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EVALUATION OF THE α-β TRACKER FOR USE WITH THE AIRBORNE TACTICAL DATA SYSTEM

By

D. A. DAWSON
Systems Evaluation Office

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U. S. NAVAL MISSILE CENTER
Point Mugu, California

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C. O. HOLMQUIST, CAPT USN
Commander

D. F. SULLIVAN
Technical Director

This report describes work accomplished under WEPTASK RA-1200015, Problem Assignment RAV-824-9, Airborne Tactical Data System Computer Programming.

Mr. G. R. Wachold, Head, Systems Evaluation Office; LCDR H. W. Jesse, Head, Airborne Tactical Data System Office; and CDR F. H. Featherston, Head, Missile Programs Department, have reviewed this report for publication.

THIS REPORT HAS BEEN PREPARED PRIMARILY FOR TIMELY PRESENTATION OF INFORMATION. ALTHOUGH CARE HAS BEEN TAKEN IN THE PREPARATION OF THE TECHNICAL MATERIAL PRESENTED, CONCLUSIONS DRAWN ARE NOT NECESSARILY FINAL AND MAY BE SUBJECT TO REVISION.

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SUMMARY

This report outlines a study whose purpose was to improve the performance of the Airborne Tactical Data System tracker, a part of the E-2A aircraft. This aircraft is designed to provide airborne early warning and automated control of intercepts.

Two types of smoothing were considered for the tracker: (1) the existing smoothing system, and (2) a proposed α-β smoothing system. Differences in performance of the two systems are small, but the α-β tracker requires less storage space in the computer and offers greater flexibility than does the smoothing scheme which is presently mechanized.

As a result of the study, a new α-β tracker was developed which can perform significantly better than the existing tracker, provided the operator is capable of a higher degree of involvement in tracking maneuvering targets.

INTRODUCTION

The Airborne Tactical Data System (ATDS) is an early-warning and control system carried aboard the E-2A aircraft. The ATDS consists of a search radar and associated digital computers which process tracking data and make tactical computations and decisions for intercept of enemy threats. The ATDS includes (1) automatic digital data links connecting the system to other Fleet units, and (2) controls and displays providing an interface between the computers and the operators.

Results of a recent series of flight tests show that a major deficiency exists in the ability of the ATDS to track targets.*

This report outlines a study which was performed at the U.S. Naval Missile Center, Point Mugu, California, under WEPTASK RA-1200015, Problem Assignment RAV-824-9, to find ways to reduce this deficiency.

There are at least three ways to ease the tracking problem:

1. By improving the accuracy, sensitivity, and reliability of the radar.
2. By improving the processing of the radar data in the tracking system.
3. By decreasing the need of the user for highly accurate tracking information.

These are all worthwhile goals. This report, however, is primarily concerned with No. 2--improving the processing of the radar data.

The α-β tracker is shown,** in certain respects, to be optimum for a track-while-scan (TWS) system such as the ATDS.

The α-β tracker has the additional advantage of requiring only four locations in the computer for each track history (two for X and two for Y), whereas the present system needs eight storage locations in the computer for each track. The α-β tracker uses two parameters, compared to the nine used by the present ATDS tracker, and thus needs less logic for mechanization.

Because there are only two stored parameters, the α-β tracker particularly lends itself to parameter switching. That is, one α-β pair can be used for track initiation, another for high-speed targets, and still another for low-speed targets. In addition, the α-β tracker lends itself to reposition computations because it is easy to modify either the position or velocity stored in the computer.

The study was carried out in three steps. The first was to determine meaningful criteria against which system performance can be measured. The second step was to compare the performance of the present ATDS tracker with an α-β tracker. The third step consisted of a re-examination of the constraints on the tracker to determine whether it is possible to significantly improve the capability of the system to track nonmaneuvering targets by requiring a higher degree of operator involvement in tracking maneuvering targets.

*U.S. Naval Air Test Center. *Board of Inspection Survey Report on the ATDS*. Patuxent River, Md., Mar 1965.

**Benedict, T.R., and G.W. Bordner. "Synthesis of an Optimal Set of Radar Track-While-Scan Smoothing Equations," *IRE Transactions on Automatic Control*, Jul 1962, Vol. AC-7, No. 4. Pp. 27-32.

FUNCTIONS OF THE ATDS TRACKING SYSTEM

The ATDS consists of an airborne radar together with computers, automatic data links, and displays. The ATDS automatically tracks targets and reports target position, speed, and heading to other Fleet units via the Link 11 data link. The ATDS automatically directs interceptors, via Link 4 data link, to intercept these targets.

The tracking system consists of:

1. A search radar which scans 360 degrees every 10 seconds and can detect targets at a maximum range of 250 miles.
2. A computer detector which automatically determines whether a particular signal received by the radar is a target and converts the resulting range and bearing measurement to X and Y for transmission to the correlator.
3. A correlator unit which stores the predicted position of each target being tracked and compares these predicted positions with the most recent X and Y target data from the computer detector in order to determine whether the new data represents additional information on a known target track, or whether it represents a new target track.
4. Related displays and controls.
5. An automatic tracking unit (ATU) that:
   a. Accepts position inputs on a target.
   b. Extrapolates target position ahead for 10 seconds. This information goes back to the correlator and forward to the ultimate users, the data links, the intercept computer, and the operators.
   c. Obtains target velocity for use by the data links and the intercept computer.

The over-all tracking system is outlined in figure 1.

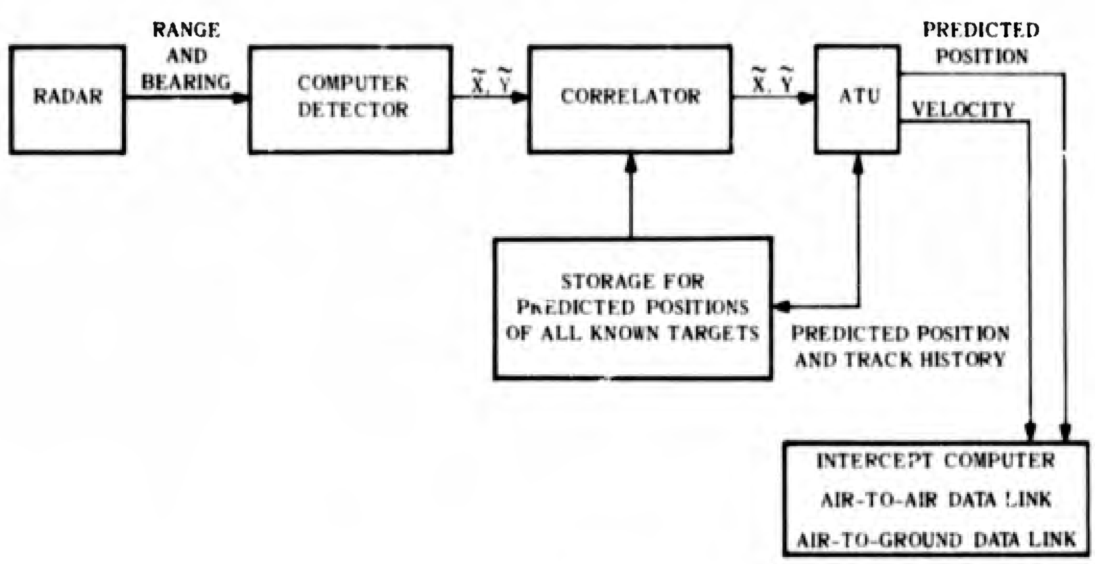

Figure 1. Functional Diagram of the ATDS.

In the ATU, the X-data (east) and Y-data (north) are processed in independent parallel channels. Subsequent description of the system will be simplified by discussing only the X-component (or X-channel), with the understanding that the Y-component is processed independently in exactly the same way.

DETAILED USES OF TRACKER OUTPUTS

Correlator

The predicted position of a target is computed in the ATU and sent to the correlator unit, along with a tracking gate size which defines the area around the predicted position within which a new target report must fall in order to be correlated with the track. A large tracking gate indicates low confidence in the predicted position and vice versa.

An incoming target report is compared with the predicted position of known targets. In general, the new target report will be correlated with that track for which the smallest error results. But in no case will the new report be correlated with a track if the difference between reported position and predicted position is greater than the tracking gate size associated with the track. If the new report fails to correlate with any of the existing tracks, it will be saved for a period of time to determine whether it represents the beginning of a new track. If not a new track, the report will eventually be dropped from the track files.

It is desirable to have the predicted position exactly at the position at which the target will appear on the next scan, thus increasing the probability that new data on a particular target track will indeed be correlated with that track. It is also desirable to have the tracking gate as small as possible in order to reduce the probability that a new piece of data from another target or from a false target will fall within the gate and correlate with the track.

Intercept Computer

Predicted position and velocity components are sent from the ATU to the intercept computer. In the intercept computer, the X- and Y-positions of the interceptor and target are combined to give range and bearing of the target from the interceptor. Velocity components are combined to give speed and heading. The intercept computer makes a "trial intercept" to determine the feasibility of the attack and generates commands to cause the interceptor to follow a predetermined flight profile in order to intercept and destroy the target.

The outputs from the ATU are of critical importance in this process. Noisy or erratic target velocity, for example, will usually lead to erratic command headings which reduce interceptor pilot confidence in the ability of the ATDS to control an intercept.

An equally serious result of a noisy track is that the intercept computer may be deceived as to certain "decision point," examples of which might be:

1. If interceptor-target range (R) is greater than R_1, command an interceptor speed of V_1; if less than R_1, command a speed of V_2.
2. If target speed (V_T) is less than Mach 1, command a speed of (V_T + 0.6); if greater than Mach 1, command a speed of 1.25 V_T.
3. If the interceptor is now forward of the beam of the target, make a head-on attack; otherwise, make a tail attack.
4. If the true situation approximately matches one of the points listed above and if the track is unreliable, the decision may become almost random and may, in fact, change from time to time even though the situation has not changed.

Data Links

The Link 11 data link transmits X- and Y-components of target position and velocity to other participating units.

The Link 4 data link transmits target speed and heading and interceptor-target relative position as determined in the intercept computer. In addition, Link 4 transmits vectoring commands. If the interceptor is using the "command" mode of data-link operation, target speed and heading are of general interest to the Radar Intercept Operator. If the "situation" mode is used, the target speed, heading, and relative position are fed into the interceptor's data-link computer, which derives a collision-course heading for presentation to the crew.

MEASURES OF TRACKER PERFORMANCE

The real problem in determining tracker performance is not so much "how to measure" as it is "what to measure." Ideally, the tracking analyst should determine how the tracker outputs are to be used (as discussed in the previous section) and develop measures of performance which are compatible with these uses. That is easier said than done: no two tracking analysts use exactly the same performance criteria and, probably, none feels completely satisfied with his own criteria.

Criteria for evaluating tracking systems responding to two broad categories of inputs are discussed below.

Noise

Figure 1 shows that only position data are fed into the ATU. The output of the ATU is predicted position and measured velocity. Since a random error, or noise, is always associated with the measured positional data, it is desirable to describe the effect of this noise on the output.

Often, the first attempt at describing this relationship is directed toward observation of the actual input and output errors on a scan-by-scan basis. This method is helpful for discovering flaws in the mechanization and, perhaps equally important, gives the analyst insight into the workings of the particular system under study.

After a while this method begins to lose its appeal because it does not lend itself to generalization. The analyst then becomes interested in finding some statistical technique for describing the system response to noise when he varies tracking coefficients, gating equations, target speeds, etc.

Two such criteria are the position noise-reduction coefficient ($K_{\bar{x}}$) and the velocity noise-reduction coefficient ($K_{\dot{x}}$).

$$K_{\bar{x}} = \frac{\sigma_{\bar{x}}}{\sigma_{\tilde{x}}} \text{ miles per mile} \tag{1}$$

$$K_{\dot{x}} = \frac{\sigma_{\dot{x}}}{\sigma_{\tilde{x}}} \text{ miles per scan per mile} \tag{2}$$

or

$$K_{\dot{x}} = 360 \left| \frac{\sigma_{\dot{x}}}{\sigma_{\tilde{x}}} \right| \text{ knots per mile} \tag{3}$$

when 1 scan = 10 seconds and where

$\sigma_{\tilde{x}}$ = standard deviation of the error in the east component of measured position, assumed to be in the range from 1 to 3 miles

$\sigma_{\bar{x}}$ = standard deviation of the error in the east component of predicted position

$\sigma_{\dot{x}}$ = standard deviation of the error in the east component of measured velocity

As will be shown in later sections of this report, typical values for ATDS are:

$0.5 \leq K_{\bar{x}} \leq 1.0$ miles per mile

$30 \leq K_{\dot{x}} \leq 100$ knots per mile

Transients

The previous comments on point-by-point observation of noise response apply also to analysis of transient response; but it is difficult, if not impossible, to arrive at any single measure of transient response.

The types of transient input which might be considered are:

1. Ramp input in velocity (longitudinal acceleration).
2. Step input in velocity. This may seem unrealistic at first glance, but for a TWS system, it is a fairly good approximation of what the system sees when a target turns for short periods of time.
3. Circular maneuvers.
4. Dog-leg maneuvers--alternating flight path from straight line to turn to straight line. Typically the straight-line portion of the flight might last from 1 to 5 minutes, and the turn might be as much as 45 degrees, or occasionally 90 degrees.

Most analyses of transient response involve generation of an error function for a particular transient input. The error function can be obtained by applying the desired transient input to the tracker, solving the tracking equations several times, and then observing the resulting error in the tracker output. Typical error functions are shown in figure 2 for a step input in velocity.

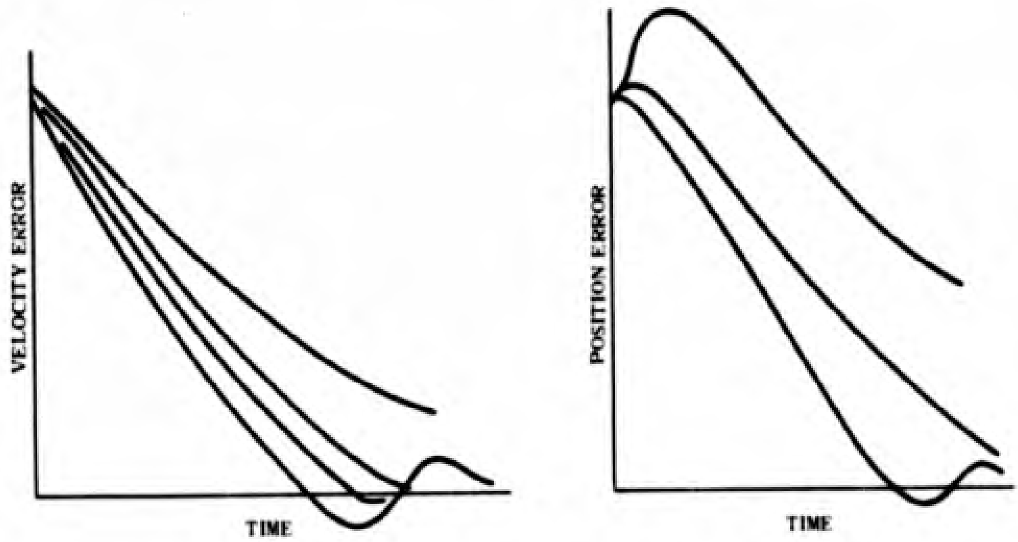

Figure 2. Typical Transient Response Curves for a Step Input in Velocity for Varying Tracking Coefficients.

With the error function generated, several characteristics can be used to describe it:

1. Sum of absolute values of position errors.
2. Sum of absolute values of velocity errors.

3. Sum of the squares of the position errors (sometimes called the position transient demerit).
4. Sum of the squares of the velocity errors (sometimes called the velocity transient demerit).
5. Time for error to decay to 10 per cent of initial error or to 10 per cent of maximum error.
6. Maximum overshoot.
7. Time for error to decay to some specified value, below which the error is considered to be insignificant.

SMOOTHING BY TRACKING SYSTEMS

Two general types of tracking equations are to be compared: the present ATDS smoothing and the α-β smoothing.

Present ATDS

$$\bar{X}_{n+1} = C_0\bar{X}_n + C_1\tilde{X}_n + C_2\tilde{X}_{n-1} + C_3\tilde{X}_{n-2} + C_4\tilde{X}_{n-3} \tag{4}$$

$$\dot{X}_n = D_0\bar{X}_{n+1} + D_1\tilde{X}_n + D_2\tilde{X}_{n-1} + D_3\tilde{X}_{n-2} \tag{5}$$

where

$\bar{X}_n$ = position which was predicted for time n

$\tilde{X}_n$ = position which was measured for time n

$\bar{X}_{n+1}$ = position predicted for time n+1

$\dot{X}_n$ = estimate of velocity at time n

$C_0 = 5/8$	$D_0 = 11/16$
$C_1 = 7/8$	$D_1 = -7/16$
$C_2 = -1/8$	$D_2 = -3/16$
$C_3 = -2/8$	$D_3 = -1/16$
$C_4 = -1/8$	

Position and velocity noise-reduction coefficients for the present ATDS smoothing are derived in appendix A.

α-β Tracker

$$\dot{X}_n = \dot{X}_{n-1} + \beta(\tilde{X}_n - \bar{X}_n) \tag{6}$$

$$\bar{X}_{n+1} = \dot{X}_n + \bar{X}_n + \alpha(\tilde{X}_n - \bar{X}_n) \tag{7}$$

Position and velocity noise-reduction coefficients for the α-β tracker are derived in appendix B.

COMPARISON OF THE α-β AND THE PRESENT ATDS TRACKERS

To make a comparative analysis of the α-β tracker and the present ATDS tracker, a set of coefficients had to be developed for the α-β tracker. Then both trackers could be analyzed under the several criteria, and comparisons could be made.

In the approach used, the velocity noise coefficient for the present ATDS tracker was determined; then an optimum α-β tracker having this same velocity noise was designed. It is shown in appendix A that the present ATDS tracker has a velocity noise coefficient of 84 knots. The optimum α-β tracking system for minimizing mean-square velocity transient response for a specified velocity noise coefficient is obtained* when

$$\beta = \frac{\alpha^2}{2 - \alpha} \tag{8}$$

The velocity noise coefficient for the α-β tracker is derived in appendix B as

$$K_{\dot{x}}^2 = \frac{2\beta^2}{\alpha(4-2\alpha-\beta)} \tag{9}$$

The position noise coefficient for predicted position is derived as

$$K_{\bar{x}}^2 = \frac{2\alpha^2 + \beta(\alpha+2)}{\alpha(4-2\alpha-\beta)} \tag{10}$$

By substituting equation (8) into equation (9) and setting $K_{\dot{x}} = \frac{84}{360}$ we obtain

$\alpha = 0.540$

$\beta = 0.200$

$K_{\bar{x}} = 0.86$ mile per mile

After multiplying $K_{\bar{x}}$ and $K_{\dot{x}}$ by $\sigma_{\bar{x}}$, the outputs of the two trackers can be compared as shown in table 1.

Table 1. Standard Deviation of Position and Velocity Noise Outputs

Criterion	Present ATDS Tracker	α-β Tracker With $\alpha = 0.54$, $\beta = 0.2$
Standard deviation of position noise output	0.98 to 2.96 miles	0.88 to 2.64 miles
Standard deviation of velocity noise output	84 to 252 knots	84 to 252 knots

* As demonstrated in the Benedict and Bordner article in *IRE Transactions on Automatic Control*, Jul 1962.

It can be seen that the α-β tracker has a slight but not important edge over the present ATDS tracker from the standpoint of position noise.

The velocity transient responses of the two systems to a step input in velocity are compared in figure 3. As expected, the α-β tracker with α equal to 0.54 and β equal to 0.2 has the better velocity response. The difference is small, however.

Figure 4 shows the position transient response of the two trackers to a step input in velocity. In this respect the response of the α-β tracker is worse by about 0.2 mile, an insignificant amount compared to other errors in the tracking system. Bear in mind that the α-β tracker was optimized with respect to velocity transient response but not with respect to position transient response.

In summary then, the particular α-β tracker under consideration has the same velocity noise characteristics as the present ATDS tracker, slightly better position noise characteristics, slightly better velocity transient response, and slightly worse position transient response.

