

MEMORANDUM

RM-4577-PR

JUNE 1965

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SOME TABLES OF
THE NEGATIVE BINOMIAL DISTRIBUTION
AND THEIR USE

Bernice Brown

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THE NEGATIVE BINOMIAL DISTRIBUTION
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Bernice Brown

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PREFACE

In this Memorandum the author presents probability tables of the negative binomial distribution for some useful sets of parameter values. This distribution may be used as a frame of reference for the study of the demand for replacement parts.

SUMMARY

This Memorandum presents tables giving the values of the individual terms of the negative binomial distribution for 130 pairs of parameter values in Part 1. Part 2, giving the cumulated terms of the same distributions, is in a form directly usable for solving problems which require the determination of probabilities of the occurrence of not more than x events.

The negative binomial distribution is described and illustrative examples are given which use this distribution as a tool in the study of demand for replacement parts.

The references indicate that the negative binomial has been used in a wide variety of applications to biological research.

SOME TABLES OF THE NEGATIVE BINOMIAL DISTRIBUTION AND THEIR USE

1. INTRODUCTION

The research worker in the field of logistics is frequently required to predict the demand for spare parts in order that effective decisions can be made in the areas of procurement, distribution, and stockage policies. Many such decisions are made on the basis of a specialized knowledge about an individual line item.

For routine decisionmaking on a large scale, it is useful to have a small number of known probability distributions which can be used as a framework to provide the basic assumptions from which predictions of future demands can be made. If demand data are available, an analyst may study the sample data and make estimates of the parameters of the underlying demand distribution. The information in the sample will often be scanty, e.g., six months experience at one base or three months of system-wide experience. A frame of reference is needed to study the limited data.

Some examples of studies in which it was necessary to make an assumption about the demand for spare parts in the population may be found in the study of procurement deferral by Petersen [1], the design of flyaway kits by Karr, Geisler, and Brown [2], and by Fort [3], the prediction of

demands by Goldman [4], and Geisler and Brown [5], the study of stockage policies by Ferguson [6], Ferguson and Fisher [7], and Petersen and Geisler [8].

There are many distribution functions which could be discussed. Only two are presented here, the Poisson and the negative binomial. These two were chosen because they are probability distributions of a discrete variable, and demand for spare parts is by nature integral, and also because we believe, hopefully, that they may describe the universe of demands from which the sample emanates. These distributions are attractive also because of their easy computation.

2. THE POISSON DISTRIBUTION

When we deal with phenomena involving events that occur randomly in time or particles that are randomly distributed in space, the Poisson process is the model used. An experiment is performed and "events" are tallied. These events can be described by a function $x = x(t)$, which gives the number, x , of events observed during the first t units of observation for all values of t from 0 through T (where T is the total number of observations). The results of such an experiment may be described by the Poisson distribution if the events occur randomly in the sense of the following definition. If any number of events x are observed in any amount of time t , and if the points of the occurrence of the x events are independently and uniformly

distributed between 0 and t , then the process may be described as random. If the probability of the event is small but a large number of independent cases are taken, the number of occurrences is likely to be distributed in the Poisson series. This distribution may be thought of as an approximation to the binomial distribution when the probability of occurrence of the event is small, that is, if Np is large relative to p and N is large relative to Np (where N is the number of trials and p is the probability of the occurrence of the event in a single trial).

Let us try to visualize what the situation might be in regard to demand for spare parts upon the Air Force Supply System. Suppose a mechanic inspects an actuator on each of the 18 planes of a squadron, once each month for two years, and requests a replacement part whenever he judges that the actuator has failed. Assume that the failure rate for this part has been found to be 25 in 1000 (i.e., 1 in 40). We may then think of this situation as being represented by a binomial distribution $(p+q)^N$, where $N = 432$ (18 inspections per month for 24 months), $p = .025$, $q = .975$ (i.e., $q = 1-p$, the probability of nonoccurrence). But the data available to us is monthly data by squadron, and we are interested in determining a squadron demand rate per month. The monthly demand data for the actuator looks like this: 0-2-0-0-1-0-0-0-1-0-0-1-0-0-2-0-0-1-1-0-0-1-0-1. These numbers are ordered in time, the first observation being January, 1956 and the last one December, 1957.

This is a "natural" for the use of the Poisson approximation to the binomial, since Np is large relative to p (432 to 1) and N is large relative to Np (40 to 1). If we assume that the Poisson law describes the data and use as the parameter of the Poisson the mean demand per squadron month, i.e., $18 \times .025 = .45$, we will find a satisfactory fit with 15 months of no demand, 7 months of demand for one part, and 2 months of demand for 2 parts.

The Poisson (like the binomial) is a distribution of a discrete variable arising from enumeration data using integral values only. Its basic characteristic is the uniform probability of the occurrence of the event. In contrast to both the binomial and the normal distribution, it is defined by a single parameter, the mean. The variance is equal to the mean. The Poisson density is represented by the probability function $P(x) = e^{-m} m^x / x!$, where $x = 0, 1, 2, 3, \dots$, and $m = \text{mean}$.

Tables of the Poisson distribution have been widely published; for example see [9], [10], [11].

3. THE NEGATIVE BINOMIAL DISTRIBUTION

But the aircraft demand data described in Sec. 2 does not present this picture for many of the parts, and the Poisson distribution often has not been a good fit for the series of observations. In many cases, the lack of fit was manifested in more months of zero demand than that described by the Poisson distribution. It was also true

that we were getting more variation than was permitted under the Poisson assumption. For these reasons and others that will be discussed later, we used a distribution known as the negative binomial to fit the observed data. It is a two-parameter distribution of a discrete variable.

The following example illustrates this distribution. The monthly demand data from a squadron of aircraft for a door assembly shows the following series of demands over a 36-month period: 0-1-0-0-0-0-0-0-0-4-0-0-, 0-0-0-1-0-0-0-0-0-0-3-0-, 0-0-0-0-1-0-0-0-0-2-0-0. The sum of the demands is 12. The mean demand per month is one-third. The assumption of a Poisson distribution, using $1/3$ as the parameter value, does not give a good fit to the data. The negative binomial distribution, using the calculation of moments of the observed distribution to estimate the parameters, gives a good fit to the data. The ratio of variance to mean is about 2.4. If we use an arbitrary ratio of variance to mean of 2, the fit is also good. The interpretation of "good fit" means that the amount of discrepancy between the observed values and the theoretical values based on the assumed distribution is not large enough to indicate the presence of anything more than the caprices of random sampling. In other words, the hypothesis is upheld that this data could have been obtained from a population in which monthly demand was described by a negative binomial distribution.

The negative binomial distribution is completely defined by two parameters, the arithmetic mean m and a positive exponent k . The distribution is written $(q - p)^{-k}$ where $p = m/k$ and $p + 1 = q$. The general term in the expansion of this binomial gives the probability P that an observation x will have values $0, 1, 2, \dots$. The general term may be written

$$P(x) = \frac{(k+x-1)!}{(k-1)!x!} \frac{p^x}{q^{k+x}}, \quad x = 0, 1, 2, \dots, P, \quad k > 0.$$

The curve defined by the value of $P(x)$ is unimodal, so that in fitting the negative binomial to an observed distribution, any apparent bimodal or multimodal tendency is attributed to random sampling. The negative binomial is an extension of the Poisson series in which the population mean m is not constant but varies continuously in a distribution which is proportional to that of Chi-square (The distribution referred to is called Pearson Type III or Gamma distribution). Thus the negative binomial may be used to represent a composite of several Poisson distributions in which the number of observations per unit time in repeated counts cannot be assumed to have the same expected value (mean) in each unit of measurement. Student [12] wrote in 1919 as follows: "If the presence of one individual in a division increases the chance of other individuals falling into that division, a negative binomial will fit best, but if it decreases the chance, a positive

binomial." Bliss reported [13] on fitting the negative binomial to biological data: "The negative binomial is the easiest to compute and the most widely applicable of the distributions for over-dispersion."

