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Smoothing and Processing of Simulated AMRAD Trajectory Data

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Lexington, Massachusetts

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LINCOLN LABORATORY

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Group 45

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ABSTRACT

In this report, an attempt is made to determine suitable smoothing intervals to be used in the smoothing of radar trajectory data. This is done by using different fitting intervals in the smoothing of simulated noisy trajectory data of different signal-to-noise ratios. The errors, or differences between the true noiseless data and the smoothed noisy data, were observed and plotted. Minimum errors were obtained when the range smoothing interval was approximately 1 to 2 seconds and the elevation and azimuth smoothing intervals 10 seconds. The plots suggest that elevation and azimuth errors can be further reduced by smoothing over intervals longer than 10 seconds but this would not be practical in many actual missile shots.

Accepted for the Air Force
Stanley J. Wisniewski
Lt Colonel, USAF
Chief, Lincoln Laboratory Office

INTRODUCTION

An attempt has been made to ascertain appropriate fitting intervals for the smoothing of AMRAD-type trajectory data. It is felt that the results obtained will be useful in processing data from ATHENA shots.

DESCRIPTION

A noiseless trajectory was generated (20 points/sec) to simulate a typical AMRAD shot. Different noise levels* were added to this basic trajectory so as to simulate the following signal-to-noise ratios: 0 db, 5 db, 10 db, ∞ db (noiseless). The resultant noisy trajectory data was then smoothed, separately in range, elevation, and azimuth, by standard quadratic least squares procedures of the "sliding arc" type and the following quantities were obtained: range, range rate, range acceleration (rate of range rate), elevation, elevation rate, azimuth, azimuth rate, and drag velocity. The corresponding noiseless trajectory values were subtracted from these and the differences, or errors, were plotted versus time. The above procedure was repeated using different smoothing intervals and the stated signal-to-noise ratios. Also, the rms errors (i.e., the standard deviation of the differences) were computed, tabulated, and, for the 5 db case, plotted versus smoothing interval. Other parameters such as drag acceleration, weight-to-drag ratio, height, path angle, etc., were computed and plotted versus time for selected cases.

RESULTS

In Figure 1, the two plots on the left describe the geometry of the

*See the Appendix for a description of the noise generation.

simulated trajectory giving latitude versus longitude and height versus surface range with AMRAD located at the origin. No tangent-plane approximations are involved in the height versus surface range plot. That is, height and surface range are measured above and along the surface of a spherical earth. The two figures on the right show range, elevation, and azimuth for the noiseless and S/N = 5 db cases.

Figures 2 through 8 show the various errors (i.e., computed value minus true value) in range, range rate, range acceleration, elevation, elevation rate, azimuth, and azimuth rate for the cases tested. Figure 9 gives the drag velocity error versus time for the cases tested. The "drag velocity" V_D is the speed of the missile relative to the radar, or equivalently, it is the airspeed, assuming a rigid rotating atmosphere with no local winds.

Plots of the standard deviation (or rms values) of the above errors versus smoothing interval for the case of 5 db signal-to-noise ratio are given in Figures 10 and 11. Tables I through III list the rms errors for all cases tested.

Figure 12 shows weight-to-drag ratio, drag acceleration, drag velocity, height, path angle, and viewing (or aspect) angle versus time for the 5 db signal-to-noise ratio case and different smoothing. The path angle GAM is the angle between the velocity vector and the local horizontal. The viewing (or aspect) angle PSI is the angle between velocity vector and the line of sight. TFITR, TFITE, and TFITA are the smoothing intervals for range, elevation and azimuth respectively. The expressions used for the computation of all parameters in Fig. 12 are as reported in 47G-6, "Determination of Weight-to-Drag Ratio from Radar Measurements," 8 March 1963, S. F. Catalano, H. Schneider. Equation (33) of the above report was used for computations of weight-to-drag ratio. Note that weight-to-drag ratio is best estimated in regions of high acceleration.

CONCLUSIONS

Referring to Tables I through III and considering the noiseless case, smallest rms errors were obtained when the shortest smoothing intervals were used. This is as expected. With no noise, smoothing is unnecessary as it only corrupts the signal. As noise is added, the smoothing should suppress the noise more than the signal, thereby improving the signal-to-noise ratio. (This will be true if the noise fluctuates much more rapidly than the signal varies.) With excessively long smoothing intervals, the signal may also be seriously distorted, thus losing any advantage gained by noise suppression.

For a signal-to-noise ratio of 5 db, minimum rms errors were obtained when the range smoothing interval was approximately 1 to 2 seconds, and the elevation and azimuth smoothing intervals 10 seconds. Figure 11 suggests that elevation and azimuth errors can be further reduced by smoothing over intervals longer than 10 seconds, but this would no be practical in many actual missile shots. From Figure 10 it is seen that the minimum errors for range, range rate, and range acceleration do not occur at the same range-smoothing interval. The minima, however, are reasonably close together and quite broad, so that very good results should be obtained using a range smoothing interval from 1 to 2 seconds.

A range smoothing interval within these limits should be quite suitable with other signal-to-noise ratios as may be seen from Table I.

Drag velocity and weight-to-drag ratio are used as initial conditions in a trajectory prediction program. Figures 9 and 12 should be useful in assuring sufficient accuracy of these quantities.

The results in this report should be considered as a useful guide rather than absolute rules.

APPENDIX

Discussion of Simulated Trajectory Data Generation

The initial conditions at the start of the generated trajectory were as follows:

Time	400.000 seconds
Range	499.791 kft
Range Rate	-20.082 kft/sec
Elevation	31.464 degrees
Elevation Rate	.016 degrees/second
Azimuth	-25.00 degrees
Azimuth Rate	0.0 degrees/second
Weight-to-Drag Ratio	110 lbs/ft^2

From the above conditions, the starting altitude was 265.160 kft, and the drag velocity 20.082 kft/sec. A trajectory generation program, using a spherical rotating earth with gravity and a realistic atmosphere, calculated range, azimuth, and elevation for the next 999 points spaced .05 seconds apart. At the last calculated data point, the range and altitude were 87.359 kft and 33.113 kft respectively. Gaussian noise, consistent with the desired signal-to-noise ratio was then added to the trajectory data and the resulting noisy data were written on magnetic tape.

The standard deviation for the Gaussian noise added to the signal was computed as described below. (These formulas are based on the assumption of a large signal-to-noise ratio. While this was not always the case here, it was felt that useful results would still be obtained).

For range data*

$$\sigma_R = \left(\frac{C}{2}\right)\delta T_R = \left(\frac{C}{2}\right) \frac{t_r}{2(S/N)}$$

where σ_R = rms range error

C = velocity of light = 9.83514×10^8 ft/sec

δT = rms error in estimating leading edge of pulse

t_r = rise time of pulse

S/N = signal-to-noise (power) ratio

The pulse rise time was assumed to be 35 nsec corresponding to a signal bandwidth of approximately 20 Mcps. For elevation and azimuth data**

$$\sigma_\theta = \frac{0.628\,\theta_B}{2(S/N)}$$

where σ_θ = rms elevation (or azimuth) error

θ_B = antenna beamwidth in elevation (or azimuth)

S/N = signal-to-noise (power) ratio

The units of σ_θ will be identical to those of θ_B. For the simulations θ_B was taken as 1° in both azimuth and elevation.

Since the noise bandwidth of the AMRAD antenna servo system is approximately 1 cps, the noise samples to be added to azimuth (or elevation) data would not be independent from point to point with a rep rate of 20 points/sec. The following formula was used to generate the azimuth (and elevation) noise.

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*Skolnik, M. I., "Introduction to Radar Systems," McGraw-Hill Book Company, 1962, pp. 462-464.

**Skolnik, op. cit., pp. 476-477.

$$n_k = \sum_{j=k}^{j=k+N-1} m_j$$

n_k = Gaussian noise to be added to k^{th} azimuth (or elevation) sample

m_j = j^{th} independent Gaussian noise sample of standard deviation σ_θ

$$N = \left(\frac{\text{Repetition Rate}}{\text{Noise Bandwidth}}\right)^{1/2}$$

This assured that the noise would be correlated over a 1 second interval.

The calculated and observed rms noise values are given below:

S/N	σ_R (feet) Calculated	Observed	σ_{EL} (degrees) Calculated	Observed	σ_{AZ} (degrees) Calculated	Observed
10 db	3.8	3.9	.14	.13	.14	.12
5 db	6.8	6.9	.25	.22	.25	.23
0 db	12.0	11.8	.44	.48	.44	.48

Signal-to-Noise Ratio	Range Smoothing Interval (seconds)	RMS Range Error (feet)	RMS Range-Rate Error (ft/sec)	RMS Range Acceleration Error (ft/sec^2)
Noiseless	.5	2.0	1.1	3.6
	2.0	1.9	14.4	5.2
	4.0	3.7	55.3	15.5
10 db	2.0	2.1	14.5	6.8
5 db	.5	4.2	13.8	194.6
	1.0	3.4	6.4	39.4
	1.5	3.1	8.8	14.1
	2.0	3.0	14.6	8.6
	4.0	4.3	55.4	15.6
0 db	2.0	3.3	15.9	13.4

TABLE I

RMS Errors of Range, Range Rate, and Range Acceleration for Different Range Smoothing Intervals and Signal-to-Noise Ratios

Signal-to-Noise Ratio	Elevation Smoothing Interval (seconds)	RMS Elevation Error (degrees)	RMS Elevation Rate Error (degrees)
Noiseless	2.0	.001	.0003
	5.0	.0002	.002
	10.0	.003	.008
10 db	2.0	.097	.092
	10.0	.045	.012
5 db	2.0	.184	.176
	5.0	.135	.056
	10.0	.108	.028
0 db	2.0	.413	.384
	10.0	.289	.051

