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DETECTION OF A PATTERN IN UNKNOWN POSITION

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ABSTRACT

In this paper, the problem of detecting an optical pattern in unknown position is considered. Two detection procedures are investigated--one which entails a search procedure, and one which does not. The false-alarm and false-dismissal probabilities for these two procedures are evaluated under certain simplifying assumptions in order to compare the two procedures with each other and with the detector which is optimum when the position of the pattern is known. It is shown that there is a trade-off between error rate and information rate. The procedure requiring a search technique processes the data less rapidly, but at the same time achieves a lower error rate for a given signal-to-noise ratio. This analysis also applies to the problem of detecting a signal with unknown arrival time provided that the assumptions stated herein are satisfied.

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INTRODUCTION

An important problem in optical data processing is that of determining whether or not a particular pattern is present in the image plane of an optical system. Various versions of this problem obtain depending on the prior knowledge available about the pattern to be detected and the noise in the system. The methods of statistical decision theory [1] can be used to derive optimum detection procedures which depend on this prior information and on the costs of the different types of errors.

In this paper we first review briefly some known results pertaining to the detection of a pattern which is exactly known in the presence of additive white Gaussian noise. We then relax the assumption that the position of the pattern in the image plane is known while retaining the assumption that its shape is known. Two detection procedures for this modified problem are considered, one which entails searching for the pattern and one which does not. Under certain simplifying assumptions, the false-alarm and false-dismissal probabilities are evaluated as functions of signal-to-noise ratio for each of these procedures. These probabilities are compared with each other and with the false-alarm and false-dismissal probabilities obtained using the optimum detector when the signal position is known. It is shown that there is a trade-off between information rate and error rate. If the procedure requiring search is used, it takes longer to process the data, but at the same time the error rate is reduced. The choice of a procedure will depend on the signal and noise parameters and on the performance required.

DETECTION OF A KNOWN PATTERN

In this section, we briefly review some known results for the detection of a known signal or pattern. These results will be useful for purposes of comparison in later sections.

The problem we treat is the following: Let $X(\xi, \eta)$ be the observed intensity distribution in the image plane, where ξ and η are the image-plane coordinates. The object of the detector is to determine whether $X(\xi, \eta)$ consists of noise alone ($X = N$) or of signal plus noise ($X = S + N$), where $S(\xi, \eta)$ is exactly known to the detector.

By employing an appropriate sampling procedure [2, 3], we can represent the functions $X(\xi, \eta)$, $N(\xi, \eta)$ and $S(\xi, \eta)$ by the finite-dimensional vectors $\underline{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$, $\underline{N} = \begin{pmatrix} n_1 \\ \vdots \\ n_m \end{pmatrix}$, and $\underline{S} = \begin{pmatrix} s_1 \\ \vdots \\ s_m \end{pmatrix}$

The noise samples $n_1 \ldots n_m$ are assumed to be independent Gaussian random variables with mean zero and variance one.

It has been shown [4] that the optimum detector for this problem computes the quantity

$$L(\underline{X}) = \underline{X}'\underline{S} \tag{1}$$

where $\underline{X}'$ is the transpose of $\underline{X}$, and compares it with a threshold. If this threshold is exceeded, the pattern is said to be present. Otherwise, it is decided that noise alone is present. This is the well-known matched filter or correlation detector.

If γ is the threshold described above, the false-alarm and false-dismissal probabilities are

$$P(FA) = P[\underline{X}'\underline{S} > \gamma \quad \underline{N} \text{ alone}] \tag{2}$$

and

$$P(FD) = P[\underline{X}'\underline{S} < \gamma \quad \underline{S} + \underline{N}] \tag{3}$$

These probabilities are easy to evaluate in the present case [4]. Curves are plotted in Figures 1 - 4 for comparison with similar curves obtained when the position of the signal is not known.

