Technical Note

1965-28

B. F. LaPage

Some Properties
of
a Scanned Circular Array

7 July 1965

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts

CSPC

AD620460

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

SOME PROPERTIES OF A SCANNED CIRCULAR ARRAY

B. F. LaPAGE

Group 61

ESEP Checked For
Public Release -ok,
17 Sept 65
LTH
Wreck

TECHNICAL NOTE 1965-28

7 JULY 1965
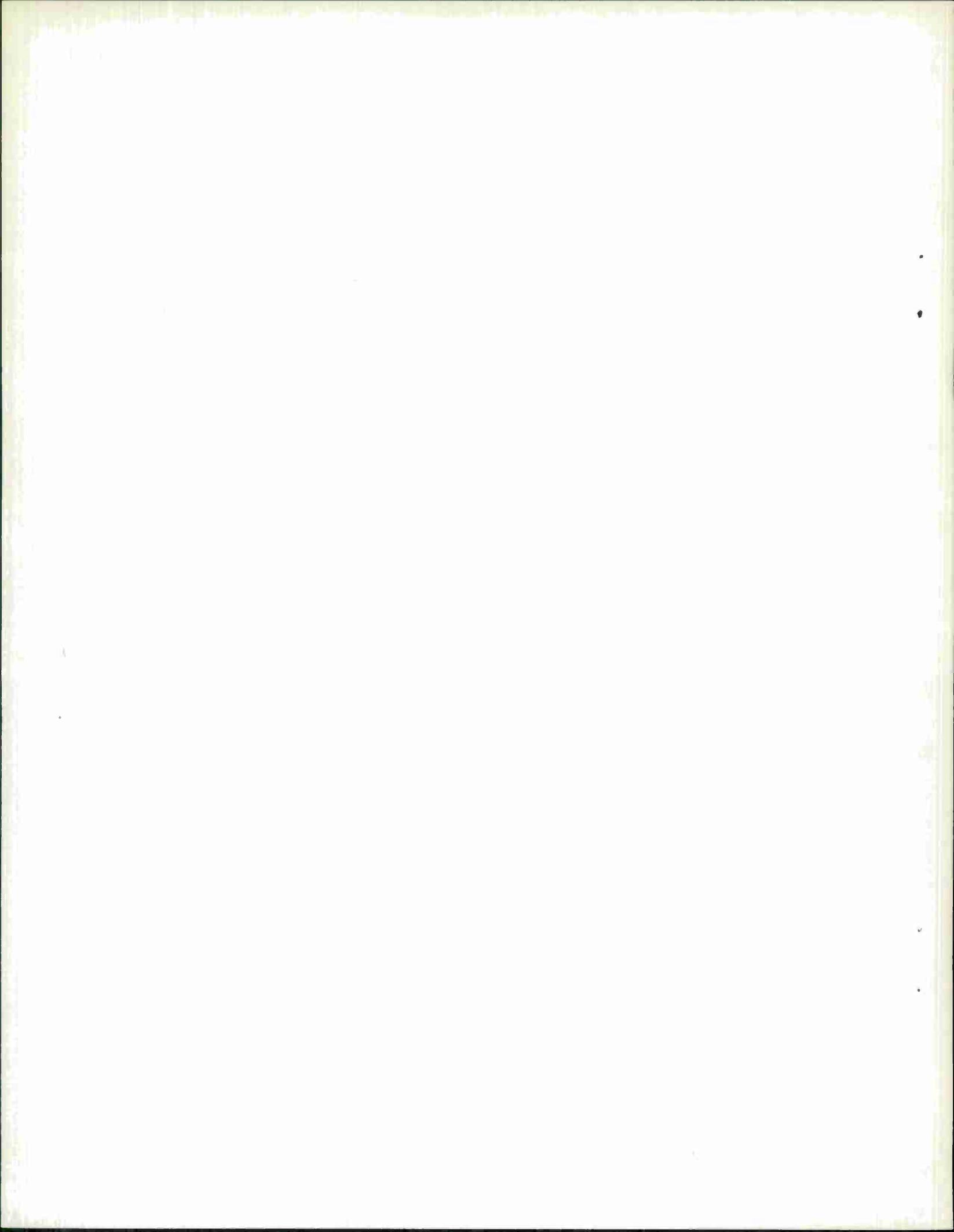
LEXINGTON   MASSACHUSETTS

ABSTRACT

The impedance, pattern and gain properties of a circular array of dipoles (or isotropic sources) is discussed. In particular the variation of these properties as the antenna beam is scanned is investigated. Computed results for typical arrays are shown and the meaning of the results are discussed, and further areas of investigation are indicated.
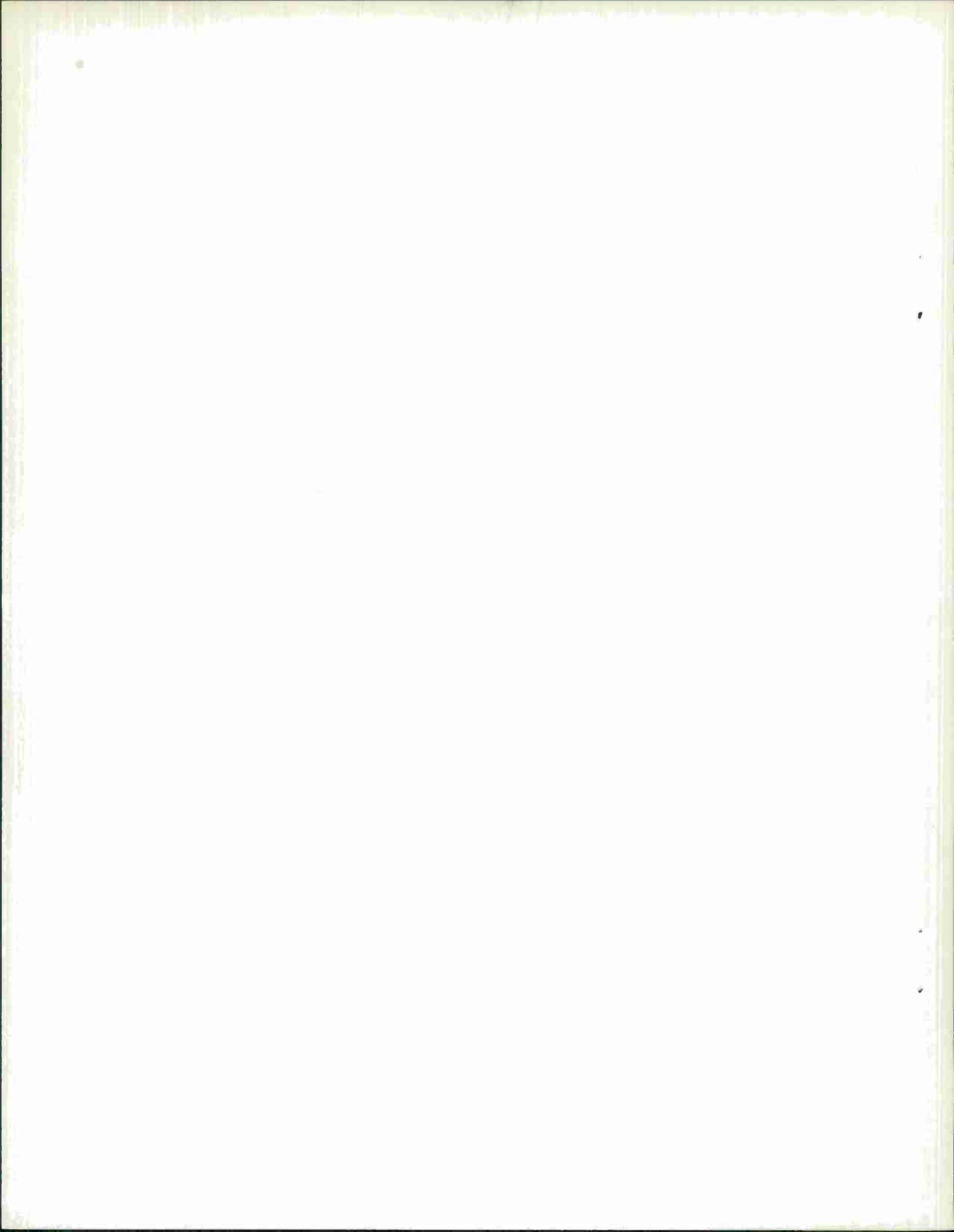
Some Properties of a Scanned Circular Array

It has been proposed that a useful satellite antenna design could be achieved by using a circular array of monopoles located about the spin axis of a short cylinder as shown in Fig. 1. The spin axis of the satellite is oriented normal to the orbital plane and the antenna beam is "de-spun" by proper phasing of the currents on the array elements. The array elements are fed in pairs with equal amplitude currents from separate sources, and phased to produce the desired beam positions. A retro-directive scheme could also be used, but in any event some basic data was needed on the behavior of scanned circular arrays and this was the motivation of this study.

There is a fair amount of literature on the subject of circular arrays; notably Knudsen's[1] exhaustive study and more recently the work of Neff,[2] Simpson[3] and Mack.[4] Unfortunately, these studies are concerned with equi-phased arrays, parasitic array systems, or other types which are not directly applicable to our problem. The problem of the input impedance variations of an element during scan and the array gain variations are also not covered, and these topics are our main interest.

The gross parameters of the array were chosen[5] from what was considered to be a reasonable size satellite based on total weight, solar cell output, booster capabilities, etc; and are typical, at least, for the near future.

The geometry of the array that is under discussion is shown in Fig. 2.

1

The N-elements are arranged on a circle of radius a, located in the x-y plane and phased to point the resultant beam at an angle $\phi_o (\theta_o = 90°)$. Each element is positioned by the angle coordinate ϕ_n where

$$\phi_n = \frac{2\pi}{N} (n - 1) \tag{1}$$

The current on the ith element is given by

$$\left. \begin{aligned} I_i &= e^{j\psi_i}, \quad |I_i| = 1 \\[2em] \text{where} \\[1em] \psi_i &= -ka \cos(\phi_o - \phi_n) \end{aligned} \right\} \tag{2}$$

Now the field due to the ith element is

$$\left. \begin{aligned} E_i &= e^{j\left[\psi_i + ka \cos(\phi - \phi_n)\right]} \\[1em] \text{or} \\[1em] E_i &= e^{jka\left[\cos(\phi - \phi_n) - \cos(\phi_o - \phi_n)\right]} \end{aligned} \right\} \tag{3}$$

The total field is obtained by summing all E_i, or

$$E(\phi) = \sum_{n=1}^{N} e^{jka\left[\cos(\phi - \phi_n) - \cos(\phi_o - \phi_n)\right]} \tag{4}$$

This can be written in the equivalent form as

2

$$E(\phi) = \sum_{n=1}^{N} e^{j u \sin \alpha_n} \quad , \tag{5}$$

$$\alpha_n = \phi_n - \frac{\phi + \phi_o}{2}$$

$$u = 2 ka \sin \frac{\phi - \phi_o}{2} \quad .$$

We see from Eq. (4) that when $\phi_o = \phi_n$, or the beam points to an element position angle, $E(\phi)$ does not depend on ϕ_o. This means that any variations that occur in $E(\phi)$ have a periodicity of the inter-element angular spacing, and we need to study only half that interval to obtain all the scan pattern variations. Now Eq. (5) may be transformed by using Bessel's expansion:

$$e^{jZ \sin \theta} = \sum_{n=-\infty}^{\infty} e^{jn\theta} J_n(Z) \quad , \tag{6}$$

putting the result in a form, that is for large N, easier to analyze. That is,

$$E(\phi) \simeq J_o(u) + 2J_N(u) \cos \frac{N}{2} (\phi + \phi_o), \text{ N even} \tag{7}$$

$$E(\phi) \simeq J_o(u) - 2jJ_N(u) \sin \frac{N}{2} (\phi + \phi_o), \text{ N odd} \tag{8}$$

$$u = 2 ka \sin \frac{\phi - \phi_o}{2} \quad .$$

Now we show that for $N \geq 6$, $E(\phi)$ is independent of N to a large degree. At

3

the null of $E(\phi)$ we have $J_0(x) = 0$ for $x = 2.404$ and $J_5(2.404) = .016$, $J_6(2.404) = .0033$, etc.

So it is evident that terms containing $J_6(u)$ are negligible, at least at the null of $E(\phi)$. We can say that for $N \geq 6$ <u>all</u> the array patterns null at nearly the same angle, given by

$$u_o = 2\,ka \sin\frac{\phi}{2} = 2.404 \quad .$$

The 3 db beamwidth can be computed similarly, i.e.,

$$(BW_3)^o = 4 \sin^{-1} (\frac{1.13}{2\,ka}) \quad .$$

Also, the first side lobe will have a level of -7.9 db located at an angle

$$(SL_\theta)^o = 2 \sin^{-1} (\frac{3.83}{2\,ka}) \quad .$$

We can now give the array plane pattern for <u>all</u> ring arrays with $N \geq 6$, as shown in Fig. 3. For $N = 4, 5$ the beamwidth and null position will be nearly the same as in Fig. 3 but the side lobe level will be completely different. It should be noted that the condition $u \leq 2\,ka$ is necessary for ϕ to be real, and hence the pattern to be in the visible region. The pattern of the array in the plane normal to the array plane, for isotropic sources, can be analyzed in a similar manner to obtain the result

4

$$E(\theta, \phi_o) \simeq J_o(u) + J_N(u) \begin{cases} 2 \cos N \left(\frac{\pi}{2} + \phi_o\right) & N \text{ even} \\ \\ 2j \sin N \left(\frac{\pi}{2} + \phi_o\right) & N \text{ odd} \end{cases} \quad (9)$$

$$u = ka\,(\sin\theta - 1) \quad .$$

This is the same form previously given except for a change in the argument of $J_o(u)$. One might ask — how can we get directivity in a plane orthogonal to the array plane for isotropic sources? The answer is seen when one "collapses" the point sources along a line and it is observed that we have an end fire array with tapered amplitude distribution. This is known to produce two-plane directivity; the well known Yagi antenna being an example of this principle. Of course, if array elements such as dipoles are considered, then the array pattern must be modified to take into account the element pattern. Figures 4 and 5 show the array 3 db beamwidth vs. array diameter (ka) for the "in-plane" and "out-of-plane" cases. We can now estimate the gain of the antenna array by using the well known expression

$$G \simeq \frac{27,000}{\theta_E^{\,o} \times \theta_H^{\,o}}$$

where $\theta_E^{\,o}$, $\theta_H^{\,o}$, are the principal plane half-power beamwidths, in degrees. Using the data in Figs. 4 and 5 we get the result shown in Fig. 6 as a gain estimate. Notice that the gain given is referenced to the gain of an element.

