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TABLE OF CONTENTS

ABSTRACT

(U) A general technique for computing the transient flow field in the nose region of a blunt body traveling at hypersonic speeds after being intercepted head-on by a shock wave is presented. The method is not limited to the assumption that the density, pressure and velocities are constant in the interaction region which has been the assumption in previous investigations of this problem. The technique can be applied to compute the flow in the entire interaction region, but is discussed here for computing the flow near the axis of symmetry.

INTRODUCTION

(U) An important problem in the field of missile defense is the determination of the damage experienced by a body upon colliding with a blast wave during re-entry. In particular it is necessary to determine the importance of the interaction of the blast wave and the bow shock of the re-entry body. It is the purpose of this report to present a technique for computing the flow field in the interaction region. Less detailed investigations of this problem have been made by Lobb and Wilson (1) as well as Bothell and Bond (2).

(U) In the present investigation it is assumed that radiation and non-equilibrium effects can be neglected and that inviscid flow equations along with the Rankine-Hugoniot shock conditions apply. The discussion however will be limited to the case where the re-entry body is intercepted head-on by the blast wave.

PROBLEM FORMULATION

(U) The equations describing the flow in the interaction zone after the blast wave collides with the bow shock and penetrates slightly (Fig. 1) are

(a) Eulers Equations (in spherical coordinates)

$$\rho \frac{Du_r}{Dt} = -\frac{\partial P}{\partial r} \tag{1}$$

$$\rho \frac{Du_\theta}{Dt} = -\frac{1}{r}\frac{\partial P}{\partial \theta} \tag{2}$$

(b) The Continuity Equation

$$\frac{D\rho}{Dt} = -\rho(\nabla \cdot \vec{V}) \tag{3}$$

(c) The Energy Equation (for equation of state $P = ZRT\rho$, with Z a constant)

$$\frac{D}{Dt}\left(\frac{P}{\rho^{\gamma}}\right) = 0 \tag{4}$$

(U) In these expressions P is the pressure, ρ the density, u_r the radial component of velocity, u_θ the angular component of velocity, γ the ratio of specific heats, r, θ and t the independent variables, D/Dt the substantial derivative and Z the compressibility factor.

(U) In order to completely determine the flow in the interaction region it is necessary to know, from independent sources, the flow conditions surrounding the shockwaves which bound the region. These conditions must then be related to the flow in the interaction region by means of the Rankine-Hugoniot relations which can be written across the boundary shock waves. These relations are

$$\rho_1 u_{N,1} = \rho_2 u_{N,2} \tag{5a}$$

$$u_{T,1} = u_{T,2} \tag{5b}$$

$$P_1 + \rho_1 u_{N,1}^2 = P_2 + \rho_2 u_{N,2}^2 \tag{5c}$$

$$h_1 + \frac{u_{N,1}^2}{2} = h_2 + \frac{u_{N,2}^2}{2} \tag{5d}$$

where

$$u_N = u_r \cos\alpha + u_\theta \sin\alpha + \frac{\frac{\partial f}{\partial t}}{|\nabla f|} \tag{6a}$$

$$u_T = -u_r \sin\alpha + u_\theta \cos\alpha + V_r \sin\alpha - V_\theta \cos\alpha \tag{6b}$$

in which V_r and V_θ are defined by

$$\frac{DR_S}{Dt} = V_r \quad \text{and} \quad \frac{\partial f}{\partial t} + V_r \frac{\partial f}{\partial r} + \frac{V_\theta}{R_s} \frac{\partial f}{\partial \theta} = 0$$

The angle α is defined by

$$\tan \alpha = \frac{1}{R_s} \left(\frac{\partial R_s}{\partial \theta} \right) \tag{7}$$

and f is the surface

$$f = r - R_s(\theta, t)$$

(U) In these expressions R_s is the shock wave surface, U_N the normal velocity, P the pressure and h the enthalpy of the gas. The subscript one refers to the fluid properties upstream of the shock and two to the downstream properties.