TABLE II

RMS Errors of Elevation and Elevation Rate
for Different Elevation Smoothing Intervals
and Signal-to-Noise Ratios

Signal-to-Noise Ratio	Azimuth Smoothing Interval (seconds)	RMS Azimuth Error (degrees)	RMS Azimuth Rate Error (degrees)
Noiseless	2.0	.00001	.00001
	5.0	.00001	.00001
	10.0	.00002	.00004
10 db	2.0	.102	.110
	10.0	.040	.010
5 db	2.0	.194	.198
	5.0	.138	.069
	10.0	.091	.019
0 db	2.0	.423	.453
	10.0	.187	.044

TABLE III

RMS Errors of Azimuth and Azimuth Rate
for Different Elevation Smoothing Intervals
and Signal-to-Noise Ratios

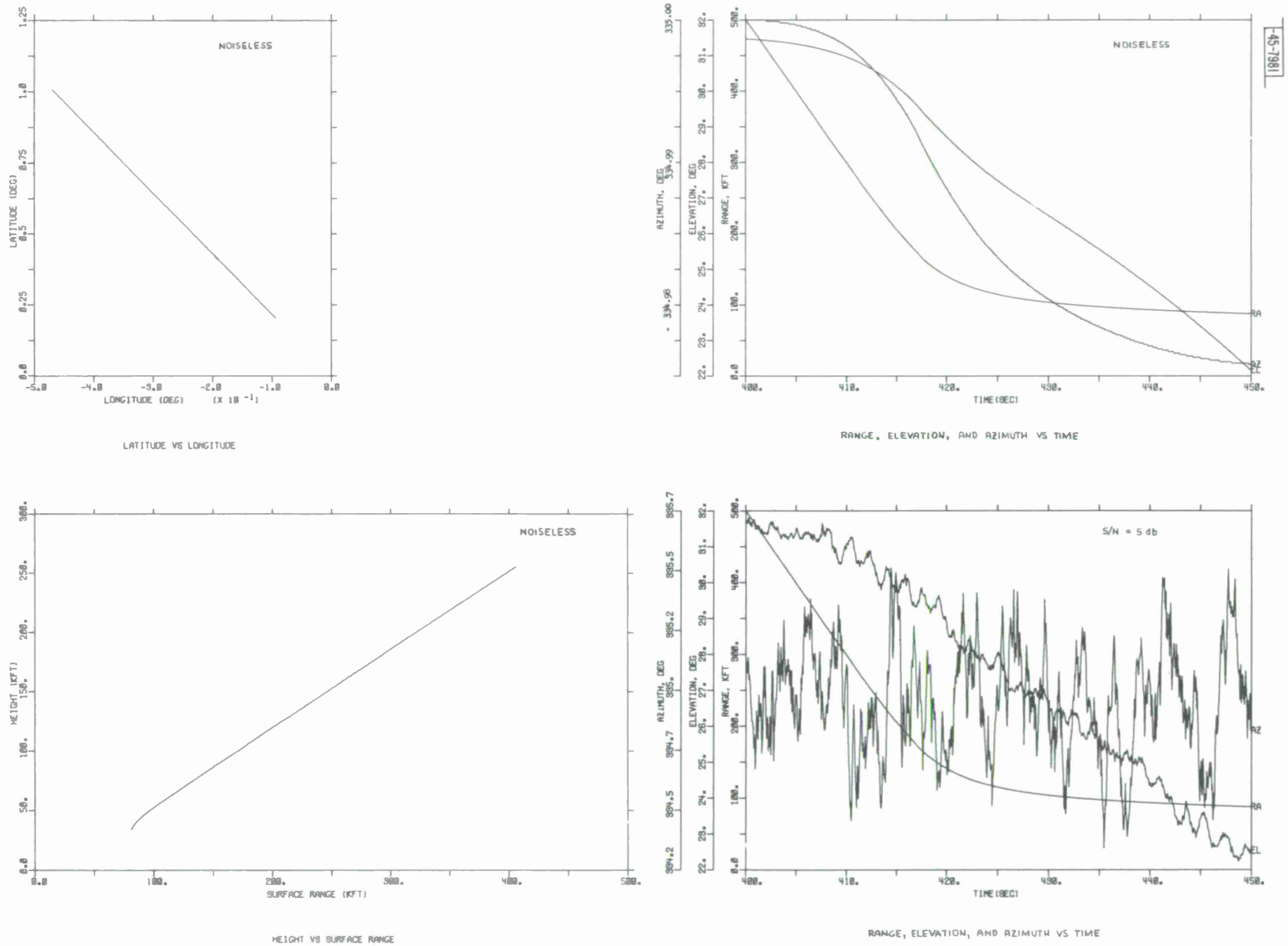

Fig. 1. Raw trajectory data vs. time for the noiseless and 5 db cases.

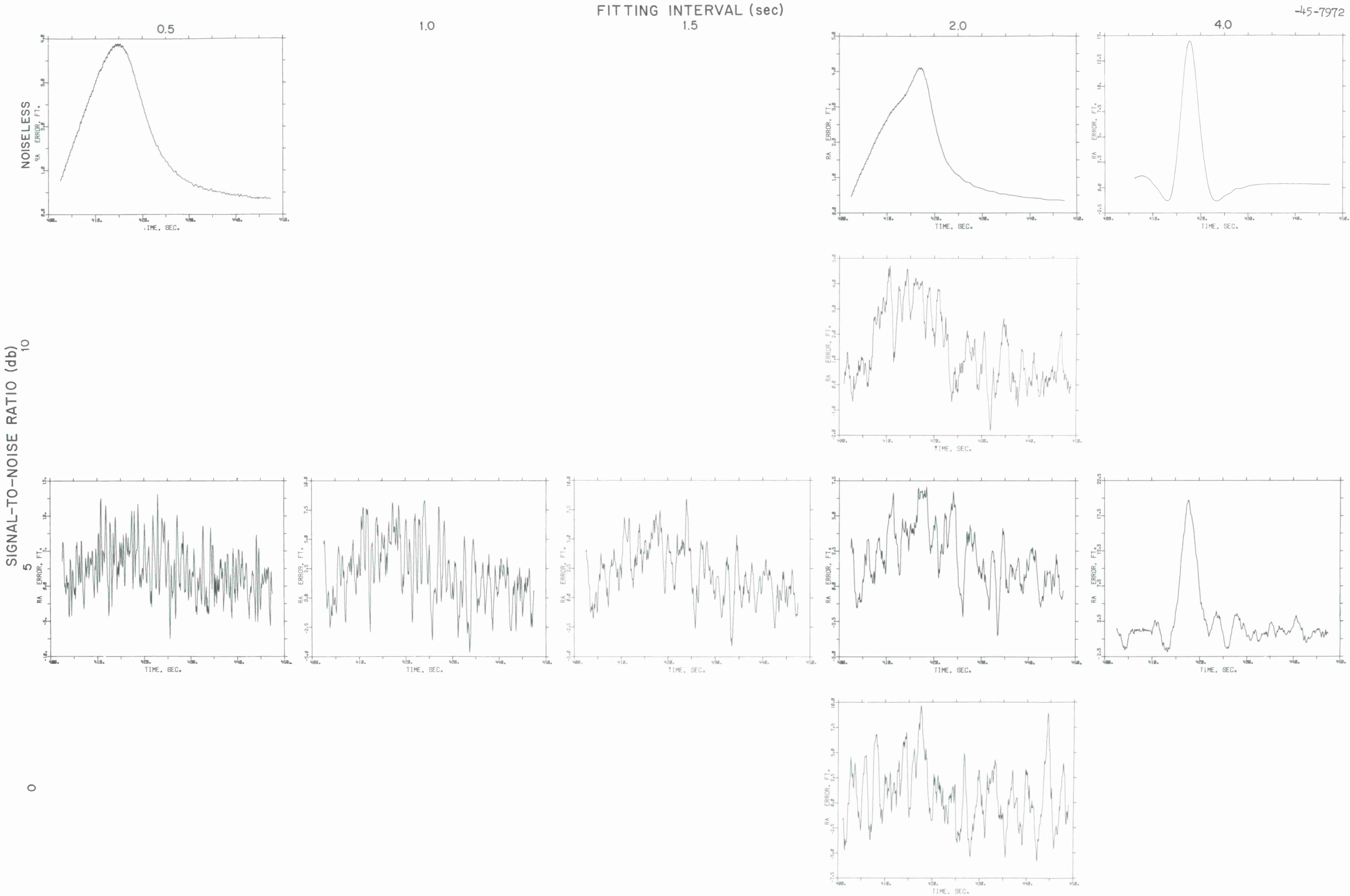

Fig. 2. Range error vs. time for different signal-to-noise ratios and smoothing intervals.