The negative binomial has been used in a variety of applications to biological data. It was used by Bliss [13] to fit a distribution function to counts of red mites on apple leaves. It was used by Morgan et al [14] to describe the distribution of bacterial clumps over a milk film. Student [15] used it for a description of the counting of yeast cells with a haemocytometer. Stirratt [16] et al. found that it was adequate to describe the distribution of corn borers in a field experiment. Greenwood and Yule [17] used a negative binomial to describe the distribution of accidents experienced by machinists. (The theory was that some machinists were more accident-prone than others. There was also the possibility that shop conditions differed from week to week in such a way as to cause accident risks to vary during successive weeks of the period covered by the data. In either case, the resulting distribution might be expected to be of the form of a negative binomial.) In plant ecology, quadrant counts which deviated from the Poisson distribution were attributed to the occurrence of plants in "clumps." Blackman [18] found that the distribution of plants per quadrant agreed very well with a negative binomial distribution, with estimates of parameters derived from the sample data. Jones, Mollison, and

Quenouille [19] used the negative binomial in fitting counts of soil bacteria. Sichel [20] made use of a negative binomial in studying psychological data on the occurrence of minor accidents in an industrial plant.

Additional references could be cited, but our purpose is merely to document the fact that the negative binomial distribution has been used for years in widely divergent fields of application.

Here we are interested in the application of this particular distribution to the demand for aircraft spare parts. We shall first discuss the rationale which leads us to believe that it may offer a reasonable description of spare-parts demand.

Suppose that two-years' data on demand for spare parts is available at a base. For illustrative purposes, we will say that it is a base with four squadrons, each consisting of 18 aircraft. That is, the data covers the demands of 72 planes for 24 months. If all the aircraft were identical, of the same age, flew identical missions, underwent identical servicing, were subject to the same maintenance practices, etc., the demand for actuator parts might be expected to be about the same as in the illustrative example on page 4. In this case, the monthly demands would follow the Poisson distribution. If the population were homogeneous, random samples of demand data would be expected to exhibit only the sampling fluctuations inherent in any well-behaved variable. But the homogeneity

described above is not present in the Air Force environment in which the demands are generated. For example, even though the planes fly equal numbers of hours, make the same number of landings, etc., some planes are nevertheless subject to greater stresses than others because of factors of speed, altitude, rate of climb, etc. The probability of failure of a particular part is no longer constant, but has increased due to the stress factor. The recorded demands at the base for this part over the 24-month period might be as follows: 1-0-7-0-0-3-1-2-0-0-6-1-, 0-1-0-4-1-1-2-0-9-2-0-3. The sum of the demands at the base is 44. There are four squadrons, so that the demand rate per squadron month is the same as in the Poisson-fitted example on p. 4.

For these demand data, the Poisson is not a good fit. If we use the χ^2 statistic to test the agreement of the observed distribution with the theoretical Poisson distribution, the computed χ^2 value for 1 d.f. is 8.3. The probability of obtaining a χ^2 value of this magnitude or greater in drawing random samples from a homogeneous population is less than .01. Such large values of chi-square may signify no more than the presence of an unusually divergent sample, but to the investigator who is none too sure of his hypothesis, the presence of repeated large chi-square values indicates that the hypothesis should be rejected.

However, if we use the negative binomial probability distribution with the same mean demand per month and assume that the ratio of variance to mean is 3, we will obtain a good fit for the sample data. (χ^2 is .17 for 1 d.f., $P = .70$.) The discrepancy between the observed frequency distribution of demands per month and the theoretical frequencies based on the negative binomial is very small indeed. The ratio of sample variance to sample mean in this case was about 3.2. A word of caution may be appropriate here concerning inferences made about the population on the basis of the chi-square test of goodness of fit. The viewpoint adopted is that the hypothesis is fixed—namely, that a negative binomial distribution describes the population of monthly demands for the part. The probability evaluated above is that of drawing from the population a sample more extreme than the one in hand. It is not a method for evaluating the probability that the hypothesis is correct. Of course, it is true that after the evidence from samples is accumulated the analyst-researcher must make a decision about the hypothesis, and his decision has some presumably high probability of being correct; but we have not presented a method of evaluating such a probability.

There are many factors in the Air Force environment that contribute to heterogeneity of demands. Among these factors we might list age of aircraft, applicability of parts, flying-program elements, nature of mission,

servicing schedules, maintenance practices, design change and modifications, changes in base personnel, etc. A crew mechanic inspecting plane number 1 may not have the same careful standards as the mechanic who inspects number 2. Some inspectors are prone to replace parts—some are not. Demands for certain spare parts have also been known to exhibit a tendency to occur in clusters—that is, the demand for a unit of part a stimulates demand for a unit of part b, which in turn creates a demand for 4 units of part c, etc. All of these factors change with time, and are reflected in the sample data.

4. TABLES OF NEGATIVE BINOMIAL DISTRIBUTION

We have found tables of the negative binomial distribution useful in our work in logistics. Since they may not be readily accessible, the complete probability distributions for a limited number of arbitrary parameter values are presented in the following pages. We have used 13 different values of the mean ($m = kp$) and 10 different ratios of variance-to-mean (q). This gives us 130 sets of parameter values for which the complete probability distribution is tabulated.

The distribution function of the negative binomial is

$$P(x) = \frac{(k+x-1)!}{(k-1)! x!} \frac{p^k q^x}{q^{k+x}}$$

where $x = 0, 1, 2, 3, \dots$, $p, k > 0$, and $q = 1 + p$.

Part 1 gives the individual terms of the distributions and Part 2 gives the cumulative probability of x demands or less. The values of the mean (kp) are .25(.25)1.0 and 1.0(1.0)10.0. The values of the ratio of variance to mean (q) are 1.5(.5)5.0 and 5.0(1.0)7.0. The choice of these values makes p vary from $1/2$ to 6 and k from $\frac{1}{24}$ to 20 .

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Probability of x demands for $q = 1.5$

[illegible]

Probability of x demands for $q = 2$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8409	.7071	.5946	.5000	.2500	.1250	.0625	.0312	.0156	.0078	.0039	.0020	.0010
1		.1051	.1768	.2230	.2500	.2500	.1875	.1250	.0781	.0459	.0273	.0156	.0088	.0049
2		.0328	.0663	.0976	.1250	.1875	.1875	.1562	.1172	.0820	.0547	.0352	.0220	.0134
3		.0123	.0276	.0447	.0625	.1250	.1563	.1562	.1367	.1094	.0820	.0586	.0403	.0269
4		.0050	.0121	.0210	.0312	.0781	.1172	.1367	.1367	.1230	.1025	.0806	.0604	.0436
5		.0021	.0054	.0100	.0156	.0469	.0820	.1094	.1230	.1230	.1128	.0967	.0786	.0611
6		.0009	.0025	.0048	.0078	.0273	.0547	.0820	.1025	.1128	.1128	.1047	.0916	.0764
7		.0004	.0012	.0023	.0039	.0156	.0352	.0586	.0806	.0967	.1047	.1047	.0982	.0873
8		.0002	.0005	.0011	.0020	.0088	.0220	.0403	.0604	.0786	.0916	.0982	.0982	.0927
9		.0001	.0003	.0005	.0010	.0049	.0134	.0269	.0436	.0611	.0764	.0873	.0927	.0927
10			.0001	.0003	.0005	.0027	.0081	.0175	.0305	.0459	.0611	.0742	.0835	.0881
11			.0001	.0001	.0002	.0015	.0048	.0111	.0208	.0331	.0472	.0607	.0721	.0801
12				.0001	.0001	.0008	.0028	.0069	.0139	.0236	.0354	.0481	.0601	.0701
13					.0001	.0004	.0016	.0043	.0091	.0163	.0259	.0370	.0485	.0593
14						.0002	.0009	.0026	.0058	.0111	.0185	.0277	.0381	.0487
15						.0001	.0005	.0016	.0037	.0074	.0129	.0203	.0292	.0390
16						.0001	.0003	.0009	.0023	.0049	.0089	.0146	.0219	.0304
17							.0002	.0005	.0014	.0031	.0060	.0103	.0161	.0233
18							.0001	.0003	.0009	.0020	.0040	.0072	.0116	.0175
19							.0001	.0002	.0005	.0013	.0026	.0049	.0082	.0129
20								.0001	.0003	.0008	.0017	.0033	.0058	.0093
21								.0001	.0002	.0005	.0011	.0022	.0040	.0067
22									.0001	.0003	.0007	.0015	.0027	.0047
23									.0001	.0002	.0004	.0009	.0018	.0033
24										.0001	.0003	.0006	.0012	.0022
25										.0001	.0002	.0004	.0008	.0015