(U) Theoretically Eulers equations could now be solved utilizing the Rankine-Hugoniot conditions as boundary conditions to determine the flow in the interaction region. The drawback however is that the positions of the reflected and transmitted shocks are a priori unknown and hence the boundaries of the flow region are unknown. Mathematically the problem is one with three independent variables and two free boundaries.

PROBLEM SOLUTION METHOD

(U) If we restrict our attention to calculating the flow only along the axis of symmetry the problem becomes simplified. Even in this case however we are troubled with the two free boundaries whose positions must be determined as part of the flow field calculation. In this case one can apply the two-dimensional method of characteristics or the artificial viscosity technique of Von Neuman and Richtmeyer (3). It is more desirable however to have a calculation procedure which is less time consuming and complex in comparison to these two methods. We will now show how to develop such a method.

(U) The ideas to be presented utilize principles set forth by Dorodnitsyn (4) and used by Belotserkovski (5), Holt (6) and Traugott (7) to calculate the steady flow in the nose region of a blunt body which is traveling at hypersonic speeds.. It has not been realized before however that Dorodnitsyn's ideas can be applied fruitfully to unsteady flow problems involving shock waves whose positions are a priori unknown. In order to illustrate this we direct our attention to solving the equations which describe the flow on the axis of symmetry. It is easily shown by expanding the dependent variables in a series about y = 0 and Z = 0 that these equations are

$$\frac{\partial \rho u_r}{\partial t} + \frac{\partial}{\partial X}\left(P + \rho u_r^2\right) = 0 \tag{8}$$

$$\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial X} = 0 \tag{9}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_r}{\partial X} = 0 \tag{10}$$

$$\frac{\partial}{\partial t}\left(\rho \frac{P}{\rho^{\gamma}}\right) + \frac{\partial}{\partial x}\left(\rho u_r \frac{P}{\rho^{\gamma}}\right) = 0 \tag{11}$$

(U) The shock conditions reduce simply to the one dimensional Rankine-Hugoniot equations. It is obvious now that equation (9) is not needed to determine the variables P, ρ, and u_r along the axis of symmetry we therefore will not deal further with it at this time.

(U) In order to set up a numerical procedure for solving for the flow variables it must first be recognized that two distinct regions of flow exist in the interaction region. These two regions are separated by a contact discontinuity (Figure 1) across which the pressure and velocity (u_r) are continuous, but not the density and temperature. Therefore we must solve the flow equations in each region and connect these solutions by the continuity conditions at the contact discontinuity.

(U) Following now an analysis which parallels that of Holt and Hoffman (8) we define two new variables by the formula

$$Z_n = \frac{(-1)^n \left(X - X_o(t)\right)}{\epsilon_n(t)} \tag{12}$$

in which n refers to the flow region, i.e., one or two, and ϵ_n is the distance measured from the C.D. to the shock wave in the region of interest. Upon introducing these variables and integrating equations (8), (10), and (11) we obtain the equations

$$-\epsilon_n' m + \frac{\partial}{\partial t}\int_0^1 \epsilon_n m \, dz_n + (-1)^n (H_1 - H_0) - (-1)^n X_0' (m_1 - m_0) = 0 \tag{13}$$

$$-\epsilon_n' \rho_1 + \frac{\partial}{\partial t}\int_0^1 \epsilon_n \rho \, dz_n + (-1)^n (m_1 - m_0) - (-1)^n X_0' (\rho_1 - \rho_0) = 0 \tag{14}$$

$$-\varepsilon_n' \rho_1 S_1 + \frac{d}{dt}\int_0^1 \varepsilon_n \rho S \, dz_n + (-1)^n (m_1 S_1 - m_0 S_0) - (-1)^n X_0' (\rho_1 S_1 - \rho_0 S_0) = 0 \qquad (15)$$

in which

$$m = \rho u_r \, , \quad H = P + \rho u_r^2 \quad \text{and} \quad S = P/\rho^\gamma$$

The subscripts one and zero denote evaluation at the shock wave and C.D., respectively. These equations are a convenient system to employ to calculate the flow in the interaction region.

