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SEMI-DIURNAL EARTH TIDAL COMPONENTS FOR VARIOUS EARTH MODELS

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Extrait des « Communications, n° 236, de l'Observatoire Royal de Belgique.
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L. E. ALSOP and J. T. KUO (*)

SEMI-DIURNAL EARTH TIDAL COMPONENTS FOR VARIOUS EARTH MODELS **

ABSTRACT

This study of the elastic deformation of the earth due to tidal-generating potentials attempts to evaluate the possible cause for the discrepancy between the accumulated observational data of earth tides for the principal semi-diurnal idal constituent, M_2, and the theoretical values. The characteristic numbers, *h*, *k*, and *l* of the earth are calculated for earth models consisting of velocities due to Jeffreys or Gutenberg with Bullen A or Bullen B density distribution. The computations were made using a modification of an earlier program for an IBM 7090 written for calculating the free periods of the earth.

The characteristic numbers of the earth are nearly independent : (1) of the presence of the solid inner core for a wide range of the possible values of the outer core rigidity; (2) of either the Jeffreys or the Gutenberg velocity models, i.e., of the presence or absence of a low velocity channel in the upper mantle. A comparison of the theoretical gravimetric factors G calculated for a continental and an oceanic earth model with the observed gravimetric factors of the semi-diurnal tidal constituent, M_2, obtained principally during the International Geophysical Year indicates that the differences in the observed values of G appear to be primarily due to the effects of the ocean and only secondarily to differences in the variation of the crustal structures.

RESUME

Cette étude de la déformation statique de la terre due à des potentiels générateurs de marées tente d'évaluer la cause possible du désaccord entre les observations accumulées de marées terrestres pour la composante principale semi-diurne, M_2, et les valeurs théoriques. Les nombres caractéristiques, *h*, *k*,

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(**) This is condensed from the paper entitled « The Characteristic Numbers of Semi-Diurnal Earth Tidal Components for Various Earth Models », which will be published by Annales de Géophysique.

et *l* de la terre sont calculés pour des modèles terrestres faits de vitesses dues à Jeffreys ou Gutenberg avec une densité de distribution Bullen A ou Bullen B. Les calculs ont été faits en employant une modification d'un programme précédent pour une IBM 7090 écrit pour calculer les périodes libres de la terre.

Les nombres caractéristiques de la terre sont presque indépendants : 1) de la présence d'un noyau intérieur solide pour une grande échelle de valeurs possibles de la rigidité du noyau extérieur; 2) des modèles de vitesse de Jeffreys ou de Gutenberg, c'est-à-dire, de la présence ou de l'absence d'un canal à faible vitesse dans le manteau supérieur. Une comparaison des facteurs gravimétriques théoriques G calculés pour un modèle terrestre continental et pour un modèle terrestre océanique avec les facteurs gravimétriques observés d'une composante de marée semi-diurne obtenue principalement durant l'Année Géophysique Internationale indique que la différence dans les valeurs observées de G semble être due premièrement aux effets de l'océan et deuxièmement à des différences dans la variation de la structure de la croûte terrestre. La présence du noyau intérieur solide n'a pratiquement pas d'influence.

INTRODUCTION

One of the major problems arising in the study of earth tides is the discrepancy between the observed values of the characteristic numbers of the earth, viz. the Love numbers *h* and *k*, and Schida's number *l*, as obtained from the accumulated data (Melchior, [7] ; Tomaschek, [18] and the theoretical values first calculated by Takeuchi [15]). The average observed values are approximately 0.48, 0.29, and 0.05 for *h, k,* and *l,* respectively, while the theoretical values for the static case are 0.60, 0.28, and 0.08. The theoretical values were obtained for a model earth with negliglible rigidity in the core at an infinite period. By increasing the core rigidity in the model, the theoretical values for *h* and *k* may be brought into agreement with the observational data, but the value of the core rigidity required, 5×10^{11} cgs units or greater corresponding to a shear wave velocity of 3.7 km/sec or greater, is too high on the basis of the present seismic evidence (Press [14]). For all of the models considered, the value of *l* obtained was much greater than the observed values.