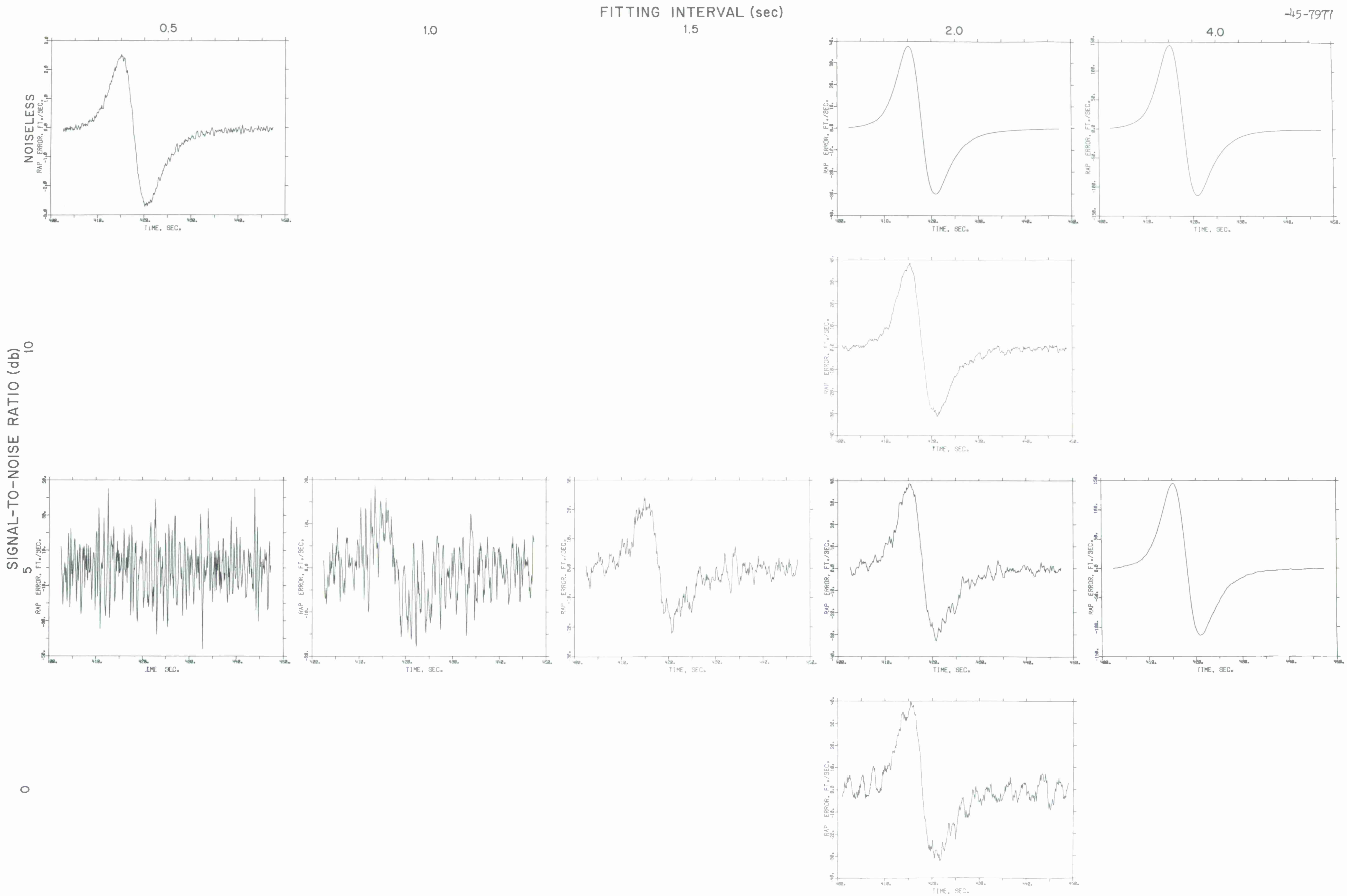

Fig. 3. Range rate error vs. time for different signal-to-noise ratios and smoothing intervals.

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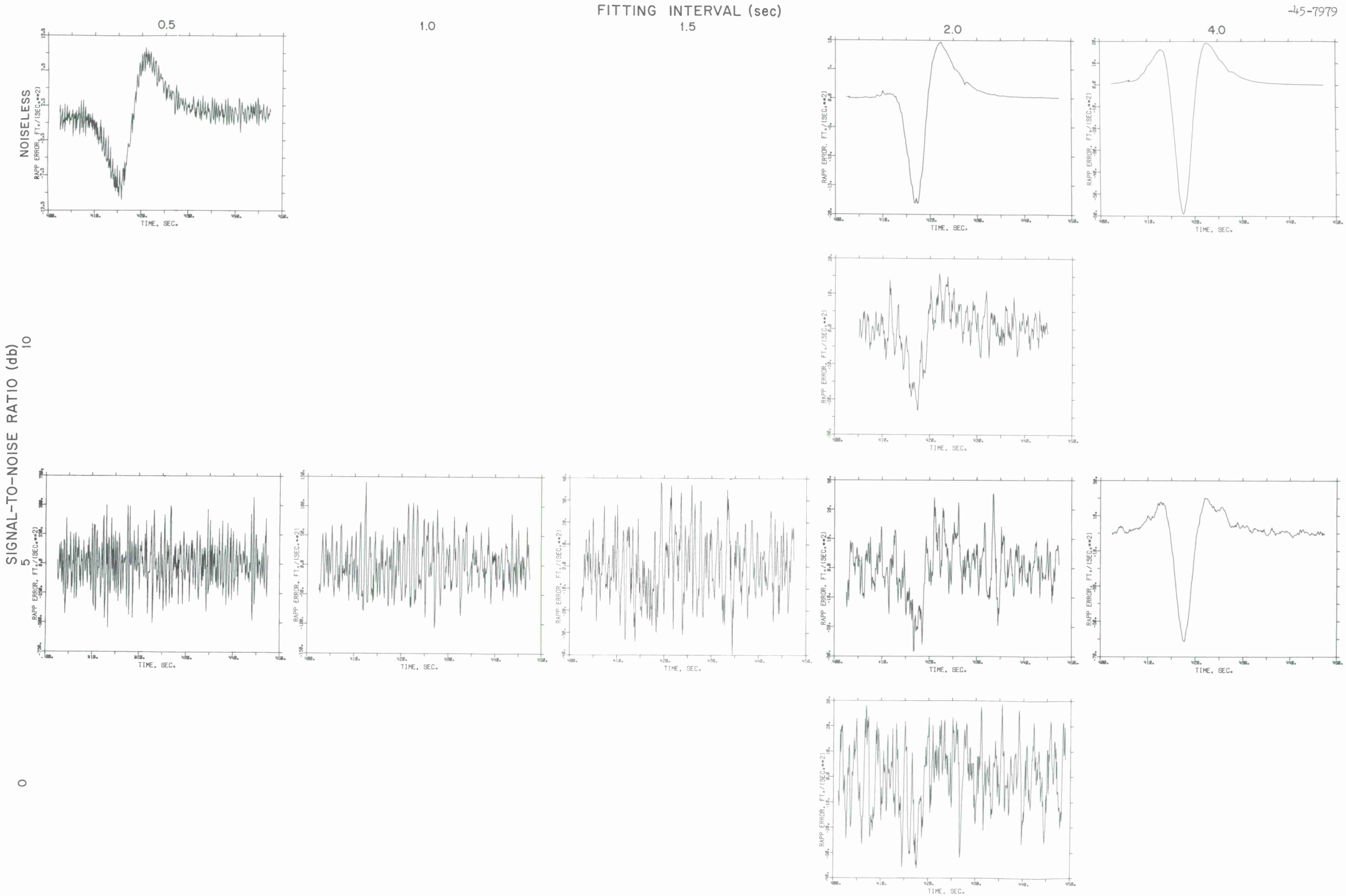

Fig. 4. Range acceleration error vs. time for different signal-to-noise ratios and smoothing intervals.

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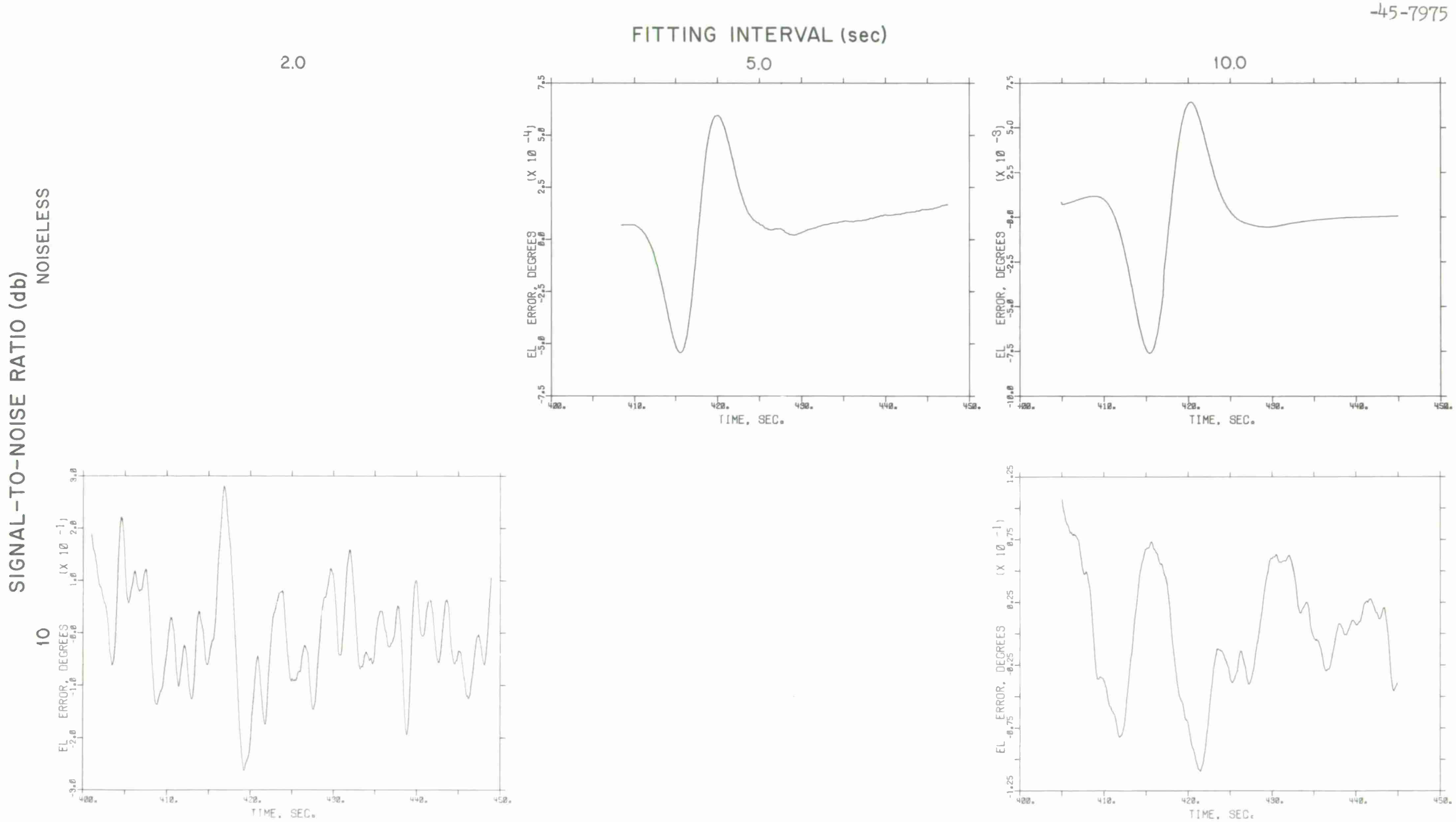

Fig. 5. Elevation error vs. time for different signal-to-noise ratios and smoothing intervals.