Probability of x demands for $q = 2.5$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8584	.7368	.6325	.5429	.2947	.1600	.0869	.0472	.0256	.0139	.0075	.0041	.0022
1		.0858	.1474	.1897	.2172	.2358	.1920	.1390	.0943	.0614	.0389	.0241	.0147	.0089
2		.0300	.0589	.0854	.1086	.1650	.1728	.1529	.1226	.0922	.0662	.0459	.0310	.0205
3		.0130	.0275	.0427	.0579	.1100	.1382	.1427	.1308	.1106	.0882	.0673	.0495	.0355
4		.0062	.0138	.0224	.0318	.0715	.1037	.1213	.1242	.1161	.1014	.0841	.0669	.0514
5		.0031	.0072	.0121	.0178	.0458	.0746	.0970	.1093	.1115	.1055	.0941	.0803	.0658
6		.0016	.0038	.0067	.0101	.0290	.0523	.0744	.0911	.1003	.1020	.0973	.0883	.0768
7		.0008	.0021	.0037	.0058	.0182	.0358	.0553	.0729	.0860	.0932	.0946	.0908	.0834
8		.0005	.0011	.0021	.0033	.0114	.0242	.0401	.0565	.0709	.0816	.0875	.0885	.0854
9		.0002	.0006	.0012	.0019	.0071	.0161	.0285	.0427	.0568	.0689	.0777	.0826	.0835
10		.0001	.0004	.0007	.0011	.0044	.0106	.0199	.0316	.0443	.0565	.0669	.0744	.0785
11		.0001	.0002	.0004	.0006	.0027	.0070	.0138	.0230	.0338	.0452	.0559	.0649	.0714
12			.0001	.0002	.0004	.0017	.0045	.0094	.0165	.0254	.0354	.0457	.0552	.0631
13			.0001	.0001	.0002	.0010	.0029	.0064	.0116	.0187	.0272	.0365	.0458	.0543
14				.0001	.0001	.0006	.0019	.0043	.0081	.0136	.0206	.0287	.0373	.0458
15					.0001	.0004	.0012	.0029	.0057	.0098	.0154	.0222	.0293	.0379
16						.0002	.0008	.0019	.0039	.0070	.0114	.0169	.0235	.0308
17						.0001	.0005	.0012	.0027	.0049	.0083	.0127	.0183	.0246
18						.0001	.0003	.0008	.0018	.0035	.0060	.0095	.0140	.0194
19						.0001	.0002	.0005	.0012	.0024	.0043	.0070	.0106	.0151
20							.0001	.0003	.0008	.0017	.0030	.0051	.0080	.0116
21							.0001	.0002	.0005	.0011	.0021	.0037	.0059	.0089
22								.0001	.0004	.0008	.0015	.0027	.0044	.0067
23								.0001	.0002	.0005	.0010	.0019	.0032	.0050
24								.0001	.0002	.0004	.0007	.0013	.0023	.0037
25									.0001	.0002	.0005	.0009	.0017	.0027

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Probability of x demands for q = 3.0

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8717	.7598	.6623	.5774	.3333	.1925	.1111	.0642	.0370	.0214	.0123	.0071	.0041
1		.0726	.1266	.1656	.1925	.2222	.1925	.1481	.1069	.0741	.0499	.0329	.0214	.0137
2		.0272	.0528	.0759	.0962	.1481	.1604	.1481	.1247	.0988	.0748	.0549	.0392	.0274
3		.0129	.0264	.0401	.0535	.0988	.1247	.1317	.1247	.1097	.0915	.0732	.0566	.0427
4		.0067	.0143	.0225	.0312	.0658	.0936	.1097	.1143	.1097	.0991	.0854	.0708	.0569
5		.0037	.0081	.0131	.0187	.0439	.0686	.0878	.0991	.1024	.0991	.0910	.0802	.0683
6		.0021	.0047	.0078	.0114	.0293	.0495	.0683	.0826	.0910	.0936	.0910	.0847	.0759
7		.0012	.0028	.0048	.0071	.0195	.0354	.0520	.0669	.0780	.0847	.0867	.0847	.0795
8		.0007	.0017	.0029	.0044	.0130	.0251	.0390	.0529	.0650	.0741	.0795	.0811	.0795
9		.0004	.0010	.0018	.0028	.0087	.0176	.0289	.0412	.0530	.0631	.0707	.0751	.0765
10		.0003	.0006	.0011	.0018	.0058	.0123	.0212	.0316	.0424	.0526	.0612	.0676	.0714
11		.0002	.0004	.0007	.0011	.0039	.0086	.0154	.0239	.0334	.0430	.0520	.0594	.0649
12		.0001	.0002	.0005	.0007	.0026	.0060	.0111	.0179	.0260	.0347	.0433	.0512	.0577
13		.0001	.0002	.0003	.0005	.0017	.0041	.0080	.0133	.0200	.0276	.0355	.0433	.0503
14			.0001	.0002	.0003	.0011	.0029	.0057	.0098	.0152	.0217	.0287	.0361	.0431
15			.0001	.0001	.0002	.0008	.0020	.0041	.0072	.0115	.0168	.0230	.0297	.0364
16				.0001	.0001	.0005	.0014	.0029	.0053	.0086	.0130	.0182	.0241	.0304
17					.0001	.0003	.0009	.0020	.0038	.0064	.0099	.0142	.0194	.0250
18					.0001	.0002	.0006	.0014	.0028	.0048	.0075	.0111	.0154	.0204
19						.0002	.0004	.0010	.0020	.0035	.0057	.0086	.0122	.0164
20						.0001	.0003	.0007	.0014	.0026	.0043	.0066	.0095	.0132
21						.0001	.0002	.0005	.0010	.0019	.0032	.0050	.0074	.0104
22							.0001	.0003	.0007	.0014	.0024	.0038	.0057	.0082
23							.0001	.0002	.0005	.0010	.0017	.0029	.0044	.0064
24							.0001	.0002	.0004	.0007	.0013	.0021	.0034	.0050
25								.0001	.0003	.0005	.0009	.0016	.0026	.0039