OPTIMIZATION OF THE α-β TRACKER

It was shown in the previous section that the present tracker has a velocity noise output of 84 to 252 knots. This has caused some difficulty in utilizing the outputs for intercept control, correlation, and reporting. It was also shown, however, that the α-β tracker offers only a slight

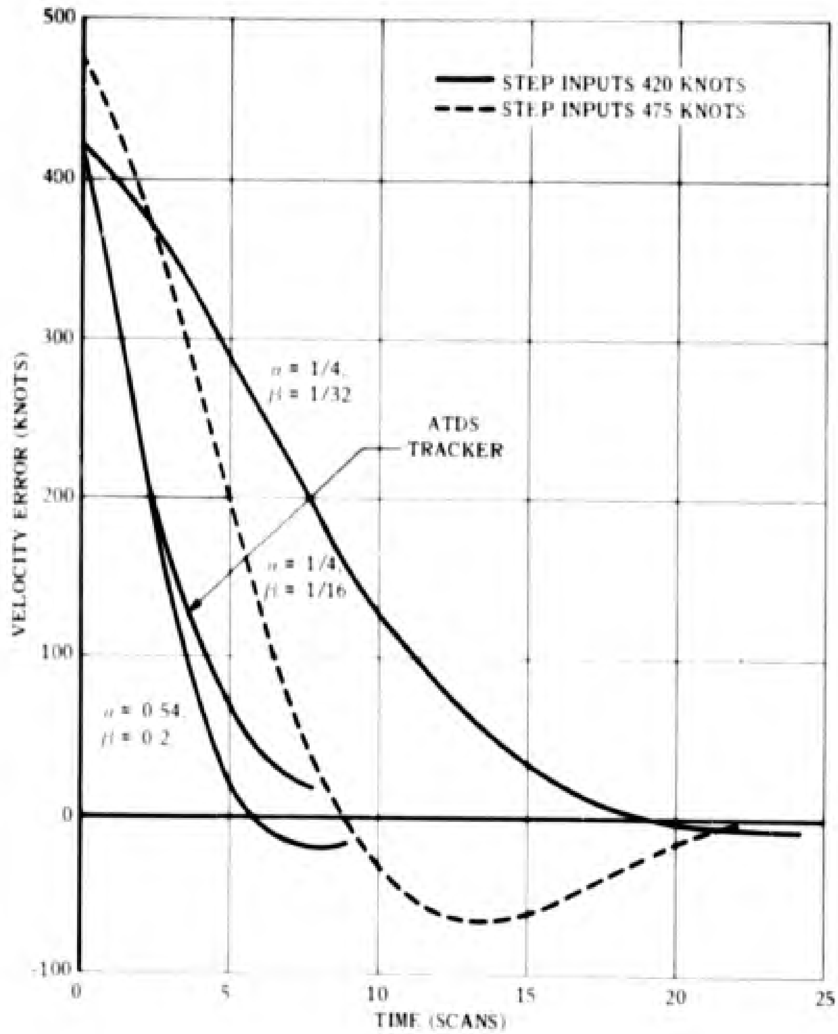

Figure 3. Velocity Transient Response to a Step Input in Velocity. When ATDS tracker is used: $C_0 = 5/8$, $C_1 = 7/8$, $C_2 = -1/8$, $D_0 = 11/16$, $D_1 = -7/16$.

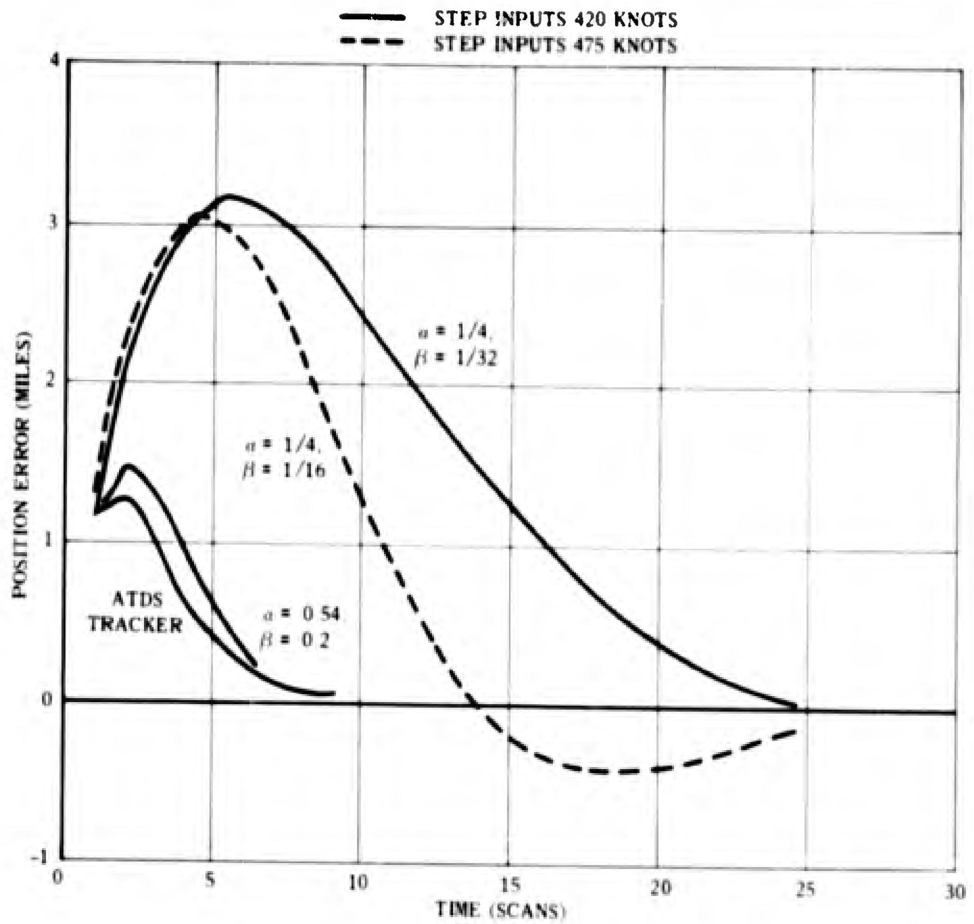

Figure 4. Position Transient Response to a Step Input in Velocity.

improvement in performance when it is required to meet the constraints which are placed on the present ATDS tracker. It appears that only two possible ways of improving the quality of the tracker output are possible: (1) improvement of the quality of the input data, and (2) change in the constraints placed on the tracker. Because solutions to the first problem fall outside the realm of this report, only the second potential change is considered.

Changing constraints usually means giving up something in one area in order to gain in another area. Before going into a detailed optimization for the α-β tracker it will be helpful to look at one example which shows that, by allowing larger position transient response, gains can be made in position noise and velocity noise without too large a penalty in velocity transient response.

For an example, the presently mechanized ATDS tracker can be compared with an α-β tracker with α equal to 1/4 and β equal to 1/16. Performance data for the two trackers can be obtained from figures 3 through 8. Comparison of the two trackers can then be summarized in table 2.

The position and velocity noise data shown in table 2 are from figures 5 and 7. Figures 6 and 8 present the same data but are based upon a simulation in which correlation gates were included. Good agreement exists between the two sets of data.

The value of 100 knots for the velocity transient threshold was arrived at rather arbitrarily by observing that 100 knots is roughly the average of the standard deviations of the velocity noise for the two trackers described in table 2. The position transient threshold of 1.5 miles was chosen for a similar reason.

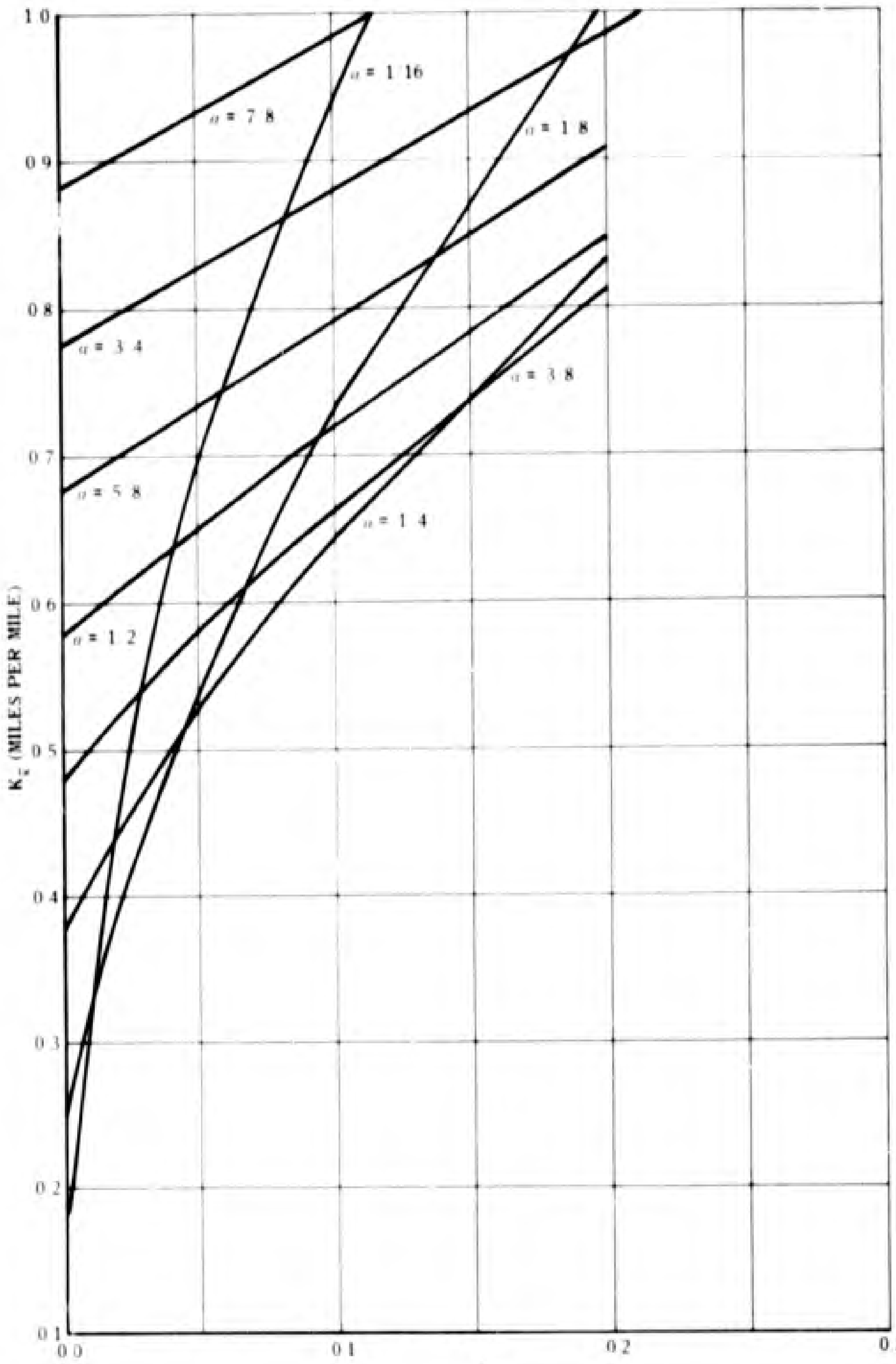

Figure 5. Theoretical Position Noise Response. Theoretical $K_{\bar{x}}$ for four-point smoothing is 0.983.

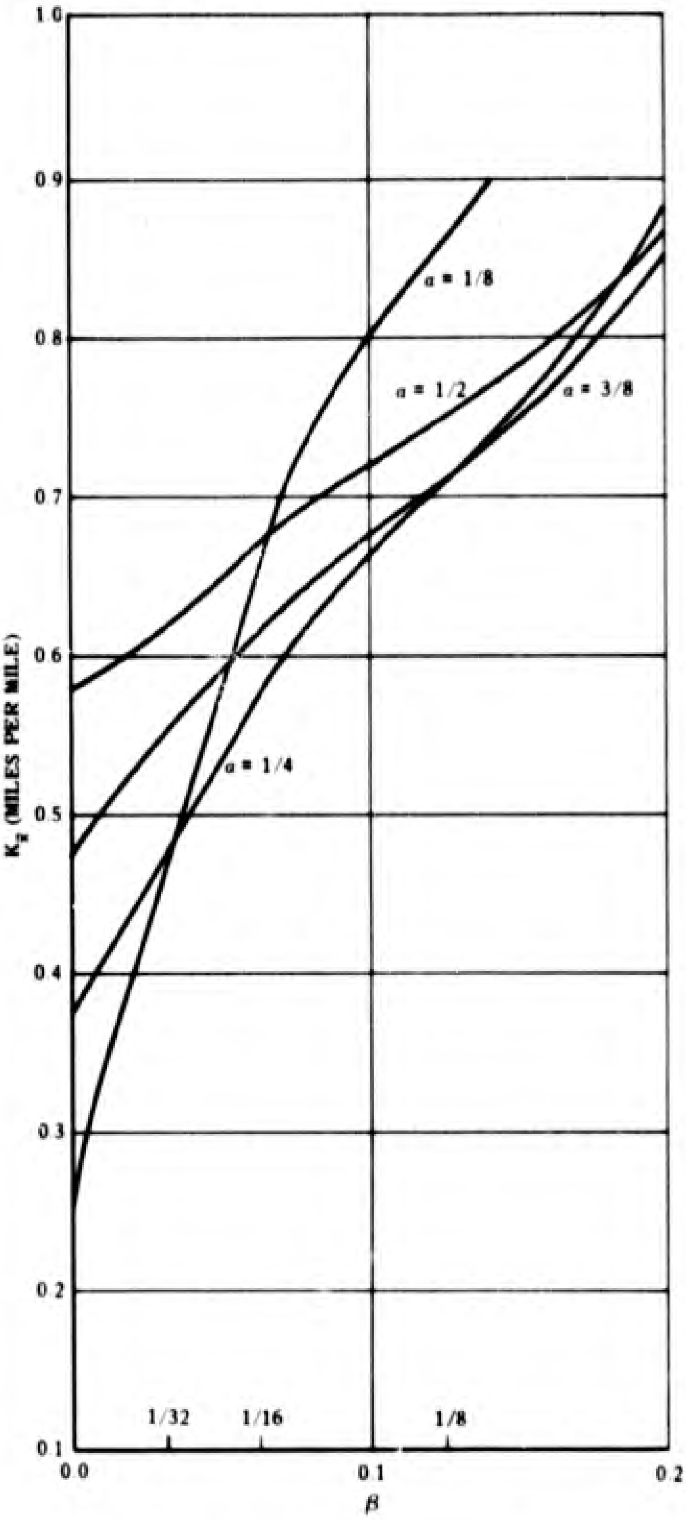

Figure 6. Position Noise Response Based on Simulation Data. Blip/scan = 1.0 with gating. Simulated $K_{\bar{x}}$ for four-point smoothing is 0.986.

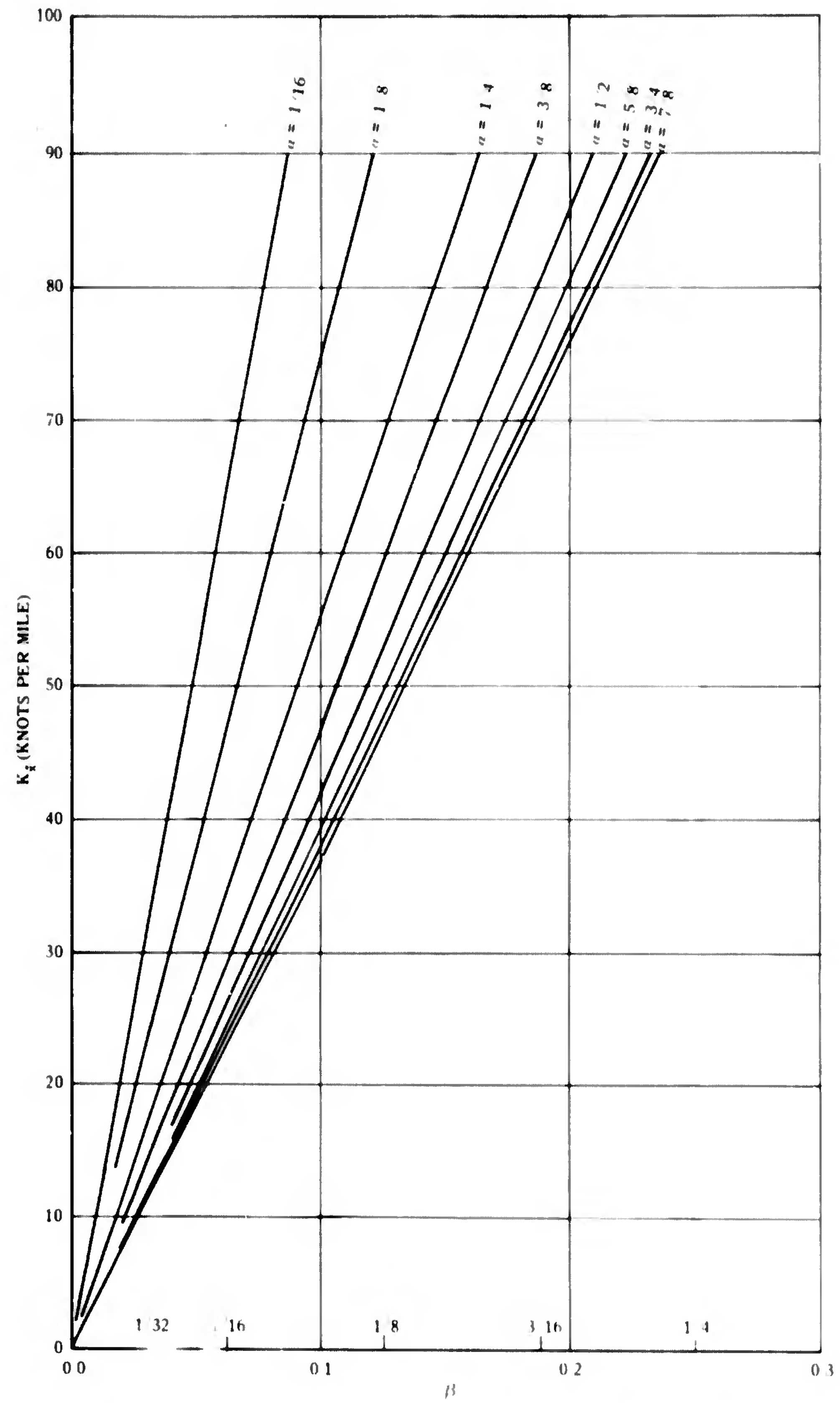

Figure 7. Theoretical Velocity Noise Response. Theoretical $K_{\dot{x}}$ for four-point smoothing is 82 knots per mile.

