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ENGINEERING GRAPHICS SEMINAR
FOUR-DIMENSIONAL DESCRIPTIVE GEOMETRY
ROTATION - DESCRIPTIVE SOLUTION

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Abstract

The paper presents the solution, by descriptive geometry methods, of some of the problems in rotation. Related problems, such as the determination of the point of intersection of a line and a 3-D space, are also discussed.

1.

ENUNCIATION OF THE PROBLEMS

In two previous papers we make some general observations on the nature of some of the problems whose descriptive solutions are presented in this paper. These two works are our "Descriptive Geometry of Four Dimensions - Appendix II"¹⁾ and our "Descriptive Geometry of Four Dimensions"²⁾.

ROTATION ABOUT A POINT

One of the problems proposes:

To rotate a point (a) of the 3-D space Γ , about the point (o) of the 3-D space Λ , until it belongs to Λ .

In the papers above mentioned we stated that the problem will always admit a solution if the point (o) belongs to the plane of intersection of the 3-D spaces Γ and Λ . If the point (o) is not on this plane, the problem may not be possible. The first assumption and its geometric solution is shown in figure 1.

1) Copies may be obtained from the Department of Graphics and Engineering Drawing, Princeton University, Princeton, New Jersey.

2) Engineering Graphics Seminar, Technical Seminar Series, Report No. 9, Department of Graphics and Engineering Drawing, Princeton University, December 5, 1963.

2.

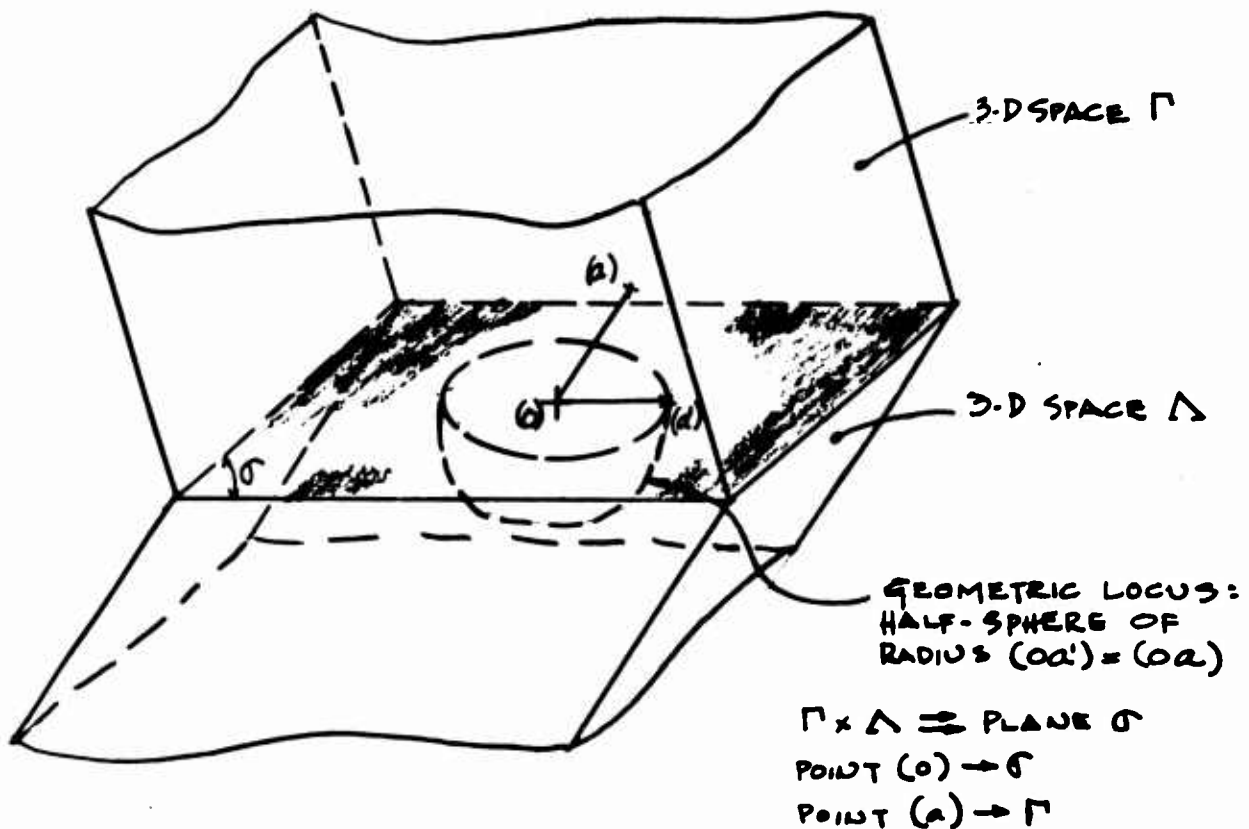


Figure 1

In figure 2 we indicate the geometric solution for the second assumption, indicating a case of possibility. The no-solution case cannot be discussed at this time since it still depends on concepts of infinity as yet to be made through studies in four-dimensional projective geometry.

3.

