AMRL-TR-65-10

TRAJECTORY VERSUS LINE-OF-SIGHT SPACE RENDEZVOUS USING OUT-OF-WINDOW VISUAL CUES

HERBERT J. CLARK, PhD

FEBRUARY 1965

BEHAVIORAL SCIENCES LABORATORY
AEROSPACE MEDICAL RESEARCH LABORATORIES
AEROSPACE MEDICAL DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

NOTICES

Qualified requesters may obtain copies from the Defense Documentation Center (DDC), Cameron Station, Alexandria, Virginia 22314. Orders will be expedited if placed through the librarian or other person designated to request documents from DDC (formerly ASTIA).

Stock quantities available, for sale to the public, from:

Chief, Input Section
Clearinghouse for Federal Scientific and Technical Information, CFSTI
Sills Building
5285 Port Royal Road
Springfield, Virginia 22151

Change of Address

Organizations and individuals receiving reports via the Aerospace Medical Research Laboratories' automatic mailing lists should submit the addressograph plate stamp on the report envelope or refer to the code number when corresponding about change of address or cancellation.

Do not return this copy. Retain or destroy.

800 - May 1965 - 448-38-812

TRAJECTORY VERSUS LINE-OF-SIGHT SPACE RENDEZVOUS USING OUT-OF-WINDOW VISUAL CUES

HERBERT J. CLARK, PhD

FOREWORD

This study was conducted by Dr. Herbert J. Clark of the Presentation of Information Branch, Human Engineering Division, Behavioral Sciences Laboratory. The research was conducted under Project 7184, "Human Performance in Advanced Systems," Task 718401, "Criteria for the Design and Arrangement of Displays." The report covers research performed between June 1964 and September 1964.

The author expresses his appreciation to Mr. L. Milton Warshawsky through whose cooperation the support of the Analog Computation Facility was made available. Acknowledgment is also made of the contribution of Mr. James Crider and Mr. Jack Capehart who developed and implemented the analog simulation. Appendix II of this report was prepared through their cooperation.

This technical report has been reviewed and is approved.

WALTER F. GRETHER, PhD
Technical Director
Behavioral Sciences Laboratory

ABSTRACT

Seven trained subjects flew simulated short range coplanar orbital rendezvous maneuvers, using direct visual cues only. Two rendezvous techniques were compared: line-of-sight and trajectory. In the former, the subject could control up-down and fore-aft thrust only; in the latter, he could, in addition, control pitch. Using either technique, all subjects were able to maneuver successfully to a position 100 ft directly in front of the target at a terminal velocity of less than 5 ft/sec. Significantly, less fuel was expended in performing the trajectory maneuver. The principal man-machine performance factors in the line-of-sight maneuver were tentatively described as (1) the ability to conserve fuel used for longitudinal and vertical translation, (2) the ability to conserve mission time, and (3) the ability to proficiently close with the target. The principal factors for the trajectory maneuver were tentatively described as (1) the ability to conserve fuel for longitudinal translation, (2) the ability to conserve mission time, (3) the ability to effectively apply longitudinal thrusts and conserve fuel used for vertical translation, and (4) the ability to match the trajectory path of a minimum fuel two impulse maneuver. Computer diagrams fully describing the analog simulation are included in the report.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

LIST OF SYMBOLS

Symbol	Description	Units
F_{XI}, F_{ZI}	Forces directed along the X_I and Z_I axes which are the result of coordinate transformation performed on T_{XB} and T_{ZB}.	lb
I_{sp}	Specific impulse of fuel used	sec
I_{YB}	Moment of inertia of interceptor about its pitch axis.	slug-ft^2
K_1, K_2	Constants which determine target size and shape. ($K_1 = 5 K_2$)	
LOS	Line-of-sight.	
m	Total mass of interceptor including fuel.	slugs
M_{YB}	Moment about the interceptor Y_B axis, produced by pitch control thrusters.	ft-lb
R	Line-of-sight distance from target to interceptor.	ft
R_e	Radius of the earth. (R_e = 3963.5 miles)	statute miles
t	Time from beginning of rendezvous.	sec
T	Trajectory. Refers to trajectory mission as opposed to LOS or two-impulse mission.	
T_{MYB}	Thrust which produces the pitch moment, M_{YB}.	lb
T_{XB}, T_{ZB}	Thrusts along interceptor body axes X_B and Z_B, respectively.	lb
W_F	Weight of fuel at beginning of rendezvous.	lb

LIST OF SYMBOLS (Continued)

Symbol	Description	Units
X, Z	Relative distances from target to interceptor measured in the X_T, Z_T coordinate system.	ft
X_B, Z_B	Coordinate system located at the center of mass of the interceptor vehicle.	
X_I, Z_I	Coordinate system located at the center of mass of the interceptor vehicle. Its axes are always parallel to X_T, Z_T axes.	
X_{Scope}, Y_{Scope}	Voltages applied to oscilloscope x and y input terminals, respectively.	
X_T, Z_T	Coordinate system located at the center of mass of the target vehicle and used for the reference coordinate system in relative motion calculations. The negative Z_T axis always points toward the center of the earth.	
Y_W	Translation displacement of target image on TV screen.	in.
α	Angle which the line-of-sight makes with the X_T or X_I axis. (α=Arctan Z/X)	rad
θ	Pitch angle; the angle between X_B and X_I axes.	rad

LIST OF SYMBOLS (Continued)

Symbol	Description	Units
μ	Gravitational parameter. Product of the universal gravitation constant and the mass of the earth. (μ = 95,825 miles^3/sec^2)	statute miles^3/sec^2
ω	Angular velocity of the target about the earth. For 200 nautical miles, ω = 0.00114 rad/sec.	rad/sec
ω_1, ω_2	Frequency of sine waves used in target image generation. (ω_1 = 60 cps, ω_2 = 15,000 cps)	

NOTE: A dot above a variable indicates the first derivative with respect to time. Two dots indicate the second derivative with respect to time.

SECTION I

INTRODUCTION

Kasten (ref 1) examined whether a pilot could accomplish short simulated coplanar transfers between orbiting space vehicles when the pilot was provided with only a minimal out-of-the-window pictorial display and limited control equipment. His subjects used two types of simulated propulsion control systems. One, the pitch-thrust (P-T) system, provided pitch and fore-aft thrust control of the interceptor vehicle. The other, the orthogonal-thrust (O-T) system, provided fore-aft and upward-downward thrust control. Under both systems, applied thrust was directly proportional to stick displacement.

When using the O-T control system, subjects attempted a line-of-sight (LOS) transfer using retrothrust to achieve a safe impact velocity. Continuous orthogonal thrust applications were required to remain on the LOS. When using the P-T system, subjects attempted an ideal two-thrust transfer which theoretically permits a minimum-fuel maneuver (ref 2). A minimum-fuel maneuver can be achieved by accelerating at some angle to the original pilot-target LOS, coasting toward the target, and applying retrothrust to decelerate to a safe impact velocity. Kasten's subjects were unsuccessful in performing the two-thrust P-T maneuver, and reverted to making various course corrections along the mission trajectory.

Results from Kasten's study showed that the O-T control system was superior to the P-T system. Subjects achieved impact with the target on 100% of the O-T trials as opposed to 58% of the P-T trials. Mean O-T velocity at impact was 4.61 ft/sec. Mean

P-T velocity at impact was 7.86 ft/sec. The O-T system also resulted in less time expenditure, and generally lower fuel consumption. The subjects unanimously preferred the O-T system.

Mueller (ref 2, p 21) has noted that ". . . Kasten's subjects had so much difficulty trying to perform a two-impulse transfer using only visual cues that they ended up using more fuel than for a comparable line-of-sight rendezvous. This result indicates that the optimum technique would involve a combination of these two methods. The astronaut could estimate the correct aiming angle and coasting velocity (if properly trained) and embark on a two-impulse transfer . . . When it becomes apparent to the astronaut that he is not going to impact the target, he could begin a line-of-sight maneuver from that point, using orthogonal thrust to correct his path to a straight line and forward or retrothrust to control his closure velocity. Fuel required for such a technique should be more than that required for a two-impulse transfer but less than that required for a completely line-of-sight maneuver."

The purpose of the present experiment was to test Mueller's prediction that trained subjects could fly this combined two-impulse-LOS rendezvous (i.e., combined P-T, O-T) as successfully as an approximately equal time LOS (O-T) rendezvous, and with greater fuel economy. A successful rendezvous was defined as coming to a point 100 ft directly in front of the target at a terminal velocity of less that 5 ft/sec, without the use of cockpit instruments. For the remainder of this report, the combined two-impulse-LOS maneuver will be referred to as the trajectory (T) maneuver, as distinguished from either the two-impulse maneuver or the LOS maneuver.

Subjects were trained and tested using two propulsion control systems. One was Kasten's O-T system and the other a combined P-T and O-T system named the pitch-orthogonal-thrust (P-O-T) system. Under the O-T system, subjects flew a LOS rendezvous, and under the P-O-T system they flew a T rendezvous. Pilot studies indicated that subjects could not perform a two-thrust maneuver using only out-of-the-window cues.

The missions that subjects were trained to fly are described in the "Instructions to Subjects" contained in appendix I. Portions of the "General Instructions" were adapted from those used by McCoy and Frost (ref 3). The initial LOS and T rendezvous positions were 10,000 ft ahead of the target at the target orbital altitude. This position was chosen to minimize mission time, and thus computer error, while still permitting the full range of variation in target size allowed by the simulator. The rendezvous point for both missions was 100 ft directly in front of the target. The recommended T mission pitch angle was 13° to 14° up from the LOS. Fourteen degrees was selected as the maximum pitch angle to allow the subject to visually estimate his attitude by referring to the position of the target on the TV screen. With a 14° pitch angle, the target appeared at the bottom of the screen (the TV screen presented a vertical field of view of 28.65°).

In flying the 9900 ft LOS mission, the subject thrusted toward the target (-X-thrust) for 8 sec, maintained the LOS, applied retrothrust, and reached the rendezvous point at as near zero ft/sec as possible. Allowing for no departure from the LOS, theoretical minimum LOS mission time was approximately 223 sec[1]; 8 sec to reach the initial range rate (Ṙ) of 46.0 ft/sec and 207 sec to travel 9532 ft before applying 8-sec retrothrust to achieve a terminal velocity of zero ft/sec. The theoretical minimum fuel expenditure was 100 lb; 80-lb fore-aft thrust and 20-lb orthogonal thrust.

In flying the 9900-ft T mission, the subject pitched up between 13° and 14°, applied 8 sec - X-thrust, reestablished zero pitch, coasted toward the target, applied retrothrust, and reached the rendezvous point as near zero ft/sec as possible. Depending on the pitch angle and deceleration procedure adopted by the subject, it was planned that approximately the last 500 ft of the mission would be flown along the LOS. In view of this leeway, the minimum time and fuel required for the maneuver could not be specified. However, it was expected that the fuel and time expended would be greater than that required for a 14°, 226 sec, two-impulse maneuver, but less than that required for the specified LOS maneuver. Disregarding pitch fuel, the fuel which would have been required for a 14° two-impulse minimum fuel maneuver is 80 lb.

Since T fuel consumption was predicted to be less than the 100 lb required for the LOS mission, but more than the 80 lb required for a two-impulse mission, it was expected to be in the neighborhood of 90 lb. Thus, T fuel consumption was expected to be about

---

[1] Unless otherwise specified, time to rendezvous was calculated from onset of initial - X-thrust. Pitch time in more realistic longer rendezvous missions is so brief by comparison with total mission time that it is inconsequential.

10 lb less than LOS fuel consumption. Trajectory mission time was predicted to be slightly longer than LOS mission time, because the theoretical minimum time required for a 14^{o} two-impulse maneuver is 226 sec, while that required for the specified LOS mission was only 223 sec.

A faster or slower LOS range rate ($\dot{R}$) could have been selected. This would have altered the experimental outcome with regard to relative fuel consumptions. On the other hand, a rendezvous of greater distance could have been selected, maintaining nearly equal LOS and T mission times. The ratio of the theoretical LOS fuel to the theoretical T fuel then would have become greater. The conditions of the present experiment led to the prediction that the T mission would be flown as successfully as the LOS mission and with greater fuel economy. Trajectory mission time was expected to be slightly longer than LOS mission time.

A major difference between this study and Kasten's study was in the display presented. Kasten's subjects viewed a grided oscilloscope which presented only a point source target. Subjects in this experiment viewed a TV monitor that presented a toroid target which interacted realistically with a star background and earth horizon. While Kasten's target varied only in size, the target in this experiment varied in size, shape and brightness.

SECTION II

METHOD

Subjects

Two male Air Force pilots and five male nonrated personnel, stationed at Wright-Patterson Air Force Base, served as experimental subjects.

Apparatus

The overall system concept of the fixed-base orbital rendezvous simulator is diagrammed in figure 1. The subject, seated in the cockpit, observed a display on a TV monitor through the cockpit window. An electromechanical planetarium produced a view of a star field and of the earth horizon. An electronic window generated a Lissajous figure representing a doughnut-shaped space station, and displayed it on an oscilloscope. Television cameras picked up the views from the planetarium and from the oscilloscope. The two pictures were then combined and presented on the subject's 21-in. TV monitor. The subject evaluated the information on display, made decisions as to his desired altitude, velocity, and direction of travel, and translated them into commands by moving the cockpit control sticks. The control inputs were transmitted to an analog computer, which, in turn, supplied the proper signals to various servos that drove the display. The stars and earth horizon moved realistically as the attitude of the vehicle changed.