Probability of x demands for $q = 3.5$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8823	.7784	.6867	.6059	.3671	.2224	.1347	.0816	.0495	.0300	.0182	.0110	.0067
1		.0630	.1112	.1472	.1731	.2098	.1906	.1540	.1166	.0848	.0599	.0415	.0283	.0190
2		.0248	.0477	.0683	.0866	.1348	.1498	.1430	.1249	.1030	.0813	.0622	.0465	.0340
3		.0124	.0296	.0374	.0495	.0899	.1141	.1226	.1190	.1079	.0930	.0771	.0620	.0486
4		.0069	.0143	.0220	.0300	.0610	.0856	.1007	.1062	.1040	.0963	.0853	.0730	.0607
5		.0040	.0086	.0135	.0189	.0418	.0636	.0805	.0911	.0951	.0935	.0878	.0793	.0694
6		.0024	.0053	.0085	.0121	.0289	.0469	.0633	.0759	.0838	.0868	.0857	.0812	.0743
7		.0015	.0034	.0055	.0079	.0200	.0345	.0491	.0620	.0718	.0780	.0804	.0795	.0759
8		.0010	.0022	.0036	.0052	.0140	.0252	.0377	.0498	.0603	.0682	.0732	.0752	.0745
9		.0006	.0014	.0024	.0035	.0097	.0184	.0287	.0395	.0497	.0585	.0651	.0693	.0710
10		.0004	.0009	.0016	.0023	.0068	.0134	.0217	.0310	.0405	.0493	.0567	.0623	.0659
11		.0003	.0006	.0010	.0016	.0048	.0098	.0164	.0242	.0326	.0410	.0487	.0551	.0599
12		.0002	.0004	.0007	.0011	.0034	.0071	.0123	.0187	.0260	.0337	.0411	.0479	.0535
13		.0001	.0003	.0005	.0007	.0024	.0051	.0092	.0144	.0206	.0274	.0343	.0410	.0470
14		.0001	.0002	.0003	.0005	.0017	.0037	.0068	.0110	.0162	.0221	.0284	.0347	.0408
15		.0001	.0001	.0002	.0003	.0012	.0027	.0051	.0084	.0126	.0176	.0232	.0291	.0350
16			.0001	.0002	.0002	.0008	.0020	.0038	.0064	.0098	.0140	.0189	.0242	.0296
17			.0001	.0001	.0002	.0006	.0014	.0028	.0048	.0076	.0111	.0152	.0199	.0249
18				.0001	.0001	.0004	.0010	.0021	.0036	.0058	.0087	.0122	.0153	.0208
19					.0001	.0003	.0007	.0015	.0027	.0045	.0068	.0097	.0132	.0172
20					.0001	.0002	.0005	.0011	.0020	.0034	.0053	.0077	.0107	.0145
21						.0001	.0004	.0008	.0015	.0026	.0041	.0061	.0086	.0115
22						.0001	.0003	.0006	.0011	.0020	.0032	.0048	.0068	.0093
23						.0001	.0002	.0004	.0009	.0015	.0024	.0037	.0054	.0075
24						.0001	.0001	.0003	.0006	.0011	.0019	.0029	.0043	.0061
25							.0001	.0002	.0005	.0009	.0014	.0023	.0034	.0049

PROBABILITY OF x DEMANDS FOR $q = 4.5$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8981	.8066	.7245	.6507	.4234	.2755	.1793	.1166	.0759	.0494	.0321	.0209	.0136
1		.0499	.0896	.1207	.1446	.1882	.1837	.1593	.1296	.1012	.0768	.0571	.0418	.0302
2		.0208	.0398	.0570	.0723	.1150	.1326	.1328	.1224	.1068	.0896	.0730	.0581	.0453
3		.0112	.0221	.0327	.0428	.0767	.0983	.1082	.1088	.1029	.0929	.0811	.0688	.0571
4		.0067	.0135	.0205	.0274	.0532	.0737	.0872	.0937	.0943	.0904	.0834	.0746	.0650
5		.0042	.0087	.0134	.0182	.0379	.0557	.0697	.0791	.0838	.0843	.0815	.0762	.0694
6		.0028	.0058	.0091	.0125	.0273	.0423	.0555	.0659	.0729	.0765	.0770	.0748	.0707
7		.0019	.0040	.0063	.0087	.0200	.0322	.0441	.0544	.0625	.0680	.0709	.0713	.0695
8		.0013	.0028	.0044	.0062	.0147	.0246	.0349	.0446	.0530	.0595	.0640	.0663	.0666
9		.0009	.0019	.0031	.0044	.0109	.0189	.0276	.0363	.0445	.0514	.0569	.0606	.0625
10		.0006	.0014	.0022	.0032	.0081	.0144	.0217	.0295	.0371	.0440	.0499	.0545	.0577
11		.0005	.0010	.0016	.0023	.0061	.0111	.0171	.0238	.0307	.0373	.0434	.0485	.0524
12		.0003	.0007	.0012	.0017	.0045	.0085	.0135	.0192	.0253	.0315	.0373	.0426	.0471
13		.0002	.0005	.0009	.0013	.0034	.0066	.0106	.0154	.0208	.0264	.0319	.0372	.0418
14		.0002	.0004	.0006	.0009	.0026	.0050	.0083	.0124	.0170	.0220	.0271	.0321	.0369
15		.0001	.0003	.0005	.0007	.0019	.0039	.0065	.0099	.0138	.0182	.0229	.0276	.0322
16		.0001	.0002	.0003	.0005	.0015	.0030	.0051	.0079	.0112	.0151	.0192	.0236	.0280
17		.0001	.0002	.0003	.0004	.0011	.0023	.0040	.0063	.0091	.0124	.0161	.0200	.0241
18			.0001	.0002	.0003	.0008	.0018	.0032	.0050	.0074	.0102	.0134	.0170	.0207
19			.0001	.0001	.0002	.0006	.0014	.0025	.0040	.0059	.0083	.0111	.0143	.0177
20			.0001	.0001	.0002	.0005	.0011	.0020	.0032	.0048	.0068	.0091	.0120	.0150
21				.0001	.0001	.0004	.0008	.0015	.0025	.0038	.0055	.0076	.0100	.0127
22					.0001	.0003	.0006	.0012	.0020	.0031	.0045	.0063	.0083	.0107
23						.0002	.0005	.0009	.0016	.0025	.0037	.0051	.0069	.0090
24						.0002	.0004	.0007	.0012	.0020	.0030	.0042	.0057	.0076
25						.0001	.0003	.0006	.0010	.0016	.0024	.0034	.0047	.0063

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.9043	.8178	.7395	.6687	.4472	.2991	.2000	.1337	.0894	.0598	.0400	.0267	.0179
1		.0452	.0818	.1109	.1337	.1789	.1794	.1600	.1337	.1073	.0837	.0640	.0481	.0358
2		.0192	.0368	.0527	.0669	.1073	.1256	.1280	.1204	.1073	.0921	.0768	.0626	.0501
3		.0106	.0209	.0307	.0401	.0716	.0921	.1024	.1043	.1002	.0921	.0819	.0709	.0601
4		.0065	.0130	.0196	.0261	.0501	.0691	.0819	.0887	.0902	.0875	.0819	.0745	.0661
5		.0042	.0086	.0131	.0177	.0361	.0525	.0655	.0745	.0793	.0805	.0786	.0745	.0688
6		.0028	.0059	.0091	.0124	.0264	.0402	.0524	.0621	.0688	.0725	.0734	.0720	.0688
7		.0020	.0041	.0064	.0089	.0196	.0311	.0419	.0514	.0589	.0642	.0671	.0679	.0668
8		.0014	.0029	.0046	.0064	.0147	.0241	.0336	.0424	.0501	.0562	.0604	.0628	.0635
9		.0010	.0021	.0034	.0047	.0111	.0187	.0268	.0349	.0423	.0487	.0537	.0572	.0592
10		.0007	.0015	.0025	.0035	.0085	.0146	.0215	.0286	.0355	.0419	.0472	.0515	.0545
11		.0005	.0011	.0018	.0026	.0065	.0114	.0172	.0234	.0297	.0358	.0412	.0459	.0495
12		.0004	.0008	.0014	.0020	.0050	.0089	.0137	.0191	.0248	.0304	.0357	.0405	.0446
13		.0003	.0006	.0010	.0015	.0038	.0070	.0110	.0156	.0206	.0257	.0308	.0355	.0398
14		.0002	.0005	.0008	.0011	.0029	.0055	.0088	.0127	.0170	.0217	.0264	.0310	.0352
15		.0002	.0004	.0006	.0008	.0023	.0043	.0070	.0103	.0141	.0182	.0225	.0268	.0310
16		.0001	.0003	.0004	.0006	.0018	.0034	.0056	.0084	.0116	.0153	.0191	.0231	.0271
17		.0001	.0002	.0004	.0005	.0014	.0027	.0045	.0068	.0096	.0127	.0162	.0199	.0236
18		.0001	.0002	.0003	.0004	.0011	.0021	.0036	.0055	.0079	.0106	.0137	.0170	.0205
19		.0001	.0001	.0003	.0003	.0008	.0017	.0029	.0045	.0065	.0088	.0115	.0145	.0177
20			.0001	.0002	.0002	.0006	.0013	.0023	.0036	.0053	.0073	.0097	.0123	.0152
21				.0002	.0002	.0005	.0010	.0018	.0029	.0043	.0061	.0081	.0104	.0130
22				.0001	.0001	.0004	.0008	.0015	.0024	.0036	.0050	.0068	.0088	.0111
23				.0001	.0001	.0003	.0007	.0012	.0019	.0029	.0041	.0057	.0075	.0095
24					.0001	.0002	.0005	.0009	.0016	.0024	.0034	.0047	.0063	.0081
25						.0002	.0004	.0008	.0013	.0019	.0028	.0039	.0053	.0068