(U) These equations can be reduced to ordinary differential equations by approximating m , ρ and S in the integrals by the first two terms of a Taylors series in Z_n, i.e.,

let

$$\rho = \rho_0 + (\rho_1 - \rho_0) Z_n, \text{ etc.}$$

This is a very good approximation for our problem since the flow variables are not expected to vary more than ten percent from shock to C.D. in each region. Upon introducing this approximation and rearranging terms, equations (13) through (15) yield the simple system

$$\frac{d}{dt}\left(\varepsilon_n \frac{(\rho_0 + \rho_1)}{2}\right) = (-1)^n (V_s \rho_1 - m_1) \qquad (16)$$

$$\frac{d}{dt}\left(\varepsilon_n \frac{(m_0 + m_1)}{2}\right) = (-1)^n (V_s m_1 - u_{r,0} m_0 - H_1 + H_0) \qquad (17)$$

$$\frac{d}{dt}\left(\varepsilon_n \left(\frac{\rho_0}{3} + \frac{\rho_1}{6}\right) S_0 + \varepsilon_n \left(\frac{\rho_0}{6} + \frac{\rho_1}{3}\right) S_1\right) = (-1)^n (V_s \rho_1 S_1 - m_1 S_1) \qquad (18)$$

which *is* easy to integrate numerically when combined with the jump conditions for the shock, the known flow conditions surrounding the interaction region and lastly the equation

$$\frac{d\varepsilon_n}{dt} = (-1)^n (V_s - u_{r,o}) \tag{19}$$

In these equations V_S denotes the velocity of the shock for the region of interest. The procedure for solving these equations along with the shock conditions will now be described.

NUMERICAL SOLUTION PROCEDURE

(U) We describe first of all how to arrive at the initial conditions for equations (16), (17), and (18). First we observe that for very small times the flow properties in interaction regions I and II can be considered constant between the C.D. and shock wave for each region. Only the pressure and velocity of each region are equal however. From these observations we conclude that by solving the problem of two shock waves colliding and reflecting into regions of constant state, these states being those immediately behind the blast wave and the bow shock, respectively, we obtain the initial conditions for our problem. These are the constant states behind the two reflected shock waves. One will also recognize that this is the solution one gets for the bow-shock-blast-wave interaction problem if it is assumed that the flow behind the bow shock is invarient. This problem is discussed in texts such as Landau and Lipsitz (9) and will not be discussed here.

(U) Knowing the initial conditions one then proceeds with the integration of equations (16), (17), (18), and (19). This can be accomplished in the following manner. Rearrange the Rankine-Hugoniot equations so that one can iterate on the velocity of blast-wave as it propagates toward the body. Upon making an assumption of the blast wave velocity, based on its initial value, we determin its position and thereby the pressure, density and velocity of the flow field at its face numerically integrating the equation

$$\frac{dX_s}{dt} = BSHV \tag{20}$$

in which X_s is the distance of the blast wave from the nose and BSHV the blast wave velocity. We then calculate the density, pressure and velocity behind the blast wave by utilizing the shock conditions which have been put in the form

$$\rho_1 = \frac{\frac{\gamma_1}{\gamma_1 - 1}\left(P_2 + \rho_2 g\right) + \left[\left(P_2 + \rho_2 g\right)^2 \left(\frac{\gamma_1}{\gamma_1 - 1}\right)^2 - 4\left(\frac{\rho_2 g}{2} + \frac{\gamma_2}{\gamma_2 - 1} P_2\right)\left(\frac{\gamma_1}{\gamma_1 - 1} - .5\right)\rho_2 g\right]^{1/2}}{2\left(\frac{g}{2} + \frac{\gamma_2}{\gamma_2 - 1}\frac{P_2}{\rho_2}\right)} \tag{21}$$

$$u_{r,1} = V_s + \frac{\rho_2}{\rho_1} g^{1/2} \tag{22}$$

$$P_1 = P_2 + \left[\rho_2 g - \rho_1 \left(u_{r,1} - V_s\right)^2\right] \tag{23}$$

where

$$g = \left(u_{r,2} - V_s\right)^2$$

Knowing these values we numerically integrate equations (16), (17), (18), and (19) for region I by using a second catagory recurrence formula (see Crandell (10)) and iterate on $u_{r,0}$ (the velocity at the C.D.) to determine the flow field in region I.