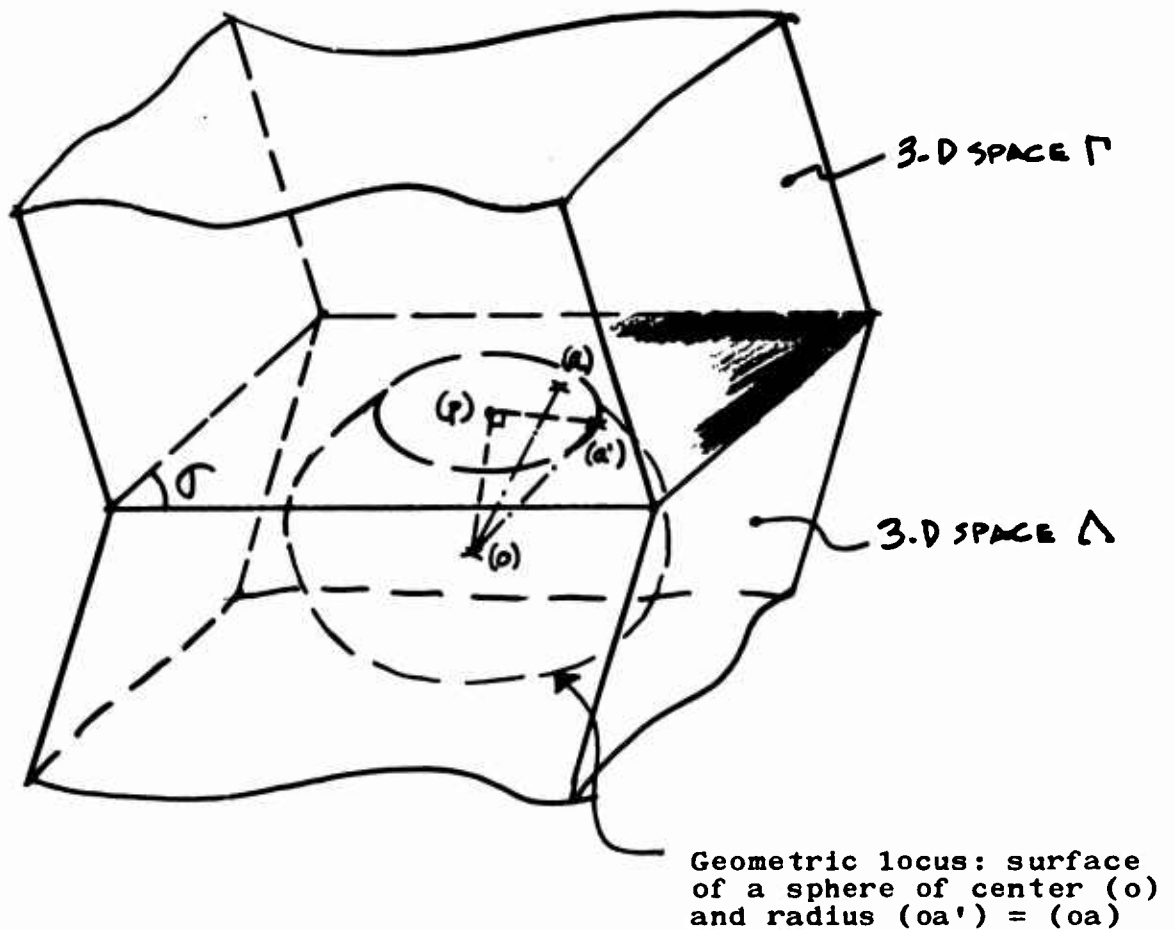


Figure 2

The descriptive solution of these two problems, follows.

Let the 3-D space Γ be parallel to one of the 3-D spaces of the system of reference. (If it is not, by applying

4.

change of space of projection³⁾ we may make that space occupy this convenient position). The plane σ , intersection of Γ and Λ is determined as shown in figure 3.

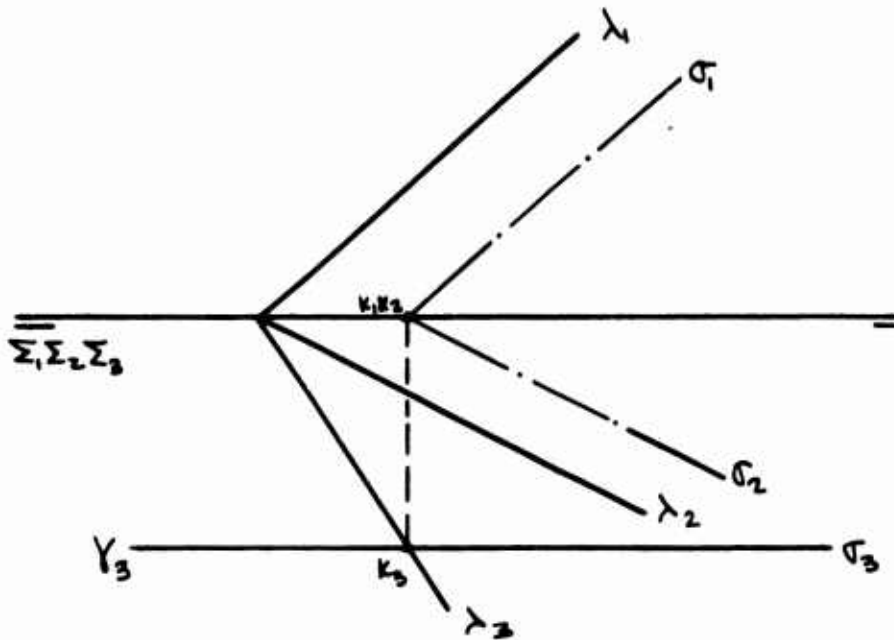


Figure 3

In figure 4 we show the two given points, (o) of plane σ and (a) of the 3-D space Γ . The first step to be taken is the determination of the true length of the distance (oa). This is also shown in figure 4, where the descriptive method of rotation is applied.

³⁾ See notes on "Methods of the Descriptive Geometry" in our "Descriptive Geometry of Four Dimensions", Technical Seminar Series, Report No. 9.

In figure 5 we indicate how to obtain the circumference of radius $(oa') = (oa)$ in the plane σ . Points of this circumference satisfy the condition of the problem. The method used is one of the three-dimensional descriptive

6.

geometry (rotation of the plane σ about its line σ_q until it becomes planar with the plane Π_q of the system of reference). Only the plane σ is shown in figure 5.

