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DETERMINATION OF THE SHAPE
OF A FREE BALLOON

Scientific Report No. 2

Balloons with Zero Superpressure
and Zero Circumferential Stress

by Justin H. Smalley

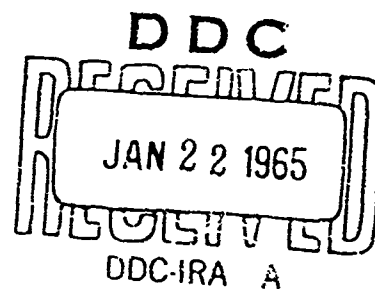
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ABSTRACT

This report, the second of a series, presents the numerical methods used and the results of the computation of the shapes of axis-symmetric free balloons. Flat-top balloons, re-entrant top balloons and double balloons, all with zero circumferential stress and zero superpressure at the bottom apex, are considered. Meridional film loads are presented. An extensive Sigma Table suitable for balloon design is included for the flat-top balloon with $\Sigma = 0(0.05) 1.0$.

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I. INTRODUCTION

This report is the second volume of a series of reports that will examine the problem of determining the shape of free balloons. Scientific Report No. 1 in this series (Reference 1) pointed out that balloons are being flown today which are outside the range of design parameters provided by the University of Minnesota (Reference 2). Therefore, in Report No. 1, a literature survey was made and the equations defining the shape of a free balloon were derived. In the current report, the methods used to calculate balloon shapes are outlined. In addition, a variety of balloon shapes are presented. They are restricted to balloons with zero circumferential stress and zero superpressure. All results and conclusions are for balloons which are fully inflated, i. e., at float altitude.

II. NUMERICAL SOLUTION OF EQUATIONS

A. Symbols

<u>Symbol</u>	<u>Definition</u>	<u>Dimension</u>
a	pressure head at bottom of balloon	length
b	difference in weight densities of air and inflation gas	force per unit volume
k	constant = $(2\pi)^{-1/3}$	
p	gas pressure across the balloon material	force per unit area
r	radial coordinate of a point on balloon, measured normal to the axis of symmetry	length
t_c	circumferential stress in balloon material	force per unit length
t_m	meridional stress in balloon material	force per unit length

<u>Symbol</u>	<u>Definition</u>	<u>Dimension</u>
s	gore coordinate of a point on the balloon, measured in the meridional direction from the bottom apex	length
w	unit weight of balloon material	force per unit area
z	height coordinate of a point on balloon, measured parallel to the axis of symmetry from the bottom apex	length
A	area of balloon surface	length squared
B	buoyant force on balloon	force
F	vertical load at top apex of balloon	force
G	gross lift of balloon = bV	force
L	payload suspended at bottom apex of balloon	force
P	balloon total payload	force
T	total film load = $2\pi r t_m$	force
V	balloon volume	length cubed
W	balloon weight	force
$\overline{rt}$	$= r t_m / P$	
α	$= a / \lambda$	
ζ	$= z / \lambda$	
θ	angle between balloon material and axis of symmetry measured in a meridional plane containing the axis of symmetry	
λ	$= (P/b)^{1/3}$	
ρ	$= r / \lambda$	
σ	$= s / \lambda$	
τ	$= t_c / b \lambda^2$	
Σ	$= w(2\pi/b^2 P)^{1/3} = w(2\pi)^{1/3} / b\lambda$	

B. Equations

In Report No. 1, Equations (8) define the shape of a free balloon. They are

$$-d\theta/d\sigma = [\rho (\zeta + \alpha) + k \Sigma \rho \rho' - \tau \zeta'] / \overline{rt}$$

$$\overline{rt} = 1/(2 \pi \cos \theta_0) + \int_0^\sigma (k \Sigma \rho \zeta' + \tau \rho') d\sigma$$

where the prime indicates a derivative with respect to σ . These derivations were based on the following assumptions:

- 1) The balloon is assumed to be rotationally symmetric about a vertical axis.
- 2) Meridional and circumferential stresses are assumed to be constant at all points on the circle lying in a plane normal to the axis of symmetry. This precludes the possibility of shear in the balloon.
- 3) The densities of the inflation gas and surrounding air are constant.
- 4) The balloon material is inextensible, thin, and incapable of supporting any bending or compressive loads.

In this report τ will be taken as equal to zero at all times.

C. Euler Method

The simplest method for the numerical solution of differential equations is the Euler Method. This involves making the assumption that for a small increment in the independent variable the rate of change in the dependent variable is constant. That is if

$$dy/dx = f(x)$$

then

$$\Delta y \approx f(x) \cdot \Delta x.$$

The smaller the value of Δx the better the approximation. The term $f(x)$ is evaluated at the beginning of the increment. A solution by this method was programmed for the Control Data Corporation G-15 computer. This method is fast but has certain inherent errors. It will always compute a balloon which is too large, since there are no inflections in the shape curve. To improve on its accuracy smaller increments are used. This, of course, increases computing time and introduces the unknown factor of round-off errors. Since only five digits are output by the computer, round-off error does become a matter of concern. For these reasons a more sophisticated solution was used.

D. Runge-Kutta Method

Both the Euler and the Runge-Kutta methods were chosen rather than finite difference methods because of the ease of starting a solution. Only initial values are needed. The somewhat higher accuracy obtainable with finite difference methods cannot justify the additional programming complications. There probably would be no time savings. The main advantage of finite difference methods is the availability of error estimates. For this relatively simple problem and with a digital computer, a precise error estimate is not important.

To apply the Runge-Kutta method to high-speed digital computers, Gill developed a calculation procedure (Reference 3) which:

- "1) requires a minimum number of storage registers;
- 2) gives the highest attainable accuracy (i. e., controls the growth of round-off errors);
- 3) requires comparatively few instructions."

If

$y' = f(x, y)$, then, quoting further from Reference 3,

"The final form of the Gill procedure is then expressed as:

$$k_1 = h f(x_0, y_0)$$

$$y_1 = y_0 + 1/2 (k_1 - 2 q_0)$$

$$q_1 = q_0 + 3 [1/2 (k_1 - 2 q_0)] - 1/2 k_1,$$

$$k_2 = h f(x_0 + h/2, y_1)$$

$$y_2 = y_1 + (1 - \sqrt{1/2}) (k_2 - q_1)$$

$$q_2 = q_1 + 3 [(1 - \sqrt{1/2}) (k_2 - q_1)] - (1 - \sqrt{1/2}) k_2$$

$$k_3 = h f(x_0 + h/2, y_2)$$

$$y_3 = y_2 + (1 + \sqrt{1/2}) (k_3 - q_2)$$

$$q_3 = q_2 + 3 [(1 + \sqrt{1/2}) (k_3 - q_2)] - (1 + \sqrt{1/2}) k_3$$

$$k_4 = h f(x_0 + h, y_3)$$

$$y_4 = y_3 + (1/6) (k_4 - 2 q_3)$$

$$q_4 = q_3 + 3 [(1/6) (k_4 - 2 q_3)] - (1/2) k_4."$$

where x_0 and y_0 are initial values of x and y , respectively

$$h = x_{n+1} - x_n;$$

$q_0 = 0$ for the first increment, thereafter

$$(q_0)_{n+1} = (q_4)_n$$

and the new values of x and y at the end of the increment are:

$$x_n = x_0 + h,$$

$$y_n = y_4.$$

Applying the foregoing to the problem at hand we have, setting $\tau = 0$ and considering the geometry,

$$- \theta' = [\rho (\zeta + \alpha) + k \Sigma \rho \sin \theta] / \overline{rt}$$

$$\overline{rt}' = k \Sigma \rho \cos \theta$$

$$\rho' = \sin \theta$$

$$\zeta' = \cos \theta$$

where the initial conditions are

$$\theta_0 = \theta_0$$

$$\overline{rt}_0 = 1/(2\pi \cos \theta_0)$$

$$\rho_0 = 0$$

$$\zeta_0 = 0$$

For application of the Gill procedure to this system of four first-order differential equations, the procedure given above is slightly modified. The flow chart in Figure 1 explains it best.

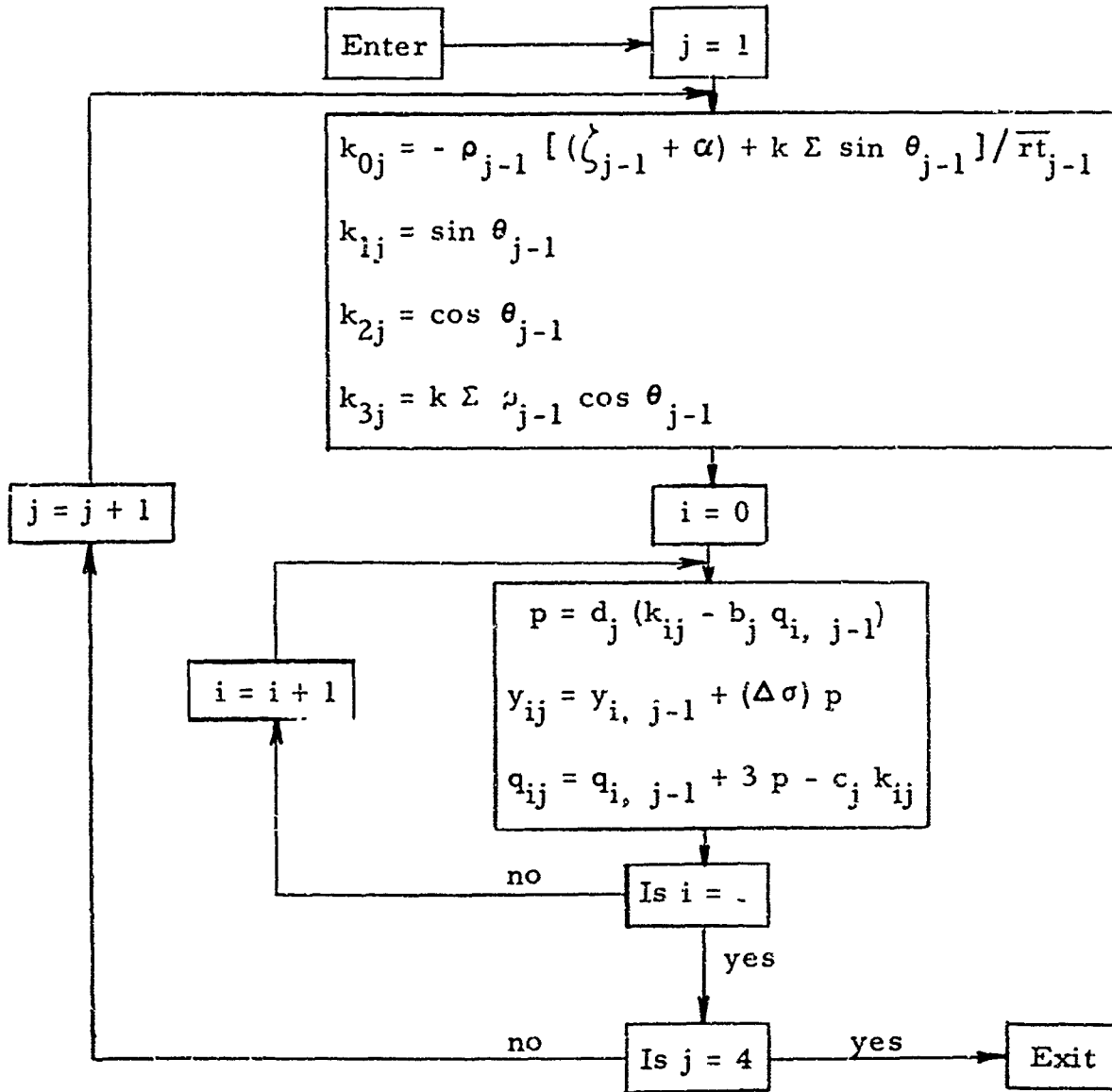
This method calculates the shape of the balloon in a very satisfactory manner. The coordinates have converged to final values in the fifth significant figure, when 20 increments in σ are used. Table I compares certain values for several numbers of increments.

Table I. Comparison of Balloon Coordinates for Several Numbers of Increments

Σ	θ_0 (deg)	No. of Increments	At Large Radius		At Top		
			ρ	ζ	ζ	σ	θ (deg)
0	50.155	10	0.64163	0.94305	1.2845	1.9953	-89.984
		20	0.64165	0.94302	1.2842	1.9946	-90.013
		40	0.64166	0.94302	1.2841	1.9946	-90.014
1.0	84.2	10	1.9962	1.2645	2.5487	5.2883	-89.710
		20	1.9961	1.2644	2.5483	5.2883	-89.709
		40	1.9961	1.2644	2.5482	5.2883	-89.709

E. Determination of Volume and Weight

At the same time that the shape was being calculated, the volume, area, and weight of the balloon was also being calculated. The first procedure involved calculating the area and volume by conical zones. It was found that these were not converging as rapidly as the coordinates. Consideration was given to quadrature formulas using several points. While more accurate, they would introduce programming difficulties because both the number of increments and the relevant length of each increment ($\Delta\zeta$) were variable.



where:

$$\begin{aligned} y_{00} &= \theta_n \\ y_{10} &= \rho_n \\ y_{20} &= \zeta_n \\ y_{30} &= \overline{rt}_n \end{aligned}$$

$$\begin{aligned} y_{04} &= \theta_{n+1} \\ y_{14} &= \rho_{n+1} \\ y_{24} &= \zeta_{n+1} \\ y_{34} &= \overline{rt}_{n+1} \end{aligned}$$

$$\begin{aligned} b_1 &= b_4 = 2 \\ b_2 &= b_3 = i \\ c_1 &= c_4 = d_1 = 1/2 \\ c_2 &= d_2 = 1 - \sqrt{1/2} \\ c_3 &= d_3 = 1 + \sqrt{1/2} \\ d_4 &= 1/6 \end{aligned}$$

Figure 1. Flow Chart of Gill Procedure for Determination of Balloon Shape

It was decided to investigate the value of the fact that the slope of the curve was known at the end of each increment. Because of this, each increment could be fitted with a polynomial. It is shown in Appendix I that if

- 1) A parabola is fitted to each increment,
- 2) The mid-point ordinate is calculated and
- 3) Integration is performed by Simpson's Rule then, for a given increment,

$$\Delta \text{Volume} = \pi (\zeta_1 - \zeta_0) [(\rho_0)^2 + 4(\rho_{1/2})^2 + (\rho_1)^2] / 6$$

$$\Delta \text{Surface Area} = \pi (\Delta \sigma) [\rho_0 + 4 \rho_{1/2} + \rho_1] / 3$$

and $\rho_{1/2} = [4(\rho_0 + \rho_1) - \Delta \sigma (\sin \theta_1 - \sin \theta_0)] / 8.$

The terms ρ_0 , ρ_1 , θ_0 , θ_1 , ζ_0 , and ζ_1 are known values at the beginning and end of the increment $\Delta \sigma$. The following table (Table II) compares volume and surface area values for several numbers of increments.

Table II. Comparison of Balloon Volumes and Surface Areas for Several Numbers of Increments

Σ	θ_0 (deg)	No. of Increments	A/λ^2	Volume	
				V/λ^3	G/P
0	50.155	10	4.9173	.99239	1.00000
		20	4.9176	.99838	1.00000
		40	4.9176	.99987	1.00000
1.0	84.2	10	39.845	22.224	22.593
		20	39.844	22.407	22.592
		40	39.844	22.455	22.592
1.0	84.274	20	40.298	22.767	22.838
		50	40.296	22.822	22.837
		100	40.296	22.830	22.837

The striking thing to be noted in this table is that the surface area has converged by the time 20 increments are used, while the volume has not converged to its final value even after 100 increments. Thus, it is better to obtain V/λ^3 from G/P where

$$\begin{aligned} V/\lambda^3 &= G/P = W/P + 1 \\ &= (A/\lambda^2) (\Sigma/k) + 1. \end{aligned}$$

The reason for the better convergence of A/λ^2 is not immediately obvious except that the integration is done with respect to σ which is the directly incremented variable in the shape determination. In light of the availability of a rapidly converging estimate of the volume, it has been deemed not fruitful to explore other methods of improving V/λ^3 .

III. RESULTS

Appendix II lists the results of calculations for $\alpha = \tau = 0$ and for $\Sigma = 0$ (0.05) 1.0. Each value of θ_0 , the one unknown initial value and which is found by trial and error, is accurate to ± 0.001 degree. Its correctness is assessed by the approach of the angle at the top to -90° . A nominal number of 50 increments were used for all calculations. These results are applicable, in the strictest sense, only to fully tailored, tapeless, seamless balloons made throughout from in-elastic, flexible material of uniform weight with all weight, other than material, concentrated at the bottom. In the practical sense these may be used in any application where the original Sigma Tables (Reference 2) have been successfully used.