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

[illegible]

x
Cumulative probability ΣP for $q = 2.0$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8409	.7071	.5946	.5000	.2500	.1250	.0625	.0312	.0156	.0078	.0039	.0020	.0010
1		.9460	.8839	.8176	.7500	.5000	.3125	.1875	.1094	.0625	.0352	.0195	.0107	.0059
2		.9789	.9502	.9151	.8750	.6875	.5000	.3438	.2266	.1445	.0898	.0547	.0327	.0193
3		.9912	.9778	.9598	.9375	.8125	.6562	.5000	.3633	.2539	.1719	.1133	.0730	.0461
4		.9962	.9899	.9808	.9688	.8906	.7734	.6367	.5000	.3770	.2744	.1938	.1334	.0898
5		.9983	.9953	.9908	.9844	.9375	.8555	.7471	.6230	.5000	.3872	.2905	.2120	.1509
6		.9992	.9978	.9955	.9922	.9648	.9102	.8281	.7256	.6128	.5000	.3953	.3036	.2272
7		.9997	.9989	.9978	.9961	.9805	.9453	.8867	.8062	.7095	.6047	.5000	.4018	.3145
8		.9998	.9995	.9989	.9980	.9893	.9673	.9270	.8666	.7880	.6964	.5982	.5000	.4073
9		.9999	.9998	.9995	.9990	.9941	.9807	.9539	.9102	.8491	.7728	.6855	.5927	.5000
10			.9999	.9997	.9995	.9968	.9888	.9713	.9408	.8949	.8338	.7597	.6762	.5881
11				.9999	.9998	.9983	.9935	.9824	.9616	.9283	.8811	.8204	.7483	.6682
12					.9999	.9991	.9963	.9894	.9755	.9519	.9165	.8684	.8083	.7333
14						.9995	.9979	.9936	.9846	.9682	.9423	.9054	.8569	.7976
15						.9997	.9988	.9962	.9904	.9793	.9608	.9331	.8950	.8463
16						.9999	.9993	.9977	.9941	.9867	.9738	.9534	.9242	.8852
17							.9996	.9987	.9964	.9915	.9827	.9680	.9461	.9157
18							.9998	.9992	.9978	.9947	.9837	.9784	.9622	.9390
19							.9999	.9996	.9987	.9967	.9927	.9855	.9739	.9564
20								.9998	.9992	.9980	.9953	.9904	.9822	.9693
21								.9999	.9995	.9988	.9970	.9937	.9879	.9786
22									.9997	.9992	.9981	.9959	.9919	.9853
23									.9998	.9995	.9988	.9974	.9947	.9900
24									.9999	.9997	.9993	.9983	.9965	.9932
25										.9998	.9996	.9989	.9977	.9955
										.9999	.9997	.9993	.9985	.9970

$\sum_0^x$ P for $q = 2.5$
Cumulative probability

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8584	.7368	.6325	.5429	.2947	.1600	.0869	.0472	.0256	.0139	.0075	.0041	.0022
1		.9442	.8842	.8222	.7600	.5305	.3520	.2258	.1415	.0870	.0528	.0317	.0188	.0111
2		.9743	.9431	.9076	.8686	.6955	.5248	.3787	.2641	.1792	.1190	.0776	.0498	.0316
3		.9873	.9706	.9503	.9265	.8056	.6530	.5214	.3948	.2898	.2072	.1448	.0994	.0670
4		.9935	.9844	.9727	.9584	.8771	.7667	.6427	.5191	.4059	.3086	.2289	.1662	.1185
5		.9965	.9915	.9848	.9762	.9229	.8414	.7397	.6284	.5174	.4141	.3231	.2465	.1843
6		.9981	.9953	.9914	.9863	.9519	.8936	.8141	.7195	.6177	.5161	.4205	.3348	.2610
7		.9990	.9974	.9951	.9921	.9701	.9295	.8694	.7924	.7037	.6093	.5150	.4256	.3444
8		.9994	.9985	.9972	.9954	.9815	.9536	.9094	.8489	.7747	.6909	.6025	.5141	.4299
9		.9997	.9992	.9984	.9973	.9886	.9698	.9379	.8916	.8314	.7598	.6802	.5968	.5134
10		.9998	.9995	.9991	.9984	.9929	.9804	.9578	.9231	.8757	.8163	.7471	.6712	.5919
11		.9999	.9997	.9995	.9991	.9957	.9874	.9716	.9461	.9095	.8615	.8030	.7361	.6633
12			.9998	.9997	.9995	.9973	.9919	.9810	.9626	.9349	.8969	.8487	.7912	.7264
13			.9999	.9998	.9997	.9984	.9948	.9874	.9742	.9536	.9241	.8852	.8371	.7807
14				.9999	.9998	.9990	.9967	.9917	.9824	.9672	.9447	.9139	.8744	.8265
15					.9999	.9994	.9979	.9945	.9881	.9770	.9601	.9361	.9043	.8644
16						.9996	.9987	.9964	.9919	.9840	.9715	.9530	.9278	.8952
17						.9998	.9992	.9977	.9946	.9890	.9797	.9658	.9460	.9198
18						.9999	.9995	.9985	.9964	.9924	.9857	.9753	.9600	.9392
19							.9997	.9990	.9976	.9948	.9900	.9822	.9706	.9543
20							.9998	.9994	.9984	.9965	.9930	.9874	.9786	.9659
21							.9999	.9996	.9990	.9976	.9952	.9910	.9845	.9748
22								.9997	.9993	.9984	.9967	.9937	.9889	.9815
23								.9998	.9996	.9989	.9977	.9956	.9920	.9865
24								.9999	.9997	.9993	.9985	.9969	.9943	.9902
25									.9998	.9995	.9990	.9979	.9960	.9930

