.0
Back-to-back coupling (same polarization)	125	10.3
Back-to-back coupling (opposite polarization)	127	10.3
Cross-polarization discrimination	28	5

A test of a similar antenna in the X-band region was conducted by the Melpar Corporation (1958) with the following results:

Frequency: 9375 Mc

Aperture: Approximately 10 × 5 in.

	<u>Vertical Polarization</u>	<u>Horizontal Polarization</u>
<u>Near Sidelobes:</u>	-23 db	-13 db
<u>Shadow Region:</u>	-50 db	-40 db

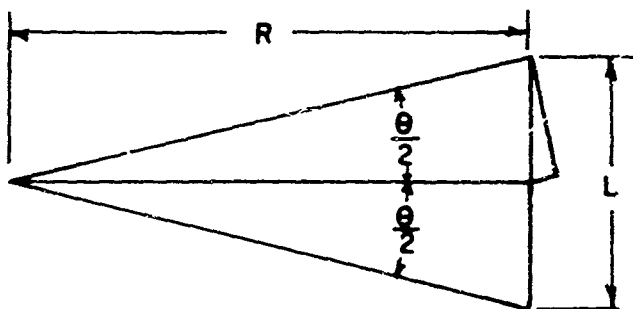
The horn parabola antenna is inherently better than a parabolic dish because of the absence of the obstructing supporting structure of the feed, higher illumination efficiency (horn parabola 0.76, typical parabolic dish 0.65), and better control of the spillover lobes.

The horn parabola is also better than a straight horn of the same flare angle and aperture because of the phase differences which

the horns exhibit in their apertures. The phase front in a horn is essentially spherical and this reduces the gain. As an example, let us consider an equivalent horn to the antenna considered here having an area of 20×20 ft. The wavelength at 2390 Mc is 0.412 ft. The one division of the aperture is therefore 48.5λ . The phase difference between the outer and the edge of the aperture is given in wavelengths by (see figure below)

$$\Delta = \frac{L}{2} \sin \frac{\theta}{2}$$

where Δ is the phase difference
 L is aperture length in wavelength
 θ is flare angle



In our case for $L = 48.5 \lambda$ and $\theta = 28^\circ$, $\Delta = 5.80 \lambda$. This difference of ~ 5 wavelengths introduces a serious reduction in gain. If, however, one restricts Δ to $\frac{\lambda}{2}$, so that no serious reduction in gain occurs, then $\theta = 2.36^\circ$, and the overall dimension of the horn is $R = 1170 \lambda$ or 480 ft.

The present dimensions of the horn parabola are only 50 ft., and its advantage therefore over a common horn is obvious. Another

possibility is to use a horn with a phase correcting lens in its aperture. Studies have shown however that for large antennas this is impractical because of the weight and cost of high quality lenses.

From all the evidence present it is believed that the shadow region radiation of the horn parabola is of the same order of magnitude as any other horn of the same dimension. This can be deduced from the fact that the shadow region radiation is predominantly by edge diffraction effects and the edge illumination conditions for both parabolic horns and ordinary horns are roughly the same.

The main advantage, therefore, of the parabolic horn is the control of the phase over the aperture and the absence of the aperture blocking supporting structures.

6.4 The Goldstone 85 Ft. Parabolic Antenna

The Goldstone parabolic dish antennas have been specifically constructed for satellite tracking applications. Its performance characteristics, as reported by P. Potter (1963), are listed below:

Frequency: 960 Mc

Diameter: 85 ft.

Gain: 45.6 db

Aperture Efficiency: ~ 50%

Near Sidelobe Levels: -13 db

Far Sidelobe Levels: Not reported

Noise Temperature: ~ 9°K

Spillover Energy: ~ 2%

The outstanding feature of this antenna is the efficient use of the Cassegrain-feed principle. This principle is superior to the horn-feed principle of commonly used parabolic antennas for two reasons: first, it solves the practical problem of placing the receiver close to the receiving horn; second, a much better control of the spill-over energy is obtained by this principle which results in a better noise figure. The Cassegrain-feed concept of illuminating parabolic reflectors is presently recognized as being one of the most efficient systems and is useful in many other applications.

6.5 Horn Excited with Higher Order Modes

In recent works of G. C. Brueckman (1958), A. W. Love (1962) and A. F. Sciambi and P. Foldes (1962), it has been shown both theoretically and experimentally that it is feasible to improve the performance of standard horns by exciting them with higher order modes. These results are in agreement with the conclusions of the first part of this report, where it was shown that arbitrarily high gains and low sidelobes can be obtained at the expense of high reactive fields and narrow-banding the system. The effect of the higher order modes in the aperture do, in fact, create large reactive fields and narrow the bandwidth. The higher mode excitation therefore is nothing more than another form of supergain synthesis. In particular, contributions appear for the diagonal horn and the conical horn. Love (1962) and Brueckman (1958) have shown that the proper superposition of a TE_{10} and TE_{01} mode produces a pattern which suppresses near-sidelobes down ~ 30 db in the E plane and ~ 20 db in the 45° plane.

Potter (1963) has also shown that the superposition of TE_{11} and TM_{11} mode produces a pattern which suppresses the first sidelobes down to 30 db in the E plane as well as in other planes.

These results are very interesting because they open the possibility of using these techniques for the construction of the large horn-parabola antennas and other applications.

6.6 Tunnel Antennas

In a series of experiments at Jet Propulsion Laboratory 1963, it has been shown that the construction of a tunnel around the rim of a parabolic dish is capable of reducing significantly the shadow region radiation without seriously damaging the front radiation pattern. The main results of this study are tabulated below:

Frequency: 2388 m.c.

Diameter: 6 feet

Gain: 30.6 db

Channel Range: (0-28) in.

Effect of Channel on Gain: It decreases by less than 1 db.

First, Second, Third Sidelobes: Increases by approximately 0-3 db.

Fourth, Fifth, Sixth Sidelobes: Increase by approximately 0-6 db.

Shadow Region: Decreases by approximately from -45 db to -55 db.

Noise Temperature: Decreases from $16^{\circ}K$ to $3.7^{\circ}K$ (calculated).

From these results it appears that this technique is perhaps one of the most practical and the cheapest for reduction of shadow region sidelobes.

6.7 Tunnel Antennas Using Microwave Material

Experiments using microwave absorbing material have been successful in reducing shadow region sidelobes. In particular, studies by L. Peters Jr. and P. C. Rudduck (1963) at Ohio State University have shown the following results:

Type of Antenna: X-band

Shadow Region Radiation: It was reduced from approximately -50 db to -60 db. It was also reported that the same technique was used successfully for pyramidal horns. Similar experiments conducted by Melpar have shown the following results:

Type of Antenna: Parabolic dish 10λ

Frequency: X-band

Shadow Region Radiation: Levels were reduced from approximately -40 db to -50 db.

The results, therefore, of these studies are positive and it seems that this is another practical way of reducing far-or shadow-region sidelobes. The only drawback to this method is the fact that a microwave absorbing element absorbs energy at 290°K and consequently does not improve the noise figure of the antenna.

6.8 Antennas with $\frac{\lambda}{4}$ Chokes in the Edges

Placing $\frac{\lambda}{4}$ chokes, as suggested by A. F. Sciambi and P. Foldes (1962), in order to reduce the radiation leakage around edges is a technique which has been used for quite some time. Recent experiments

performed by Sciambi have shown that $\frac{\lambda}{4}$ chokes in the edges of a parabolic cylinder reflector are effective and in fact reduce the back lobes by 6 to 9 db.

7.0 Experimental Work

In studying techniques to suppress the sidelobes, two methods have been tried and verified experimentally in the antenna testing range of the University of Pennsylvania: (1) by using Microwave Absorbing Material (MAM), and (2) by placing chokes around the aperture of the horn. The horn used is an "S"-band exponentially tapered rectangular horn. The test setup is shown in Fig. 5.