Each table is headed by the Σ value and θ_0 . The input value θ_0 is given only for information purposes. It is not used with other parts of the tables. The second and third columns give corresponding radial and height coordinates for the series of equally spaced points along the gore listed in the first column. (The spacing of the last point, the top of the balloon, is usually different.) All

coordinates are given from the bottom apex. The fourth column gives the total meridional load in the balloon material. Stresses are not given since these results are independent of material thickness or thickness distribution. At the bottom of each table the resulting surface area, volume, and weight are listed. All results are in non-dimensional terms. Thus, they are quite general--being independent of altitude, payload, inflation gas, or choice of balloon material.

To use the tables, λ is determined first from knowledge of the design altitude and inflation gas, and the payload. An estimate of the material weight will then permit calculation of Σ . From the proper Sigma Table, all physical characteristics are now available. A check on material stress should be made from T/P.

The balloon shapes given in the Sigma Tables all have flat tops. This has been done deliberately. A balloon with a light payload and a heavy top-end fitting, e. g. , weighing 10 percent of the load, will cause the top to re-enter by only 3 to 4 degrees on a $\Sigma = 0$ balloon, only 1 to 2 degrees on a $\Sigma = 0.4$ balloon, and even less on larger Σ balloons. Thus, in many cases, a flat top is a better approximation than some other value which, of necessity, will have to be arbitrary.

IV. BALLOONS WITHOUT A FLAT TOP

There are two classes of balloons with zero circumferential stress which do not have a flat top.

- 1) Balloons with part of the load applied at the top. These will be re-entrant there. A portion of the payload may be physically located at the top, or part of the payload may be supported by tension members from the payload to the top.
- 2) Balloons which have additional lift at the top. These will be conical there. The additional lift would usually be supplied by one or more balloons.

Both of these classes of balloons have been investigated. For balloons with either payload or additional lift applied at the top and at the axis of symmetry, the equations and computer program are unchanged. There is one minor exception: it is convenient to have as an output a term giving the load at the top,

$$F/P = - (T/P) \cos \theta_{\text{Top}}.$$

The minus sign is used so that F/P is positive for balloons with part of the payload at the top.

A. Part of the Payload at the Top

These balloon shapes are calculated by entering L/P and an estimate of θ_0 into the computer, such that

$$(L/P) + (F/P) = 1.$$

The computation is repeated until a value of θ_0 has been found which yields F/P to the desired accuracy.

Representative values of θ_0 for several values of F/P and for $0 \leq \Sigma \leq 1$ are shown in Figure 2. It will be noted that θ_0 is larger than for the corresponding flat-top case. The angle of the re-entrant top is shown in Figure 3.

In the limit, of course, θ_0 would be 90° (i. e., a flat bottom) for $L/P = 0$ and $F/P = 1$. Representative shapes of top-loaded balloons are shown in Figure 4. Corresponding meridional film loads are presented in Figure 5. Comparison with the Sigma Tables in Appendix II will show that for the same Σ , a top-loaded balloon will have slightly lower meridional loads. This means that, for the same payload, a slightly lighter material may be used in a top-loaded balloon.

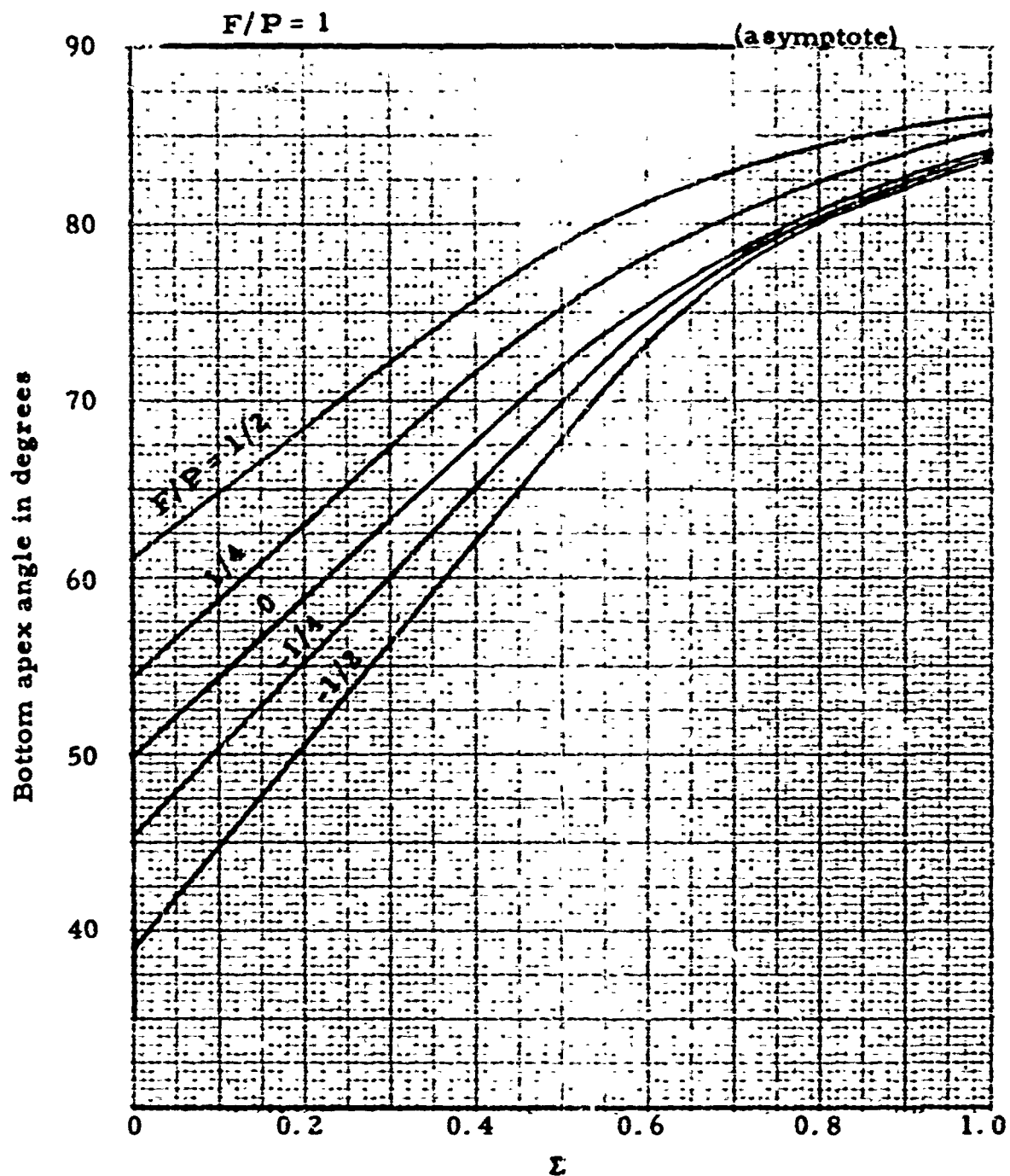


Figure 2. Variation of the bottom apex angle with the design parameter, Σ , for several portions of payload at the top of the balloon (Zero superpressure, zero circumferential stress)

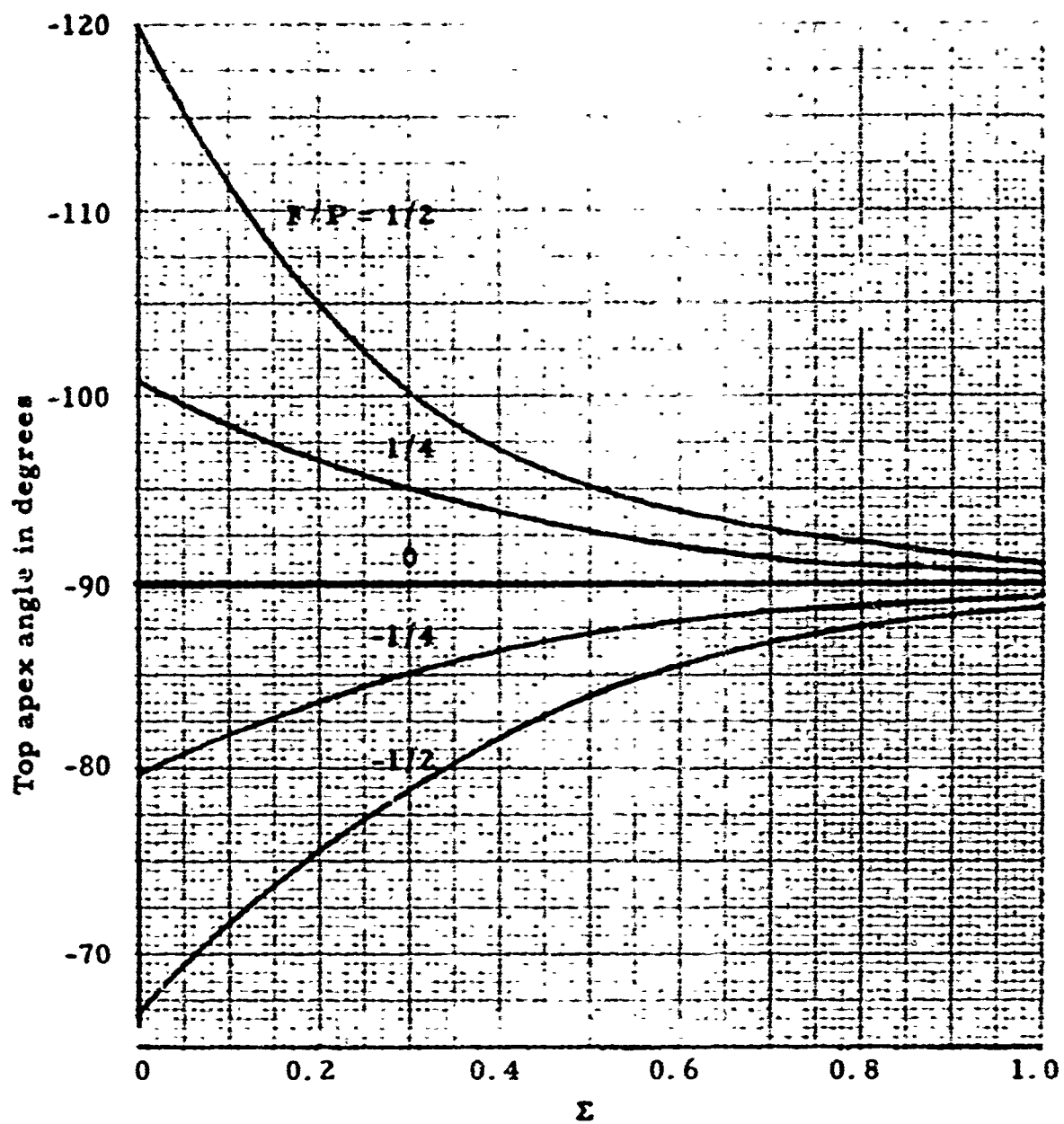


Figure 3. Variation of the top apex angle with the design parameter, Σ , for several portions of the payload at the top of the balloon
(Zero superpressure, zero circumferential stress)

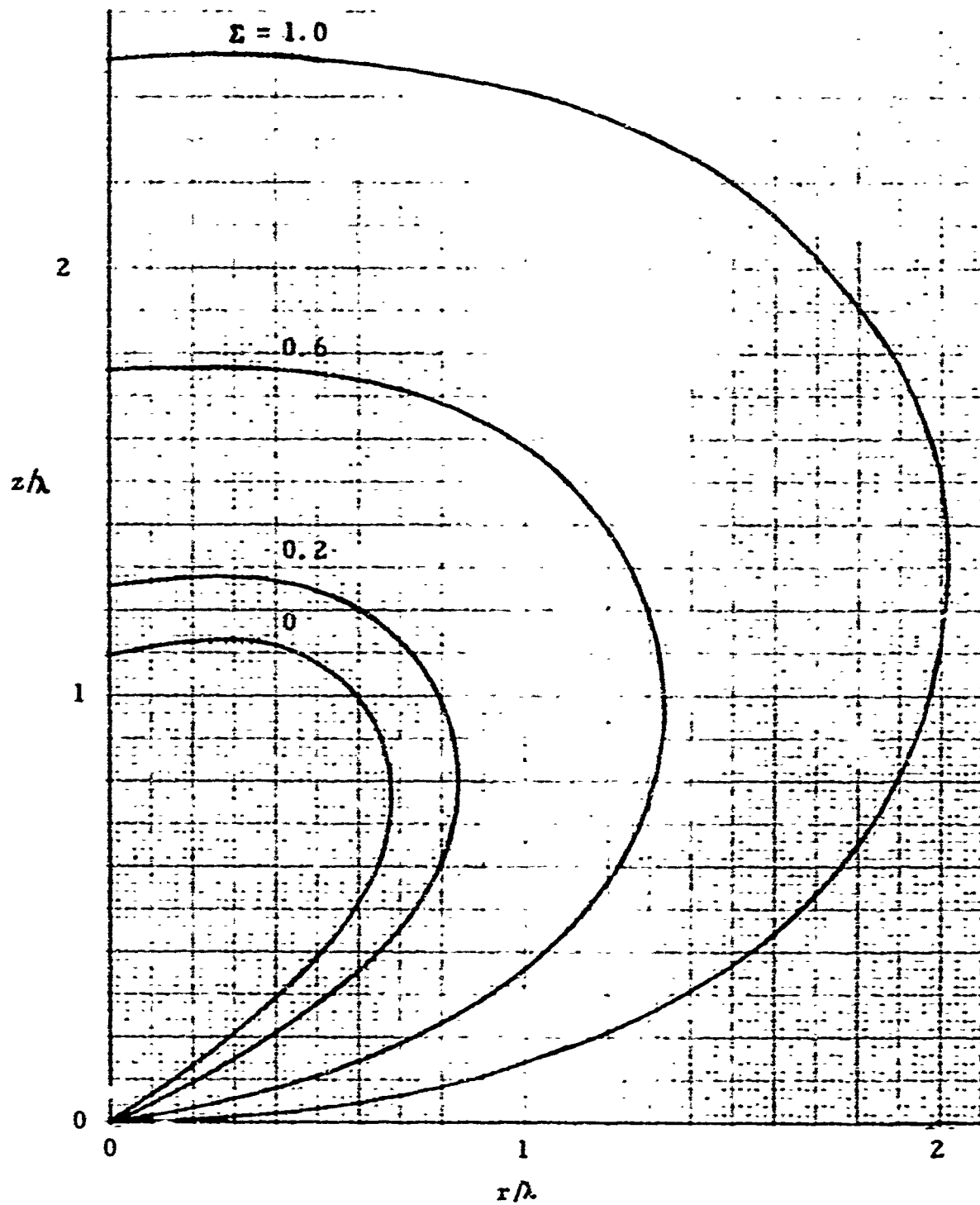


Figure 4. Shape of balloons with a portion of the payload at the top
(Zero superpressure, zero circumferential stress)

(a) one-fourth the payload at the top

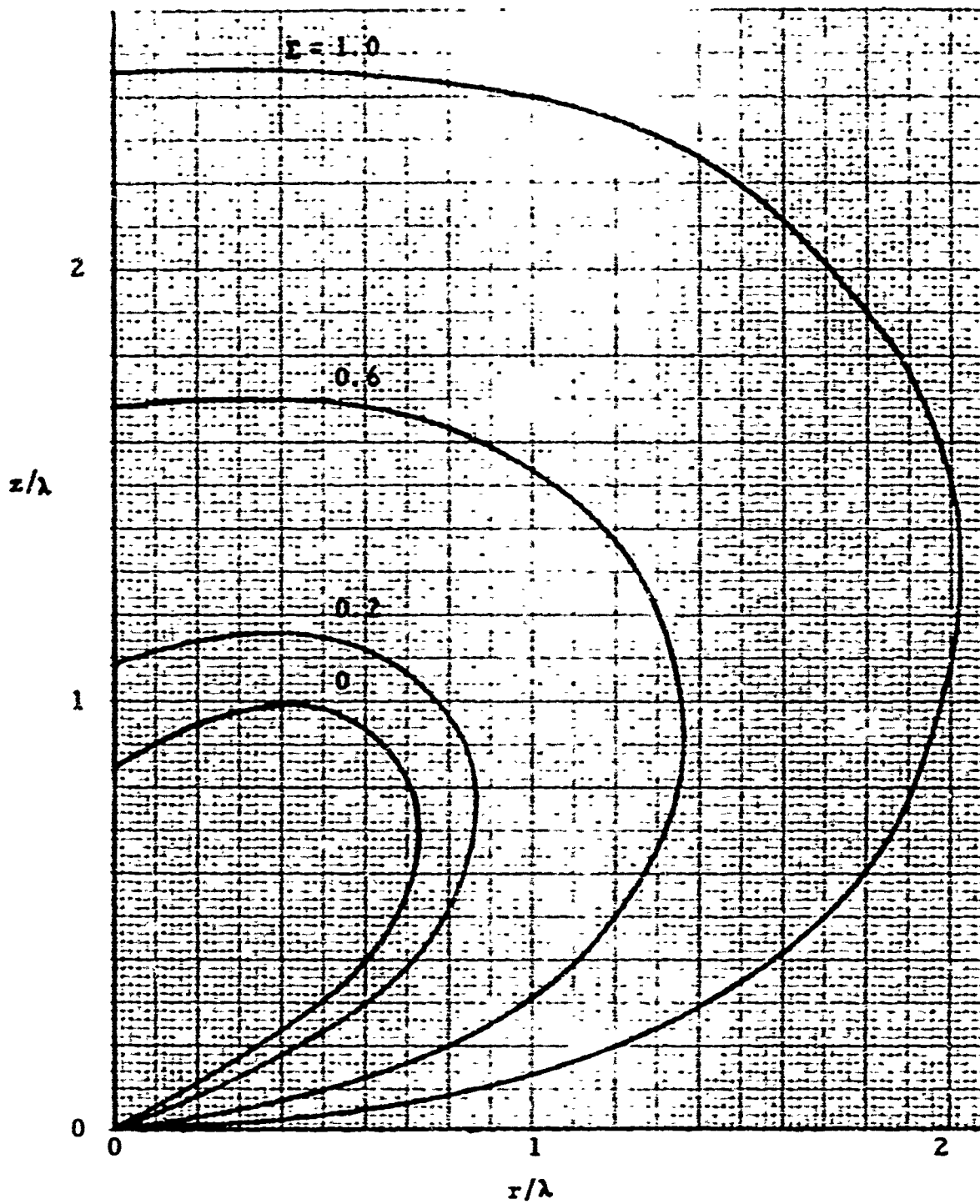


Figure 4. (concluded)