5

These curves tell us that for a satellite of about 1λ diameter, we can expect beamwidths not less than 60° wide and total gains of 10 - 12 db over isotropic. (This assumes that monopoles are used and the top and bottom arrays are spaced 0.7λ.) More exact gain calculations presented later show that the gain estimates of Fig. 6 are quite good.

Equation (4) has been used to machine compute a series of patterns as a function of ka, N, ϕ, and scan angle ϕ_o. The following combinations of values were used

$$Ka = 2, 4, 6, 8, 10 \qquad\qquad N = 4, 6, 7, 8, 10$$

$$k = 0, .1, .2, .3, .4, .5, \qquad 0 \le \phi \le 180°, \Delta\phi = 10°$$

k is a parameter that tells us where the beam is pointed, i.e.,

$$\phi_o = k\phi_n = k\frac{2\pi}{N}$$

A range of $0 \le k \le .5$ scans the beam one-half an inter-element angular spacing and this includes all the desired pattern data as mentioned before. Both in-plane and out-of-plane patterns were computed, but our main interest is in the array plane patterns. Some of this data is given in Fig. 7 (a through d). The patterns for $N \ge 6$ show the general characteristics for the main beam given by the approximate formulas, but as expected the side and back lobe levels are considerably different. The patterns for N = 4, Ka = 2.28 (Fig. 8) for example, show a back lobe for some scan angles whose intensity

6

equals that of the main beam. With a beamwidth of 58° compared to 56° from Fig. 3, we see that the beamwidth approximation is still good for N = 4. For the detailed study of an array pattern, the more exact formula (Eq. 5) is necessary, but the approximations are still generally quite useful.

We now turn our attention to the problem of the variation of the input impedance of each dipole element during scan and also a calculation of the gain of the array. Figure 9 shows the dipole array geometry. Since the impedance of a monopole over a infinite ground plane is one-half that of a dipole the results of the calculations here are easily modified to include the monopole case. The input impedance of an array element is given by the circuit equations

$$Z_1 = \frac{V_1}{I_1} = Z_{11} + Z_{12} \frac{I_2}{I_1} + \ldots + Z_{1n} \frac{I_n}{I_1}$$

$$Z_2 = Z_{21} \frac{I_1}{I_2} + Z_{22} + \ldots + Z_{2n} \frac{I_n}{I_2} \qquad (10)$$

$$\cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot$$

$$Z_n = Z_{n1} \frac{I_1}{I_n} + \ldots + Z_{nn}$$

or

$$Z_s = \sum_{j=1}^{n} \frac{I_j}{I_s} Z_{sj}$$

Let the power input to element s be P_s and the far-field intensity generated be E_s. We assume $E_s = k I_s$ where I_s is the element current.

Then

$$P_T = \sum_s |I_s|^2 R_e(Z_s)$$

and for a single element

$$P_o = |I_o|^2 R_e(Z_{oo}) \qquad .$$

The field intensities are related by

$$E_o = k|I_o|$$

$$E_T = \sum_s k|I_s| = k \sum_s |I_s| \qquad .$$

Since the fields are "phased" to produce a maximum, the power gain is defined as

$$G = \frac{|E_T|^2}{|E_o|^2} \text{ for } P_o = P_T$$

and we have

$$G = \frac{k^2 \left(\sum_s |I_s| \right)^2}{k^2 |I_o|^2} = \frac{R_e(Z_{oo}) \left(\sum_s |I_s| \right)^2}{\sum \left\{ |I_s|^2 R_e(Z_s) \right\}} \qquad .$$

Now in our case $I_s = e^{j\phi_s}$, since $|I_s| = 1$.

8

Then

$$G = \frac{N^2 R_e(Z_{oo})}{\sum_s R_e(Z_s)} \quad .$$

Since

$$R_e(Z_s) = \sum_j \left\{ |Z_{sj}| \cos(\alpha_{sj} + \phi_j - \phi_s) \right\}$$

where

$$\alpha_{sj} = \tan^{-1} \frac{X_{sj}}{R_{sj}}, \quad Z_{sj} = R_{sj} + jX_{sj} \quad ,$$

$$\phi_j = \text{phase of the current in } j^{th} \text{ element.}$$

The final form of the gain expression is:

$$G \text{ (over an element)} = \frac{N^2 R_e(Z_{oo})}{\sum_{s=1}^{N} \sum_{j=1}^{N} \left\{ |Z_{sj}| \cos(\alpha_{sj} + \phi_j - \phi_s) \right\}} \quad . \quad (11)$$

We also have the important relations:

$$Z_{oo} = Z_{ss} \quad \text{(identical elements)} \quad ,$$

$$Z_{mn} = Z_{nm} \quad \text{(reciprocity of mutual impedance)} \quad ,$$

9

$$\phi_i = ka \cos \frac{2\pi}{N} (p - i + 1) \left(0 \le p \le \frac{1}{2} \text{ scan factor}\right) \quad .$$

Now Eq. (11) reduces to the form $G = N$ if all $Z_{mn} = 0$ for $m \ne n$, as it should. The formula also agrees exactly with some results given in Kraus[6] for two elements spaced one-half wavelength, with in phase and out-of-phase excitations. In order to provide more flexibility, a separate computer subroutine is used to compute the mutual impedances, given the length of an element L/λ and the spacing d/λ. The induced EMF method[7] is used, which assumes sinusoidal antenna currents and infinitely thin dipoles. The mutual impedance of parallel spaced dipoles can be expressed by using the Ci and Si functions,[8] but for machine computation more effort is expended in calculating these functions than in a direct calculation of the mutual impedance itself. The input parameters to the main computer program are as follows:

Parameter	Values Used
N - number of elements	4, 5, 6
ka - ring size	2.28, 2.50, 2.75, 3.0
L/λ - dipole length, wavelengths	.475, .500, .525

The program computes the following:

1. All mutual impedances Z_{sj}.

2. All self impedances Z_{oo}.

3. Input impedance of each element vs. scan angle.

4. Gain of the array vs. scan angle.

A typical calculation result is shown in Fig. 10. The gain variation is seen to be very small. However, the input impedance variation is quite large. In fact, some combinations of parameters even lead to negative real values of input impedance. This indicates that more power is flowing towards the generator for that element, than away from it. An operating condition that one would certainly wish to avoid. To show the variations in a more readily understandable format, we have normalized the dipole input impedance values to 100 ohms and plotted them on a Smith Chart. This is equivalent to normalizing a monopole input impedance to a 50 ohm transmission line, a practical consideration in any actual system. Figures 11 (a through m) shows some typical data along with the gain variations and self impedance values. The impedance variation causes the VSWR to vary between 2 and 3. If we are willing to give up the continuous beam scanning in favor of a step-scanned system, we can avoid the ends of the impedance variations curve and reduce the variation to about a VSWR of 1.7. This step-scanning procedure causes discrete overlapping beams and a subsequent loss in gain under some conditions. For the N = 6 case, for instance, the stepping would be in 60° increments and would produce a 3.0 db ripple in the scanned pattern. Another possible way to minimize the impedance variation is to use isolators in each element feed line. A VSWR of 2.0 is effectively reduced to 1.23 for 10 db isolation loss and for 20 db loss the effective VSWR is 1.07. Recent advances

in the design of coaxial isolators has made available units that have 20 db reverse loss and 0. 7 db forward loss and weigh less than one pound. These values are typical of commercial units and have not been optimized, especially as regards the weight of the unit, for satellite use.

It is the author's opinion that an analysis of the properties of a ring array of dipoles much more rigorous than that presented here is of little immediate value. The reason for this is that the mathematical model of monopoles on an infinite ground plane bears a questionable resemblance to monopoles arrayed on the top and bottom of a small cylinder. Until an exact analysis of the actual geometry is made and then checked with measured data, the correlation between model and real world is not really known. It is felt that a survey type look at a large number of possible satellite antenna schemes is more desirable, so that the most promising of these may receive more intensive scrutiny from the analytical and the experimental viewpoint.

Some computations concerned with the allowable tolerance on the excitation currents necessary to keep gain variations within some arbitrary bound have been made by the author, and the results are given in Lincoln Laboratory Technical Memorandum 61 L-0016.

Mr. Robert Peck (Group 61) programmed the IBM-7094 for the computations given in this report.

REFERENCES

1. H. L. Knudsen, "Radiation from Ring Quasi-Arrays," Trans. IRE, PGAP AP-4, 452-472 (1956).

2. H. P. Neff and J. D. Tillman, "An Electronically Scanned Circular Antenna Array," IRE International Convention Record, Part I, 41-47 (1960).

3. T. L. Simpson and J. D. Tillman, "Parasitic Excitation of Circular Antenna Arrays," IRE, PGAP AP-9, 263-267 (1961).

4. R. B. Mack, "A Study of Circular Arrays," TR-381 and TR-386, Harvard University, Cruft Laboratory, Cambridge, Mass. (1963).

5. B. Reiffen, M. I. T., Lincoln Laboratory (private communication).

6. J. D. Kraus, Antennas (McGraw-Hill, New York, 1950) 283-292.

7. Ibid., 254-271.

8. Ibid., 265.
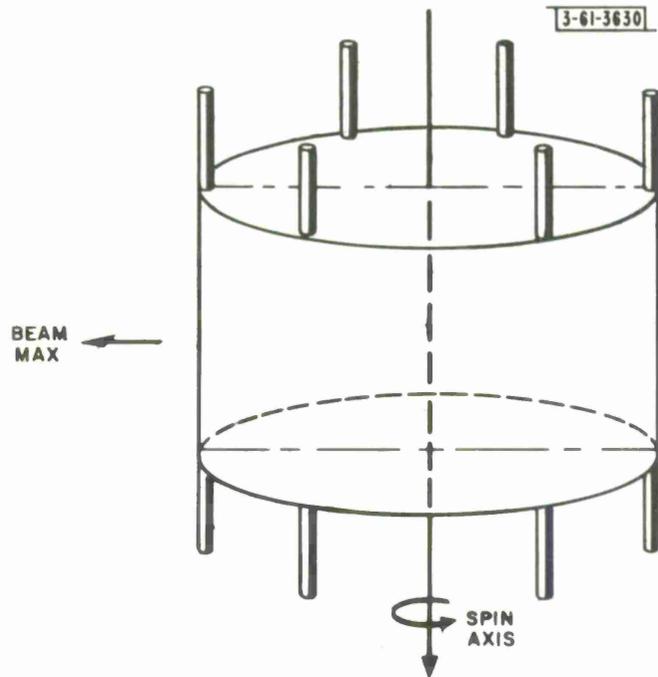
Fig. 1.   Satellite antenna configuration.
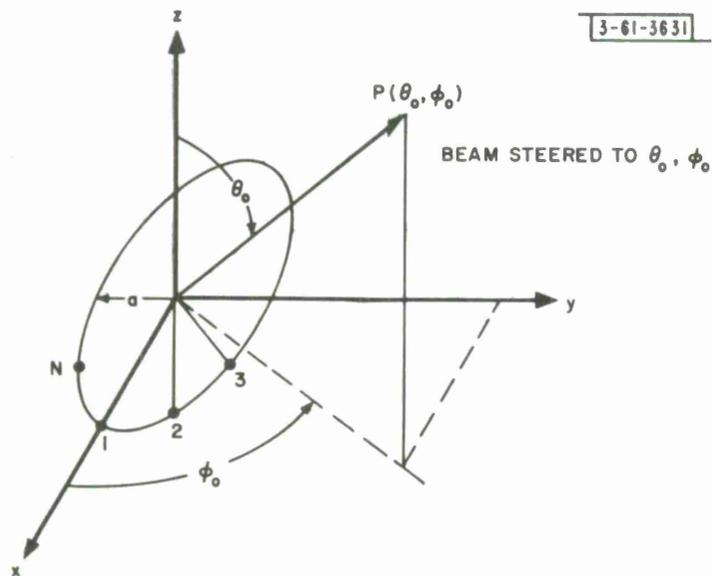
Fig. 2.   Array geometry.