DETECTION OF A SIGNAL IN UNKNOWN POSITION--SEARCH TECHNIQUE

In many applications, it is realistic to assume that the shape of the pattern to be detected is known, but that its exact position is not. In this paper, we assume that the orientation as well as the shape is known but allow an unknown translation. Any detection scheme for this situation will, of course, suffer a certain amount of degradation due to the additional noise which the position uncertainty allows to enter the system. Two questions occur in connection with this problem. First, how much degradation do various schemes suffer relative to the optimum known-position detector? Second, how do these schemes compare with one another? In this section and the next, we consider and compare two such schemes.

In optical data processing, the correlation detector or matched filter can take the form of a mask with the same shape as the pattern to be detected. The amount of light which passes through the mask is the quantity on which the decision is based. If the position of the pattern (if it is present) in the image plane is known, the mask is simply placed in that position and the transmitted light compared with the threshold.

If the position of the pattern is not known, the above detection procedure must be modified. The optimum procedure under certain reasonable assumptions about the prior knowledge is not difficult to obtain mathematically, but it is rather complicated to instrument and extremely difficult to evaluate. Hence, this optimum procedure is not considered in this paper. Instead, two reasonable sub-optimum detection procedures are considered and compared.

When the position of the pattern in the image plane is not known, a reasonable way to modify the matched-filter detection procedure is to move the mask around and compare successive outputs with threshold. If the output exceeds the threshold in any one position, say the pattern is present. Otherwise, say noise alone is present.

If the dimension of the signal vector is greater than one and if all possible signal positions are searched, there will be overlap in the matched-filter outputs. The random variables involved are then dependent, and evaluation of system performance becomes very complicated. The essential features of the problem are preserved, and performance evaluation is made tractible, if it is assumed that the signal vector consists of just one non-zero sample. The output random variables in different positions will then be independent, and the false-alarm and false-dismissal probabilities can be evaluated quite easily.

To be more specific, we consider the following problem. A vector $\underline{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$ is presented to a detector whose objective is to determine whether X consists of noise alone or of signal plus noise. If noise alone is present, we have $x_i = n_i$ for all $i = 1, \ldots, m$, where the n_i are independent Gaussian random variables with zero mean and unit variance. If signal is present, we have $x_i = n_i$ for all $i = 1, \ldots, m$, except $i = j$, and $x_j = s + n_j$. s is a known scalar, but j is not known. That is, the position of the signal in the image plane is not known.

The following detection scheme is used. Compare each x_i with a threshold γ. If all of the x_i are less than γ, choose noise alone; if at least one $x_i > \gamma$, choose signal plus noise. Note that this procedure is equivalent to moving the matched-filter mask around as described above, since matched filtering in this case corresponds to scalar multiplication.

The false-alarm and false-dismissal probabilities for this detection procedure are quite easy to evaluate. We have

$$\begin{aligned} P(FA) &= P\{\text{at least one } x_i > \gamma \mid \text{noise alone}\} \\ &= 1 - P\{\text{all } x_i \leq \gamma \mid \text{noise alone}\} \\ &= 1 - [P(n \leq \gamma)]^m \end{aligned} \tag{4}$$

where n is a Gaussian random variable with zero mean and unit variance. For a given m, the threshold γ can be chosen to yield any preassigned false-alarm probability. γ having thus been determined, the false-dismissal probability can then be calculated as a function of the signal strength s as follows.

$$P(FD) = P\left\{\text{all } x_i \leq \gamma \;\middle|\; \underline{S} + \underline{N}\right\}$$

$$= P(u \leq \gamma)\,[P(n < \gamma)]^{m-1} \qquad (5)$$

where $n \sim N(0, 1)$ and $u \sim N(s, 1)$. This can also be written

$$P(FD) = P(n \leq \gamma - s)\,[P(n \leq \gamma)]^{m-1} \qquad (6)$$

These probabilities can be easily evaluated using a table of the unit normal distribution function. Plots of P(FD) as a function of s for P(FA) = .01 and P(FA) = 0.1 and various values of m are shown in Figures 1-4.

DETECTION OF A SIGNAL IN UNKNOWN POSITION WITHOUT SEARCH

One disadvantage of the procedure described in the previous section is the time required to search for the signal. This reduces the rate at which data can be processed. It would be desirable to have available a method of detection which is independent of the exact position of the pattern and which does not entail a search procedure. One such method has been considered by Horwitz and Shelton [5]. This procedure is briefly described here, and its performance is evaluated for a special case and compared with those of the optimum known-position detector and the search procedure.