In the first method, the MAM was shaped and placed on the horn as shown in Fig. 6. Going inside from the edge of the horn, the MAM was tapered gradually along the internal surface of the horn to increase the attenuating wave on the surface and thus make a more tapered illumination on the aperture. The patterns (E-plane) of the horn with and without MAM on it are shown in Fig. 9. It is seen explicitly that the sidelobes are suppressed through using MAM. The intensity of the backlobe of the horn has been cut down by 26 db. The beamwidth is increased slightly, hence the gain is reduced because of more tapered illumination.

In the second method, sidelobes were suppressed by making quarter wavelength chokes and placing them around the edge of the horn. This is illustrated in Fig. 7.

The choke used is similar to a short-circuited quarter wavelength transmission line; hence the impedance looking from Z direction

is infinite. This will prevent the diffracted wave at the edge from radiating to the back of the horn and consequently suppress the sidelobes and backlobe. The result is shown in Fig. 10. The intensity of the backlobe of the horn is suppressed by 16 db. Actual suppression is less than 16 db because the area of aperture is increased as the choke is placed; however, since $\frac{\lambda}{4} = 1.875$ cm is small compared with the dimensions of the aperture of the horn, the gain increase is not significant. Therefore, the backlobe suppression is nearly 16 db, which is a satisfactory result. Placing a double choke on (shown in Fig. 8) the resultant E-plane pattern (Fig. 11) also shows an improvement. The new backlobe is 21 db down, with almost no change of beamwidth.

Both the ways used in studying suppression of sidelobes show effective results. However, the method of using chokes seems to have some advantages over the method of using MAM. First, the loss is much less than the latter because the MAM absorbs much more energy from the radiating wave and, second, the beamwidth almost remains unchanged, whereas the MAM broadens the beamwidth.

8.0 Conclusions

In the theoretical part of this study the existing techniques for antenna pattern synthesis of arrays and aperture antennas was briefly discussed. In addition to this, contributions were made in the following two areas.

First, it was shown that the criterion of signal to noise ratio leads to a new technique of designing antenna arrays and opens new possibilities of mechanizing such systems into adaptive antennas.

Second, the prolate spheroidal functions aperture synthesis method has shown that it is always possible to obtain arbitrarily large gains and suppress the sidelobes at the expense of a large reactive power present in the aperture. This work can be considered as being an extension of Chu's work on supergain phenomena of spherical antennas. This analysis enables one to find a sequence of aperture distributions which give a sequence of patterns with increasing gain. This result can serve as a guide for the design of more sophisticated aperture illuminations. The only disadvantage of this approach is the limited availability of tables for the prolate spheroidal function.

A study of the many applications reported in the literature shows the following results:

Arrays

Sidelobe levels of -35 db have been obtained with considerable difficulty by making great efforts to control the excitation of the elements.

Near Sidelobes of Aperture Antennas

Near sidelobes of aperture antennas for nearly uniform distributions, which is the case in most radio astronomy and satellite tracking antennas, is of the order of magnitude of -20 db. Taylor tapered distributions can provide lower sidelobes at the expense of a

thicker beam. The theoretical studies of this report and other independent experimental work have shown that more sophisticated aperture distributions (such as obtained by superimposing higher order modes in horns) are capable of reducing sidelobes down to approximately -30 db.

Shadow Region Radiation of Aperture Antennas

The shadow region radiation of aperture antennas is essentially an edge diffraction phenomena.

Although present results are far from being complete because all of this information has been collected from various sources, the indications are that the $\frac{\lambda}{4}$ chokes are the most promising methods.

Finally, as a conclusion, it can be pointed out that from present evidence it seems that a high performance antenna can be built with the following characteristics:

- Physical construction: Horn parabola without obstructing feed and a good control of the phase front.
- Excitation: A combination of higher order modes in order to decrease the near sidelobes.
- Edge Control: Some type of choking device, perhaps more efficient than $\frac{\lambda}{4}$ chokes.

The methods of sidelobe reduction which have been discussed in this report are what might be called conventional techniques. Recently, a number of new approaches have been reported in the literature, such as: time modulation, cross-correlation and surface wave antennas. These techniques are expected to be examined at a future date in continuation of this effort.

Appendix A

PROLATE SPHEROIDAL WAVE FUNCTIONS

In a paper, Slepian and Pollak (1961) have put forth, for the first time, prolate spheroidal wave functions as the mathematical link that allows the exact determination of the aperture field distribution for a specified radiation pattern. The methods that were used in the past have, to the best of our knowledge, been approximation methods or methods that applied only to specific radiation patterns. In order to obtain a better understanding of many of the mathematical manipulations involved, some of the basic properties of the prolate spheroidal functions are presented. The discussion begins with the scalar wave equation. It should be clear, however, that a diffraction problem is not solved here. The wave equation is used only to derive some fundamental properties of the spheroidal functions. Flammer's (1957) notation is followed.

The scalar-wave equation

$$\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0 \quad (\text{A1})$$

can be separated in the time coordinate for sinusoidal time dependence by the substitution

$$\psi(x, y, z, t) = \psi(x, y, z) e^{-i\omega t} \quad (\text{A1.a})$$

to give the scalar Helmholtz equation

$$\nabla^2 \psi + k^2 \psi = 0$$

where $k = \omega/c$, $c =$ velocity of light. The prolate spheroidal coordinate system is one of the eleven coordinate systems in which the scalar Helmholtz equation

$$(\nabla^2 + k^2) \psi = 0 \quad (\text{A1.b})$$

is separable. To express this equation in spheroidal coordinates, the metrical coefficients h_η , h_ζ and h_φ are defined by

$$dx^2 + dy^2 + dz^2 = h_\eta^2 d\eta^2 + h_\zeta^2 d\zeta^2 + h_\varphi^2 d\varphi^2 \quad (\text{A2})$$

These scale factors are

$$h_\eta = \frac{d}{2} \left(\frac{\zeta^2 - \eta^2}{1 - \eta^2} \right)^{1/2}$$

$$h_\zeta = \frac{d}{2} \left(\frac{\zeta^2 - \eta^2}{\eta^2 - 1} \right)^{1/2} \quad (\text{A3})$$

$$h_\varphi = \frac{d}{2} \left(\left[(1 - \eta^2)(\zeta^2 - 1) \right] \right)^{1/2}$$

The prolate spheroidal coordinates are related to rectangular coordinates by the transformation

$$x = \frac{d}{2} \left[(1 - \eta^2)(\zeta^2 - 1) \right]^{1/2} \cos \varphi$$

$$y = \frac{d}{2} \left[(1 - \eta^2)(\zeta^2 - 1) \right]^{1/2} \sin \varphi \quad (\text{A4})$$

$$z = \frac{d}{2} \eta \zeta$$

where

$$-1 \leq \eta \leq 1, \quad 1 \leq \zeta < \infty, \quad 0 \leq \varphi \leq 2\pi.$$

An illustration of these coordinates is given in Fig. 12. Furthermore, the surfaces $\eta = \text{constant}$ are confocal hyperboloids of revolution of two sheets. The surfaces $\zeta = \text{constant}$ are confocal prolate spheroids. The surfaces $\varphi = \text{constant}$ are half planes containing the z-axis and terminated by it.