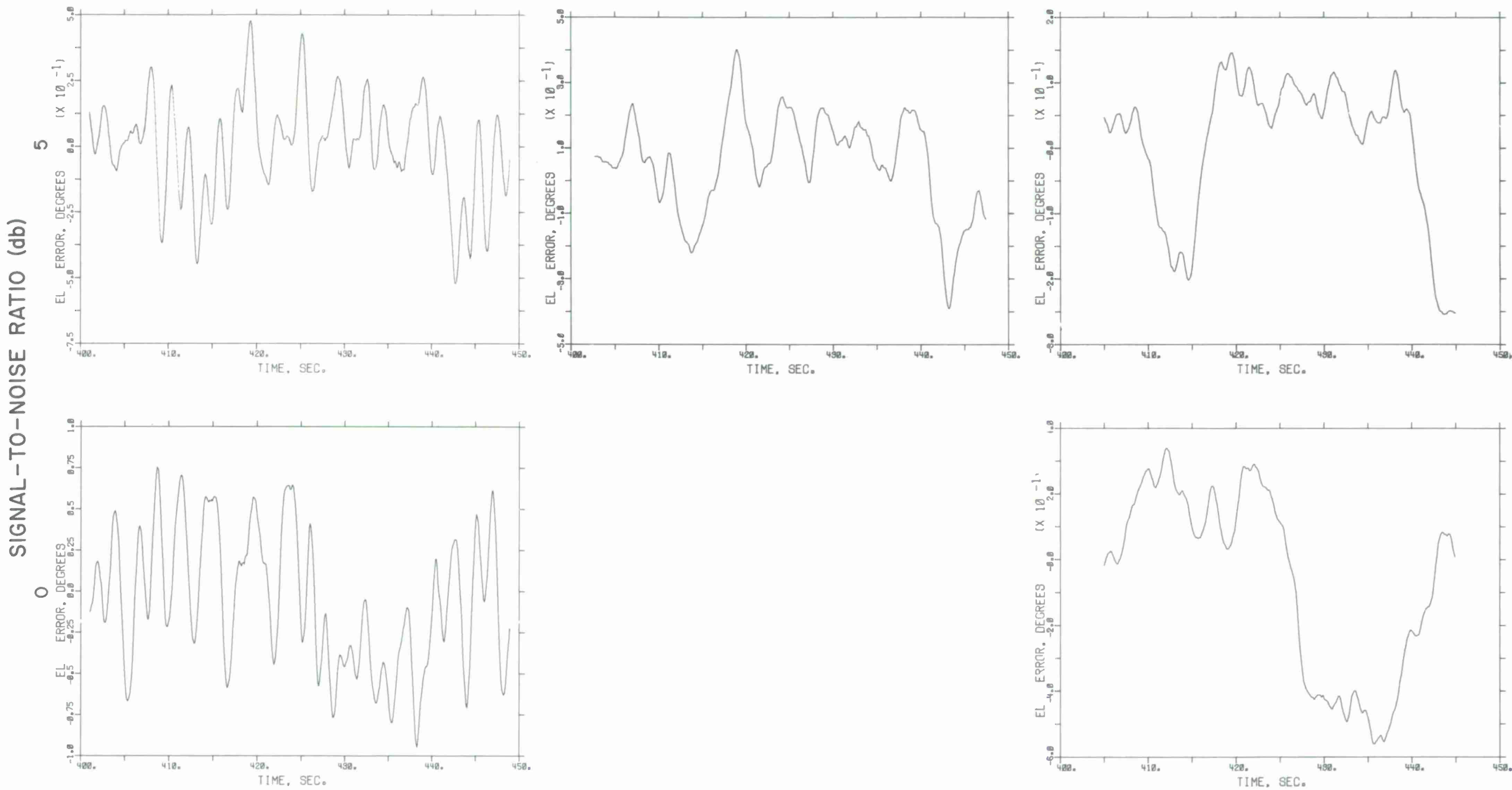

Fig. 5 continued

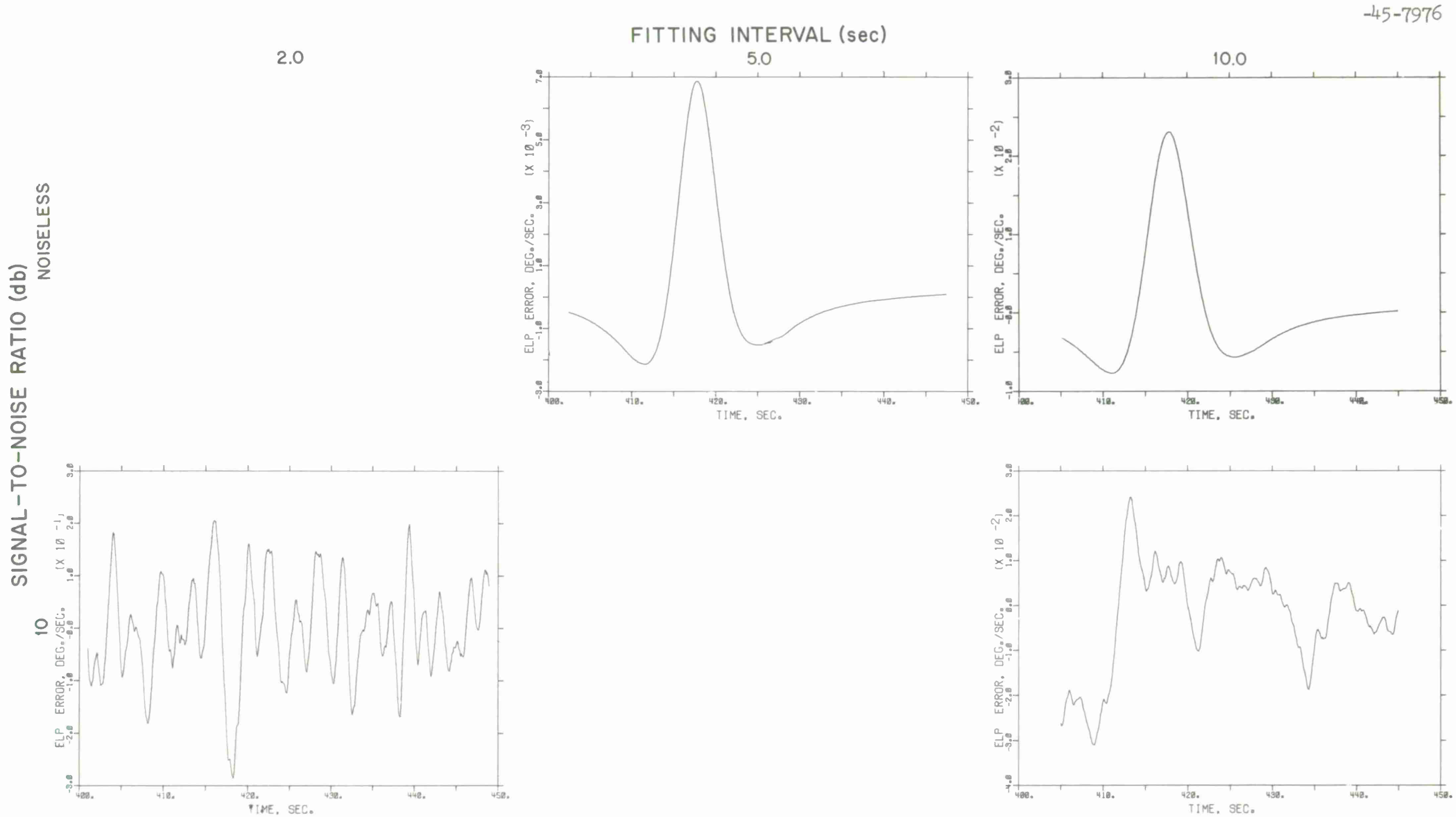

Fig. 6. Elevation rate error vs. time for different signal-to-noise ratios and smoothing intervals.