To motivate this detection procedure, consider a one-dimensional signal $s(\xi - \xi_o)$. Suppose that $s(\xi)$ is an exactly-known signal having Fourier transform $S(f)$ but that ξ_o is unknown. For any fixed ξ_o, the Fourier transform of $s(\xi - \xi_o)$ is $e^{-j\omega\xi_o} S(f)$. Clearly, the square of the magnitude of this quantity, which is just the energy spectral density of the signal $s(\xi - \xi_o)$, is independent of the unknown position ξ_o. Hence, this energy spectrum can be used in various ways to detect a signal in unknown position.

In the detection procedure considered in [5], the detector simply cross-correlates the energy spectrum of $s(\xi)$, $|S(f)|^2$, th the energy spectrum of the observed waveform, $|X(f)|^2$. If the pattern is present, $|X(f)|^2$ will consist of $|S(f)|^2$ plus a noise term regardless of the position of the signal. Hence, this scheme is effective for essentially the same reasons that the ordinary correlation receiver is effective. The noise term is increased due to the uncertainty in position, however, and this noise is no longer Gaussian.

In order to evaluate this procedure, it is convenient to obtain an expression for the detector output in terms of autocorrelation functions rather than energy spectra. By Parseval's theorem, the output of the detector can be expressed as [5]

$$V = \sum_{i=-m+1}^{m-1} R_s(i)\, R_x(i) \tag{7}$$

where the discrete autocorrelation functions $R_s(i)$ and $R_x(i)$ are given by

$$R_s(i) = \sum_{j=1}^{m} s_j s_{i+j} \tag{8}$$

and

$$R_x(i) = \sum_{j=1}^{m} x_j x_{i+j} \tag{9}$$

The detector output can be expressed as a quadratic form in the variables x_i as follows:

$$V = \sum_{i=-m+1}^{m-i} R_s(i) \sum_{j=1}^{m} x_j x_{i+j} \tag{10}$$

In matrix form,

$$V = \underline{X}' \underline{\underline{R}} \underline{X} \tag{11}$$

where

$$\underline{\underline{R}} = (r_{ij}), \; r_{ij} = R_s(|i-j|) \tag{12}$$

Quadratic forms of this type in Gaussian random variables have been treated by Middleton [6]. Although the characteristic function can be found for a general quadratic form of this type, the probability density function can be found only for certain special cases. If m is large, V becomes approximately Gaussian, but it is dangerous to use this approximation to compute error probabilities. The approximation is at its worst in the tails of the distribution, and it is precisely in these tails that the information regarding error probability lies. Rather than resort to this questionable approximation, we consider a special case for which the probability density functions can be found exactly.

We assume that the signal $\underline{S}$ is such that $R_s(i)$ is zero or approximately zero for $i \neq 0$. This will be true in particular if $\underline{S}$ consists of a single signal sample. Since this is precisely the case for which the search procedure was evaluated, we will be able to compare the two procedures directly in similar circumstances.

With this as background, we now state explicitly the problem to be solved. Let $X = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$ be the observed vector. The purpose of the detector is to use this observation to determine whether or not a known scalar signal s is present. If noise alone is present, $x_1, \ldots, x_m$ are independent Gaussian random variables with mean zero and unit variance. If signal is present, one of the x_i is Gaussian with mean s and variance unity, and the rest, as before, are Gaussian with mean zero and unit variance. As in the case of noise alone, all the x_i are assumed to be independent.

The detector computes the quantity $V = \underline{X}' \, \underline{X}$ and compares it with a threshold γ. If $V > \gamma$, the pattern is said to be present. Otherwise, it is said to be absent. Hence

$$P(FA) = P\left\{ V > \gamma \mid \underline{N} \right\} \tag{13}$$

$$P(FD) = P\left\{ V \leq \gamma \mid \underline{S} + \underline{N} \right\} \tag{14}$$

In order to evaluate these probabilities, we must first obtain the conditional probability density functions $f(V \mid \underline{N})$ and $f(V \mid \underline{S} + \underline{N})$.