(b) one-half the payload at the top

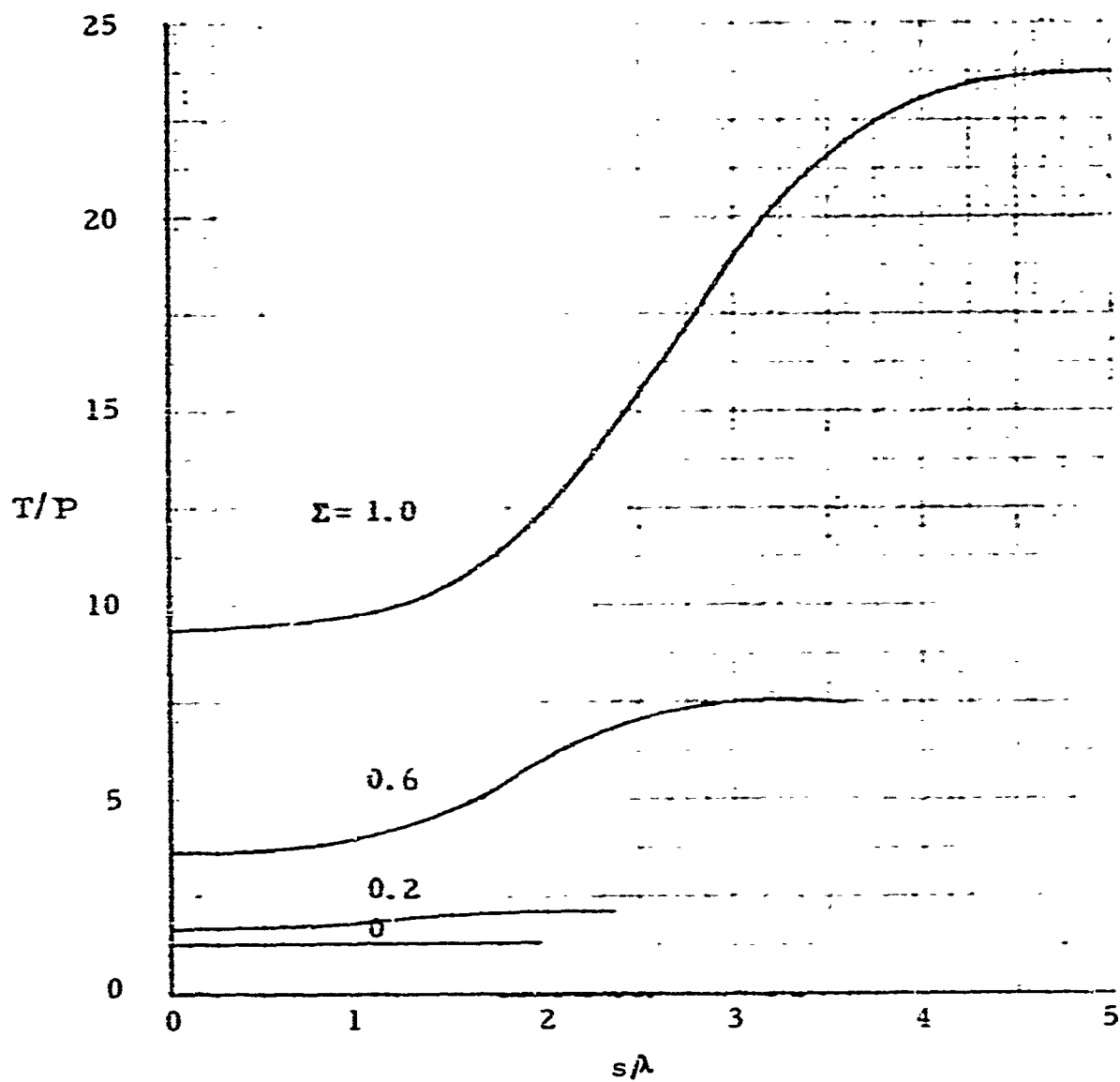


Figure 5. Variation of meridional film load with gore position for balloons with a portion of the payload at the top (Zero superpressure, zero circumferential stress)

(a) one-fourth the payload at the top

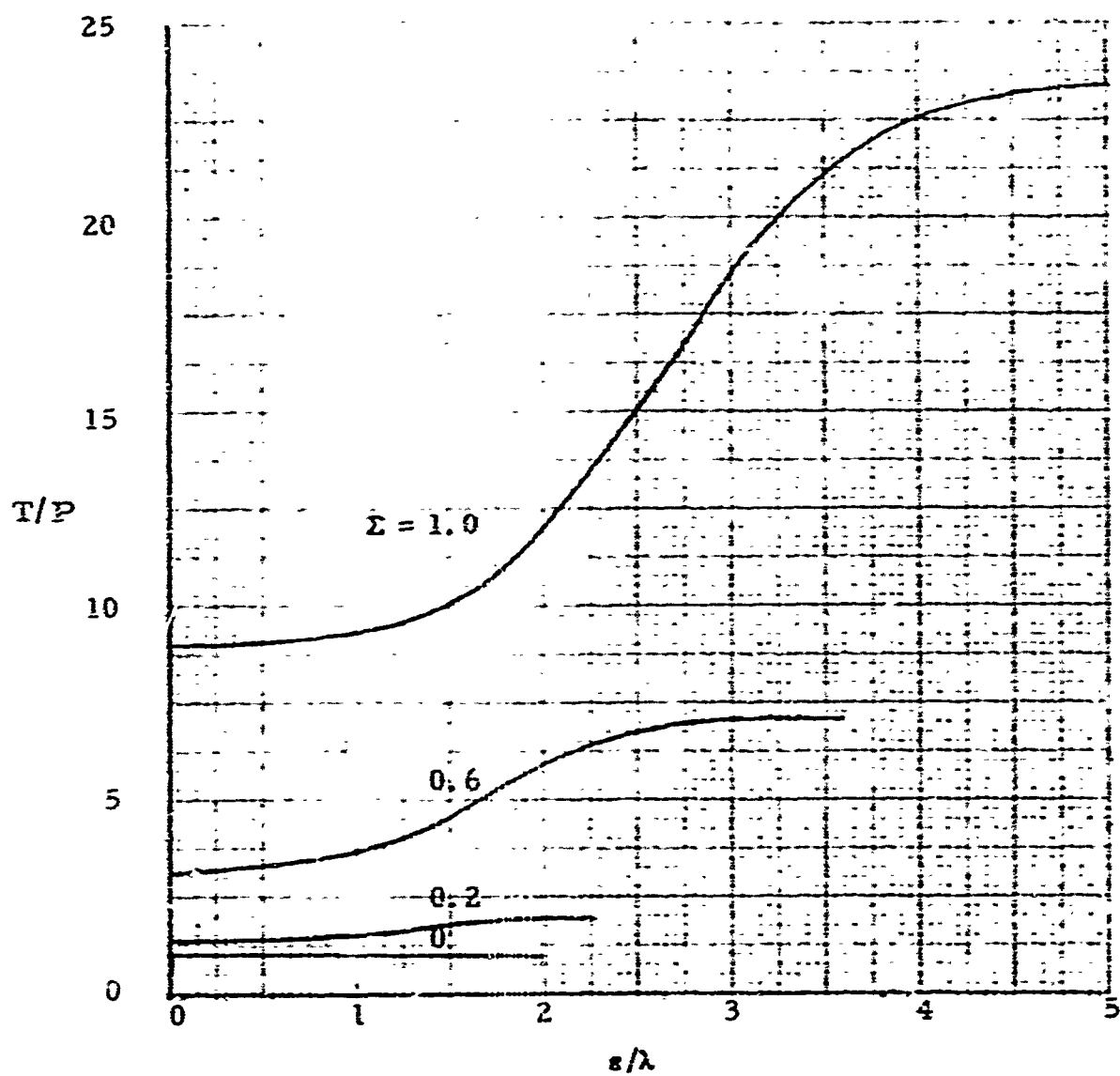


Figure 5. (concluded)

(b) one-half the payload at the top

B. Additional Lift at the Top

The investigations were confined to consideration of double balloons only. The methods and equations are equally applicable to multiple balloons.

There are several ways one could design a double balloon. If there is no gas passage between the two balloons, the upper balloon would be identical to the zero superpressure balloons given in the Sigma Tables. If there is a gas passage, the upper balloon then is subject to a superpressure. The upper balloon in either case has a flat top. The apportionment of load-carrying capacity between the two balloons can be arbitrary. In addition, the lift supplied by the upper balloon may be carried by the film of the lower balloon or by an axial structural member. After these items have been determined, the computation of the bottom balloon is accomplished with the same computer program. The solution is complete when the desired F/P (negative in this case) has been achieved. Representative values of the initial angle, θ_0 , are shown in Figure 2. The angle of the conical top on the lower balloon is shown in Figure 3. Representative shapes of double balloons are presented in Figure 6. Corresponding meridional film loads are given in Figure 7.

In Figure 8, the shape and meridional film load are presented for a natural-shape double balloon and compared with a natural-shape single balloon. The term "natural-shape double balloon" has been chosen because, in such a design, the top apex angle of the lower balloon is equal to the bottom apex angle of the upper balloon. Furthermore, the meridional loads and internal pressures are equal at the adjoining apexes. The upper balloon is simply an extension of the lower balloon. A gas passage is assumed to exist between the two balloons. This shape is interesting academically. In practice, a mechanical structure or end fitting would be located between the two balloons and equality of angle, pressure, and meridional load is not necessary. Meridional loads are somewhat less for double balloons but, for the same Σ , the weight is higher than for a single balloon.

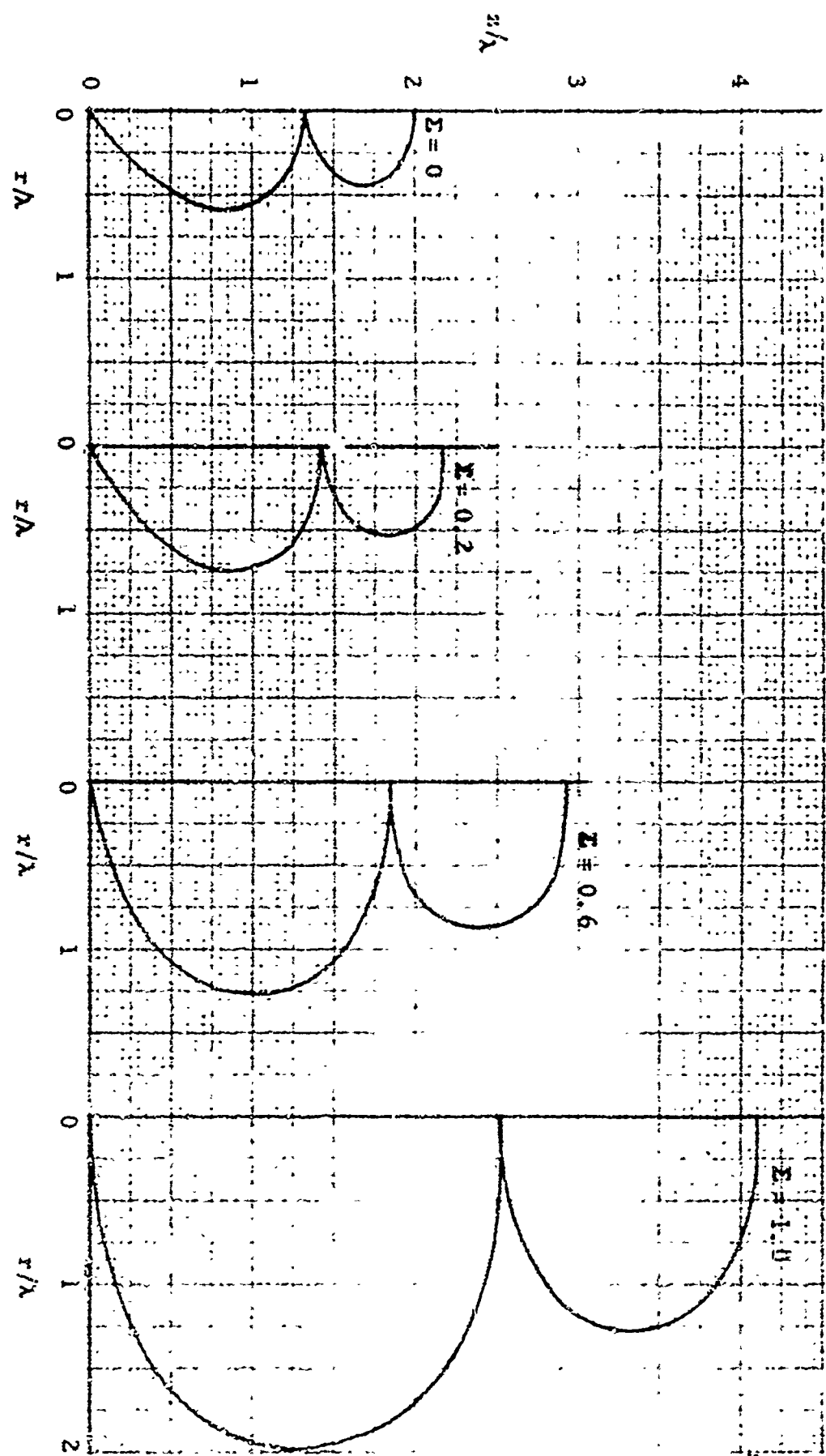


Figure 6. Shape of double balloons with the superpressure of the upper balloon equal to the height of the lower balloon (Zero superpressure, zero circumferential stress)

(a) one-fourth of the net lift supplied by the upper balloon

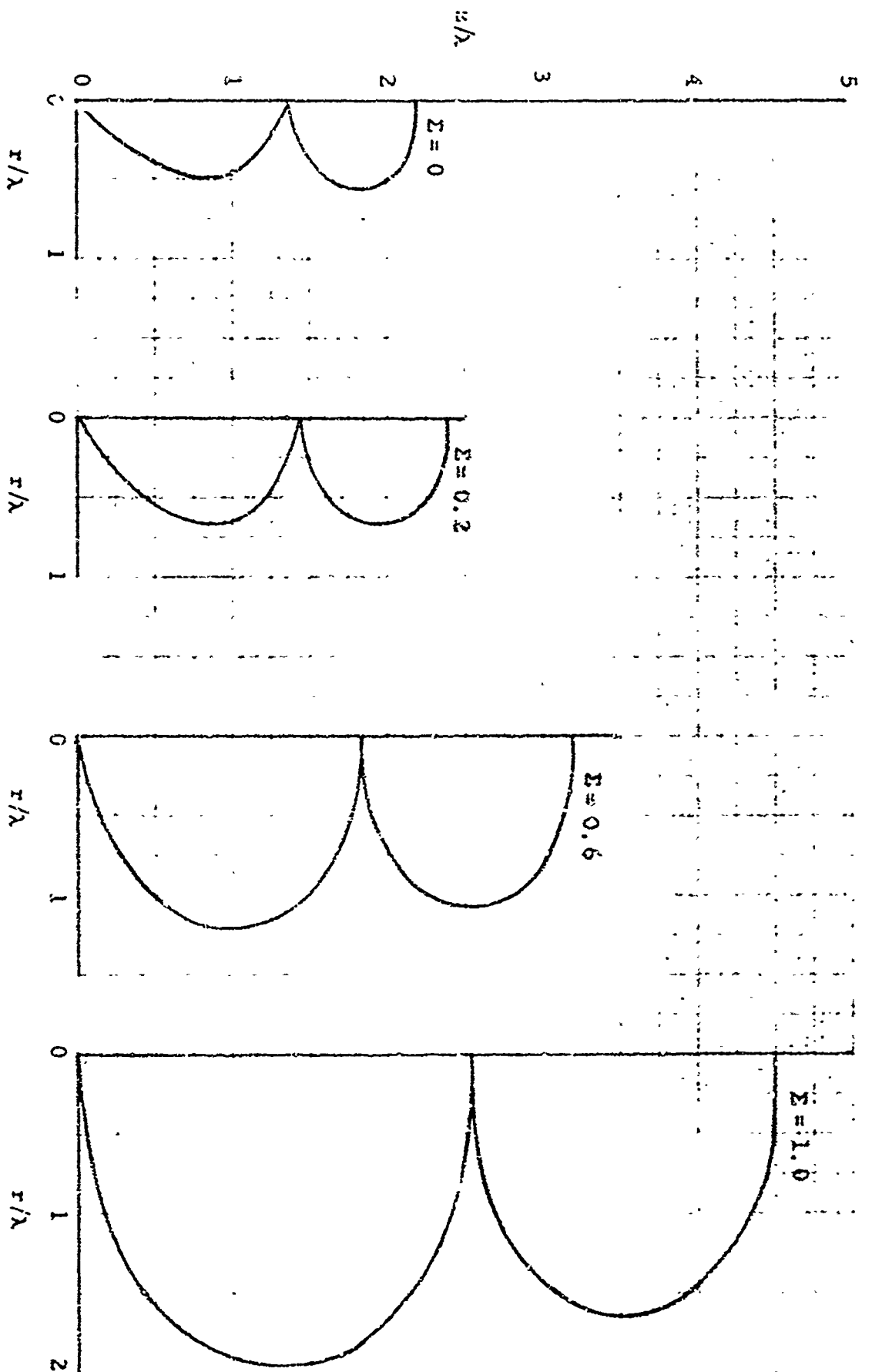


Figure 6. (concluded)