As the majority of the existing earth tidal data have been obtained by stations located near to large bodies of water, it should be noted that some of the more recent measurements of tidal gravity of the semi-diurnal tide constituent obtained at inland stations agree quite well with the theoretical values (Pariisky [12]).

The present study of elastic deformation of the earth due to tidal generating potentials attempts to evaluate the possible causes for the still existing discrepancy between the observed data and theoretical values for semi-diurnal tides. The possibilities considered are : (1) the presence of a solid inner core; (2) differences in the structure of the crust and upper mantle; (3) the proximity of stations to large bodies of water. It is found that the last of the three possibilities has the greatest effect, and that the presence of a solid inner core has very little effect.

METHOD OF COMPUTING CHARACTERISTIC NUMBERS

The computer program used here to compute the values of the characteristic numbers of the earth, *h*, *k*, and *l*, is a modification of an earlier program (Alsop, [1, 2]) written for calculating the free periods of oscillation of the earth.

For the present problem of the forced vibration of a sphere by an external potential of order n, the derivation of the appropriate boundary conditions at the surface, by matching internal and external potentials and their derivatives, is given by Takeuchi, Saito, and Kobyashi [17].

If the external potential is chosen, the expressions for the characteristic numbers *h*, *k* and *l* may be obtained.

In this paper only external forcing potentials with a period of 12 hours and of order 2, corresponding to the semi-diurnal tide-producing potentials of the moon and the sun acting on the earth, and applicable to all semi-diurnal tidal constituents, will be considered. Neither the inner nor the outer core of the earth is ever assumed to be a perfect liquid for these calculations; they always have some rigidity which may be very small, so that the complications introduced into the problem by a solid-liquid boundary at the coremantle interface are eliminated.

EFFECT OF THE RIGIDITY OF THE OUTER AND INNER CORE ON THE CHARACTERISTIC NUMBERS

Figure 1 shows the values of *h*, *k* and *l* for four earth models as a function of the rigidity of the core. These models use either Bullen's model A or B for the density distribution within the earth. The actual values used were taken from Alterman, Jarosch, and Pekeris [3]. The model here called Bullen A corresponds to what Bullen [4] calls Hypothesis (i); the actual model Bullen A is defined by Bullen as lying midway between his Hypothesis (i) and (ii), and therefore has higher densities within the core than the model used for these calculations. Two of the models use compressional velocities throughout the earth and shear velocities within the mantle as given by Jeffreys [6]. The other two models use similar velocities as given by Gutenberg. Both of these velocity models are tabulated in Alterman, Jarosch, and Pekeris [3]. The rigidity of the entire core is assumed to be uniform for these calculations and is varied between almost zero and 10^{13} cgs units.