When noise alone is present, $V = \underline{X}'\underline{X} = x_1^2 + x_2^2 + \ldots + x_m^2$ is just the sum of squares of m independent Gaussian random variables with zero mean and unit variance. Hence, V is a chi-square random variable with m degrees of freedom. The conditional PDF of V when noise alone is present is

$$f(V|\underline{N}) = \begin{cases} (V/2)^{(m/2)-1} \dfrac{e^{-V/2}}{2\,(m/2)}, & V \geq 0 \\ 0, & V < 0 \end{cases} \tag{15}$$

The false-alarm probability is

$$P(FA) = \int_{\gamma}^{\infty} f(V|\underline{N})\, dV \tag{16}$$

a quantity which may be obtained from tables of the chi-square distribution [7]. As before, we choose γ such that P(FA) = .01 and P(FA) = 0.1 for various values of m and then plot P(FD) as a function of signal strength for these values of γ and m.

When signal is present, the quadratic form $V = \underline{X}'\underline{X}$ computed by the detector is a special case of a form treated by Middleton [6]. From his (17.32b) and problem 17.9, we see that in our notation the conditional characteristic function of V in the presence of signal plus noise is

$$\varphi(i\xi \mid \underline{S} + \underline{N}) = \frac{1}{(1-2i\xi)^{m/2}} \exp\left\{\frac{i\xi s^2}{1-2i\xi}\right. \qquad (17)$$

The conditional probability density function $f(V|\underline{S} + \underline{N})$ is obtained from $\varphi(i\xi|\underline{S} + \underline{N})$ by taking the Fourier transform. This quantity can be obtained from pair 650.0 of Campbell and Foster [8] upon making the change of variable $P = -2i\xi$. This yields

$$f(V|\underline{S} + \underline{N}) = \begin{cases} \frac{1}{2} e^{-s^2/2} (V/s^2)^{(m-2)/4} e^{-V/2} I_{m/2-1}(\sqrt{Vs^2}), & V > 0 \\ 0, & V < 0 \end{cases} \qquad (18)$$

Where I_ν is a modified Bessel function of the first kind, order ν.

The false-dismissal probability for this detection scheme is

$$P(FD) = \int_{-\infty}^{\gamma} f(V|\underline{S} + \underline{N})\, dV$$

$$= \frac{1}{2} e^{-s^2/2} \int_0^{\gamma} (V/s^2)^{(m-2)/4} e^{-V/2} I_{m/2-1}(\sqrt{Vs^2})\, dV \qquad (19)$$

Integrals of this type have been treated by Marcum and Swerling [9] in connection with radar detection. From (100b) of [9], we see that P(FD) can be expressed in terms of the incomplete Toronto function.

In our notation,

$$P(FD) = T_{\sqrt{\gamma/2}}(m-1,\ m/2-1,\ s/\sqrt{2}) \qquad (20)$$

Curves giving values of the incomplete Toronto function for various values of γ, s, and m are contained in [9], Figures 13-32. Using these curves, we obtain P(FD) as a function of signal strength for various values of m and for values of γ corresponding to P(FA) = 0.01 and P(FA) = 0.1. These values of P(FD) are plotted in Figures 1-4, along with corresponding values for the optimum known-position detector and the search detector.

CONCLUSIONS

The essential conclusions are in agreement with intuition. Lack of position information inevitably leads to a higher error rate. Searching for the signal yields a lower information rate, but also a lower error rate, than a system which processes the information instantaneously. Figures 1-4 provide quantitative information regarding this trade-off.