The rendezvous target was represented by the toroidal shaped figure shown in figure 2. Its apparent size was an inverse function of range and its ellipticity was a function of the vertical displacement of the interceptor relative to the target. The target

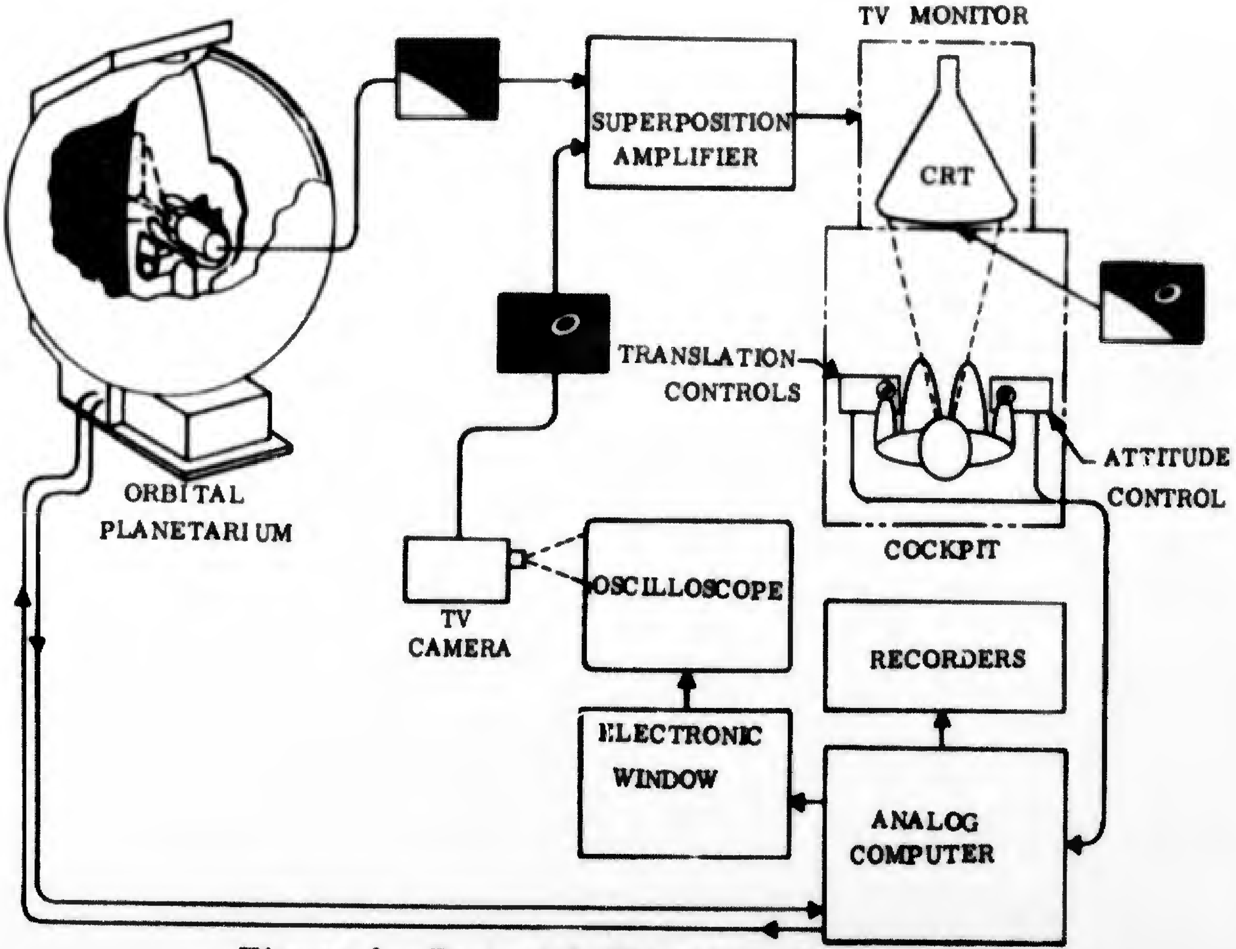


Figure 1. Pictorial Diagram of the Simulator

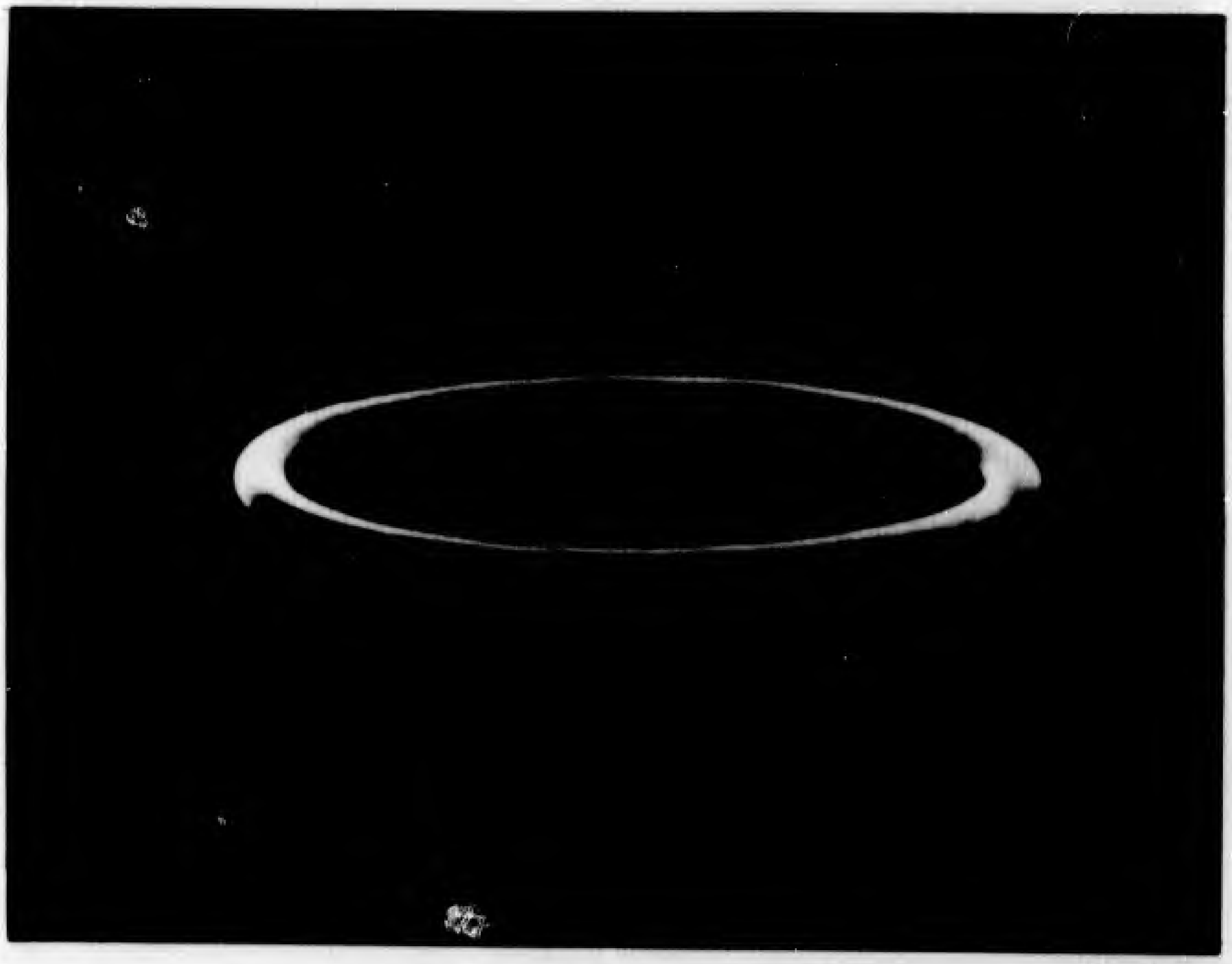


Figure 2. The Rendezvous Target Viewed from above the LOS at a Range of 200 Feet.

image moved up and down on the TV screen in accordance with variations in either interceptor altitude or pitch angle. "Sunlight" came from directly above the target. The computer was programmed to produce target shading, which varied realistically with changes in interceptor altitude, and apparent target brightness, which varied inversely with the range squared.

The target and interceptor described coplanar 200-nautical-mile west-to-east equatorial circular earth orbits. Because the star sphere was rotated by a motor and drive system that caused one revolution in 90 min, the orbital period was necessarily 90 min, which is approximately 1.9 min short of the proper period for a 200-nautical-mile orbit. The stars seen during the rendezvous were representative of those which would be seen in an equatorial orbit, however, only stars up to the fourth magnitude were represented. In all, 463 stars were plotted on the sphere of the orbital planetarium (fig. 3). The names and locations of these stars are listed in ref 4. The planetarium TV camera presented a horizontal view of the star background of 43.80° and a vertical view of 28.65°. As many as 27 stars per scene could be observed on the monitor. The monitor was placed 19 in. behind the cockpit window, and the subject sat with his eyes positioned approximately 5 in. from the window. The window was trapezoidal, having a 7-3/8 in. base, a 6-1/4 in. top, and 4-1/8 in. sides. The entire TV screen could be seen through the window.

The interceptor vehicle was presumed to be cylindrical, 12 ft long and 6 ft in diameter. There was no roll, yaw, or lateral (left-right) thrust control available to the subject, and the vehicle

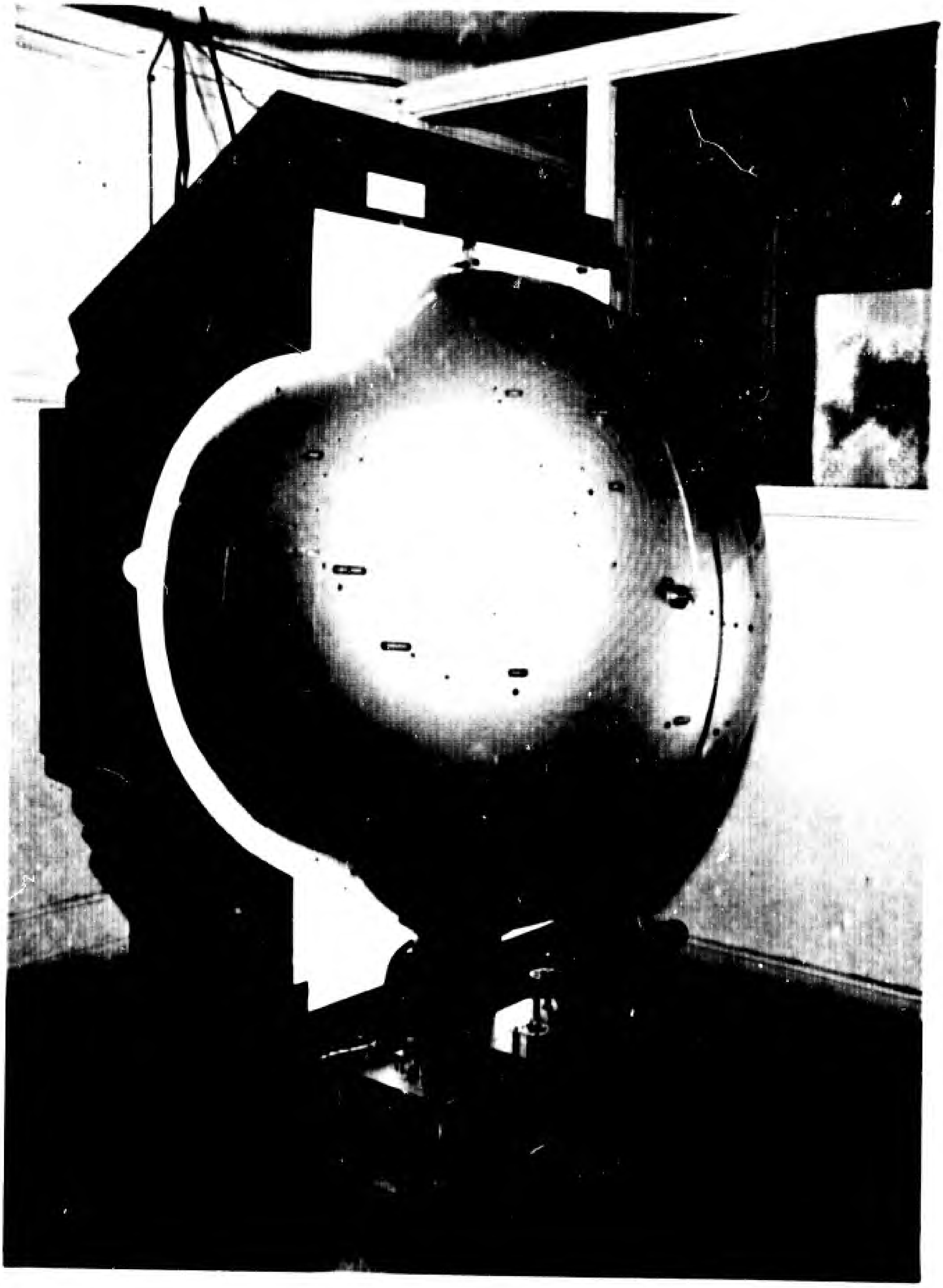


Figure 3. Orbital Planetarium

was pitch stabilized. The parameters used in its simulation are listed in table I. The coordinate system within which the simulation was analyzed is shown in figure 4.

TABLE I

INTERCEPTOR PARAMETERS

Interceptor Length = 12 ft

Interceptor Diameter = 6 ft

m = 261 slugs

I_{sp} = 300 sec (each thruster)

M_F = 200 lb at beginning of rendezvous*

M_{YB} = 240 ft-lb (2 thrusters, 20 lb each, at 6 ft fore and aft of center of mass)

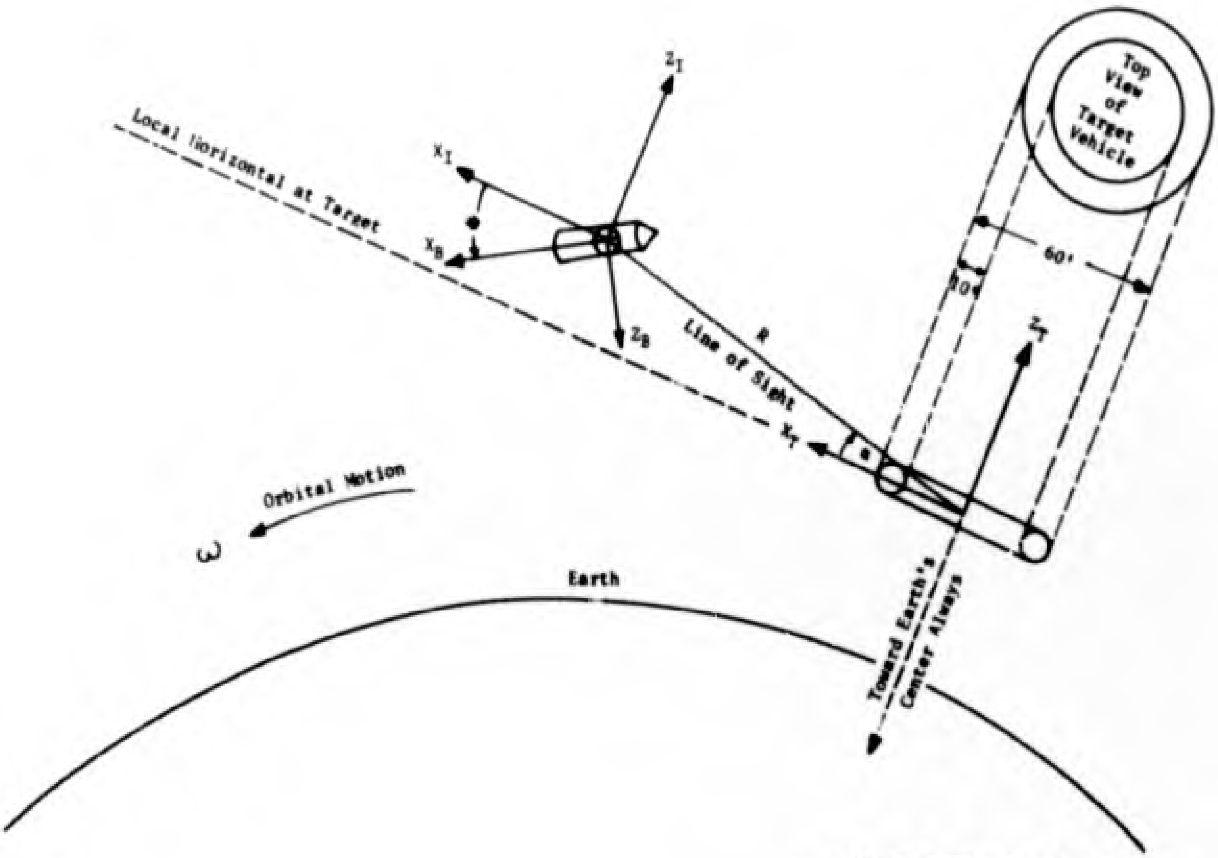
I_{YB} = 5000 slug-ft^2

T_{XB} = 1500 lb**

T_{ZB} = 300 lb

*Decrease in interceptor mass, as a function of fuel consumption during rendezvous, was not accounted for in the simulation. Fuel consumption was instantaneous (rise and decay times were zero).