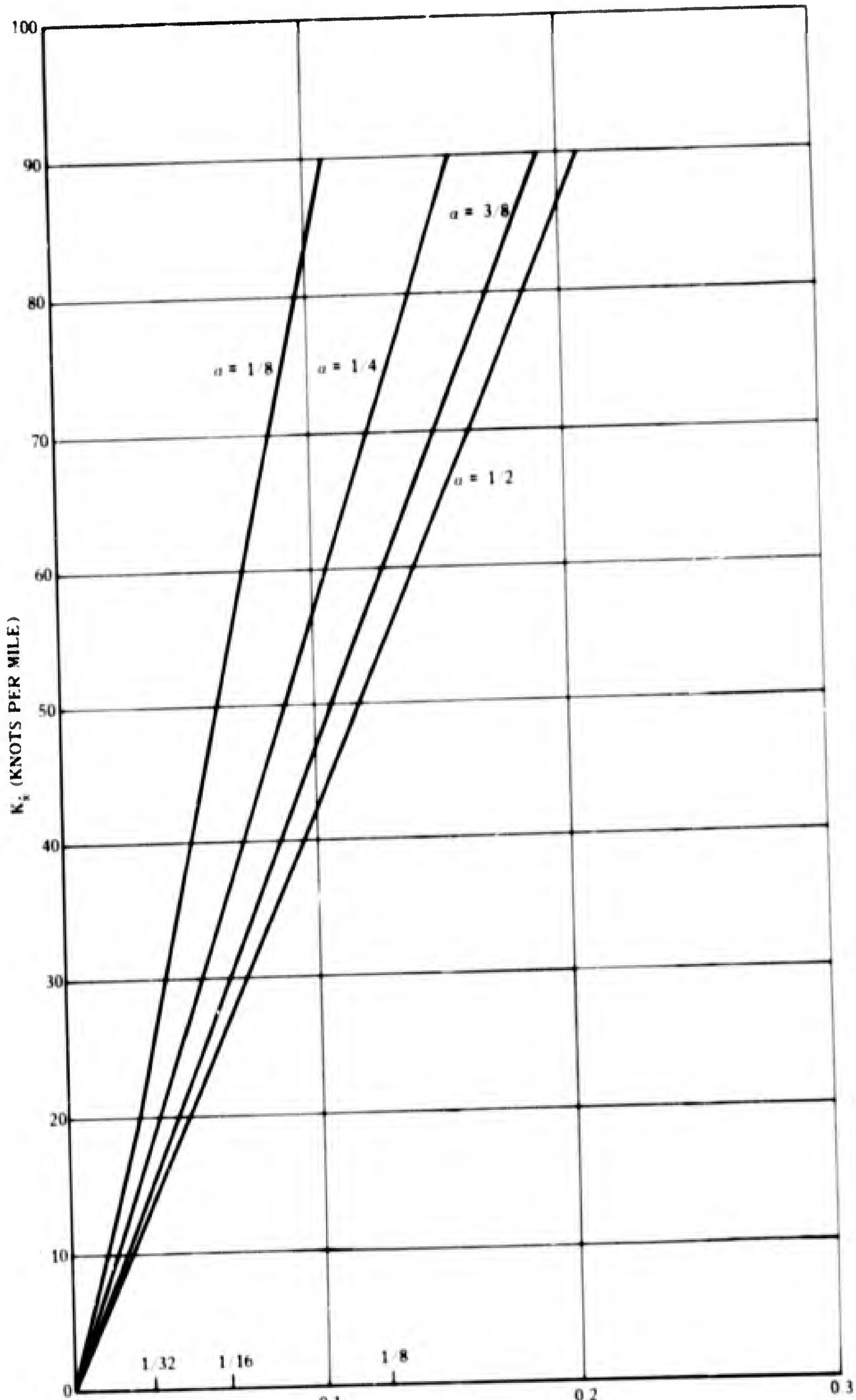

Figure 8. Velocity Noise Response Based on Simulation Data. Blip/scan ratio equal to 1.0 with gates. Simulated $K_{\dot{x}}$ for four-point smoothing (without gates) $K_{\dot{x}}$ equal to 84 knots per mile.

Table 2. Theoretical Position and Velocity Data
(Standard deviation of position noise input is from 1 to 3 miles.)

Criterion	ATDS Tracker	$\alpha = 1/4$ $\beta = 1/16$
Standard deviation of velocity noise output (knots)	84 252	35 105
Standard deviation of position noise output (miles)	0.98 2.94	0.56 1.68
Time for velocity error to decay to 100-knot threshold (scans)	4	6
Velocity transient overshoot (knots)	0	60
Time for position error to decay to 1.5-mile threshold (scans)	0	9
Position transient overshoot (miles)	1.3	3.0

Referring to table 2, it can be seen that by using the α-β tracker with the specified coefficients, the standard deviation of the velocity noise is cut by more than half and the time needed for the velocity error to decay to 100 knots increases from 4 to 6 scans.

The standard deviation of the position noise has been reduced by almost half, which should reduce the problem of miscorrelation with other targets or false targets. The position settling time and the position overshoot have both increased significantly. Thus, improvements have been made in velocity noise and position noise at the expense of poorer position settling time and position overshoot.

Figures 9 through 18 have been included to show how wide variations in α and β influence the position and velocity transient response to a velocity step input.

Determination of Constraints

Significant gains having been shown by changing the constraints, the next step is to determine a reasonable set of constraints.

Because the α-β tracker takes up significantly less computer space than the present tracker, it should be possible to mechanize sets of coefficients which are optimum for more than one target speed. Two representative target speeds have been chosen: (1) 540 knots (Mach 0.93), nominal subsonic speed, and (2) 900 knots (Mach 1.56), nominal supersonic speed.

Next, reasonable values for $K_{\dot{x}}$ and for the transient input must be determined.

It seems reasonable to restrict the velocity noise level to less than 10 per cent of the true velocity for input position errors up to 3 miles standard deviation. Thus,

$K_{\dot{x}} = 0.1 \times 540 \times 1/3 = 18$ knots per mile for subsonic targets

$K_{\dot{x}} = 0.1 \times 900 \times 1/3 = 30$ knots per mile for supersonic targets

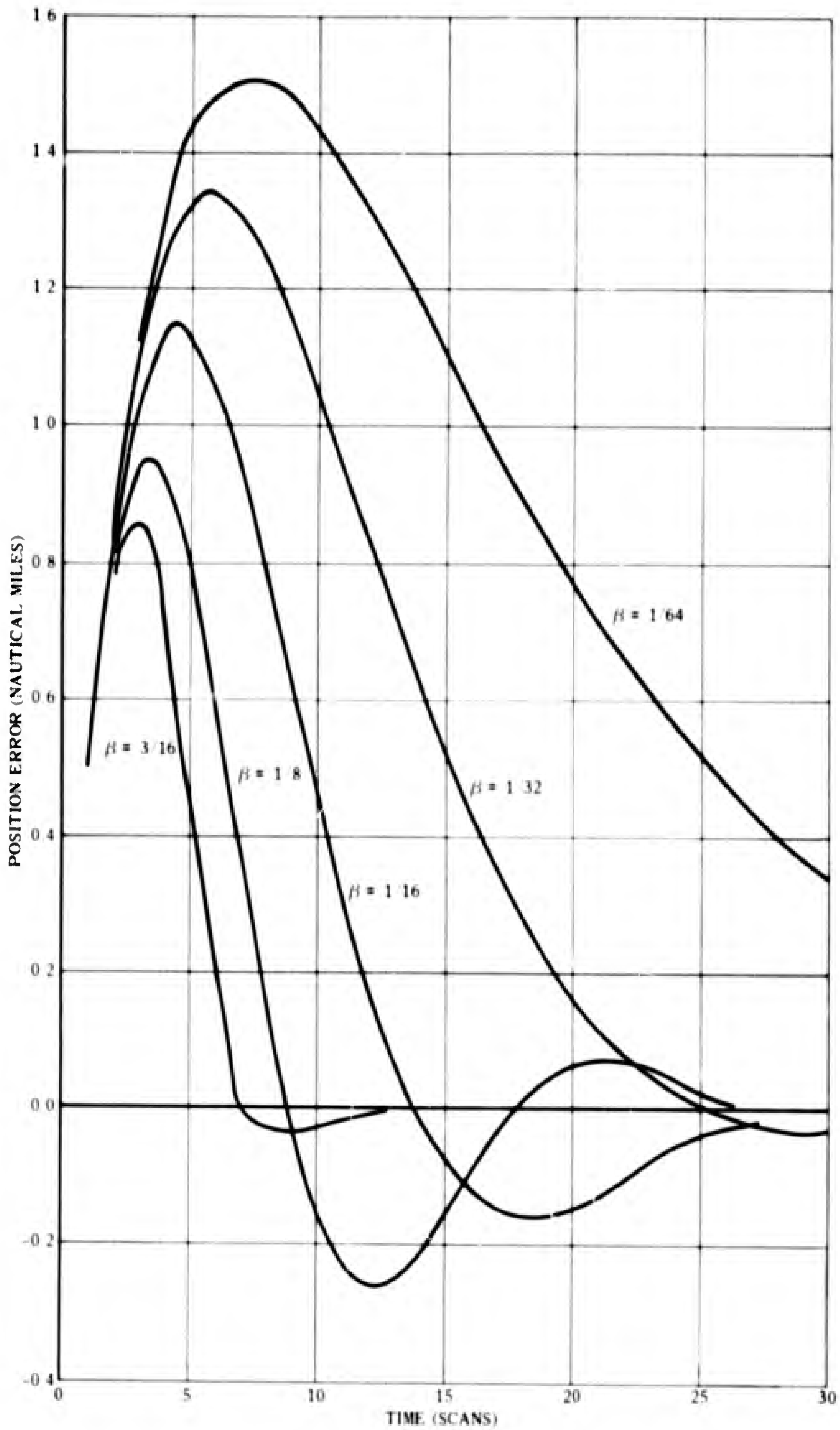

Figure 9. Position Transient Response to a 180-Knot Step Input in Velocity When α = 1/4.

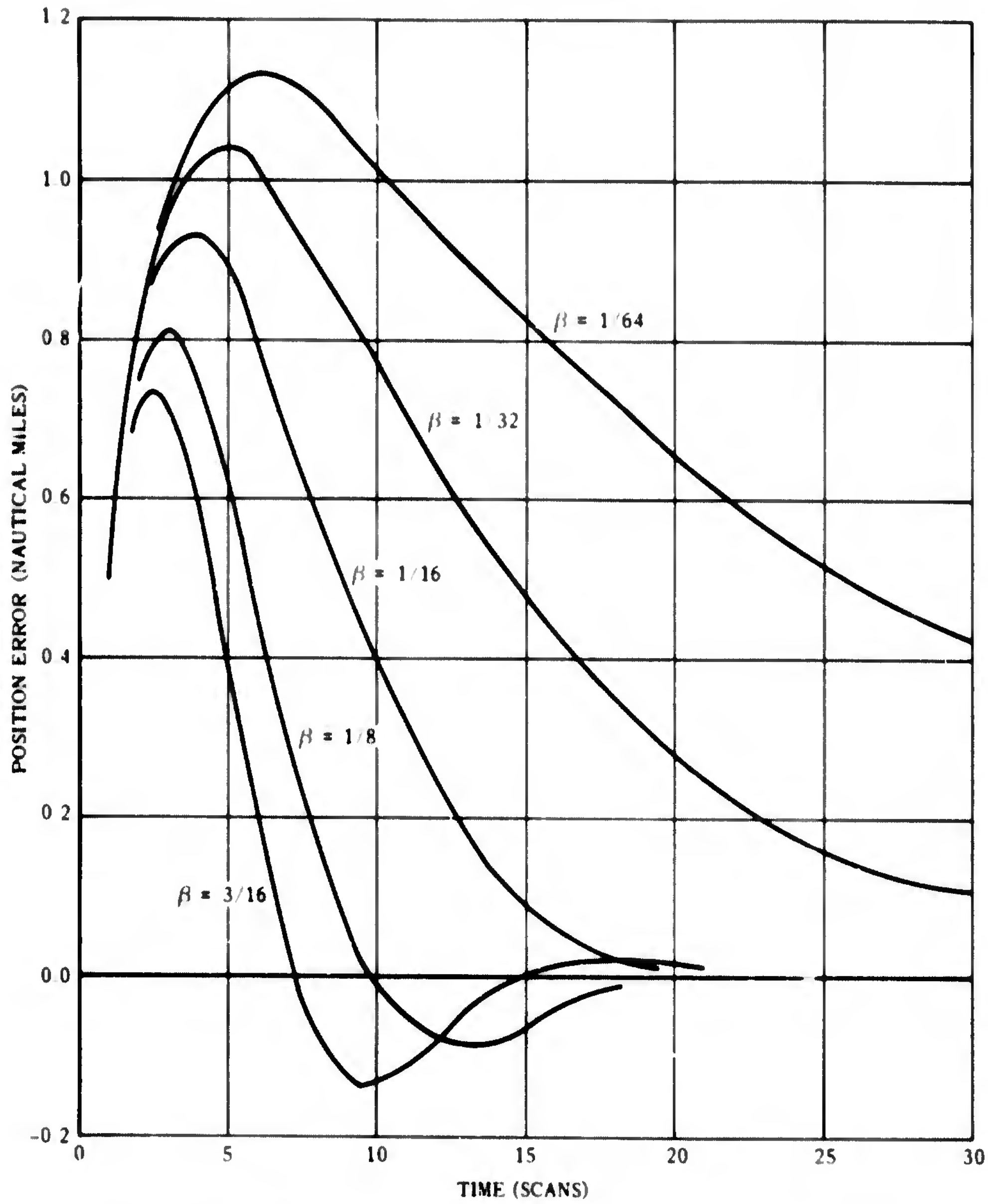

Figure 10. Position Transient Response to a 180-Knot Step Input in Velocity When α = 3/8.

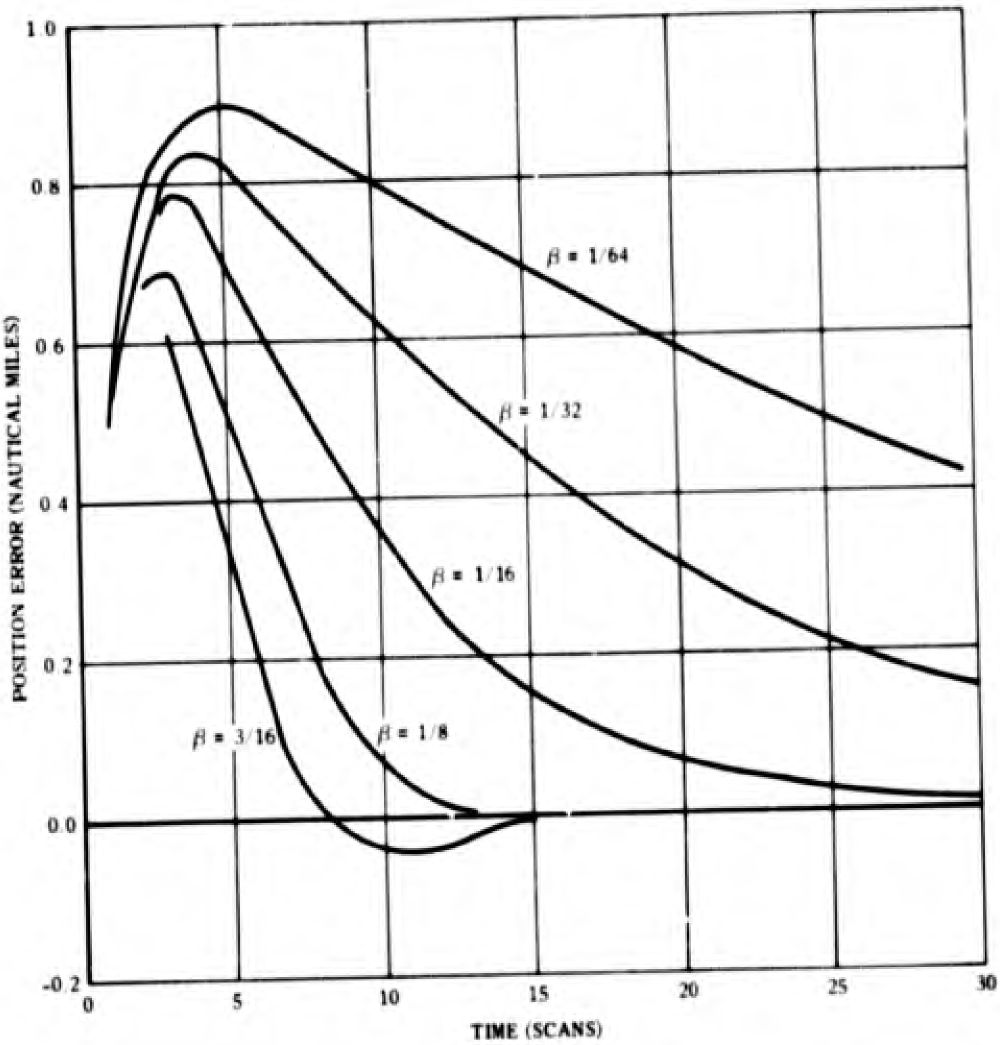

Figure 11. Position Transient Response to a 180-Knot Step Input in Velocity When α = 1/2.

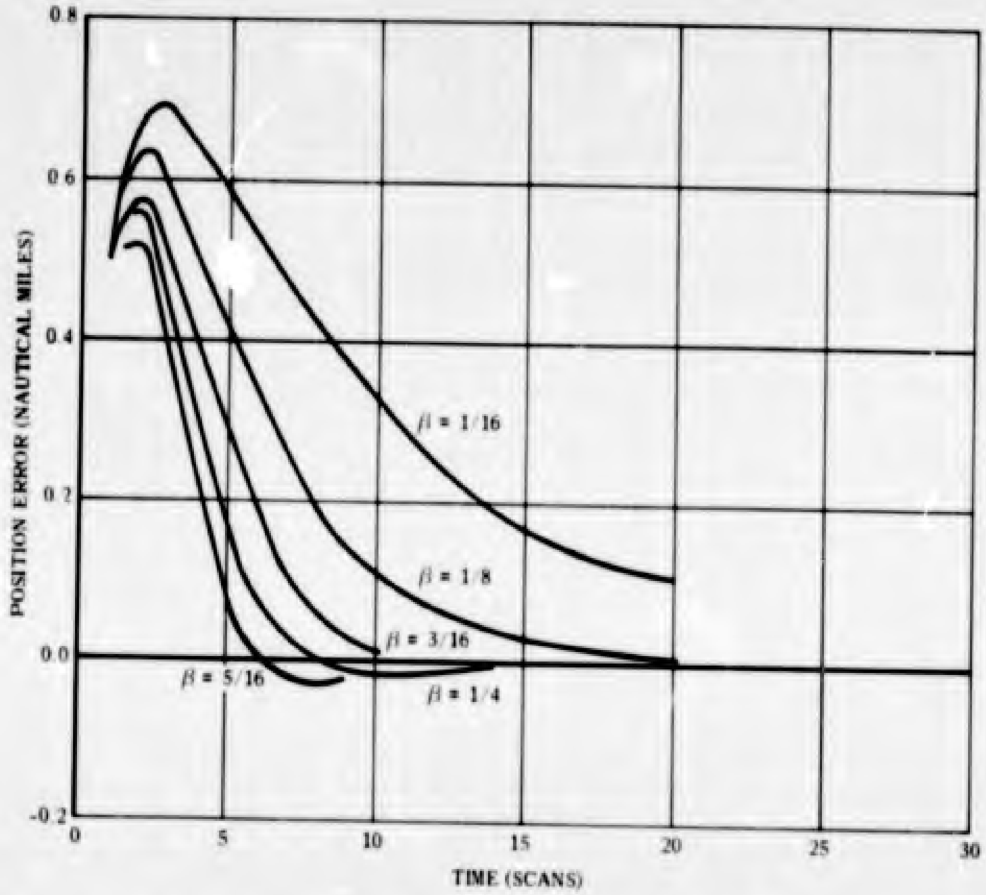

Figure 12. Position Transient Response to a 180-Knot Step Input in Velocity When α = 5/8.

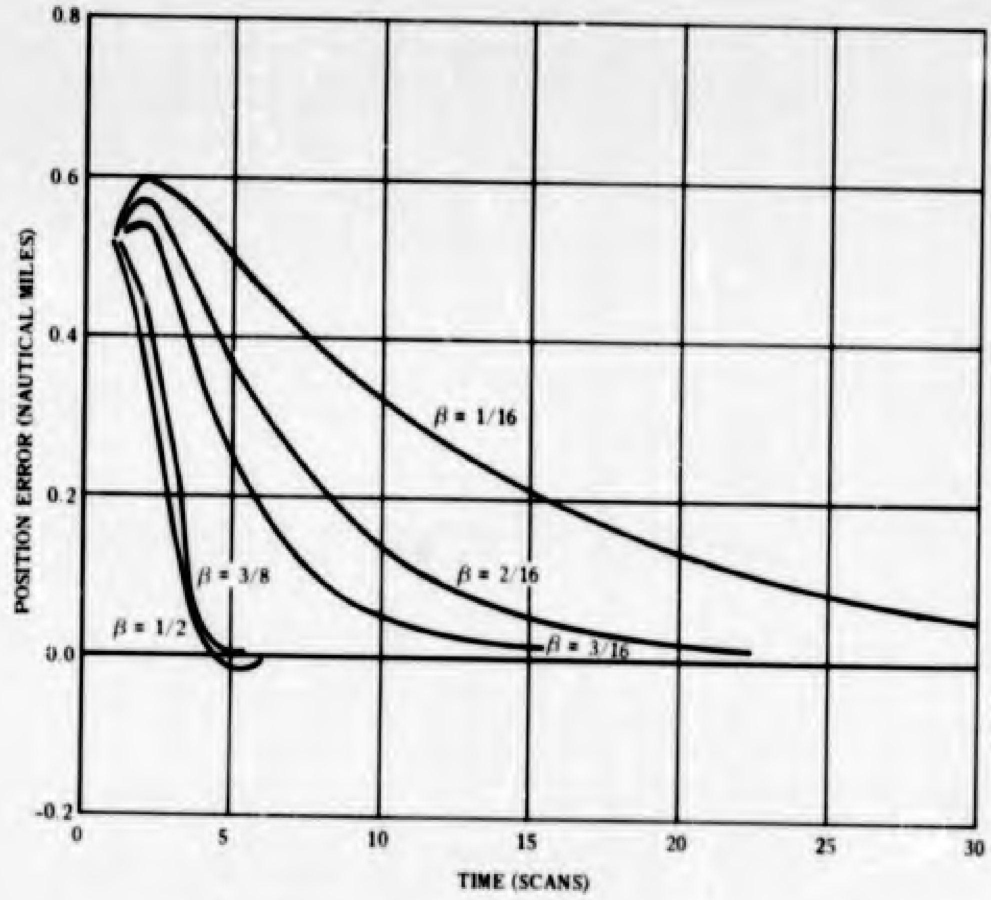

Figure 13. Position Transient Response to a 180-Knot Step Input in Velocity When α = 3/4.