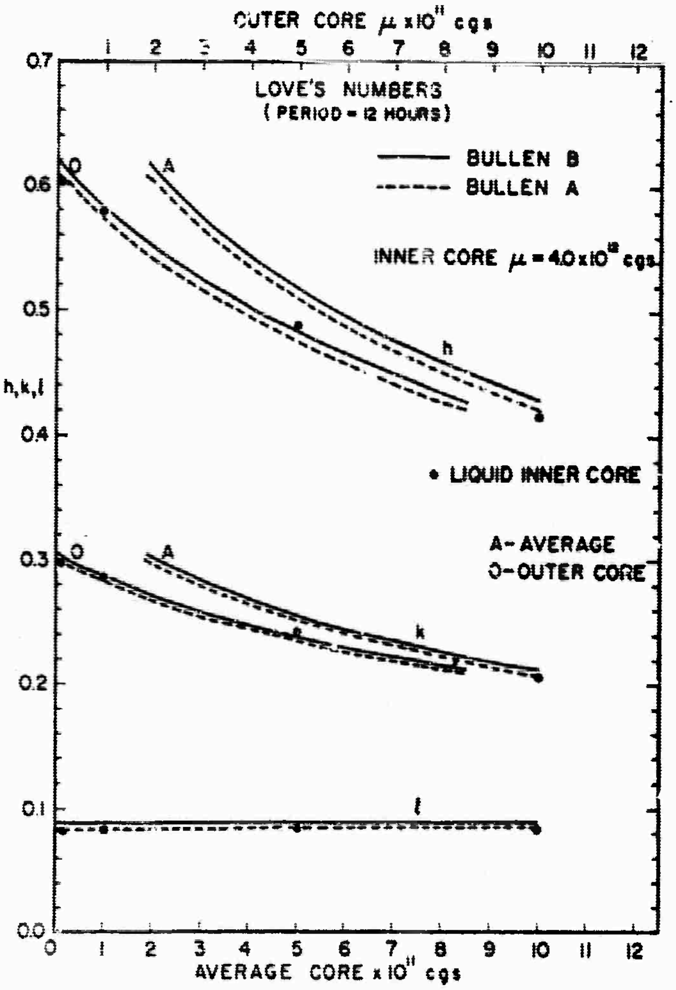

Fig. 1. — Values of *h*, *k*, and *l* as a function of the rigidity of the earth's core for several earth models. The points designated Takeuchi are from Takeuchi [16].

The velocity model used has very little effect on the values of *h*, *k* and *l*. The values for the same density model but different velocity models almost coincide, and cannot be drawn separately in the figure. For this reason the curves are identified only by the appropriate density model. As a comparison the values previously obtained by Takeuchi [15] are indicated on Figure 1. The apparent discrepancy may be explained by : (1) Takeuchi's calculations were made for a density model which is slightly different from either of the two models used here; (2) Takeuchi's calculations were made for an infinite period.

To determine the effect of a solid inner core on the characteristic numbers, calculations were made for the same four models assuming an overall rigidity for the inner core of 4 × 10^{12} cgs units. The volume average rigidity of the total core was allowed to vary between the values of $\mu = 1.86 \times 10^{11}$ and 1.00 × 10^{13} cgs units, corresponding to an overall rigidity of the outer

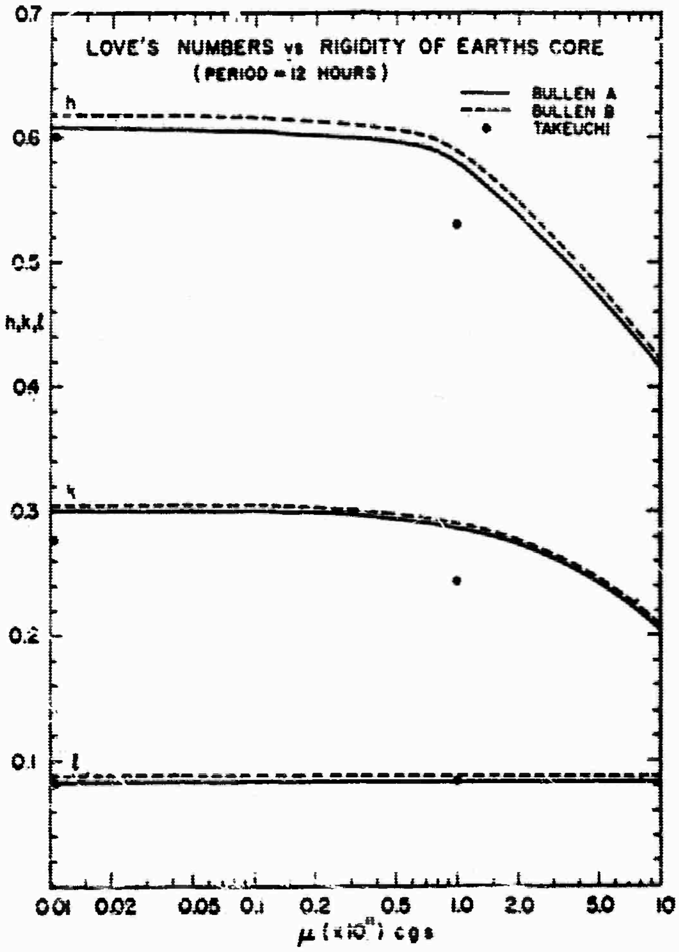

Fig. 2. — Values of *h*, *k*, and *l* as a function of the rigidity of the earth's outer core for several earth models all having an assumed solid inner core with a rigidity of 4.0 x 10^{12} cgs units.