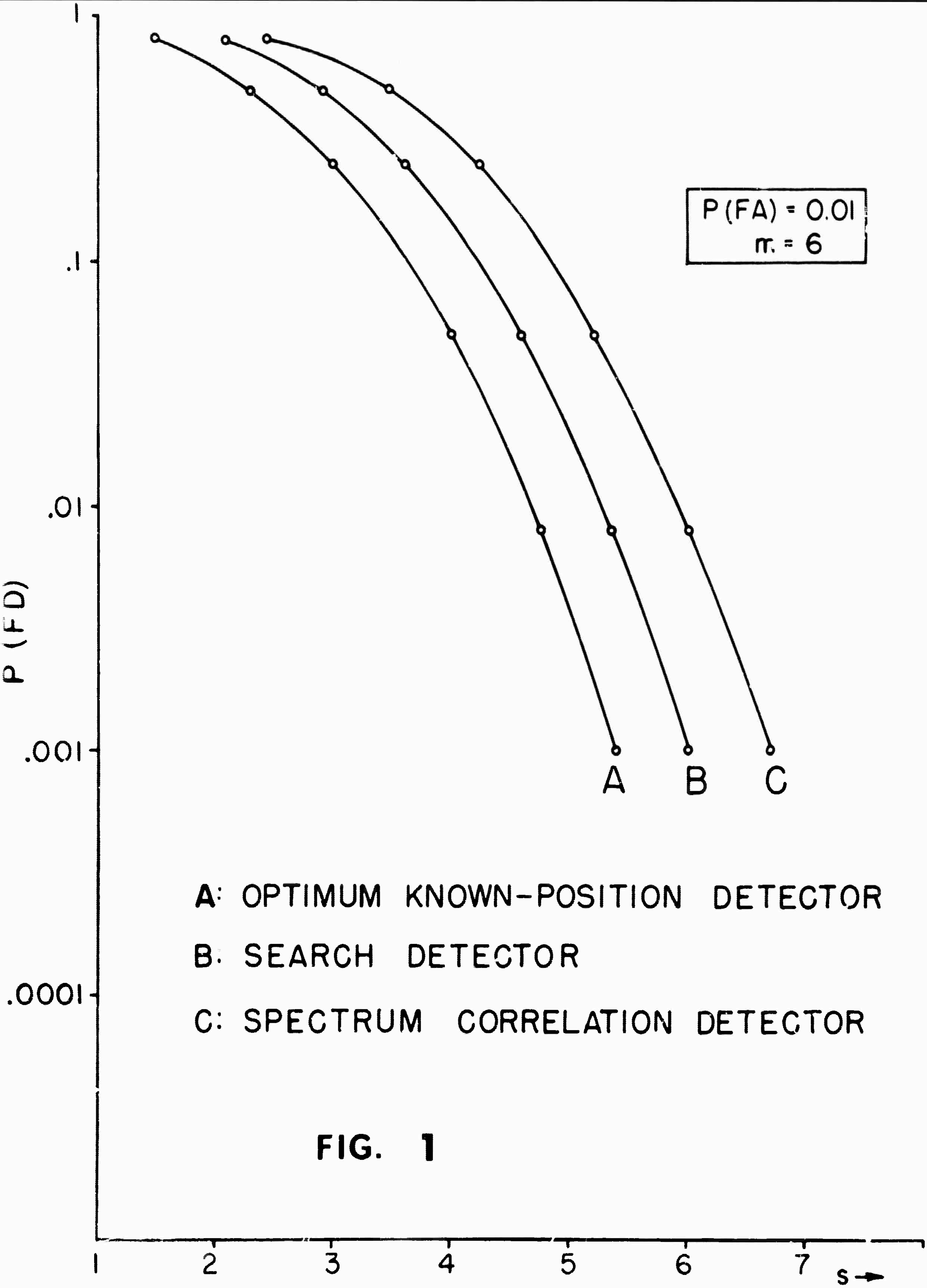

P(FA) = 0.01
m = 6
P (FD)
1
.1
.01
.001
.0001
A
B
C
A: OPTIMUM KNOWN-POSITION DETECTOR
B: SEARCH DETECTOR
C: SPECTRUM CORRELATION DETECTOR
1
2
3
4
5
6
7
s

