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APPLICATION OF SIMILITUDE THEORY TO THE PROBLEM OF FUEL SLOSHING IN RIGID TANKS

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H. Norman Abrameon
R. J. Martin
Guido E. Ransleben, Jr.

Technical Report No. 1
Contract DA-23-072-ORD-1251
SwRI Project 43-768-2

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Department of Engineering Mechanics

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ABSTRACT

Similitude theory is applied to the problem of fuel sloshing in accelerated tanks to establish criteria for the design of model experiments. It is found that dynamic modeling is possible even if liquid viscosity is considered. The ranges of significant parameters and the selection of model liquids are discussed.

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1. INTRODUCTION

Sloshing motions of fuel in partially filled tanks may be established by essentially lateral accelerations of the vehicle. Such fuel motions are of importance for there is the possibility of extreme oscillations if the frequency of the excitation is in the neighborhood of one of the natural frequencies of the liquid fuel. The forces exerted by the fuel on the tank walls may then lead to perturbations in the flight trajectory, or may even impose severely high stresses on structural components.

Suppression of sloshing modes has been attempted by means of various mechanical devices, and, therefore, it is of some importance to evaluate the effectiveness of such devices. Purely mathematical approaches to this problem are exceedingly difficult because of the boundary conditions introduced by such suppression devices, and are, therefore, restricted essentially to very simple cases. On the other hand, it may be possible to evaluate suppression devices by means of suitably designed and conducted model tests.

The problem of concern in the present study is the specification of criteria for model tests which will give useful quantitative information concerning sloshing and slosh suppressors in full-scale fuel tanks. This is to be accomplished by the application of Similitude Theory to obtain appropriate expressions for the model - prototype relationships. These

may in turn be employed to yield a rational design of model tests. This is in contrast to previous experimental studies of the fuel sloshing problem (Refs. 1-4), which have apparently been conducted without regard for similitude requirements and consequently are believed to be of limited usefulness.

The purpose of the present report is to discuss the basic requirements for model studies of fuel sloshing by application of similitude theory.

II. SIMILITUDE RELATIONS FOR FUEL SLOSHING

A. General

We consider the vehicle in vertical flight with constant acceleration directed along the flight path (Fig. 1). Sloshing is assumed to be excited by time-dependent accelerations acting essentially normal to the flight path. For purposes of the present study, the exciting accelerations will be considered either as purely translational, or purely rotational about an axis normal to the flight path.

The essential features of the sloshing motion are further governed by the physical properties of the liquid and the pressurizing gas. The importance of the liquid density and total mass are obvious; however, viscosity and surface tension require additional comment. If suppression devices are incorporated it is clear that their effectiveness must, to a very substantial degree, depend upon the damping forces they can provide as the result of viscous action of the liquid. If models of very small size are employed, it is conceivable that the effects of sloshing may be substantially modified by surface tension forces acting on the suppression devices and at the tank walls. As will be discussed later, it does not seem possible to provide rigorous modeling if surface tension forces are included, and further it is believed that the total contribution of such forces would be small compared to the inertial and viscous forces. Consequently, surface tension forces will be omitted in the analysis.

Similarly, it is supposed that the forces produced by the pressurizing gas are negligible compared with the other forces, and will, therefore, be omitted. The gas pressure may be accounted for implicitly, however, as it is included in the total pressure at the tank wall by direct superposition since the gas volume remains essentially constant during sloshing.

Further restrictions to the analysis are provided by the assumption of small excitations, rigid tank, and the existence of geometrical similarity in all respects between model and prototype. The suppression devices themselves will be considered geometrically similar in all respects between model and prototype, and the corresponding inertial and apparent mass forces associated with them will be neglected in comparison with the viscous forces.

B. Similitude Relations - Translational Excitation

The relevant parameters, based on the preceding discussion, will be taken to be (Fig. 1).