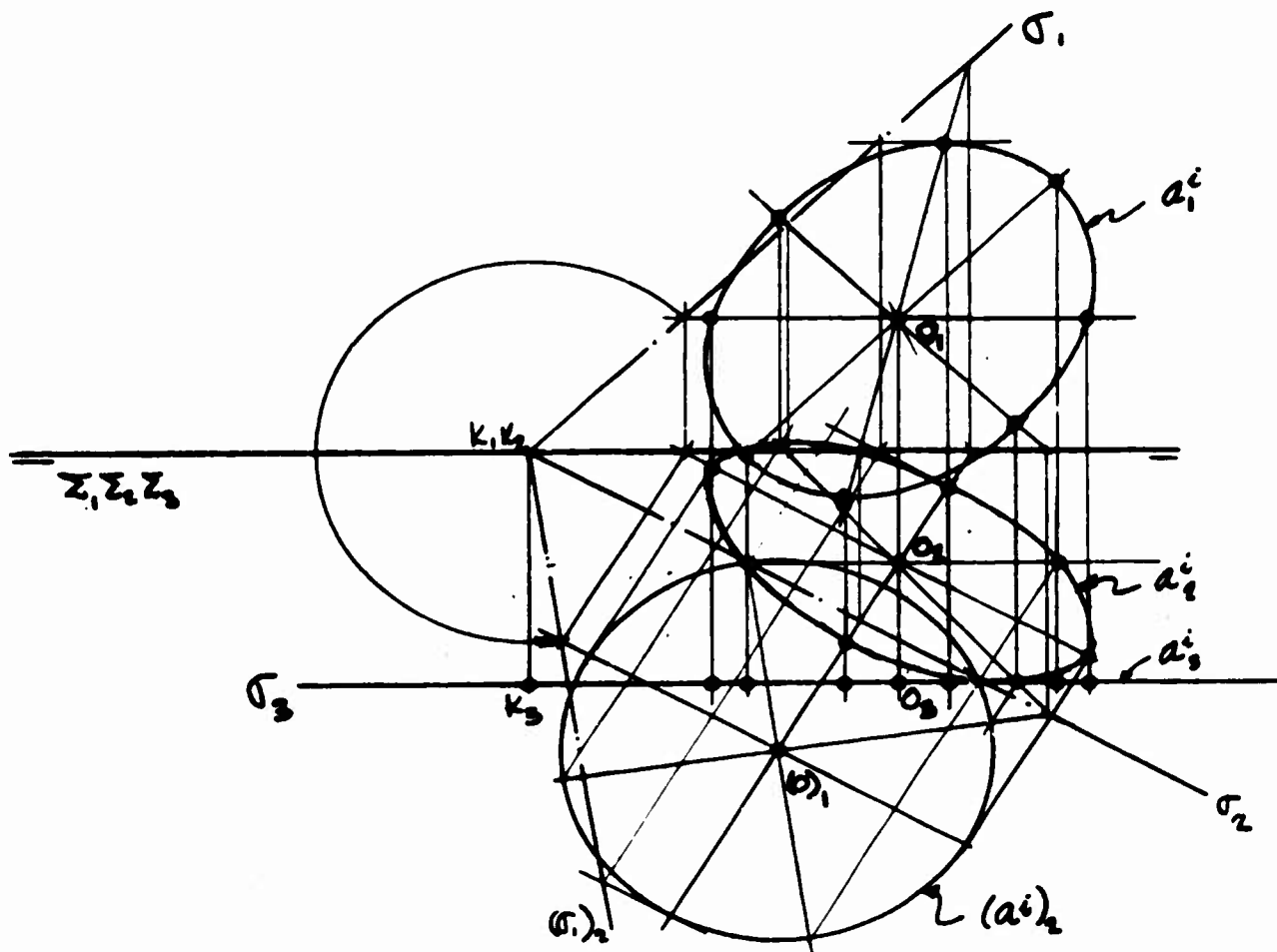


Figure 5

In the second problem, let α be a plane of the 3-D space Λ belonging to the given point (o). The plane α , shown in figure 6 is parallel to the plane of intersection

7.

of the 3-D spaces Γ and Δ . Let (a) be the given point, in the 3-D space Γ .

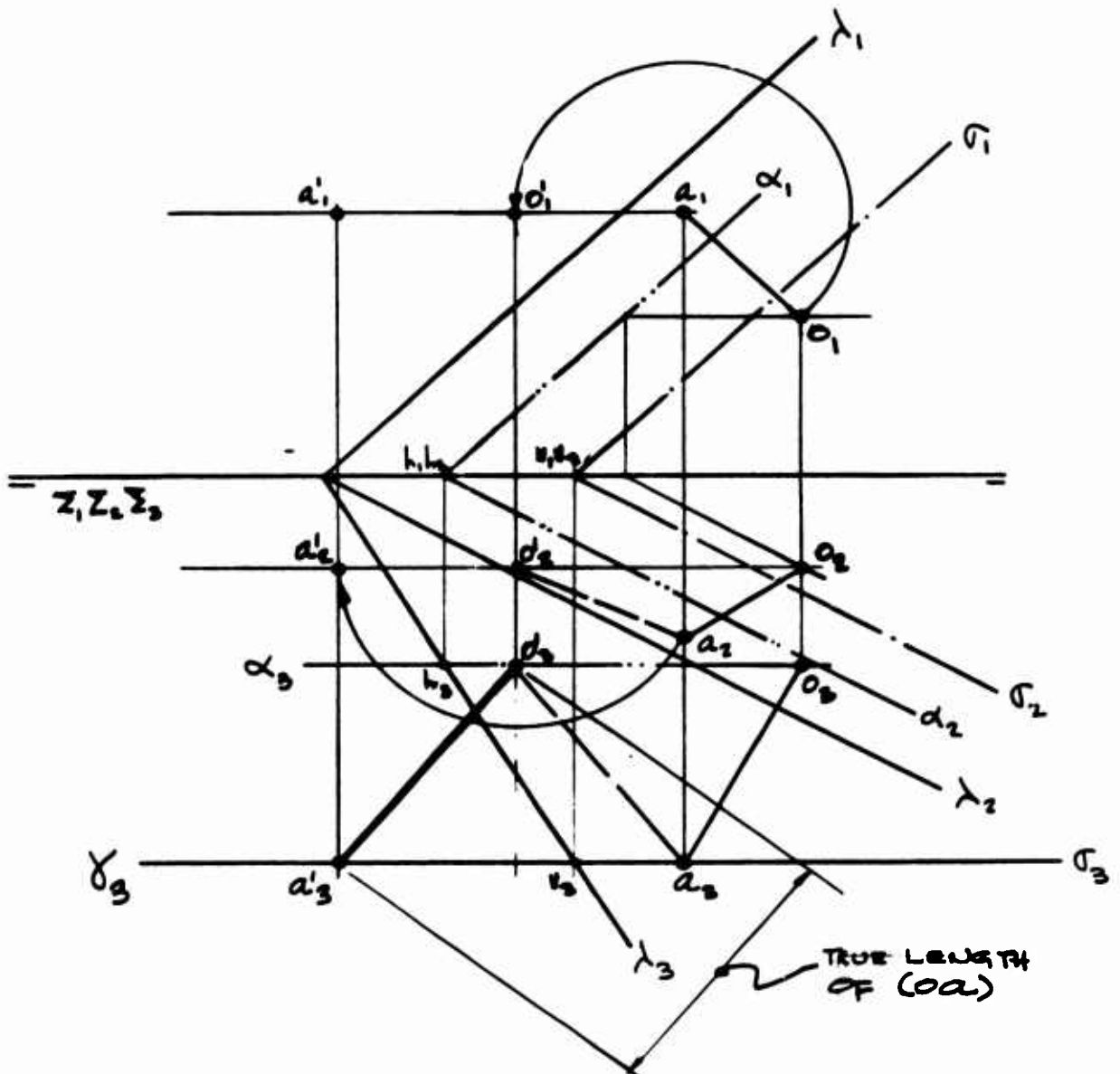


Figure 6

In figure 6 we also determine the true length of (oa).

In order to determine the geometric locus of points in the plane $\mathcal{G}$ satisfying the condition of the problem, we will have to determine first, the foot (p) of the perpendicular through (o) to $\mathcal{G}$ and second, the radius (pa') of the circumference which is such a geometric locus. The point (p) is the center of the circumference.