With the use of the expression for the Laplacian ∇^2 in orthogonal curvilinear coordinates

$$\nabla^2 \psi = \frac{1}{h_1 h_2 h_3} \sum_i \frac{\partial}{\partial \xi_i} \left[\frac{h_1 h_2 h_3}{h_i^2} \frac{\partial \psi}{\partial \xi_i} \right] \quad (\text{A5})$$

we obtain the equation

$$\left[\frac{\partial}{\partial \eta} (1 - \eta^2) \frac{\partial}{\partial \eta} + \frac{\partial}{\partial \zeta} (\zeta^2 - 1) \frac{\partial}{\partial \zeta} + \frac{\zeta^2 - \eta^2}{(\zeta^2 - 1)(1 - \eta^2)} \frac{\partial^2}{\partial \varphi^2} + c^2 (\zeta^2 - \eta^2) \right] \psi = 0 \quad (\text{A6})$$

where

$$c = \frac{1}{2} kd \quad (\text{A7})$$

By the usual separation of variables, solutions of eq. (A6) may be obtained in the form of the Lamé products

$$\psi_{mn} = S_{mn}(c, \eta) R_{mn}(c, \zeta) \frac{\cos}{\sin} m\varphi \quad (\text{A8})$$

The two functions $S_{mn}(c, \eta)$ and $R_{mn}(c, \zeta)$ satisfy the ordinary differential equations

$$\frac{d}{d\eta} \left[(1 - \eta^2) \frac{d}{d\eta} S_{mn}(c, \eta) \right] + \left[\lambda_{mn} - c^2 \eta^2 - \frac{m^2}{1 - \eta^2} \right] S_{mn}(c, \eta) = 0 \quad (A9)$$

$$\frac{d}{d\zeta} \left[(\zeta^2 - 1) \frac{d}{d\zeta} R_{mn}(c, \zeta) \right] - \left[\lambda_{mn} - c^2 \zeta^2 + \frac{m^2}{\zeta^2 - 1} \right] R_{mn}(c, \zeta) = 0 \quad (A10)$$

The separation constants λ_{mn} and m are the same in the above equations. Their values will be determined presently.

The only concern here is with those functions from which a single valued wave function of the form given by eq. (A8) can be found. This requires that m be an integer which can be restricted without any loss in generality, to positive or zero values.

Those values $\lambda_{mn}(c)$ for which eq. (A9) admits solutions that are finite at $\eta = \pm 1$ are the eigenvalues of the differential eq. (A9). The associated eigenfunctions $S_{mn}(c, \eta)$ are the prolate spheroidal angle functions of the first kind, of order m and degree n . They are given by the infinite sum of the form

$$S_{mn}(c, \eta) = \sum_{r=0,1}^{\infty'} d_r^{mn}(c) P_{m+r}^m(\eta) \quad (A11)$$

where $P_{m+r}^m(\eta)$ are the associated Legendre functions and $d_r^{mn}(c)$ are constant coefficients. The prime over the summation sign indicates that the summation is over only even values of r when $n-m$ is even, and over only odd values of r when $n-m$ is odd.

The eigenvalues λ_{mn} which occur in eq. (A10) are those to which the angle functions S_{mn} belong. In eq. (A10) the $R_{mn}(c, \zeta)$ are the prolate spheroidal radial functions. Since both the radial function $R_{mn}(c, Z)$ and the angular function $S_{mn}(c, Z)$ satisfy the same differential equations (cf. eqs. (A9) and (A10)), they must be proportional to one another. So that

$$S_{mn}(c, Z) = K_{mn}(c) R_{mn}(c, Z). \quad (\text{A12})$$

In this equation $S_{mn}(c, Z)$ and $R_{mn}(c, Z)$ are to be continued beyond the regions in which they were originally defined by proper adjustment of the phase of $(Z^2 - 1)^{\frac{1}{2m}}$. That is, in going from the region $Z > 1$ to $Z < 1$, we replace $(Z^2 - 1)^{\frac{1}{2m}}$ by $(1 - Z^2)^{\frac{1}{2m}}$, and vice versa. The joining factor $K_{mn}(c)$ may be found by putting Z equal to zero in eq. (A12) and in the derivative of this equation with respect to Z , in the even and odd cases respectively. In particular, in evaluation of the main problem the following relations are needed:

$$S_{on}(c, Z) = K_{on}(c) R_{on}(c, Z) \quad (\text{A13})$$

where in this case

$$K_{on}(c) = \frac{S_{on}(c, 1)}{R_{on}(c, 1)} \quad (\text{A14})$$

Some other properties that will be needed in later evaluations are the orthogonality property,

$$\int_{-1}^{+1} S_{on}(c, \eta) S_{om}(c, \eta) d\eta = \begin{cases} N_{on} & n=m \\ 0 & n \neq m \end{cases} \quad (A15)$$

where N_{on} is the normalization factor.

Since the prolate spheroidal functions are bandlimited functions, orthogonal and complete on the real axis, i.e.

$$\int_{-\infty}^{\infty} S_{on}(c, \eta) S_{om}(c, \eta) d\eta = \begin{cases} \frac{N_{on}}{\lambda_{on}} & n=m \\ 0 & n \neq m \end{cases} \quad (A16)$$

where λ_{on} are the eigenvalues given by

$$\lambda_{on} = \frac{2c}{\pi} \left[R_{on}(c, 1) \right]^2 \quad (A17)$$

which is the first iterate of the integral

$$2i^n R_{on}(c, 1) S_{on}(c, \eta) = \int_{-1}^{+1} e^{ic\eta s} S_{on}(c, s) ds \quad (A18)$$

namely of

$$\lambda_{on} S_{on}(c, \eta) = \int_{-1}^{+1} \frac{\sin c(\eta-s)}{\pi(\eta-s)} S_{on}(c, s) ds \quad (A19)$$

The angular prolate spheroidal functions are continuous functions of c for $c = 0$. And since they are orthogonal in $(-1,1)$ and complete in L_1^2 , it follows that the eigenvalues in eq. (A17) are the only eigenvalues of (A19) and that if these quantities are distinct, the S_{on} are (apart from multiplicative constants) the unique L_1^2 solutions of eq. (A19). L_1^2 are the class of all complex valued functions $f(t)$ defined for $-1 \leq t \leq 1$ and integrable in absolute square in the interval $(-1,1)$.

APPENDIX B

DETERMINATION OF THE EXPANSION COEFFICIENT a_n

The constant a_n in eq. (26) was obtained by the use of Schwarz's inequality for summations,

$$\left| \sum_{n=0}^N a_n b_n \right|^2 \leq \sum_{n=0}^N a_n^2 \cdot \sum_{n=0}^N b_n^2 \quad (B1)$$

In order for the right-hand side of eq. (24) to be a maximum, the equality sign in eq. (B1) must hold. So that

$$\left| \sum_{n=0}^N a_n S_{on}(c, 0) \right|^2 = \sum_{n=0}^N a_n^2 N_{on} \cdot \sum_{n=0}^N \frac{S_{on}^2(c, 0)}{N_{on}} \quad (B2)$$

And consequently,

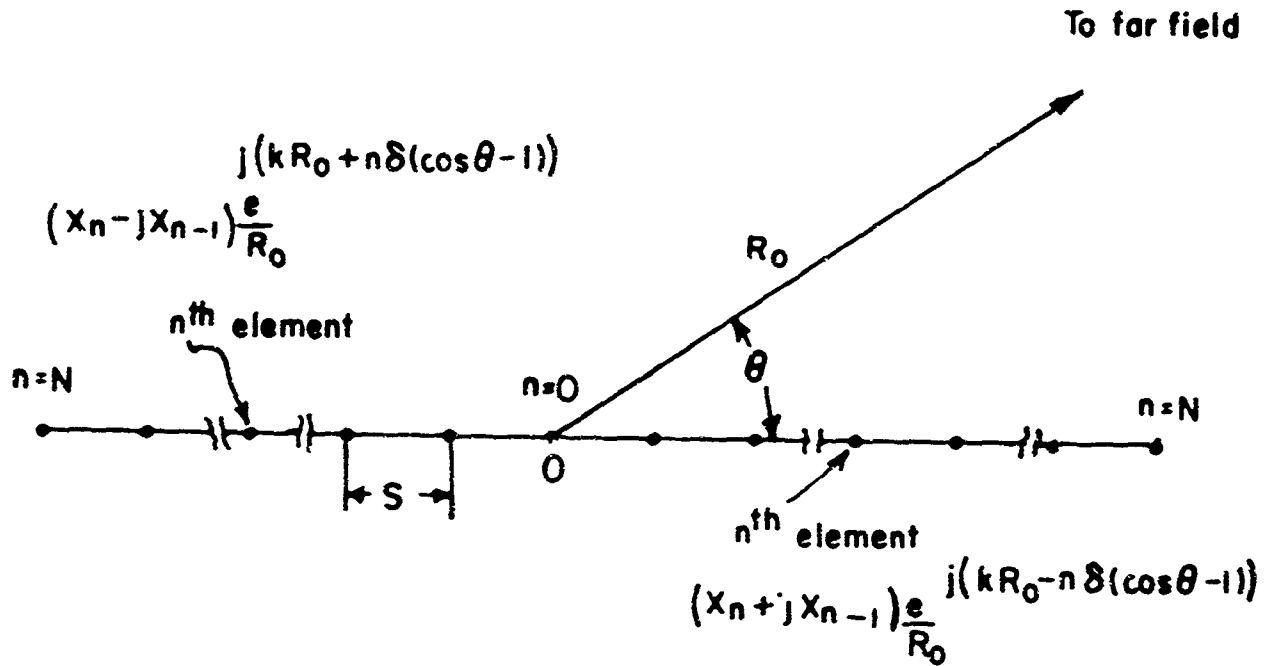
$$a_n = \frac{S_{on}(c, 0)}{N_{on}} \quad (B3)$$