core of between zero and 8.53×10^{11} cgs units. The results are illustrated in Figure 2. Each model is represented by two plots on Figure 2, one as a function of the overall core rigidity and the other as a function of the outer core rigidity only. The values for the Bullen A model, without a solid inner core are also plotted on Figure 2 for comparison. For rigidities of the outer core which are much less than the assumed rigidity of the inner core, the values of *h* and *k* are determined by the rigidity of the outer core only, and are not affected by the rigidity of the inner core. As the rigidity of the outer core becomes close to that of the inner core there is a slight effect on the values of the Love numbers. The values of the Shida's number *l* are very nearly independent of the core rigidity.

EFFECT OF THE CRUST AND OCEANS ON THE LOVE NUMBERS

All of the earth models considered so far have continental crusts. In order to determine the effect of the crust and of the ocean on the values of the characteristic numbers the following modifications were made to the Jeffreys-Bullen B model. The upper fifty kilometers of the model were replaced with ten kilometers of a pseudo-crust having a compressional velocity of 4.0 km/sec, a shear velocity of 1.5 km/sec, and a density of 1.9 gm/cm^3 plus forty, kilometers of mantle with a compressional velocity of 7.8 km/sec, a shear velocity of 4.38 km/sec, and a density of 3.34 gm/cm^3. In this way it was hoped to be able to study the effects of an oceanic crust without considering the effects of the ocean layer itself. This model is referred to as Bullen B modified.

The effect of the ocean upon the observed earth tides is complicated. The program for a liquid layer on top of a solid sphere considers only a non-viscous ocean, completely covering the earth, but it may give a first approximation to the effect of the ocean. For this purpose the top five kilometers of the model Bullen B modified were replaced with five kilometers of ocean water; compressional velocity 1.52 km/sec; shear velocity 0.0 km/sec; density 1.03 gm/cm^3. This model is referred to as Bullen B Oceanic. The remaining pseudo-crust is not a good approximation to an oceanic crust, but since its effect in the Bullen B modified model was small, it was retained for reasons of convenience. The values of the characteristic numbers calculated for this model are for the top of the crust, not for the top of the ocean.

Figure 3 compares the values of *h*, *k* and *l*, and Figure 4 compared the gravimetric factor G, and the diminishing factor D for the Jeffreys-Bullen B model, the Bullen B modified model, an the Bullen B Oceanic model for various values of the rigidity of the outer core and assuming a solid inner core.