Because the plane $\mathcal{G}$ belongs to a 3-D space Γ parallel to a 3-D space of reference and so does the point (o), in a 3-D space Λ parallel to the same 3-D space of reference, we may obtain two projections of the point (p) by operating only with two of the projections of indexes 1 and 2 (for our particular case, for Γ and Λ are parallel to the 3-D space of reference Σ_3). In this case, the methods of the three-dimensional descriptive geometry can be applied, both in the construction of the perpendicular and in the determination of its foot. This is shown in figure 7.

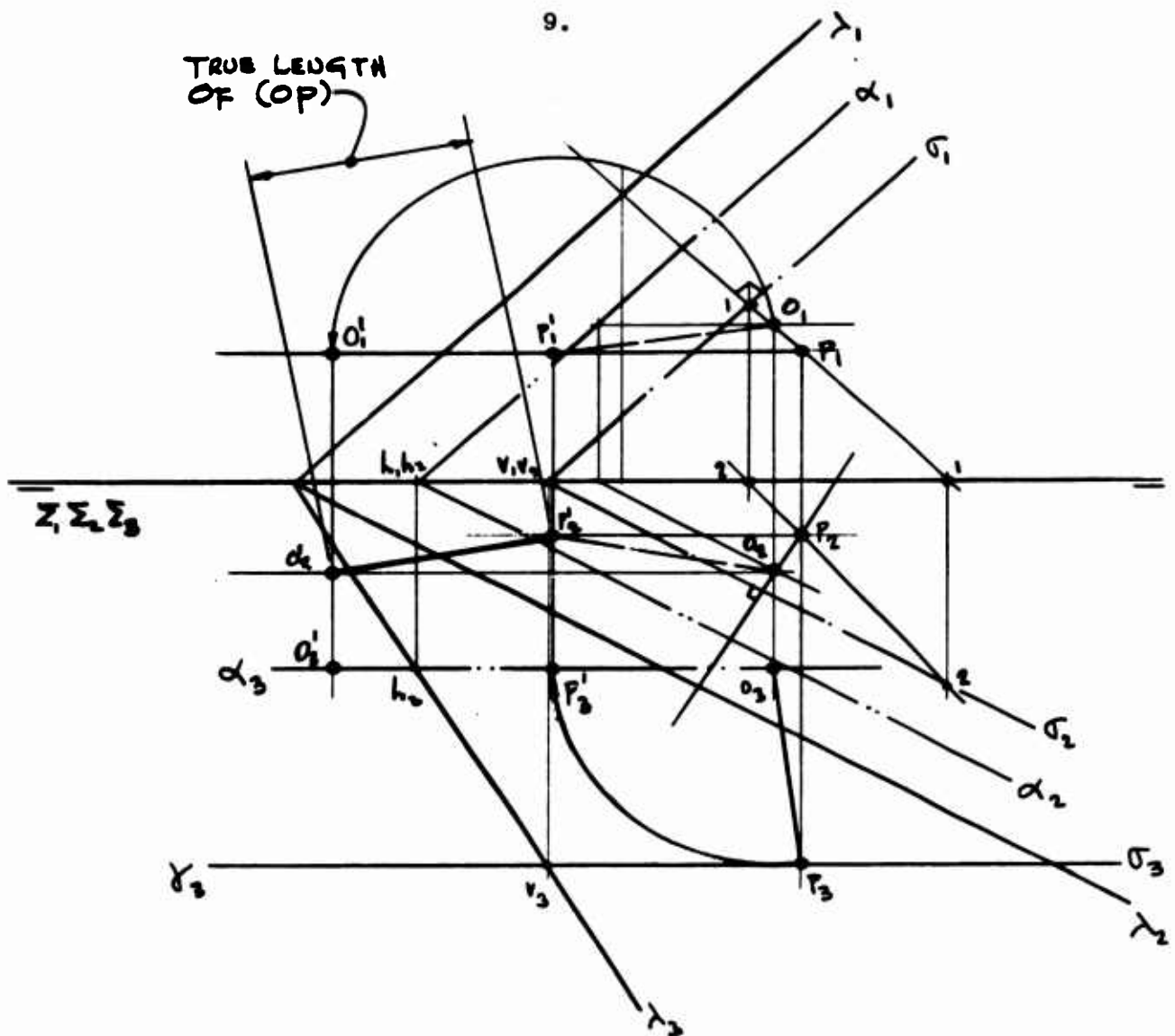


Figure 7

Since we will also need the true length of (op), we obtain this length (shown in figure 7), by taking into account all three projections of points (o) and (p).

10.

The determination of the radius (pa') can be now made, observing that (oa') is the hypotenuse of the right triangle (opa') . See figure 8.

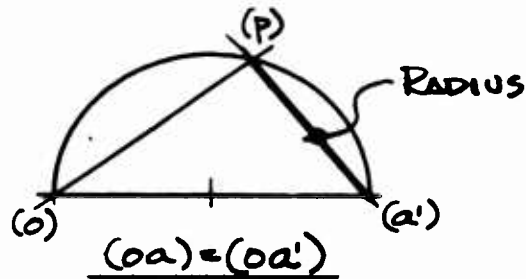


Figure 8

The determination of the projections of the circumference can now be made with constructions similar to those outlined in the making of figure 5.

The general enunciation of another problem in rotation is as follows.

To rotate a point (a) of the 3-D space Γ about the point (o) of the 3-D space Λ , until it belongs to the 3-D space χ .

11.

Let these three 3-D spaces be those shown in figure 9.
The three planes of intersection are:

$\Gamma \times \Lambda \equiv \text{PLANE } \sigma (\sigma_1, \sigma_2, \sigma_3)$