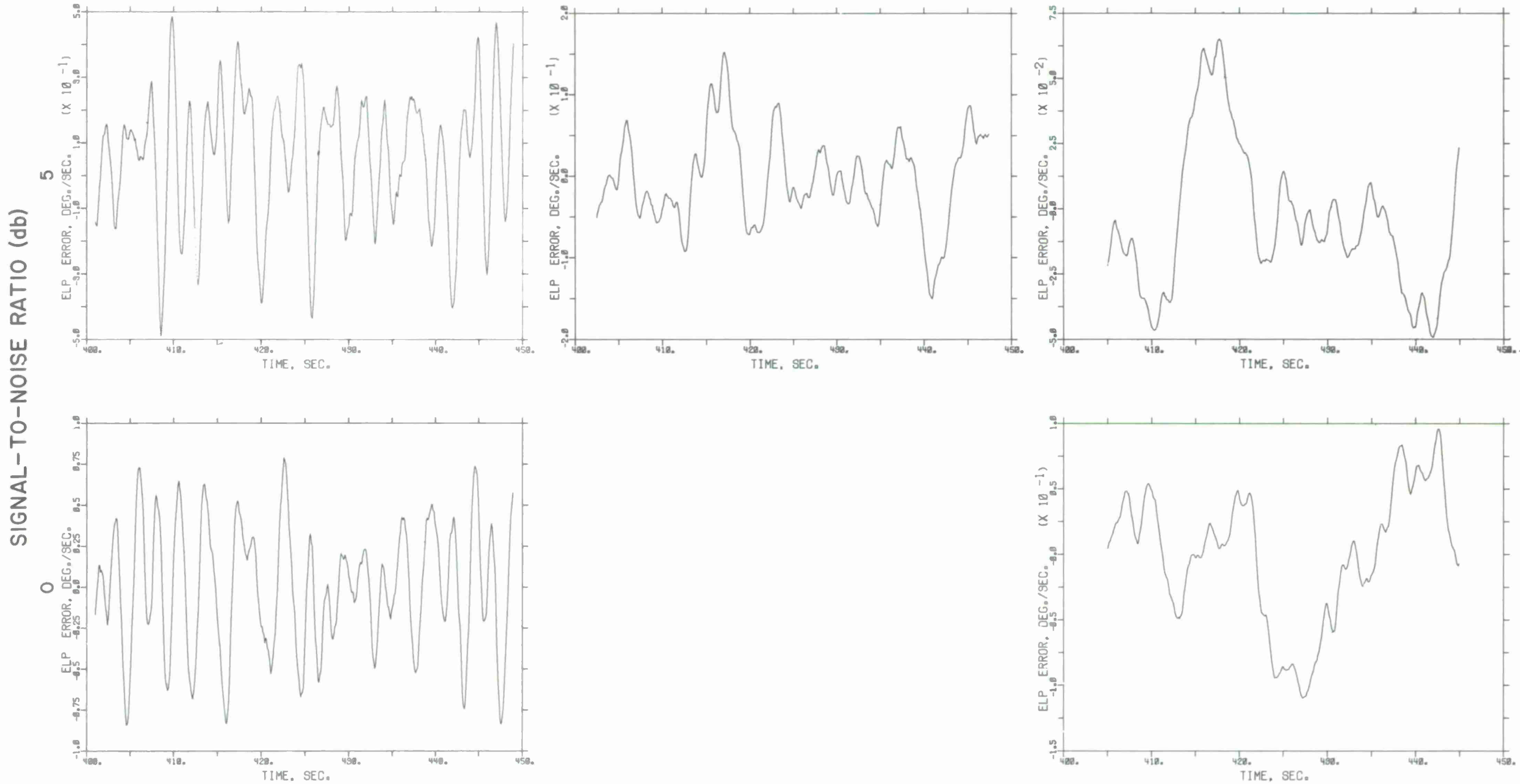

Fig. 6 continued

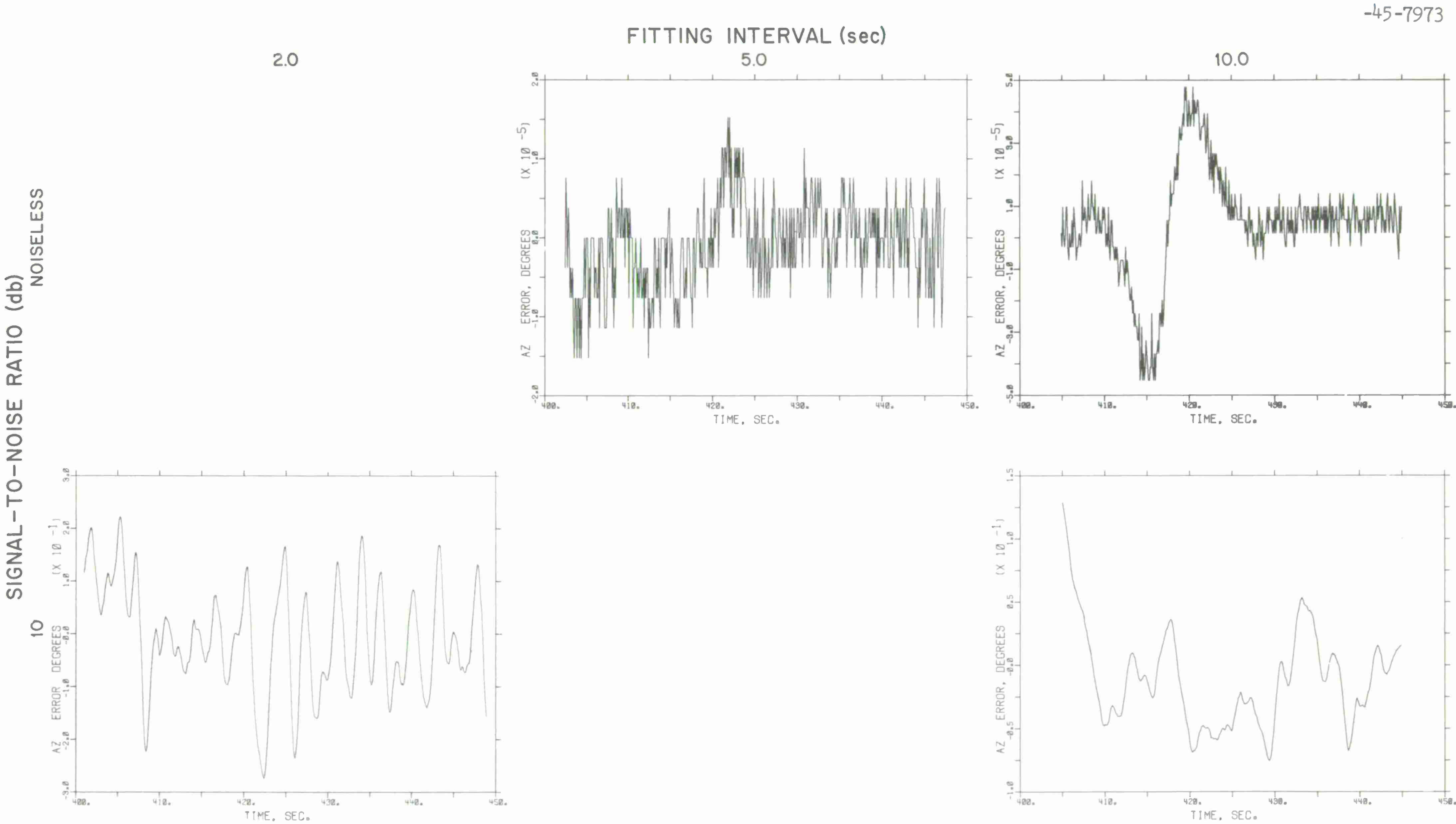

Fig. 7. Azimuth error vs. time for different signal-to-noise ratios and smoothing intervals.