**All interceptor body thrusts were perfectly aligned with the interceptor center of gravity.



NOTE: North pole of earth is out of plane of paper.

Figure 4. Simulation Coordinate Systems

Figure 5 shows a subject seated in the simulator cockpit. The cockpit area was 67 in. high, 52 in. wide, and 42 in. deep. It was dimly illuminated by two 7-watt white lights, each covered by a yellow filter. A cockpit curtain kept out room light, and a canvas cover placed over the monitor and attached to the cockpit kept the TV screen in darkness.

The pitch control stick was located at the subject's right hand. It could be moved fore and aft for down and up pitch, respectively.

Figure 5. Subject Seated in Cockpit

Two translation control sticks were located at the subject's left hand. One was located on a vertical panel and could be moved up or down to give corresponding vertical thrust. The other, which was similar to the attitude control, could be moved fore and aft for corresponding longitudinal thrust. Both the attitude and translation controls were of the finger-tip type, which actuate microswitches to give on-off impulses.

Four Simpson galvanometers, located between the cockpit controls, were used as interceptor navigation instruments during a training period. They served to quickly acquaint subjects with the dynamics of the TV scene. The top left meter registered interceptor velocity with respect to the target. The top right meter registered interceptor pitch angle, and the right and left bottom meters registered, respectively, interceptor altitude and range with respect to the target. Velocity was calibrated in ft/sec, pitch in degrees, altitude in feet above or below the target, and range in feet from the target. A small white light on the front of the instrument panel automatically lighted when the interceptor range was 1000 ft, and went out when the rendezvous was completed. During the experimental test period, the entire instrument panel was removed from the cockpit.

Three consoles of Electronic Associates 16-31R equipment were programed to simulate the orbital mechanics of rendezvous, to record subjects' control inputs and to plot the course flown. Mission duration, fuel consumption, R, $\dot{R}$, θ, and interceptor altitude with respect to the target were also recorded. The computer equipment consisted of 83 amplifiers, 73 potentiometers, 6 servomultipliers, 11 diode multipliers, 18 function relays, 17 function switches, 2 six-channel Brush recorders, and 1 Mosely X-Y recorder. A block diagram of the analog simulation is shown in figure 6. The equations used in the simulation are listed in table II, and the analog computer circuit diagrams are shown in appendix II.

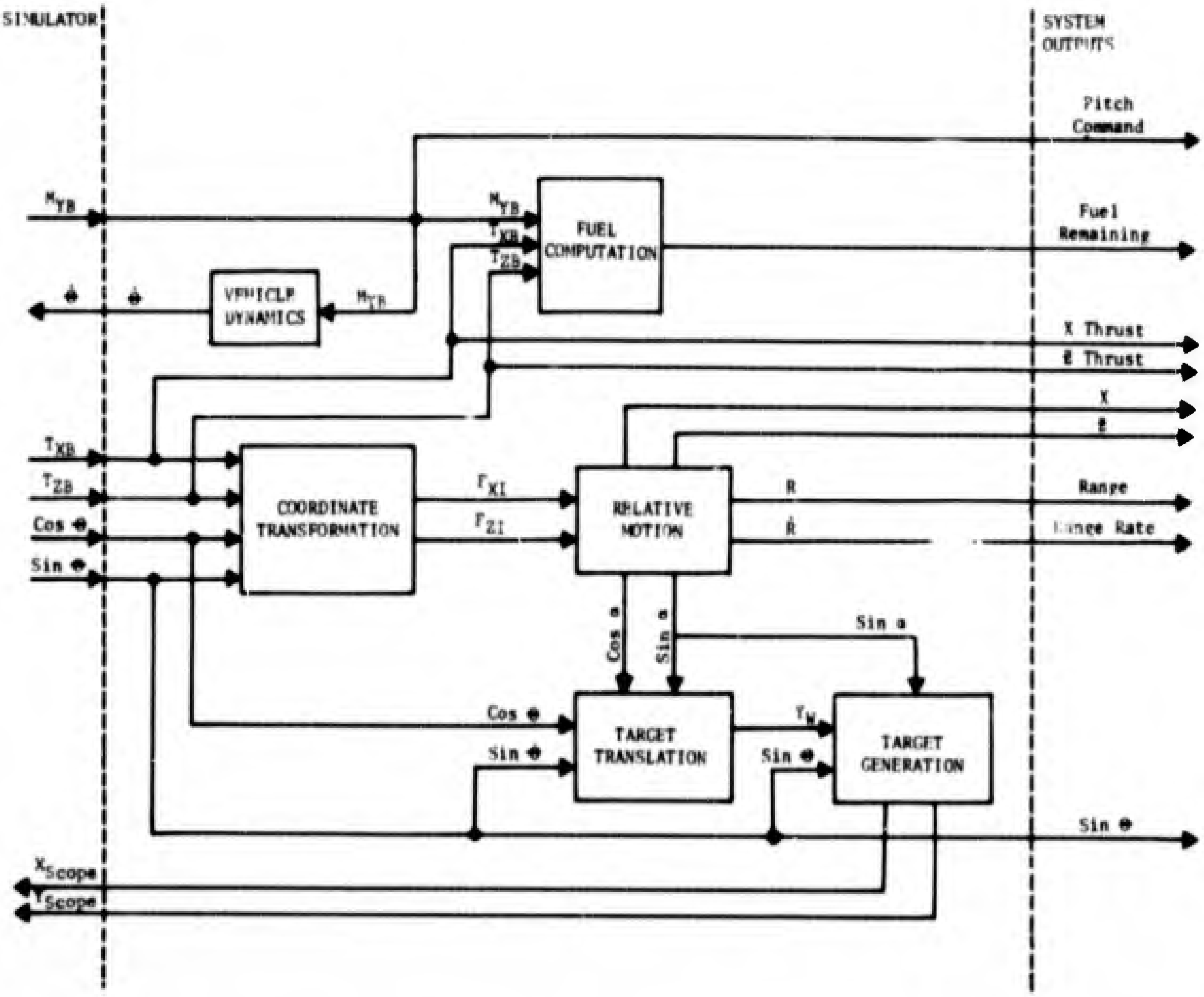


Figure 6. Block Diagram of the Computer Simulation

TABLE II

EQUATIONS USED IN THE SIMULATION

$\ddot{\theta} = \frac{M_{YB}}{I_{YB}}$	VEHICLE DYNAMICS
$F_{XI} = T_{XB} \operatorname{Cos} \theta + T_{ZB} \operatorname{Sin} \theta$	COORDINATE TRANSFORMATION
$F_{ZI} = T_{XB} \operatorname{Sin} \theta - T_{ZB} \operatorname{Cos} \theta$	
$W_F = W_F(0) - \frac{1}{I_{sp}} \int_o^t (T_{XB} + T_{YB} + T_{MYB})dt$	FUEL COMPUTATION
$\ddot{X} = \frac{F_{XI}}{m} - 2\omega\dot{Z}$	RELATIVE MOTION
$\ddot{Z} = \frac{F_{ZI}}{m} + 2\omega\dot{X} + 3\omega^2 Z$	
$R = \frac{X^2}{R} + \frac{Z^2}{R}$	
$\dot{R} = \dot{X} \operatorname{Cos}\alpha + \dot{Z} \operatorname{Sin}\alpha$	
$\operatorname{Sin}\alpha = \frac{Z}{R}$	
$\operatorname{Cos}\alpha = \frac{X}{R}$	
$y_W = d(\operatorname{Sin}\theta \operatorname{Cos}\alpha - \operatorname{Cos}\theta \operatorname{Sin}\alpha)$	TARGET TRANSLATION
$X_{Scope} = \frac{K_1}{R} \operatorname{Sin} \omega_1 t + \frac{K_2}{R} \operatorname{Cos} \omega_2 t$	TARGET GENERATION INCLUDING TRANSLATION
$Y_{Scope} = y_W + \frac{K_1}{R} \operatorname{Sin}\alpha \operatorname{Cos} \omega_1 t + \frac{K_2}{R} \operatorname{Sin} \omega_2 t$	

Quality of the Simulation

At the simulated distance of 10,000 ft, the TV target image should have been 0.14 in. wide. The actual size was 0.24 in. Table III shows the relationship between the theoretical and actual image widths.

The correlation (agreement between actual and theoretical values) becomes higher at ranges less than 5000 ft. Actual image size, as designated in table III, was the same for all subjects.

Although the computer was programed so that apparent target brightness would vary inversely with the range squared, and target shading would vary realistically with changes in interceptor altitude, only changes in brightness were apparent on the TV screen. Other aspects of the simulation, such as accuracy of fuel consumption and dynamic effects of control movements, were checked daily. No measured value varied more that 1% from its expected theoretical value.

Often, the fuel used for achieving rendezvous is interpreted as a ΔV, where ΔV is the velocity increment that would have been produced in a gravity-free field if the fuel had been consumed by one of the engines (ref 5). This ΔV is given as $\Delta V = g_o I_{sp} \log_e \frac{m_i}{m_f}$,

where I_{sp} = specific impulse, m_i= initial mass of the vehicle, and m_f= final mass of the vehicle. The symbol g_o = 32.2 ft/sec^2.

TABLE III

THEORETICAL VERSUS ACTUAL TARGET IMAGE WIDTH

Distance	Theoretical	Actual
10000 ft.	0.14 in	0.24 in
9000	0.16	0.24
8000	0.18	0.28
7000	0.21	0.28
6000	0.24	0.32
5000	0.29	0.35
4000	0.36	0.39
3000	0.48	0.51
2000	0.72	0.67
1000	1.44	1.38
900	1.60	1.61
800	1.80	1.81
700	2.06	2.13
600	2.40	2.44
500	2.88	2.91
400	3.60	3.62
300	4.80	4.80
200	7.20	7.09
100	14.40	14.53

Since in this simulation, the mass term, m, in the equations of motion was treated as a constant, it is not appropriate to use the above equation for converting fuel consumed to ΔV. The following derivation provides a measure, $\Delta V'$, which specifies the velocity increment for an interceptor of unchanging mass.

$$\begin{aligned}\Delta V' &= at \\ &= \frac{T}{m}\,\frac{W_f}{\dot{W}_f} \\ &= \frac{T}{m}\,\frac{W_f}{T/I_{sp}} \\ &= \frac{W_f\ I_{sp}}{m}\end{aligned}$$

where

a = acceleration in ft/sec^2

t = time in sec

T = thrust in lb

W_f= weight of fuel in lb

Since in this simulation m = 261 slugs and I_{sp} = 300 sec,

$$\Delta V' = \frac{300\ \text{sec}}{261\ \text{slugs}}\ (W_f) = \frac{W_f}{0.87}\ \text{lb/sec/ft}$$

The error involved in assuming no mass change during rendezvous is reflected in the difference between ΔV and $\Delta V'$. Using 100 lb as W_f, $\Delta V'$ = 114.9 ft/sec and ΔV = 115.6 ft/sec. The difference, 0.7 ft/sec, reflects a simulation fuel error of 0.6%.

Although the subject was seated at a distance from the TV monitor so that the retinal image projection corresponded in size to that which would be normal if the subject were actually viewing a 60-ft-wide target at 10,000 ft, no presumption was made that the TV view corresponded to a precise out-of-the-window view of the rendezvous scene. Realistic image size, perspective, brightness, contrast, color, detail resolution, and parallax were all compromised or absent to varying degrees. Therefore, the experimental results obtained from the simulation can be most accurately generalized to the visual scene perceived by an astronaut viewing an on-board TV monitor depicting the rendezvous scene. Results are also relevant to an astronaut's ability to control a remotely maneuvered TV unit from a parent satellite.

Procedure

All subjects familiarized themselves with the conditions of the experiment by reading the "Instructions to Subjects" (appendix I). Individual training periods followed during which the specified LOS and T rendezvous maneuvers were practiced to a criterion of five consecutive successful maneuvers of each type. Following the training period, instruments were removed from the cockpit, and the subject flew alternating LOS and T missions to a criterion of 10 consecutive successful missions. These missions were analyzed as the test data.

Before each training or test trial, a cockpit buzzer sounded for 0.5 sec. Immediately after the buzzer, a small red light located near the attitude control came on, indicating the computer was in the "operate" mode and the subject should begin the rendezvous. When the rendezvous was completed, the computer went into "hold," and the red light went out. Time between each rendezvous mission was 90 sec, with a 5-min rest period following each fifth mission.

SECTION III

RESULTS

Table IV is a resume of the 70 test missions. Each recorded value is a mean based on five LOS trials or five T trials per subject. All test missions were successful. Terminal velocities were less than 5 ft/sec, and vertical displacement from the LOS at rendezvous was minimal. The absolute mean vertical displacement (i.e., absolute error without regard to direction) for the LOS mission was 1.8 ft and for the T mission, 2.3 ft.

Figure 7 shows that, on the average, all subjects flew their five T missions with greater fuel economy than their five LOS missions. Comparison theoretical values in figure 7 are based on a terminal $\dot{R}$ of zero ft/sec. Subject 6 had an actual fuel consumption less than the theoretical, but his average terminal velocity was comparatively high.

The mean difference between LOS and T total fuel consumption was 11.57 lb. The probability is .95 that the difference between the population means lies between 9.45 and 13.69 lb. Table V is the summary of a subjects'-by-treatments analysis of variance on actual fuel used. The significant F for conditions indicates that total T fuel consumption was significantly less than total LOS fuel consumption. The significant F for subjects reflects individual differences in total fuel consumption. The nonsignificant interaction indicates that the absolute difference between LOS and T fuel consumption was approximately the same for all subjects.