$\Gamma \times X \equiv \text{PLANE } \eta (\eta_1, \eta_2, \eta_3)$

$\Delta \times X \equiv \text{PLANE } \alpha (\alpha_1, \alpha_2, \alpha_3)$

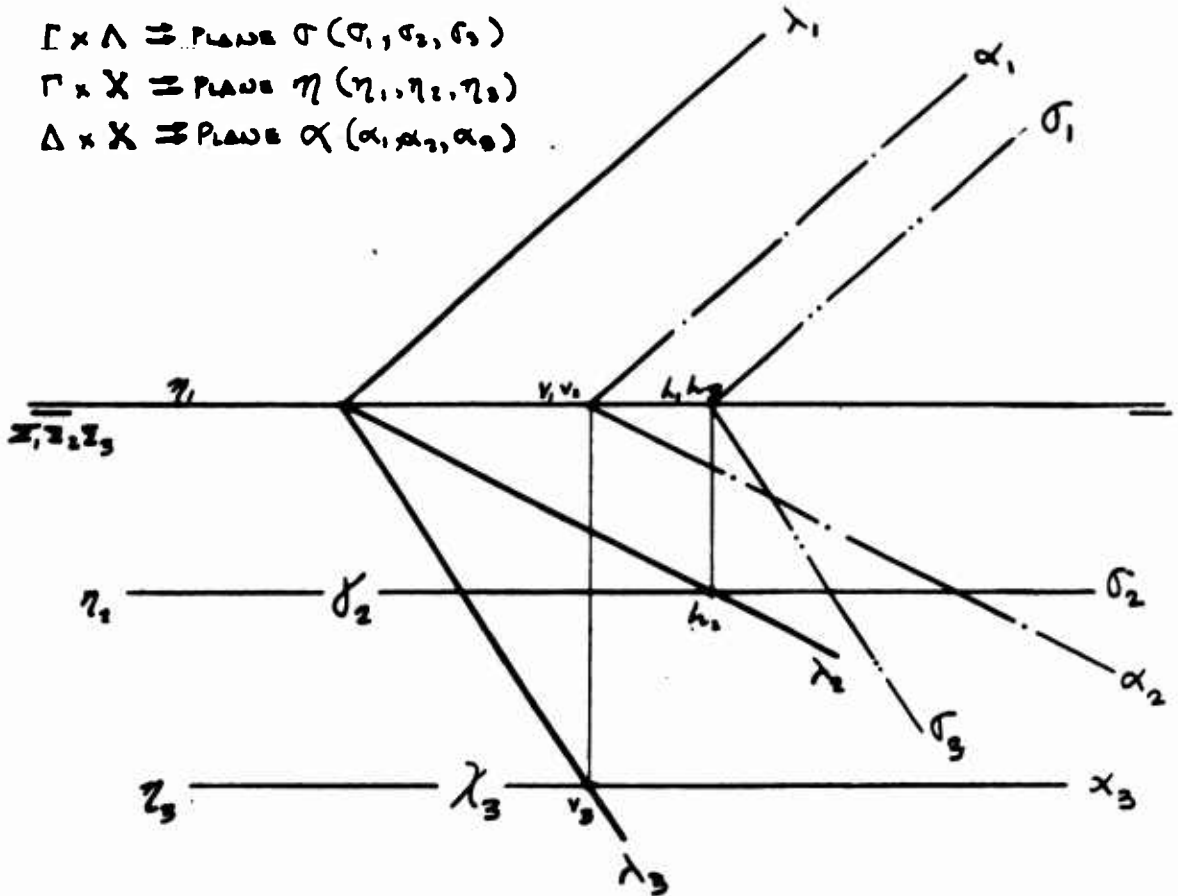


Figure 9

A pictorial representation is indicated in figure 10.
It should be kept in mind that the relationship among the
various elements cannot be analyzed and understood on the
basis of our experiences with three-dimensional relations.
Consult the references mentioned for notes on this matter.

12.

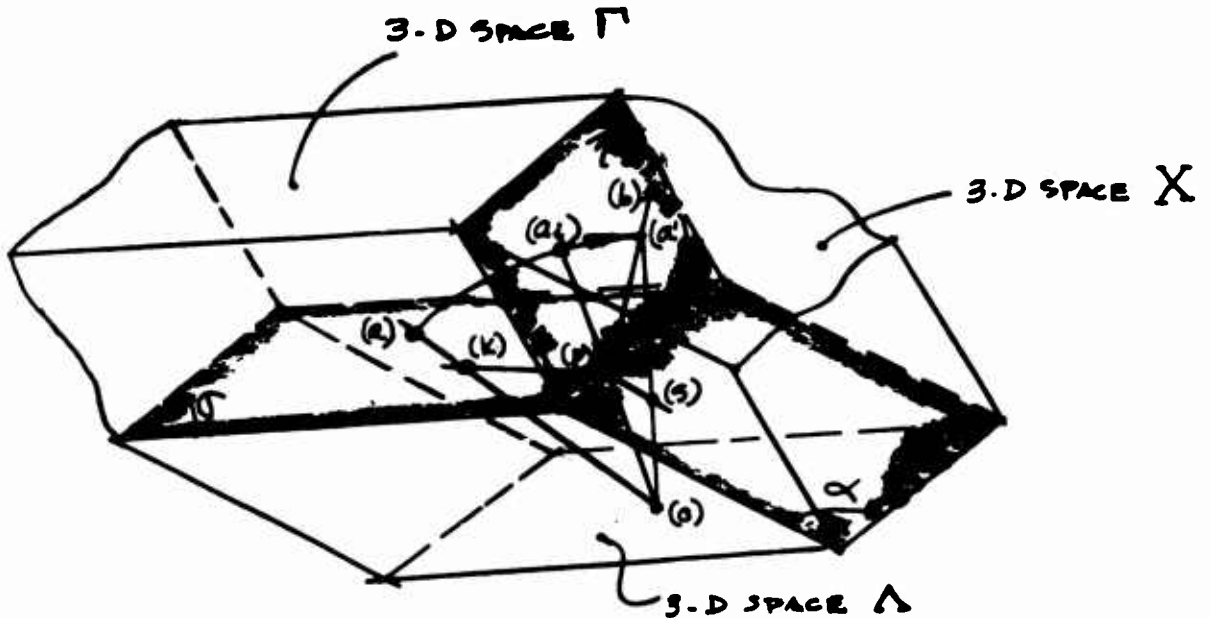


Figure 10

As indicated in figure 10, the point (a) is rotated about point (o) until it belongs to the plane η .

One solution may be obtained by imposing condition of belonging between point (a') - the solution sought - and a plane (apb) or (kpb) , selected "a priori".

We can also determine the geometric locus of the points (a_i) when in the plane η . That will be a circumference of radius (ma_i) where (m) is the foot of the perpendicular through (o) to plane η . (See figure 11).

13.

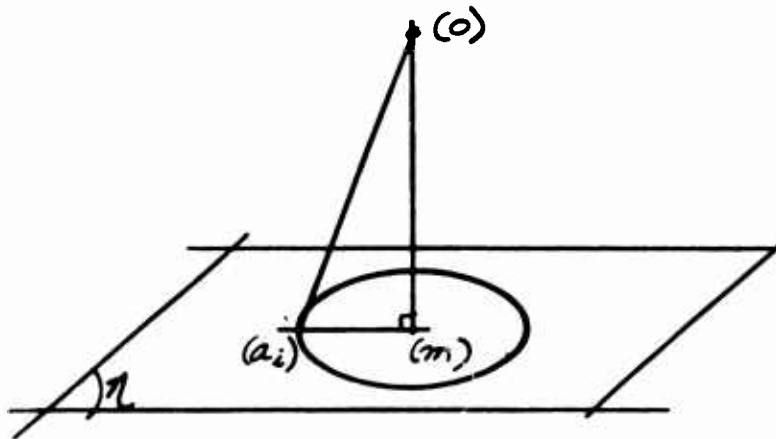


Figure 11

By determining the true lengths of (oa) and (om) we can determine the true length of (ma_1) , and operating in the plane η , obtain the projections of the geometric locus.