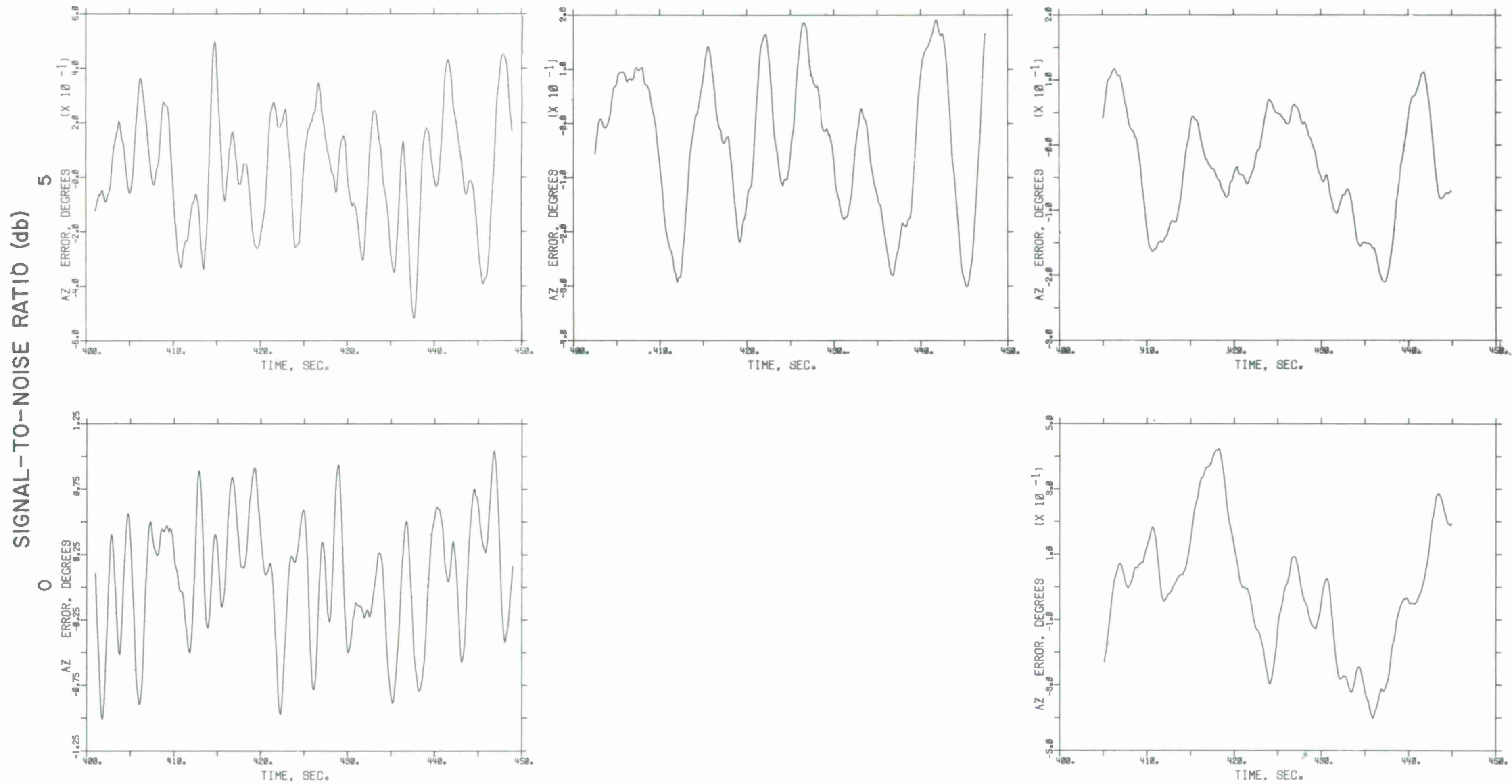

Fig. 7 continued

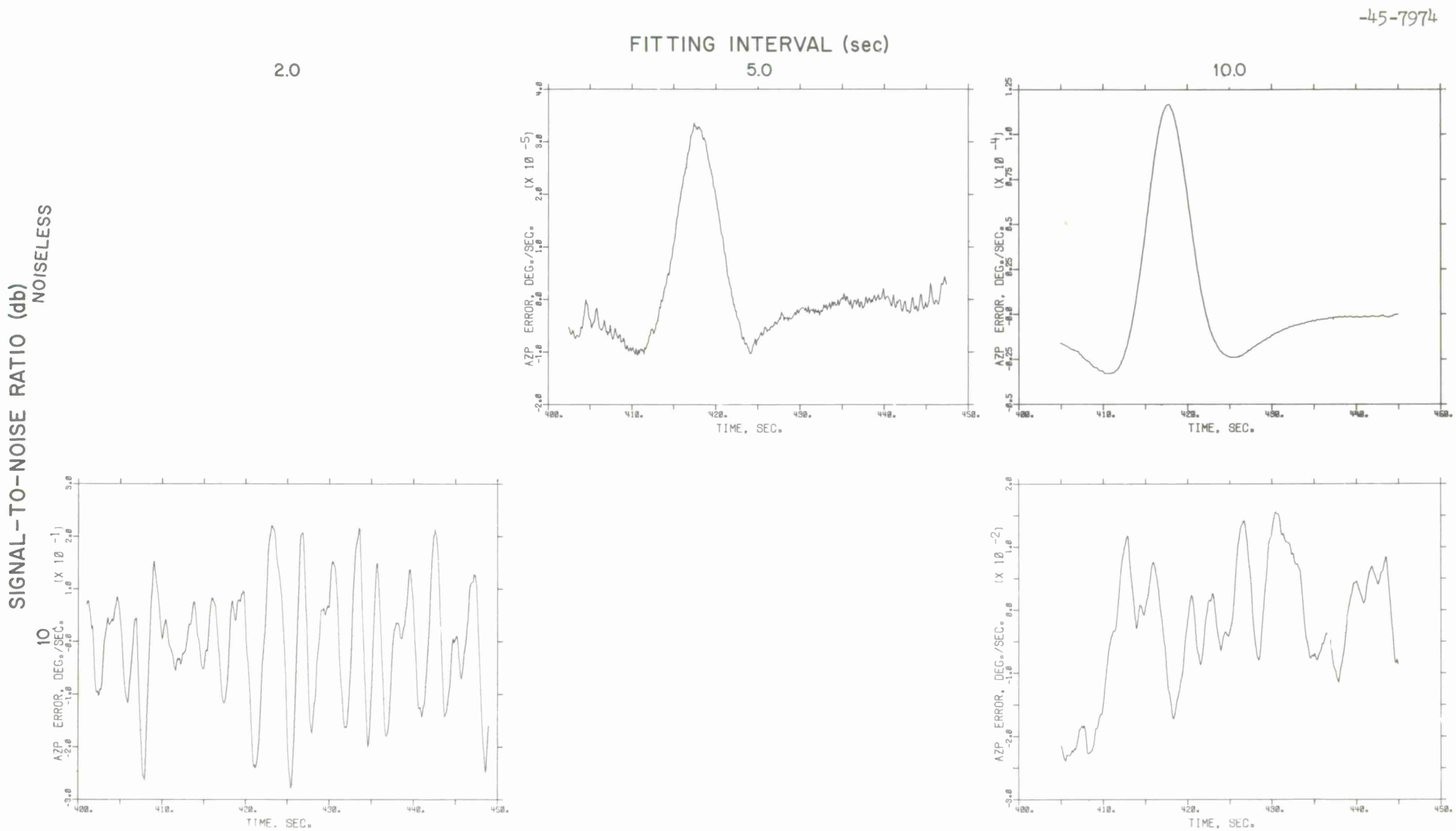

Fig. 8. Azimuth rate error vs. time for different signal-to-noise ratios and smoothing intervals.