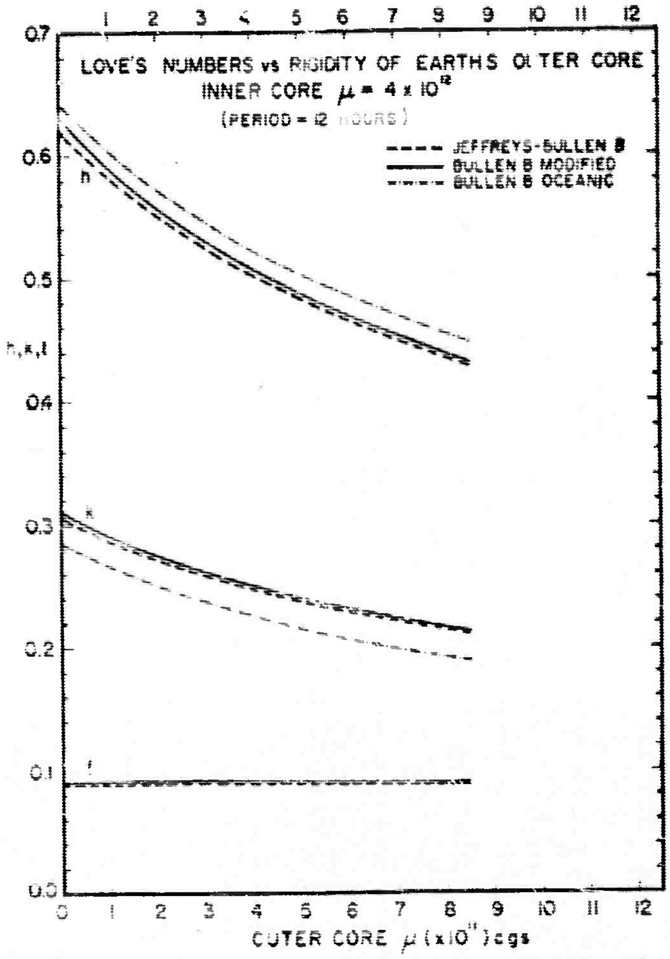

Fig. 3. — Comparison of the values of *h, k,* and *l* as a function of rigidity of the earth's outer core for a continental model, for a model with an oceanic crust but no ocean, and for an oceanic model.

The three values of *l* are not shown in Figure 3 because of insufficient separation. Replacing a continental crust by an oceanic crust raises the values of *h, k,* and *l* slightly, but since all are raised by approximately the same ratio, the factors G and D do not differ appreciably between the two models. The effect of adding an ocean layer is more striking. The value of *h* is larger than for the model with the oceanic crust, but the value of *k* is much less even than for the model with the continental crust. Therefore, there is a large effect upon the values of G and D. It must be stressed that the values obtained for the Bullen B Oceanic model, which assumes an idealized ocean, can give only an approximation to the effect of the ocean on the Love numbers.

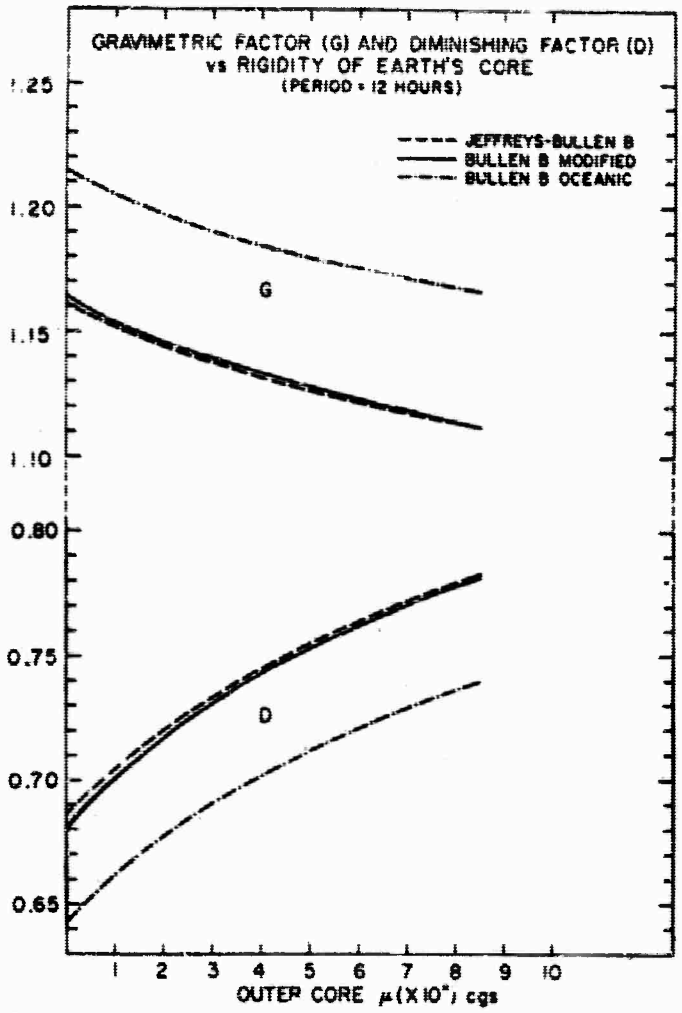

Fig. 4. — Comparison of the values of the gravimetric factor (G) and the diminishing factor (D) as a function of rigidity of the earth's outer core for a continental model, for a model with an oceanic crust but no ocean, and for an oceanic model.