(b) one-half of the net lift supplied by each balloon

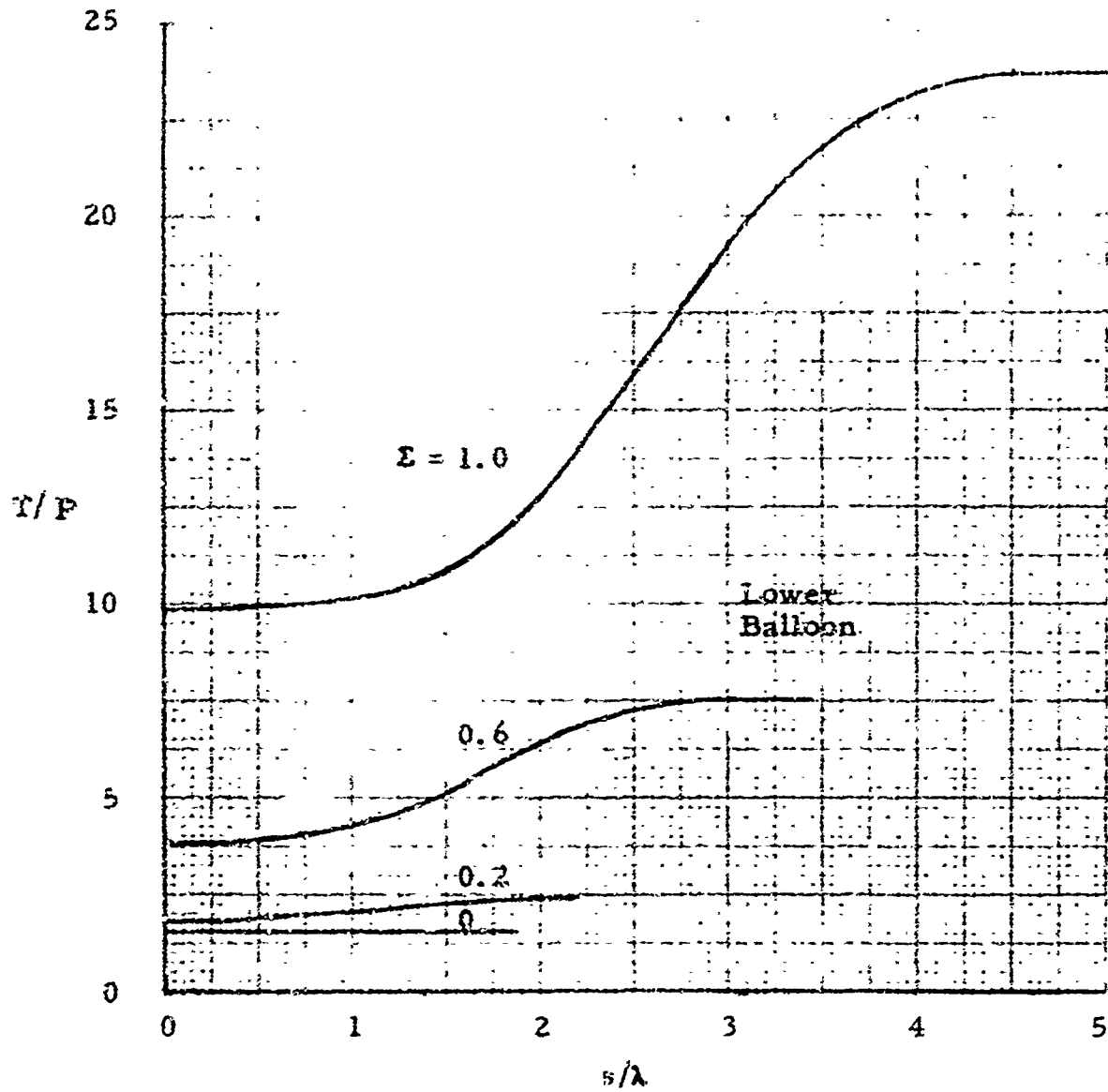


Figure 7. Variation of meridional film load with gore position for double balloons with the superpressure of the upper balloon equal to the height of the lower balloon (Zero superpressure, zero circumferential stress)

(a) one-fourth of the net lift supplied by the upper balloon

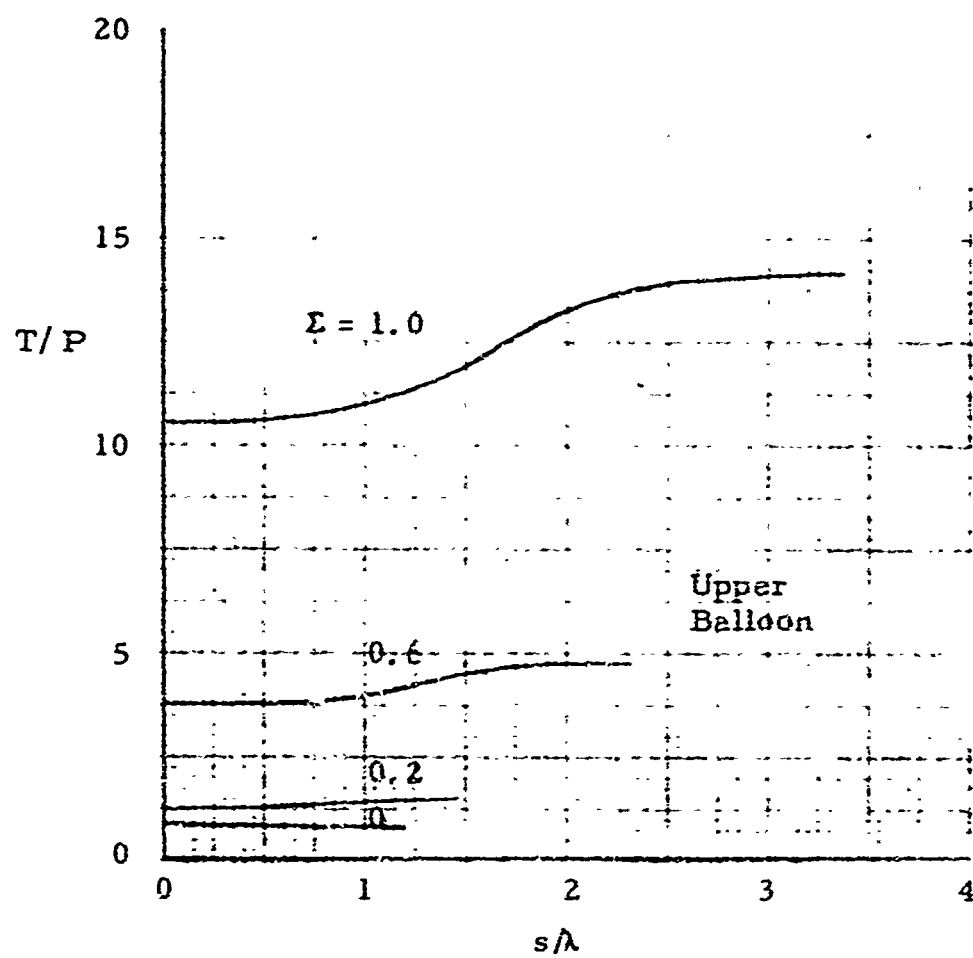


Figure 7. (continued)

(a) (continued)

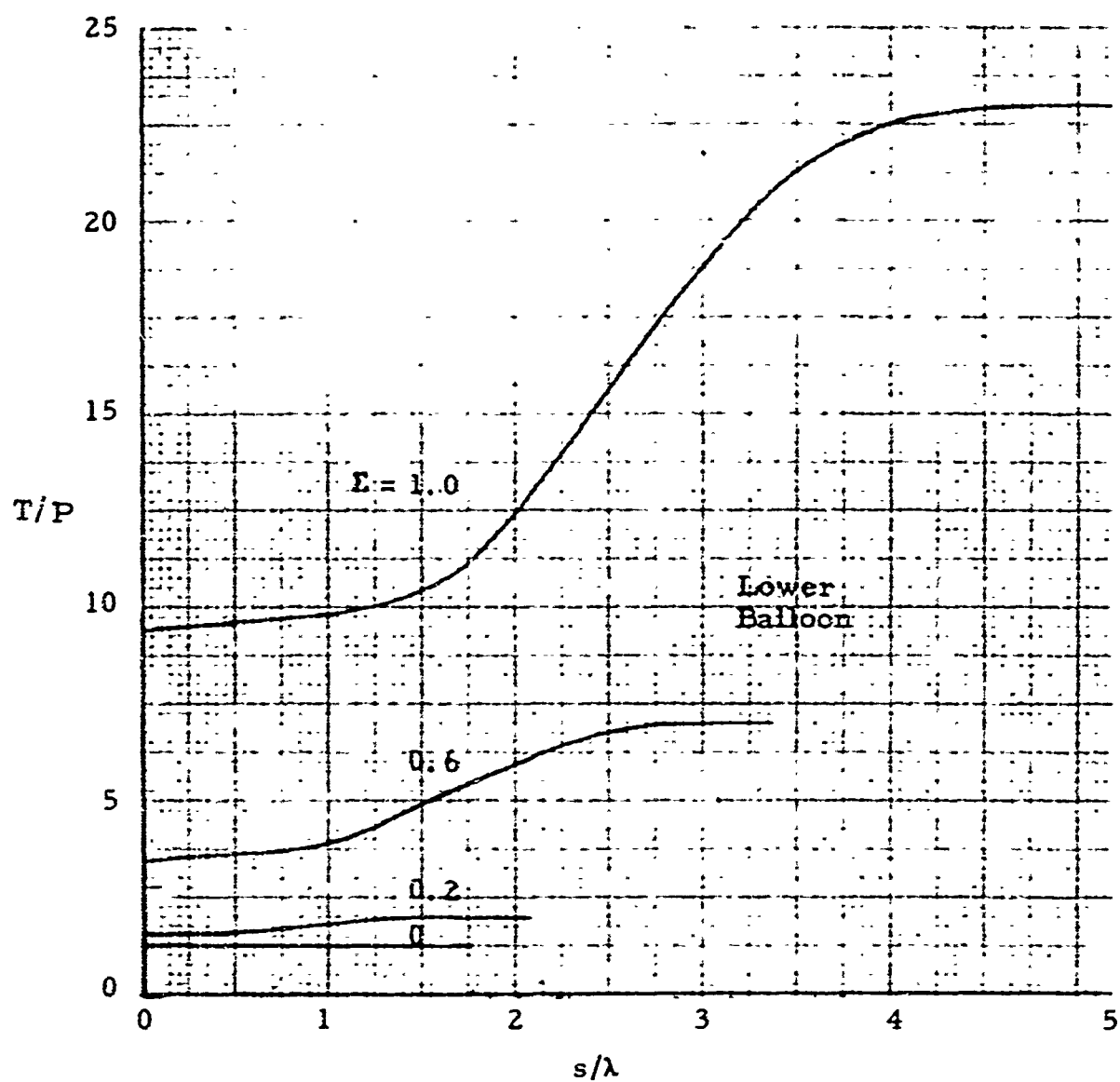


Figure 7. (continued)

(b) one-half of the net lift supplied by each balloon

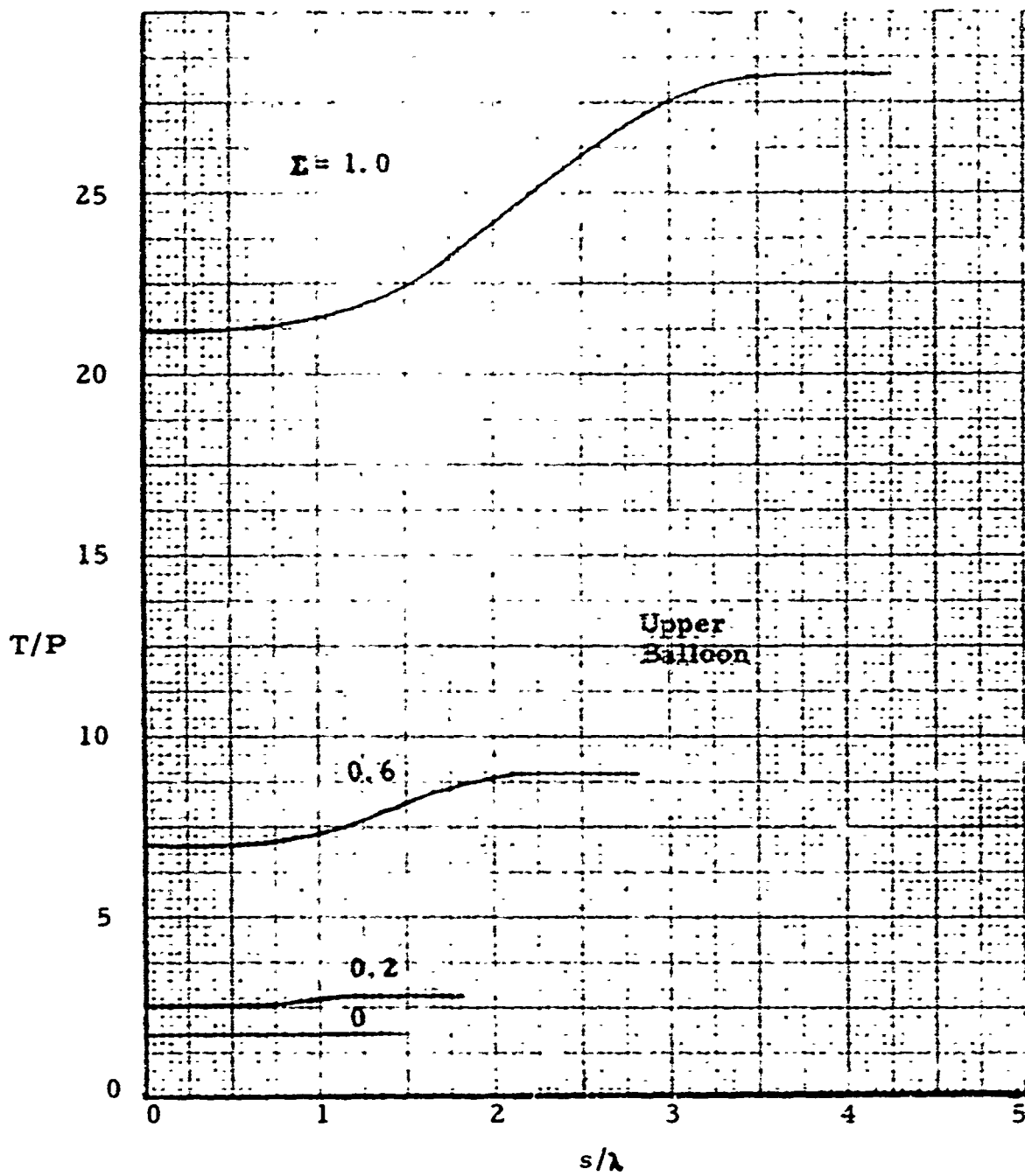


Figure 7. (concluded)

(b) (concluded)