14
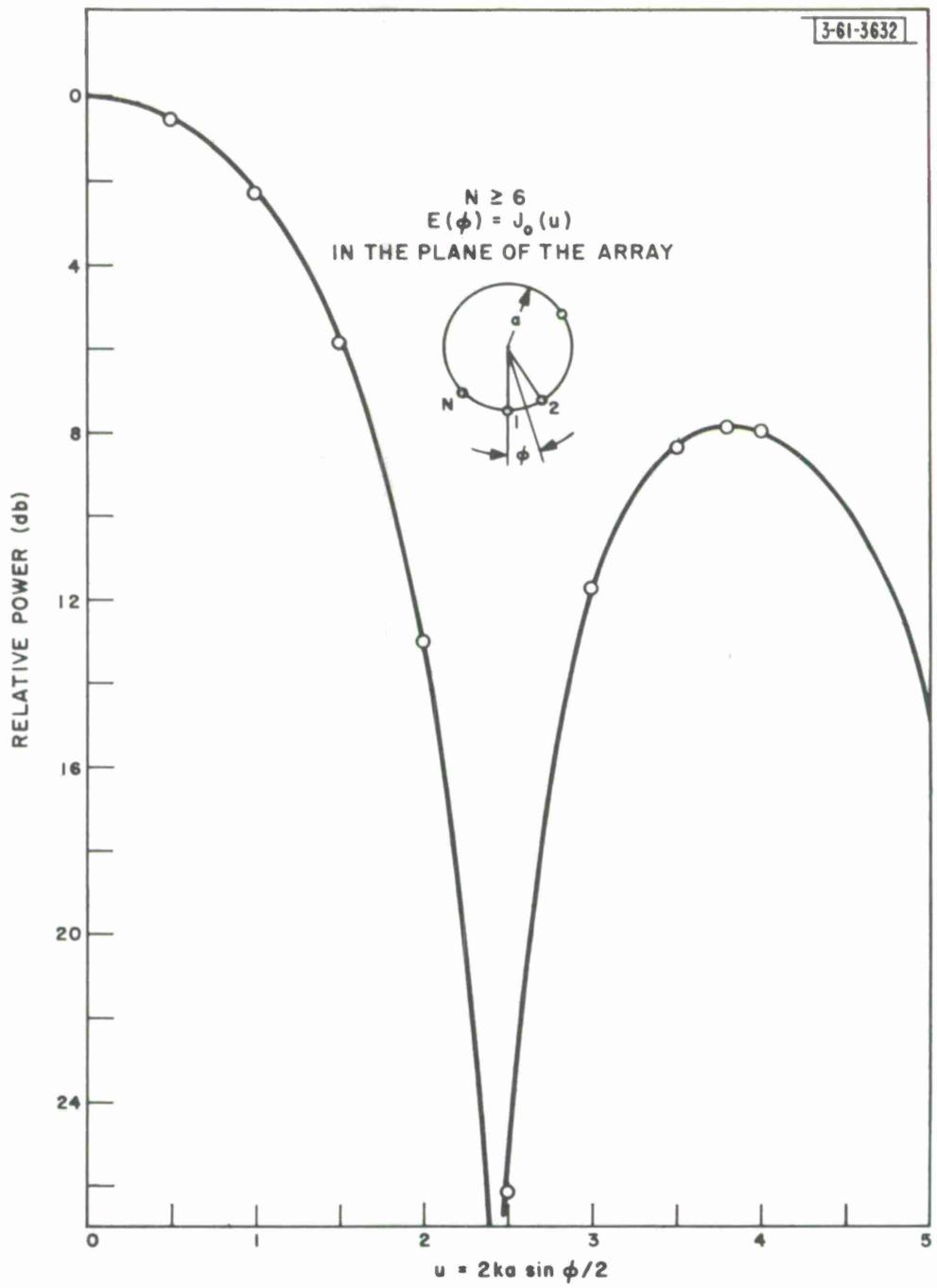
Fig. 3.   Ring array pattern.

15
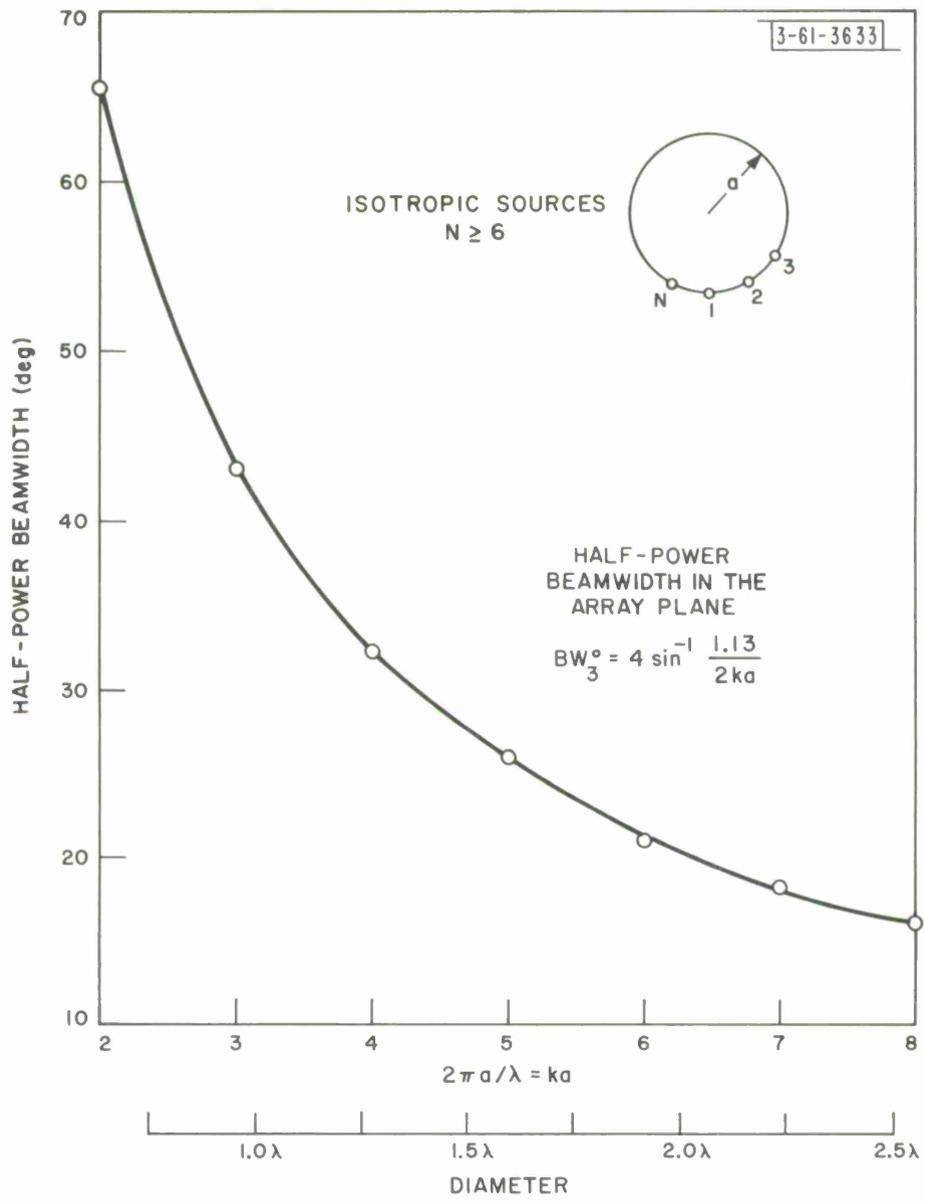
Fig. 4.   Ring array beamwidth vs. diameter.
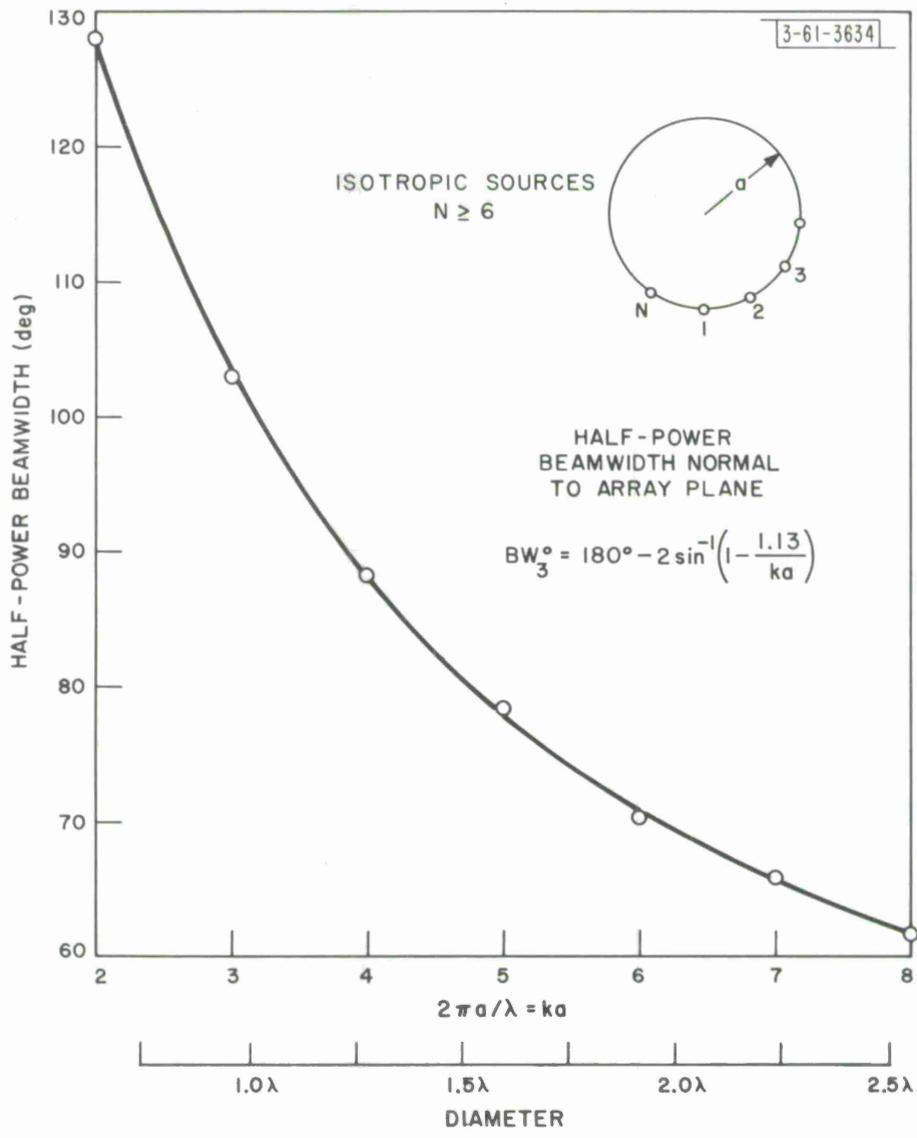
Fig. 5.   Ring array beamwidth vs. diameter.