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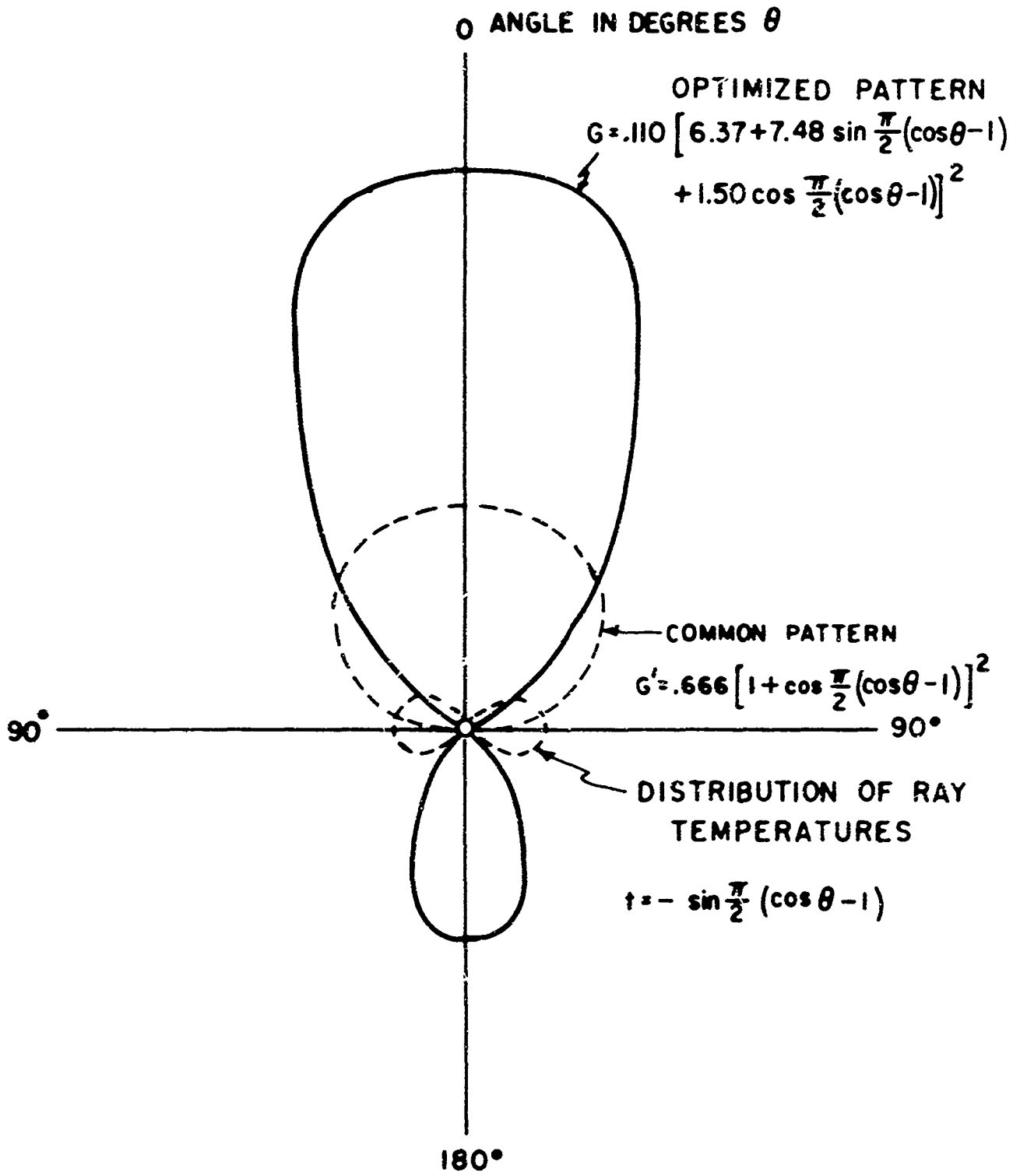
Elements are isotropic sources

Spacing is $S = \frac{\lambda \delta}{2\pi}$ ($\delta = \frac{2\pi}{\rho}$)