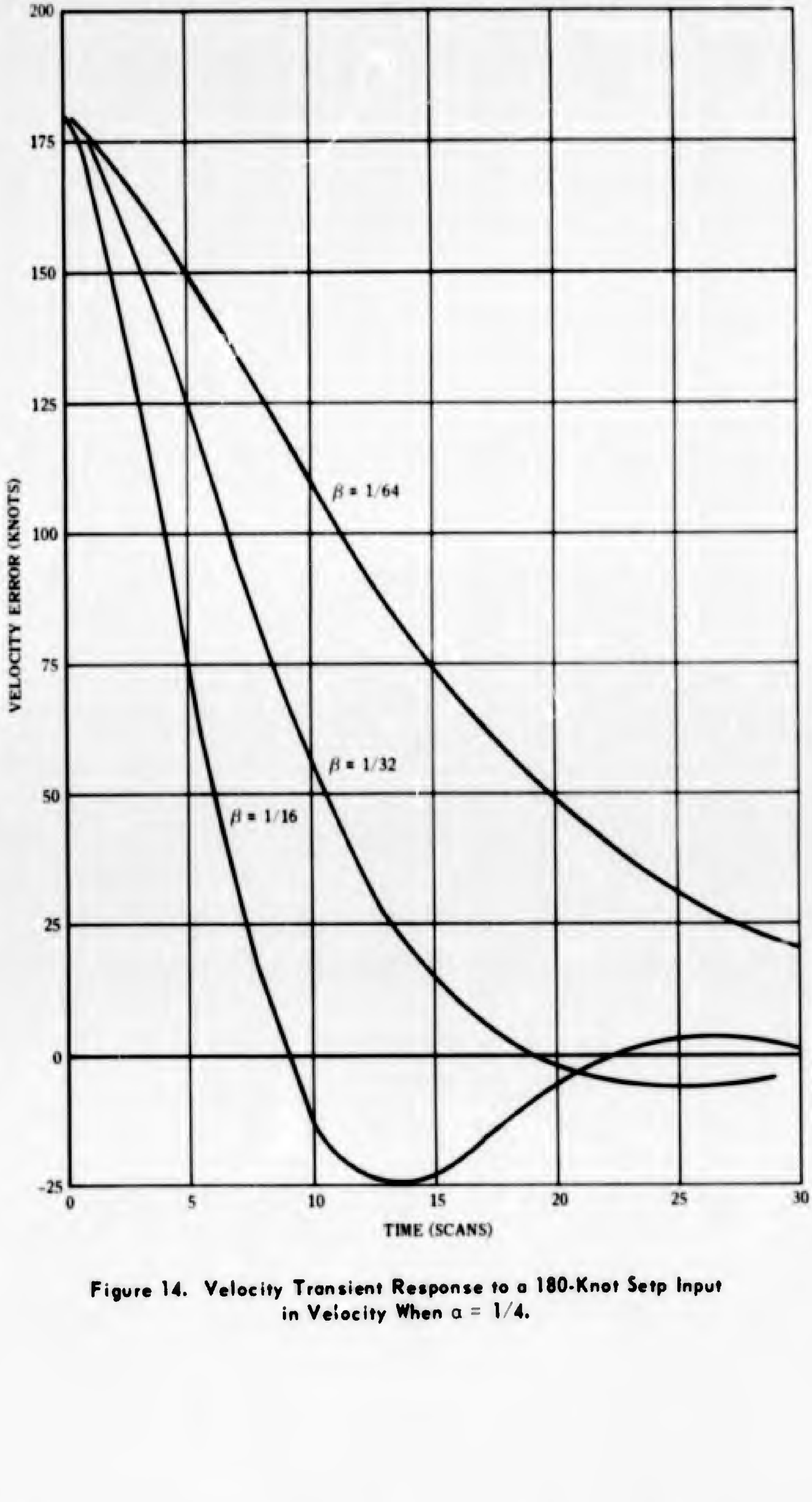

Figure 14. Velocity Transient Response to a 180-Knot Setp Input in Velocity When α = 1/4.

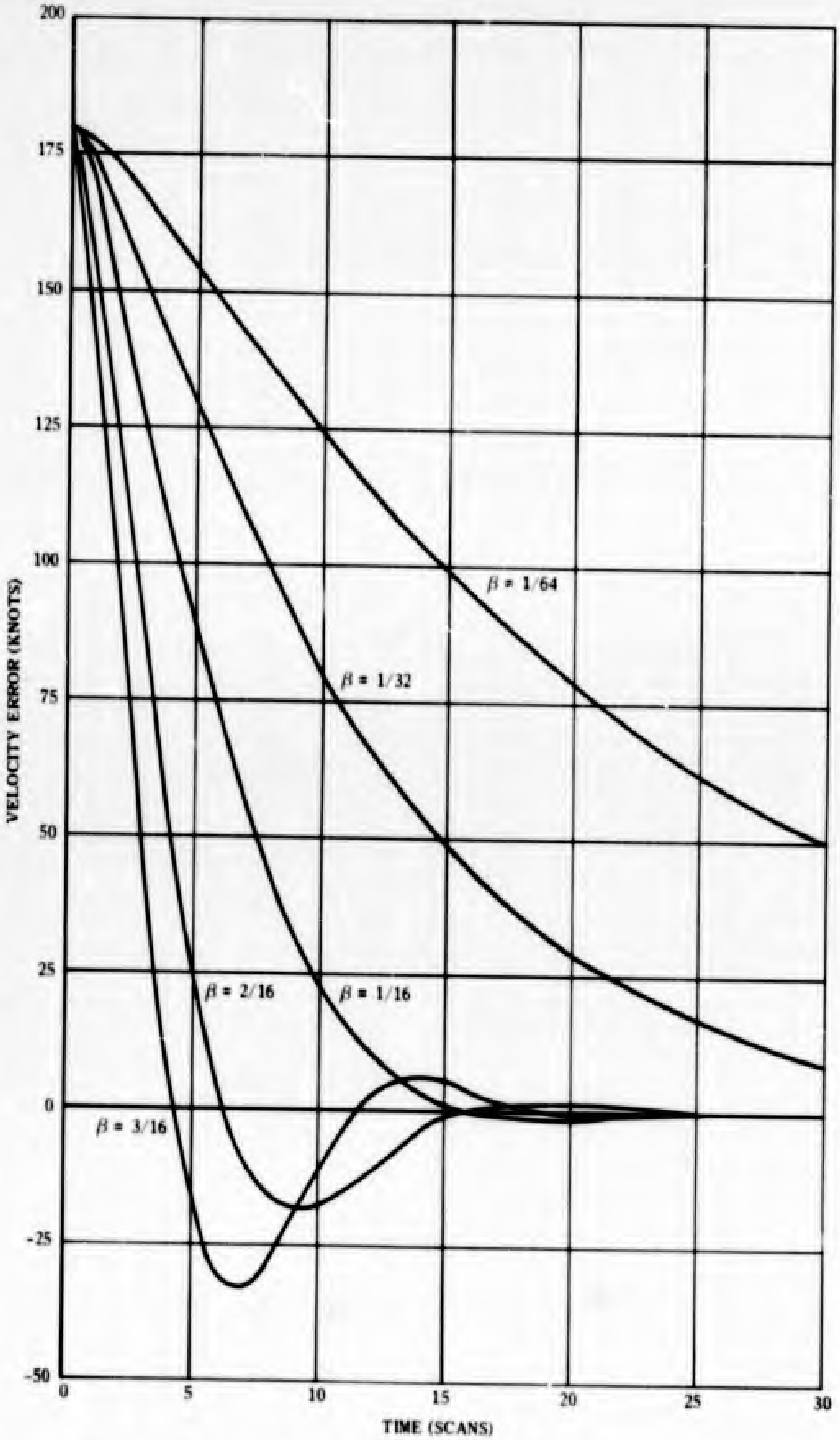

Figure 15. Velocity Transient Response to a 180-Knot Step Input in Velocity When α = 3/8.

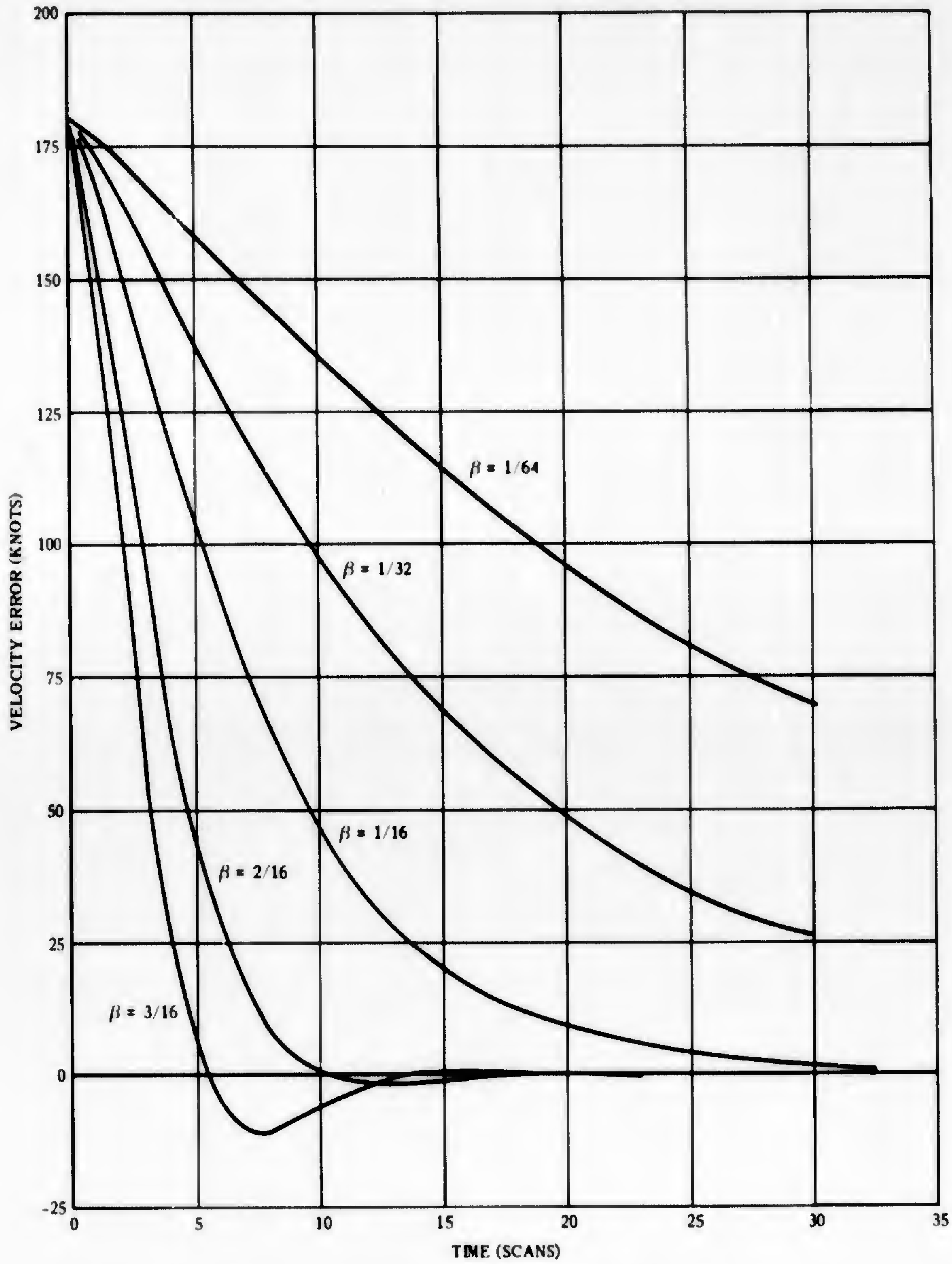

Figure 16. Velocity Transient Response to a 180-Knot Step Input in Velocity When $\alpha = 1/2$.

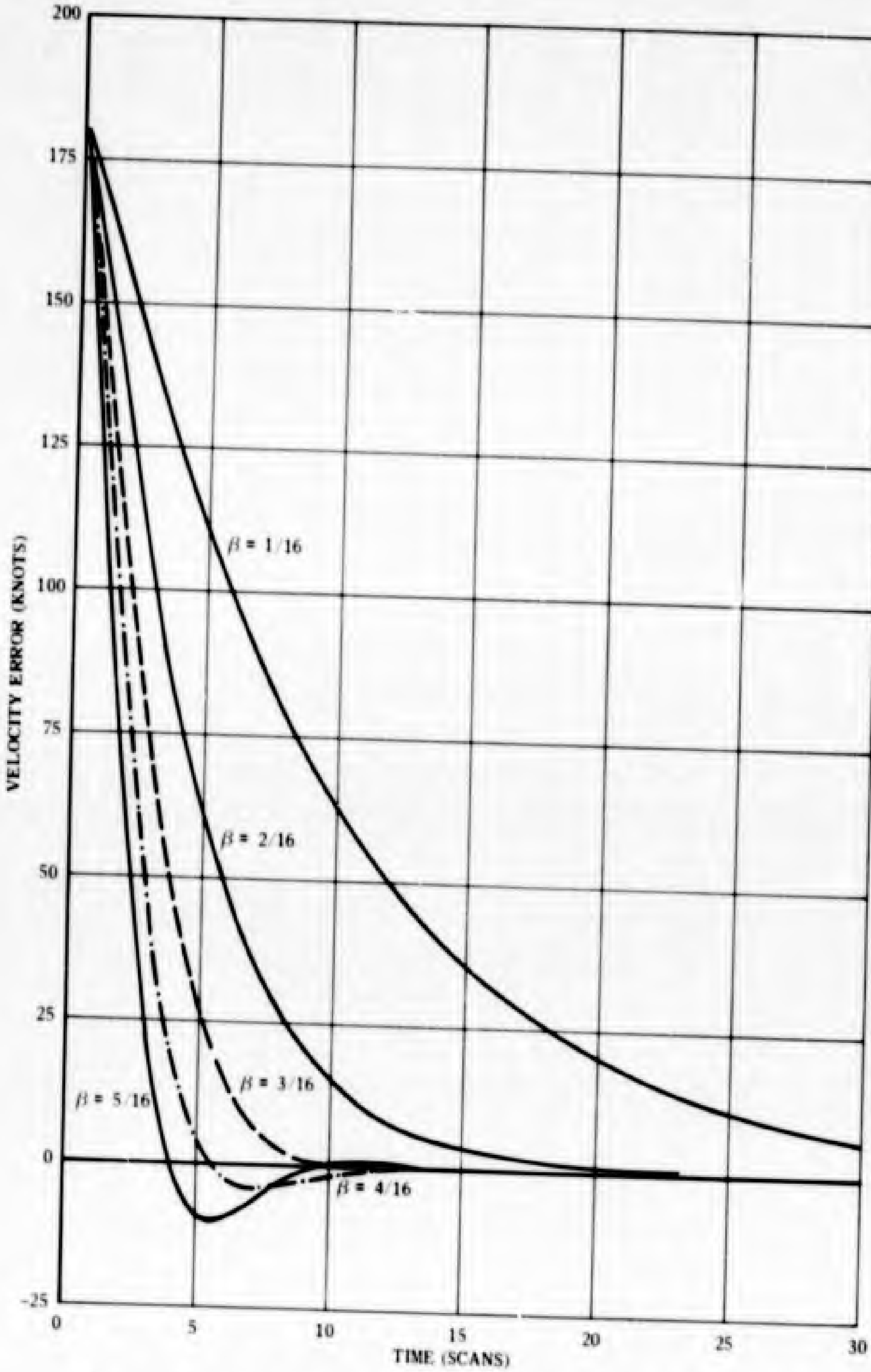

Figure 17. Velocity Transient Response to a 180-Knot Step Input in Velocity When α = 5/8.

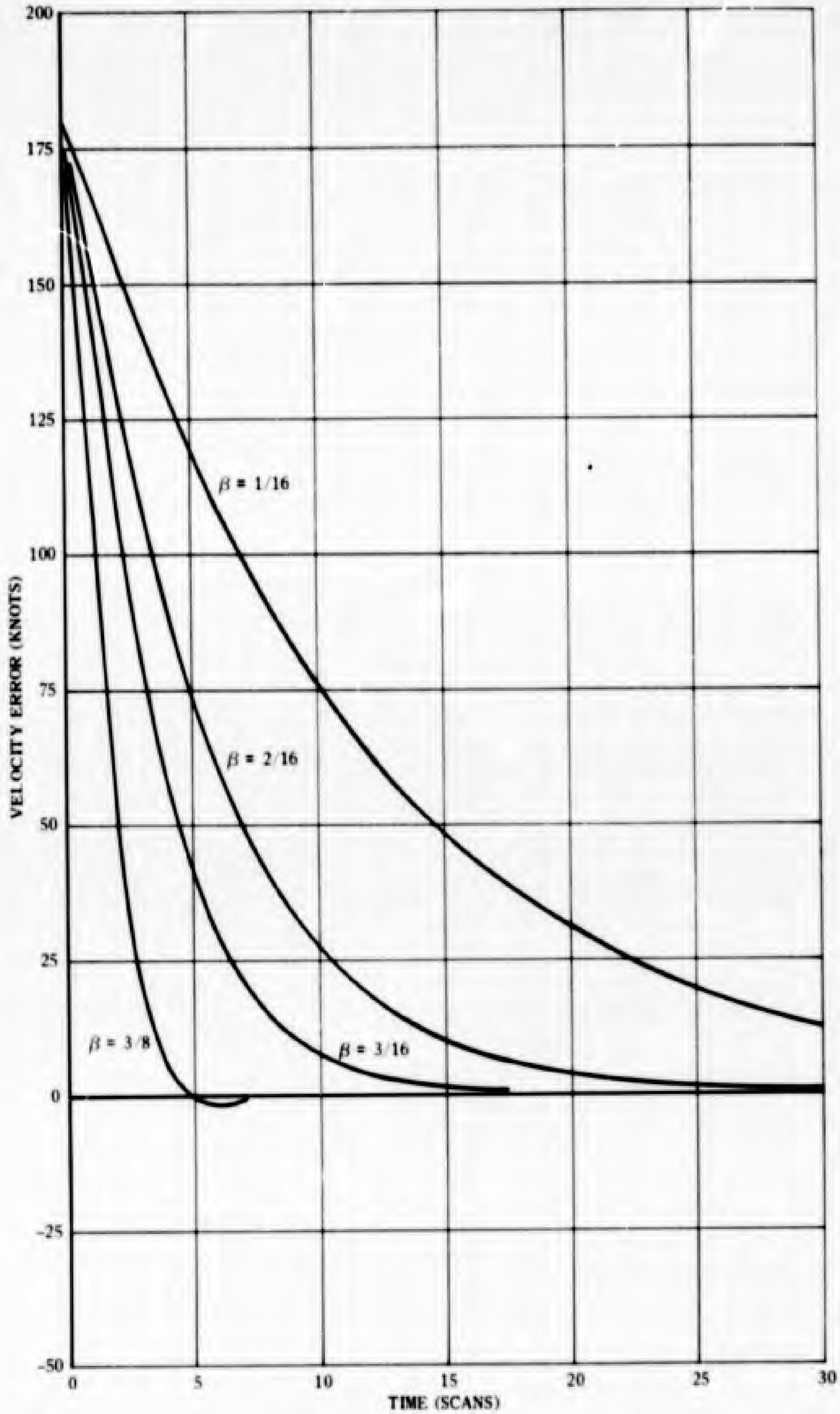

Figure 18. Velocity Transient Response to a 180-Knot Step Input in Velocity When α = 3/4.