17
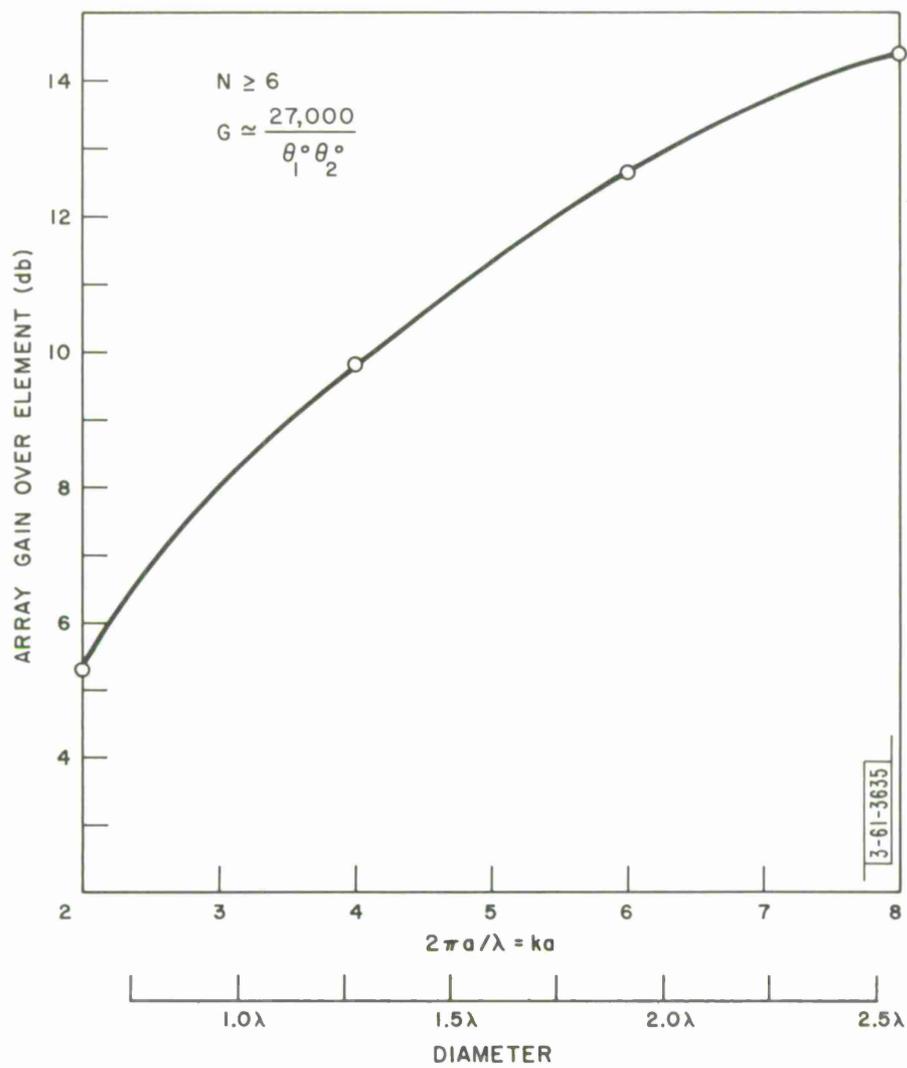
Fig. 6.   Ring array gain vs. diameter.
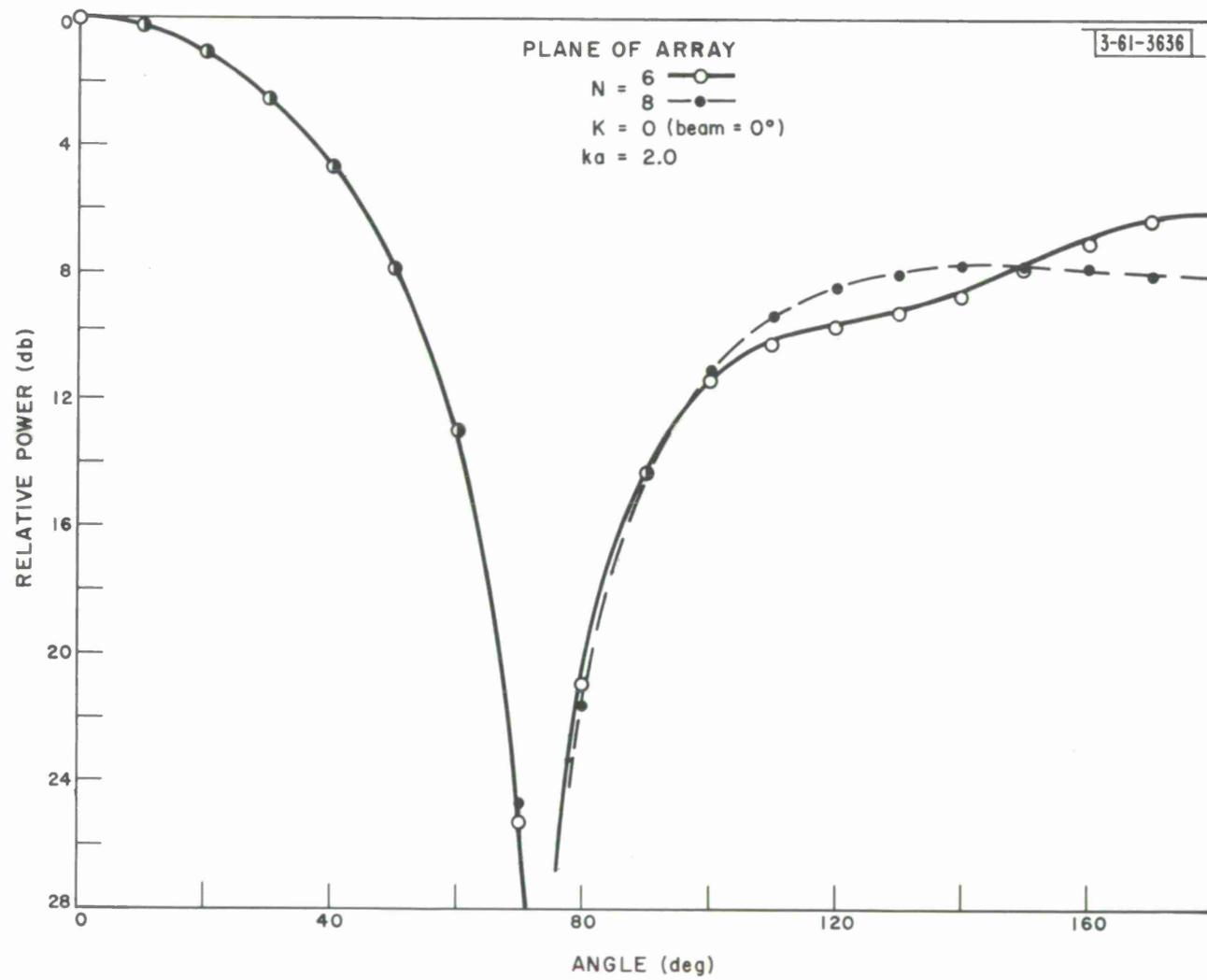
Fig. 7-a.   Pattern of circular array.
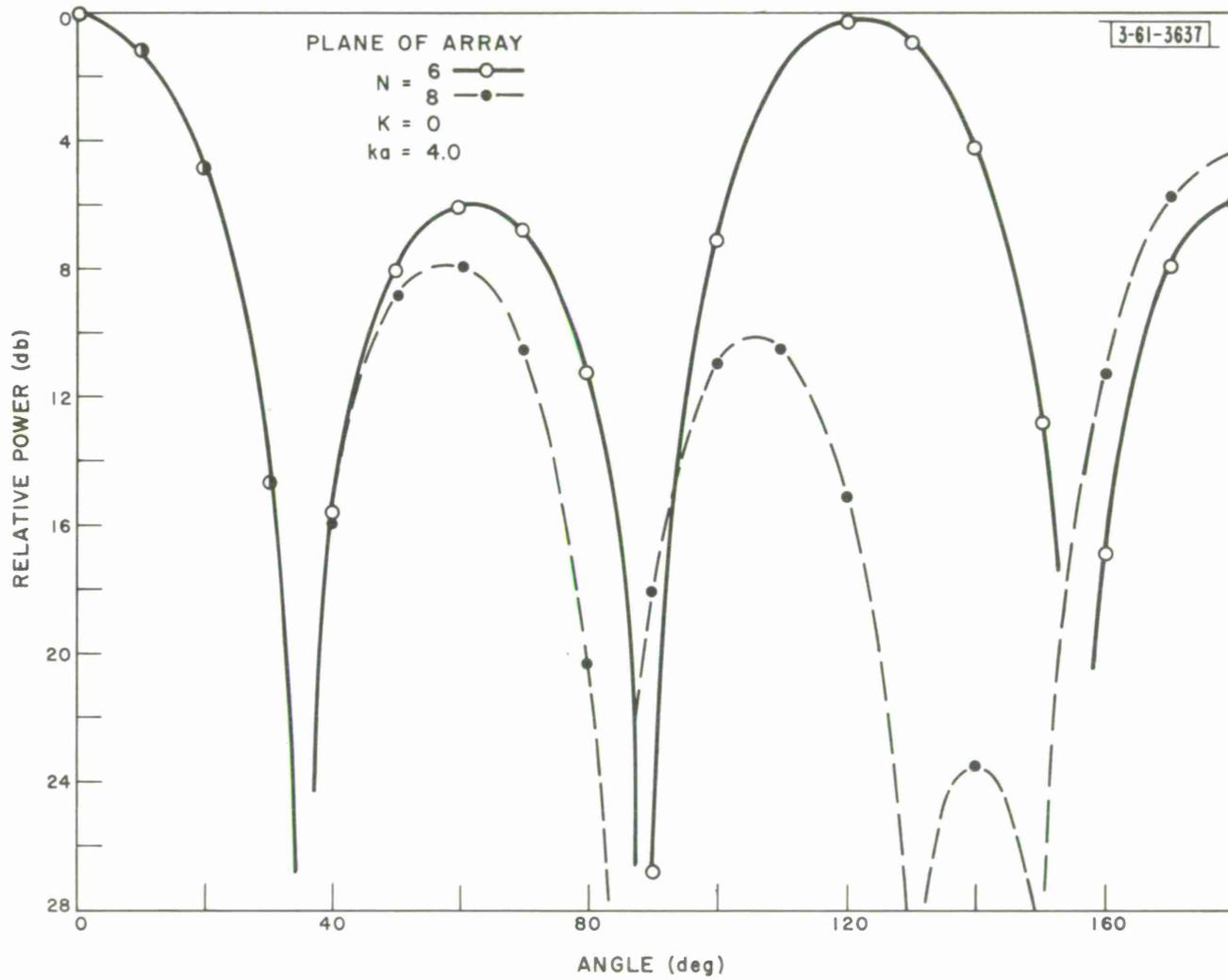
Fig. 7-b.   Pattern of circular array.
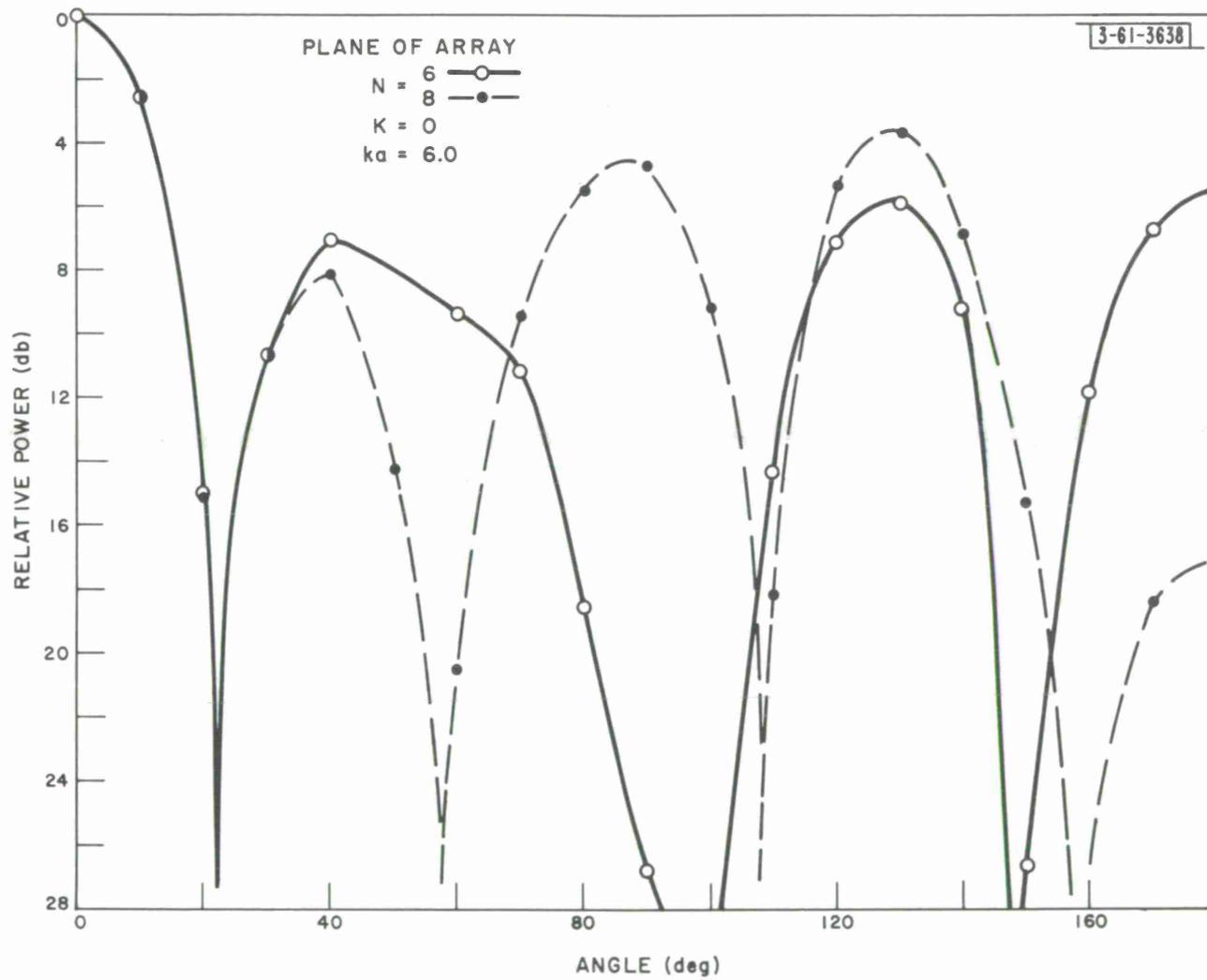
Fig. 7-c. Pattern of circular array.
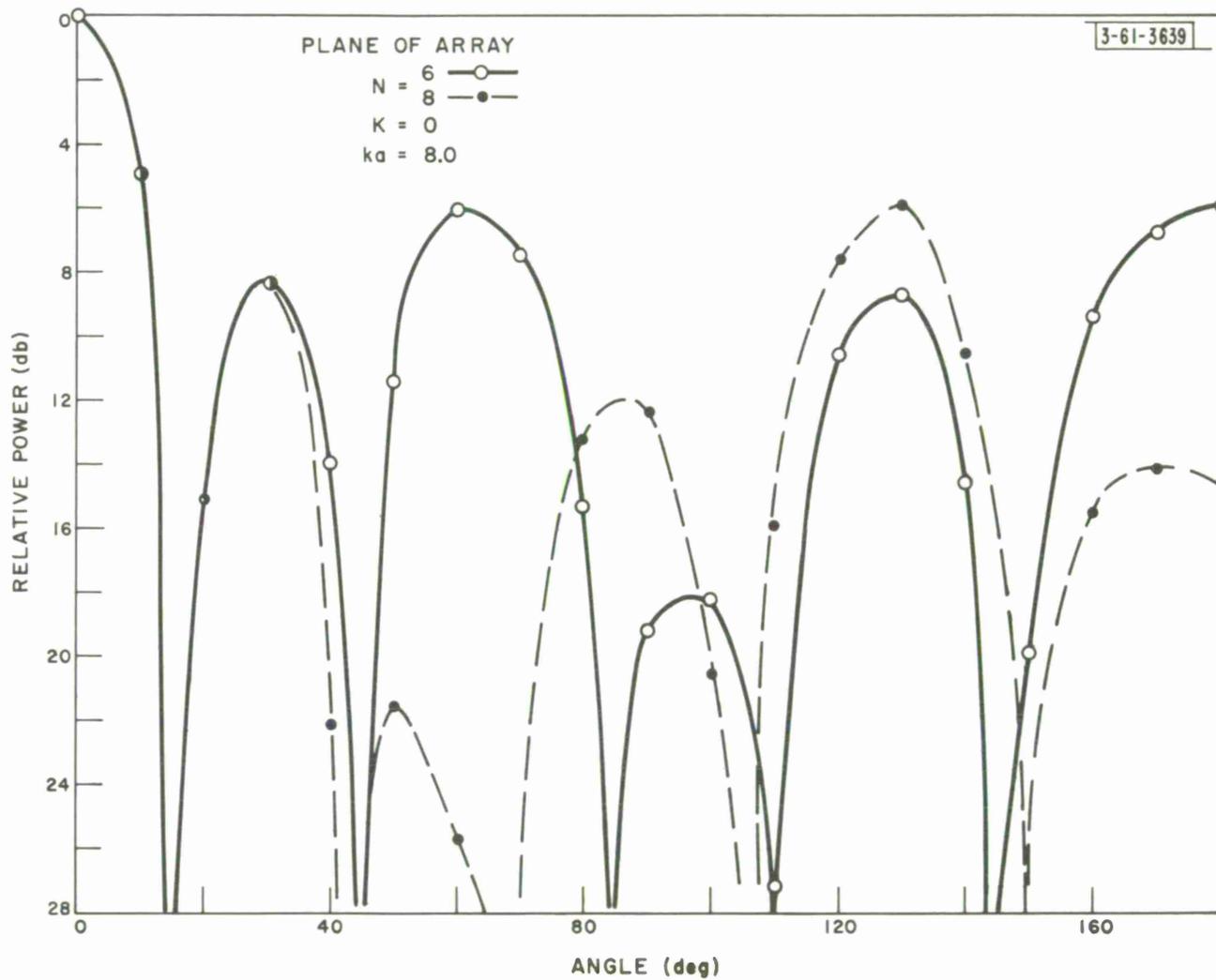
Fig. 7-d.   Pattern of circular array.
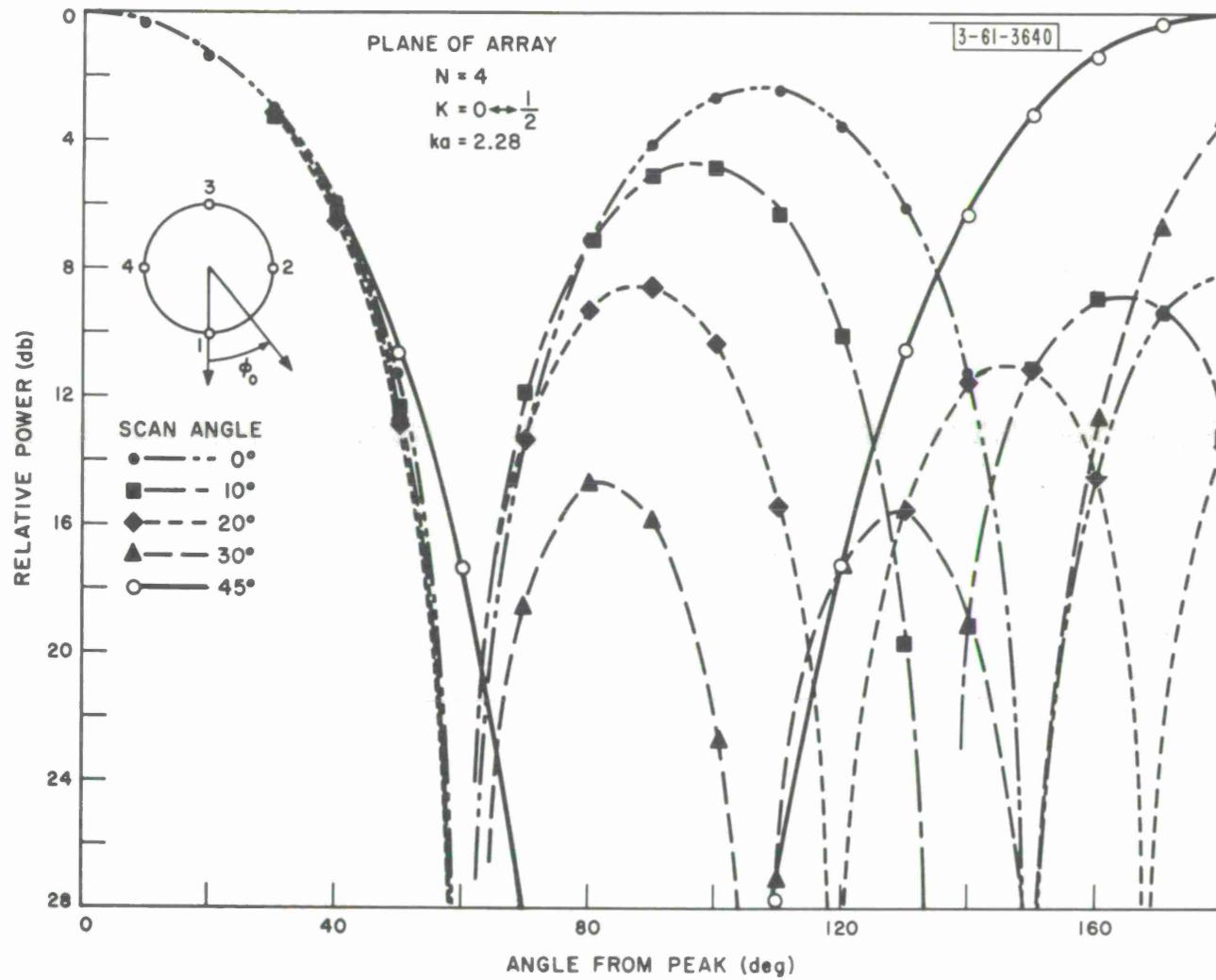
Fig. 8.   Pattern of circular array.
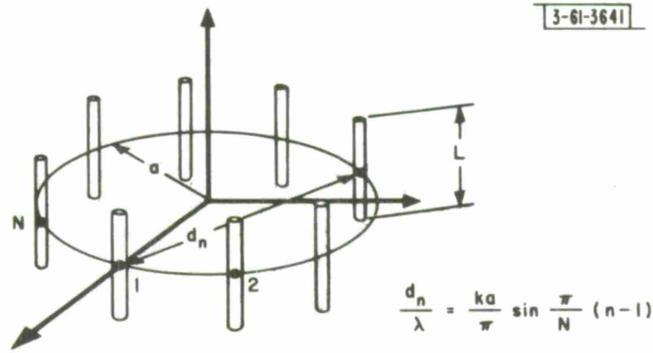
Fig. 9. Dipole array geometry.