Individual differences in total fuel consumption are reflected in

TABLE IV
RESUME OF THE TEST DATA

Ss	Mission	Mission Time from -X Thrust (sec)	Time to -X Thrust (sec)	Initial $\dot{R}$ (ft/sec)	Terminal $\dot{R}$ (ft/sec)	Initial θ (deg)	Total θ Thrusts	Total ± Z Thrusts	Z Fuel (lb)	Total ± X Thrusts	X Fuel (lb)	Total θ, X, Z Fuel (lb)	$\Delta V'$ (ft/sec)	R at 1st Retro-thrust (ft)	Duration of 1st Retro-thrust (sec)	Vertical Displacement from LOS (ft)
1	LOS	251.02	1.24	46.9	1.1	0	0	18.2	22.6	9.2	81.6	105	125	550	3.4	1.6
	T	245.54	15.36	45.4	2.0	13.77	5	9.2	12.4	7.4	77.2	90	103	585	3.4	5.8
2	LOS	295.54	1.80	46.7	0.4	0	0	22.4	24.0	9.0	81.0	105	121	548	6.9	2.3
	T	306.96	20.82	45.6	0.4	13.75	13.4	11.8	11.2	7.6	84.0	96	110	645	6.8	2.4
3	LOS	298.48	0.72	45.8	0.5	0	0	28.8	25.0	7.8	79.2	105	121	670	3.8	0.8
	T	301.18	17.62	44.8	0.5	12.80	8.6	18.4	13.6	6.4	78.0	92	106	664	3.3	1.0
4	LOS	265.66	2.74	46.0	2.0	0	0	30.4	23.6	4.4	79.8	103	118	604	5.0	3.1
	T	274.56	24.42	44.9	2.0	13.07	9.6	14.4	13.4	6.4	80.0	94	108	653	4.7	2.7
5	LOS	312.08	9.38	46.1	0.7	0	0	35.8	23.8	8.8	81.2	105	121	1030	5.2	0.7
	T	314.28	22.66	44.6	1.4	13.13	8.6	18.0	12.0	8.6	79.6	92	106	884	5.3	0.7
6	LOS	260.34	1.00	46.2	2.1	0	0	20.6	21.0	8.4	77.2	98	113	728	1.3	2.4
	T	275.60	22.26	45.5	1.8	13.65	10.2	7.8	9.8	9.0	75.6	86	99	1245	1.7	1.5
7	LOS	260.30	0.20	46.3	1.2	0	0	25.4	22.6	8.0	80.0	103	118	573	3.8	1.4
	T	264.10	14.80	45.3	0.9	12.68	6.4	15.6	11.2	8.4	81.4	93	107	541	3.7	1.9

TABLE V

ANALYSIS OF VARIANCE ON FUEL USED

LOS (vs) T	1	2285.71	2285.71	192.40***
Subjects	6	428.14	71.36	4.35*
LOS (vs) T x Subjects	6	71.29	11.88	
Error	56	917.86	16.39	
Total	69	3703.00		

*** P < .001

* P < .05

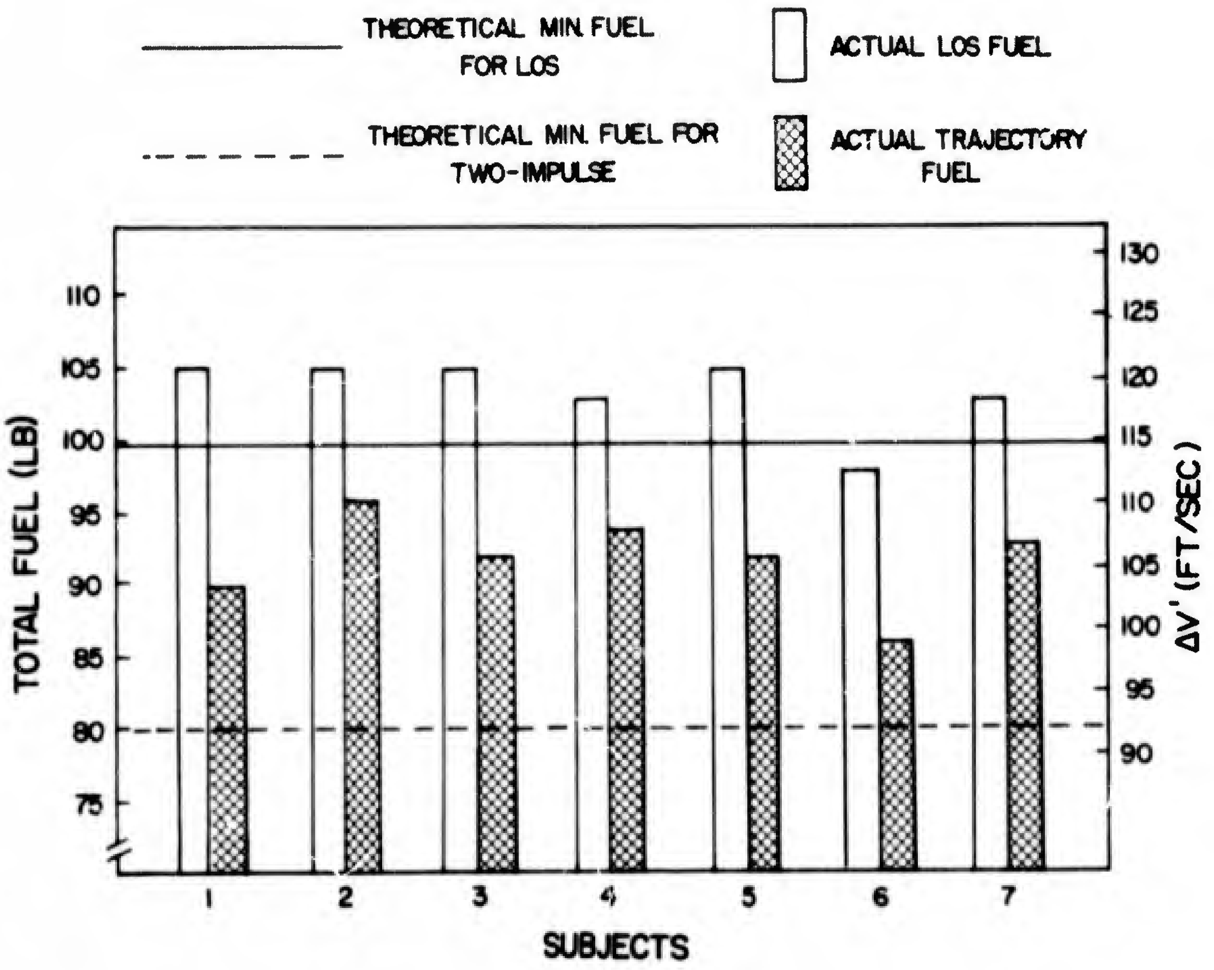


Figure 7. Average Fuel and $\Delta V'$ Required

the significant F for subjects. The nonsignificant interaction indicates that the absolute difference between LOS and T fuel consumption was approximately the same for all subjects.

Table VI contains the overall means and standard deviations for each of the performance measures listed in table IV except Time to - X Thrust, Total θ Thrusts, and $\Delta V'$. Overall means and standard deviations were based on the seven LOS or T means contained in each column of table IV. There was only a 5. 6 sec difference between LOS and T mission time, and initial velocities were approximately equal to those recommended. The specified LOS intital $\dot{R}$ was 46. 0 ft/sec. Recommended T initial $\dot{R}$ was 44. 6 ft/sec. Average terminal $\dot{R}$ for both missions was well within the 5 ft/sec criterion, and the T Z-fuel requirement was approximately half that required for the LOS mission. Respective X-fuel consumptions were nearly equal.

Using as data the seven means obtained on each LOS performance measure (see table IV), 11 of the LOS performance measures were intercorrelated. A principal factor analysis was performed on the results, using the BIMD 17 digital computer program (ref 6). The number of factors rotated was equal to the number of eigenvalues of the correlation matrix (with ones in the diagonal) which were greater than one (ref 7). Table VII is the rotated LOS factor matrix. Table VIII is a corresponding factor matrix for the T mission. With only seven independent observations on each performance measure, results of the factor analysis are only suggestive of the human abilities associated with successful rendezvous performance. A more reliable determination of the principal factors required for successful rendezvous must await the collection of more data. The following

TABLE VI

PERFORMANCE MEASURES AVERAGED OVER SUBJECTS (N=7)

	LOS		T	
	Mean	Standard Deviation	Mean	Standard Deviation
Mission Time from - X Thrust (sec)	277.6	23.8	283.2	25.1
Initial $\dot{R}$ (ft/sec)	46.3	0.4	45.2	0.4
Terminal $\dot{R}$ (ft/sec)	1.1	0.7	1.3	0.7
Initial θ (deg)	0	0	13.3	0.5
Total ± Z Thrusts	25.9	6.1	13.6	4.1
Z Fuel (lb)	23.2	1.3	11.9	1.3
Total ± X Thrusts	7.9	1.6	7.7	1.0
X Fuel (lb)	80.0	1.5	79.4	2.8
Total θ, X, Z Fuel (lb)	103.4	2.6	91.9	3.2
R at 1st Retrothrust (ft)	671.9	171.3	745.3	245.5
Duration of 1st Retrothrust (sec)	4.2	1.7	4.1	1.6
Vertical Displacement from LOS (ft)	1.8	0.9	2.3	1.7

TABLE VII

ROTATED LOS FACTOR MATRIX

	A	B	C
Mission Time from - X Thrust	.52	.67	.36
Initial $\dot{R}$	.20	-.90	.25
Terminal $\dot{R}$	-.75	-.08	-.58
Total ± Z-Thrusts	.31	.90	-.09
Z Fuel	.83	.44	-.03
Total ± X-Thrusts	.01	-.33	.92
X-Fuel	.82	-.25	.22
Total X, Z, θ Fuel	.94	.03	.21
R at 1st Retrothrust	-.12	.75	.44
Duration of 1st Retrothrust	.91	.07	-.15
Vertical Displacement from LOS	-.21	-.37	-.80

TABLE VIII

ROTATED TRAJECTORY FACTOR MATRIX

	A	B	C
Mission Time from - X Thrust	.39	-.84	.07
Initial $\dot{R}$	.19	.69	.64
Terminal $\dot{R}$	-.67	.42	-.07
Total ± Z-Thrusts	.26	-.72	-.56
Z Fuel	.05	-.09	-.99
Total ± X-Thrusts	-.18	-.13	.85
X Fuel	.97	-.06	.00
Total X, Z, θ Fuel	.90	-.08	-.39
R at 1st Retrothrust	-.60	-.34	.64
Duration of 1st Retrothrust	.87	-.14	-.13
Vertical Displacement from LOS	-.03	.94	-.21

results of the factor analysis are presented with these limitations in mind.

Factor A for the LOS mission had its highest loadings on Terminal $\dot{R}$, Z-Fuel, X-Fuel, Total Fuel, and Duration of 1st Retrothrust. It was considered to be primarily a measure of the subjects' ability to conserve X- and Z-Fuel.

Factor B had its highest loadings on Mission Time, Initial $\dot{R}$, and Total ± Z-Thrusts. Total ± Z Thrusts and Mission Time were positively related which suggested that many Z-thrusts were associated with time consuming missions which deviated from the LOS. Initial $\dot{R}$ was negatively related to Mission Time as might be expected. Factor B was considered to be primarily a measure of the ability to conserve mission time.

Factor C had its highest loadings on Terminal $\dot{R}$, Total ± X-Thrusts, and Vertical Displacement from LOS at rendezvous. Each of these measures reflects the subjects *closing proficiency* (i. e. ability to slow down and effectively maneuver the vehicle during the final stage of rendezvous). Total ± Thrusts reflects closing proficiency because—except for the initial - X-thrust—all X-thrusts were applied in an effort to achieve a terminal $\dot{R}$ of zero. Terminal $\dot{R}$ and Vertical Displacement from LOS, which were positively related to each other, are also measures of closing proficiency. Factor C was considered to be a measure of closing proficiency. The three LOS factors account for 87% of the total common factor variance associated with the 11 LOS performance measures.

Factor A for the T mission had its highest loadings on X-Fuel and Total Fuel. Terminal $\dot{R}$ and Duration of 1st Retrothrust also had high loadings, and both were related to closing proficiency and X-fuel consumption. These results suggested that factor A was a measure of the subjects' ability to conserve X-fuel during the final stage of rendezvous.

Factor B had its highest loadings on Mission Time, Total ± Z-Thrusts, and Vertical Displacement from LOS. These loadings reflect performance during the final stage of rendezvous. Mission Time reflects terminal rendezvous performance, because it was variable among subjects primarily as a function of the subjects closing proficiency. Total ± Z-Thrusts reflects terminal rendezvous performance, because all Z-thrusts were applied during the closing maneuver. Vertical Displacement from LOS also reflects terminal rendezvous performance, because it was during the closing procedure that subjects attempted to minimize vertical displacement. While factor A was considered to be a measure of the subjects' ability to conserve fuel during the closing procedure, factor B was considered to be a measure of the subjects' ability to conserve time during the closing maneuver.

Factor C for the T mission had its highest loadings on Z-Fuel and Total ± X-Thrusts. All Z-fuel was used during the closing maneuver and all X-thrusts, except one, were applied during the closing maneuver. Therefore, factor C was considered to be a measure of the subjects ability to conserve Z-fuel and effectively apply X-thrusts while performing the closing maneuver. The three T factors account for 87% of the total common factor variance associated with the 11 T performance measures.

In summary, there were three factors underlying LOS rendezvous performance: the ability to conserve X- and Z-fuel, the ability to conserve mission time, and the ability to close proficiently with the target. There were also three ability factors associated with the T rendezvous maneuver: the ability to conserve X-fuel while closing with the target, the ability to conserve mission time while closing with the target, and the ability to conserve Z-fuel and effectively apply X-thrusts while closing with the target.

The factor analysis revealed the principal man-machine factors underlying successful LOS and T rendezvous performance from the time of the first X-thrust to the time of rendezvous. While the three LOS factors adequately describe the abilities associated with LOS rendezvous performance, the three T factors do not adequately describe the abilities associated with T rendezvous performance; the subjects' ability to establish the proper pitch angle was not included in the T-factor analysis. To more fully describe the T mission, the consequences of establishing slightly different pitch angles were determined. Pitch Angle was correlated with all other performance measures. It correlated significantly with only total ± Z-thrusts ($r = .83$, $p < .01$). The smaller the interceptor pitch angle, the longer the subject flew along the LOS while closing with the target. Many Z-thrusts were applied to remain on the LOS. This result suggested a forth T performance factor; the ability to match the trajectory path of a 14° two-impulse trajectory maneuver. The closer the subject came to approximating a 14° pitch angle, the closer his trajectory path came to matching the path for a two-impulse trajectory mission.

SECTION IV

DISCUSSION

The principal experimental prediction was supported. The T maneuver was executed as successfully as the LOS maneuver, and with significantly less fuel consumption. Thus, given the conditions of this experiment, the T rendezvous technique is superior to the LOS technique.

Results of the factor analysis indicated major man-machine performance factors to be considered in evaluating space-flight performance. Although different performance measures would have produced different factors, the performance measures selected were deemed satisfactory for describing rendezvous performance. Future research might focus on examining the interrelationships among performance factors, such as fuel economy, time economy, and closing proficiency. Subjects in this experiment tended to sacrifice fuel and time for closing proficiency or vice versa. However, since individual differences in fuel and time economy were slight, the tradeoffs involved were of little practical significance.