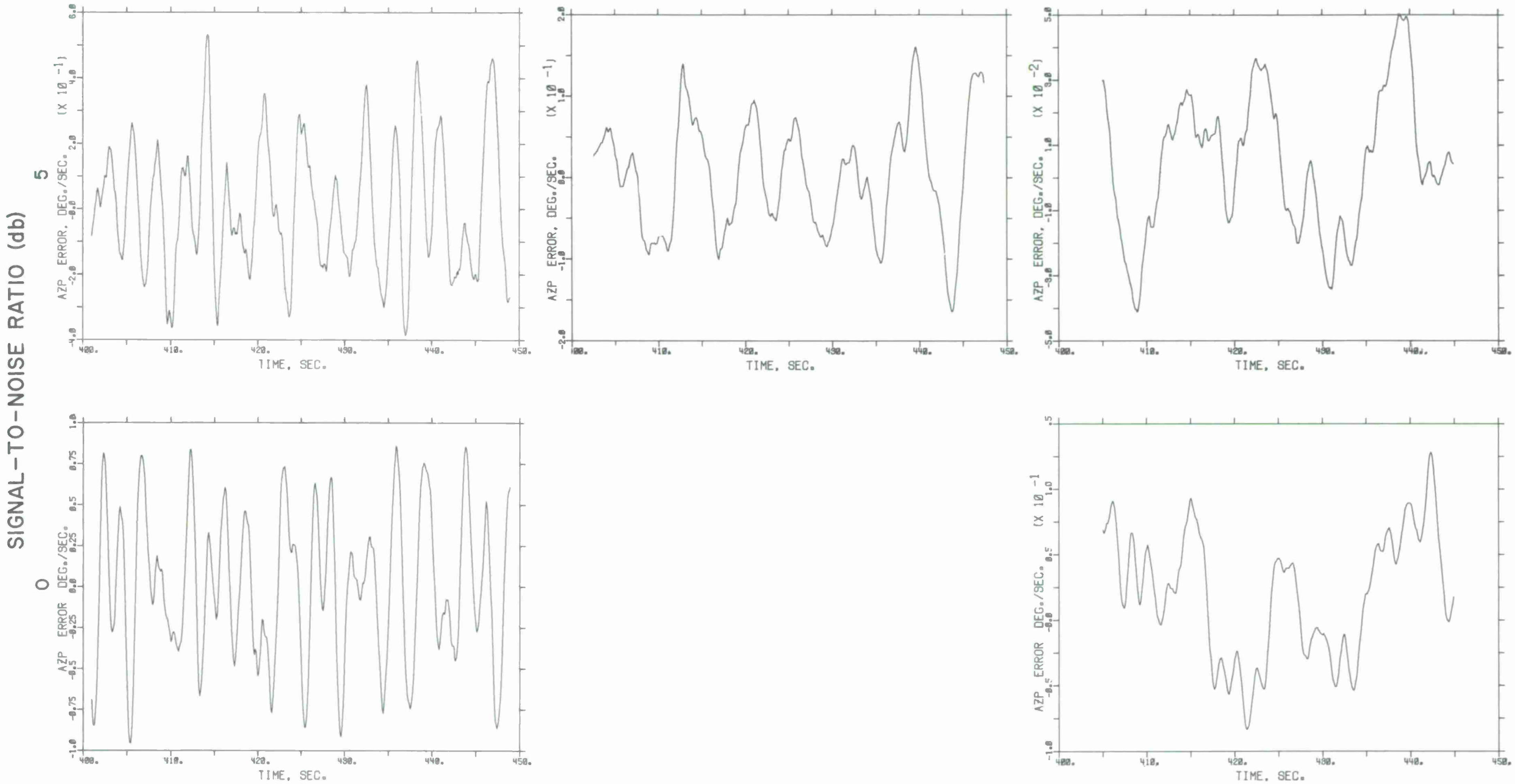

Fig. 8 continued

-45-7978

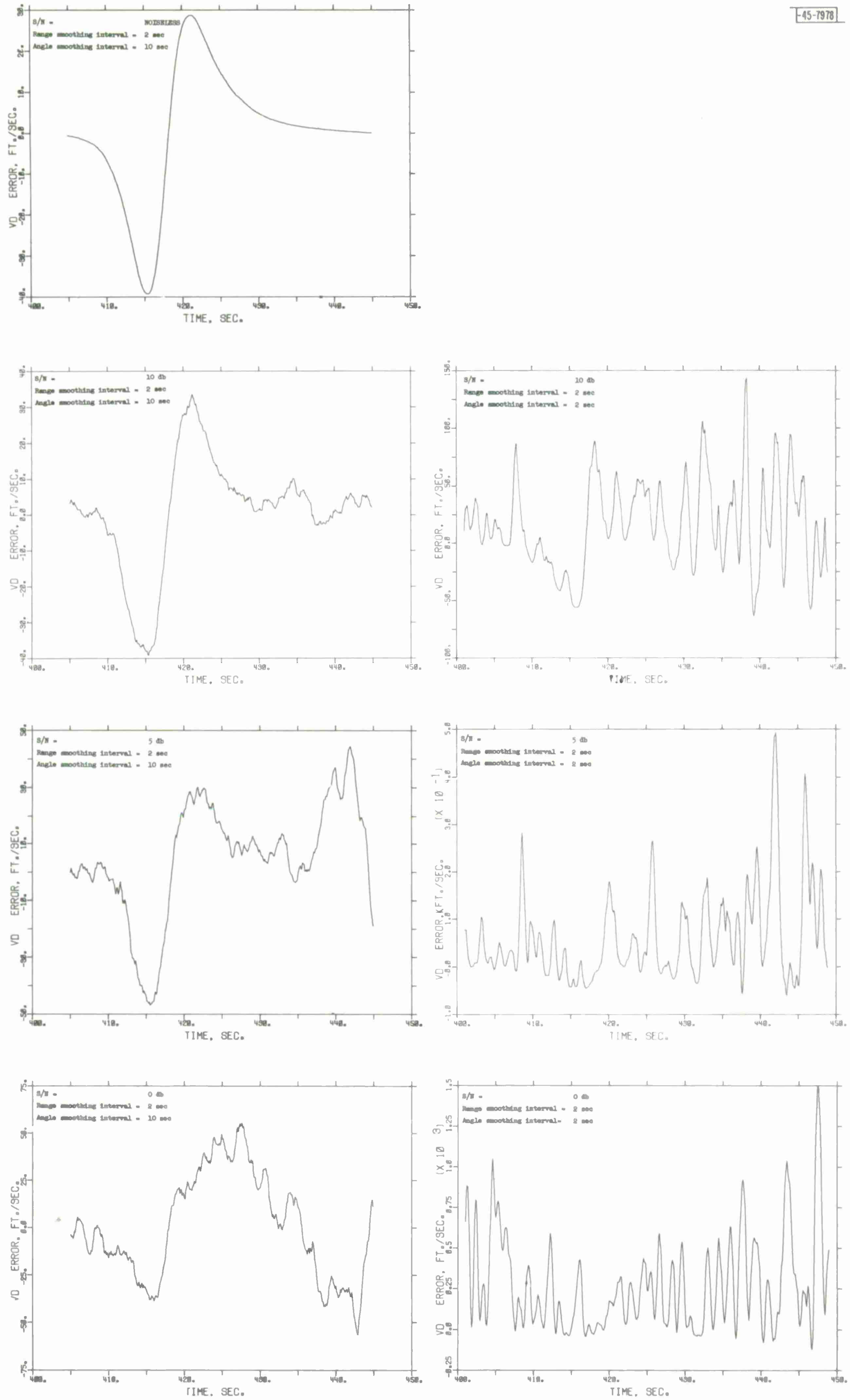

Fig. 9. Drag velocity error vs. time for different smoothing intervals and signal-to-noise ratios.