$$\frac{d_n}{\lambda} = \frac{ka}{\pi} \sin \frac{\pi}{N} (n-1)$$
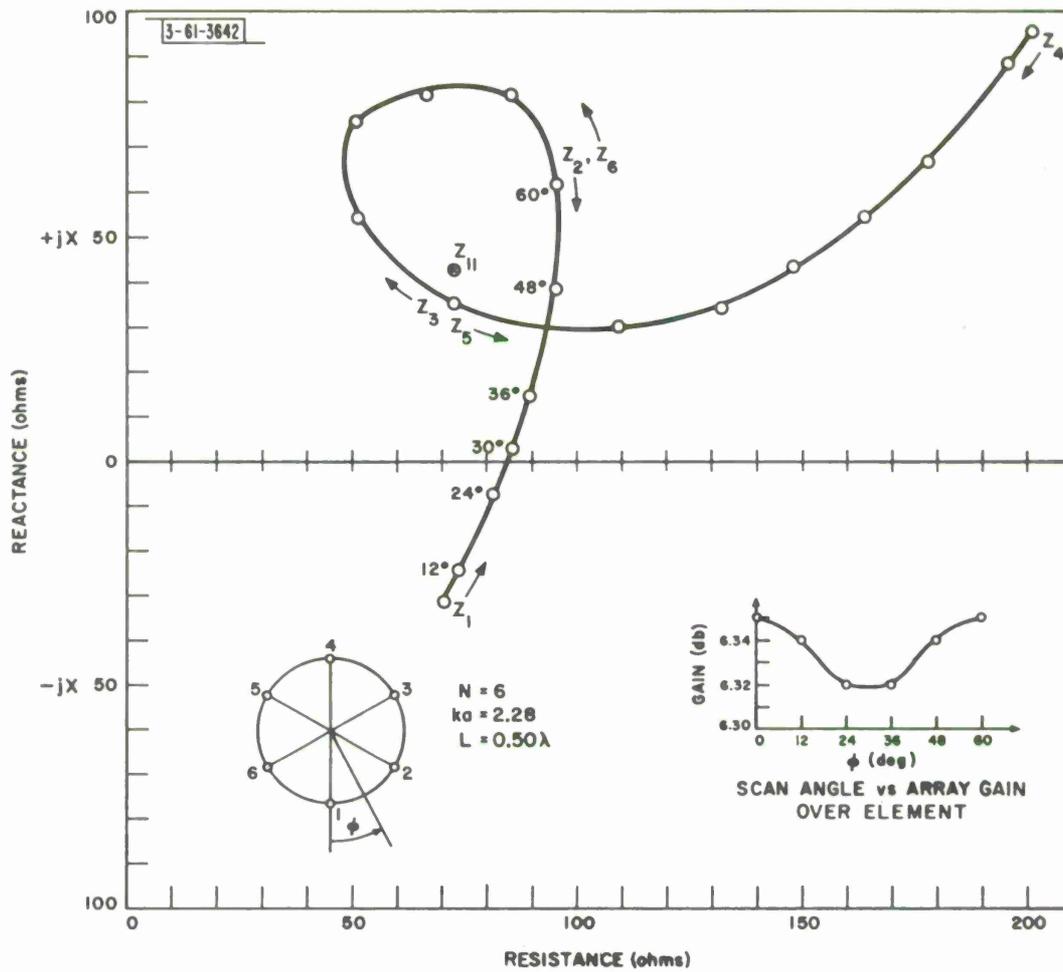
Fig. 10. Input impedance vs. scan angle.