In figure 12 we show the points (o) and (a) , and the determination of the true length of (oa) .

14.

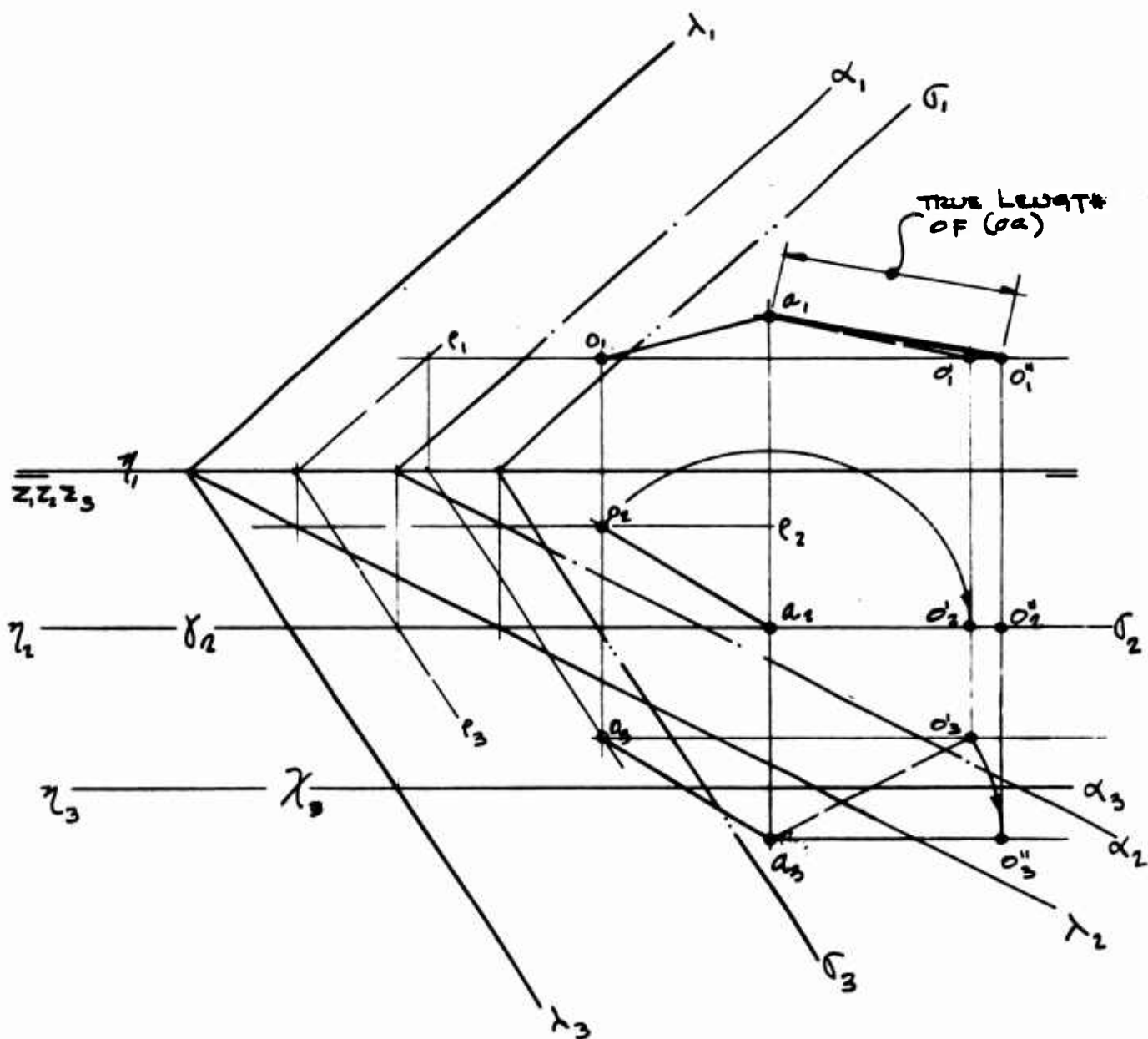


Figure 12

15.

In figure 13 we isolated the point (o) and the plane η , in order to facilitate the reading of the drawing, showing the perpendicular through (o) to η and its foot, point (m).

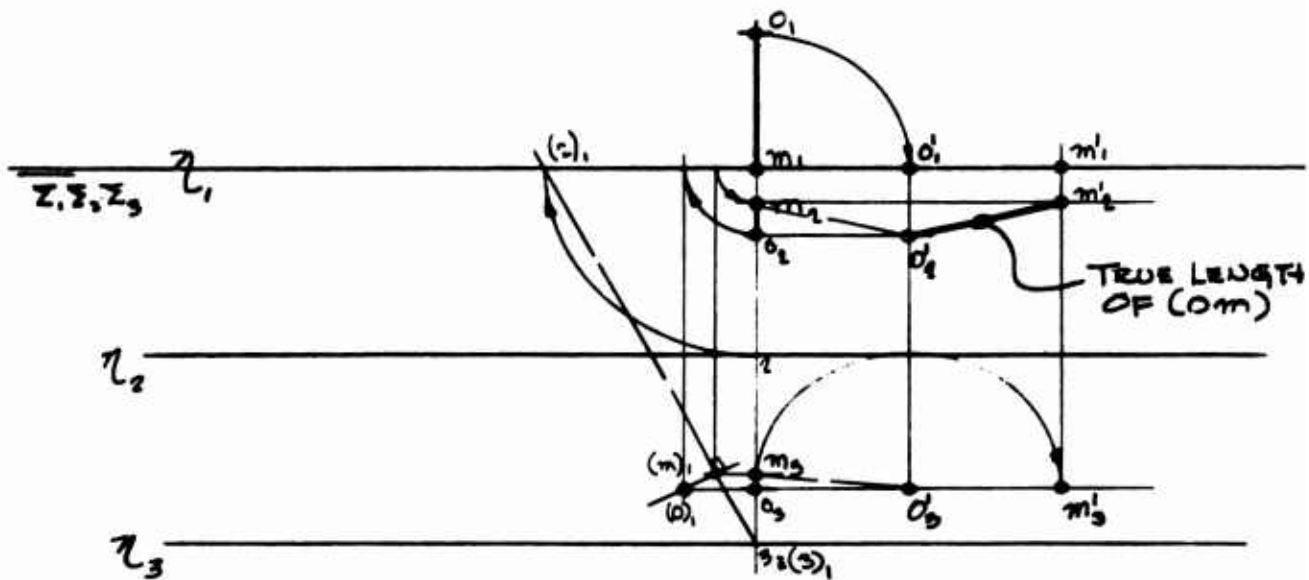


Figure 13

16.