The transient input will be defined as that change in velocity which the specified target can achieve in 10 seconds with a lateral acceleration of 2.5g. In 10 seconds the 540-knot target can turn through an angle of 52 degrees, changing its velocity vector by 420 knots; and in the same time the 900-knot target can turn through 31 degrees, changing its velocity vector by 475 knots.

Selection of Coefficients for the α-β Tracker

A set of coefficients must be selected for each of the nominal target speeds. In choosing the coefficients it will be assumed that the track has been well established and that the blip/scan ratio is unity. Next, a third set of coefficients will be selected for track initiation, and finally the performance of the resulting tracker will be checked to determine the significance of the assumption of unity blip/scan ratio.

The coefficients to be used for established target tracks at the two nominal speeds can be obtained from figure 19, which shows the minimum velocity transient demerit corresponding to the selected values of $K_{\dot{x}}$ for the two maneuvering targets. It is seen that the velocity transient demerit is minimized in the range $\alpha = 0.25$ to 0.4. Values of α and β which can easily be mechanized in the ATU are shown as points on the figure. It appears that $\alpha = 1/4$, $\beta = 1/32$ is most reasonable for the low-speed target, while $\alpha = 1/4$, $\beta = 1/16$ is best for the high-speed target. Note that position transient response, shown in figure 20, has not been minimized.

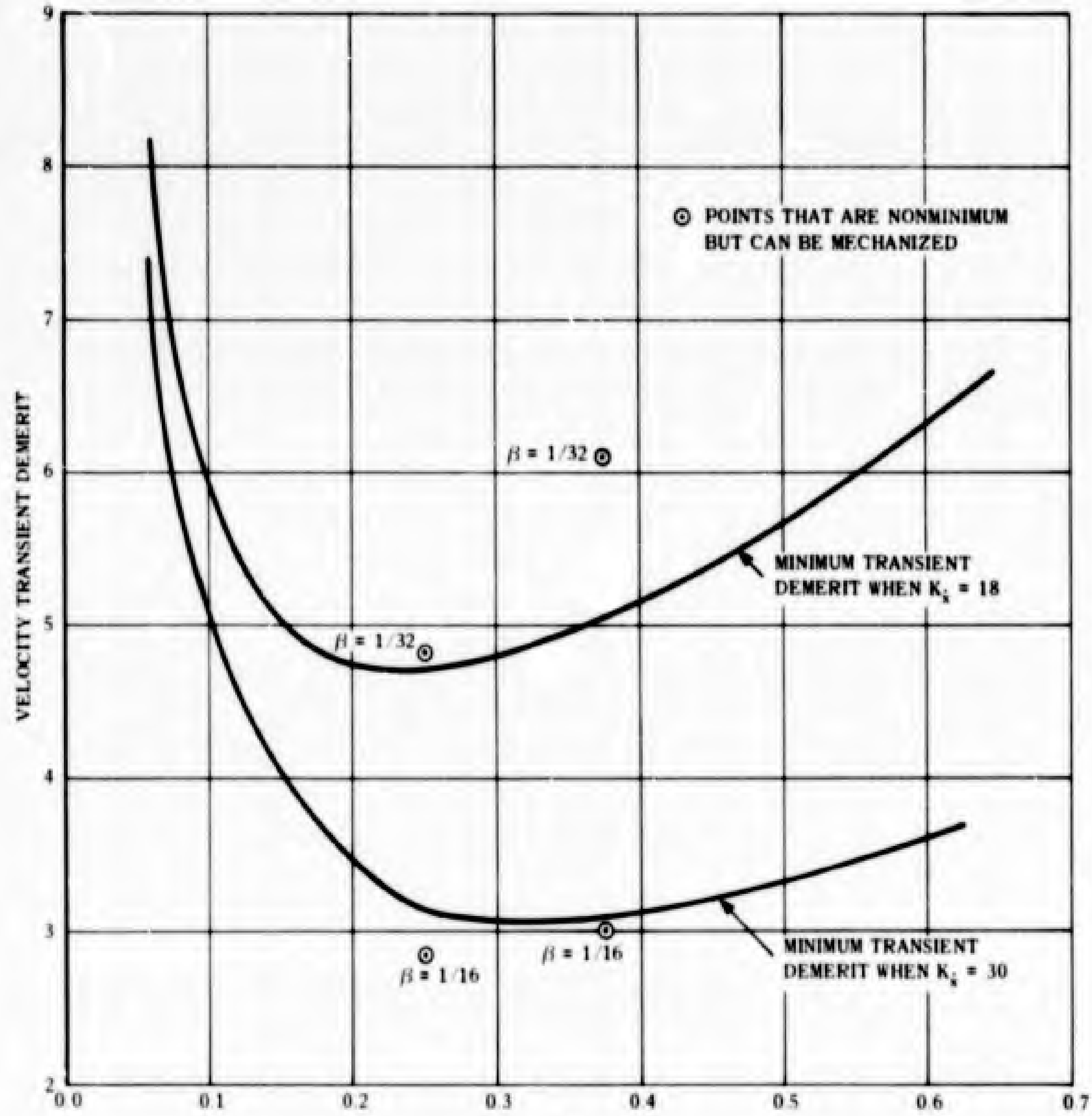

Figure 19. Velocity Transient Demerit Versus α for a 180-Knot Step Input in Velocity.

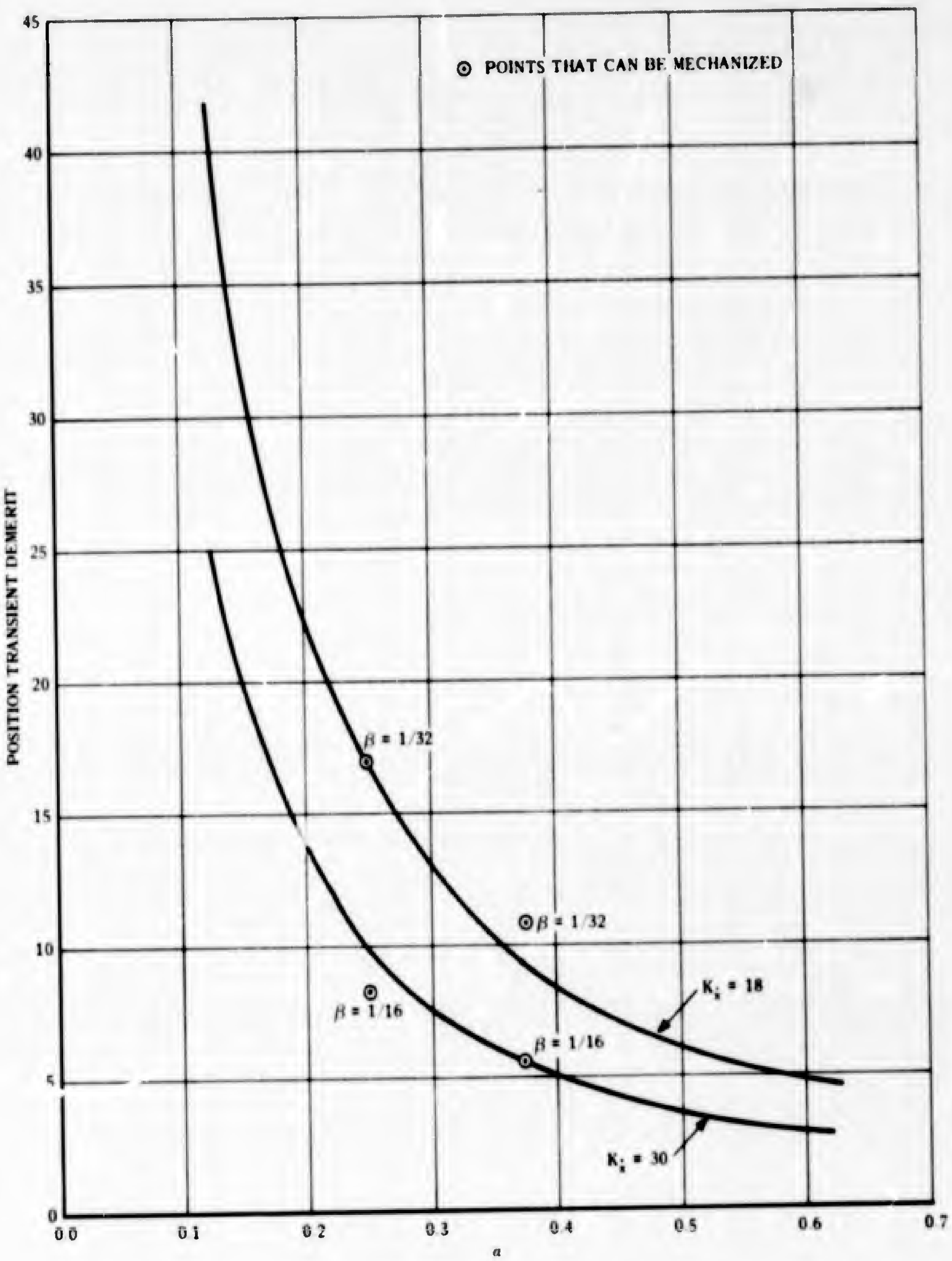

Figure 20. Position Transient Demerit Versus α for a 180-Knot Step Input in Velocity.

Figure 21 shows the position noise as a function of a, with $K_{\dot{x}}$ held constant. The position noise coefficient does not appear to be significantly affected by small variations in a.

Referring back now to figure 7, it is seen that the velocity noise coefficients actually achieved are 17 knots per mile for the subsonic target and 35 knots per mile for the supersonic target.

In appendix B it is shown that the track at the time of initiation is particularly susceptible to noisy input data because only two target positions are available on which to base an estimate of velocity and predicted position. As time goes by and more tracking data becomes available the noise response decreases to a steady-state value.

The two sets of coefficients previously chosen produce heavy smoothing, but they require unreasonably long times to achieve the heavy smoothing. For the first few scans after track initiation the position noise-reduction coefficient exceeds 3.0 miles per mile, which could lead to such large errors in the predicted position that the correlation process could fail, causing the track to be dropped before it ever became firmly established. Therefore, it is desirable to use a third set of coefficients for track initiation.

Figures 22 and 23 show K_x and $K_{\dot{x}}$ for several sets of coefficients. These figures were obtained by repeated solutions of equations (B-9), (B-10), (B-13), and (B-14) in appendix B. One set ($a = 5/8$, $\beta = 3/16$) clearly gives the fastest response of any of the sets considered. Therefore, this set should be used until the rate of decrease of $K_{\bar{x}}$ and $K_{\dot{x}}$ is less than the rate of decrease which can be obtained by switching to the sets of coefficients previously chosen for established tracks. The switching should occur about 4 or 5 scans after track initiation.

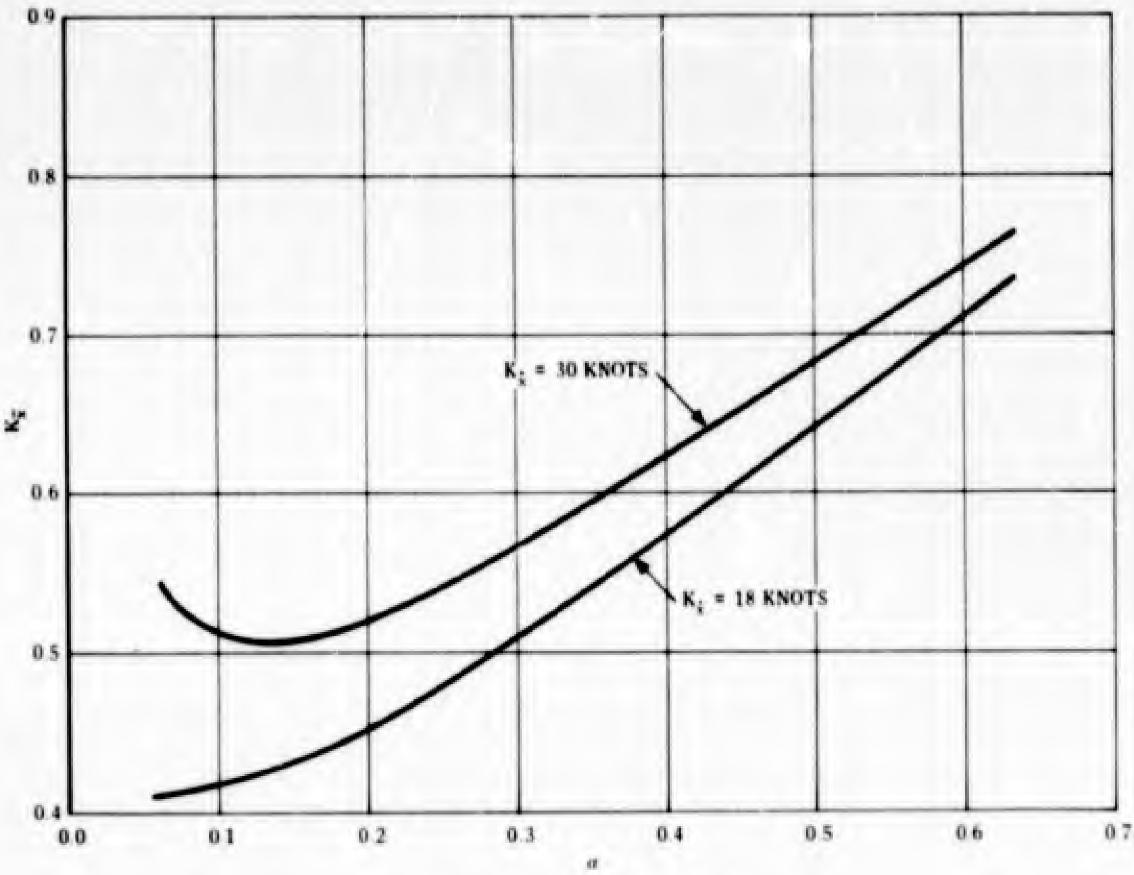

Figure 21. $K_{\bar{x}}$ Versus a for Constant $K_{\dot{x}}$.

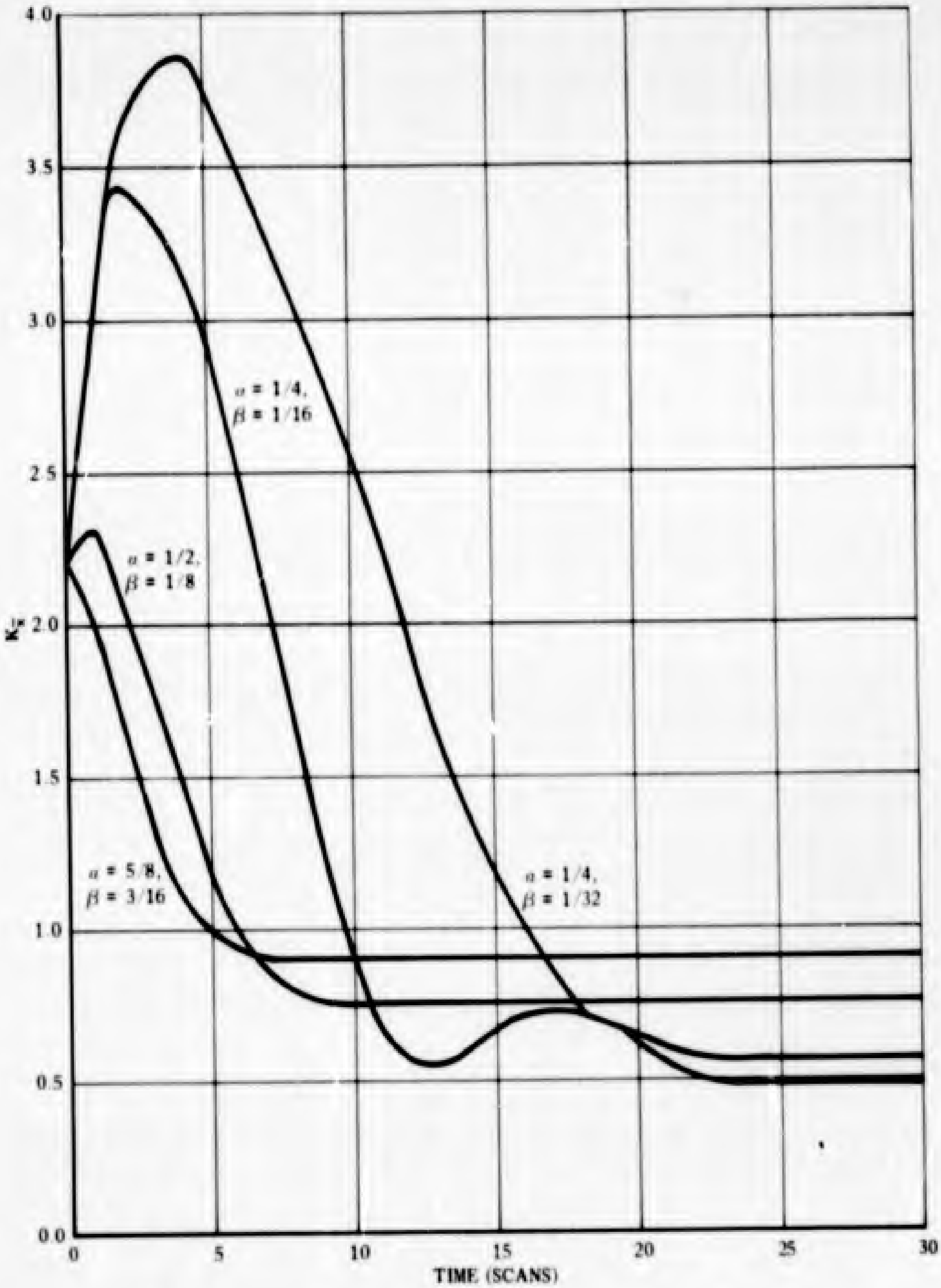

Figure 22. $K_{\bar{x}}$ Versus Time.