(U) Next we solve for the flow region II by assuming a bow-shock velocity and applying equations (21), (22), and (23) to compute the flow inside the shock bounding region II. Then we numerically integrate equations (16), (17), and (19) for region II by making use of $u_{r,0}$ and P from the solution in region I, since they are continuous across the C.B., and iterate on the bow-shock velocity until we arrive at a flow field for region II.

(U) Lastly we check our initial assumption of the blast wave velocity by requiring that equation (18) for region II be satisfied. If it is not we go back and assume a new blast wave velocity and go through the outlined procedure once again. One can however reduce the iteration time in all of the steps by using a systematic method of iteration such as the method of double false position. The overall calculation method which has been described may at first glance appear too time consuming, but this is not generally true since large time steps can be taken and thereby reduce the overall computation time down to approximately a minute on the IBM 7090 for calculating the flow along the axis of symmetry in the initial interaction region. Subsequent interactions occur however after the reflection of the blast from the nose of the body. We will now discuss how to compute these as well as the reflection of the blast wave from the nose.

REFLECTION OF THE BLAST WAVE

(U) The density and pressure behind the reflected blast-wave and its shock velocity can be calculated from the equations (see Figure 2)

$$\frac{P_1}{P_0} = \frac{(3\gamma-1) - (\gamma-1)P_s/P_0}{(\gamma+1)P_s/P_0 + (\gamma-1)} \tag{24}$$

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma+1)P_1/P_0 + (\gamma-1)}{(\gamma-1)P_1/P_0 + (\gamma+1)} \tag{25}$$

$$V_s = \left(\frac{\gamma P_0}{\rho_0}\right)^{1/2}\left[\frac{(\gamma+1)P_1/P_0 + (\gamma-1)}{2\gamma}\right]^{1/2} - u_0 \tag{26}$$

(U) The subscript zero refers to the conditions in front of the reflected blast wave and the subscript one to the flow behind it. V_S in these equations denotes the reflected shock velocity and P_S the nose pressure before reflection. One should note here that the velocity behind the reflected blast-wave will be zero.

SECONDARY INTERACTIONS

(U) Upon being reflected from the nose of the body the blast wave collides with either the contact discontinuity, if present, or the inward moving bow shock. The C.D. will be present at all times except in the rare instance that the blast wave and the flow field behind it is a mirror image of the bow-shock and the flow between it and the body nose. In the latter case the reflected blast wave interacts with the bow shock with the result that a near stationary shock wave is produced along with a right running rarefaction wave. This rarefaction wave propagates to the nose of the body and is then reflected with the result that the pressure at the nose is greatly reduced. It also should be noted that any left running shock wave which interacts with the bow shock will produce a shock wave and a right running rarefaction wave. This phenomena occurs even if the C.D. is present since when the reflected blast wave interacts with the C.D. the result is either right and left running shock waves or a left running shock wave and a right running rarefaction wave. The criteria for choosing which of these results occur is set forth in Landau and Lipshitz (9) and will not be discussed here. The left running shock wave or rarefaction wave produced at the C.D. interaction will of course increase or decrease, respectively, the nose pressure.

(U) For calculating the secondary interactions it is reasonable to assume that constant state interaction results can be applied. The solution of C.D. reflected shock interaction then reduces to a straight forward problem. In the case where the interaction produces reflected and transmitted shocks one simply solves the Rankine-Hugoniot equations for the flow conditions behind each shock and requires that the pressure and velocity be equal throughout the region behind the shock waves.