- a = longitudinal acceleration acting on tank (LT^{-2})
- d = tank diameter (L)
- F = resultant liquid force on tank wall (MLT^{-2})
- h = depth of liquid in tank (L)
- X_0 = excitation amplitude (L)
- μ = liquid viscosity ($ML^{-1}T^{-1}$)
- ρ = liquid density (ML^{-3})
- τ = excitation period (T)

An equation relating these eight parameters analytically can be written in the general form (Ref. 5)

$$\phi'(\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}) = 0 \quad (1)$$

where n is the number of physical parameters involved and m is the number of fundamental dimensions (in this case $n-m = 8-3 = 5$). The π 's are dimensionless combinations of the parameters listed. The general expression

$$\pi = a^{\alpha} d^{\beta} F^{\gamma} h^{\delta} X_{\mu}^{\epsilon} \xi^{\eta} \rho^{\nu} \quad (2)$$

or

$$\pi = (LT^{-2})^{\alpha} (L)^{\beta} (MLT^{-2})^{\gamma} (L)^{\delta} (L)^{\epsilon} (ML^{-1}T^{-1})^{\eta} (ML^{-3})^{\nu} (T)^{\nu} \quad (3)$$

is first formed. In order for each π group to be dimensionless, the final exponent on each group must be zero. Considering the fundamental dimensions, we find

$$\left. \begin{aligned} \alpha + \beta + \gamma + \delta + \epsilon - \xi - 3\eta &= 0 \text{ (condition on L)} \\ -2\alpha - 2\gamma - \xi + \nu &= 0 \text{ (condition on T)} \\ \gamma + \xi + \eta &= 0 \text{ (condition on M)} \end{aligned} \right\} \quad (4)$$

From these three equations in eight unknowns, we are to obtain five dimensionless groups. For each of these groups five of the unknowns must be set arbitrarily. Since the resultant liquid force on the tank wall is the dependent variable of the problem, we may assign γ the value of unity in one solution and zero in all others, thus insuring that F occurs only once and only to the first power.

Setting $\gamma = 1, a = \delta = \epsilon = \xi = 0$, Eq. (4) yields $\beta = -4, \nu = 2, \eta = -1$

so that

$$\Pi_1 = \frac{F\tau^2}{\rho d^3} \quad (5)$$

and for $a = 1, \gamma = \delta = \epsilon = \xi = 0$, we find $\beta = -1, \nu = 2, \eta = 0$, so that

$$\Pi_2 = \frac{a\tau^2}{d} \quad (6)$$

Proceeding in a similar manner, the remaining dimensionless groups may be found to be

$$\Pi_3 = \frac{h}{d} \quad (7)$$

$$\Pi_4 = \frac{X_0}{d} \quad (8)$$

$$\Pi_5 = \frac{\rho d^2}{\mu\tau} \quad (9)$$

A general solution of the form of Eq.(1) may now be written in terms of the Π 's as

$$\frac{F}{(\rho d^3)(d/\tau^2)} = \phi \left[\frac{a}{d/\tau^2}, \frac{h}{d}, \frac{X_0}{d}, \frac{\rho d (d/\tau)}{\mu} \right] \quad (10)$$

We note from Π_3 and Π_4 that all linear dimensions are scaled in the same ratio as the diameter. Further, should other shape parameters be considered in this analysis, we note that the effect would be only to add corresponding shape ratios (with the diameter) in Eq.(10). Also, we note that Π_2 is equivalent to Froude's Number and that Π_5 is equivalent to Reynold's Number.

Should interest reside not in the resultant liquid force on the tank wall, but rather in the resultant liquid pressure, a similar analysis would yield

$$p = f\left(\frac{F}{d^2}\right) \quad (11)$$

so that

$$\Pi'_1 = \frac{p\tau^2}{\rho d^2} = \frac{p}{\rho (d/\tau)^2} \quad (12)$$

which is in the form of a pressure coefficient (Euler Number).