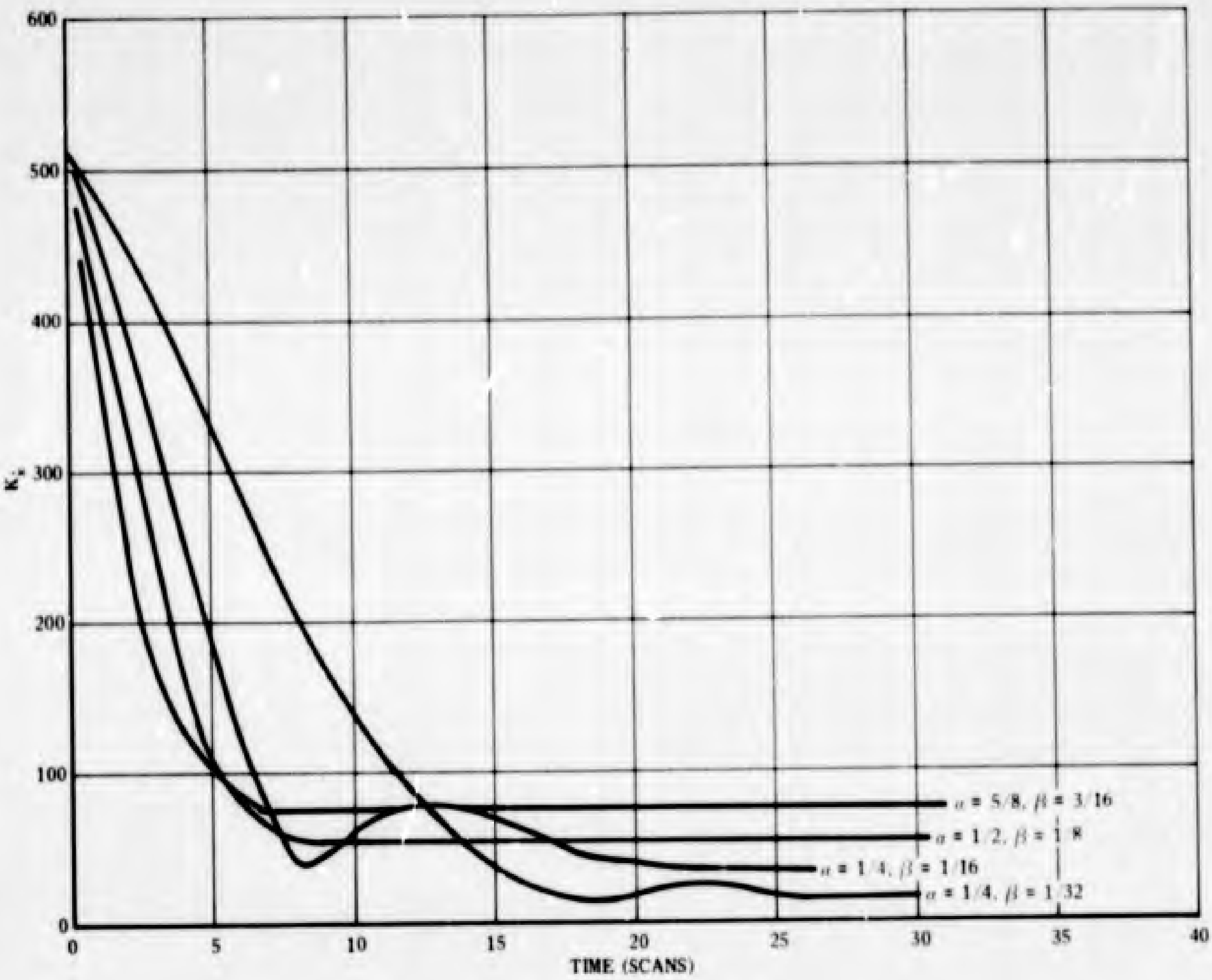

Figure 23. $K_{\dot{x}}$ Versus Time.

Blip/Scan Effects

The blip/scan ratio is defined as the probability that a non-noise report will be sent from the computer detector to the correlator on a given scan. The previous analysis was performed under the assumption that the blip/scan ratio was unity. However, the blip/scan ratio normally is not unity, and it was suspected that this would have some effect on the amount of noise coming out of the tracker.

A simulation of the tracker is described in appendix C. This simulation was used to study the effect of nonunity blip/scan ratios on $K_{\dot{x}}$ and $K_{\bar{x}}$. The results of the study are summarized in figures 24 and 25 for the existing ATDS tracker and for the α-β tracker with the two selected sets of coefficients.

It can be seen that, for the α-β tracker, the maximum increase in $K_{\dot{x}}$ is about 5 knots, and the maximum increase in $K_{\bar{x}}$ is 0.2 mile as the blip/scan ratio varies from 1.0 to 0.5. Thus, the effect of nonunity blip/scan ratios is not significant for the conditions considered here and can be ignored.

The steady-state noise characteristics of the three recommended sets of coefficients are summarized in table 3 for comparison with the present ATDS tracker.

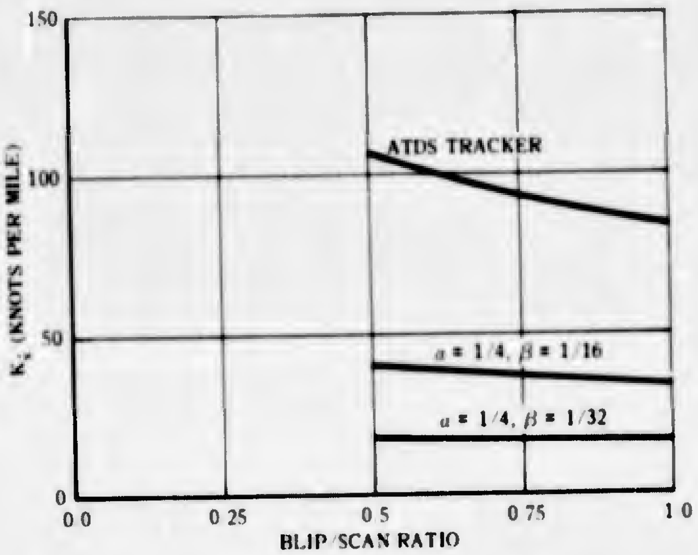

Figure 24. $K_{\dot{x}}$ Versus Blip/Scan Ratio.

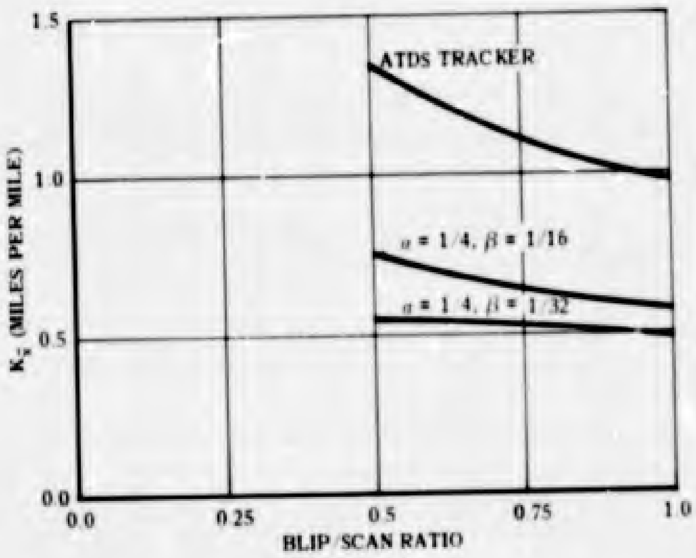

Figure 25. $K_{\bar{x}}$ Versus Blip/Scan Ratio.

Table 3. Summary of Steady-State Noise Characteristics of the Three Recommended Sets of Coefficients

Tracker Coefficients	Circumstances Under Which Tracker Would Be Used	Standard Deviations	
		Velocity Noise (Knots)	Position Noise (Miles)
Those of the present ATDS	Those of the present ATDS	84-252	0.98-2.94
$\alpha = 5/8$ $\beta = 3/16$	Track initiation	75-225	0.89-2.67
$\alpha = 1/4$ $\beta = 1/16$	Supersonic target	35-105	0.56-1.68
$\alpha = 1/4$ $\beta = 1/32$	Subsonic target	17-51	0.48-1.44

CONCLUSIONS

Based on an analytical and simulation analysis of two tracking systems, it is concluded that:

1. No significant difference exists between the performance of the present ATDS tracker and that of an α-β tracker which is designed to have the same velocity noise response as the present tracker.
2. The α-β tracker has an advantage in that it requires only four computer storage locations per target track (versus eight for the present system) and it uses two parameters (versus nine for the present system). Thus, the α-β tracker is easier to mechanize and requires less computer storage space.
3. Because of the simplicity of the α-β tracker, it is feasible to use one set of coefficients for high-speed targets, a second set for low-speed targets, and a third set for track initiation.
4. Adoption of the α-β tracker with the three recommended sets of coefficients would reduce the velocity noise of the tracker from its present high level to a level which the system should be able to accept. This would result, however, in a significant decrease in the ability of the system to automatically track rapidly maneuvering targets and would place a greater load on the operator. Only if an ATDS computer were mechanized* with the α-β tracker as developed in this report, would it be possible to determine whether the operator has the ability to perform manual correlations on heavily smoothed and maneuvering tracks.
5. An effort should be made to improve the capability of the operator to perform manual correlations, either during or after proving the feasibility of the α-β tracker.

* Either in a laboratory or in an aircraft.

APPENDIX A

DERIVATION OF NOISE RESPONSE OF PRESENT ATDS

The position-prediction equation is of the form

$$\overline{X}_{n+1} = C_0 \overline{X}_n + C_1 \widetilde{X}_n + C_2 \widetilde{X}_{n-1} + C_3 \widetilde{X}_{n-2} + C_4 \widetilde{X}_{n-3} \qquad \text{(A-1)}$$

where

$\overline{X}_{n+1}$ = position predicted for time n + 1.

$\overline{X}_n$ = position which was predicted for time n.

$\widetilde{X}_n$ = position which was measured for time n.

Noise inputs into the system are assumed to be uncorrelated from scan to scan, to have constant mean, and to be stationary for at least a few scans.

$\overline{X}_n$ is predicted before $\widetilde{X}_n$ is measured so

$$\sigma_{\bar{x}_n, \tilde{x}_n} = 0$$

Also

$$\sigma_{\tilde{x}_n, \tilde{x}_{n-1}} = \sigma_{\tilde{x}_n, \tilde{x}_{n-2}} = \sigma_{\tilde{x}_n, \tilde{x}_{n-3}} = \sigma_{\tilde{x}_{n-1}, \tilde{x}_{n-2}} = \sigma_{\tilde{x}_{n-1}, \tilde{x}_{n-3}}$$

$$= \sigma_{\tilde{x}_{n-2}, \tilde{x}_{n-3}} = 0 \qquad \text{(A-2)}$$

and

$$\sigma^2_{\tilde{x}_n} = \sigma^2_{\tilde{x}_{n-1}} = \sigma^2_{\tilde{x}_{n-2}} = \sigma^2_{\tilde{x}_{n-3}} \qquad \text{(A-3)}$$

It can be shown that

$$\sigma^2_{\bar{x}_{n+1}} = \left(\frac{\partial \overline{X}_{n+1}}{\partial \overline{X}_n}\right)^2 \sigma^2_{\bar{x}_n} + \left(\frac{\partial \overline{X}_{n+1}}{\partial \widetilde{X}_n}\right)^2 \sigma^2_{\tilde{x}_n} + \left(\frac{\partial \overline{X}_{n+1}}{\partial \widetilde{X}_{n-1}}\right)^2 \sigma^2_{\tilde{x}_{n-1}}$$

$$+ \left(\frac{\partial \overline{X}_{n+1}}{\partial \widetilde{X}_{n-2}}\right)^2 \sigma^2_{\tilde{x}_{n-2}} + \left(\frac{\partial \overline{X}_{n+1}}{\partial \widetilde{X}_{n-3}}\right)^2 \sigma^2_{\tilde{x}_{n-3}}$$

$$+ 2\left(\frac{\partial \overline{X}_{n+1}}{\partial \overline{X}_n}\right)\left(\frac{\partial \overline{X}_{n+1}}{\partial \widetilde{X}_n}\right) \sigma_{\bar{x}_n, \tilde{x}_n} + 2\left(\frac{\partial \overline{X}_{n+1}}{\partial \overline{X}_n}\right)\left(\frac{\partial \overline{X}_{n+1}}{\partial \widetilde{X}_{n-1}}\right) \sigma_{\bar{x}_n, \tilde{x}_{n-1}}$$

$$+ 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \bar{X}_n}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-2}}\right)\sigma_{\bar{x}_n, \tilde{x}_{n-2}} + 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \bar{X}_n}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-3}}\right)\sigma_{\bar{x}_n, \tilde{x}_{n-3}}$$

$$+ 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_n}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-1}}\right)\sigma_{\tilde{x}_n, \tilde{x}_{n-1}} + 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_n}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-2}}\right)\sigma_{\tilde{x}_n, \tilde{x}_{n-2}}$$

$$+ 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_n}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-3}}\right)\sigma_{\tilde{x}_n, \tilde{x}_{n-3}} + 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-1}}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-2}}\right)\sigma_{\tilde{x}_{n-1}, \tilde{x}_{n-2}}$$

$$+ 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-1}}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-3}}\right)\sigma_{\tilde{x}_{n-1}, \tilde{x}_{n-3}}$$

$$+ 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-2}}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-3}}\right)\sigma_{\tilde{x}_{n-2}, \tilde{x}_{n-3}} \qquad \text{(A-4)}$$

Placing the values of equations (A-2) and (A-3) into equation (A-4), we obtain

$$\sigma^2_{\bar{x}_{n+1}} = \sigma^2_{\bar{x}_n}\left(\frac{\partial \bar{X}_{n+1}}{\partial \bar{X}_n}\right)^2 + \sigma^2_{\tilde{x}}\left[\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_n}\right)^2 + \left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-1}}\right)^2\right.$$

$$\left. + \left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-2}}\right)^2 + \left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-3}}\right)^2\right]$$

$$+ 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \bar{X}_n}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-1}}\right)\sigma_{\bar{x}_n, \tilde{x}_{n-1}} + 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \bar{X}_n}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-2}}\right)\sigma_{\bar{x}_n, \tilde{x}_{n-2}}$$

$$+ 2\left(\frac{\partial \bar{X}_{n+1}}{\partial \bar{X}_n}\right)\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-3}}\right)\sigma_{\bar{x}_n, \tilde{x}_{n-3}} \qquad \text{(A-5)}$$

Using equation (A-1) we obtain

$$\left(\frac{\partial \bar{X}_{n+1}}{\partial \bar{X}_n}\right) = C_0$$

$$\left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_n}\right) = C_1$$

$$\left(\frac{\partial \overline{X}_{n+1}}{\partial \widetilde{X}_{n-1}}\right) = C_2$$

$$\left(\frac{\partial \overline{X}_{n+1}}{\partial \widetilde{X}_{n-2}}\right) = C_3$$

$$\left(\frac{\partial \overline{X}_{n+1}}{\partial \widetilde{X}_{n-3}}\right) = C_4$$

Placing these values in equation (A-5) we obtain

$$\begin{aligned} \sigma^2_{\bar{x}_{n+1}} &= C_0^2\,\sigma^2_{\bar{x}_n} + \sigma^2_{\tilde{x}}\,(C_1^2 + C_2^2 + C_3^2 + C_4^2) \\ &+ 2\,C_0\,C_2\,\sigma_{\bar{x}_n, \tilde{x}_{n-1}} + 2\,C_0\,C_3\,\sigma_{\bar{x}_n, \tilde{x}_{n-2}} \\ &+ 2\,C_0\,C_4\,\sigma_{\bar{x}_n, \tilde{x}_{n-3}} \end{aligned} \qquad \text{(A-6)}$$

To evaluate the remaining covariance terms, write

$$\overline{X}_n = C_0\,\overline{X}_{n-1} + C_1\,\widetilde{X}_{n-1} + C_2\,\widetilde{X}_{n-2} + C_3\,\widetilde{X}_{n-3} + C_4\,\widetilde{X}_{n-4} \qquad \text{(A-7)}$$

Thus,

$$\sigma_{\bar{x}_n, \tilde{x}_{n-1}} = \left(\frac{\partial \overline{X}_n}{\partial \widetilde{X}_{n-1}}\right)\left(\frac{\partial \widetilde{X}_{n-1}}{\partial \widetilde{X}_{n-1}}\right)\sigma^2_{\tilde{x}_{n-1}} = C_1\,\sigma^2_{\tilde{x}} \qquad \text{(A-8)}$$

Then write

$$\begin{aligned} \overline{X}_n &= C_0\,(C_0\,\overline{X}_{n-2} + C_1\,\widetilde{X}_{n-2} + C_2\,\widetilde{X}_{n-3} + C_3\,\widetilde{X}_{n-4} + C_4\,\widetilde{X}_{n-5}) \\ &+ C_1\,\widetilde{X}_{n-1} + C_2\,\widetilde{X}_{n-2} + C_3\,\widetilde{X}_{n-3} + C_4\,\widetilde{X}_{n-4} \end{aligned} \qquad \text{(A-9)}$$

Thus,

$$\sigma_{\bar{x}_n, \tilde{x}_{n-2}} = \left(\frac{\partial \overline{X}_n}{\partial \widetilde{X}_{n-2}}\right)\left(\frac{\partial X_{n-2}}{\partial \widetilde{X}_{n-2}}\right)\sigma_{\tilde{x}_{n-2}} = (C_0\,C_1 + C_2)\,\sigma^2_{\tilde{x}} \qquad \text{(A-10)}$$

Then write

$$\begin{aligned} \overline{X}_n &= C_0\,[C_0\,(C_0\,\overline{X}_{n-3} + C_1\,\widetilde{X}_{n-3} + C_2\,\widetilde{X}_{n-4} + C_3\,\widetilde{X}_{n-5} + C_4\,\widetilde{X}_{n-6}) \\ &+ C_1\,\widetilde{X}_{n-2} + C_2\,\widetilde{X}_{n-3} + C_3\,\widetilde{X}_{n-4} + C_4\,\widetilde{X}_{n-5}] \end{aligned}$$

$$+ C_1 \tilde{X}_{n-1} + C_2 \tilde{X}_{n-2} + C_3 \tilde{X}_{n-3} + C_4 \tilde{X}_{n-4} \qquad \text{(A-11)}$$

Thus,

$$\sigma_{\bar{x}_n, \tilde{x}_{n-3}} = \left(\frac{\partial \bar{X}_n}{\partial \tilde{X}_{n-3}}\right)\left(\frac{\partial \tilde{X}_{n-3}}{\partial \tilde{X}_{n-3}}\right) \sigma^2_{\tilde{x}_{n-3}}$$

$$= (C_0^2 C_1 + C_0 C_2 + C_3)\, \sigma_{\tilde{x}} \qquad \text{(A-12)}$$

Using equations (A-8), (A-10), and (A-12) in equation (A-6), we obtain

$$\sigma^2_{\bar{x}_{n+1}} = C_0^2 \sigma^2_{x_n} + \sigma^2_{\tilde{x}} [C_1^2 + C_2^2 + C_3^2 + C_4^2 + 2 C_0 C_1 C_2$$

$$+ 2 C_0 C_3 (C_0 C_1 + C_2) + 2 C_0 C_4 (C_0^2 C_1 + C_0 C_2 + C_3)] \qquad \text{(A-13)}$$

After the system has been operating for approximately ten scans, we can say that

$$\sigma^2_{\bar{x}_{n+1}} = \sigma^2_{\bar{x}_n} \qquad \text{(A-14)}$$

Thus,

$$\sigma^2_{\bar{x}_n} = C_0^2 \sigma^2_{\bar{x}_n} + \sigma^2_{\tilde{x}} [C_1^2 + C_2^2 + C_3^2 + C_4^2$$

$$+ 2 (C_0 C_1 C_2 + C_0^2 C_1 C_3 + C_0 C_2 C_3 + C_0^3 C_1 C_4$$

$$+ C_0^2 C_2 C_4 + C_0 C_3 C_4)] \qquad \text{(A-15)}$$

Define

$$K^2_{\bar{x}} = \frac{\sigma^2_{\bar{x}_n}}{\sigma^2_{\tilde{x}}} \qquad \text{(A-16)}$$

to be the position noise-reduction coefficient, then

$$K^2_{\bar{x}} = \frac{C_1^2 + C_2^2 + C_3^2 + C_4^2 + 2 (C_0 C_1 C_2 + C_0^2 C_1 C_3 + C_0 C_2 C_3 + C_0^3 C_1 C_4 + C_0^2 C_2 C_4 + C_0 C_3 C_4)}{1 - C_0^2} \qquad \text{(A-17)}$$