(U) In the case where the C.D. reflected shock interaction produces a rarefaction wave the problem is not quite so easy, but still it is straight forward.

(U) By modifying the analysis of the flow in a shock tube appearing in Liepmann and Roshko (11), which is identical to the phenomena of a constant state C.D. shock wave interaction or a shock-shock interaction, we can develop an equation for the pressure in rarefaction wave. Following a procedure identical to that of the aforementioned authors, but noting that the velocity in front of the left running shock wave produced at the interaction is not zero, we arrive at the equation

$$\frac{P_4}{P_1} = \frac{P_2}{P_1}\left[1 + \frac{(\gamma_4 - 1)u_1}{2a_4} - \frac{(\gamma_4-1)(a_1/a_4)(P_2/P_1 - 1)}{\sqrt{2\gamma_1}\sqrt{2\gamma_1 + (\gamma_1+1)(P_2/P_1 - 1)}}\right]^{\frac{-2\gamma_4}{(\gamma_4-1)}} \tag{27}$$

where $a_n = \sqrt{\gamma_n P_n/\rho_n}$

The subscripts denote the flow variables in the regions shown in Figure (3). Also "a" denotes the speed of sound and γ_n the adiabatic gas constant.

(U) The pressure at the nose of the body after the reflection of the rarefaction wave can be calculated with the equation.

$$\left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{2\gamma}} = 2\left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{2\gamma}} - 1 \tag{28}$$

The subscripts in this equation once again refer to the regions shown in Figure (3).

(U) For the case where a reflected shock wave interacts with the bow shock equation (27) still holds and all that one has to do is interpret the subscripts properly.

(U) The interactions which have been discussed thus far are the only ones which can be approximated as constant state phenomena. If for instance one desires to compute what happens when the rarefaction wave produced by the bow shock interaction with the transmitted shock from the C. D. blast wave interaction interacts with the C. D. one must employ Eulers equations to solve for the flow field. This entails a tedious procedure and will not be discussed here.

CONCLUDING REMARKS

(U) A computer program for the IBM 709 which incorporates the ideas and formulae set forth in the previous discussion has been developed and is currently being used to compute the impulse produced on the nose of a re-entry body upon being intercepted by a blast wave. The results of these investigations will soon be issued in a report and will in part appear in the proceedings of the 1964 Army Science Conference (12).

(U) The technique described herein for computing the flow in the interaction region along the axis of symmetry can easily be extended to calculate the flow away from the axis of symmetry. It is obvious that the solution along the axis of symmetry will serve as the initial conditions for determining the flow off the axis of symmetry since the equations describing the flow possess only first order derivatives in the coordinate y. Also the general equations describing the reflection of a curved shock wave from a curved aurface must be derived before one can compute the pressure pulses experienced at points on a body which are away from the axis of symmetry.