The preceding analysis has considered the excitation to be a displacement, in anticipation of actual model experiments. The excitation could, however, have been taken equally well as a force τ (or even an acceleration), in which case the appropriate dimensionless group would become

$$\Pi'_4 = \frac{P}{(\rho d^3) (d/\tau^2)} \quad (13)$$

Consideration of the surface tension in the foregoing analysis would have resulted in a new Π group of the form $\rho d^3 / \tau^2$, which corresponds to a Weber Number. As will be seen from later considerations, the simultaneous solution of this new group and Π_5 would be exceedingly difficult, in view of the model liquids readily available. Further, we assume that surface tension forces are small compared with the inertial

and viscous forces, and consequently we shall omit any additional consideration of surface tension.

C. Similitude Relations - Rotational Excitation

In this case the pertinent parameters remain the same as in the translational case except that the excitation is now defined by two parameters, the angular rotation θ_0 and the location of the rotational axis b (Fig. 2). Since we consider only small excitation amplitudes, it may be noted that $b\theta_0 \approx X_0$ and therefore the general functional Equation (10) applies to both kinds of excitation. The use of a moment $M = Pb$ as the excitation would result in a group formed by Ω_4' multiplied by the ratio b/d .

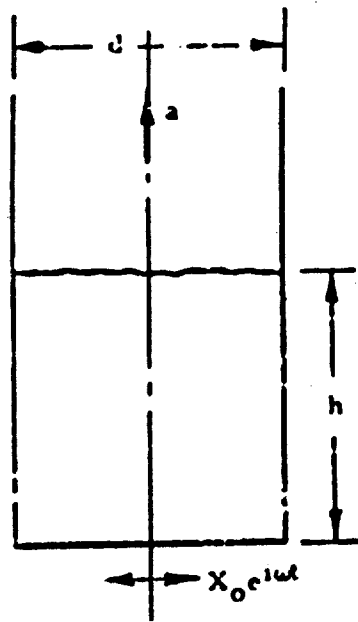


FIG. 1

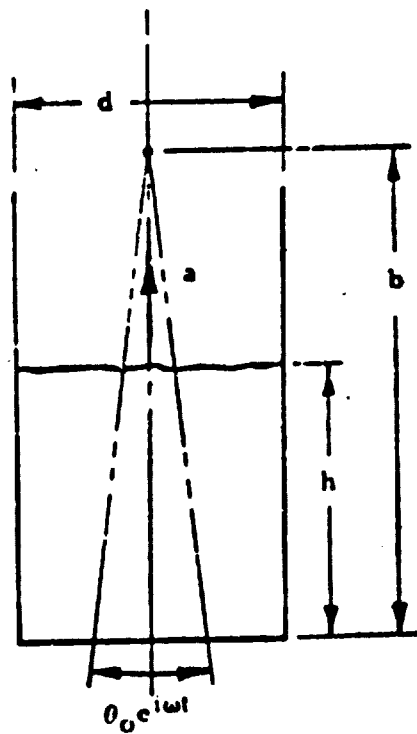


FIG. 2

III. MODELING CONSIDERATIONS BASED ON THE SIMILITUDE RELATIONS

A. General

Equation (10), and the associated alternate expressions for special cases, governs the design of models to simulate a given prototype. If all dimensionless groups in this equation have the same numerical value for both model and prototype, then the forces and pressures measured on the model are directly applicable to the prototype.

Denoting the ratio of model to prototype parameters by the subscript r , we note that fluid depth and excitation displacement amplitude are scaled in the same proportion as the geometrical scale. Thus

$$d_r = d_m/d_p \quad (14)$$

and,

$$h_r = X_{Or} = b_r = d_r, \theta_r = 1 \quad (15)$$

The remaining model parameters are to be established by further study of Eq. (10).