FIG. 1

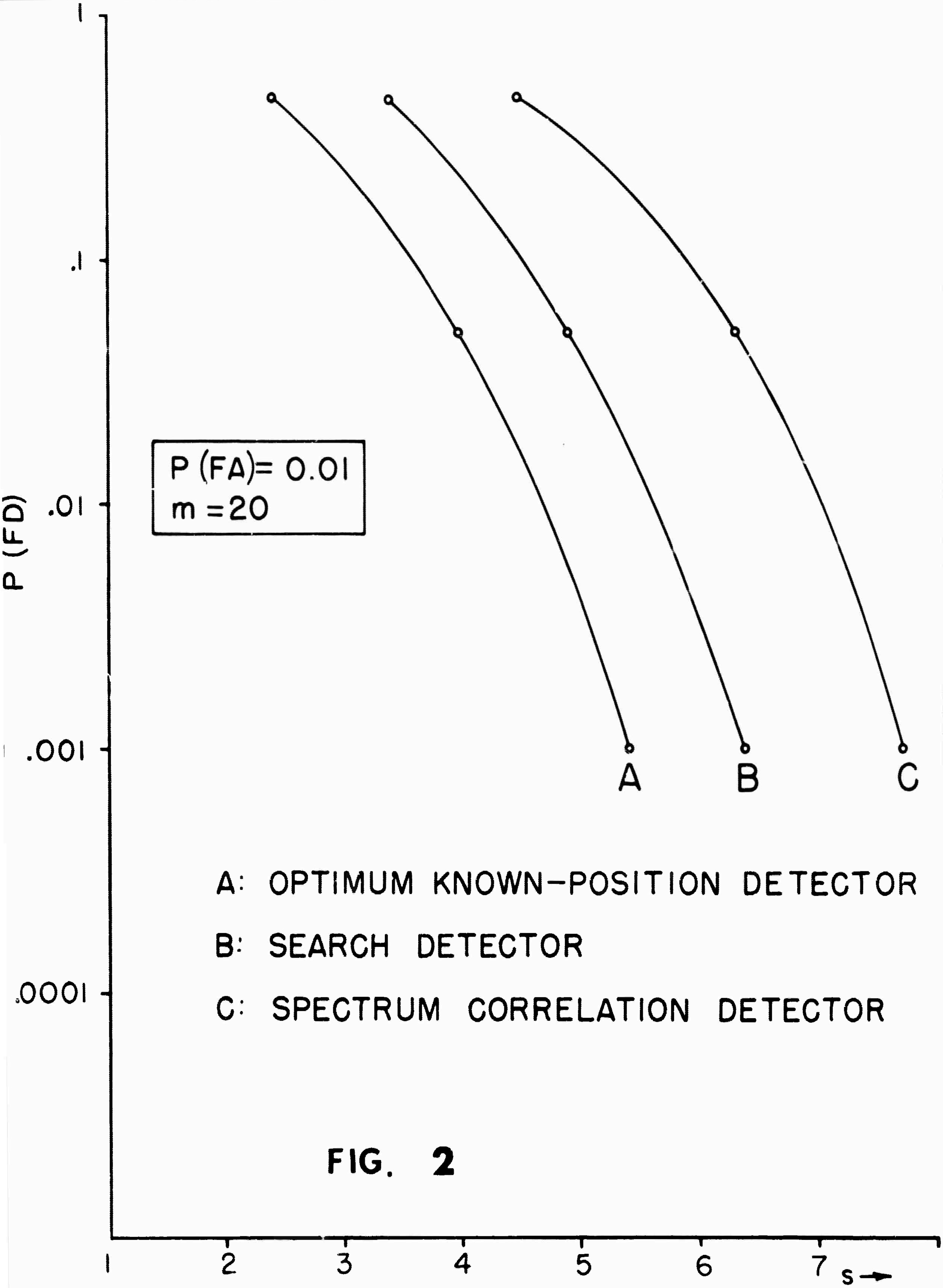