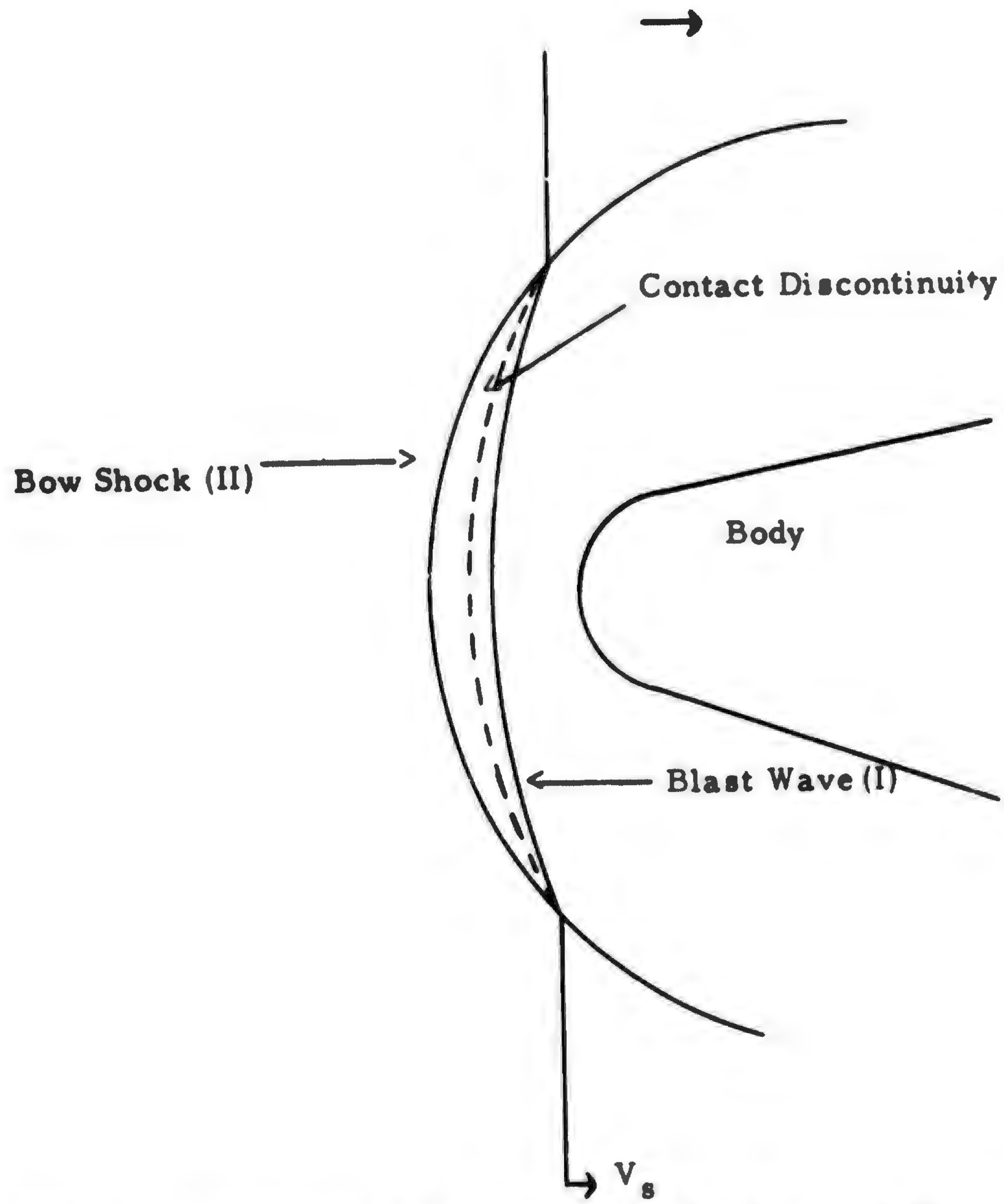

Fig. 1 Interaction of the Bow Shock of a Re-entry Body and a Blast Wave

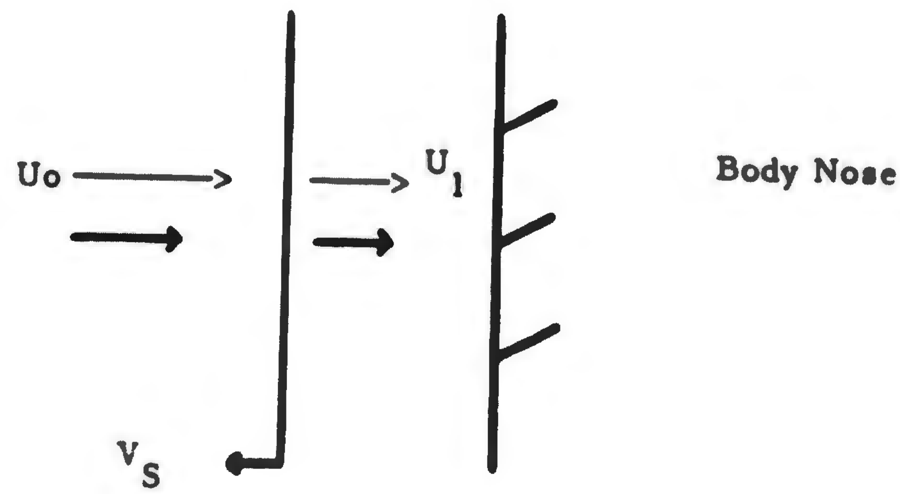

Fig. 2 Reflection of a Shock Wave from a Rigid Wall.