B. Modeling Considerations - Viscosity Neglected

It is instructive to consider the modeling parameters in the case that viscous effects are neglected throughout. This means that Π_5 drops out of the previous analysis, and the relation between model and prototype depends on the geometric factors discussed above and the time scale factor

$$\tau_r^2 = d_r/a_r \quad (16)$$

This result means that any size model in a $1g$ acceleration field can be used to simulate any size prototype in any acceleration field by varying the time scale, and would further permit the use of any model liquid since the density appears only in the force (or pressure) parameter.

In presenting measured data, the parameter ω^2/d may be used as the primary independent variable. Since frequency is a more commonly used variable, it may be introduced for the period to give $d\omega^2/a$. The force (or pressure) group may also be modified by introduction of Eq. (16) to give

$$\frac{F}{\rho d^3} \text{ or } \frac{p}{\rho d} \quad (17)$$

the measured data may then be presented with $F/\rho d^3$ and $d\omega^2/a$ as the primary variables and the remaining quantities as parameters.

C. Modeling Considerations - Viscosity Included

With viscous forces included, the data may still be presented as indicated above; however, the model design is now subject to certain restrictions arising from Π_5 , the Reynold's Number. Introducing Eq. (16), we find

$$\frac{\rho d^2}{\mu \tau} = \frac{d^{3/2} a^{1/2}}{\mu} \quad (18)$$

from which

$$d_r = \left(\frac{\mu_r}{\rho_r} \right)^{2/3} a_r^{-1/3} \quad (19)$$

This important result shows that the geometrical scale is determined by the acceleration, liquid density, and liquid viscosity ratios. If it is desired to test models so as to cover a range of prototype accelerations, we may either fix the model diameter and vary the liquid properties, or fix the model liquid and vary the model size. Since the range of liquid properties is very restricted, the latter course is chosen.

In order to employ models of reasonably small size to study large prototypes inspection of Eq. (19) indicates that the model liquids must have high density or low viscosity, or both, compared to the prototype liquid. Mercury, of course, has a very high density, but is undesirable for a variety of reasons; some of the organic solvents have very low viscosity, therefore offer some promise.

As an example of model simulation, assume the following typical case:

<u>Parameter</u>	<u>Prototype</u>	<u>Model</u>
Liquid	Kerosene	Methylene Chloride
Specific gravity	.83	1.336
Viscosity	2.5 c. p.	.465 c. p.
Acceleration	1-5g	1g
Diameter	9 ft	-

For a prototype acceleration of 1g,

$$d_r = (1)^{-1/3} \left(\frac{1}{8.65} \right)^{2/3} = \frac{1}{4.21}$$

$$d_m = d_p d_r = \frac{9}{4.21} = 2.14 \text{ ft}$$

For any other prototype acceleration, the model size is 2.14 divided by the cube root of the acceleration ratio. Thus

$$(d_m)_{5g} = \frac{(2.14)}{(5)^{1/3}} = 3.66 \text{ ft}$$

Therefore, a 9-ft diameter prototype with kerosene fuel and undergoing accelerations from 1-5g may be simulated by a series of models of 2.14 to 3.66-ft diameter with methylene chloride as the model liquid. The model frequency range is determined from Eq. (16) as

$$\omega_r = 1/\tau_r = (a_r/d_r)^{1/2} = 2.052 \text{ to } 0.701$$

It is clear from the foregoing that the inclusion of an additional group to account for surface tension would have rendered the model design virtually impossible because of the inability to find a model liquid of appropriate properties to satisfy all dimensionless groups simultaneously, in a small model.

It may be noted in passing that the experiments reported in Refs. 1-3 were restricted to small models with water as the model liquid. Based on the considerations of the present report, those results cannot adequately simulate sloshing with suppressors because of the failure to model viscous forces. While liquids other than water were also employed in the experiments of Ref. 4, the liquid properties were not properly fitted to the model sizes and hence the results cannot be used to evaluate suppression devices.

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