A more difficult experimental task would likely produce greater individual differences, in which case tradeoffs of the sort mentioned above would be more meaningful. Where large individual differences are found, it would be helpful to express a subject's performance in terms of factor scores (ref 7). It is usually less difficult to interpret 3 or 4 factor scores than 11 or 12 performance measures.

Individual differences in this experiment were slight, because a training period was employed and the experimental task was not especially difficult. Approximately 15 T training trials and 10 LOS training trials were required for a naive nonrated subject to fly five consecutive successful missions of each type. Principal changes in performance accompanying training were: (1) decrease in X- and Z- fuel consumption (2) decrease in mission time, and (3) decrease in range at which the first retrothrust was applied.

Experienced pilots reached the criterion for mission success sooner than nonrated personnel. Seven Appollo-Gemini astronauts flew both missions successfully and two achieved success on their first attempt. These two believed that the maneuvers would have been more difficult if the interceptor vehicle had been undergoing random perturbations. Both astronauts had previous rendezvous training. Use of a spherical target or a tumbling target would also have increased task difficulty.

A principal cue used by subjects for ascertaining their vertical displacement from the target was the center of the TV screen. A grided screen would have enhanced this cue. A second cue used for determining vertical displacement was change in target shape. This cue was utilized primarily at ranges less than 800 ft.

Other possible references for orientation were the earth horizon and the stars. The earth horizon was not used, because it was not in the field of view unless the interceptor was pitched down, which it never was. Reference to the stars was seldom employed because they moved from the top to the bottom of the TV screen at the orbital rate. Finally, it was thought that the experimental results would not have changed significantly if subjects had viewed a three-dimensional display. Research is being designed to test that contention.

APPENDIX I

INSTRUCTIONS TO SUBJECTS

I. General Instructions

You are going to participate in a study designed to determine how accurately an astronaut can bring his interceptor vehicle into rendezvous with a target vehicle in coplanar orbit. Presume you are in a space vehicle circling the earth 10,000 feet ahead of the target. Both you and the target are at an altitude of 200 nautical miles. The picture on the TV screen before you simulates how this target and its surrounds would appear to you looking out the window of your interceptor. As you manipulate the controls of your interceptor, the visual scene will continue to be realistic by changing in accordance with your control movements. For example, if you decrease your altitude, the doughnut-shaped target will move toward the top of the screen. Before describing your task in detail, however, we want to acquaint you with some orbital dynamics.

Look at figure A, which shows a space vehicle in orbit around the earth. Orbit 1 is a circular orbit. If a slowing-down thrust is applied as shown in the figure, elliptical trajectories result. Orbit 2 is such an ellipse. As the vehicle falls toward the earth it gains velocity. The increase in velocity is sufficient to cause it to regain altitude, but, as it climbs, it slows down again resulting in the elliptic path. Orbit 3 results from enough deceleration to cause the vehicle to re-enter the atmosphere before regaining sufficient velocity to climb.

Now look at figure B. Orbit 1 is again a circular orbit. The thrust applied as shown would cause the vehicle to accelerate and

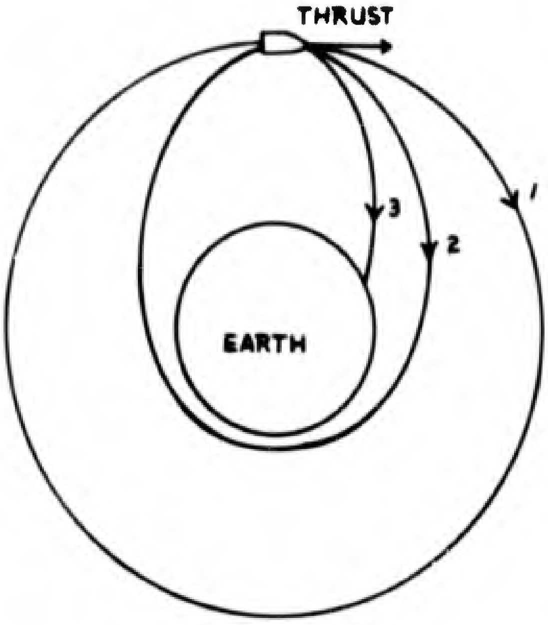


Figure A
Effects of Deceleration

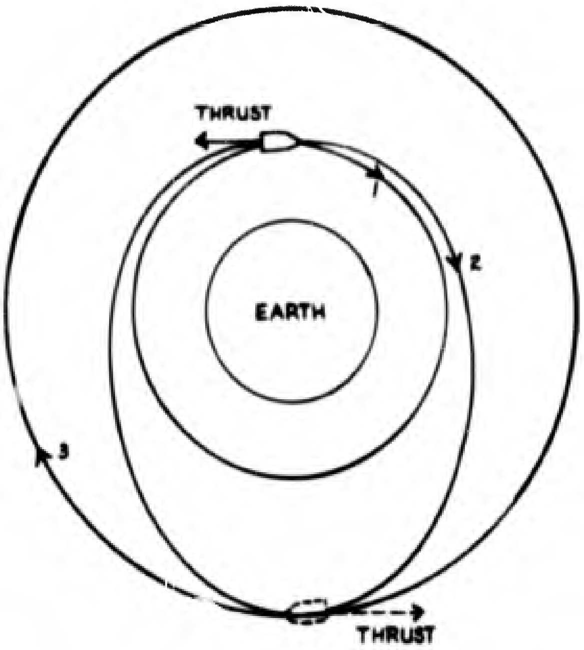


Figure B
Effects of Acceleration

move into Orbit 2. This orbit is also an ellipse, since as the vehicle gains altitude, it slows down and begins to fall. As the vehicle falls it gains velocity and begins to climb as in the first case. If a second thrust is applied at the highest point in the orbit (apogee), shown by the dotted line, Orbit 3 is attained. This is a circular orbit higher than Orbit 1. Thrust is used only twice; the remainder of the time is spent in coasting. Similar two-impulse transfers exist for any orbit change.

The purpose of the above descriptions was to show what happens when thrust is applied to an orbiting vehicle. Any questions?

Consider the path of a second vehicle attempting to rendezvous with the target vehicle. If the interceptor vehicle is initially directly ahead of the target at the same altitude and speed, it must slow down to allow the target to catch up. If the interceptor simply slows up it will lose altitude and follow path 1 shown in figure C.

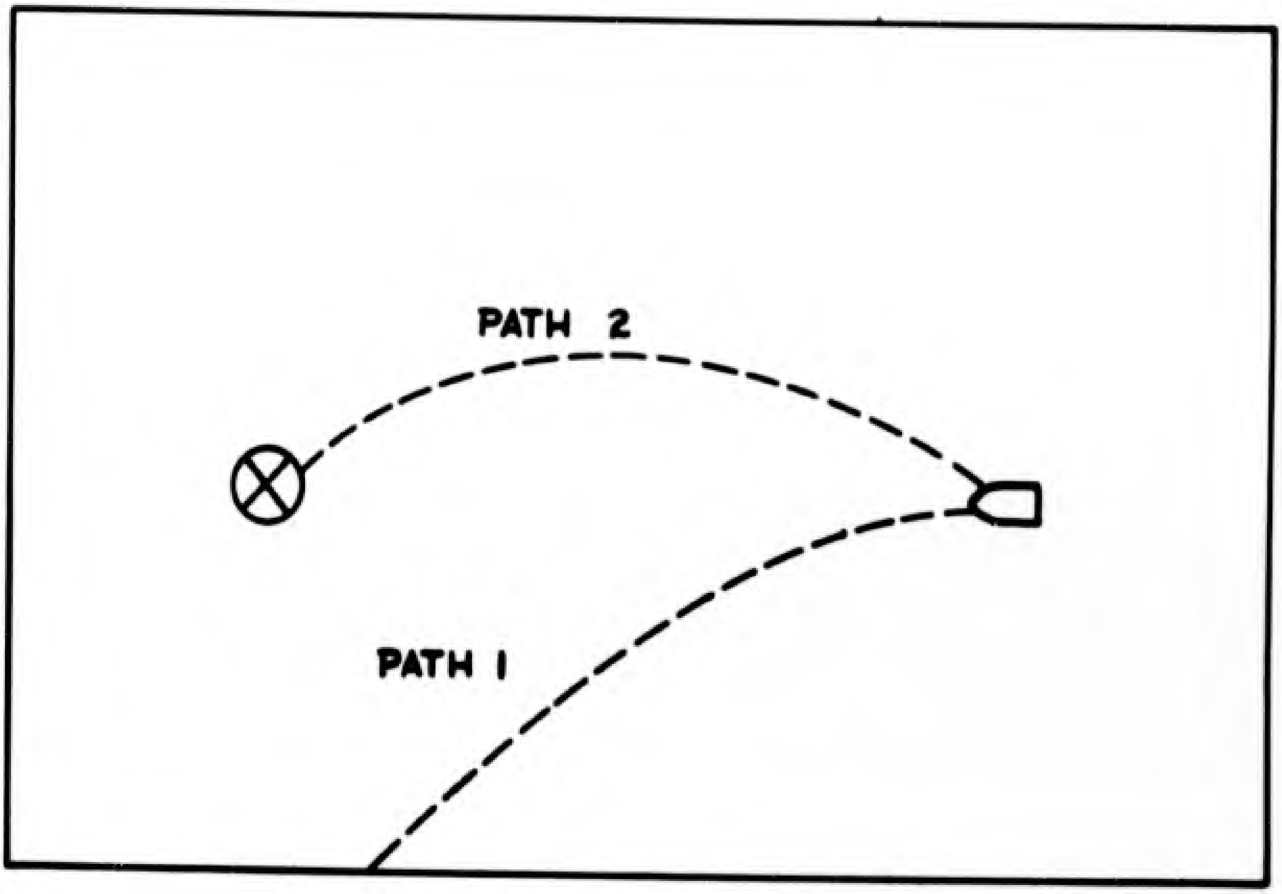


Figure C. Thrusts Required for Rendezvous

Since this path will not result in a rendezvous with the target, upward thrust must be applied to maintain altitude as the vehicle loses velocity. If the proper combination of thrusts is applied, path 2 will result. All that remains to be done is to thrust to a safe closing velocity with the target.

Now, back to your specific task. In this study you will rendezvous with the target vehicle shown on the TV screen before you. You will fly two types of rendezvous using two different propulsion control systems. One control system is called the pitch-orthogonal-thrust (P-O-T) system. When using it you will fly a trajectory-type mission to the target like that shown in figure C. Three degrees of interceptor control will be available: (1) fore and aft thrust, (2) up and down thrust, and (3) pitch. You will pitch your vehicle up, apply forward thrust, and coast to the target, using whatever vehicle control is necessary to achieve a successful rendezvous.

The second control system is called the orthogonal-thrust (O-T) system. You will fly a line-of-sight (LOS) type mission when using this control system. That is, you will fly directly to the target without putting your vehicle into a trajectory. When using the O-T system you will have available only fore and aft and up and down thrusts.

You will learn to fly the two types of rendezvous missions described above using the four instruments before you. When you are able to come within 100 ft of the target in five min at a terminal velocity of as near zero ft/sec as possible, you will then fly the mission using no instruments.

In flying either the LOS or trajectory-type mission, remember that:

1. The initial conditions for rendezvous are always the same. You are 10,000 ft. ahead of the target in a coplanar orbit. You are at the same altitude as the target.

2. You always lose altitude as you thrust toward the target.

3. You must apply retrothrust in order to slow down. There is no friction to slow you down.

4. Do your best in flying all missions. Any questions?

II. Specific Instructions

A. Flying the 9900 ft LOS Mission

1. Apply forward thrust for 8.0 sec using the stop watch. Your closing velocity ($\dot{R}$) is now 46 ft/sec.

2. Maintain the LOS using up-thrust. If you go above the LOS, wait until you drift back to it if you are far (e.g., 2000 ft) from the target. Do not waste fuel by applying down-thrust to reach the LOS when you are far from the target.

3. Reach the rendezvous point at as near zero ft/sec as possible by applying retrothrust. Maintain the LOS by applying up or down thrust.

Thrust Accelerations*	Fuel Expenditure
$\pm X = 5.75$ ft/sec^2	5 lb/sec
$\pm Z = 1.15$ ft/sec^2	1 lb/sec

You have 200 lb of fuel on board. At a 100 ft radius from the target in any direction, the computer automatically enters the "hold" mode and your mission is terminated.

The Ideal Mission

Minimum time to target = approximately 223 sec.

Minimum fuel = 100 lb

Your main task is to successfully rendezvous with the target, using as little fuel as possible, in less than five min. Above all, come to a point 100 ft directly in front of the target at a terminal velocity of as near zero ft/sec as possible. Time between trails is 90 sec. Use the stop watch only for timing the initial forward thrust.

*with respect to the target

B. Flying the 9900 Trajectory Mission

1. Pitch your vehicle up to an angle of from 13° to 14°. Notice that the target is not quite out of sight (i.e., off the bottom of the TV screen).

2. Apply forward thrust for 8.0 sec using the stop watch. Your closing velocity ($\dot{R}$) is now 44.6 ft/sec, and your trajectory velocity 46.0 ft/sec.

3. Immediately pitch to zero degrees by bringing the target to the center of the screen. A change in pitch does not change your trajectory direction or velocity.

4. Coast toward the target. Depending on the pitch angle you adopted, with no further control inputs, your trajectory will cross the LOS short of the target. With a 13° pitch and 8 sec -X-thrust, you would cross the LOS 808 ft from the target, which is 708 ft from the point of rendezvous. Before you cross the LOS, however, apply up-thrust so as to maintain the LOS once you reach it. Once on the LOS, fly directly to the target.

5. Reach the rendezvous point at as near zero ft/sec as possible by applying retrothrust. Maintain the LOS by applying up- or down-thrust. Do not use pitch.

Thrust Accelerations*	Fuel Expenditure
$\pm X = 5.75$ ft/sec^2	5 lb/sec
$\pm Z = 1.15$ ft/sec^2	1 lb/sec
$\pm$ pitch = 2.75 deg/sec^2	0.1333 lb/sec

You have 200 lb of fuel on board. At a 100 ft radius from the target in any direction, the computer automatically enters the "hold" mode and your mission is terminated.

*with respect to the target

The Ideal Mission

Your minimum time and fuel to target depend on the pitch angle and braking procedure you adopt. A 14° 226-sec minimum fuel two-thrust maneuver would require 80 lb of fuel, ignoring fuel used in pitching, which is < 1 lb. The 226 sec time specified was calculated from the moment of the initial - X-thrust application.

Your main task is to successfully rendezvous with the target, using as little fuel as possible, in less than five min. Above all, come to a point 100 ft directly in front of the target at a terminal velocity of as near zero ft/sec as possible.