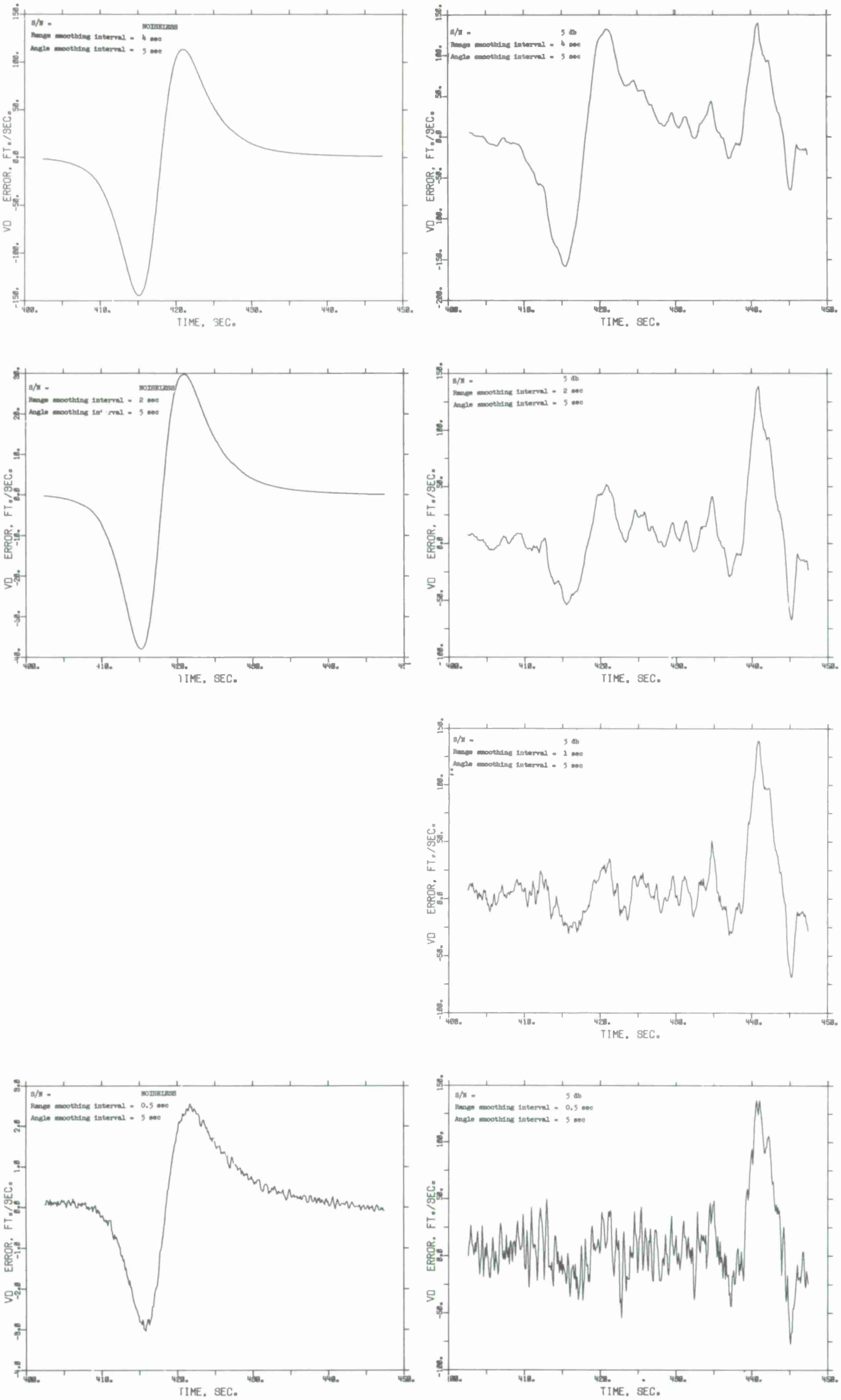

Fig. 9 continued

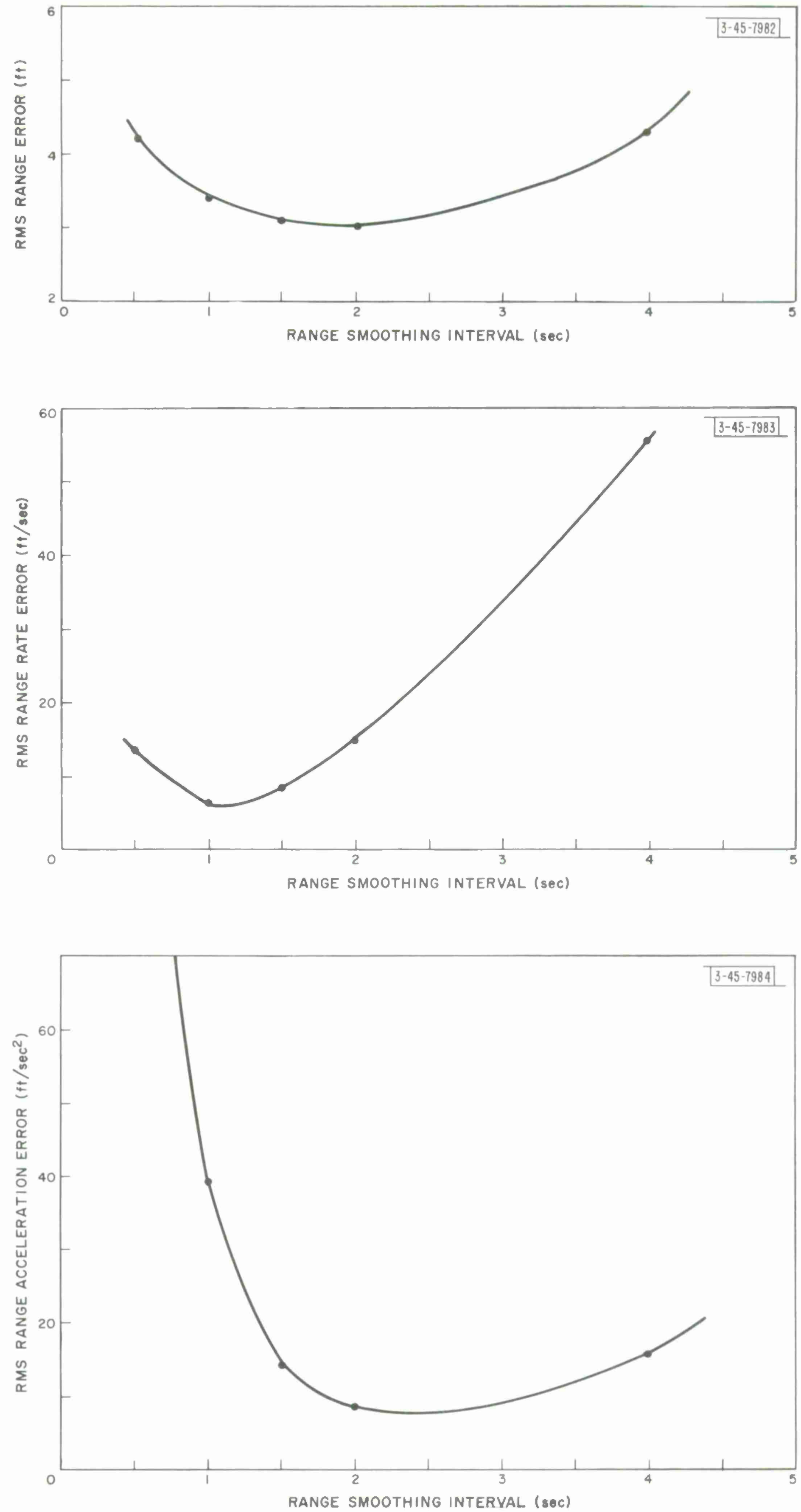

Fig. 10. RMS range errors vs. smoothing interval for the S/N = 5 db case.

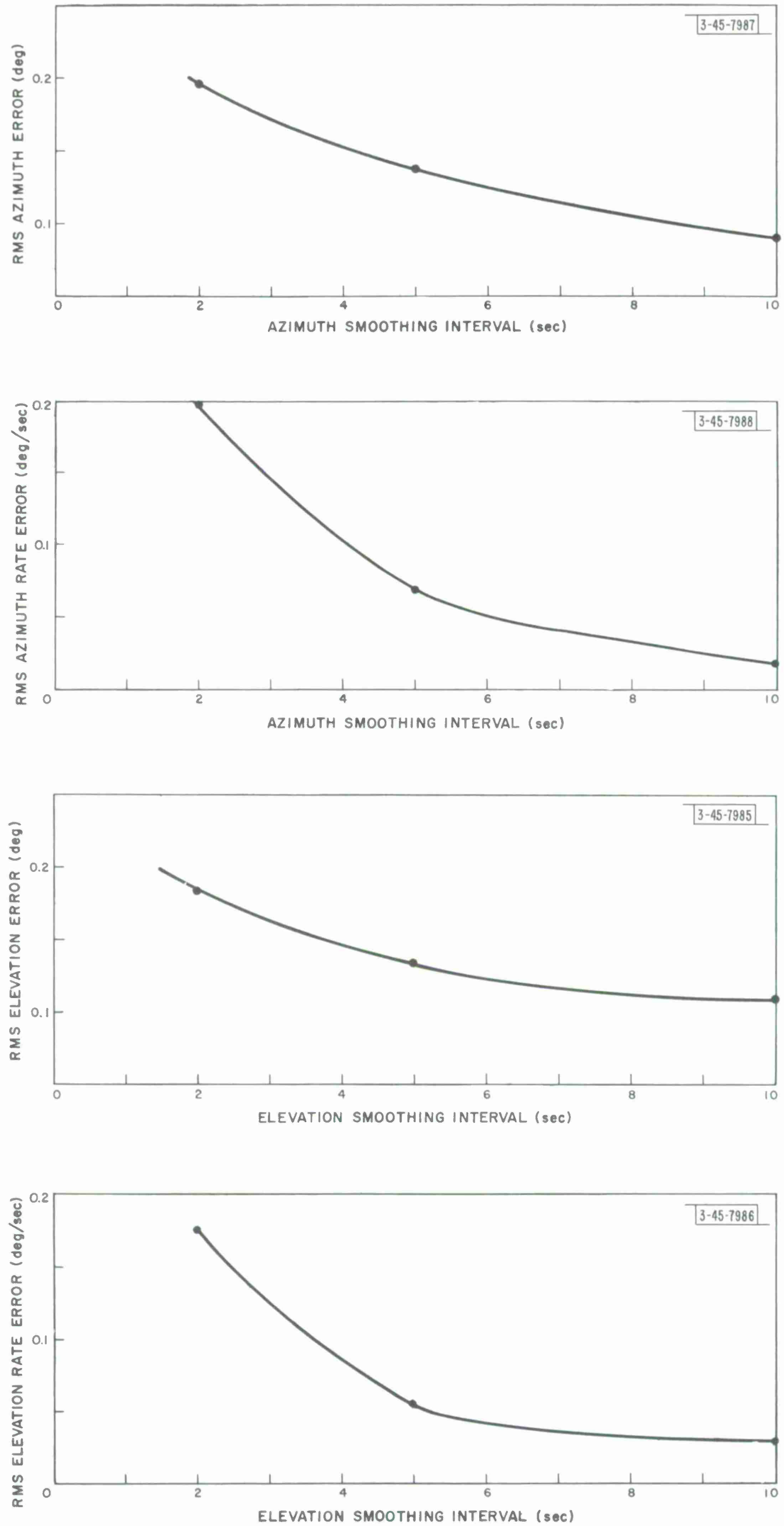

Fig. 11. RMS angle errors vs. smoothing interval for the S/N = 5 db case.

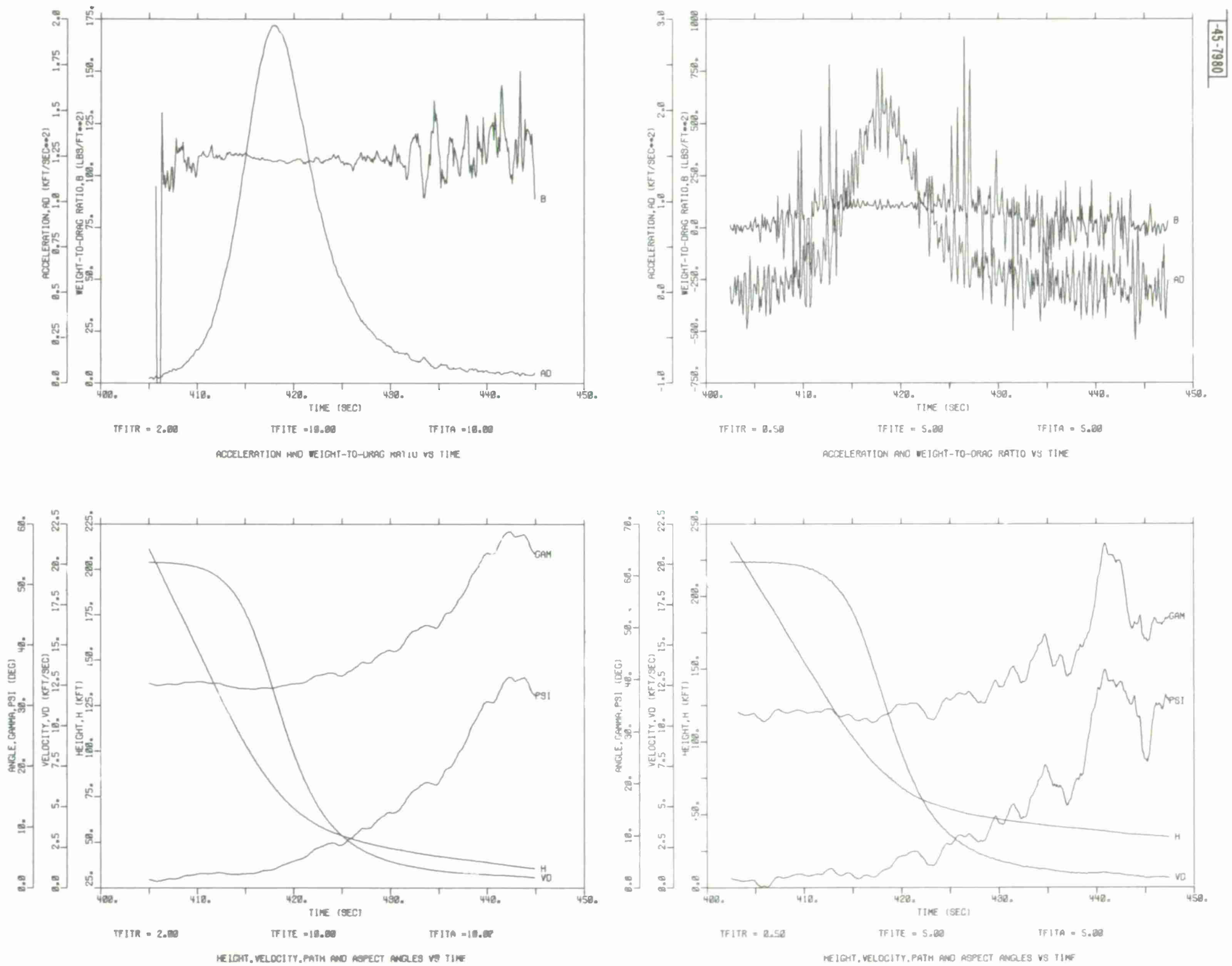

Fig. 12. Trajectory parameters vs. time for the S/N = 5 db case with different smoothing.

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13. ABSTRACT

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