In figure 14 we show the determination of the radius (ma_1) .



Figure 14

In figure 15 we show the construction of the circumference in the plane η . The method used is that of the three-dimensional descriptive geometry.

the line in relation to the elements of the system of reference.

As it can be observed the procedure is divided in two steps: first we make a rotation as if the point (a) belongs to one of the 3-D spaces to which the line (xy) is perpendicular; next we rotate the point obtained through the first rotation, as if it belongs to the second 3-D space to which (xy) is perpendicular. The problem of rotation thus presented

19.

is characterized by two angles of rotation and two radii. They correspond to two spheres which are sections of the hyper-sphere generated by the rotation of the point.

This procedure can be better appreciated in the rotation of the line. (Figure 17). The determination of the true length of a segment is made by use of this procedure.

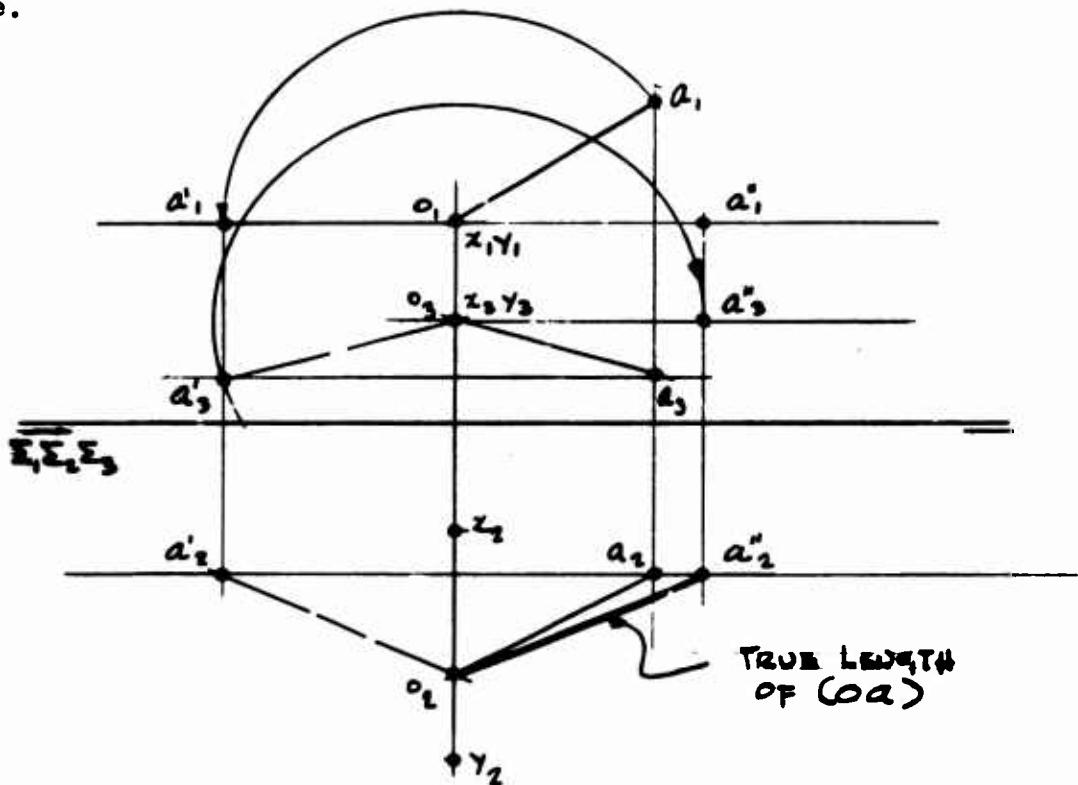


Figure 17

20.

The rotation of a plane or of a 3-D space may be discussed by rotating three points of the plane or four points of the 3-D space.

When the 3-D space is given by its traces, the problem may be solved as follows.

First, we should determine the intersection of the axis of rotation with the 3-D space. This will permit the consideration of only three other points of the 3-D space, since the point of intersection remains in the same position, and being a point of the 3-D space, enable us to re-determine the traces of the 3-D space in its new position.

Figure 18 shows the necessary constructions. It should be noticed that these are very similar to the problem of rotation of a plane, given by its traces, in three-dimensional descriptive geometry.