COMPARISON WITH OBSERVATIONAL DATA

The Love number k can be obtained directly from the ratio of the actual period of the free nutation of the earth to that for a rigid earth. Combinations of the Love numbers k and h can be obtained from the values of the diminishing factor, $D = 1 + k - h$, and the gravimetric factor, $G = 1 + h - 3/2\,k$. Tomaschek [18] thus obtains from the polar movement a value of $k = 0.28$, which agrees with the theoretical value of $k = 0.29$, calculated by Takeuchi [15] for the Jeffreys-Bullen earth model, and which Tomaschek considers a most reliable value. He also obtains a value of $h = 0.46$ from accumulated observations of gravity and tilt measurements with D = 0.72 and G = 1.20. However, the close agreement of the observed value of k from the free nutation of the earth with the theoretical value for the Jeffreys-Bullen earth model is somewhat misleading, as pointed out by Munk and MacDonald [8]. The Love number k critically depends upon the surface density ρ_s. For the same model, Takeuchi [16] obtains.

k = 0.281 for ρ_s = 3.0 gm/cm^3
and k = 0.256 for ρ_s = 2.7 gm/cm^3.

Furthermore, dynamic effects of the re and oceans, and even, perhaps, of convection on earth tides, particularly the diurnal tide, must be taken into account. Therefore the value of $k = 0.28$ obtained from the polar movement given by Tomaschek still seems tentative. Melchior [7], on the other hand, considers that D = 0.706 ± 0.01 and G = 1.20 ± 0.02 are the most probable values for the earth. The values for the Love numbers k and h thus deduced are 0.188 ± 0.06 and 0.482 ± 0.07, respectively, and are smaller than the most probable theoretically calculated values of $K = 0.28$ and $h = 0.60$. The most recent data obtained, for instance, by Pariisky [13] on the diminishing factor D, indicate more clearly that tidal tilts are strongly influenced by « shallow, small scale » heterogenieties of the earth's surface layers. The difference of the diminishing factor between the N-S and the E-W components is more than 20 per cent. Precise and systematic evaluation of such effects on earth tidal tilts would add greatly to the understanding of earth tides.

So far, the values of L for the combination of the Love number k and Shida's number l in latitude variation still lack consistency. The problem of determining a definite value of L from latitude variation analyses cannot yet be regarded as solved. The value of Shida's number l quoted in the literature to date is principally based on strain observations obtained by use of superinvar type extensometers (Ozawa [10 and 11]), and is found to be about 0.05.

The most recent values of G from various tidal gravity measurements obtained principally during the International Geophysical Year are now available, and are generally consistent with those obtained earlier as shown in Table 1. The significant fact is that the values of G for the semi-diurnal tidal constituent M_2 at inland stations are definitely lower, around the value of 1.14-1.16, than the values of 1.7-1.21 at stations close to oceans or to large bodies of water. A comparison of the values of G for the semi-diurnal tidal constituent M_2 at various stations, as a function of the distance from the nearest ocean or large body of water, with the theoretical values for the tidal period of 12 hours for the earth models of the Jeffreys-Bullen B oceanic and continental structure is shown in Figure 5. These two earth models assume (1) the range of average core rigidity 2.0-3.0 × 10^{11} cgs; (2) the inner core rigidity 4.0 × 10^{12} cgs; and (3) the outer core rigidity 0.14-1.19 × 10^{11} cgs. Figure 5 indicates that stations far inland have values of G close to that predicted for the continental model of Jeffreys-Bullen B, while stations near large bodies of water have values of G near to that predicted for the oceanic model of Jeffreys-Bullen B. It must be remembered, however, that these calculations are for a non-viscous fluid completely covering the earth. Furthermore, the relationship between the values of the characteristic numbers observed on the floor of the ocean and those observed at a coastal station is for from clear. Therefore, the agreement