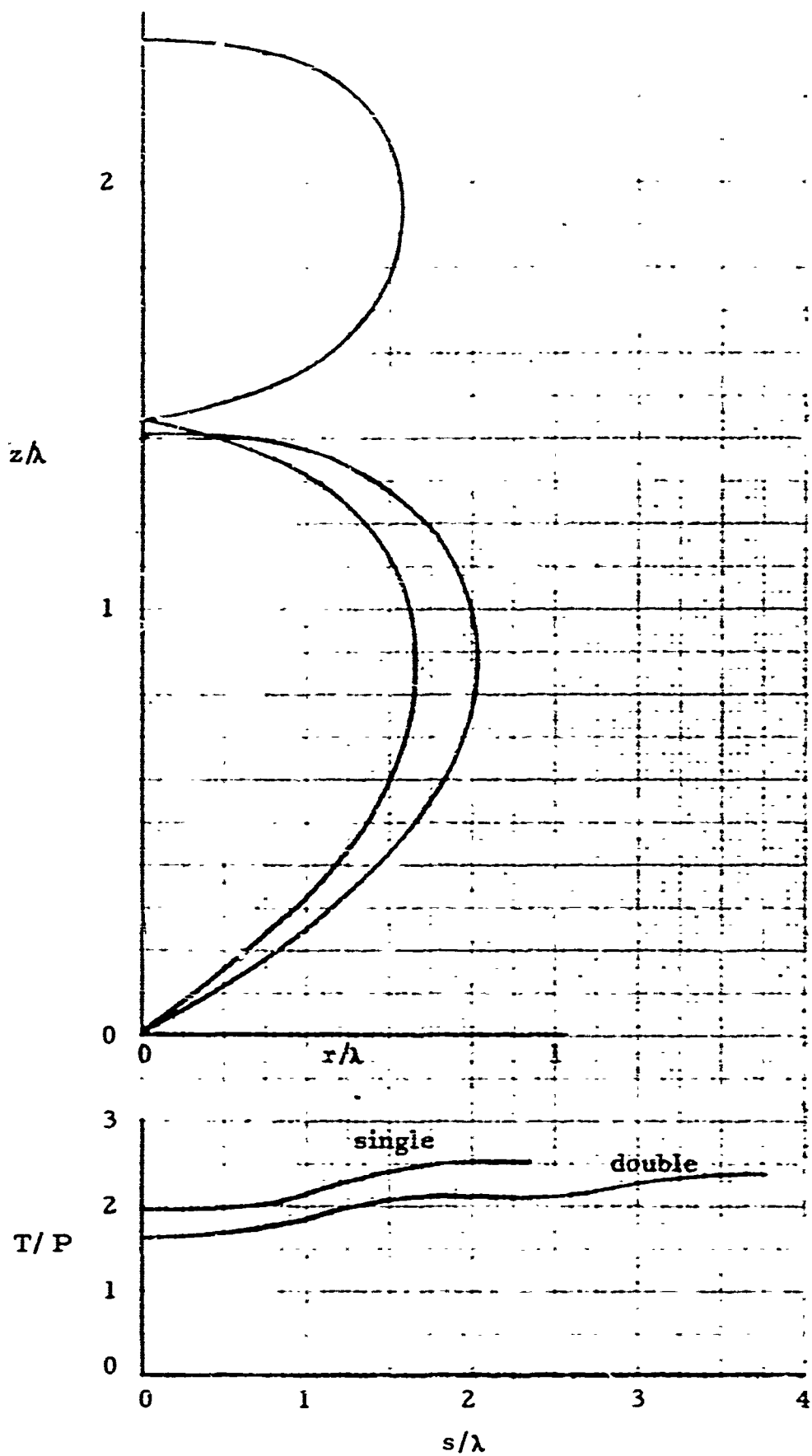


Figure 8. Comparison of the shape and meridional film load for a single natural-shape and double natural-shape balloon ($\Sigma = 0.2$, zero superpressure, zero circumferential stress)

The discontinuity in meridional loads in Figure 7(a) indicates it would not be necessary to use the same Σ for both the upper and lower balloons. A weight savings would be possible if meridional stress were matched rather than Σ .

In summary, Figure 9 shows the variation of balloon weight as a function of F/P . When F/P is negative, the weight given is for a double-balloon system. In this case, $|F/P|$ is the portion of lift provided by the upper balloon. [It should be noted that P is defined as the total payload, not the payload of either single balloon in a two-balloon system. Obviously, P in this case (as with the flat-top single cell) is thus equivalent to L , the load suspended at the nadir of the lower balloon.] Note that the flat-top single cell ($F/P = 0$) has minimal weight for any given payload.

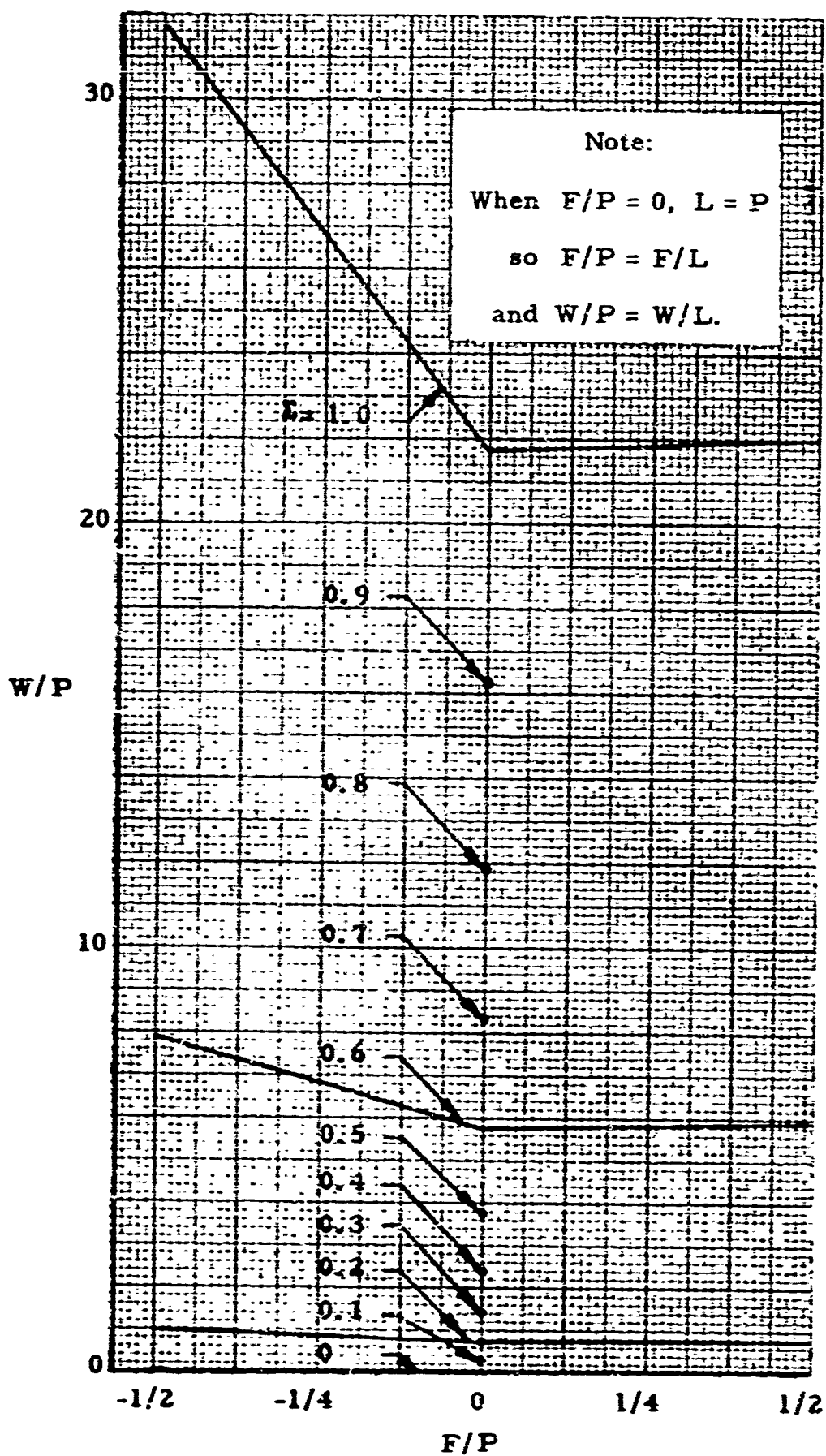


Figure 9. Balloon weight as a function of top load for a range of E values
(Zero superpressure, zero circumferential stress)

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APPENDIX I

DEVELOPMENT OF EXPRESSIONS FOR BALLOON AREA AND VOLUME

APPENDIX I

DEVELOPMENT OF EXPRESSIONS FOR BALLOON AREA AND VOLUME

During the calculation of balloon shapes, the problem of accurately computing the volume and area presented itself. Since the slope of the shape curve was known at both ends of each increment, it was decided to fit each increment with a polynomial and integrate over each increment separately. Probably the simplest formula for numerical integration with other than a straight line is Simpson's Rule. Use of this formula requires at least three equally spaced points. Thus, a method of determining the ordinate of the midpoint of the increment was investigated. The equations used are derived below.

If each increment is fitted with a second-degree curve, we may write

$$y = A x^2 + B x + C$$

and $y' = 2A x + B.$

Then, if $y = y_1$ and $y' = y'_1$ when $x = x_1$ and $y = y_0$ and $y' = y'_0$

when $x = x_0$, we have

$$A = (y'_1 - y'_0) / 2(x_1 - x_0)$$

$$B = \frac{y_1 - y_0}{x_1 - x_0} - A (x_0 + x_1)$$

$$C = A x_0 x_1 + (x_1 y_0 - x_0 y_1) / (x_1 - x_0).$$

Now let $h = x_1 - x_0$

and $x = x_0 + k h.$

Substituting $y = y_0 (1 - k) + y_1 (k) - k (1 - k) (y'_1 - y'_0) (h/2).$

For the midpoint, $k = 1/2$, so

$$y_{1/2} = \frac{y_0 + y_1}{2} - (h/8) (y_1' - y_0').$$

Now, making a transformation to balloon coordinates by referring to the sketch below, the above formula is

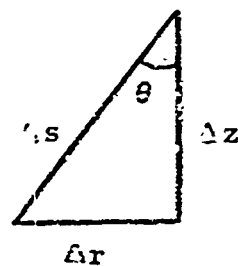
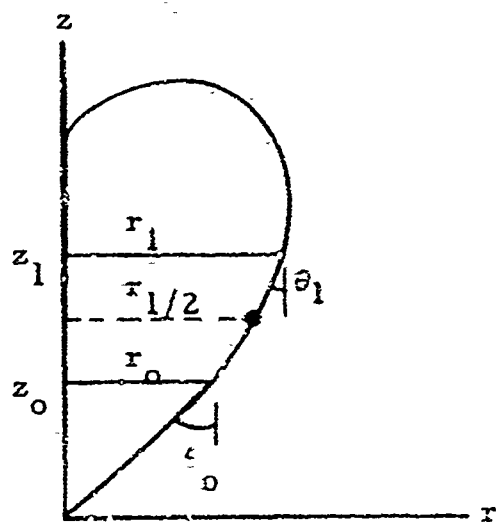
$$r_{1/2} = \frac{r_0 + r_1}{2} - (\Delta z/8) [(dr/dz)_1 - (dr/dz)_0]$$

but

$$\Delta z (dr/dz) = \Delta z \tan \theta = \Delta r = \Delta s \sin \theta$$

so

$$r_{1/2} = \frac{r_0 + r_1}{2} - (\Delta s/8) (\sin \theta_1 - \sin \theta_0)$$



The terms r_0 , r_1 , z_0 , z_1 , and s are known from the shape calculations.

Simpson's Rule* for the area under a curve is

$$\int_{x_0}^{x_0+nh} y \, dx = h/3 [y_0 + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) + y_n]$$

Application of the rule here requires care in that the increment h will refer to one half of the increment of interest.

We finally have

$$\begin{aligned} \Delta \text{ Volume} &= \pi \int_{z_0}^{z_1} r^2 \, dz \\ &= \pi \left(\frac{\Delta z/2}{3} \right) [(r_0)^2 + 4(r_{1/2})^2 + (r_1)^2] \end{aligned}$$

and

$$\begin{aligned} \Delta \text{ Area} &= 2\pi \int_{s_0}^{s_1} r \, ds \\ &= 2\pi \left(\frac{\Delta s/2}{3} \right) [r_0 + 4r_{1/2} + r_1] \end{aligned}$$

The additional term Δz is also known from the shape calculations. The derivation of the term $r_{1/2}$ has been shown in this appendix.

This method is useful for computation with a digital computer because there are no terms which might become very large (even though there may be infinite slopes) and introduce programming problems due to a possible overflow.

* Scarborough J. B. Numerical mathematical analysis, 3rd ed. Baltimore. Johns Hopkins Press, 1955. p. 132.

APPENDIX II

SIGMA TABLES
FOR
ZERO CIRCUMFERENTIAL STRESS,
ZERO SUPERPRESSURE AND
A FLAT TOP

s/λ	r/λ	z/λ	T/P
0.	0.	0.	1.56057
0.03392	0.03063	0.02557	1.56057
0.07980	0.06126	0.05114	1.56057
0.11970	0.09187	0.07673	1.56057
0.15960	0.12246	0.10235	1.56057
0.19950	0.15299	0.12804	1.56057
0.23940	0.18344	0.15382	1.56057
0.27930	0.21378	0.17974	1.56057
0.31920	0.24395	0.20585	1.56057
0.35910	0.27391	0.23220	1.56057
0.39900	0.30360	0.25886	1.56057
0.43890	0.33294	0.28589	1.56057
0.47880	0.36188	0.31337	1.56057
0.51870	0.39030	0.34136	1.56057
0.55860	0.41812	0.36997	1.56057
0.59850	0.44522	0.39925	1.56057
0.63840	0.47147	0.42930	1.56057
0.67830	0.49673	0.46018	1.56057
0.71820	0.52084	0.49197	1.56057
0.75810	0.54361	0.52473	1.56057
0.79800	0.56487	0.55849	1.56057
0.83790	0.58438	0.59329	1.56057
0.87780	0.60193	0.62912	1.56057
0.91770	0.61726	0.66595	1.56057
0.95760	0.63013	0.70370	1.56057
0.99750	0.64028	0.74228	1.56057
1.03740	0.64746	0.78152	1.56057
1.07730	0.65143	0.82121	1.56057
1.11720	0.65192	0.86109	1.56057
1.15710	0.64894	0.90086	1.56057
1.19700	0.64218	0.94017	1.56057
1.23690	0.63165	0.97864	1.56057
1.27680	0.61736	1.01588	1.56057
1.31670	0.59940	1.05149	1.56057
1.35660	0.57794	1.08510	1.56057
1.39650	0.55321	1.11639	1.56057
1.43640	0.52550	1.14508	1.56057
1.47630	0.49515	1.17096	1.56057
1.51620	0.46253	1.19392	1.56057
1.55610	0.42801	1.21369	1.56057
1.59600	0.39193	1.23092	1.56057
1.63590	0.35485	1.24511	1.56057
1.67580	0.31646	1.25662	1.56057
1.71570	0.27766	1.26567	1.56057
1.75560	0.23830	1.27251	1.56057
1.79550	0.19871	1.27744	1.56057
1.83540	0.15895	1.28075	1.56057
1.87530	0.11910	1.28275	1.56057
1.91520	0.07921	1.28378	1.56057
1.95510	0.03932	1.28416	1.56057
1.99442	0.00000	1.28421	1.56057

$$\Sigma = 0$$

$$\phi_0 = 50.149$$

$$A/\lambda^2 = 4.9163$$

$$V/\lambda^3 = 1 + (W)$$

$$W/P = 0$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	1.63274
0.04140	0.03273	0.02536	1.63282
0.08280	0.06544	0.05072	1.63303
0.12420	0.09814	0.07611	1.63338
0.16560	0.13080	0.10156	1.63388
0.20700	0.16340	0.12708	1.63452
0.24840	0.19589	0.15273	1.63530
0.28980	0.22825	0.17856	1.63623
0.33120	0.26042	0.20462	1.63732
0.37260	0.29234	0.23099	1.63856
0.41400	0.32395	0.25772	1.63996
0.45540	0.35517	0.28491	1.64153
0.49680	0.38592	0.31262	1.64328
0.53820	0.41610	0.34096	1.64522
0.57960	0.44560	0.37001	1.64735
0.62100	0.47430	0.39985	1.64969
0.66240	0.50204	0.43057	1.65224
0.70380	0.52866	0.46226	1.65502
0.74520	0.55403	0.49499	1.65804
0.78660	0.57791	0.52880	1.66130
0.82800	0.60009	0.56376	1.66480
0.86940	0.62034	0.59986	1.66856
0.91080	0.63843	0.63709	1.67255
0.95220	0.65408	0.67541	1.67677
0.99360	0.66703	0.71472	1.68119
1.03500	0.67702	0.75489	1.68579
1.07640	0.68378	0.79572	1.69052
1.11780	0.68710	0.83697	1.69533
1.15920	0.68677	0.87836	1.70018
1.20060	0.68263	0.91954	1.70498
1.24200	0.67460	0.96013	1.70967
1.28340	0.66264	0.99975	1.71418
1.32480	0.64680	1.03798	1.71845
1.36620	0.62720	1.07443	1.72240
1.40760	0.60406	1.10874	1.72600
1.44900	0.57762	1.14058	1.72921
1.49040	0.54821	1.16969	1.73200
1.53180	0.51619	1.19590	1.73438
1.57320	0.48191	1.21910	1.73635
1.61460	0.44577	1.23926	1.73794
1.65600	0.40811	1.25643	1.73919
1.69740	0.36926	1.27072	1.74014
1.73880	0.32953	1.28232	1.74083
1.78020	0.28915	1.29143	1.74131
1.82160	0.24833	1.29833	1.74163
1.86300	0.20723	1.30330	1.74182
1.90440	0.16597	1.30665	1.74193
1.94580	0.12462	1.30868	1.74198
1.98720	0.08324	1.30973	1.74200
2.02860	0.04184	1.31011	1.74201
2.07000	0.00044	1.31017	1.74201
2.07044	0.00000	1.31017	1.74201