Phase difference between adjacent elements is δ ,
 ρ is a constant

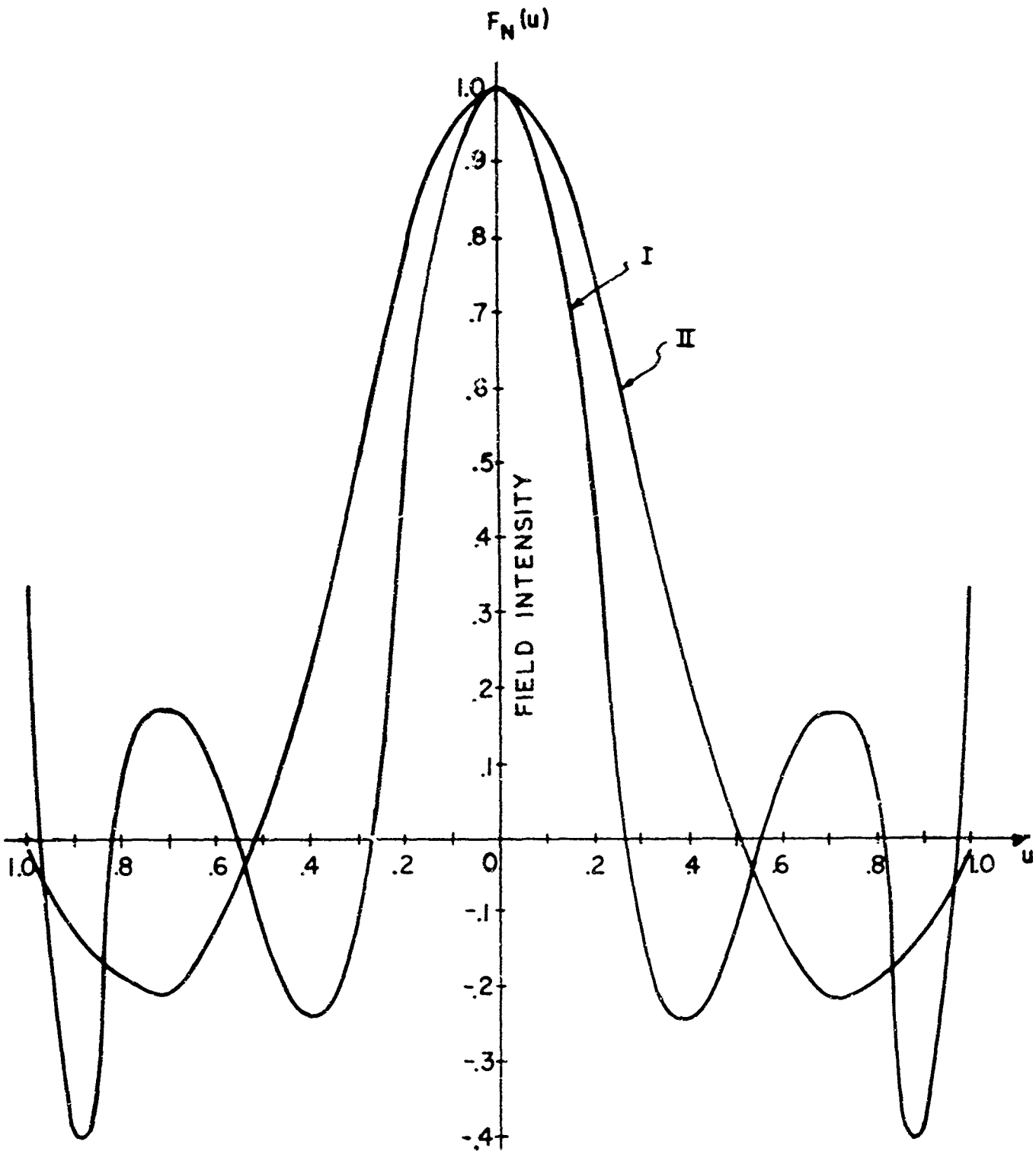
THE ARRAY

FIGURE 1



GAIN PATTERNS

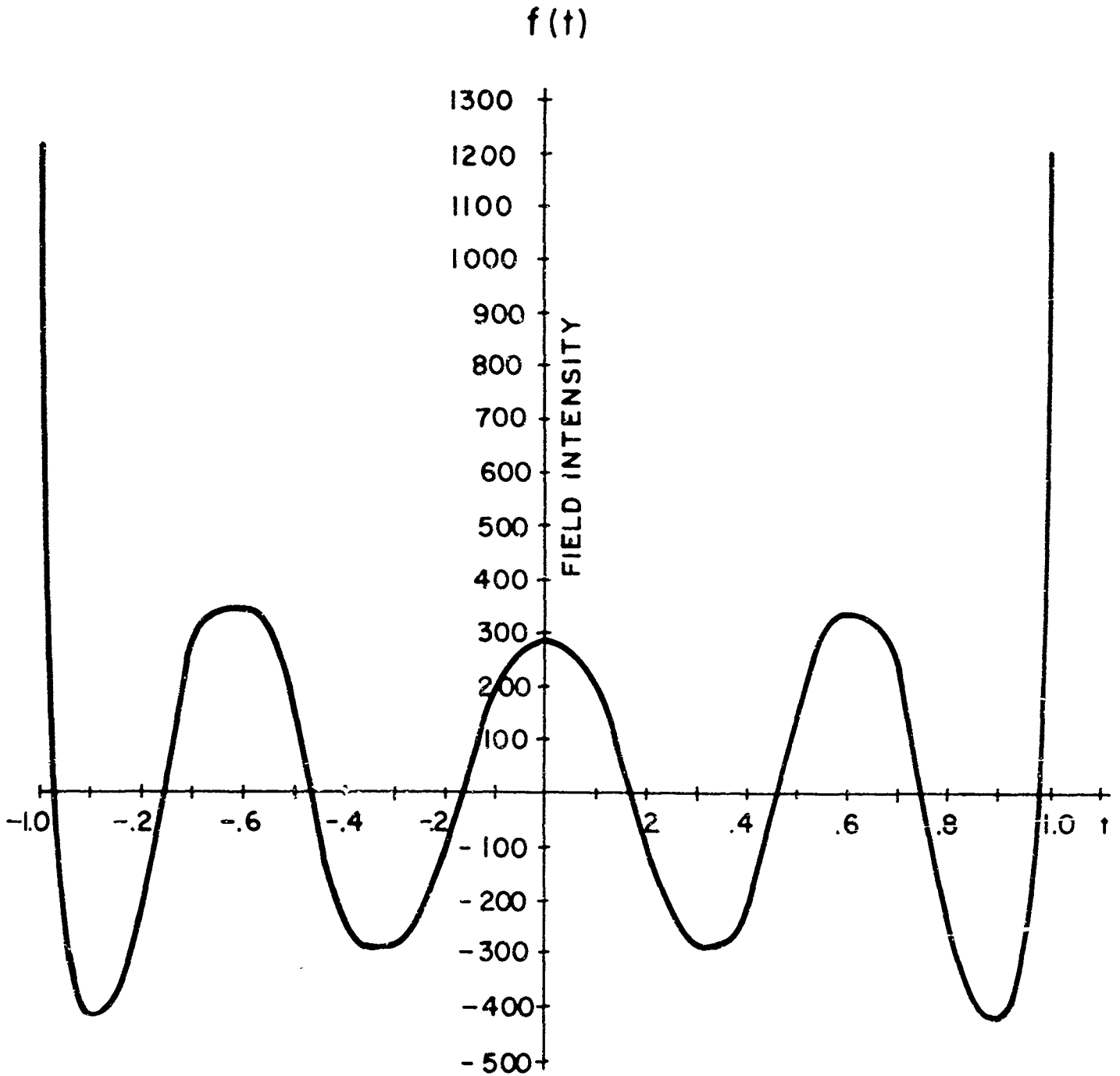
FIGURE 2



I. APPROXIMATE RADIATION PATTERN SPECTRUM OF THE DELTA FUNCTION
($C=6, N=8$)

II APPROXIMATE RADIATION PTTTERN FOR UNIFORM APERTURE DISTRIBUTION
($C=6, N=8$)

FIG. 3



THE APPROXIMATE APERTURE FIELD FOR THE DELTA FUNCTION
($C=6, N=8$)