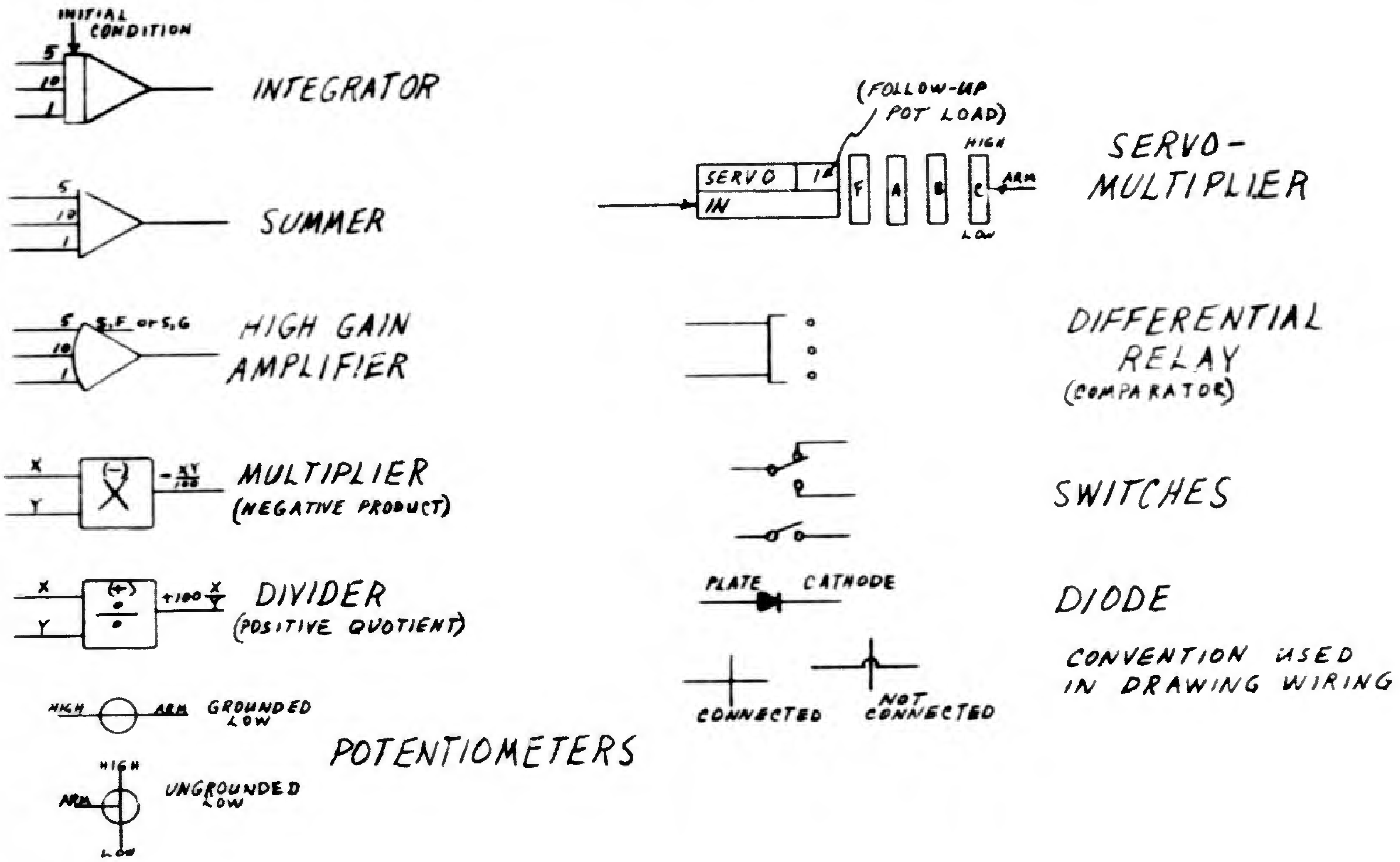
Time between trials is 90 sec. Use the stop watch only for timing the initial forward thrust.

APPENDIX II

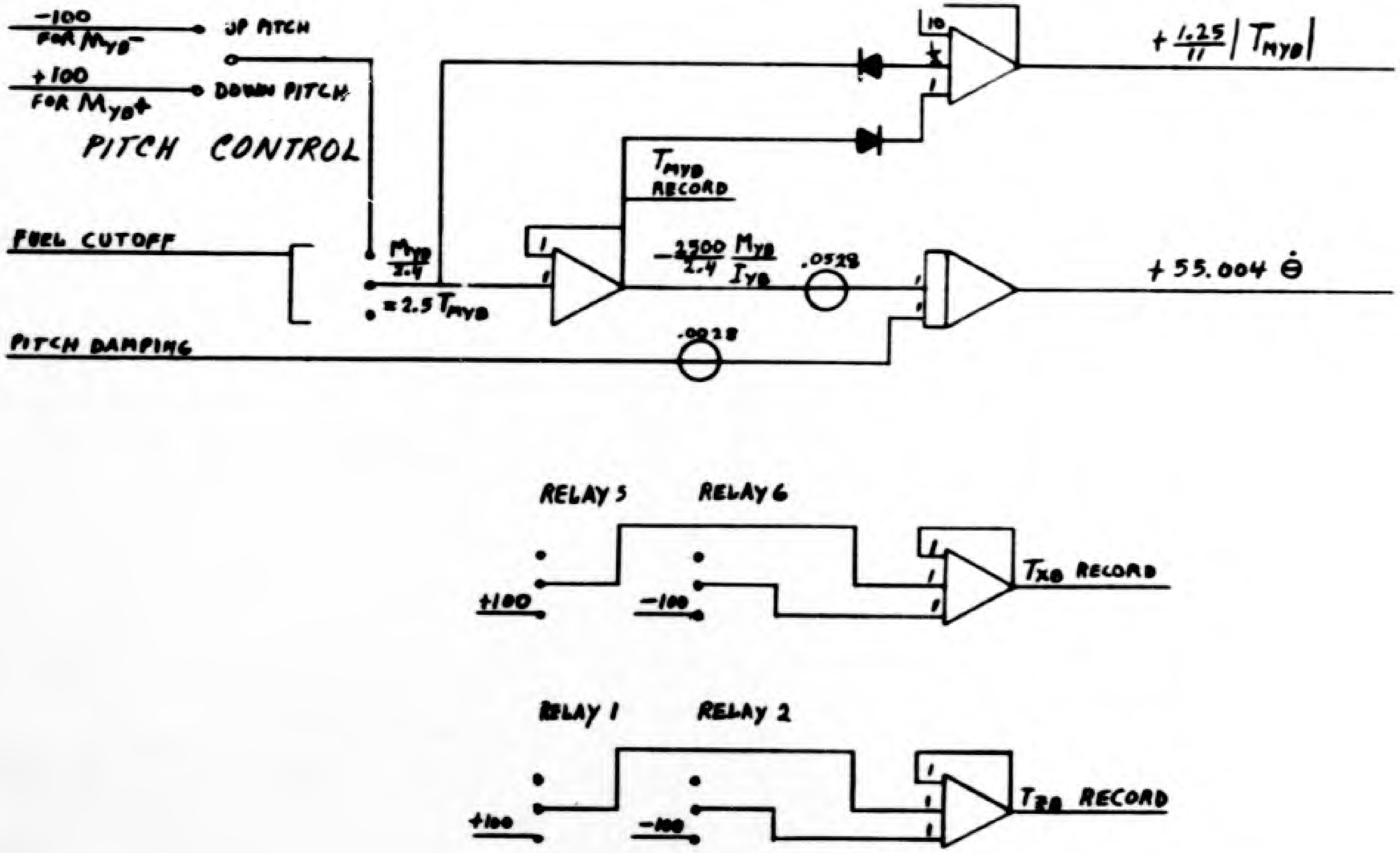
Computer Diagram

This appendix contains computer diagrams which describe the simulation. Except for the last diagram entitled "Resetting Circuit and Servo Compensation for $\dot{\theta}$," the diagrams correspond to, and are arranged in the same order as, the equations of table II. For example, the diagram entitled "Fuel Computation" shows the analog simulation of the equation called Fuel Computation given in table II.

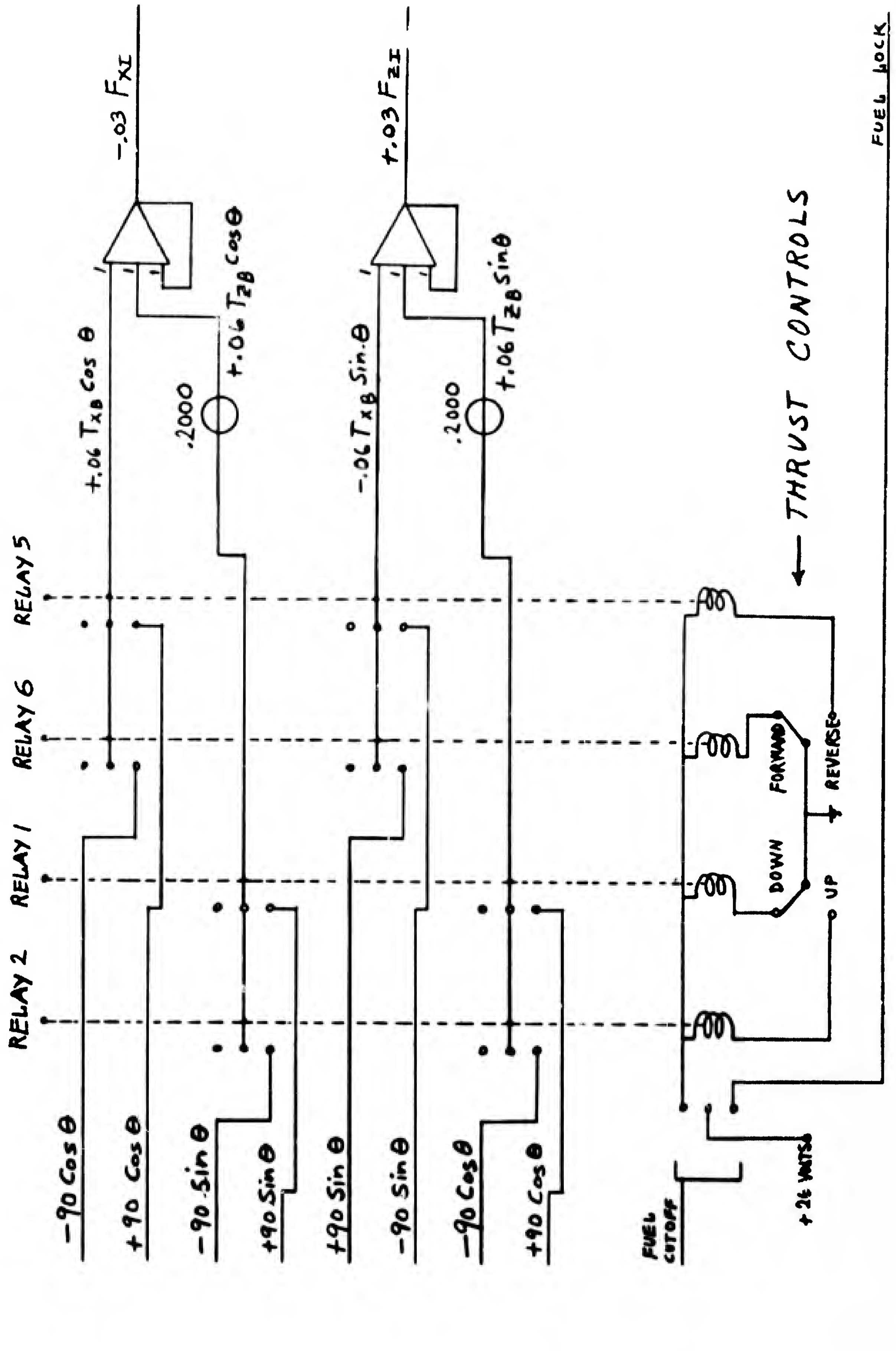
SYMBOLS FOR COMPONENTS USED IN COMPUTER DIAGRAMS