$$\varepsilon = 0.05$$

$$\phi_0 = 52.232$$

$$A/\lambda^2 = 5.2$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 0.1458$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	1.71747
0.04309	0.03503	0.02509	1.71762
0.08618	0.07005	0.05020	1.71807
0.12927	0.10505	0.07534	1.71882
0.17236	0.13999	0.10055	1.71987
0.21545	0.17485	0.12587	1.72123
0.25854	0.20960	0.15136	1.72290
0.30163	0.24418	0.17707	1.72489
0.34472	0.27854	0.20308	1.72720
0.38781	0.31261	0.22945	1.72986
0.43090	0.34634	0.25627	1.73287
0.47399	0.37962	0.28363	1.73625
0.51708	0.41237	0.31164	1.74003
0.56017	0.44448	0.34037	1.74422
0.60326	0.47582	0.36994	1.74885
0.64635	0.50626	0.40044	1.75395
0.68944	0.53564	0.43196	1.75955
0.73253	0.56377	0.46459	1.76566
0.77562	0.59047	0.49841	1.77230
0.81871	0.61552	0.53346	1.77951
0.86180	0.63869	0.56979	1.78727
0.90489	0.65971	0.60739	1.79558
0.94798	0.67833	0.64624	1.80443
0.99107	0.69427	0.68627	1.81379
1.03416	0.70725	0.72735	1.82360
1.07725	0.71699	0.76931	1.83378
1.12034	0.72324	0.81193	1.84423
1.16343	0.72577	0.85493	1.85484
1.20652	0.72440	0.89799	1.86548
1.24961	0.71900	0.94072	1.87599
1.29270	0.70951	0.98273	1.88621
1.33579	0.69593	1.02361	1.89599
1.37888	0.67834	1.06293	1.90520
1.42197	0.65692	1.10030	1.91370
1.46506	0.63190	1.13535	1.92140
1.50815	0.60356	1.16779	1.92823
1.55124	0.57225	1.19737	1.93415
1.59433	0.53834	1.22394	1.93918
1.63742	0.50221	1.24740	1.94334
1.68051	0.46424	1.26774	1.94669
1.72360	0.42479	1.28505	1.94931
1.76669	0.38418	1.29943	1.95130
1.80978	0.34271	1.31109	1.95275
1.85287	0.30061	1.32025	1.95375
1.89596	0.25808	1.32718	1.95441
1.93905	0.21528	1.33216	1.95482
1.98214	0.17233	1.33551	1.95504
2.02523	0.12929	1.33755	1.95515
2.06832	0.08621	1.33859	1.95519
2.11141	0.04312	1.33898	1.95519
2.15450	0.00003	1.33904	1.95520
2.15453	0.00000	1.33904	1.95520

$$\Sigma = 0.10$$

$$\theta_0 = 54.391$$

$$A/\lambda^2 = 5.9142$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 0.3205$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

r/λ	r/λ	z/λ	T/P
0.	0.	0.	1.81741
0.04496	0.03754	0.02474	1.81765
0.08992	0.07507	0.04950	1.81836
0.13488	0.11256	0.07432	1.81955
0.17984	0.14999	0.09923	1.82122
0.22480	0.18732	0.12428	1.82338
0.26976	0.22451	0.14954	1.82604
0.31472	0.26151	0.17509	1.82921
0.35968	0.29826	0.20099	1.83291
0.40464	0.33468	0.22734	1.83717
0.44960	0.37071	0.25424	1.84202
0.49456	0.40624	0.28179	1.84749
0.53952	0.44117	0.31010	1.85362
0.58448	0.47537	0.33927	1.86045
0.62944	0.50872	0.36942	1.86803
0.67440	0.54106	0.40066	1.87640
0.71936	0.57221	0.43308	1.88562
0.76432	0.60197	0.46677	1.89573
0.80928	0.63013	0.50181	1.90676
0.85424	0.65645	0.53826	1.91874
0.89920	0.68068	0.57613	1.93168
0.94416	0.70252	0.61541	1.94556
0.98912	0.72171	0.65606	1.96035
1.03408	0.73794	0.69798	1.97598
1.07904	0.75092	0.74101	1.99235
1.12400	0.76035	0.78495	2.00932
1.16896	0.76605	0.82954	2.02670
1.21392	0.76773	0.87445	2.04430
1.25888	0.76525	0.91933	2.06188
1.30384	0.75850	0.96376	2.07918
1.34880	0.74745	1.00732	2.09594
1.39376	0.73216	1.04958	2.11192
1.43872	0.71273	1.09011	2.12688
1.48368	0.68937	1.12850	2.14064
1.52864	0.66236	1.16442	2.15304
1.57360	0.63200	1.19756	2.16401
1.61856	0.59867	1.22771	2.17349
1.66352	0.56275	1.25472	2.18151
1.70848	0.52463	1.27853	2.18813
1.75344	0.48468	1.29914	2.19345
1.79840	0.44328	1.31665	2.19760
1.84336	0.40075	1.33119	2.20074
1.88832	0.35736	1.34296	2.20302
1.93328	0.31337	1.35220	2.20461
1.97824	0.26896	1.35918	2.20565
2.02320	0.22428	1.36420	2.20629
2.06816	0.17945	1.36757	2.20664
2.11312	0.13454	1.36961	2.20681
2.15808	0.08959	1.37066	2.20687
2.20304	0.04463	1.37105	2.20688
2.24767	0.00000	1.37110	2.20688

$$\Sigma = 0.15$$

$$\phi_0 = 56.617$$

$$\lambda/\lambda^2 = 6.5275$$

$$v/\lambda^3 = 1 + (W/P)$$

$$W/P = 0.5306$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

r/λ	r/λ	r/λ	T/P
0.	0.	0.	1.93514
0.04701	0.04024	0.02430	1.93548
0.09402	0.08047	0.04862	1.93648
0.14103	0.12065	0.07302	1.93815
0.18804	0.16076	0.09754	1.94050
0.23505	0.20075	0.12225	1.94354
0.28206	0.24058	0.14722	1.94729
0.32907	0.28019	0.17254	1.95178
0.37608	0.31952	0.19830	1.95705
0.42309	0.35848	0.22460	1.96312
0.47010	0.39699	0.25156	1.97006
0.51711	0.43495	0.27930	1.97792
0.56412	0.47223	0.30793	1.98676
0.61113	0.50871	0.33758	1.99667
0.65814	0.54423	0.36836	2.00771
0.70515	0.57862	0.40041	2.01997
0.75216	0.61169	0.43382	2.03352
0.79917	0.64321	0.46869	2.04842
0.84618	0.67295	0.50509	2.06474
0.89319	0.70064	0.54307	2.08251
0.94020	0.72600	0.58265	2.10175
0.98721	0.74873	0.62378	2.12241
1.03422	0.76852	0.66641	2.14444
1.08123	0.78597	0.71040	2.16772
1.12824	0.79896	0.75557	2.19208
1.17525	0.80722	0.80166	2.21729
1.22226	0.81229	0.84838	2.24306
1.26927	0.81308	0.89537	2.26908
1.31628	0.80944	0.94222	2.29497
1.36329	0.80129	0.98850	2.32037
1.41030	0.78864	1.03375	2.34488
1.45731	0.77156	1.07753	2.36815
1.50432	0.75023	1.11940	2.38986
1.55133	0.72488	1.15897	2.40974
1.59834	0.69580	1.19588	2.42761
1.64535	0.66334	1.22986	2.44335
1.69236	0.62789	1.26071	2.45693
1.73937	0.58984	1.28830	2.46838
1.78638	0.54960	1.31258	2.47781
1.83339	0.50756	1.33358	2.48537
1.88040	0.46406	1.35139	2.49128
1.92741	0.41845	1.36617	2.49573
1.97442	0.37399	1.37814	2.49897
2.02143	0.32793	1.38753	2.50122
2.06844	0.28147	1.39462	2.50270
2.11545	0.23474	1.39973	2.50360
2.16246	0.18786	1.40316	2.50410
2.20947	0.14089	1.40524	2.50434
2.25648	0.09390	1.40631	2.50442
2.30349	0.04689	1.40671	2.50444
2.35038	0.00000	1.40676	2.50445

$$\Sigma = 0.20$$

$$\theta_0 = 56.885$$

$$A/\lambda^2 = 7.2517$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 0.7838$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

r/λ	r/λ	z/λ	T/P
0.	0.	0.	2.07450
0.04928	0.04317	0.02376	2.07494
0.09856	0.08632	0.04756	2.07525
0.14784	0.12942	0.07146	2.07844
0.19712	0.17243	0.09551	2.08153
0.24640	0.21531	0.11960	2.08554
0.29568	0.25800	0.14442	2.09050
0.34496	0.30045	0.16946	2.09646
0.39424	0.34257	0.19504	2.10346
0.44352	0.38428	0.22128	2.11156
0.49280	0.42549	0.24830	2.12090
0.54208	0.46609	0.27623	2.13150
0.59136	0.50593	0.30523	2.14350
0.64064	0.54489	0.33541	2.15701
0.68992	0.58277	0.36692	2.17214
0.73920	0.61940	0.39968	2.18901
0.78848	0.65455	0.43441	2.20774
0.83776	0.68799	0.47061	2.22843
0.88704	0.71944	0.50854	2.25117
0.93632	0.74861	0.54825	2.27599
0.98560	0.77520	0.58973	2.30290
1.03488	0.79888	0.63294	2.33186
1.08416	0.81931	0.67777	2.36275
1.13344	0.83616	0.72406	2.39538
1.18272	0.84917	0.77158	2.42949
1.23200	0.85799	0.82004	2.46472
1.28128	0.86241	0.86911	2.50066
1.33056	0.86223	0.91837	2.53684
1.37984	0.85734	0.96739	2.57273
1.42912	0.84766	1.01569	2.60760
1.47840	0.83331	1.06281	2.64153
1.52768	0.81434	1.10827	2.67343
1.57696	0.79097	1.15163	2.70308
1.62624	0.76346	1.19251	2.73013
1.67552	0.73218	1.23055	2.75437
1.72480	0.69746	1.26550	2.77565
1.77408	0.65972	1.29716	2.79395
1.82336	0.61937	1.32542	2.80936
1.87264	0.57682	1.35026	2.82202
1.92192	0.53247	1.37172	2.83216
1.97120	0.48668	1.38990	2.84006
2.02048	0.43977	1.40498	2.84602
2.06976	0.39203	1.41717	2.85034
2.11904	0.34369	1.42674	2.85335
2.16832	0.29495	1.43396	2.85532
2.21760	0.24595	1.43915	2.85652
2.26688	0.19679	1.44264	2.85718
2.31616	0.14756	1.44476	2.85750
2.36544	0.09829	1.44584	2.85761
2.41472	0.04901	1.44624	2.85764
2.46373	0.00000	1.44629	2.85764

$$\Sigma = 0.25$$

$$\theta_0 = 61.181$$

$$\lambda/\lambda^2 = 8.3431$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 1.0897$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

r/λ	r/λ	z/λ	T/P
.00000	.00000	.00000	2.23863
.05175	.04630	.02313	2.23918
.10352	.09258	.04631	2.24082
.15528	.13880	.06962	2.24358
.20704	.18492	.09312	2.24747
.25880	.23088	.11692	2.25252
.31056	.27663	.14111	2.25880
.36232	.32211	.16583	2.26536
.41408	.36723	.19119	2.27329
.46584	.41190	.21734	2.28170
.51760	.45601	.24441	2.29171
.56935	.49945	.27257	2.30145
.62112	.54205	.30196	2.32709
.67288	.58366	.33273	2.34479
.72464	.62410	.36504	2.36473
.77640	.66314	.39902	2.38708
.82816	.70054	.43479	2.41201
.87992	.73683	.47245	2.43965
.93168	.76933	.51208	2.47013
.98344	.80009	.55369	2.50350
1.03520	.82800	.59727	2.53976
1.08696	.85270	.64274	2.57981
1.13872	.87383	.68998	2.62048
1.19048	.89105	.73877	2.66448
1.24224	.90404	.78886	2.71042
1.29400	.91252	.83990	2.75780
1.34576	.91625	.89151	2.80602
1.39752	.91507	.94323	2.85443
1.44928	.90888	.99460	2.90230
1.50104	.89767	1.04511	2.94893
1.55280	.88151	1.09426	2.99362
1.60456	.86057	1.14157	3.03574
1.65632	.83508	1.18660	3.07475
1.70808	.80535	1.22894	3.11025
1.75984	.77173	1.26827	3.14195
1.81160	.73463	1.30433	3.16972
1.86336	.69446	1.33695	3.19355
1.91512	.65166	1.36603	3.21356
1.96688	.60665	1.39156	3.22998
2.01864	.55982	1.41359	3.24312
2.07040	.51155	1.43224	3.25335
2.12216	.46216	1.44770	3.26105
2.17392	.41194	1.46020	3.26664
2.22568	.36113	1.47001	3.27053
2.27744	.30990	1.47741	3.27307
2.32920	.25842	1.48273	3.27463
2.38096	.20672	1.48631	3.27548
2.43272	.15507	1.48848	3.27589
2.48448	.10333	1.48960	3.27604
2.53524	.05157	1.49001	3.27608
2.58701	.00000	1.49007	3.27608

$$\epsilon = 0.30$$

$$\phi_0 = 63.468$$

$$\lambda/\lambda^2 = 8.9719$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 1.4586$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
.00000	.00000	.00000	2.43238
.05447	.04965	.02240	2.43304
.10894	.09927	.04488	2.43903
.16341	.14882	.06750	2.43838
.21788	.19825	.09038	2.44311
.27235	.24751	.11361	2.44929
.32682	.29655	.13733	2.45698
.38129	.34528	.16167	2.46629
.43576	.39361	.18679	2.47735
.49023	.44145	.21283	2.49032
.54470	.48868	.23996	2.50536
.59917	.53516	.26835	2.52269
.65364	.58074	.29818	2.54254
.70811	.62522	.32961	2.56513
.76258	.66840	.36280	2.59073
.81705	.71004	.39791	2.61958
.87152	.74987	.43506	2.65191
.92599	.78759	.47434	2.68792
.98046	.82287	.51583	2.72775
1.03493	.85536	.55954	2.77148
1.08940	.88469	.60542	2.81907
1.14387	.91049	.65338	2.87040
1.19834	.93237	.70324	2.92518
1.25281	.94998	.75477	2.98300
1.30728	.96299	.80765	3.04330
1.36175	.97112	.86149	3.10537
1.41622	.97414	.91585	3.16842
1.47069	.97192	.97025	3.23153
1.52516	.96438	1.02418	3.29378
1.57963	.95154	1.07709	3.35421
1.63410	.93352	1.12847	3.41196
1.68857	.91052	1.17782	3.46621
1.74304	.88281	1.22469	3.51632
1.79751	.85072	1.26867	3.56179
1.85198	.81464	1.30946	3.60229
1.90645	.77500	1.34680	3.63768
1.96092	.73225	1.38052	3.66799
2.01539	.68682	1.41054	3.69340
2.06986	.63915	1.43687	3.71423
2.12433	.58965	1.45958	3.73088
2.17880	.53870	1.47879	3.74361
2.23327	.48661	1.49471	3.75356
2.28774	.43369	1.50757	3.76063
2.34221	.38017	1.51766	3.76553
2.39668	.32624	1.52528	3.76875
2.45115	.27205	1.53076	3.77072
2.50562	.21770	1.53444	3.77180
2.56009	.16328	1.53667	3.77231
2.61456	.10883	1.53782	3.77250
2.66903	.05436	1.53824	3.77255
2.72339	.00000	1.53830	3.77255