24
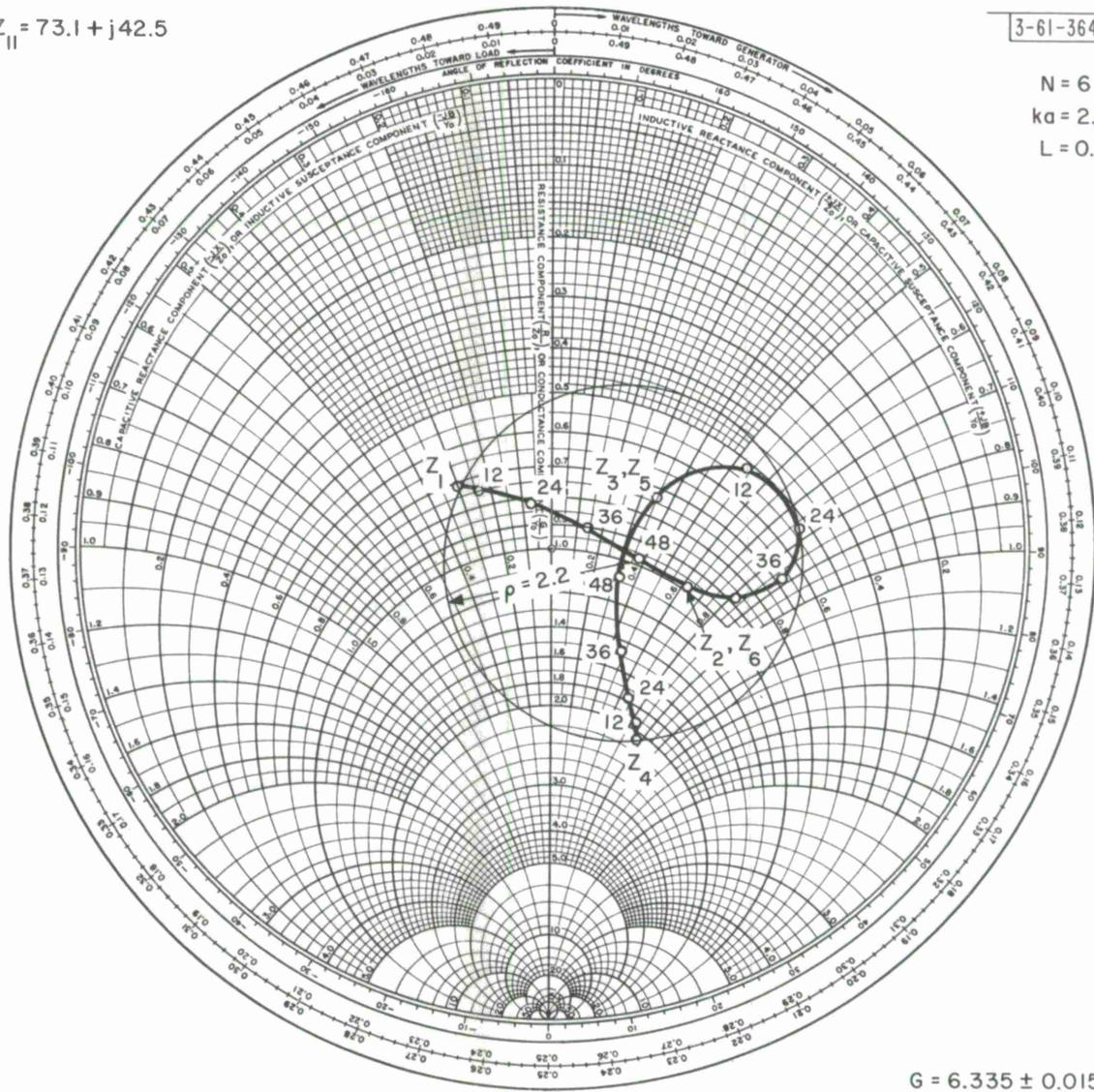
Fig. 11-a.   Ring array impedance.
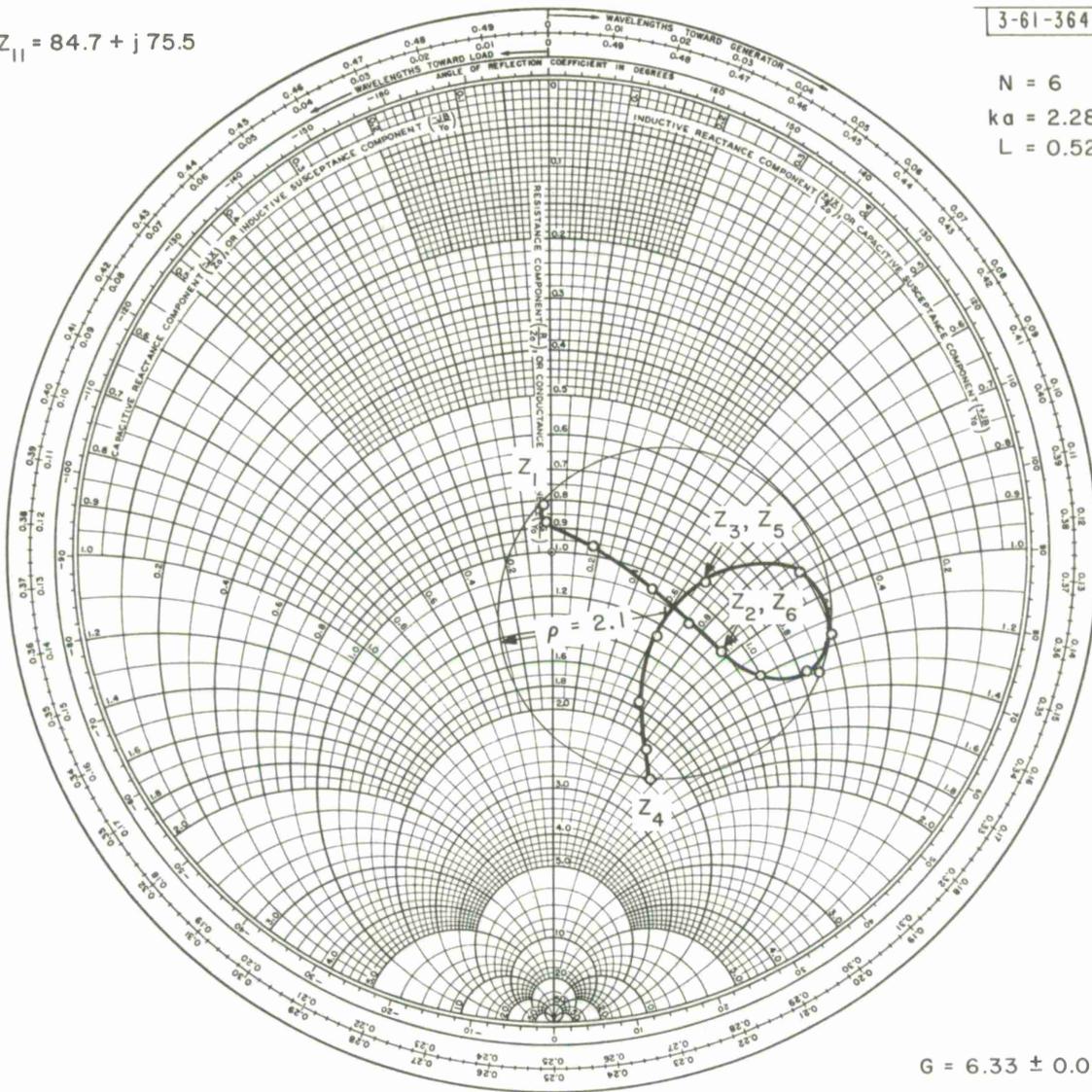
Fig. 11-b.   Ring array impedance.
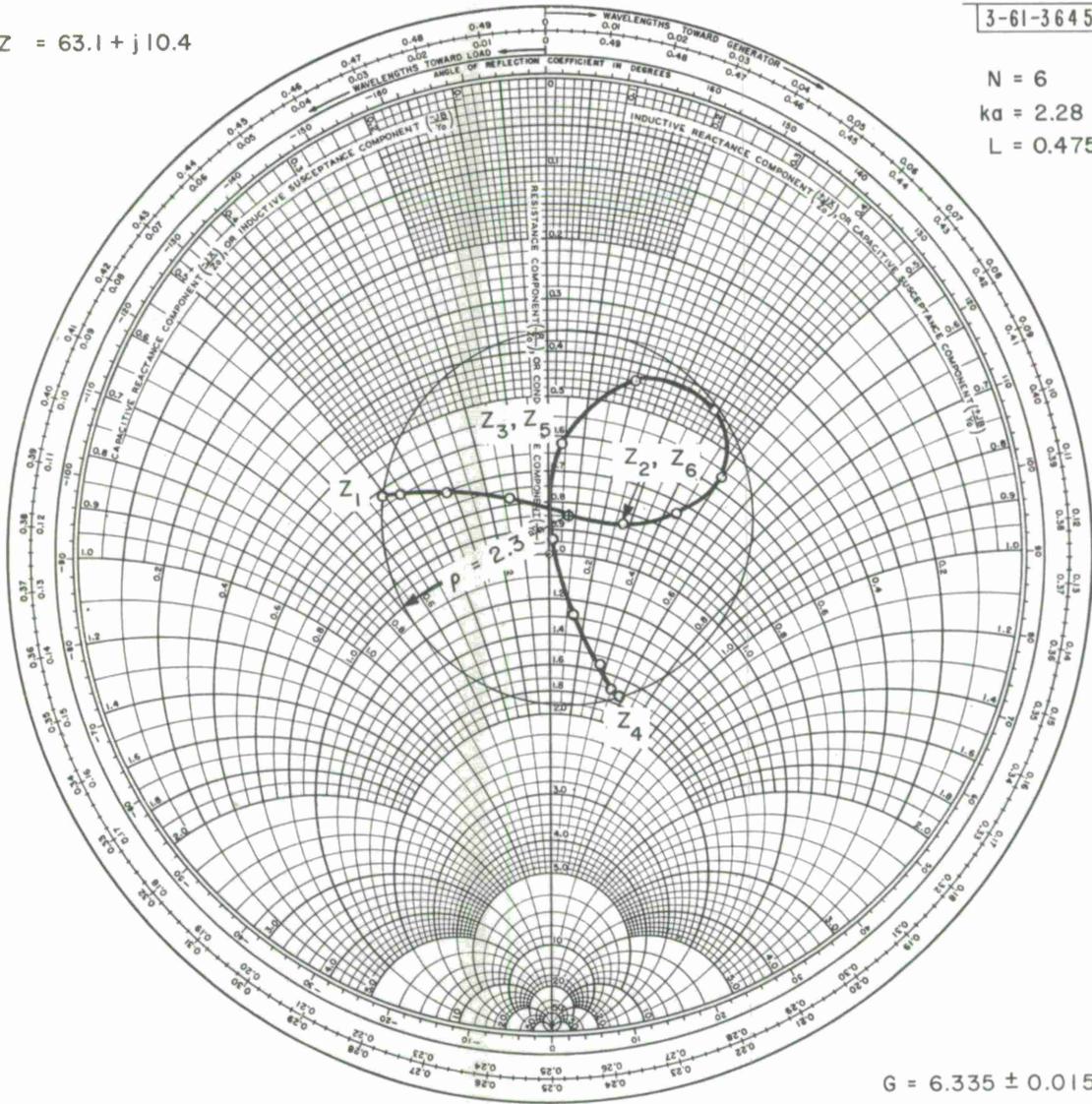
Fig. 11-c.   Ring array impedance.
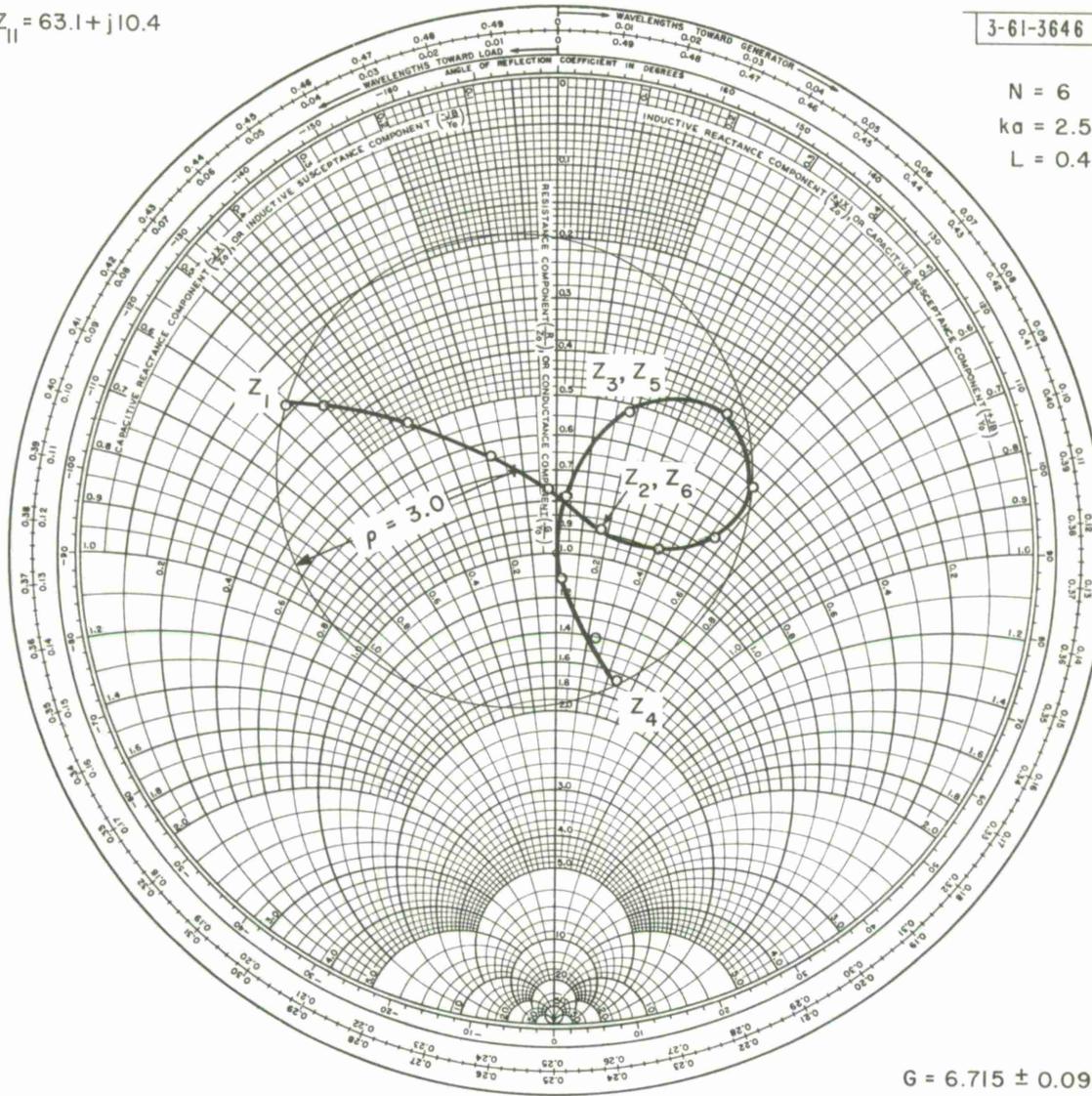
Fig. 11-d.   Ring array impedance.

28

Fig. 11-e.   Ring array impedance.
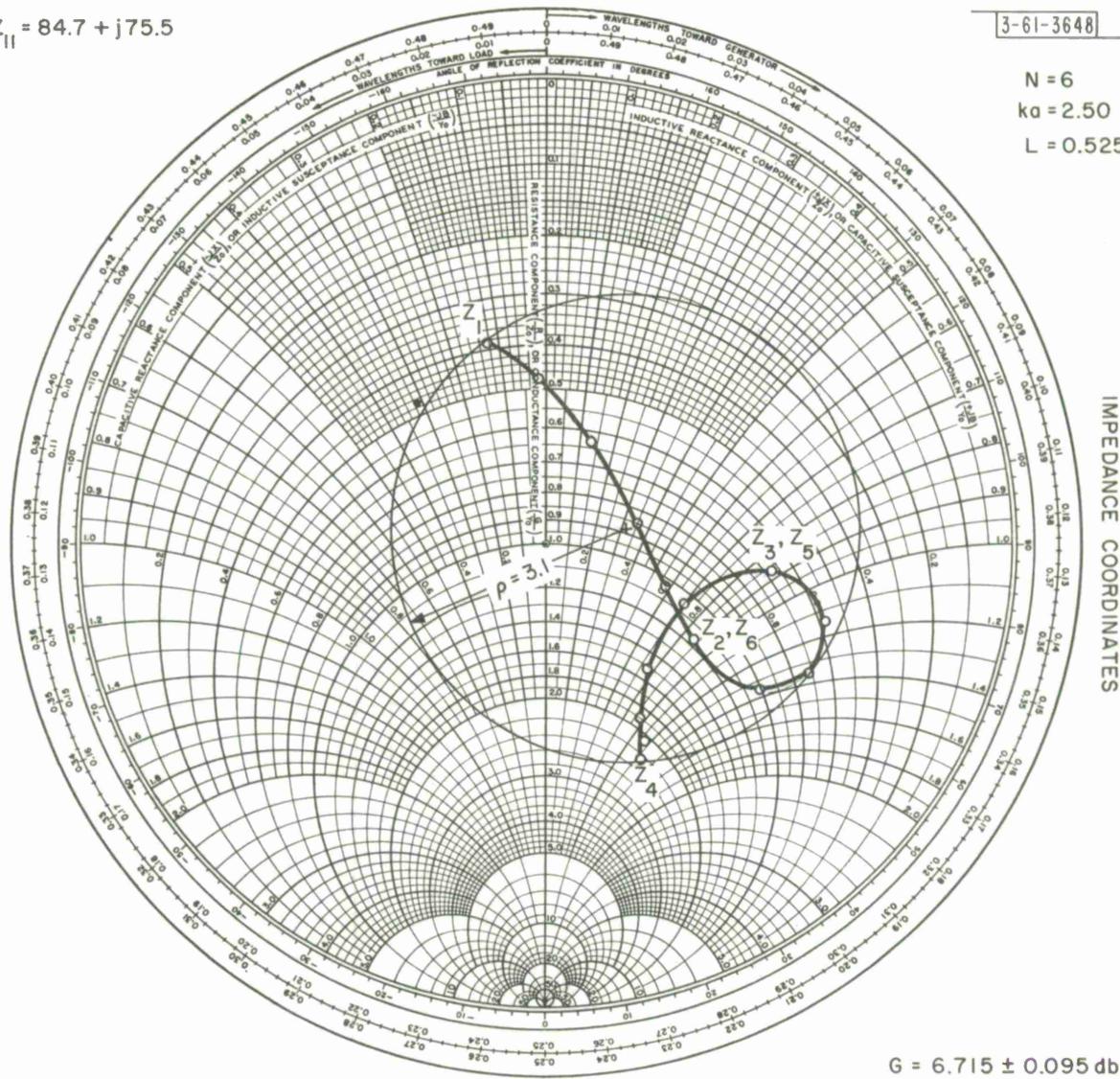
Fig. 11-f.   Ring array impedance.