Cumulative probability $\sum_{0}^x P$ for $q = 3.0$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8717	.7598	.6623	.5774	.3333	.1925	.1111	.0642	.0370	.0214	.0123	.0071	.0041
1		.9443	.8865	.8279	.7698	.5556	.3849	.2593	.1711	.1111	.0713	.0453	.0285	.0178
2		.9716	.9392	.9038	.8660	.7037	.5453	.4074	.2958	.2099	.1461	.1001	.0677	.0453
3		.9844	.9656	.9439	.9195	.8025	.6700	.5391	.4205	.3196	.2376	.1733	.1243	.0879
4		.9911	.9799	.9664	.9407	.8683	.7636	.6488	.5349	.4294	.3367	.2586	.1951	.1448
5		.9948	.9880	.9795	.9694	.9122	.8322	.7366	.6340	.5318	.4358	.3497	.2753	.2131
6		.9969	.9927	.9874	.9808	.9415	.8817	.8049	.7166	.6228	.5294	.4407	.3600	.2890
7		.9981	.9955	.9922	.9879	.9610	.9171	.8569	.7834	.7009	.6141	.5274	.4447	.3685
8		.9989	.9972	.9951	.9923	.9740	.9422	.8960	.8363	.7659	.6881	.6069	.5258	.4480
9		.9993	.9983	.9969	.9951	.9827	.9598	.9249	.8775	.8189	.7513	.6776	.6010	.5245
10		.9996	.9989	.9980	.9969	.9884	.9722	.9460	.9091	.8613	.8039	.7388	.6686	.5959
11		.9997	.9993	.9988	.9980	.9923	.9808	.9615	.9330	.8947	.8469	.7908	.7280	.6609
12		.9998	.9996	.9992	.9987	.9949	.9868	.9726	.9509	.9206	.8816	.8341	.7792	.7186
13		.9999	.9997	.9995	.9992	.9966	.9909	.9806	.9642	.9406	.9091	.8696	.8225	.7689
14			.9998	.9997	.9995	.9977	.9937	.9863	.9741	.9558	.9308	.8983	.8586	.8121
15			.9999	.9998	.9996	.9985	.9957	.9904	.9813	.9674	.9476	.9213	.8883	.8485
16				.9999	.9998	.9990	.9971	.9932	.9865	.9760	.9606	.9396	.9124	.8788
17					.9999	.9993	.9980	.9953	.9904	.9824	.9705	.9538	.9317	.9038
18						.9995	.9986	.9967	.9931	.9872	.9780	.9649	.9472	.9242
19						.9997	.9991	.9977	.9951	.9907	.9837	.9735	.9594	.9406
20						.9998	.9994	.9984	.9965	.9935	.9880	.9801	.9689	.9538
21						.9999	.9996	.9989	.9975	.9951	.9912	.9851	.9763	.9642
22							.9997	.9992	.9983	.9965	.9935	.9889	.9821	.9725
23							.9998	.999	.9988	.9975	.9953	.9918	.9865	.9789
24							.9999	.9996	.9991	.9982	.9966	.9939	.9898	.9839
25								.9997	.9994	.987	.9975	.995	.9974	.990

$\sum_0^x$ Cumulative probability ΣP for $q = 3.5$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8823	.7784	.6867	.6059	.3671	.2224	.1347	.0816	.0495	.0300	.0182	.0110	.0067
1		.9453	.8896	.8339	.7790	.5768	.4130	.2887	.1983	.1342	.0899	.0597	.0393	.0257
2		.9700	.9372	.9022	.8655	.7117	.5628	.4317	.3232	.2372	.1712	.1219	.0857	.0597
3		.9824	.9622	.9396	.9150	.8016	.6769	.5543	.4422	.3451	.2642	.1990	.1477	.1083
4		.9893	.9764	.9617	.9450	.8626	.7625	.6549	.5484	.4491	.3604	.2843	.2207	.1690
5		.9933	.9850	.9752	.9639	.9044	.8261	.7355	.6395	.5441	.4540	.3720	.3000	.2384
6		.9957	.9903	.9837	.9760	.9333	.8730	.7988	.7154	.6279	.5408	.4577	.3812	.3127
7		.9972	.9937	.9892	.9839	.9533	.9075	.8478	.7774	.6997	.6188	.5381	.4607	.3883
8		.9982	.9958	.9922	.9892	.9673	.9327	.8855	.8271	.7600	.6870	.6114	.5359	.4631
9		.9988	.9972	.9952	.9927	.9770	.9511	.9142	.8667	.8097	.7455	.6765	.6052	.5340
10		.9992	.9981	.9967	.9950	.9838	.9646	.9360	.8977	.8502	.7948	.7332	.6675	.5999
11		.9995	.9987	.9978	.9966	.9886	.9743	.9524	.9219	.8828	.8358	.7818	.7226	.6598
12		.9996	.9992	.9985	.9977	.9920	.9814	.9646	.9406	.9088	.8694	.8229	.7704	.7133
13		.9998	.9994	.9990	.9984	.9943	.9866	.9738	.9550	.9294	.8968	.8573	.8115	.7603
14		.9998	.9996	.9993	.9989	.9960	.9903	.9806	.9660	.9456	.9189	.8856	.8462	.8011
15		.9999	.9997	.9995	.9992	.9972	.9930	.9857	.9744	.9582	.9365	.9089	.8753	.8361
16			.9998	.9997	.9995	.9950	.9949	.9895	.9808	.9680	.9505	.9278	.8995	.8657
17			.9999	.9998	.9996	.9986	.9964	.9925	.9856	.9756	.9616	.9430	.9194	.8906
18				.9998	.9997	.9990	.9973	.9943	.9892	.9815	.9703	.9552	.9357	.9114
19				.9999	.9998	.9993	.9981	.9958	.9920	.9859	.9771	.9649	.9489	.9285
20					.9999	.9995	.9986	.9969	.9940	.9894	.9824	.9727	.9595	.9427
21						.9996	.9990	.9978	.9956	.9920	.9865	.9787	.9681	.9542
22						.9997	.9993	.9984	.9967	.9940	.9897	.9835	.9749	.9635
23						.9998	.9995	.9988	.9976	.9955	.9921	.9873	.9804	.9710
24						.9999	.9996	.9991	.9982	.9966	.9940	.9902	.9847	.9771
25							.9997	.9994	.9987	.9974	.9955	.9925	.9881	.9820

Cumulative probability $\sum_{0}^x P$ for $q = 4.0$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8909	.7937	.7071	.6300	.3969	.2500	.1575	.0992	.0625	.0394	.0248	.0156	.0098
1		.9466	.8929	.8397	.7875	.5953	.4375	.3150	.2232	.1562	.1083	.0744	.0508	.0344
2		.9692	.9363	.9018	.8662	.7193	.5781	.4528	.3472	.2617	.1944	.1426	.1035	.0744
3		.9810	.9598	.9368	.9121	.8020	.6836	.5676	.4609	.3672	.2877	.2222	.1694	.1278
4		.9878	.9738	.9581	.9408	.8588	.7627	.6609	.5604	.4661	.3810	.3067	.2436	.1911
5		.9920	.9825	.9717	.9595	.8986	.8220	.7356	.6449	.5551	.4697	.3913	.3215	.2607
6		.9946	.9881	.9806	.9719	.9268	.8665	.7947	.7154	.6329	.5509	.4723	.3993	.3333
7		.9963	.9919	.9866	.9804	.9469	.8999	.8411	.7733	.6997	.6235	.5476	.4744	.4058
8		.9975	.9944	.9906	.9862	.9614	.9249	.8774	.8203	.7560	.6809	.6157	.5447	.4761
9		.9983	.9961	.9934	.9902	.9718	.9437	.9056	.8582	.8025	.7416	.6764	.6093	.5425
10		.9988	.9972	.9953	.9930	.9794	.9578	.9274	.8885	.8416	.7881	.7294	.6674	.6039
11		.9992	.9981	.9967	.9950	.9849	.9683	.9443	.9126	.8733	.8271	.7752	.7189	.6597
12		.9994	.9986	.9976	.9964	.9889	.9762	.9574	.9317	.8990	.8597	.8143	.7639	.7097
13		.9996	.9990	.9983	.9974	.9919	.9822	.9674	.9467	.9198	.8866	.8474	.8029	.7539
14		.9997	.9993	.9988	.9982	.9940	.9866	.9751	.9586	.9365	.9087	.8752	.8363	.7926
15		.9998	.9995	.9991	.9987	.9956	.9900	.9810	.9678	.9499	.9268	.8984	.8647	.8262
16		.9998	.9996	.9994	.9990	.9968	.9925	.9855	.9750	.9605	.9415	.9175	.8887	.8550
17		.9999	.9997	.9995	.9993	.9976	.9944	.9889	.9807	.9690	.9533	.9333	.9037	.8796
18			.9998	.9997	.9995	.9982	.9958	.9916	.9851	.9757	.9629	.9463	.9255	.9004
19			.9999	.9998	.9996	.9987	.9968	.9936	.9885	.9810	.9706	.9568	.9394	.9179
20				.9998	.9997	.9990	.9976	.9951	.9911	.9851	.9767	.9654	.9508	.9326
21				.9999	.9998	.9993	.9982	.9963	.9932	.9884	.9816	.9723	.9602	.9449
22					.9999	.9995	.9987	.9972	.9947	.9910	.9855	.9779	.9679	.9550
23						.9996	.9990	.9979	.9960	.9930	.9886	.9824	.9742	.9634
24						.9997	.9992	.9984	.9969	.9945	.9910	.9861	.9793	.9703
25						.9998	.9994	.9988	.9976	.9958	.9930	.9890	.9834	.9760