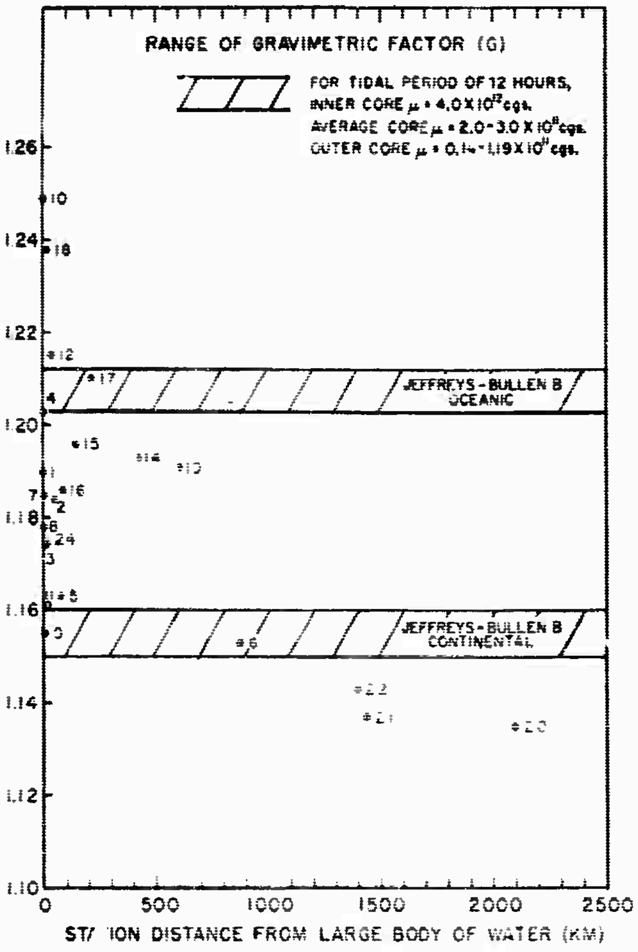

Fig. 5. — Observed values of the gravimetric factor (G) as a function of the observing stations' distance from large bodies of water.

between the value of G for the oceanic model and that observed by stations near large bodies of water may be fortuitous, but the result does indicate that differences in observed values of G with respect to the theoretical values are primarily due to the effects of the ocean and only secondarily to differences in the thickness of the crust.

CONCLUSIONS

The major results arising from this study are :

1. The effect of a solid inner core on the characteristic numbers is negligible for reasonable values of the rigidity of the outer core.

2. There are slight variations in the Love numbers for the two density models used, and practically no variation between velocity models. This indicates that the presence or absence of a low velocity channel in the upper mantle has no effect on the value of the characteristic numbers.

3. The assumption of an ocean completely covering the earth results in theoretical values for the Love numbers which are closer to the observed values than for the same model without the ocean. This indicates that the presence of large bodies of water near the observing station will have an effect on the Love numbers and therefore on the gravimetric and diminishing factors. Figure 5 shows this relationship for the existing data.

4. The thickness of the crust also has effect upon the Love numbers, but it is small compared to the effect of the ocean.

Finally, two important matters still remain to be resolved. They are a calculation of the effect of the ocean upon the earth tides as observed at a coastal station and the effect of discontinuities in the local geological structure upon the observed values of the Love numbers.