$$\bar{z} = 0.35$$

$$\theta_c = 65.725$$

$$A/\lambda^2 = 10.036$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 1.9035$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	2.66005
0.05751	0.05328	0.02163	2.66083
0.11500	0.10652	0.04335	2.66320
0.17250	0.15966	0.06525	2.66717
0.23000	0.21272	0.08746	2.67280
0.28750	0.26557	0.11011	2.68019
0.34500	0.31817	0.13335	2.68943
0.40250	0.37043	0.15733	2.70068
0.46000	0.42226	0.18222	2.71412
0.51750	0.47355	0.20820	2.72998
0.57500	0.52418	0.23546	2.74851
0.63250	0.57398	0.26420	2.77001
0.69000	0.62278	0.29459	2.79480
0.74750	0.67039	0.32684	2.82321
0.80500	0.71655	0.36111	2.85560
0.86250	0.76101	0.39757	2.89230
0.92000	0.80345	0.43635	2.93364
0.97750	0.84356	0.47754	2.97987
1.03500	0.88095	0.52121	3.03118
1.09250	0.91524	0.56735	3.08764
1.15000	0.94604	0.61589	3.14920
1.20750	0.97292	0.66670	3.21563
1.26500	0.99550	0.71956	3.28652
1.32250	1.01339	0.77419	3.36129
1.38000	1.02627	0.83021	3.43913
1.43750	1.03385	0.88718	3.51910
1.49500	1.03593	0.94462	3.60010
1.55250	1.03239	1.00199	3.68094
1.61000	1.02319	1.05873	3.76040
1.66750	1.00840	1.11427	3.83728
1.72500	0.98817	1.16806	3.91047
1.78250	0.96273	1.21961	3.97898
1.84000	0.93241	1.26843	4.04203
1.89750	0.89757	1.31415	4.09904
1.95500	0.85864	1.35644	4.14965
2.01250	0.81607	1.39506	4.19374
2.07000	0.77033	1.42968	4.23138
2.12750	0.72187	1.46081	4.26284
2.18500	0.67116	1.48788	4.28855
2.24250	0.61861	1.51118	4.30904
2.30000	0.56459	1.53086	4.32492
2.35750	0.50945	1.54713	4.33684
2.41500	0.45347	1.56024	4.34546
2.47250	0.39690	1.57050	4.35142
2.53000	0.33992	1.57821	4.35531
2.58750	0.28269	1.58374	4.35766
2.64500	0.22531	1.58743	4.35895
2.70250	0.16785	1.58964	4.35955
2.76000	0.11037	1.59076	4.35977
2.81750	0.05287	1.59116	4.35982
2.87037	0.00000	1.59121	4.35982

$$r = 0.40$$

$$\theta_0 = 67.918$$

$$A/\lambda^2 = 11.247$$

$$v/\lambda^3 = 1 + (W/P)$$

$$W/P = 2.4380$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	2.92717
0.06058	0.05693	0.02071	2.92808
0.12118	0.11382	0.04153	2.93080
0.18174	0.17063	0.06258	2.93539
0.24232	0.22729	0.08399	2.94192
0.30290	0.28376	0.10593	2.95052
0.36348	0.33996	0.12855	2.96133
0.42406	0.39580	0.15204	2.97458
0.48464	0.45118	0.17659	2.99052
0.54522	0.50599	0.20240	3.00946
0.60580	0.56007	0.22968	3.03176
0.66638	0.61326	0.25865	3.05781
0.72696	0.66540	0.28951	3.08806
0.78754	0.71623	0.32246	3.12296
0.84812	0.76549	0.35771	3.16300
0.90870	0.81290	0.39541	3.20862
0.96928	0.85811	0.43572	3.26025
1.02986	0.90077	0.47872	3.31822
1.09044	0.94047	0.52447	3.38278
1.15102	0.97678	0.57294	3.45402
1.21160	1.00926	0.62404	3.53181
1.27218	1.03753	0.67761	3.61586
1.33276	1.06111	0.73340	3.70559
1.39334	1.07962	0.79106	3.80020
1.45392	1.09274	0.85018	3.89864
1.51450	1.10018	0.91028	3.99965
1.57508	1.10176	0.97081	4.10182
1.63566	1.09737	1.03121	4.20362
1.69624	1.08700	1.09087	4.30351
1.75682	1.07075	1.14920	4.39999
1.81740	1.04880	1.20564	4.49168
1.87798	1.02141	1.25965	4.57739
1.93856	0.98894	1.31076	4.65616
1.99914	0.95179	1.35859	4.72731
2.05972	0.91040	1.40280	4.79042
2.12030	0.86526	1.44316	4.84538
2.18088	0.81684	1.47954	4.89230
2.24146	0.76562	1.51187	4.93133
2.30204	0.71208	1.54017	4.96361
2.36262	0.65663	1.56455	4.98920
2.42320	0.59967	1.58515	5.00906
2.48378	0.54156	1.60221	5.02401
2.54436	0.48257	1.61599	5.03484
2.60494	0.42297	1.62679	5.04236
2.66552	0.36294	1.63495	5.04728
2.72610	0.30265	1.64081	5.05029
2.78668	0.24220	1.64474	5.05195
2.84726	0.18167	1.64713	5.05273
2.90784	0.12110	1.64836	5.05302
2.96842	0.06032	1.64881	5.05309
3.02894	0.00000	1.64887	5.05310

$$\Sigma = 0.45$$

$$\phi_0 = 70.024$$

$$A/\lambda^2 = 12.621$$

$$V/\lambda^3 = 1 + (W/F)$$

$$W/P = 3.0778$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	3.23811
0.06400	0.06086	0.01979	3.23913
0.12800	0.12169	0.03970	3.24223
0.19200	0.18242	0.05989	3.24746
0.25600	0.24301	0.08051	3.25493
0.32000	0.30336	0.10175	3.26482
0.38400	0.36346	0.12379	3.27734
0.44800	0.42317	0.14684	3.29278
0.51200	0.48238	0.17111	3.31151
0.57600	0.54098	0.19684	3.33394
0.64000	0.59881	0.22425	3.36055
0.70400	0.65569	0.25359	3.39190
0.76800	0.71140	0.28508	3.42857
0.83200	0.76570	0.31895	3.47118
0.89600	0.81829	0.35540	3.52036
0.96000	0.86886	0.39462	3.57672
1.02400	0.91702	0.43675	3.64081
1.08800	0.96237	0.48189	3.71306
1.15200	1.00448	0.53007	3.79377
1.21600	1.04287	0.58126	3.88302
1.28000	1.07709	0.63532	3.98063
1.34400	1.10666	0.69206	4.08615
1.40800	1.13114	0.75117	4.19880
1.47200	1.15012	0.81227	4.31750
1.53600	1.16327	0.87487	4.44085
1.60000	1.17032	0.93846	4.56721
1.66400	1.17109	1.00243	4.69477
1.72800	1.16550	1.06616	4.82158
1.79200	1.15358	1.12901	4.94572
1.85600	1.13545	1.19036	5.06532
1.92000	1.11134	1.24962	5.17871
1.98400	1.08154	1.30623	5.28445
2.04800	1.04645	1.35973	5.38140
2.11200	1.00651	1.40970	5.46879
2.17600	0.96218	1.45583	5.54615
2.24000	0.91397	1.49790	5.61339
2.30400	0.86240	1.53577	5.67069
2.36800	0.80796	1.56939	5.71854
2.43200	0.75113	1.59879	5.75760
2.49600	0.69235	1.62409	5.78873
2.56000	0.63204	1.64546	5.81286
2.62400	0.57054	1.66313	5.83098
2.68800	0.50815	1.67739	5.84411
2.75200	0.44514	1.68856	5.85320
2.81600	0.38170	1.69698	5.85915
2.88000	0.31799	1.70303	5.86277
2.94400	0.25412	1.70708	5.86476
3.00800	0.19017	1.70953	5.86570
3.07200	0.12618	1.71078	5.86605
3.13600	0.06219	1.71124	5.86613
3.19819	0.00000	1.71130	5.86613

$$\Sigma = 0.50$$

$$\theta_0 = 72.012$$

$$A/\lambda^2 = 14.165$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 3.8380$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	3.59883
0.06756	0.06489	0.01880	3.59997
0.13512	0.12974	0.03775	3.60343
0.20268	0.19449	0.05702	3.60929
0.27024	0.25909	0.07680	3.61769
0.33780	0.32347	0.09729	3.62888
0.40536	0.38754	0.11871	3.64315
0.47292	0.45122	0.14128	3.66089
0.54048	0.51438	0.16525	3.68259
0.60804	0.57689	0.19087	3.70879
0.67560	0.63859	0.21840	3.74014
0.74316	0.69927	0.24810	3.77737
0.81072	0.75870	0.28022	3.82126
0.87828	0.81660	0.31501	3.87260
0.94584	0.87267	0.35269	3.93224
1.01340	0.92654	0.39345	4.00095
1.08096	0.97780	0.43744	4.07945
1.14852	1.02600	0.48476	4.16828
1.21608	1.07067	0.53542	4.26781
1.28364	1.11130	0.58938	4.37810
1.35120	1.14739	0.64646	4.49890
1.41876	1.17844	0.70644	4.62958
1.48632	1.20399	0.76896	4.76911
1.55388	1.22362	0.83358	4.91607
1.62144	1.23698	0.89978	5.06867
1.68900	1.24381	0.96696	5.22481
1.75656	1.24395	1.03450	5.38220
1.82412	1.23735	1.10170	5.53842
1.89168	1.22405	1.16791	5.69109
1.95924	1.20422	1.23247	5.83795
2.02680	1.17810	1.29475	5.97694
2.09436	1.14604	1.35419	6.10637
2.16192	1.10846	1.41030	6.22489
2.22948	1.06582	1.46267	6.33158
2.29704	1.01862	1.51098	6.42595
2.36460	0.96741	1.55502	6.50789
2.43216	0.91271	1.59463	6.57770
2.49972	0.85504	1.62980	6.63596
2.56728	0.79490	1.66055	6.68352
2.63484	0.73275	1.68701	6.72142
2.70240	0.66901	1.70936	6.75080
2.76996	0.60404	1.72786	6.77289
2.83752	0.53816	1.74279	6.78804
2.90508	0.47163	1.75449	6.80000
2.97264	0.40465	1.76333	6.80728
3.04020	0.33739	1.76968	6.81171
3.10776	0.26997	1.77394	6.81416
3.17532	0.20246	1.77653	6.81532
3.24288	0.13492	1.77786	6.81575
3.31044	0.06736	1.77835	6.81585
3.37780	0.00000	1.77843	6.81586

$$\Sigma = 0.55$$

$$\theta_0 = 73.867$$

$$A/\lambda^2 = 15.890$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 4.7363$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

P)

θ/λ	r/λ	z/λ	T/P
0.	0.	0.	4.01507
0.07134	0.06906	0.01750	4.01632
0.14268	0.13812	0.03577	4.02013
0.21402	0.20706	0.05412	4.02661
0.28536	0.27584	0.07306	4.03596
0.35670	0.34440	0.09240	4.04848
0.42804	0.41263	0.11361	4.06459
0.49938	0.48046	0.13573	4.08478
0.57072	0.54774	0.15943	4.10970
0.64206	0.61434	0.18499	4.14006
0.71340	0.68002	0.21269	4.17672
0.78474	0.74474	0.24282	4.22061
0.85608	0.80807	0.27566	4.27274
0.92742	0.86976	0.31147	4.33416
0.99876	0.92947	0.35049	4.40593
1.07010	0.98680	0.39293	4.48906
1.14144	1.04130	0.43894	4.58444
1.21278	1.09249	0.48861	4.69276
1.28412	1.13984	0.54195	4.81446
1.35546	1.18281	0.59887	4.94958
1.42680	1.22085	0.65919	5.09777
1.49814	1.25345	0.72263	5.25817
1.56948	1.28010	0.78877	5.42944
1.64082	1.30038	0.85714	5.60974
1.71216	1.31394	0.92715	5.79679
1.78350	1.32053	0.99816	5.98796
1.85484	1.32000	1.06946	6.18039
1.92618	1.31231	1.14036	6.37111
1.99752	1.29754	1.21012	6.55719
2.06886	1.27590	1.27807	6.73589
2.14020	1.24765	1.34355	6.90476
2.21154	1.21319	1.40598	7.06178
2.28288	1.17295	1.46486	7.20537
2.35422	1.12744	1.51976	7.33448
2.42556	1.07720	1.57038	7.44854
2.49690	1.02276	1.61648	7.54750
2.56824	0.96474	1.65793	7.63173
2.63958	0.90363	1.69471	7.70199
2.71092	0.83996	1.72686	7.75931
2.78226	0.77422	1.75451	7.80497
2.85360	0.70682	1.77787	7.84037
2.92494	0.63816	1.79720	7.86697
2.99628	0.56856	1.81281	7.88625
3.06762	0.49828	1.82503	7.89962
3.13896	0.42754	1.83426	7.90838
3.21030	0.35651	1.84090	7.91372
3.28164	0.28532	1.84536	7.91667
3.35298	0.21403	1.84807	7.91807
3.42432	0.14270	1.84946	7.91859
3.49566	0.07137	1.84998	7.91871
3.56700	0.00003	1.85005	7.91872
3.56703	0.00000	1.85005	7.91872

$$\Sigma = 0.60$$

$$\theta_0 = 75.578$$

$$A/\lambda^2 = 17.806$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 5.7898$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
.00000	.00000	.00000	4.49219
.07530	.07340	.01650	4.49355
.15060	.14676	.03380	4.49770
.22590	.22001	.05123	4.50478
.30120	.29311	.06932	4.51507
.37650	.36596	.08834	4.52896
.45180	.43850	.10855	4.54697
.52710	.51061	.13024	4.56977
.60240	.58216	.15370	4.59816
.67770	.65299	.17923	4.63310
.75300	.72292	.20716	4.67566
.82830	.79170	.23778	4.72703
.90360	.85907	.27141	4.78851
.97890	.92469	.30832	4.86143
1.05420	.98818	.34879	4.94715
1.12950	1.04911	.39302	5.04692
1.20480	1.10698	.44116	5.16187
1.28010	1.16127	.49332	5.29286
1.35540	1.21141	.54947	5.44038
1.43070	1.25682	.60951	5.60447
1.50600	1.29690	.67323	5.78462
1.58130	1.33111	.74028	5.97971
1.65660	1.35892	.81022	6.18803
1.73190	1.37990	.88251	6.40722
1.80720	1.39369	.95651	6.63443
1.88250	1.40005	1.03150	6.86641
1.95780	1.39884	1.10676	7.09961
2.03310	1.39006	1.18152	7.33042
2.10840	1.37380	1.25501	7.55530
2.18370	1.35030	1.32651	7.77096
2.25900	1.31987	1.39536	7.97449
2.33430	1.28292	1.46094	8.16347
2.40960	1.23994	1.52273	8.33609
2.48490	1.19146	1.58031	8.49112
2.56020	1.13805	1.63336	8.62796
2.63550	1.08030	1.68164	8.74658
2.71080	1.01878	1.72504	8.84747
2.78610	.95408	1.76353	8.93156
2.86140	.88673	1.79716	9.00015
2.93670	.81722	1.82608	9.05475
3.01200	.74600	1.85050	9.09707
3.08730	.67347	1.87071	9.12887
3.16260	.59997	1.88702	9.15190
3.23790	.52577	1.89980	9.16787
3.31320	.45109	1.90944	9.17833
3.38850	.37612	1.91637	9.18472
3.46380	.30097	1.92103	9.18823
3.53910	.22572	1.92386	9.18990
3.61440	.15044	1.92531	9.19053
3.68970	.07514	1.92585	9.19067
3.76484	.00000	1.92593	9.19068

$$\Sigma = 0.65$$

$$\Theta_0 = 77.138$$

$$A/\lambda^2 = 19.916$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 7.0155$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

θ/λ	r/λ	z/λ	T/P
0	0	0	5.03659
.07941	.07782	.01580	5.03806
.15882	.15559	.03184	5.04253
.23823	.23327	.04837	5.05012
.31764	.31077	.06564	5.06141
.39705	.38805	.08395	5.07667
.47646	.46499	.10358	5.09666
.55587	.54149	.12467	5.12222
.63528	.61742	.14812	5.15437
.71469	.69260	.17367	5.19430
.79410	.76683	.20186	5.24337
.87351	.83987	.23302	5.30311
.95292	.91139	.26749	5.37511
1.03233	.98106	.30558	5.46108
1.11174	1.04846	.34756	5.56268
1.19115	1.11310	.39365	5.68151
1.27056	1.17447	.44403	5.81893
1.34997	1.23198	.49876	5.97600
1.42938	1.28501	.55784	6.15330
1.50879	1.33295	.62111	6.35083
1.58820	1.37517	.68834	6.56791
1.66761	1.41107	.75914	6.80311
1.74702	1.44013	.83301	7.05424
1.82643	1.46188	.90935	7.31838
1.90584	1.47596	.98747	7.59201
1.98525	1.48215	1.06660	7.87112
2.06466	1.48031	1.14596	8.15141
2.14407	1.47045	1.22472	8.42851
2.22348	1.45273	1.30209	8.69818
2.30289	1.42737	1.37731	8.95648
2.38230	1.39475	1.44967	9.19998
2.46171	1.35529	1.51855	9.42586
2.54112	1.30953	1.58342	9.63197
2.62053	1.25803	1.64383	9.81623
2.69994	1.20139	1.69945	9.98006
2.77935	1.14022	1.75005	10.12138
2.85876	1.07514	1.79552	10.24151
2.93817	1.00674	1.83582	10.34161
3.01758	.93558	1.87104	10.42321
3.09699	.86219	1.90131	10.48817
3.17640	.78702	1.92688	10.53850
3.25581	.71049	1.94803	10.57631
3.33522	.63294	1.96510	10.60370
3.41463	.55467	1.97847	10.62268
3.49404	.47591	1.98857	10.63513
3.57345	.39684	1.99583	10.64272
3.65286	.31758	2.00070	10.64691
3.73227	.23823	2.00366	10.64889
3.81168	.15883	2.00518	10.64963
3.89109	.07942	2.00574	10.64980
3.97050	.00001	2.00582	10.64981
3.97051	0	2.00582	10.64981

$$\bar{\epsilon} = 0.70$$

$$\phi_0 = 78.548$$

$$A/\lambda^2 = 22.227$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 8.4319$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