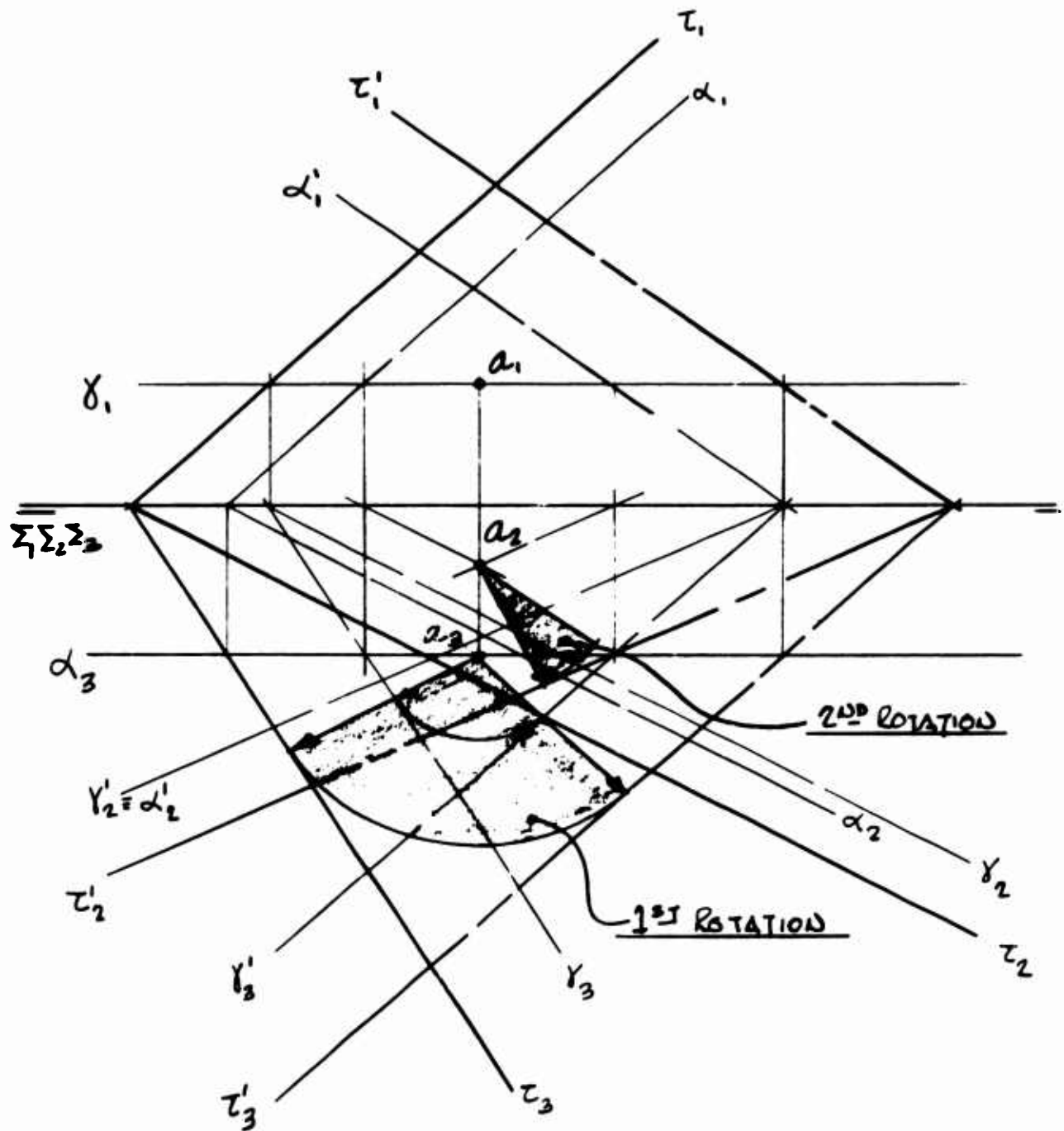


Figure 18

Indicated in Figure 18 are the constructions relative to the first rotation only. The radius, for the second rotation, with center in a_2 is shown. The constructions are the same as for the first rotation and have not been completed in order to avoid over-crowding of the drawing. The radius, for this second rotation is, necessarily, perpendicular to $\mathcal{U}_2'$.

ROTATION ABOUT A PLANE

Let α be a plane of the 3-D space $\mathcal{T}$. (Figure 19). Consider the point (p) of the line (mn), belonging to the plane, and draw perpendiculars through (p), to the plane α and to the 3-D space $\mathcal{T}$. The two perpendiculars are (po) and (pr).

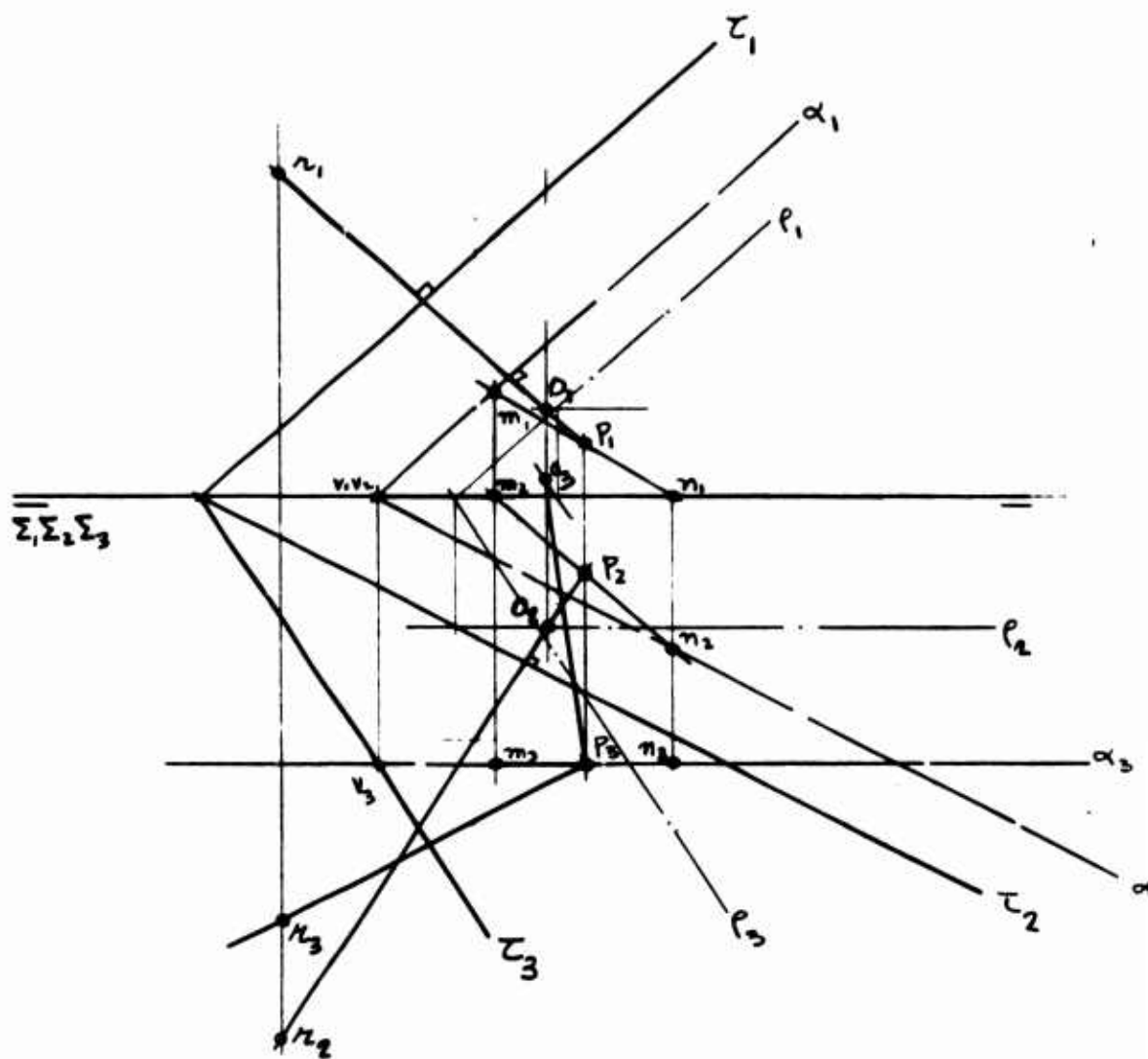


Figure 19

The plane $(po - pr) \equiv \gamma$ is absolutely perpendicular to α . Thus, $\alpha \not\perp \gamma$, $\alpha \times \gamma \Rightarrow \text{POINT}(p)$.

24.

The rotation of a point about a plane is made under the conditions above described: the plane - "axis" of rotation is absolutely perpendicular to a plane belonging to the point.

As analyzed in the literature previously referred to, the radius of rotation is the distance from the point to the point of intersection of the two planes. In figure 19, such a point is (p). Therefore, the procedure to be followed is the same as that presented in the study of rotation about a point.