Cumulative probability $\sum_0^x P$ for $q = 4.5$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.8981	.8066	.7245	.6507	.4234	.2755	.1793	.1166	.0759	.0494	.0321	.0209	.0136
1		.9480	.8963	.8452	.7953	.6116	.4591	.3386	.2462	.1771	.1262	.0893	.0627	.0438
2		.9688	.9361	.9022	.8676	.7265	.5918	.4714	.3686	.2839	.2156	.1622	.1208	.0892
3		.9800	.9582	.9350	.9104	.8032	.6900	.5796	.4774	.3868	.3088	.2434	.1896	.1463
4		.9867	.9718	.9554	.9378	.8564	.7637	.6667	.5711	.4810	.3991	.3267	.2642	.2113
5		.9909	.9805	.9688	.9560	.8943	.8194	.7364	.6502	.5649	.4835	.4082	.3404	.2807
6		.9937	.9863	.9779	.9685	.9216	.8617	.7920	.7162	.6378	.5600	.4852	.4152	.3514
7		.9955	.9903	.9842	.9773	.9416	.8939	.8360	.7706	.7003	.6280	.5560	.4865	.4209
8		.9968	.9930	.9886	.9835	.9563	.9185	.8709	.8152	.7533	.6875	.6200	.5528	.4875
9		.9977	.9949	.9917	.9879	.9672	.9373	.8985	.8515	.7978	.7390	.6769	.6134	.5500
10		.9983	.9963	.9939	.9911	.9753	.9518	.9202	.8810	.8348	.7830	.7268	.6679	.6077
11		.9988	.9973	.9955	.9934	.9814	.9629	.9374	.9048	.8655	.8203	.7702	.7163	.6601
12		.9991	.9980	.9967	.9951	.9859	.9714	.9508	.9240	.8908	.8518	.8075	.7590	.7072
13		.9993	.9985	.9976	.9964	.9893	.9779	.9614	.9394	.9116	.8781	.8394	.7961	.7491
14		.9995	.9989	.9982	.9973	.9919	.9830	.9698	.9517	.9285	.9001	.8665	.8283	.7859
15		.9996	.9992	.9986	.9980	.9938	.9869	.9763	.9616	.9424	.9183	.8894	.8559	.8182
16		.9997	.9994	.9990	.9985	.9953	.9898	.9814	.9695	.9536	.9334	.9086	.8795	.8461
17		.9998	.9996	.9992	.9989	.9964	.9922	.9855	.9758	.9627	.9458	.9247	.8995	.8703
18		.9999	.9997	.9994	.9991	.9973	.9939	.9886	.9808	.9701	.9559	.9381	.9165	.8910
19			.9998	.9996	.9994	.9979	.9953	.9911	.9848	.9760	.9643	.9493	.9308	.9086
20			.9998	.9997	.9995	.9984	.9964	.9930	.9880	.9808	.9711	.9585	.9428	.9237
21			.9999	.9998	.9996	.9988	.9972	.9946	.9905	.9846	.9756	.9661	.9528	.9364
22				.9998	.9997	.9991	.9978	.9957	.9925	.9877	.9811	.9724	.9611	.9471
23				.9999	.9998	.9993	.9983	.9967	.9941	.9902	.9848	.9775	.9680	.9561
24					.9998	.9995	.9987	.9974	.9953	.9922	.9878	.9817	.9738	.9637
25					.9999	.9996	.9990	.9980	.9963	.9938	.9902	.9852	.9785	.9700

Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0	.9043	.8178	.7395	.6687	.4472	.2991	.2000	.1337	.0894	.0598	.0400	.0267	.0179
1	.9495	.8995	.8504	.8025	.6261	.4785	.3620	.2675	.1968	.1436	.1040	.0749	.0537
2	.9687	.9363	.9031	.8694	.7334	.6041	.4880	.3879	.3041	.2357	.1808	.1375	.1038
3	.9793	.9572	.9339	.9095	.8050	.6962	.5904	.4922	.4043	.3278	.2627	.2084	.1639
4	.9858	.9702	.9535	.9356	.8551	.7653	.6723	.5809	.4444	.4153	.3446	.2829	.2300
5	.9900	.9788	.9666	.9533	.8911	.8178	.7378	.6554	.5738	.4958	.4233	.3574	.2987
6	.9928	.9847	.9757	.9657	.9176	.8581	.7903	.7174	.6425	.5683	.4967	.4294	.3675
7	.9948	.9888	.9821	.9746	.9372	.8891	.8322	.7689	.7015	.6324	.5638	.4973	.4343
8	.9962	.9918	.9867	.9810	.9520	.9132	.8658	.8113	.7516	.6886	.6242	.5601	.4977
9	.9972	.9939	.9901	.9857	.9631	.9319	.8926	.8462	.7939	.7372	.6779	.6173	.5570
10	.9979	.9954	.9925	.9892	.9716	.9465	.9141	.8748	.8294	.7791	.7251	.6688	.6115
11	.9984	.9966	.9944	.9918	.9780	.9579	.9313	.8982	.8591	.8149	.7664	.7147	.6610
12	.9988	.9974	.9957	.9938	.9830	.9669	.9450	.9173	.8839	.8453	.8021	.7552	.7056
13	.9991	.9980	.9967	.9952	.9868	.9739	.9560	.9329	.9045	.8710	.8329	.7907	.7454
14	.9993	.9985	.9975	.9964	.9898	.9794	.9648	.9456	.9215	.8927	.8593	.8217	.7806
15	.9995	.9989	.9981	.9972	.9920	.9837	.9719	.9559	.9356	.9109	.8818	.8486	.8116
16	.9996	.9991	.9985	.9978	.9938	.9871	.9775	.9643	.9472	.9261	.9009	.8717	.8387
17	.9997	.9993	.9989	.9983	.9951	.9898	.9820	.9711	.9568	.9389	.9171	.8916	.8623
18	.9998	.9995	.9991	.9987	.9962	.9920	.9856	.9766	.9647	.9495	.9308	.9086	.8828
19	.9998	.9996	.9993	.9990	.9970	.9936	.9885	.9811	.9712	.9583	.9424	.9231	.9005
20	.9999	.9997	.9995	.9992	.9977	.9950	.9908	.9847	.9765	.9657	.9520	.9354	.9157
21		.9998	.9996	.9994	.9982	.9960	.9926	.9877	.9808	.9717	.9602	.9459	.9287
22		.9998	.9997	.9995	.9986	.9968	.9941	.9900	.9843	.9767	.9669	.9547	.9398
23		.9999	.9998	.9996	.9989	.9975	.9953	.9920	.9873	.9809	.9726	.9622	.9493
24			.9998	.9997	.9991	.9980	.9962	.9935	.9896	.9843	.9773	.9684	.9573
25			.9999	.9998	.9993	.9984	.9970	.9943	.9916	.9871	.9813	.9737	.9642