ϕ/λ	r/λ	z/λ	T/P
0.	0.	0.	5.65359
0.08370	0.08237	0.01485	5.65516
0.16740	0.16470	0.02996	5.65993
0.25110	0.24692	0.04561	5.66817
0.33480	0.32898	0.06210	5.68032
0.41850	0.41080	0.07974	5.69700
0.50220	0.49228	0.09886	5.71908
0.58590	0.57332	0.11979	5.74760
0.66960	0.65377	0.14290	5.78384
0.75330	0.73344	0.16853	5.82931
0.83700	0.81212	0.19707	5.88568
0.92070	0.88953	0.22887	5.95485
1.00440	0.96536	0.26429	6.03881
1.08810	1.03921	0.30367	6.13968
1.17180	1.11063	0.34729	6.25951
1.25550	1.17910	0.39539	6.40026
1.33920	1.24406	0.44815	6.56361
1.42290	1.30487	0.50564	6.75081
1.50660	1.36087	0.56781	6.96256
1.59030	1.41140	0.63450	7.19879
1.67400	1.4576	0.70544	7.45860
1.75770	1.49339	0.78017	7.74019
1.84140	1.52368	0.85817	8.04080
1.92510	1.54616	0.93875	8.35683
2.00880	1.56049	1.02118	8.68397
2.09250	1.56642	1.10464	9.01732
2.17620	1.56386	1.18826	9.35172
2.25990	1.55284	1.27120	9.68192
2.34360	1.53351	1.35260	10.00287
2.42730	1.50616	1.43167	10.30991
2.51100	1.47118	1.50767	10.59901
2.59470	1.42906	1.57996	10.86686
2.67840	1.38034	1.64798	11.11101
2.76210	1.32562	1.71129	11.32986
2.84580	1.26555	1.76953	11.52270
2.92950	1.20076	1.82249	11.68960
3.01320	1.13191	1.87004	11.83136
3.09690	1.05961	1.91218	11.94937
3.18060	0.98445	1.94897	12.04550
3.26430	0.90696	1.98058	12.12195
3.34800	0.82764	2.00726	12.18114
3.43170	0.74691	2.02922	12.22556
3.51540	0.66513	2.04710	12.25771
3.59910	0.58261	2.06103	12.27996
3.68280	0.49957	2.07153	12.29452
3.76650	0.41622	2.07906	12.30338
3.85020	0.33267	2.08411	12.30825
3.93390	0.24903	2.08717	12.31056
4.01760	0.16534	2.08873	12.31140
4.10130	0.08165	2.08930	12.31160
4.18295	0.00000	2.08938	12.31161

$$\Sigma = 0.75$$

$$\phi_0 = 79.812$$

$$A/\lambda^2 = 24.739$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 10.055$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	6.34838
0.08803	0.08692	0.01391	6.35003
0.17606	0.17380	0.02812	6.35509
0.26409	0.26057	0.04293	6.36386
0.35212	0.34719	0.05866	6.37691
0.44015	0.43356	0.07565	6.39591
0.52818	0.51960	0.09427	6.41921
0.61621	0.60518	0.11487	6.45081
0.70424	0.69016	0.13784	6.49139
0.79227	0.77434	0.16357	6.54278
0.88030	0.85749	0.19245	6.60705
0.96833	0.93932	0.22488	6.68650
1.05636	1.01946	0.26125	6.78360
1.14439	1.09755	0.30189	6.90089
1.23242	1.17305	0.34713	7.04092
1.32045	1.24542	0.39722	7.20603
1.40848	1.31404	0.45253	7.39827
1.49651	1.37823	0.51253	7.61915
1.58454	1.43731	0.57776	7.86945
1.67257	1.49053	0.64784	8.14906
1.76060	1.53721	0.72244	8.45685
1.84863	1.57669	0.80106	8.79057
1.93666	1.60837	0.88318	9.14687
2.02469	1.63178	0.96800	9.52137
2.11272	1.64655	1.05474	9.90888
2.20075	1.65244	1.14254	10.30354
2.28878	1.64937	1.23048	10.69918
2.37681	1.63739	1.31765	11.08960
2.46484	1.61660	1.40317	11.46881
2.55287	1.58756	1.48621	11.83137
2.64090	1.55044	1.56599	12.17253
2.72893	1.50583	1.64184	12.48846
2.81696	1.45433	1.71320	12.77630
2.90499	1.39657	1.77958	13.03423
2.99302	1.33320	1.84066	13.26145
3.08105	1.26493	1.89616	13.45807
3.16908	1.19240	1.94603	13.62507
3.25711	1.11628	1.99021	13.76410
3.34514	1.03717	2.02878	13.87736
3.43317	0.95564	2.06194	13.96747
3.52120	0.87219	2.08993	14.03725
3.60923	0.78727	2.11307	14.08966
3.69726	0.70125	2.13175	14.12761
3.78529	0.61446	2.14638	14.15390
3.87332	0.52713	2.15742	14.17113
3.96135	0.43946	2.16535	14.18163
4.04938	0.35159	2.17067	14.18742
4.13741	0.26362	2.17390	14.19016
4.22544	0.17561	2.17556	14.19118
4.31347	0.08758	2.17617	14.19142
4.40195	0.00000	2.17626	14.19143

$$\Sigma = 0.80$$

$$\theta_0 = 80.937$$

$$A/\lambda^2 = 27.450$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 11.901$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	7.12776
0.09250	0.09156	0.01303	7.12949
0.18500	0.18311	0.02638	7.13481
0.27750	0.27454	0.04039	7.14411
0.37000	0.36581	0.05542	7.15806
0.46250	0.45684	0.07182	7.17762
0.55500	0.54754	0.08999	7.20408
0.64750	0.63778	0.11032	7.23900
0.74000	0.72739	0.13322	7.28431
0.83250	0.81619	0.15912	7.34223
0.92500	0.90392	0.18843	7.41528
1.01750	0.99026	0.22158	7.50624
1.11000	1.07486	0.25897	7.61810
1.20250	1.15725	0.30099	7.75394
1.29500	1.23692	0.34796	7.91682
1.38750	1.31326	0.40015	8.10957
1.48000	1.38562	0.45774	8.33464
1.57250	1.45328	0.52078	8.59380
1.66500	1.51547	0.58921	8.88797
1.75750	1.57144	0.66252	9.21697
1.85000	1.62044	0.74123	9.57937
1.94250	1.66178	0.82394	9.97241
2.03500	1.69486	0.91029	10.39203
2.12750	1.71917	0.99949	10.83298
2.22000	1.73433	1.09070	11.28902
2.31250	1.74013	1.18298	11.75320
2.40500	1.73649	1.27537	12.21823
2.49750	1.72346	1.36691	12.67678
2.59000	1.70126	1.45667	13.12184
2.68250	1.67026	1.54377	13.54704
2.77500	1.63087	1.62743	13.94687
2.86750	1.58366	1.70693	14.31689
2.96000	1.52923	1.78168	14.65382
3.05250	1.46826	1.85120	14.95559
3.14500	1.40146	1.91514	15.22129
3.23750	1.32952	1.97325	15.45113
3.33000	1.25317	2.02542	15.64626
3.42250	1.17306	2.07163	15.80867
3.51500	1.08985	2.11199	15.94096
3.60750	1.00411	2.14666	16.04617
3.70000	0.91638	2.17593	16.12764
3.79250	0.82711	2.20013	16.18881
3.88500	0.73671	2.21966	16.23309
3.97750	0.64549	2.23495	16.26377
4.07000	0.55371	2.24649	16.28388
4.16250	0.46159	2.25478	16.29612
4.25500	0.36926	2.26034	16.30287
4.34750	0.27682	2.26372	16.30607
4.44000	0.18434	2.26545	16.30725
4.53250	0.09184	2.26608	16.30752
4.62434	0.00000	2.26617	16.30754

$$\Sigma = 0.85$$

$$\theta_0 = 81.935$$

$$A/\lambda^2 = 30.366$$

$$v/\lambda^3 = 1 + (W/P)$$

$$W/P = 13.987$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	7.99641
0.09702	0.09625	0.01219	7.99821
0.19404	0.19246	0.02473	8.00377
0.29106	0.28857	0.03799	8.01356
0.38808	0.38452	0.05235	8.02841
0.48510	0.48023	0.06821	8.04946
0.58212	0.57561	0.08597	8.07825
0.67914	0.67052	0.10607	8.11669
0.77616	0.76480	0.12894	8.16706
0.87318	0.85824	0.15503	8.23204
0.97020	0.95058	0.18480	8.31464
1.06722	1.04147	0.21869	8.41821
1.16424	1.13054	0.25714	8.54630
1.26126	1.21729	0.30054	8.70260
1.35828	1.30117	0.34926	8.89075
1.45530	1.38154	0.40357	9.11415
1.55232	1.45770	0.46354	9.37568
1.64934	1.52887	0.52954	9.67745
1.74636	1.59425	0.60118	10.02048
1.84338	1.65303	0.67832	10.40453
1.94040	1.70444	0.76056	10.82784
2.03742	1.74774	0.84734	11.28711
2.13444	1.78230	0.93796	11.77747
2.23146	1.80760	1.03157	12.29268
2.32848	1.82327	1.12728	12.82536
2.42550	1.82909	1.22408	13.36736
2.52252	1.82497	1.32097	13.91009
2.61954	1.81102	1.41694	14.44500
2.71656	1.78745	1.51101	14.96394
2.81358	1.75465	1.60228	15.45950
2.91060	1.71308	1.68990	15.92531
3.00762	1.66333	1.77315	16.35625
3.10464	1.60605	1.85141	16.74853
3.20166	1.54194	1.92419	17.09979
3.29868	1.47174	1.99112	17.40903
3.39570	1.39618	2.05194	17.67651
3.49272	1.31602	2.10654	17.90361
3.58974	1.23194	2.15491	18.09263
3.68676	1.14462	2.19715	18.24662
3.78378	1.05467	2.23346	18.36912
3.88080	0.96263	2.26411	18.46401
3.97782	0.86899	2.28946	18.53528
4.07484	0.77417	2.30992	18.58692
4.17186	0.67849	2.32596	18.62272
4.26888	0.58223	2.33803	18.64619
4.36590	0.48561	2.34677	18.66052
4.46292	0.38877	2.35252	18.66842
4.55994	0.29181	2.35618	18.67218
4.65696	0.19481	2.35801	18.67358
4.75398	0.09775	2.35868	18.67390
4.85100	0.00077	2.35878	18.67392
4.85177	0.00000	2.35878	18.67392

$$\Sigma = 0.90$$

$$\theta_0 = 82.816$$

$$\lambda/\lambda^2 = 33.480$$

$$v/\lambda^3 = 1 + (v/p)$$

$$w/p = 16.329$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	8.95996
0.10170	0.10166	0.01141	8.96183
0.20340	0.20207	0.02321	8.96762
0.30510	0.30299	0.03578	8.97792
0.40680	0.40375	0.04955	8.99369
0.50850	0.50428	0.06492	9.01633
0.61020	0.60448	0.08236	9.04766
0.71190	0.70420	0.10231	9.08996
0.81360	0.80327	0.12524	9.14596
0.91530	0.90148	0.15163	9.21884
1.01700	0.99854	0.18197	9.31218
1.11870	1.09411	0.21672	9.42996
1.22040	1.18775	0.25637	9.57641
1.32210	1.27896	0.30132	9.75591
1.42380	1.36714	0.35195	9.97278
1.52550	1.45161	0.40855	10.23102
1.62720	1.53161	0.47130	10.53402
1.72890	1.60632	0.54026	10.88423
1.83060	1.67489	0.61532	11.28283
1.93230	1.73647	0.69621	11.72942
2.03400	1.79022	0.78256	12.22188
2.13570	1.83539	0.87357	12.75620
2.23740	1.87132	0.96867	13.32659
2.33910	1.89749	1.06690	13.92566
2.44080	1.91349	1.16729	14.54472
2.54250	1.91912	1.26879	15.17418
2.64420	1.91432	1.37033	15.80404
2.74590	1.89919	1.47085	16.42435
2.84760	1.87400	1.56934	17.02566
2.94930	1.83914	1.66483	17.59943
3.05100	1.79512	1.75647	18.13833
3.15270	1.74255	1.84349	18.63649
3.25440	1.68213	1.92525	19.08964
3.35610	1.61460	2.00125	19.49510
3.45780	1.54072	2.07110	19.85181
3.55950	1.46128	2.13455	20.16013
3.66120	1.37704	2.19149	20.42171
3.76290	1.28875	2.24190	20.63929
3.86460	1.19708	2.28590	20.81649
3.96630	1.10269	2.32370	20.95718
4.06800	1.00613	2.35560	21.06614
4.16970	0.90792	2.38196	21.14790
4.27140	0.80848	2.40322	21.20706
4.37310	0.70816	2.41987	21.24901
4.47480	0.60724	2.43242	21.27483
4.57650	0.50595	2.44144	21.29114
4.67820	0.40443	2.44748	21.30012
4.77990	0.30280	2.45114	21.30436
4.88160	0.20112	2.45302	21.30593
4.98330	0.09942	2.45371	21.30629
5.08272	0.00000	2.45360	21.30631

$$\Sigma = 0.95$$

$$\Theta_0 = 83.592$$

$$A/\lambda^2 = 36.792$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 18.942$$

Zero Circumferential Stress
Zero Superpressure
Flat Top

s/λ	r/λ	z/λ	T/P
0.	0.	0.	10.02816
0.10640	0.10586	0.01068	10.03009
0.21280	0.21168	0.02177	10.03610
0.31920	0.31741	0.03369	10.04688
0.42560	0.42299	0.04690	10.06357
0.53200	0.52833	0.06184	10.08782
0.63840	0.63334	0.07898	10.12180
0.74480	0.73757	0.09880	10.16617
0.85120	0.84175	0.12182	10.23017
0.95760	0.94474	0.14852	10.31152
1.06400	1.04654	0.17943	10.41645
1.17040	1.14679	0.21506	10.54964
1.27680	1.24503	0.25588	10.71606
1.38320	1.34072	0.30236	10.92087
1.48960	1.43323	0.35488	11.16911
1.59600	1.52184	0.41374	11.46549
1.70240	1.60574	0.47914	11.81396
1.80880	1.68405	0.55112	12.21736
1.91520	1.75589	0.62955	12.67702
2.02160	1.82035	0.71416	13.19244
2.12800	1.87657	0.80445	13.76105
2.23440	1.92373	0.89977	14.37812
2.34080	1.96116	0.99932	15.03686
2.44720	1.98834	1.10215	15.72860
2.55360	2.00484	1.20721	16.44321
2.66000	2.01046	1.31342	17.16956
2.76640	2.00514	1.41964	17.89608
2.87280	1.98902	1.52476	18.61126
2.97920	1.96236	1.62772	19.30424
3.08560	1.92560	1.72753	19.96518
3.19200	1.87929	1.82327	20.58571
3.29840	1.82406	1.91417	21.15911
3.40480	1.76064	1.99955	21.68052
3.51120	1.68980	2.07890	22.14692
3.61760	1.61236	2.15182	22.55713
3.72400	1.52912	2.21805	22.91160
3.83040	1.44089	2.27746	23.21229
3.93680	1.34843	2.33007	23.46234
4.04320	1.25247	2.37598	23.66586
4.14960	1.15367	2.41542	23.82762
4.25600	1.05262	2.44869	23.95278
4.36240	0.94985	2.47619	24.04670
4.46880	0.84579	2.49836	24.11465
4.57520	0.74083	2.51572	24.16168
4.68160	0.63524	2.52881	24.19246
4.78800	0.52926	2.53820	24.21119
4.89440	0.42305	2.54450	24.22148
5.00080	0.31672	2.54831	24.22635
5.10720	0.21034	2.55026	24.22814
5.21360	0.10394	2.55096	24.22855
5.31754	0.00000	2.55106	24.22857

$$\Sigma = 1.00$$

$$\phi_0 = 84.277$$

$$\lambda/\lambda^2 = 40.317$$

$$V/\lambda^3 = 1 + (W/P)$$

$$W/P = 21.849$$

Zero Circumferential Stress
Zero Superpressure
Flat Top