FIG. 4

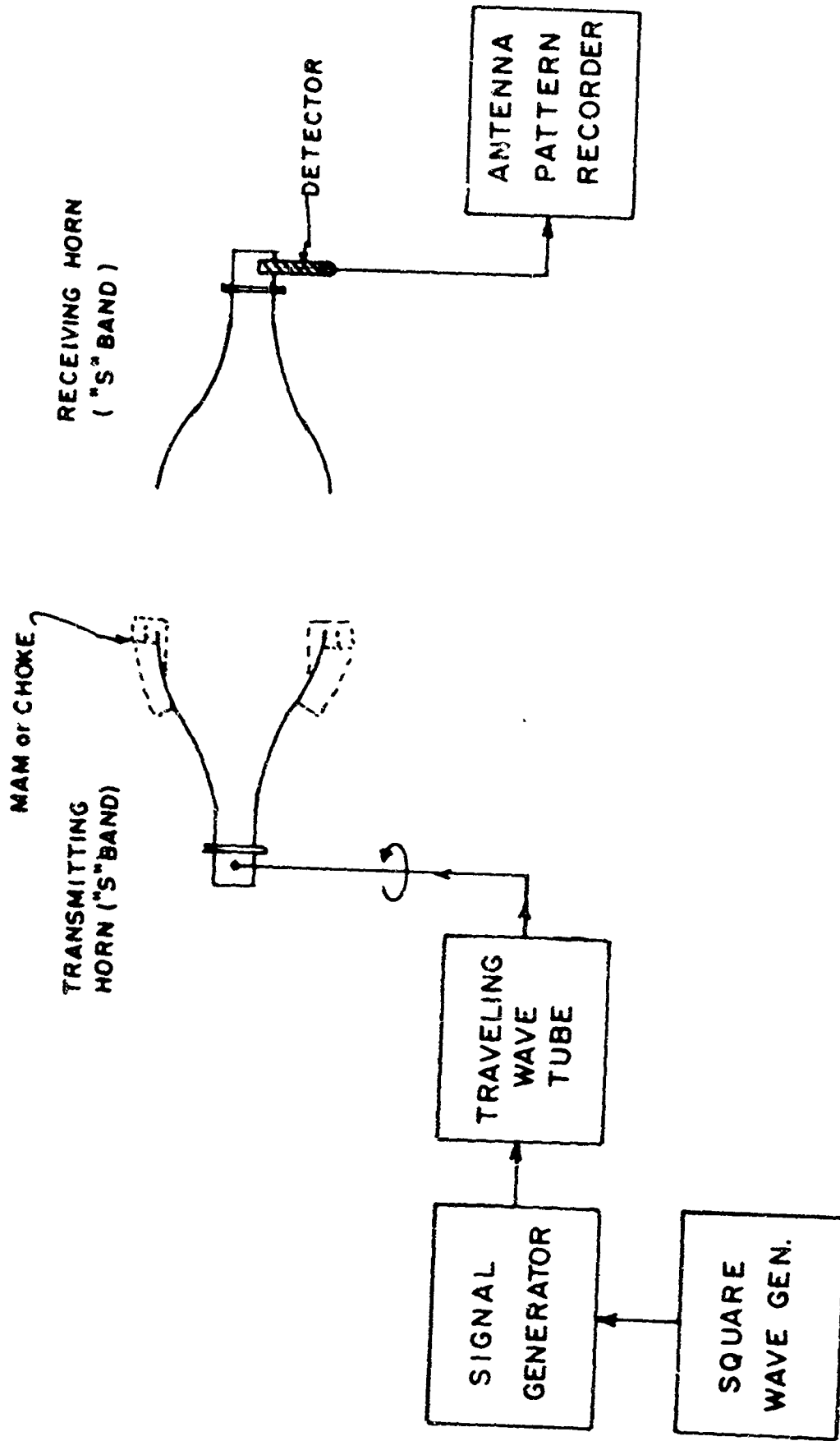


FIGURE 5. TEST SET UP

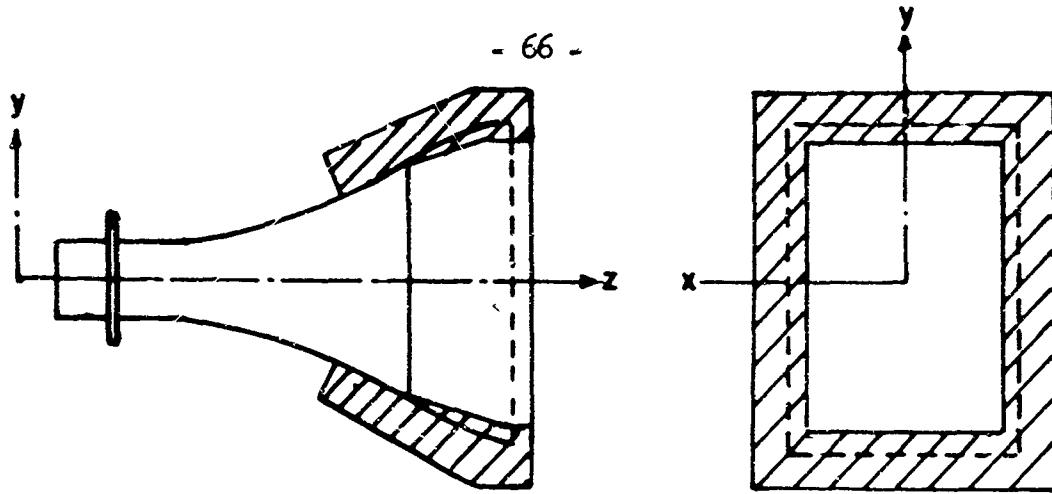


FIGURE 6. HORN (S' BAND) WITH MAM

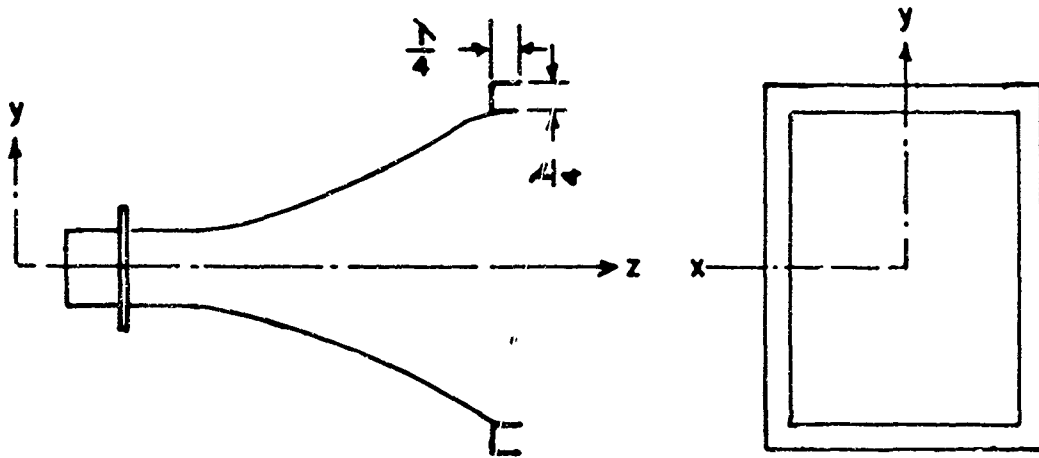


FIGURE 7. HORN (S' BAND) WITH SINGLE CHOKE

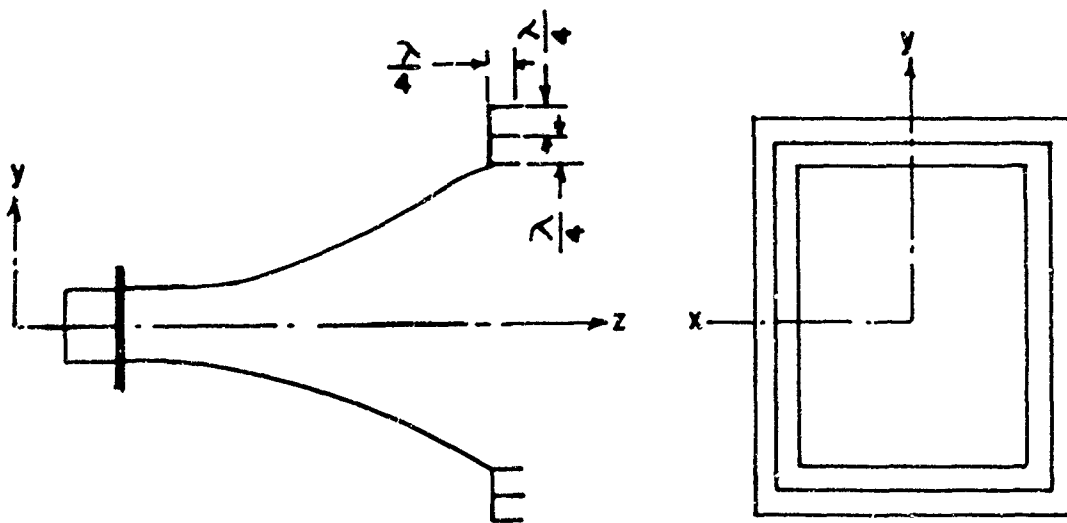


FIGURE 8. HORN (S' BAND) WITH DOUBLE CHOKE

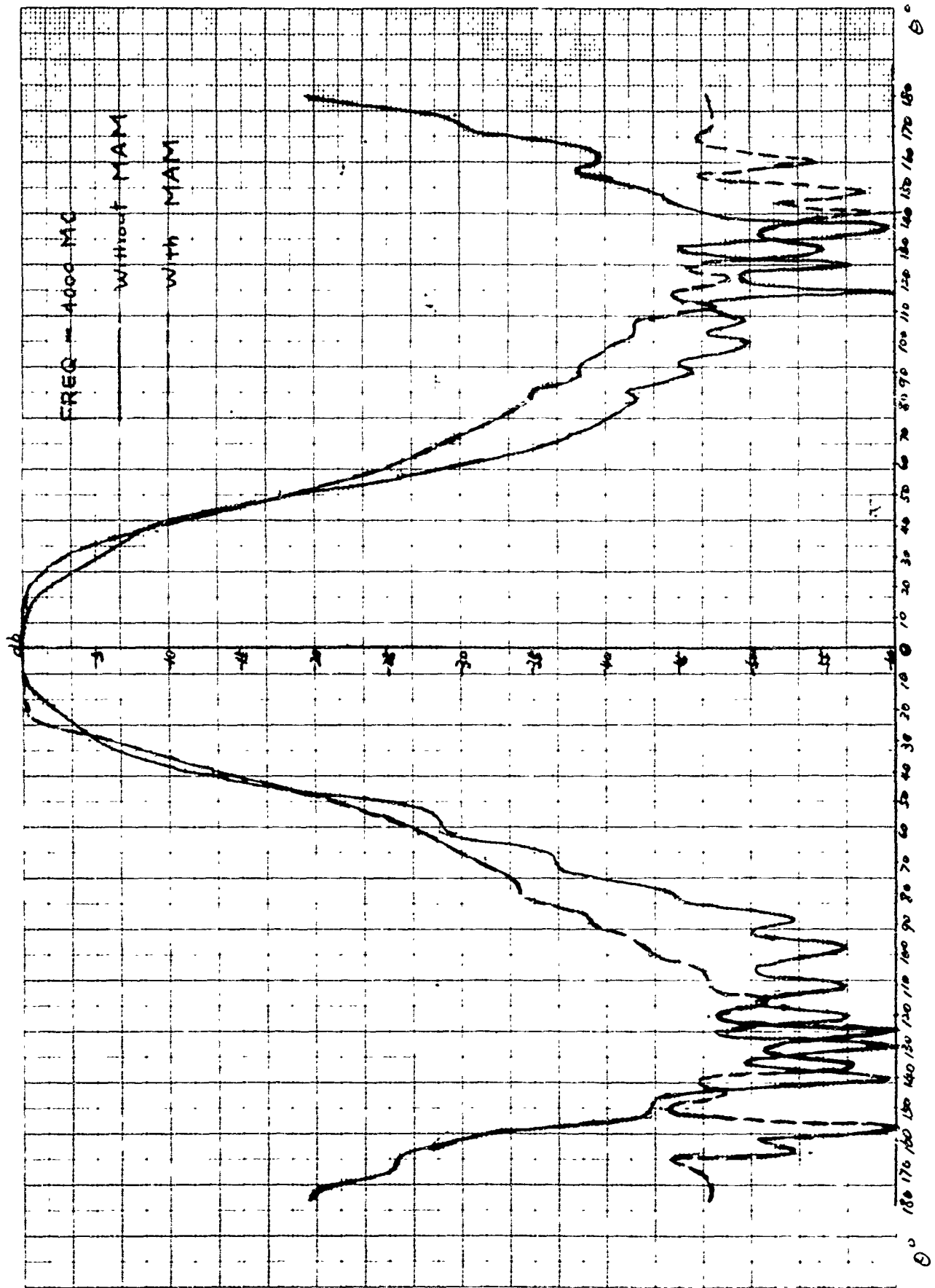


FIGURE 9

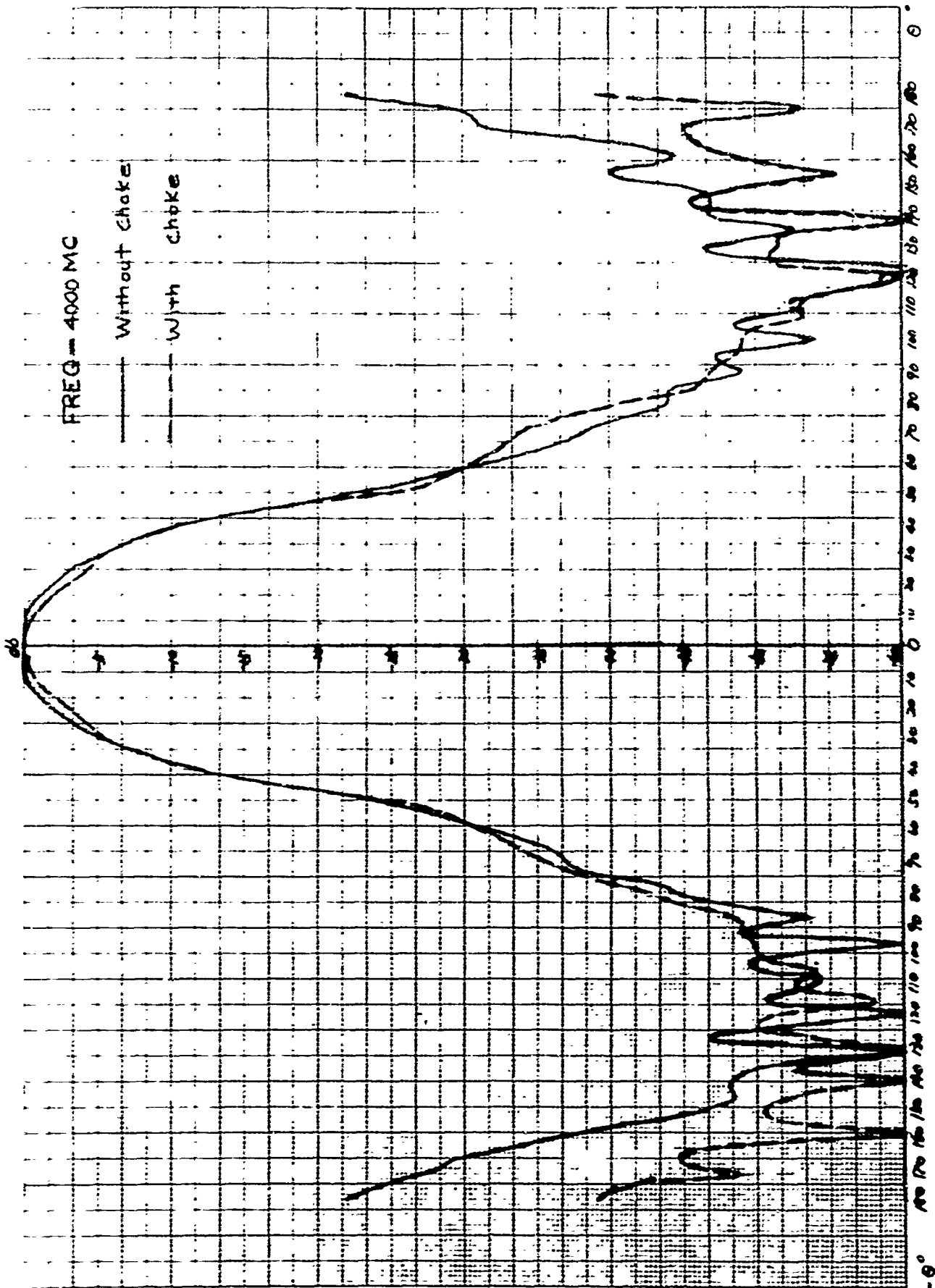


FIGURE 10

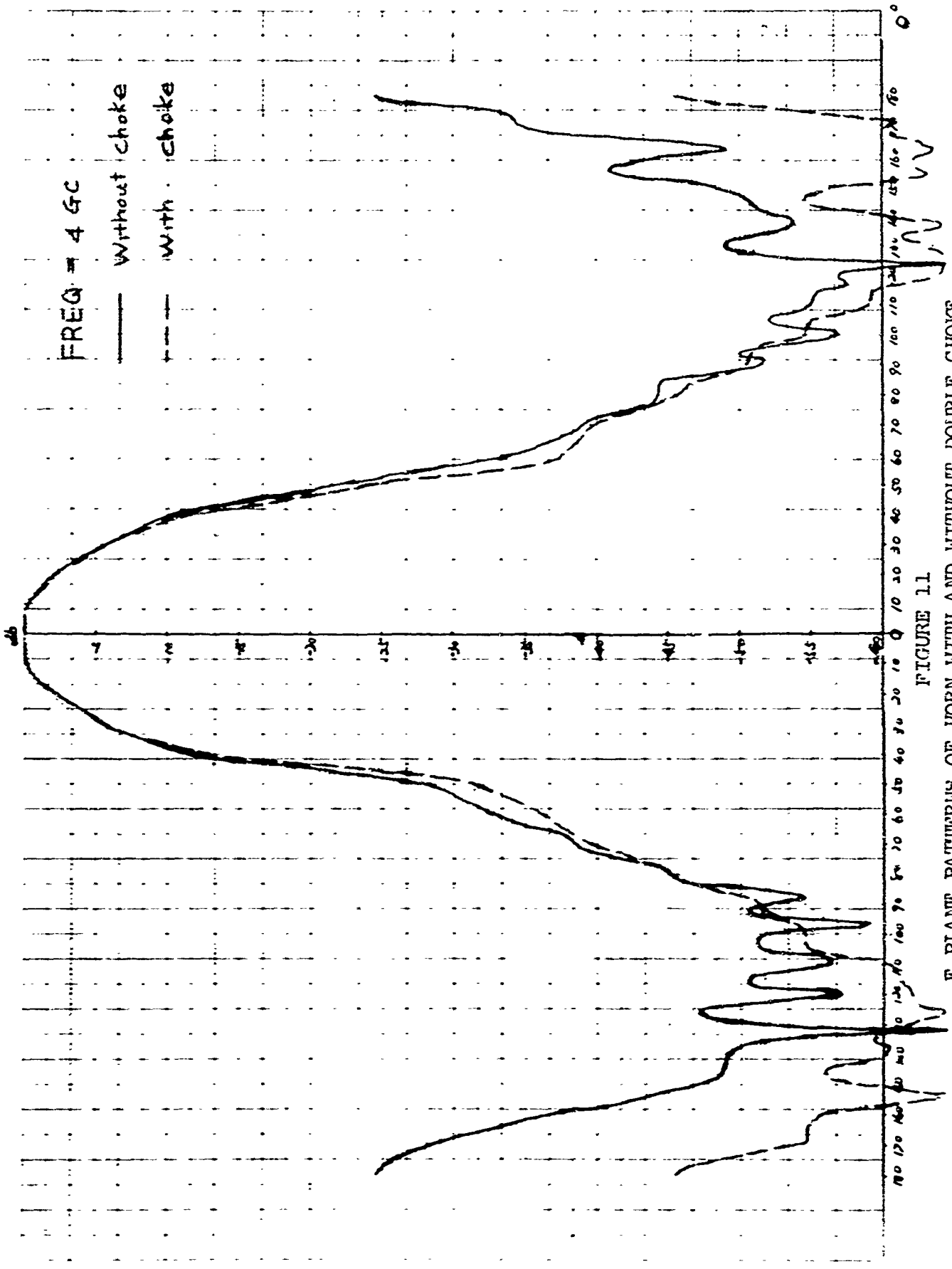
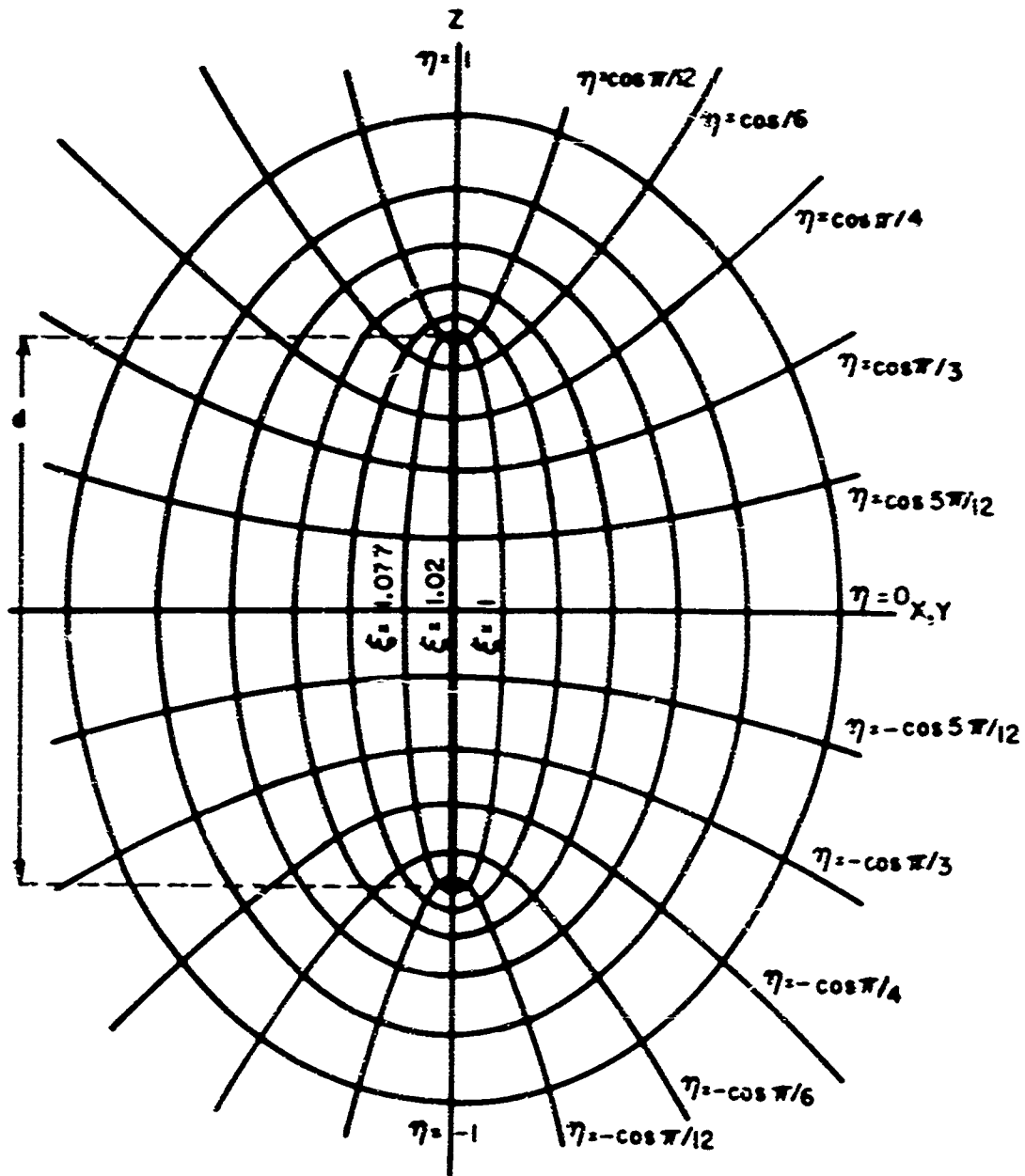


FIGURE 11
E-PLANE PATTERNS OF HORN WITH AND WITHOUT DOUBLE CHOKE



THE PROLATE SPHEROIDAL COORDINATE SYSTEM

FIG. 12

UNCLASSIFIED

Security Classification

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13. ABSTRACT This report consists of six volumes: (a) Volume I: Studies of Antenna Side-Lobe Reduction This consisted of a theoretical study and a discussion of actual applications. Two novel approaches were suggested. First, the use of the signal-to-noise ratio as a criterion for design of arrays and secondly, the use of the prolate spheroidal functions