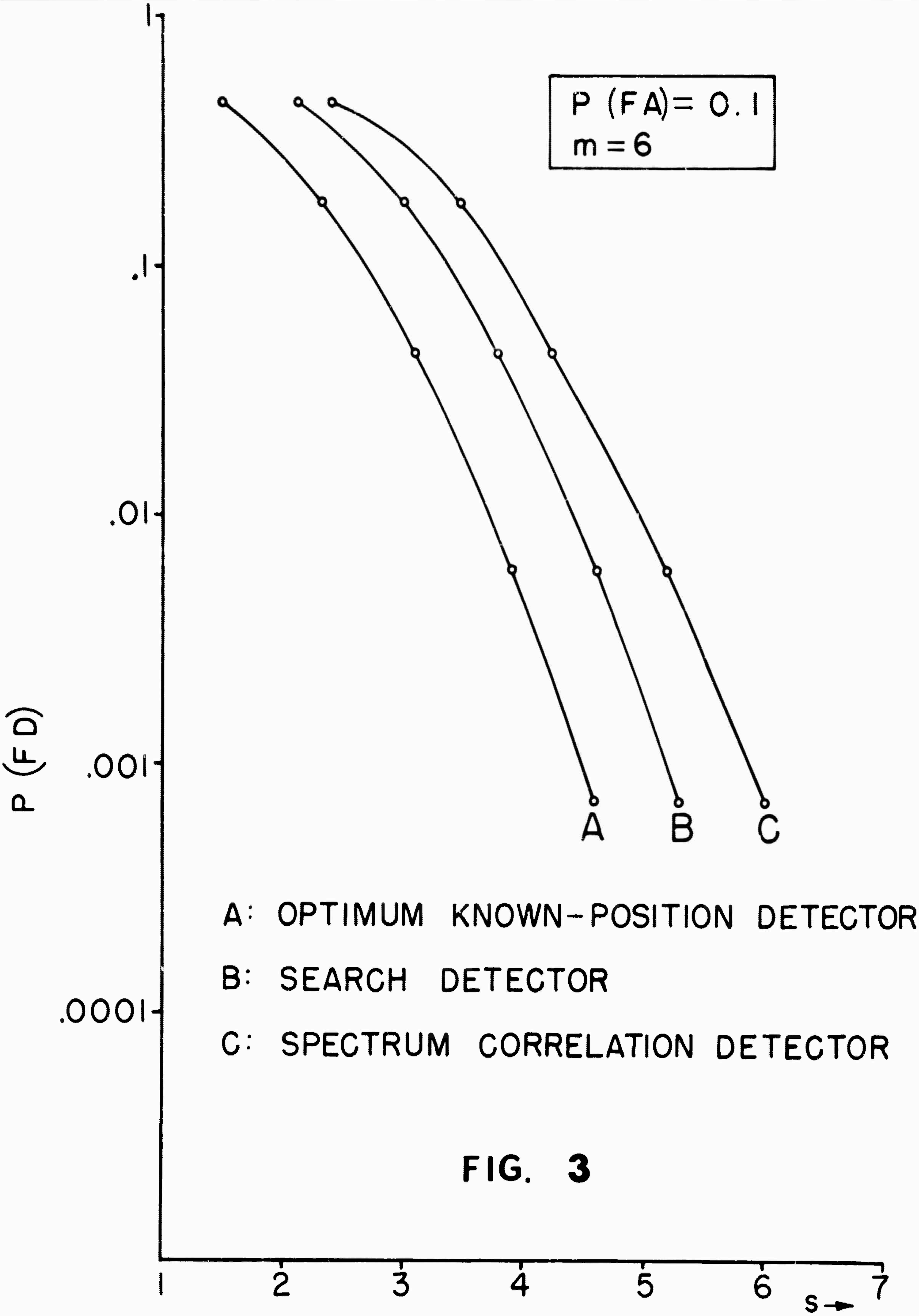