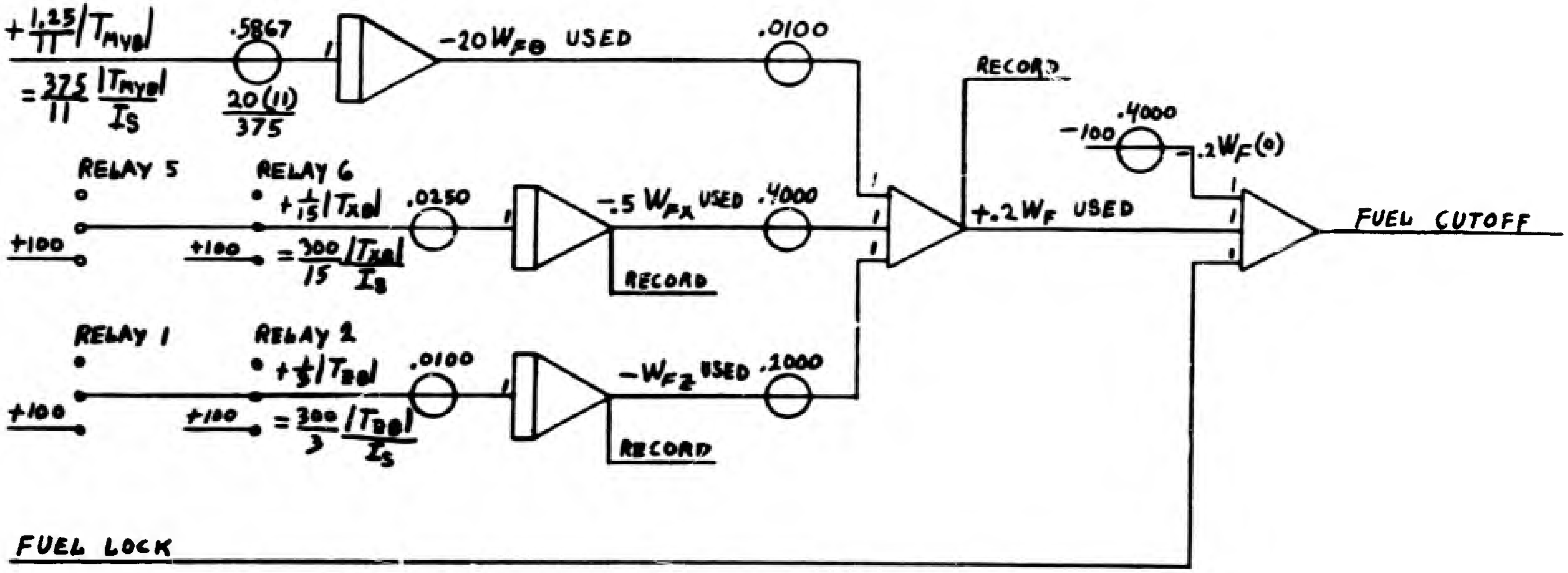
VEHICLE DYNAMICS AND THRUST INDICATIONS



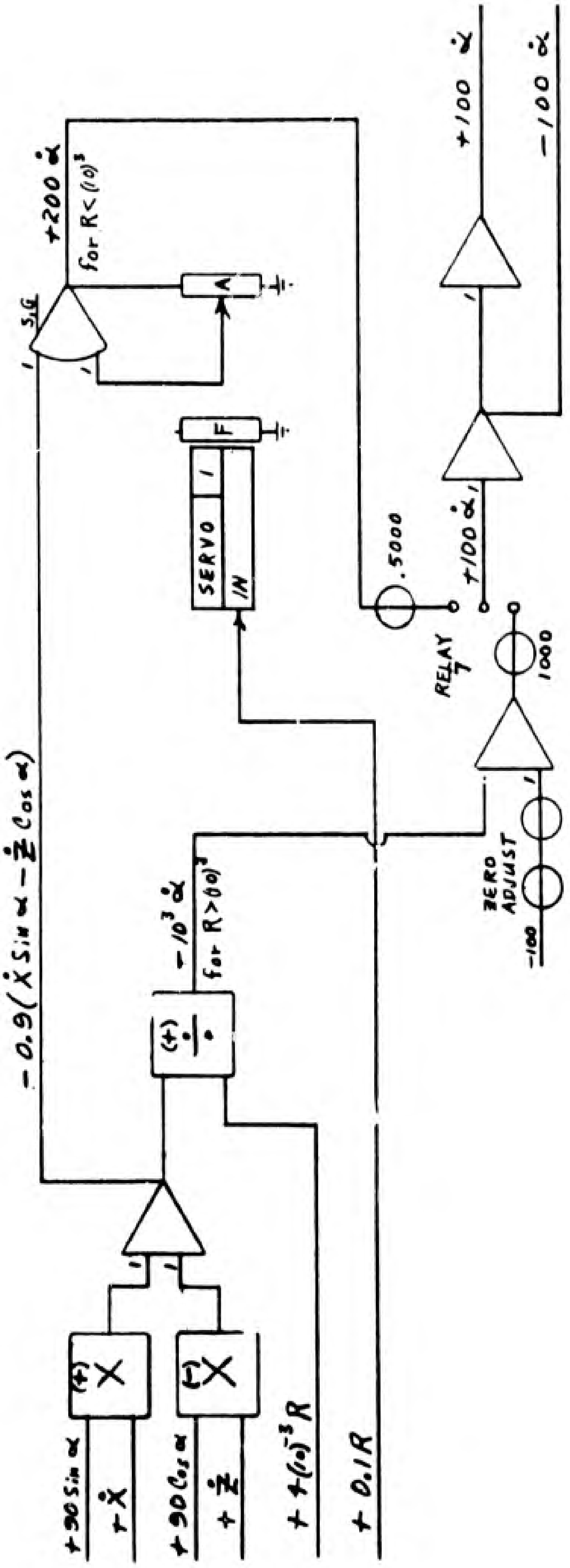
COORDINATE TRANSFORMATION



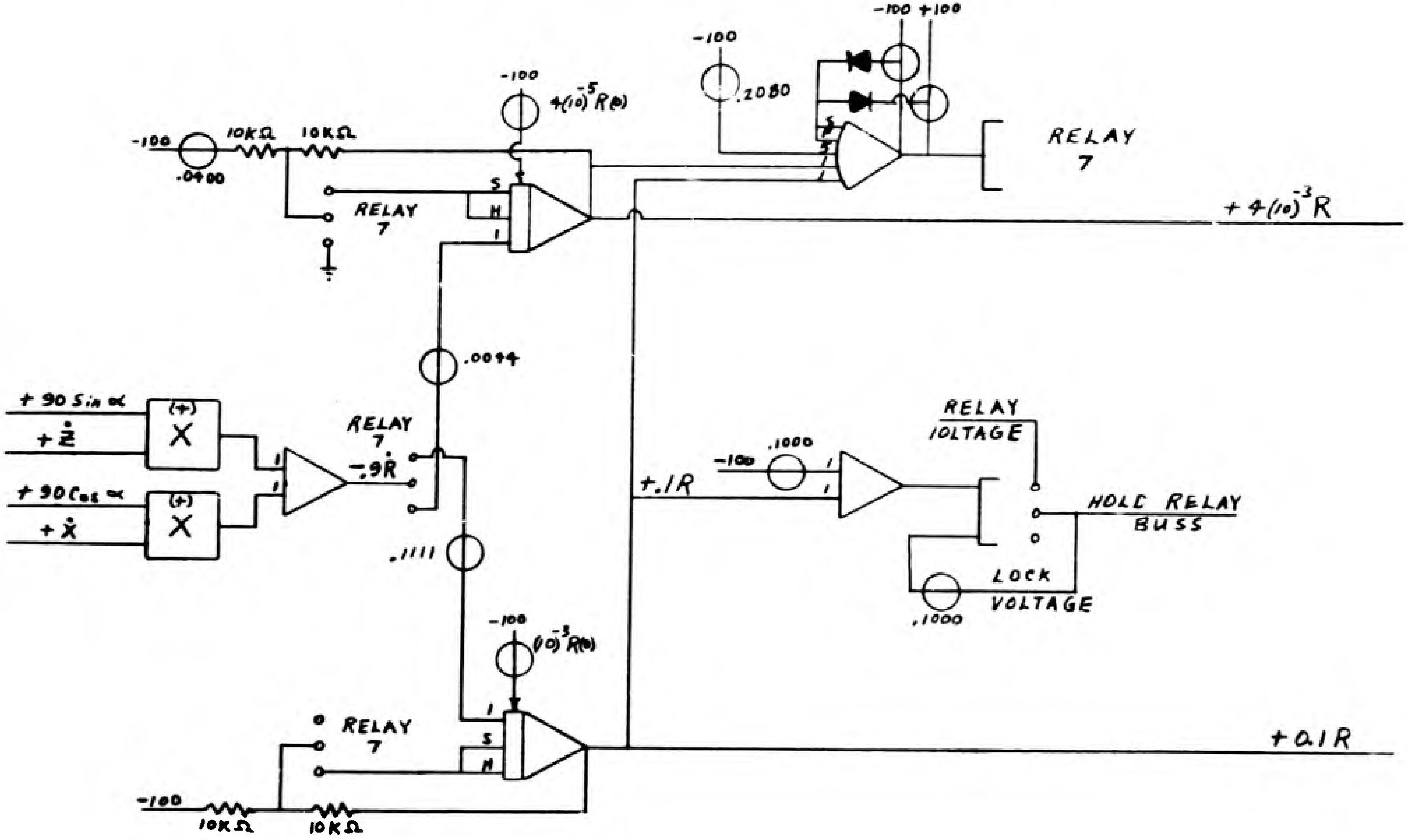
FUEL COMPUTATION



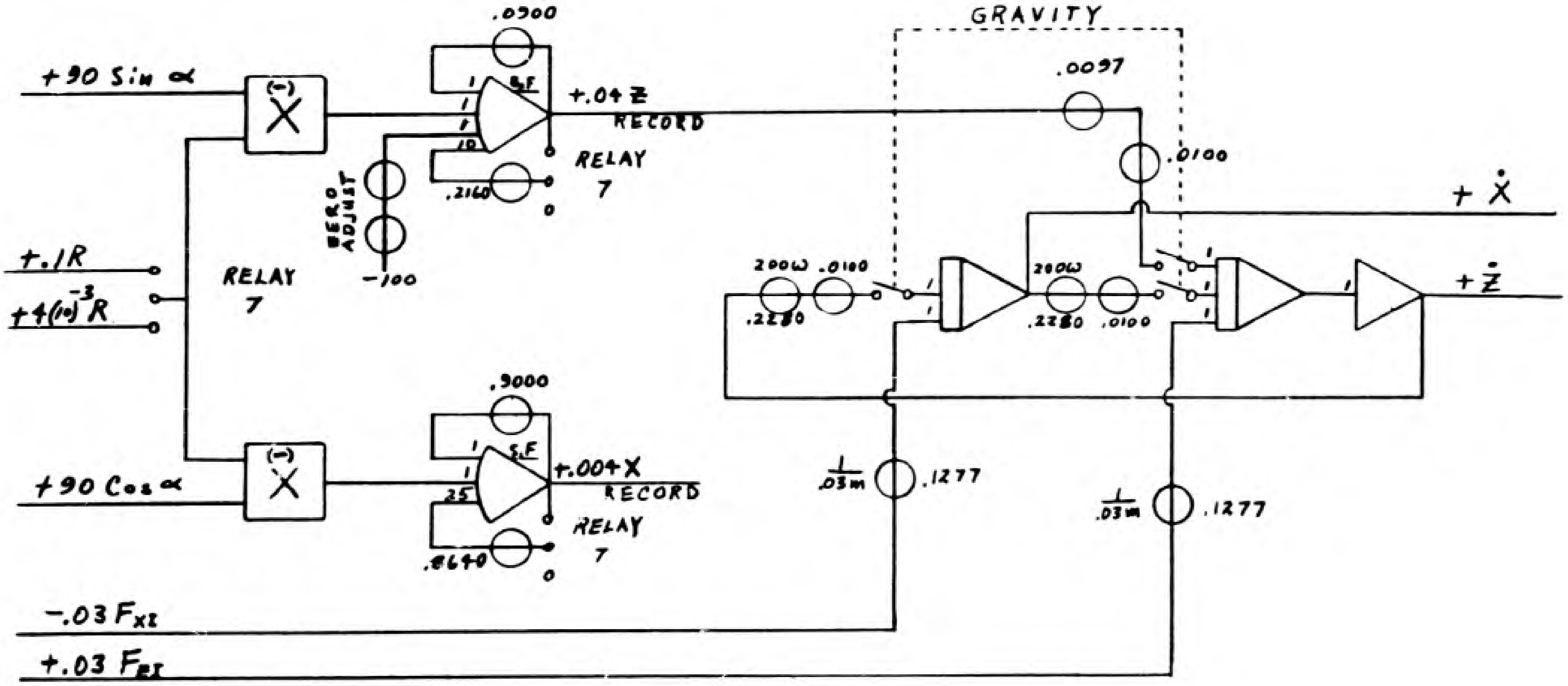
RELATIVE MOTION



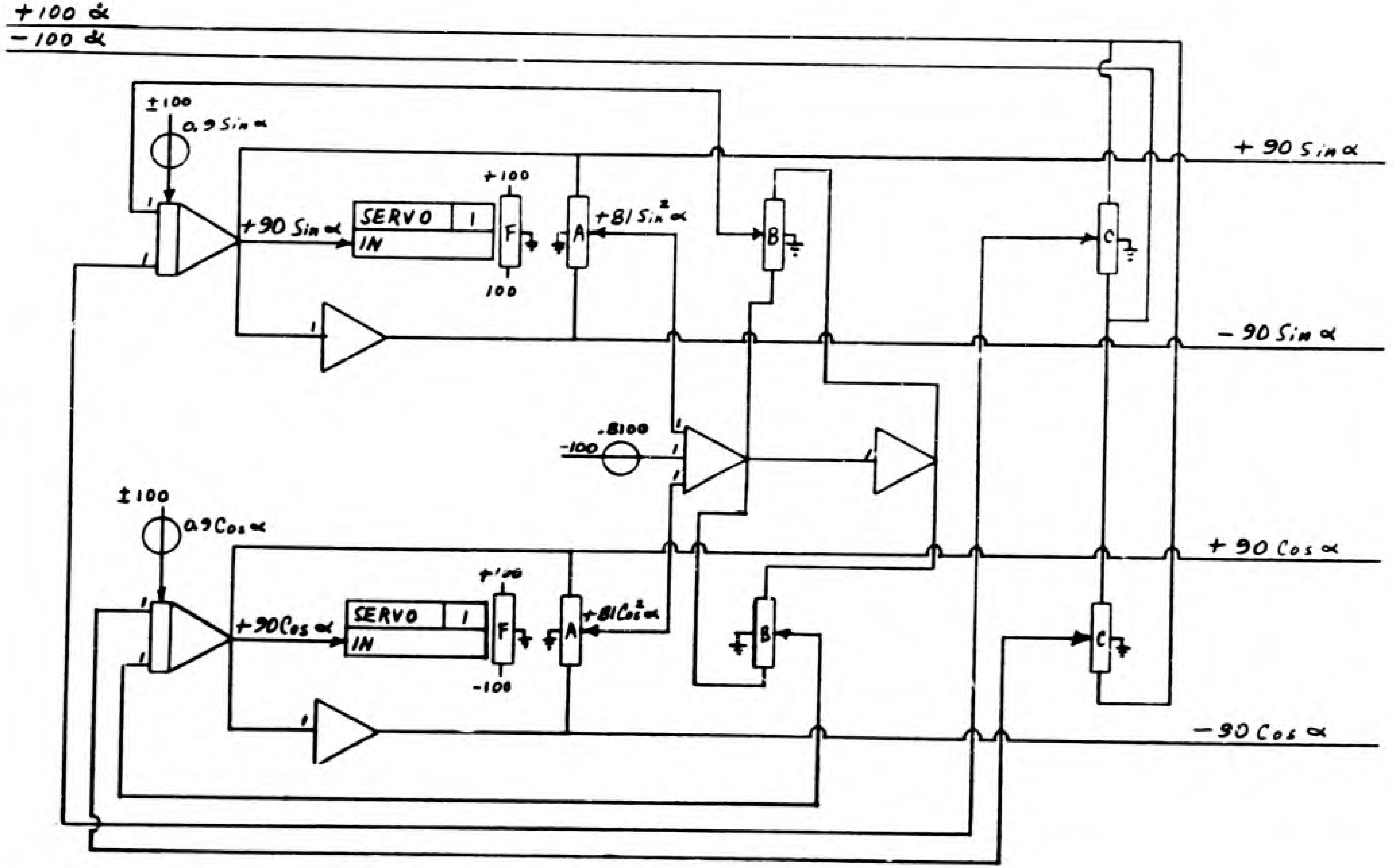
RELATIVE MOTION



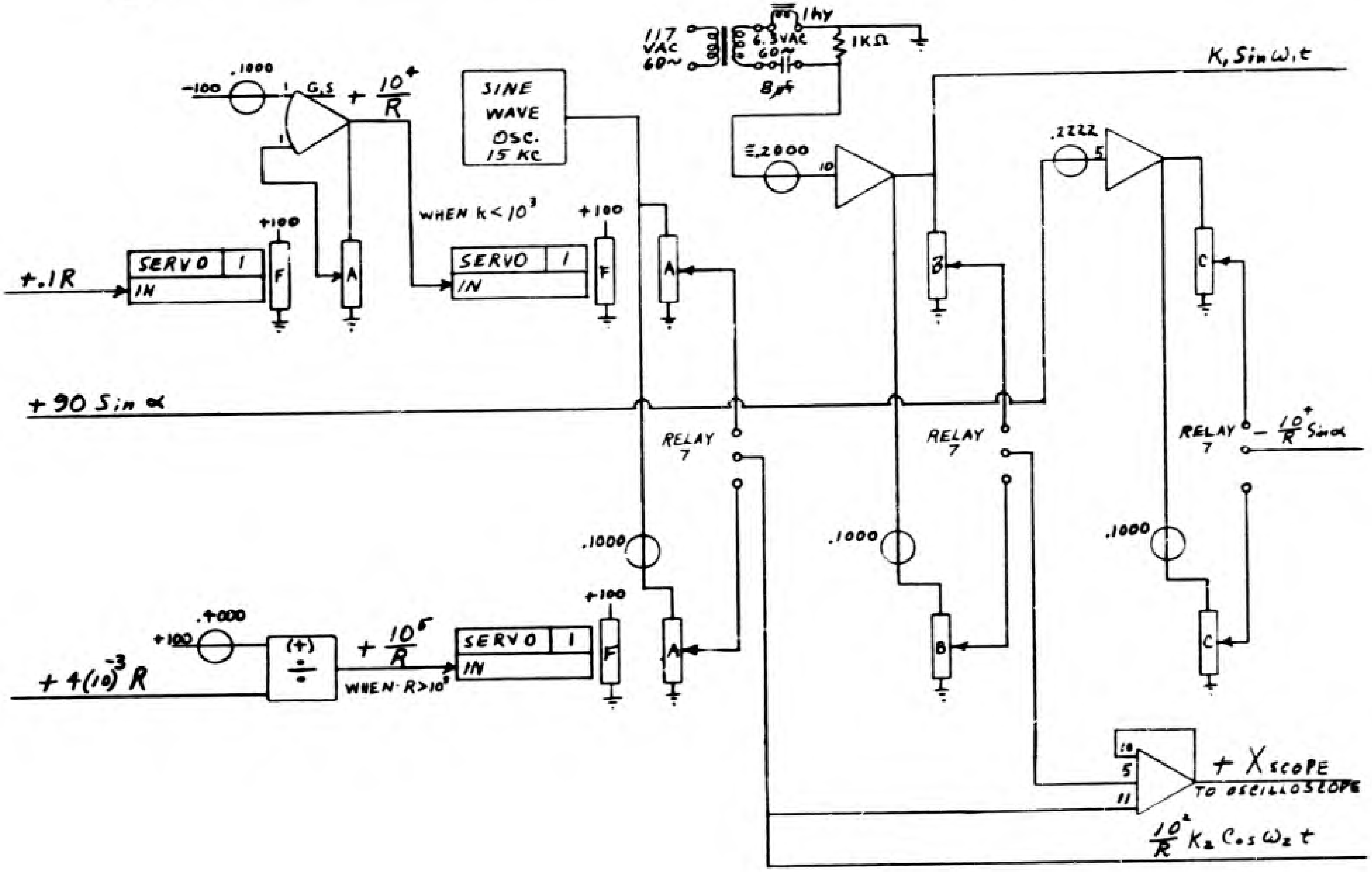
RELATIVE MOTION



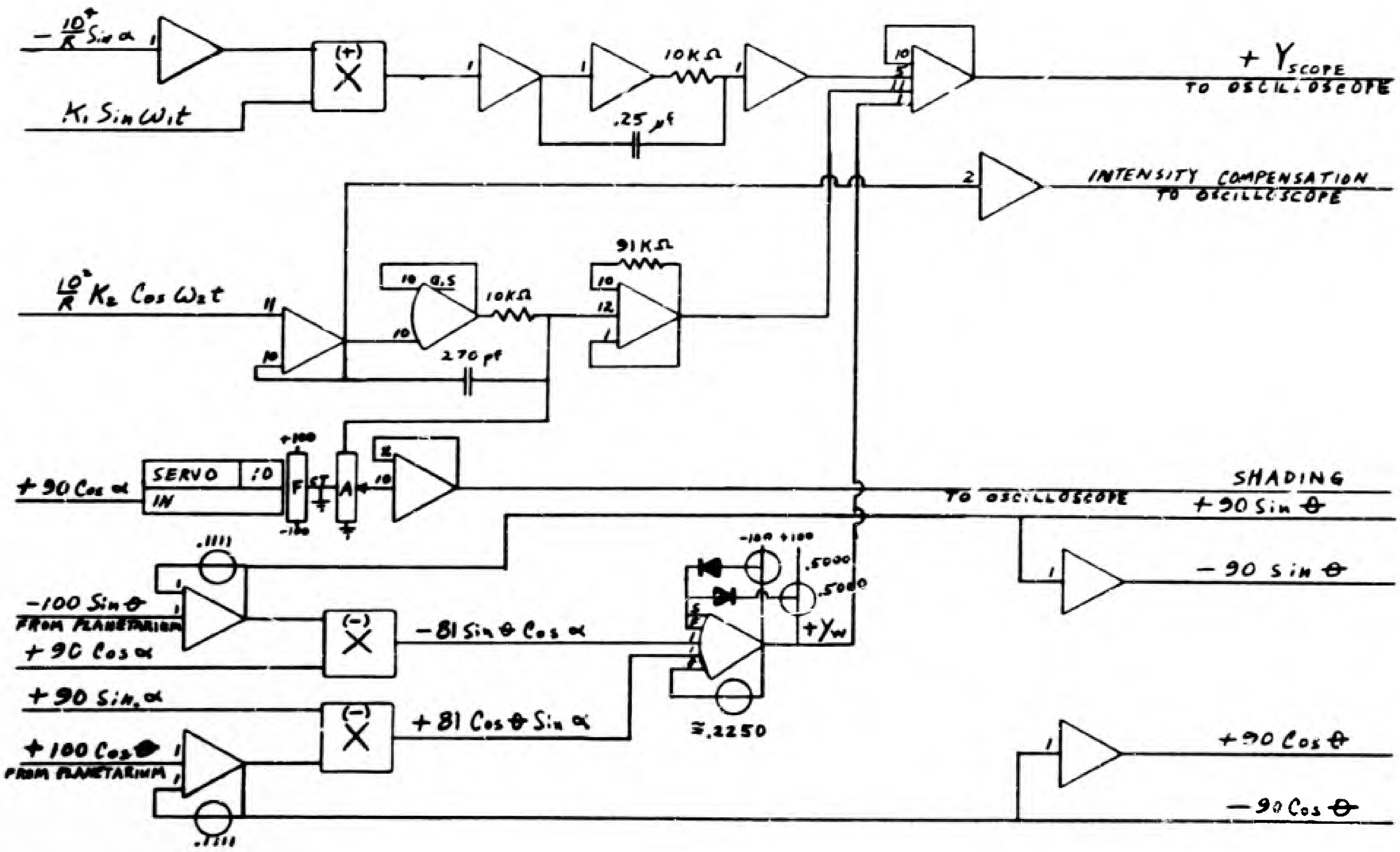
$\dot{\alpha}$ CONTINUOUS RESOLVER



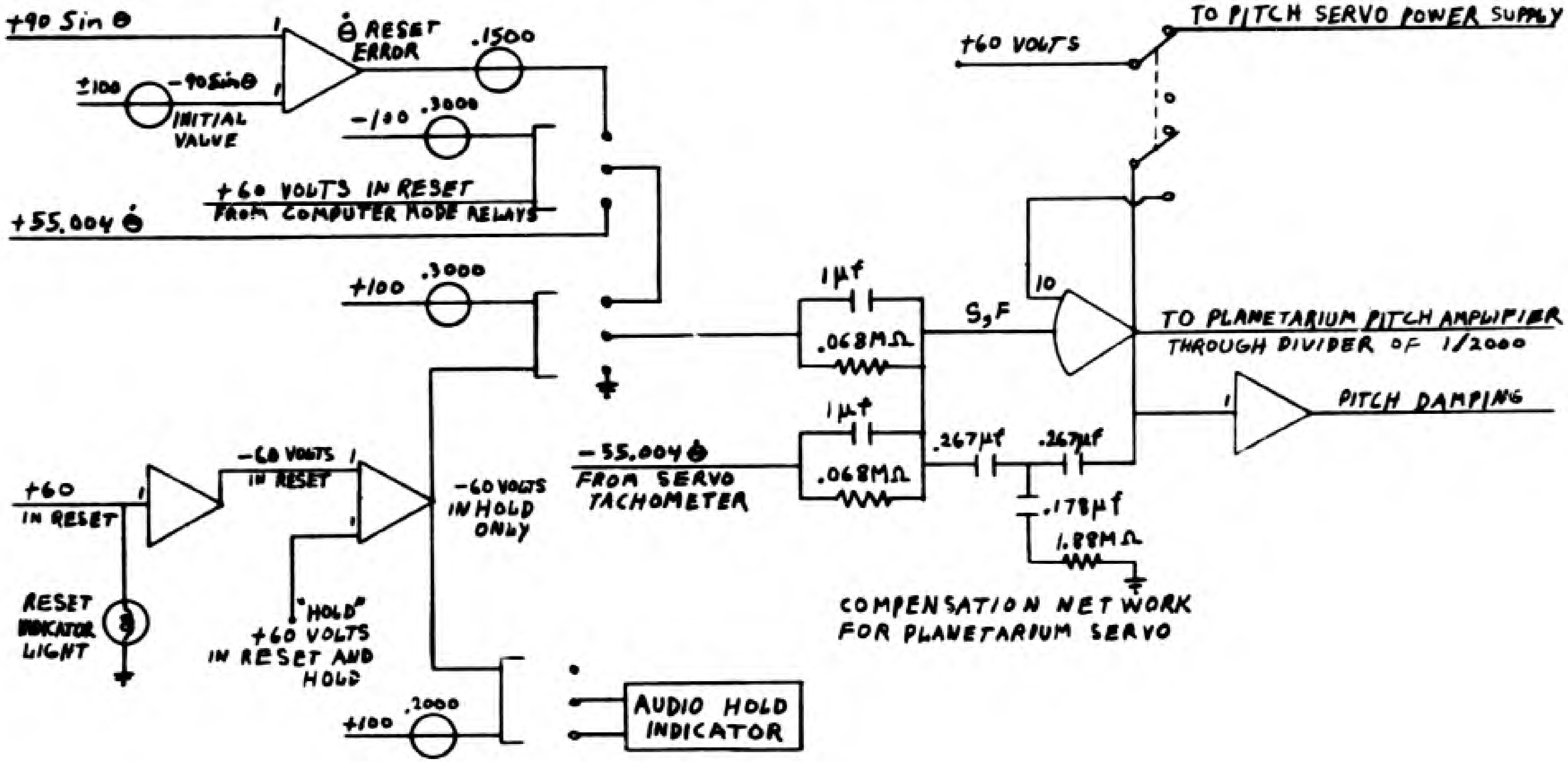
TARGET IMAGE GENERATION INCLUDING TRANSLATION



TARGET IMAGE GENERATION INCLUDING TRANSLATION



RESETTING CIRCUIT AND SERVO COMPENSATION FOR $\dot{\theta}$



REFERENCES

1. Kasten, D. F., Human Performance in a Simulated Short Orbital Transfer. Aerospace Medical Research Laboratories Technical Documentary Report 62-138 (AD 400 484). Wright-Patterson Air Force Base, Ohio, December 1962.

2. Mueller, D. D., Relative Motion in the Docking Phase of Orbital Rendezvous, Aerospace Medical Research Laboratories Technical Documentary Report 62-124 (AD 402 384), Wright-Patterson Air Force Base, Ohio, December 1962.

3. McCoy, W. K., Jr., Frost, G. G., Time History as an Informational Source for Operator Control of Orbital Rendezvous. Aerospace Medical Research Laboratories Technical Documentary Report 64-55 (AD 603 597), June 1964.

4. Bell Aerosystems Company, Operational and Maintenance Handbook for Visual Space Simulator for Rendezvous Missions. Bell Aerosystems Report, No. 60012-059, Buffalo, New York, September 1963.

5. Ryken, J. M., Emerson, J. E., and Biltz, J. L., Computor Equations for a Visual Space Simulator for Rendezvous Missions. Bell Aerosystems Report, No. 60009-185, Buffalo, New York, November 1962.

6. Dixon, W. J. (ed). BIMD Computer Programs. Health Sciences Computing Facility, Department of Preventative Medicine and Public Health, School of Medicine, University of California, Los Angeles, 1961.