$Z_{11} = 73.1 + j\,42.5$

3-61-3649

N = 5
ka = 2.28
L = 0.500

IMPEDANCE COORDINATES

$\rho = 2.6$

Z_1

Z_4, Z_3

Z_2, Z_5

G = 6.05 db
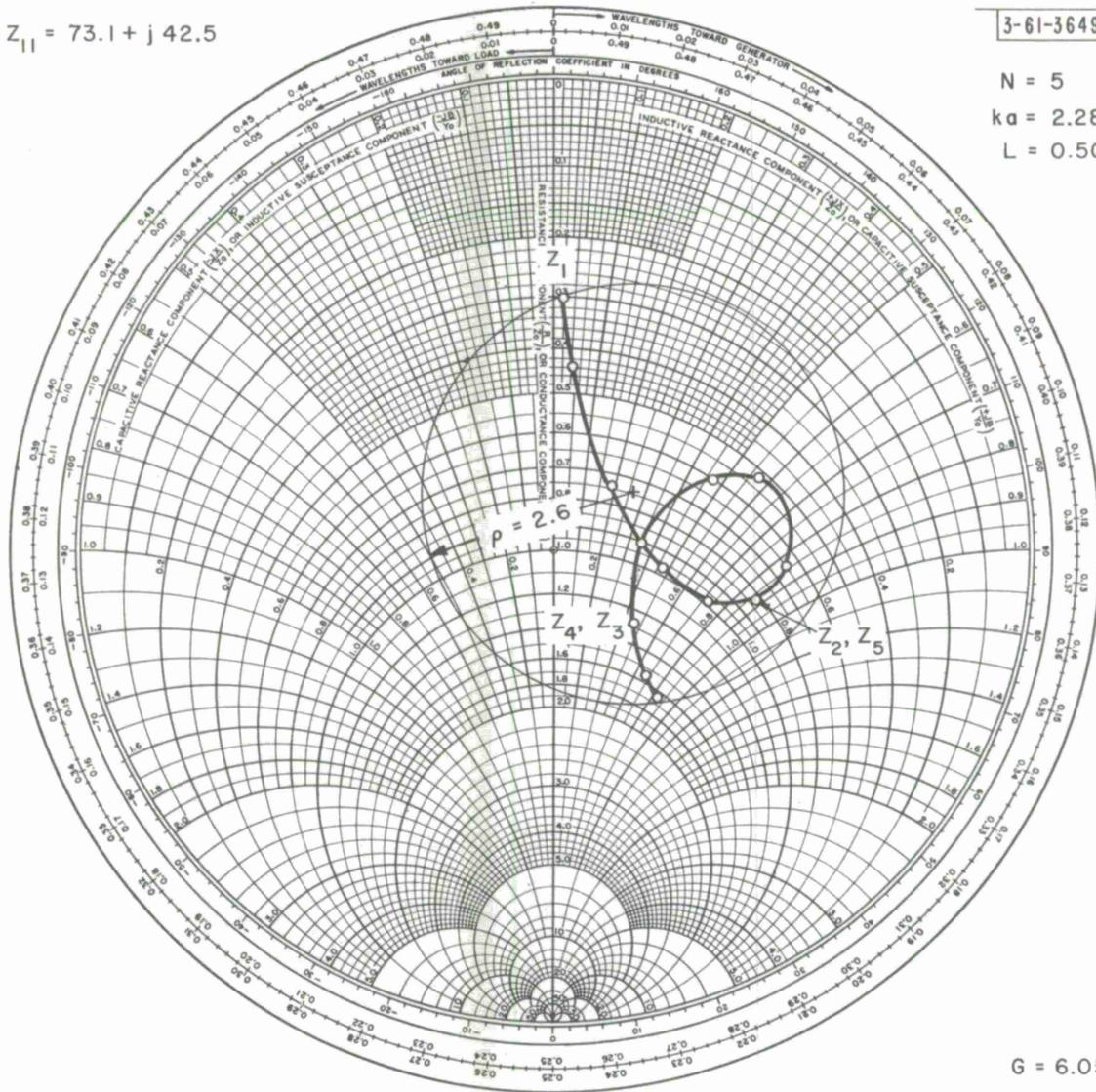
Fig. 11-g.   Ring array impedance.

31

$Z_{11} = 63.08 + j10.4$

N = 5
ka = 2.28
L = 0.475

IMPEDANCE COORDINATES

G = 6.055 ± 0.005 db
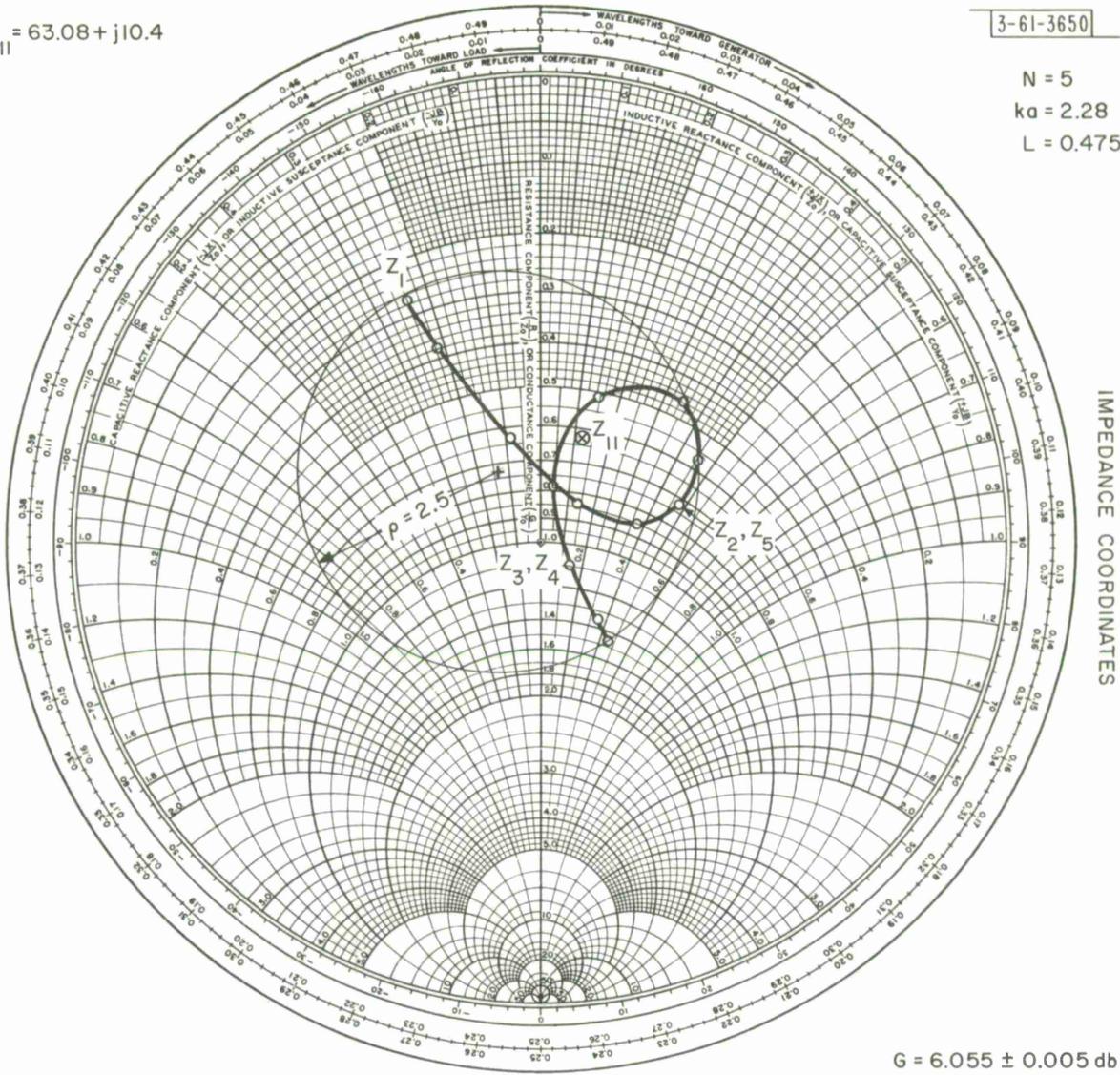
Fig. 11-h.   Ring array impedance.

32

$Z_{11} = 84.7 + j75.5$

3-61-3651

N = 5
ka = 2.28
L = 0.525

IMPEDANCE COORDINATES

$\rho = 2.1$

Z_1

Z_3, Z_4

Z_2, Z_5
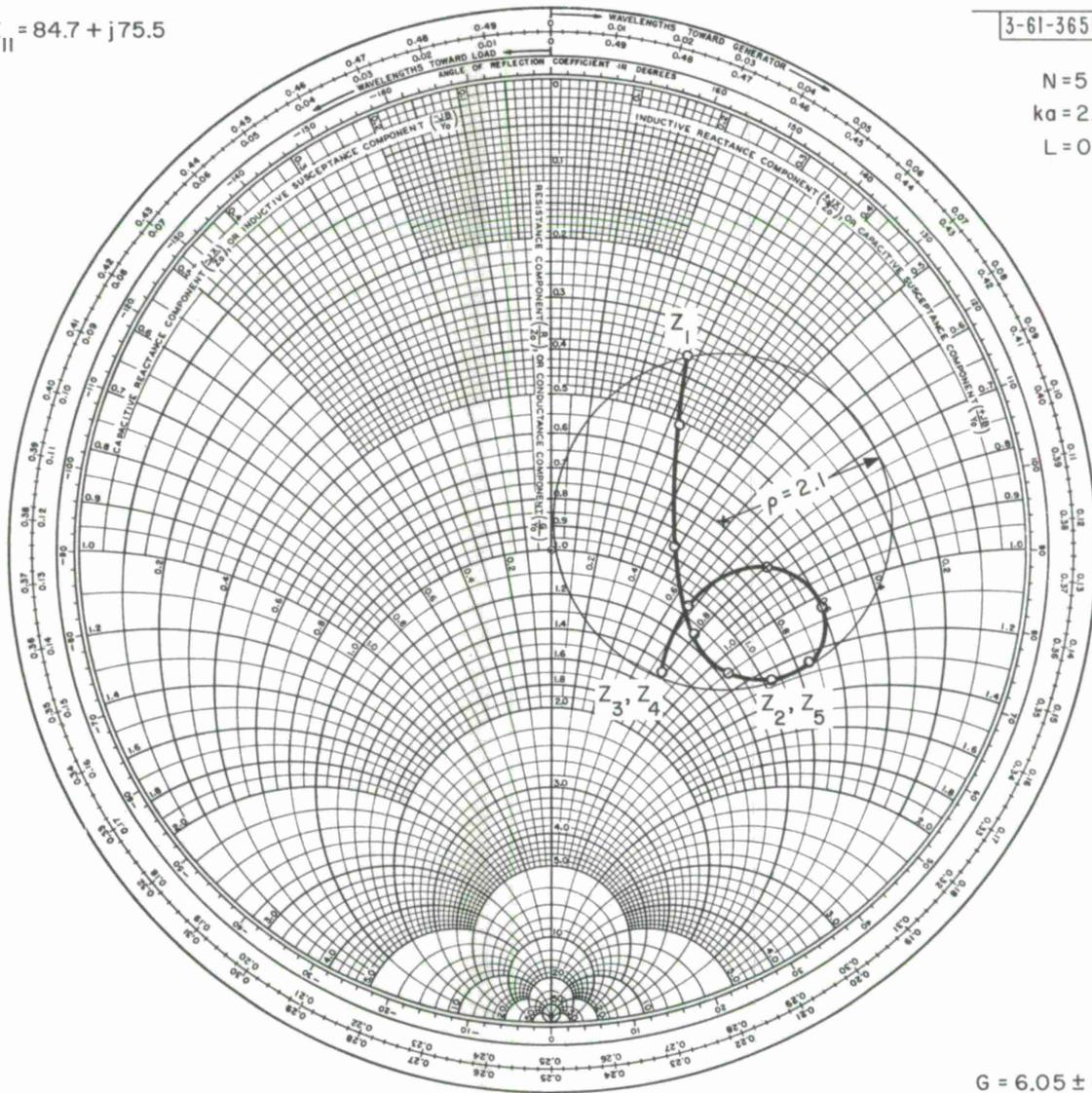
G = 6.05 ± 0 db

Fig. 11-i.   Ring array impedance.
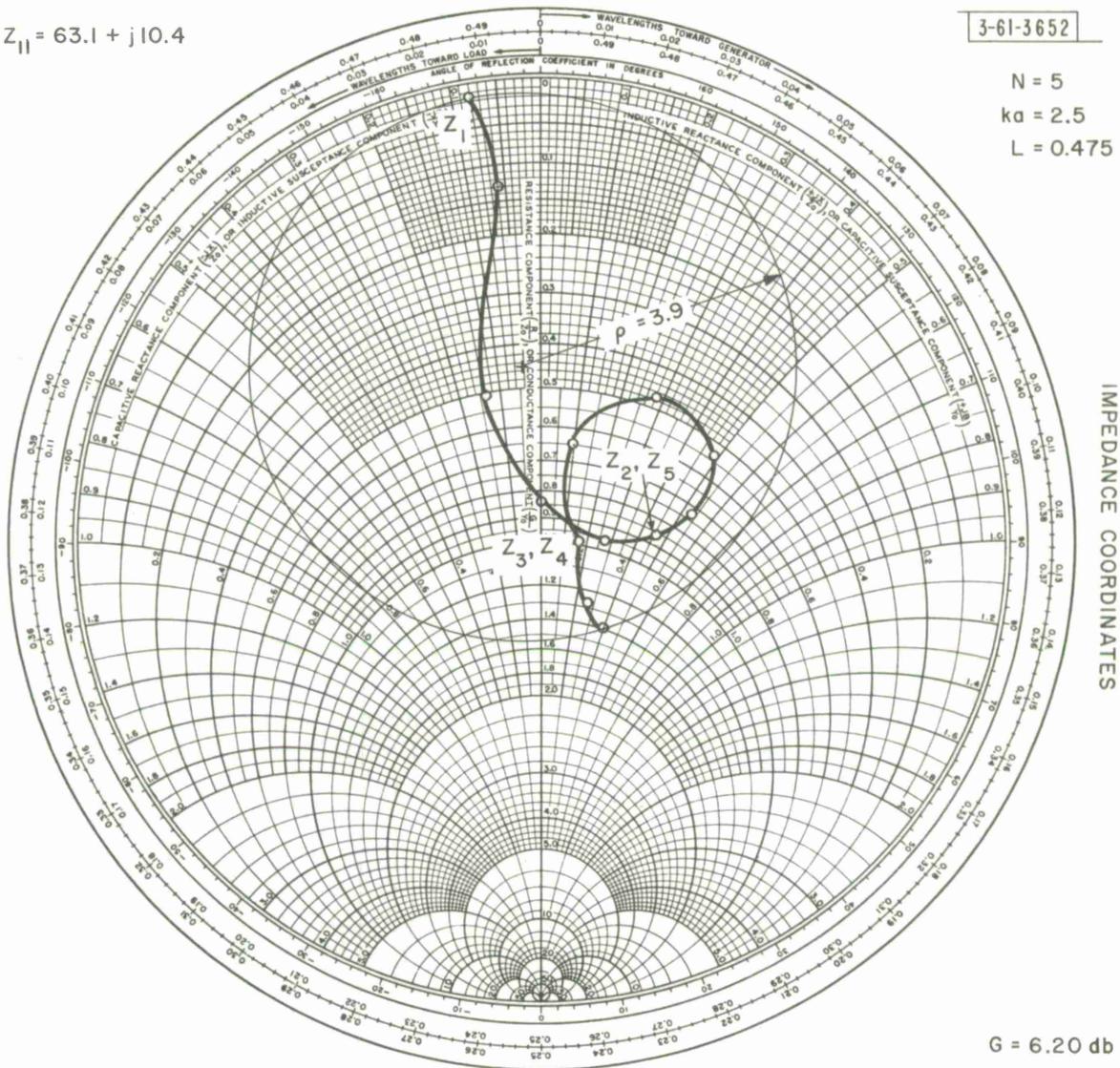
Fig. 11-j.   Ring array impedance.