P (FD)
1
.1
.01
.001
.0001
1
2
3
4
5
6
7
s
P (FA)= 0.01
m =20
A
B
C
A: OPTIMUM KNOWN-POSITION DETECTOR
B: SEARCH DETECTOR
C: SPECTRUM CORRELATION DETECTOR

FIG. 2



P (FA) = 0.1
m = 6
1
.1
.01
.001
.0001
P (FD)
A
B
C
A: OPTIMUM KNOWN-POSITION DETECTOR
B: SEARCH DETECTOR
C: SPECTRUM CORRELATION DETECTOR
1
2
3
4
5
6
7
s →

FIG. 3

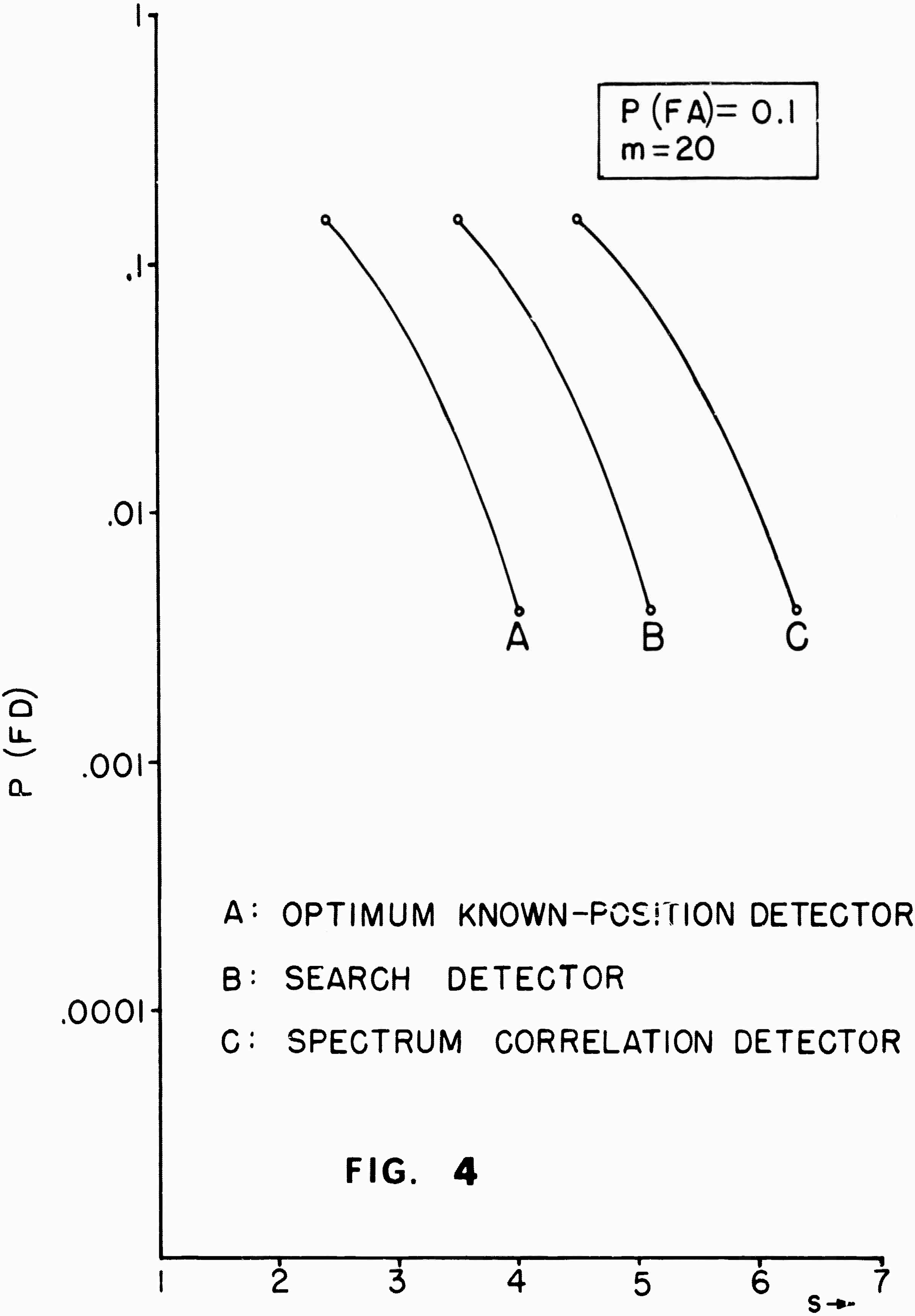

P (FA)= 0.1
m = 20
1
.1
.01
.001
.0001
P (FD)
A
B
C
A: OPTIMUM KNOWN-POSITION DETECTOR
B: SEARCH DETECTOR
C: SPECTRUM CORRELATION DETECTOR
1
2
3
4
5
6
7
s →

FIG. 4

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