for aperture synthesis. (b) Volume II: Multiple Use of Antennas and Antenna Stacking The feasibility of the multiple use of a common antenna and stacking of several antennas on a small site is studied. It is concluded that a combination of multiplexing and stacking of antennas is feasible and desirable. (c) Volume III: Noise and Interference Monitoring Studies This effort is concerned with the establishment of a signal density monitoring facility which is designed to measure ambient noise and signal densities over the frequency range from 100 mcs to 20 gc. (d) Volume IV: Directional Mode Couplers This effort includes a re-evaluation of the rectangular waveguide couplers previously reported in Volume I, Final Report, Task 1, of Contract AF30(602)-1615, entitled, Interference Studies, and Final Report, RADC-TDR-63-359, of contract AF30(602)-1785, entitled, Interference Studies. (e) Volume V: Studies of Transmitter Bandwidth Limiting and Spurious Output Suppression This is a review of existing techniques for reducing intermodulation and harmonic distortion in radio transmitters.		

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Antenna Radars Radio Transmitters Communication Equipment						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

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3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parentheses immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

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8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).

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There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

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UNCLASSIFIED

Security Classification

DD Form 1473 - 13. Abstract - CONTINUED

(f) Volume VI: Study of Techniques for Spectrum Signature Measurements

Some special features of a spread spectrum communication transceiver, a GCA radar and the two BMEWS radars are summarized to describe suggested changes to Mil-Std-449 recommended test procedures. This is not to modify the Mil-Std, but to show how it applies to these specific equipments.