$Z_{11} = 73.1 + j42.5$

3-61-3653

N = 5
ka = 2.5
L = 0.500

IMPEDANCE COORDINATES

$\rho = 3.9$

Z_1

Z_3, Z_4

Z_2, Z_5
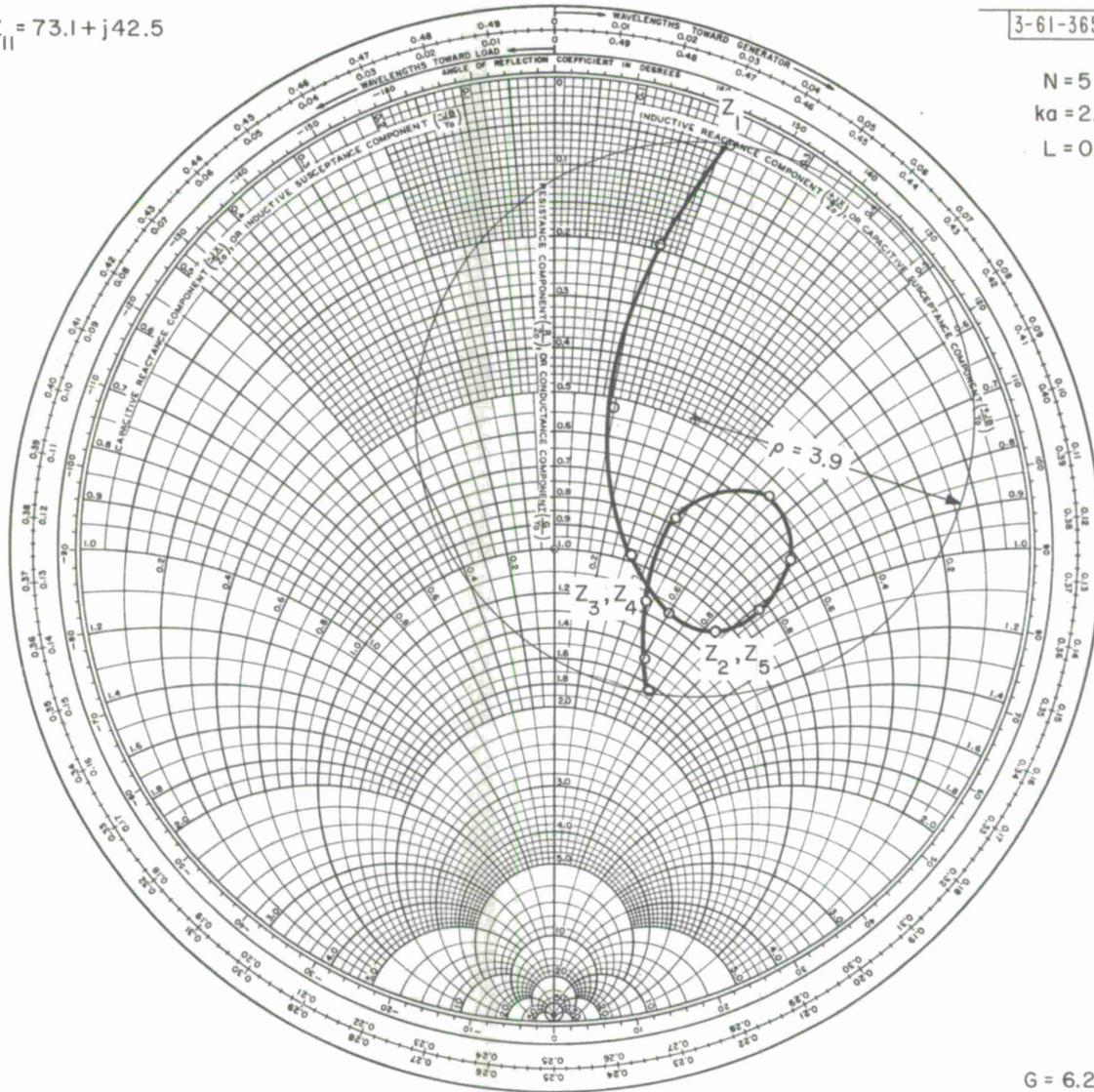
G = 6.20 db

Fig. 11-k. Ring array impedance.

35

$Z_{11} = 63.1 + j10.4$

N = 4
ka = 2.28
L = 0.475

G = 5.195 ± 0.345 db
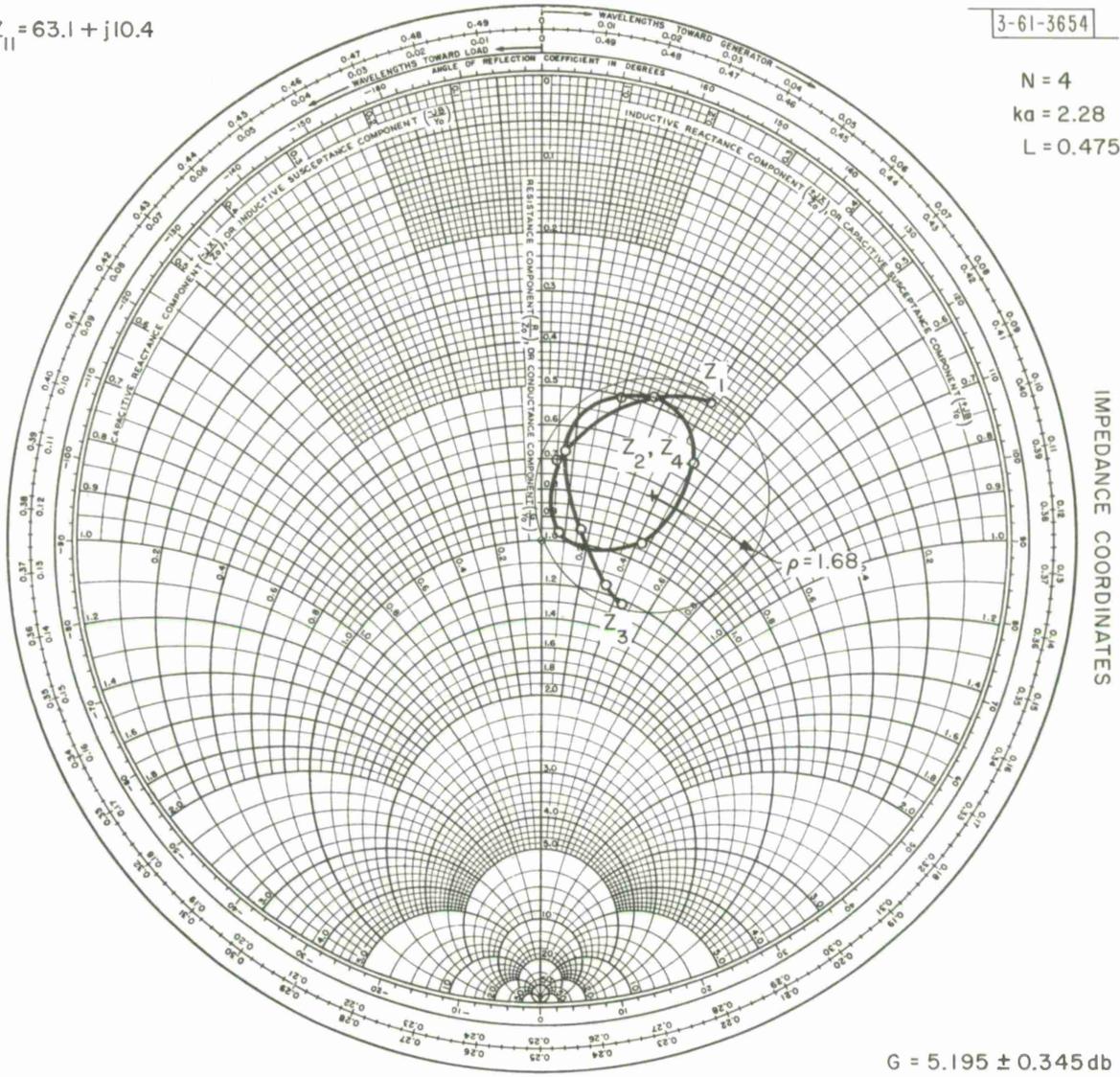
Fig. 11-1.   Ring array impedance.
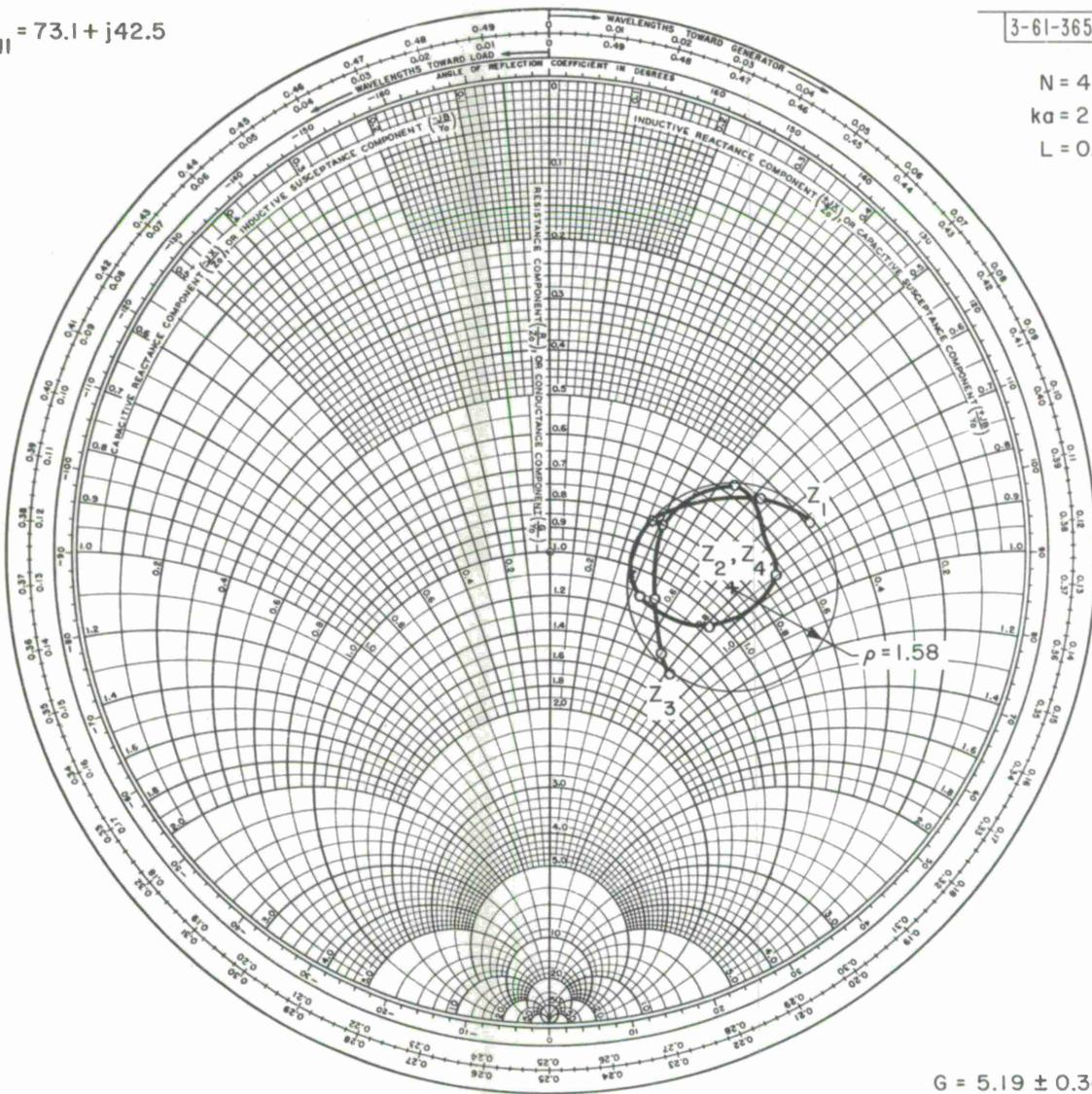
Fig. 11-m. Ring array impedance.

DISTRIBUTION LIST

Division 6

Dinneen, G. P.
Morrow, W. E.

Group 61

Ricardi, L. J. (20)
Assaly, R. N.
Burrows, M. L.
Crane, R. K.
Devane, M. E.
LaPage, B. F.
Lindberg, C. A.
Malbon, R. M.
Niro, L.
Peck, R. J.
Rankin, J. B.
Rosenthal, M. L.
Sotiropoulos, A.

Group 62

Rosen, P. (5)

Group 63

Sherman, H. (5)

Group 64

Green, P. E.

Group 65

Wood, R. V.

Group 66

Reiffen, B.

Division 3

Chisholm, J. H.
Ruze, J.

Division 4

Weiss, H. G.

Group 44

Allen, J. L.

Group 45

Ward, W. W.

Group 46

Jones, C. W.
Keeping, K. J.

Division 7

Hutzenlaub, J. F.

Group 71

Blaisdell, E. W.

DOCUMENT CONTROL DATA – R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Lincoln Laboratory, M.I.T.	Unclassified
	2b. GROUP None

3. REPORT TITLE

Some Properties of a Scanned Circular Array

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Technical Note

5. AUTHOR(S) *(Last name, first name, initial)*

LaPage, Bernard F.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
1 July 1965	44	8

8a. CONTRACT OR GRANT NO. AF 19 (628)-5167	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	Technical Note 1965-28
c. None	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	ESD-TDR-65-274

10. AVAILABILITY/LIMITATION NOTICES

None

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Air Force Systems Command, USAF

13. ABSTRACT

The impedance, pattern and gain properties of a circular array of dipoles (or isotropic sources) are discussed. In particular, the variation of these properties as the antenna beam is scanned is investigated. Computed results for typical arrays are shown and the meaning of the results is discussed, and further areas of investigation are indicated.

14. KEY WORDS

circular array	satellite
impedance	monopoles
dipoles	