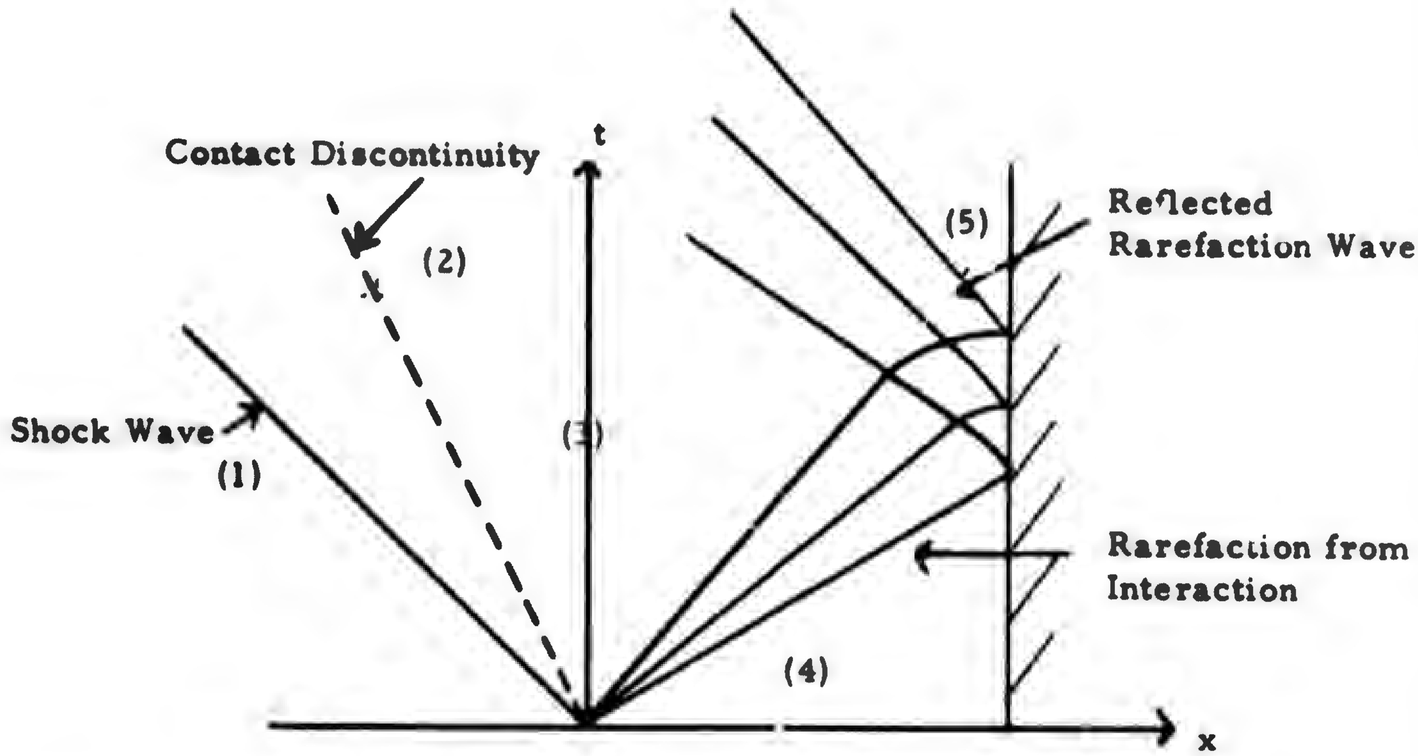

Fig. 3 Interaction of Contact Discontinuity and Reflected Blast Wave

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