The ATDS tracker now uses

$$C_0 = \frac{5}{8},\ C_1 = \frac{7}{8},\ C_2 = -\frac{1}{8},\ C_3 = -\frac{2}{8},\ C_4 = -\frac{1}{8}$$

so that

$$K_{\bar{x}}^2 = 0.965$$

$$K_{\bar{x}} = 0.983$$

Velocity estimation is accomplished by using the equation

$$V_x = D_0 \bar{X}_{n+1} + D_1 \tilde{X}_n + D_2 \tilde{X}_{n-1} + D_3 \tilde{X}_{n-2} + D_4 \tilde{X}_{n-3} \tag{A-18}$$

therefore,

$$\begin{aligned}
\sigma_{V_x}^2 = {} & \left(\frac{\partial V_x}{\partial \bar{X}_{n+1}}\right)^2 \sigma_{\bar{x}_{n+1}}^2 + \left(\frac{\partial V_x}{\partial \tilde{X}_n}\right)^2 \sigma_{\bar{x}_n}^2 + \left(\frac{\partial V_x}{\partial \tilde{X}_{n-1}}\right)^2 \sigma_{\bar{x}_{n-1}}^2 \\
& + \left(\frac{\partial V_x}{\partial \tilde{X}_{n-2}}\right)^2 \sigma_{\bar{x}_{n-2}}^2 + \left(\frac{\partial V_x}{\partial \tilde{X}_{n-3}}\right)^2 \sigma_{\bar{x}_{n-3}}^2 \\
& + 2\left(\frac{\partial V_x}{\partial \bar{X}_{n+1}}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_n}\right) \sigma_{\bar{x}_{n+1},\bar{x}_n} + 2\left(\frac{\partial V_x}{\partial \bar{X}_{n+1}}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_{n-1}}\right) \sigma_{\bar{x}_{n+1},\bar{x}_{n-1}} \\
& + 2\left(\frac{\partial V_x}{\partial \bar{X}_{n+1}}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_{n-2}}\right) \sigma_{\bar{x}_{n+1},\bar{x}_{n-2}} + 2\left(\frac{\partial V_x}{\partial \bar{X}_{n+1}}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_{n-3}}\right) \sigma_{\bar{x}_{n+1},\bar{x}_{n-3}} \\
& + 2\left(\frac{\partial V_x}{\partial \tilde{X}_n}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_{n-1}}\right) \sigma_{\bar{x}_n,\bar{x}_{n-1}} + 2\left(\frac{\partial V_x}{\partial \tilde{X}_n}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_{n-2}}\right) \sigma_{\bar{x}_n,\bar{x}_{n-2}} \\
& + 2\left(\frac{\partial V_x}{\partial \tilde{X}_n}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_{n-3}}\right) \sigma_{\bar{x}_n,\bar{x}_{n-3}} + 2\left(\frac{\partial V_x}{\partial \tilde{X}_{n-1}}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_{n-2}}\right) \sigma_{\bar{x}_{n-1},\bar{x}_{n-2}} \\
& + 2\left(\frac{\partial V_x}{\partial \tilde{X}_{n-1}}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_{n-3}}\right) \sigma_{\bar{x}_{n-1},\bar{x}_{n-3}} \\
& + 2\left(\frac{\partial V_x}{\partial \tilde{X}_{n-2}}\right)\left(\frac{\partial V_x}{\partial \tilde{X}_{n-3}}\right) \sigma_{\bar{x}_{n-2},\bar{x}_{n-3}}
\end{aligned} \tag{A-19}$$

As in the derivation of the position noise-reduction coefficient,

$$\sigma_{\bar{x}_{n+1},\bar{x}} = \sigma_{\bar{x}_n,\bar{x}_{n-1}} = \sigma_{\bar{x}_n,\bar{x}_{n-2}} = \sigma_{\bar{x}_n,\bar{x}_{n-3}} = \sigma_{\bar{x}_{n-1},\bar{x}_{n-2}} = \sigma_{\bar{x}_{n-1},\bar{x}_{n-3}} = \sigma_{\bar{x}_{n-2},\bar{x}_{n-3}} = 0$$

and

$$\sigma^2_{\tilde{x}_n} = \sigma^2_{\tilde{x}_{n-1}} = \sigma^2_{\tilde{x}_{n-2}} = \sigma^2_{\tilde{x}_{n-3}}$$

which we set equal to $\sigma^2_{\tilde{x}}$

If we evaluate the indicated partials, we obtain

$$\frac{\partial V_x}{\partial \overline{X}_{n+1}} = D_0$$

$$\frac{\partial V_x}{\partial \widetilde{X}_n} = D_1$$

$$\frac{\partial V_x}{\partial \widetilde{X}_{n-1}} = D_2$$

$$\frac{\partial V_x}{\partial \widetilde{X}_{n-2}} = D_3$$

$$\frac{\partial V_x}{\partial \widetilde{X}_{n-3}} = D_4$$

Thus, equation (A-19) becomes

$$\begin{aligned}\sigma^2_{v_x} &= D_0^2 \, \sigma^2_{\overline{x}_{n+1}} + \sigma^2_{\tilde{x}} \left(D_1^2 + D_2^2 + D_3^2 + D_4^2\right) \\ &+ 2\, D_0\, D_1\, \sigma_{\overline{x}_{n+1}, \tilde{x}_n} + 2\, D_0\, D_2\, \sigma_{\overline{x}_{n+1}, \tilde{x}_{n-1}} \\ &+ 2\, D_0\, D_3\, \sigma_{\overline{x}_{n+1}, \tilde{x}_{n-2}} + 2\, D_0\, D_4\, \sigma_{\overline{x}_{n+1}, \tilde{x}_{n-3}} \end{aligned} \qquad \text{(A-20)}$$

By rewriting equations (A-8), (A-10), and (A-12), we obtain

$$\sigma_{\overline{x}_{n+1}, \tilde{x}_n} = C_1 \, \sigma^2_{\tilde{x}} \qquad \text{(A-21)}$$

$$\sigma_{\overline{x}_{n+1}, \tilde{x}_{n-1}} = (C_0\, C_1 + C_2)\, \sigma^2_{\tilde{x}} \qquad \text{(A-22)}$$

$$\sigma_{\overline{x}_{n+1}, \tilde{x}_{n-2}} = (C_0^2\, C_1 + C_0\, C_2 + C_3)\, \sigma^2_{\tilde{x}} \qquad \text{(A-23)}$$

To obtain the last covariance term, write

$$\bar{X}_{n+1} = C_0 \{ C_0 [C_0 (C_0 \bar{X}_{n-3} + C_1 \tilde{X}_{n-3} + C_2 \tilde{X}_{n-4} + C_3 \tilde{X}_{n-5} + C_4 \tilde{X}_{n-6})$$

$$+ C_1 \tilde{X}_{n-2} + C_2 \tilde{X}_{n-3} + C_3 \tilde{X}_{n-4} + C_4 \tilde{X}_{n-5}]$$

$$+ C_1 \tilde{X}_{n-1} + C_2 \tilde{X}_{n-2} + C_3 \tilde{X}_{n-3} + C_4 \tilde{X}_{n-4} \}$$

$$+ C_1 \tilde{X}_n + C_2 \tilde{X}_{n-1} + C_3 \tilde{X}_{n-2} + C_4 \tilde{X}_{n-3} \qquad \text{(A-24)}$$

Then

$$\sigma_{\bar{x}_{n+1}, \bar{x}_{n-3}} = \left(\frac{\partial \bar{X}_{n+1}}{\partial \tilde{X}_{n-3}} \right) \left(\frac{\partial \tilde{X}_{n-3}}{\partial \tilde{X}_{n-3}} \right) \sigma^2_{\tilde{x}_{n-3}}$$

$$= (C_0^3 C_1 + C_0^2 C_2 + C_0 C_3 + C_4) \sigma^2_{\tilde{x}} \qquad \text{(A-25)}$$

Placing these covariance terms in equation (A-20), we obtain

$$\sigma^2_{v_x} = D_0^2 \sigma^2_{\bar{x}_{n+1}} + \sigma^2_{\tilde{x}} \{ D_1^2 + D_2^2 + D_3^2 + D_4^2 + 2 D_0 [D_1 C_1$$

$$+ D_2 (C_0 C_1 + C_2) + D_3 (C_0^2 C_1 + C_0 C_2 + C_3)$$

$$+ D_4 (C_0^3 C_1 + C_0^2 C_2 + C_0 C_3 + C_4)] \} \qquad \text{(A-26)}$$

Define

$$K^2_{\dot{x}} = \frac{\sigma^2_{v_x}}{\sigma^2_{\tilde{x}}}$$

to be the velocity noise reduction coefficient, then

$$K^2_{\dot{x}} = D_0^2 K^2_{\bar{x}} + \{ D_1^2 + D_2^2 + D_3^2 + D_4^2 + 2 D_0 [D_1 C_1$$

$$+ D_2 (C_0 C_1 + C_2) + D_3 (C_0^2 C_1 + C_0 C_2 + C_3)$$

$$+ D_4 (C_0^3 C_1 + C_0^2 C_2 + C_0 C_3 + C_4)] \} \qquad \text{(A-27)}$$

The ATDS tracker now uses

$$D_0 = \frac{11}{16}, \; D_1 = -\frac{7}{16}, \; D_2 = -\frac{3}{16}, \; D_3 = -\frac{1}{16}, \; D_4 = 0$$

so that

$K_{\dot{x}}$ = 0.225 mile per scan per mile

$K_{\dot{x}}$ = 84 knots per mile

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APPENDIX B

DERIVATION OF NOISE RESPONSE OF α-β TRACKER

Problems: (1) Derive the variance of X_s, $\dot{X}_n$, $\bar{X}_{n+1}$ as a function of time

(2) Derive the steady-state variance of X_s, $\dot{X}$, $\bar{X}$

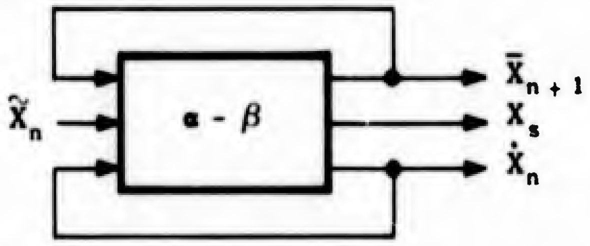

Figure 26. Block Diagram of α - β Tracker Operation.

The $\alpha - \beta$ tracking equations are:

$$X_s = \bar{X}_n + \alpha(\tilde{X}_n - \bar{X}_n) = \alpha X_n + (1-\alpha)\,\bar{X}_n \quad \text{(B-1)}$$

$$\dot{X}_n = \dot{X}_{n-1} + \beta(\tilde{X}_n - \bar{X}_n) \quad \text{(B-2)}$$

$$\bar{X}_{n+1} = \dot{X}_n + X_s \quad \text{(B-3)}$$

where

$\tilde{X}_n$ = Measured position at time t_n

X_s = Calculated smooth position at time t_n

$\bar{X}_n$ = Position which was previously predicted for time t_n

$\bar{X}_{n+1}$ = Position which is predicted for time t_{n+1}

$\dot{X}_n$ = Calculated velocity at time t_n

VARIANCE AS FUNCTION OF TRACK AGE

The tracking process begins when only two points are available

$$\dot{X}_1 = \tilde{X}_1 - \tilde{X}_o \quad \text{(B-4)}$$

$$\bar{X}_2 = \tilde{X}_1 + \dot{X}_1 = 2\tilde{X}_1 - \tilde{X}_o \quad \text{(B-5)}$$

If $\tilde{X}_n$ is nontime varying, then

$$\sigma^2_{\dot{x}_1} = \sigma^2_{\tilde{x}_1} + \sigma^2_{\tilde{x}_o} = 2\sigma^2_{\tilde{x}} \quad \text{(B-6)}$$

$$\sigma^2_{\bar{x}_2} = 4\sigma^2_{\bar{x}_1} + \sigma^2_{\bar{x}_o} = 5\sigma^2_{\bar{x}} \qquad \text{(B-7)}$$

$$\sigma_{\dot{x}_1, \bar{x}_2} = 2\sigma^2_{\bar{x}_1} + \sigma^2_{\bar{x}_o} = 3\sigma^2_{\bar{x}} \qquad \text{(B-8)}$$

Referring back to equations (B-1), (B-2), and (B-3), it is possible to determine the noise response at time t_n as a function of the response at time t_{n-1}

$$\sigma^2_{x_s} = \alpha^2\sigma^2_{\bar{x}_n} + (1-\alpha)^2\,\sigma^2_{\bar{x}_n} \qquad \text{(B-9)}$$

$$\sigma^2_{\dot{x}_n} = \sigma^2_{\dot{x}_{n-1}} + \beta^2(\sigma^2_{\bar{x}_n} + \sigma^2_{\bar{x}_n}) - 2\beta\sigma_{\bar{x}_n, \dot{x}_{n-1}} \qquad \text{(B-10)}$$

$$\sigma_{x_s, \dot{x}_n} = \sigma_{\bar{x}_n, \dot{x}_{n-1}} - \beta\sigma^2_{\bar{x}_n} + \alpha\beta\sigma^2_{\bar{x}_n} + \alpha\beta\sigma^2_{\bar{x}_n} - \alpha\sigma_{\bar{x}_n, \dot{x}_{n-1}}$$

$$= (1-\alpha)\,\sigma_{\bar{x}_n, \dot{x}_{n-1}} + \alpha\beta\sigma^2_{\bar{x}_n} - \beta(1-\alpha)\,\sigma^2_{\bar{x}_n} \qquad \text{(B-11)}$$

$$\sigma^2_{\bar{x}_{n+1}} = \sigma^2_{\dot{x}_n} + \sigma^2_{x_s} + 2\sigma_{x_s, \dot{x}_n} \qquad \text{(B-12)}$$

$$\sigma^2_{\bar{x}_{n+1}} = \sigma^2_{\dot{x}_{n-1}} + (\alpha+\beta)^2\,\sigma^2_{\bar{x}_n} + (1-\alpha-\beta)^2\,\sigma^2_{\bar{x}_n} + 2(1-\alpha-\beta)\,\sigma_{\bar{x}_n, \dot{x}_{n-1}} \qquad \text{(B-13)}$$

$$\sigma_{\bar{x}_{n+1}, \dot{x}_n} = (1-\alpha-2\beta)\,\sigma_{\bar{x}_n, \dot{x}_{n-1}} + \sigma^2_{\dot{x}_{n-1}} + \beta(\alpha+\beta)\,\sigma^2_{\bar{x}_n} - \beta(1-\alpha-\beta)\,\sigma^2_{\bar{x}_n} \qquad \text{(B-14)}$$

Now the results of equations (B-6), (B-7), and (B-8) can be substituted into equations (B-9), (B-10), (B-13), and (B-14) to give the noise response at time t_2. Then the response at time t_2 can be substituted into equations (B-9), (B-10), (B-13), and (B-14) to obtain the response at time t_3, etc.

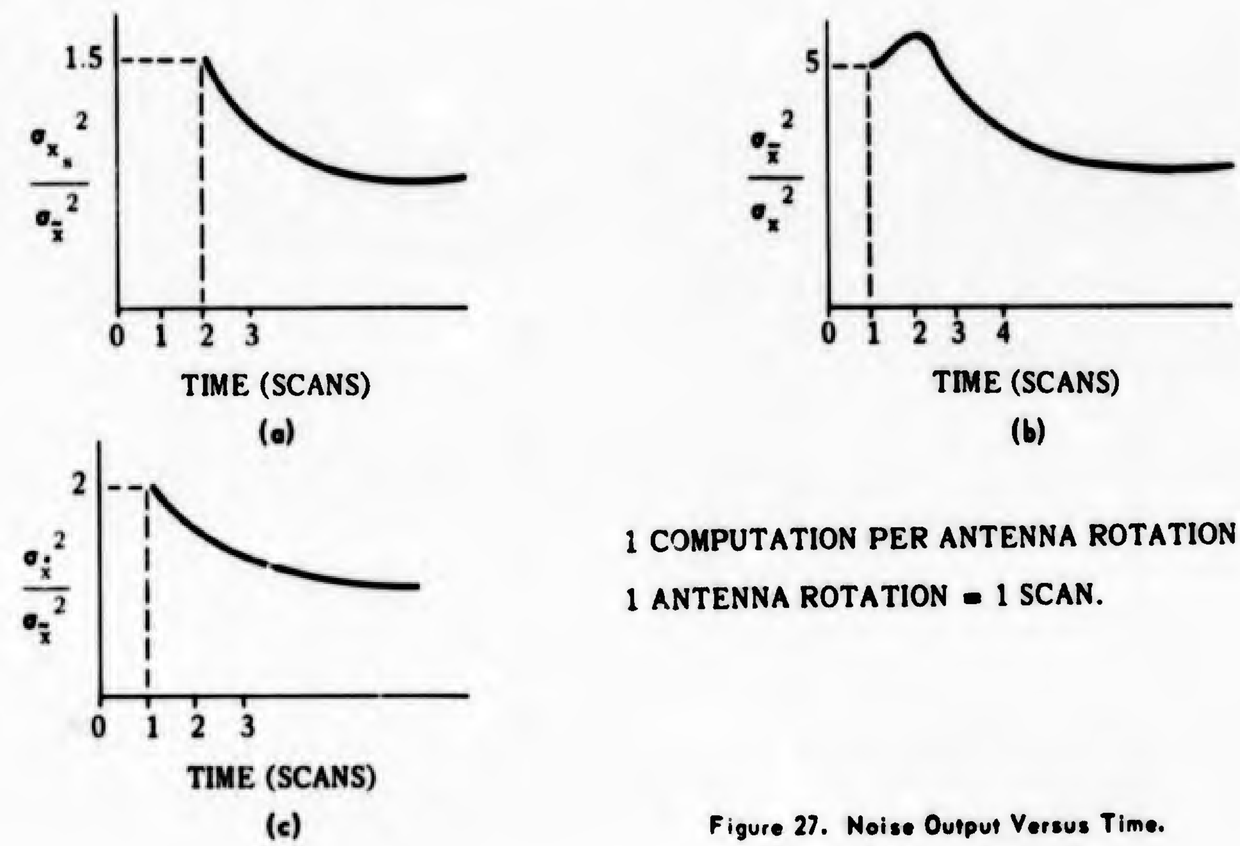

Figure 27. Noise Output Versus Time.

Note that the initial conditions will be the same for any tracking system, provided the same track initiation scheme is used.

$\sigma^2_{\dot{x}_s}$ is undefined until t_2.

STEADY-STATE NOISE RESPONSE

The steady-state noise response of the tracking system could be obtained by the process previously described, simply by solving the problem enough times so that the solution converges to a steady-state valve.

If only the steady-state solution is needed, however, a simpler process would be desirable.

When t is large enough, a steady state is reached and if $\sigma^2_{\tilde{x}}$ is nontime-varying,

$$\sigma^2_{\bar{x}_n} = \sigma^2_{\bar{x}_{n+1}} \quad \text{(B-15)}$$

$$\sigma^2_{\dot{x}_n} = \sigma^2_{\dot{x}_{n-1}} \quad \text{(B-16)}$$

$$\sigma_{\bar{x}_n, \dot{x}_{n-1}} = \sigma_{\bar{x}_{n+1}, \dot{x}_n} \quad \text{(B-17)}$$

By substituting equations (B-15), (B-16), and (B-17) into equations (B-10), (B-13), and (B-14), and dropping all subscripts denoting time,

$$\sigma^2_{\dot{x}} = \sigma^2_{\dot{x}} + \beta^2 (\sigma^2_{\tilde{x}} + \sigma^2_{\bar{x}}) - 2\beta\, \sigma_{\bar{x},\dot{x}} \quad \text{(B-18)}$$

$$\sigma^2_{\bar{x}} = \sigma^2_{\dot{x}} + (\alpha+\beta)^2 \sigma^2_{\tilde{x}} + (1-\alpha-\beta)^2 \sigma^2_{\bar{x}} + 2(1-\alpha-\beta)\, \sigma_{\bar{x},\dot{x}} \quad \text{(B-19)}$$

$$\sigma_{\dot{x},\bar{x}} = (1-\alpha-\beta)\, \sigma_{\bar{x},\dot{x}} + \sigma^2_{\dot{x}} + \beta(\alpha+\beta)\, \sigma^2_{\tilde{x}} - \beta(1-\alpha-\beta)\, \sigma^2_{\bar{x}} \quad \text{(B-20)}$$

From equation (B-18), we get

$$\sigma_{\bar{x},\dot{x}} = \frac{\beta}{2}(\sigma^2_{\tilde{x}} + \sigma^2_{\bar{x}}) \quad \text{(B-21)}$$

Substituting equation (B-21) into equations (B-19) and (B-20),

$$\sigma^2_{\bar{x}} [1-(1-\alpha-\beta)^2 - \beta(1-\alpha-\beta)] = \sigma^2_{\dot{x}} + \sigma^2_{\tilde{x}} (\alpha^2 + \alpha\beta + \beta) \quad \text{(B-22)}$$

$$\sigma^2_{\bar{x}}\left(\beta - \frac{\alpha\beta}{2}\right) = \sigma^2_{\dot{x}} + \sigma^2_{\tilde{x}} \left(\frac{\alpha\beta}{2}\right) \quad \text{(B-23}$$

Subtracting equation (B-23) from equation (B-22) gives

$$\sigma^2_{\bar{x}}\left(2\alpha - \alpha^2 - \frac{\alpha\beta}{2}\right) = \sigma^2_{\tilde{x}}\left(\alpha^2 + \frac{\alpha\beta}{2} + \beta\right) \quad \text{(B-24)}$$

$$\frac{\sigma^2_{\bar{x}}}{\sigma^2_{\tilde{x}}} = \frac{2\alpha^2 + \beta(\alpha+2)}{\alpha(4-2\alpha-\beta)} \quad \text{(B-25)}$$

Substituting equation (B-25) into equations (B-23) and (B-9) gives

$$\frac{\sigma^2_{\dot{x}}}{\sigma^2_{\tilde{x}}} = \frac{2\beta^2}{\alpha(4-2\alpha-\beta)} \tag{B-26}$$

$$\frac{\sigma^2_{x_s}}{\sigma^2_{\tilde{x}}} = \frac{2\alpha^2+\beta(-3\alpha+2)}{\alpha(4-2\alpha-\beta)} \tag{B-27}$$

APPENDIX C

SIMULATION OF THE AUTOMATIC TRACKING UNIT

Two subroutines for the 7090 computer were developed to simulate the functioning of the automatic tracking unit (ATU). One subroutine simulates the four-point smoothing of the present ATDS tracking unit, while the other simulates the α-β tracker. Figures 28 and 29 are flow diagrams for the two simulations.