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0	.9143	.8360	.7643	.6988	.6413	.5884	.5413	.4985	.4667	.4367	.4014	.3769	.3537	.3278
1	.9524	.9056	.8599	.8153	.7719	.7301	.6895	.6517	.6167	.5896	.5612	.5375	.5168	.4941
2	.9691	.9376	.9056	.8735	.8461	.8236	.8061	.7917	.7801	.7701	.7613	.7536	.7469	.7419
3	.9786	.9562	.9330	.9091	.8894	.8743	.8629	.8536	.8461	.8401	.8353	.8316	.8286	.8262
4	.9846	.9682	.9509	.9328	.9143	.8994	.8895	.8829	.8781	.8746	.8721	.8701	.8686	.8672
5	.9887	.9764	.9633	.9495	.9371	.9271	.9195	.9147	.9111	.9086	.9069	.9056	.9046	.9032
6	.9915	.9822	.9722	.9615	.9518	.9436	.9366	.9313	.9271	.9239	.9216	.9196	.9179	.9163
7	.9936	.9865	.9787	.9703	.9636	.9582	.9539	.9501	.9467	.9436	.9409	.9386	.9366	.9347
8	.9951	.9896	.9836	.9769	.9711	.9661	.9619	.9584	.9551	.9521	.9496	.9476	.9456	.9437
9	.9962	.9919	.9872	.9820	.9776	.9739	.9701	.9666	.9633	.9601	.9576	.9556	.9536	.9517
10	.9971	.9937	.9900	.9858	.9821	.9789	.9759	.9733	.9709	.9686	.9666	.9646	.9626	.9607
11	.9977	.9951	.9921	.9888	.9858	.9833	.9811	.9791	.9771	.9751	.9731	.9711	.9691	.9672
12	.9982	.9961	.9938	.9912	.9883	.9858	.9836	.9816	.9796	.9776	.9756	.9736	.9716	.9697
13	.9986	.9969	.9951	.9930	.9905	.9880	.9858	.9838	.9819	.9801	.9781	.9761	.9741	.9722
14	.9989	.9976	.9961	.9944	.9923	.9898	.9876	.9856	.9836	.9816	.9796	.9776	.9756	.9737
15	.9991	.9981	.9969	.9955	.9933	.9908	.9883	.9858	.9833	.9808	.9783	.9758	.9733	.9708
16	.9993	.9985	.9971	.9964	.9951	.9936	.9919	.9901	.9883	.9866	.9849	.9833	.9816	.9799
17	.9994	.9988	.9980	.9971	.9961	.9951	.9941	.9931	.9921	.9911	.9901	.9891	.9881	.9871
18	.9996	.9990	.9984	.9977	.9969	.9961	.9953	.9945	.9937	.9929	.9921	.9913	.9905	.9897
19	.9996	.9992	.9987	.9981	.9974	.9967	.9960	.9953	.9946	.9939	.9932	.9925	.9918	.9911
20	.9997	.9994	.9990	.9985	.9979	.9973	.9967	.9961	.9955	.9949	.9943	.9937	.9931	.9925
21	.9998	.9995	.9992	.9988	.9984	.9979	.9975	.9971	.9967	.9963	.9959	.9955	.9951	.9947
22	.9998	.9996	.9993	.9990	.9987	.9984	.9981	.9978	.9975	.9972	.9969	.9966	.9963	.9960
23	.9999	.9997	.9995	.9992	.9989	.9987	.9985	.9983	.9981	.9979	.9977	.9975	.9973	.9971
24		.9997	.9996	.9994	.9992	.9990	.9988	.9986	.9984	.9982	.9980	.9978	.9976	.9974
25		.9998	.9997	.9995	.9993	.9991	.9989	.9987	.9985	.9983	.9981	.9979	.9977	.9975

Cumulative probability $\hat{\Sigma}_0^x P$ for $q = 7.0$

x	Mean	.25	.50	.75	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0		.9221	.8503	.7841	.7230	.5228	.3780	.2733	.1976	.1429	.1033	.0747	.0540	.0390
1		.9551	.9110	.8681	.8263	.6721	.5399	.4294	.3387	.2653	.2066	.1600	.1234	.0948
2		.9698	.9392	.9086	.8800	.7575	.6441	.5410	.4496	.3703	.3025	.2454	.1978	.1585
3		.9783	.9560	.9332	.9099	.8144	.7185	.6260	.5394	.4602	.3893	.3267	.2722	.2253
4		.9839	.9671	.9497	.9316	.8550	.7742	.6927	.6131	.5373	.4667	.4021	.3439	.2921
5		.9878	.9749	.9613	.9471	.8852	.8173	.7461	.6742	.6034	.5354	.4711	.4115	.3570
6		.9906	.9805	.9698	.9586	.9082	.8511	.7894	.7251	.6601	.5958	.5336	.4743	.4187
7		.9926	.9847	.9762	.9672	.9260	.8780	.8247	.7677	.7086	.6489	.5896	.5320	.4767
8		.9942	.9879	.9811	.9738	.9401	.8926	.8537	.8035	.7503	.6953	.6397	.5845	.5306
9		.9954	.9904	.9849	.9790	.9512	.9171	.8776	.8336	.7859	.7358	.6842	.6320	.5802
10		.9963	.9923	.9878	.9830	.9601	.9314	.8975	.8589	.8165	.7711	.7236	.6748	.6255
11		.9970	.9938	.9902	.9862	.9672	.9431	.9139	.8803	.8427	.8019	.7584	.7131	.6667
12		.9976	.9950	.9920	.9888	.9730	.9526	.9277	.8984	.8652	.8286	.7891	.7473	.7040
13		.9981	.9959	.9935	.9909	.9778	.9605	.9391	.9137	.8845	.8518	.8160	.7778	.7376
14		.9984	.9967	.9947	.9925	.9816	.9671	.9487	.9267	.9010	.8719	.8397	.8048	.7677
15		.9987	.9973	.9957	.9939	.9848	.9725	.9568	.9376	.9151	.8893	.8604	.8288	.7947
16		.9990	.9978	.9964	.9949	.9874	.9770	.9635	.9470	.9272	.9044	.8785	.8500	.8189
17		.9991	.9982	.9971	.9958	.9895	.9807	.9692	.9549	.9376	.9174	.8944	.8686	.8403
18		.9993	.9985	.9976	.9966	.9913	.9838	.9740	.9616	.9465	.9287	.9082	.8851	.8594
19		.9994	.9988	.9980	.9971	.9927	.9864	.9780	.9673	.9542	.9385	.9203	.8996	.8764
20		.9995	.9990	.9983	.9976	.9939	.9886	.9814	.9722	.9607	.9470	.9308	.9123	.8914
21		.9996	.9991	.9986	.9980	.9949	.9904	.9843	.9763	.9663	.9543	.9400	.9234	.9046
22		.9997	.9993	.9989	.9984	.9957	.9919	.9867	.9798	.9711	.9606	.9479	.9332	.9164
23		.9997	.9994	.9991	.9986	.9964	.9932	.9887	.9828	.9753	.9660	.9549	.9418	.9257
24		.9998	.9995	.9992	.9989	.9970	.9943	.9904	.9853	.9788	.9707	.9609	.9493	.9358
25		.9998	.9996	.9993	.9991	.9975	.9952	.9919	.9875	.9818	.9747	.9661	.9558	.9438

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