7. Harman, H. Modern Factor Analysis. The University of Chicago Press, 1960.

AMRL-TR-65-10

February 1965

ERRATA - May 1965

The following corrections apply to Technical Report No. AMRL-TR-65-10, <u>Trajectory Versus Line-of-Sight Space Rendezvous Using Out-of-Window Visual Cues</u>.

Page 4

Second line, change "(R)" to "($\dot{R}$)"

Page 10

In Table I, change "M_F" to "W_F"

Page 15

Delete all of Table II and replace with Table II on back of this page.

Page 45

Schematic entitled "Fuel Computation," change "Is" to "Isp"

Page 46

Schematic entitled "Relative Motion," change signs of the outputs of the two multipliers and change -0.9 to +0.9. Insert gain 1 on amplifier input from divider output.

BEHAVIORAL SCIENCES LABORATORY
AEROSPACE MEDICAL RESEARCH LABORATORIES
AEROSPACE MEDICAL DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

TABLE II

EQUATIONS USED IN THE SIMULATION

$\ddot{\theta} = \frac{M_{YB}}{I_Y}$	VEHICLE DYNAMICS
$F_{XI} = T_{XB}\ \text{Cos}\ \theta + T_{ZB}\ \text{Sin}\ \theta$	COORDINATE TRANSFORMATION
$F_{ZI} = T_{XB}\ \text{Sin}\ \theta - T_{ZB}\ \text{Cos}\ \theta$	
$W_F = W_F(0) - \frac{1}{I_{sp}} \int_o^t (T_{XB} + T_{YB} + T_{MYB})dt$	FUEL COMPUTATION
$\dot{\alpha} = \frac{\dot{Z}\ \text{Cos}\ \alpha - \dot{X}\ \text{Sin}\ \alpha}{R}$	RELATIVE MOTION
$\dot{R} = \dot{X}\ \text{Cos}\ \alpha + \dot{Z}\ \text{Sin}\ \alpha$	
$\ddot{X} = \frac{F_{XI}}{m} - 2\omega \dot{Z}$	
$\ddot{Z} = \frac{F_{ZI}}{m} + 2\omega \dot{X} + 3\omega^2 Z$	
$X = R\ \text{Cos}\ \alpha$	
$Z = R\ \text{Sin}\ \alpha$	

NOTE: Sin α and Cos α come from locked loop continuous resolver driven by $\dot{\alpha}$.

$Y_W = d(\text{Sin}\ \theta\ \text{Cos}\ \alpha - \text{Cos}\ \theta\ \text{Sin}\ \alpha)$	TARGET TRANSLATION
$X_{Scope} = \frac{K_1}{R}\ \text{Sin}\ \omega_1 t + \frac{K_2}{R}\ \text{Cos}\ \omega_2 t$	TARGET IMAGE GENERATION INCLUDING TRANSLATION
$Y_{Scope} = Y_W + \frac{K_1}{R}\ \text{Sin}\ \alpha\ \text{Cos}\ \omega_1 t + \frac{K_2}{R}\ \text{Sin}\ \omega_2 t$	