ACKNOWLEDGMENTS

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TABLE 1

VALUES OF G = 1 + *h* — 3/2 *k* FROM VARIOUS OBSERVATIONS (INCLUDING IGY)

No.	Author	Year	Locality	G		Distance from nearest large body of water (km)
				Semi-Diurnal M_2	Diurnal O_1	
1	Harrison, Ness Longman, Forbes, Kraut, Slichter (1963) [5]	1956-53	Honolulu, Hawaiian Islands 21°18'N 157°49'W	1.190 ± 0.009	1.181 ± 0.054	1
2	as above		Glendora, California 34°10'N 117°49'W	1.184 ± 0.013	1.221 ± 0.054	55
3	»		Wake Island 19°18'N 166°39'E	1.174 ± 0.0061	1.315 ± 0.044	5
4	»		Manila Obs. 16°25'N 120°35'E	1.203 ± 0.0028	1.205 ± 0.028	2
5	»		Saigon 10°47'N 106°42'E	1.163 ± 0.0045	1.088 ± 0.08	75
6	»		New Delhi 28°38'N 77°11'E	1.153 ± 0.0073	1.132 ± 0.029	875
7	»		Bunia, Republic of the Congo 1°32'N 30°11'E	1.185 ± 0.0036	—	1
8	»		Trieste 45°42'N 13°46'E	1.178 ± 0.017	1.137 ± 0.049	2

No.	Author	Year	Locality	G Semi-Diurnal M_2	G Diurnal O_1	Distance from nearest large body of water (km)
9	»		Bidston 53°24'N 3°04'W	1.155 $\pm$ 0.012	1.118 $\pm$ 0.033	5
10	»		Bermuda 32°23'N 64°41'W	1.249 $\pm$ 0.0063	1.198 $\pm$ 0.031	1
11	»		Lwiro, Republic of the Congo 2°14'S 28°48'E	1.161 $\pm$ 0.0070	—	5
12	»		Winsford, England 53°13'N 2°31'W	1.215 $\pm$ 0.011	1.197 $\pm$ 0.020	40
13	M. Bossolasco and G. Cicconi (1961)	1960-61	Genoa, Italy 44°24'N 8°56'E	1.196	1.150	1
14	R. Lecolazet	1957-60	Strasbourg, France 43°35'N 7°47'E	1.193 $\pm$ 0.002	1.168 $\pm$ 0.004	430
15	G. Jobert	1958-59	Paris, France 48°50'N 2°20'E	1.196	1.169	150
16	P. Melchior	1958-59	Uccle, Belgium 50°48'N 4°21'E	1.186	1.183	90
17	J. Witkowski	1959	Borowiec, Poland 52°16'N 17°04'E	1.210	1.184	220
18	J.S. Dobrokhotov, et al	1958	Poulkovo, U.S.S.R. 59°46'N 30°19'E	1.238 $\pm$ 0.017	1.180 $\pm$ 0.008	20
19	N.N. Pariisky, et al	1958-59	Kransnaya, U.S.S.R. Pakhra 55°28'N 27°19'E	1.191 $\pm$ 0.026	1.160 $\pm$ 0.014	610
20	N.N. Pariisky, et al	1958-59	Alma-Ata, U.S.S.R. 43°11'N 76°58'E	1.135 $\pm$ 0.005	1.143 $\pm$ 0.012	2100

No.	Author	Year	Locality	G Semi-Diurnal M_2	G Diurnal O_1	Distance from nearest large body of water (km)
21	N.N. Pariisky	1959-60	Tashkent, U.S.S.R. 41°20'N 69°18'E	1.137 ± 0.004	1.148 ± 0.003	1440
22	N.N. Pariisky	1959	Langchou, China 36°03'N 103°50'E	1.1483 ± 0.012	1.1387 ± 0.012	1400
23	Geographical Survey Ins.	1957-58	Chiba, Japan 35°38'N 140°38'E	1.143	1.137	50
	* Geographical Survey Ins.	1958	Kanozan, Japan 35°15'N 139°58'E	1.449*	1.174*	—
24	G. Fieldler and J. Perez	1958-60	Caracas, Venezuela 10°30'S 66°55'W	1.174	1.139	15
25	Nakagawa, I.	1959-60	Kyoto, Japan 35°01.8'N 135°47.2'E	1.138 ± 0.005*		70

* excluded
* corrected for ocean tidal load
No. 13-25 are extracted from Nakagawa [9]

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