Definitions of the various quantities used in this appendix are as follows:

D, E	Numbers from a population of normally distributed random numbers with a mean of zero and standard deviation of one.
G_x	Half gate size for east coordinate.
G_y	Half gate size for north coordinate.
N	Current number of successive times the target is not detected or the target position does not correlate with the existing track.
R	Range from E-2A aircraft to the target.
R_s	Measured range.
TQN	Track quality number.
V	True target speed.
$\tilde{X}$	East component of measured target position.
$\bar{X}$	East component of predicted target position.
$\dot{X}_n$	East component of measured target velocity.
X_w	East coordinate of E-2A aircraft.
$\tilde{Y}$	North component of measured target position.
$\bar{Y}$	North component of predicted target position.
$\dot{Y}_n$	North component of measured target velocity.
Y_w	North coordinate of E-2A aircraft.
α-β	Smoothing coefficients used in α-β tracker.
γ	True bearing of target from E-2A aircraft, measured from north.
γ_s	Measured bearing of target from E-2A aircraft, measured from north.
θ	Target heading, measured from north.
σ_r	One sigma value of radar range error.
σ_γ	One sigma value of radar azimuth error.

Input requirements to both subroutines include the range from the E-2A aircraft to the target, the bearing of the target from the E-2A aircraft, and initial values for target heading and target velocity. Range and bearing are modified to simulate tracking error as follows:

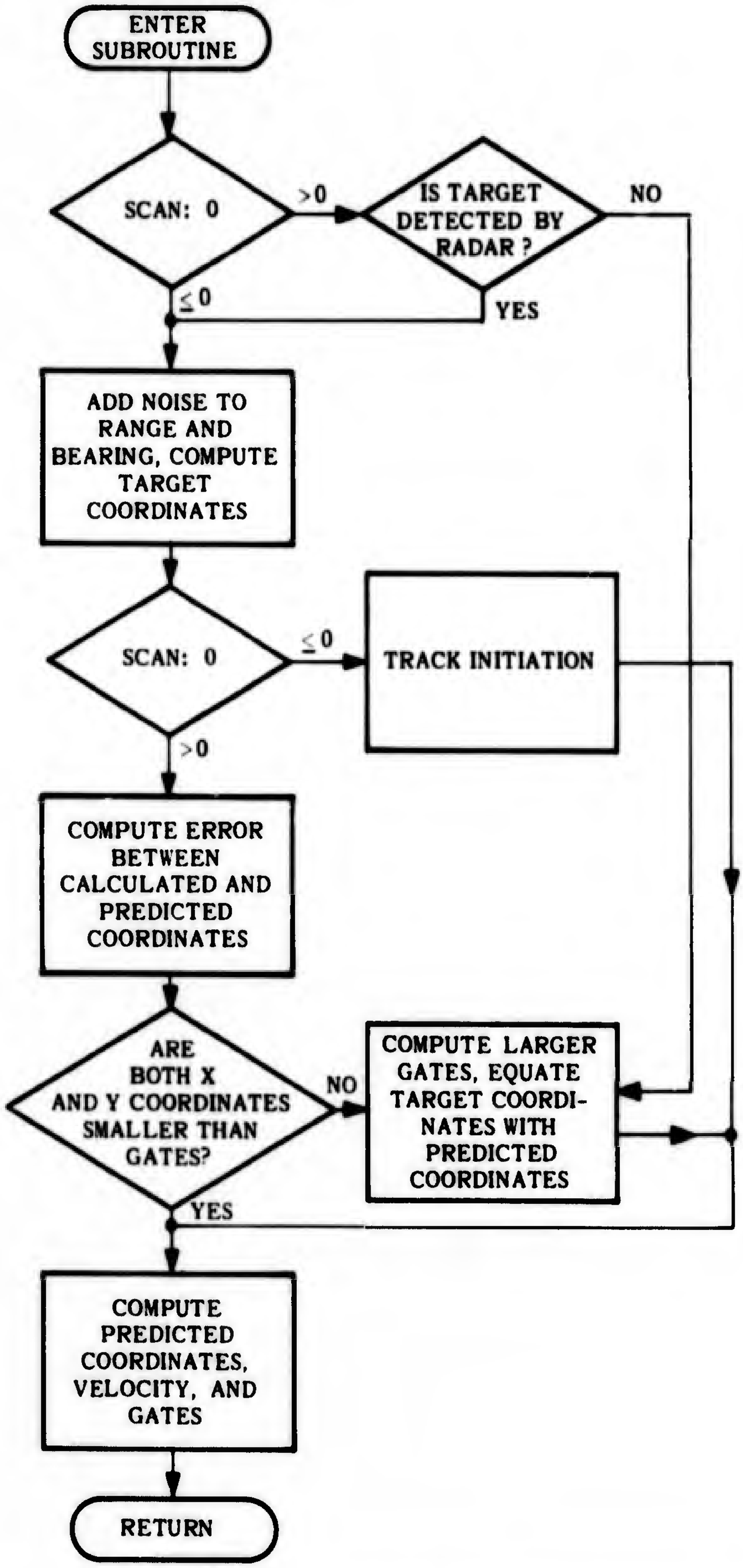

Figure 28. Flow Diagram of Four-Point Smoothing Simulation.

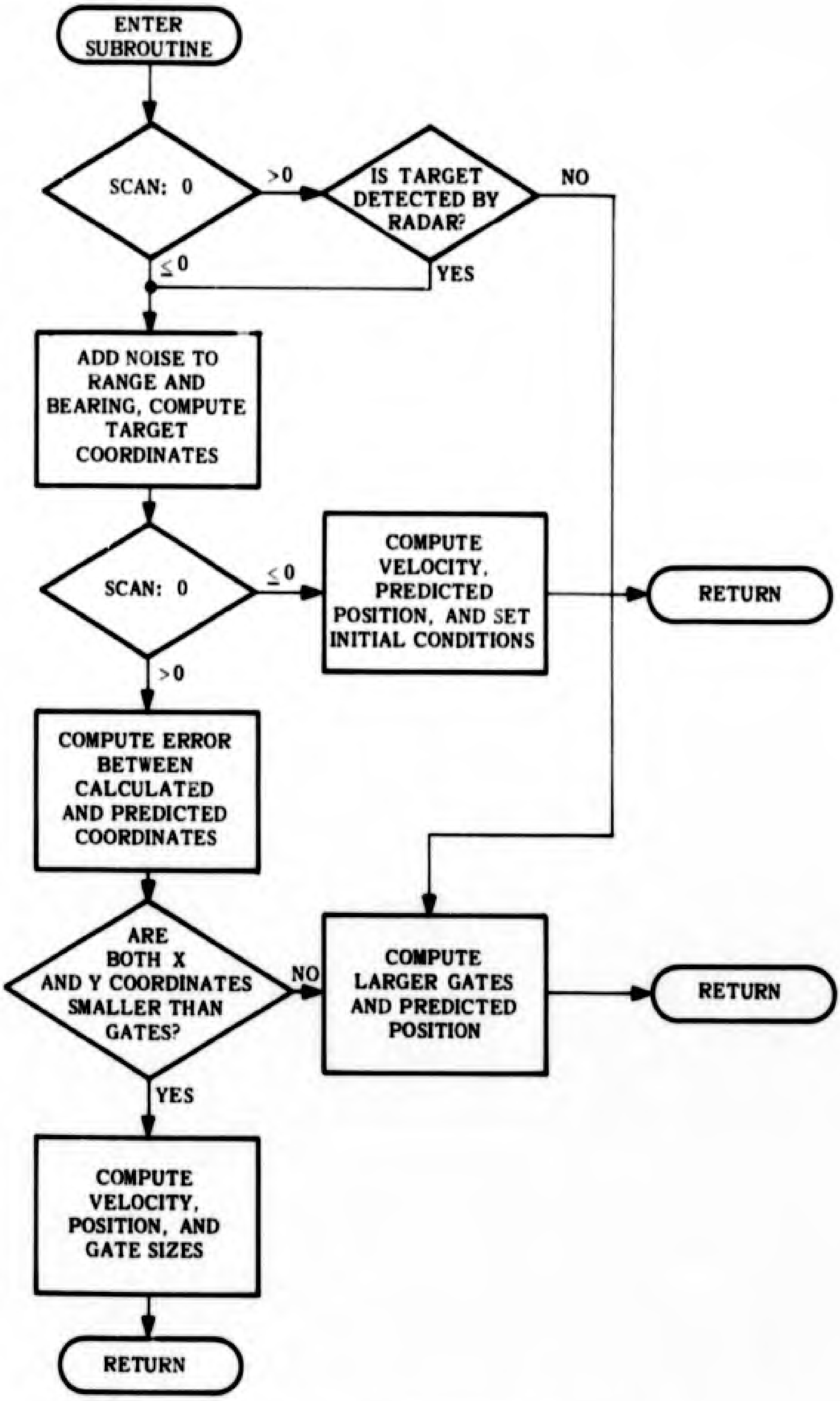

Figure 29. Flow Diagram of $\alpha-\beta$ Tracker Simulation.

$$R_s = R + \sigma_r \cdot D$$

$$\gamma_s = \gamma + \sigma_\gamma \cdot E$$

The coordinates of the target are then calculated from the measured values of range and bearing.

$$\tilde{X} = R_s \sin \gamma_s + X_w$$

$$\tilde{Y} = R_s \cos \gamma_s + Y_w$$

TRUNCATION

In general, both simulations quantize position in increments of 1/4 data mile and velocity into increments of 1/16 data mile per scan. Range is truncated to the next lower 1/4 data mile, and bearing is rounded to the next 0.6-degree increment. Because the computer stores negative numbers in 2's complement form, truncation rounds negative numbers to the next lower, or more negative, increment. In the velocity and prediction position computations, the ATU simulations add a correction term before truncation so that the mean round-off error is approximately zero. In all cases an attempt was made to truncate so that the results of the simulation and the actual ATU would be approximately the same. An extensive discussion about the truncation method is not presented here, however.

TRACK QUALITY NUMBER

In both simulations, the track quality number is a measure of the history of the track.

Initially the track is tentative, and both TQN and N are equal to zero.

If the track fails to correlate with previous data or if the radar fails to detect the target, the following conditions prevail:

N = N + 1 where N has a maximum value of 7.

TQN = TQN-1 where TQN has a minimum value of zero.

If the target position is accepted

TQN = TQN + 2 where TQN has a maximum value of 15.

N = 0.

When TQN > 4, the track is an established track.

When TQN $\leq$ 4, the track is a tentative track.

If the track is still tentative 6 scans after initiation or if the track is tentative at any time and TQN = 0, the tentative track is cancelled.

When TQN = 0 and N = 7, the established track has been lost.

BLIP/SCAN RATIO

In an actual tracking situation, the radar does not always detect the target. The blip/scan ratio, which is the probability that the radar detects the target on any one scan, is used to simulate this effect. The ATU subroutine generates a random number from a population of uniform random numbers and compares this random number with the blip/scan ratio. If the blip/scan ratio is less than this random number, the target is not detected.

GATING

The absolute value of the difference between the target position and the position predicted for that point is computed for each range component. If either or both of these errors are greater than their respective half gate size, the new target position fails to correlate with the previous target data. Gate size is computed by the following equations.

Four-Point Smoothing

The minimum half gate size for both components is 2 miles; however, the maximum half gate size depends upon the target position relative to the E-2A aircraft. If the target is within 26.5 degrees of the east coordinate axis, the maximum half gate size is 5 by 12*; if it is within 26.5 degrees of the north coordinate axis, the maximum half gate size is 12 by 5. Otherwise, the maximum half gate size is 8 by 8. Initially both half gate sizes are 5 miles. If the target is not detected or if the target position does not correlate with previous data, larger gates are computed by

$$G_{x_{n+1}} = 2G_{x_n} + 1$$

$$G_{y_{n+1}} = 2G_{y_n} + 1$$

otherwise, smaller gates are computed.

$$G_{x_{n+1}} = \frac{3}{4}\left| \tilde{X}_n - \bar{X}_n \right| + \frac{1}{2} G_{x_n} + 1$$

$$G_{y_{n+1}} = \frac{3}{4}\left| \tilde{Y}_n - \bar{Y}_n \right| + \frac{1}{2} G_{y_n} + 1$$

α-β Tracker

Both gate components have a minimum value of 2 miles and a maximum value of 8 miles. The minimum value is inherent in the gating equations. Initially both half gates are 4 miles. If the target is not detected or if the target data does not correlate with previous data, new gates are computed as follows:

$$G_{x_{n+1}} = \frac{1}{2}(3G_{x_n} + 1)$$

$$G_{y_{n+1}} = \frac{1}{2}(3G_{y_n} + 1)$$

*G_x = 5 miles, G_y = 12 miles.

If the target data is valid, however, the following equations are used:

$$G_{x_{n+1}} = \frac{3}{4}\left|\tilde{X}_n - \bar{X}_n\right| + \frac{1}{2}G_{x_n} + 1$$

$$G_{y_{n+1}} = \frac{3}{4}\left|\tilde{Y}_n - \bar{Y}_n\right| + \frac{1}{2}G_{y_n} + 1$$

TRACKING EQUATIONS

Both simulations compute the initial velocity vectors by the following equations:

$$\dot{X}_o = V \sin \theta$$

$$\dot{Y}_o = V \cos \theta$$

The rest of the tracking equations are given for the east component only.

Four-Point Smoothing

(1) Track initiation (n = 0)

$$\bar{X}_n = \tilde{X}_n$$

$$\tilde{X}_{n-1} = \tilde{X}_n - \dot{X}_o$$

$$\tilde{X}_{n-2} = \tilde{X}_{n-1} - \dot{X}_o$$

$$\tilde{X}_{n-3} = \tilde{X}_{n-2} - \dot{X}_o$$

The velocity and predicted position are then calculated from the regular equations in (3) below.

(2) If the target is not detected or if the target data does not correlate with previous data, the present position is equated to the predicted position, and the velocity and predicted position are computed from the equations in (3).

(3) If the target is detected and correlated, the following equations are used:

$$\bar{X}_{n+1} = \frac{5\bar{X}_n + 7\tilde{X}_n - \tilde{X}_{n-1} - 2\tilde{X}_{n-2} - \tilde{X}_{n-3}}{8}$$

$$\dot{X}_n = \frac{11\bar{X}_{n+1} - 7\tilde{X}_n - 3\tilde{X}_{n-1} - \tilde{X}_{n-2}}{16}$$

α-β Tracker

(1) Track initiation (n = 0)

$$\bar{X}_{n+1} = \tilde{X}_n + \dot{X}_o$$

(2) If the target is not detected or if the target position fails to correlate with the track.

$$\bar{X}_{n+1} = \bar{X}_n + \dot{X}_n$$

(3) If the target is detected and correlated the following equations are used:

$$\dot{X}_n = \dot{X}_{n-1} + \beta(\hat{X}_n - \bar{X}_n)$$

$$\bar{X}_{n+1} = \dot{X}_n + \bar{X}_n + \alpha(\tilde{X}_n - \bar{X}_n)$$

Figure 30 is a flow diagram of a program using either ATU subroutine. In this program, the target flight and the E-2A flight are simulated by subroutines. The data is then used to compute the range and bearing for input into the ATU.

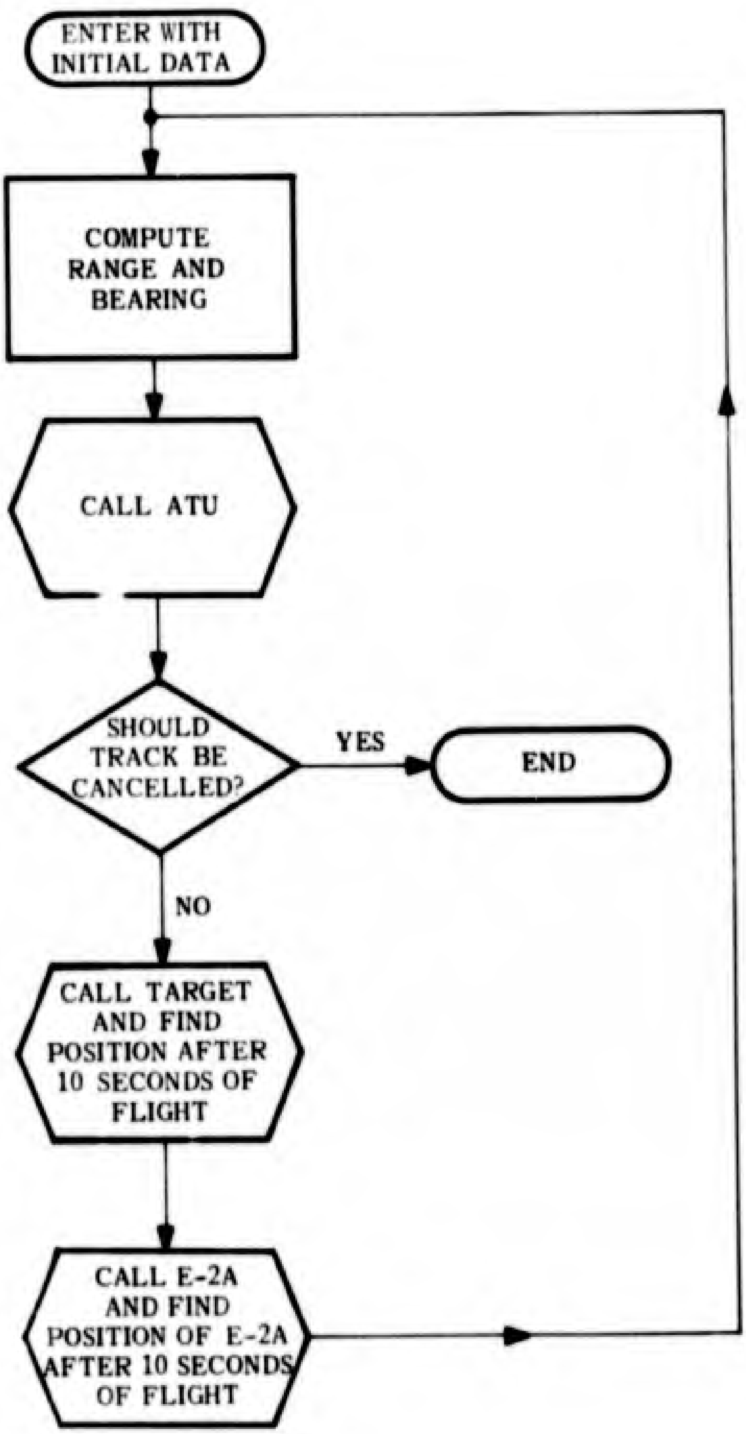

Figure 30. An Example of a Program Using the ATU Simulation.