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RESEARCH IN AIRCRAFT STRUCTURES ANALYSIS AND DESIGN

FINAL REPORT

By

W. H. Horton
J. Mayers

September 1965

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

CONTRACT DA 44-177-AMC-115(T)

DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS
STANFORD UNIVERSITY

DEPARTMENT OF DEFENSE

DEPARTMENT OF THE ARMY
U S ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA 23604

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Prepared by

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for

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FORT EUSTIS, VIRGINIA

SUMMARY

This paper presents a condensation of a wide range of research efforts carried out at Stanford University during the period November 1963 - February 1965 on contract DA 44-177-AMC-115(T). In so doing, it outlines new approaches to the experimental study of the instability of shell bodies and provides fresh analyses in this field. It abstracts from recent extensions to the maximum strength analysis for flat plates and gives an outline of a generalization in the theory of buckling of sandwich plates. It refers to research efforts on the stability of conical shells and indicates progress in studies of the influence of higher order linear and nonlinear effects on the lateral vibrations of solid and sandwich beams.

CONTENTS

ILLUSTRATIONS

SYMBOLS

a	panel dimension, x direction	inch
b	panel dimension, y direction	inch
c	core thickness	inch
D_t	$= 2D_f + D_s$	lb. inch
D_f	$= E_f t^3 / 12(1 - \nu^2)$	lb. inch
D_s	$= E_f (c + t)^2 t/2(1 - \nu^2)$	lb. inch
e	unit shortening applied to plate	
e_{cr}	buckling strain or shortening	
E	Young's modulus of elasticity	
E_f	face extensional modulus	psi
G_x, G_y	$= G_{xz}$, G_{yz} core shear moduli	psi
k_n	$= \frac{2b^2 N_x}{\pi^2 D_t}$ plate buckling coefficient	
n	= 1, 2, 3, ...	

N_x	in-plane force	lb./inch
P_{cr}	critical compressive load	lb.
R	mean radius of circular cylindrical shell	inch
t	uniform wall thickness of shell	inch
x, y	coordinate axes	
ϵ	end shortening parameter	
ν	Poisson's ratio = 0.3	
σ	mean axial compressive stress	psi
σ_{av}	average compressive stress	
σ_{cy}	compressive yield stress of material	
σ_{max}	maximum average compressive stress	
φ_y	angle of rotation	

Subscript:

cl	classical value (corresponding to an infinitely long or square plate)

INTRODUCTION

A well-designed primary aerospace structure must possess mechanical strength sufficient to sustain a given system of external forces without fracturing or crushing of the material, and, in addition, it must be sufficiently stable to sustain the applied forces without excessive alteration in the geometric form in which it has been designed.

In aeronautical applications these conditions must be met with efficiency, reliability, and repeatability. Low weight, long life, and simplicity of inspection and maintenance are of the greatest importance in this field of endeavor. To achieve optimum or near optimum design, we cannot depend upon repeated tests, redesigns, and continued experimental evaluation of the final product, but we must develop analytical procedures which can describe the facts of experience adequately and which can be adapted to project this experience into the realm of the unestablished. We need to devise procedures of computation which deal in a realistic way with actuality. We need true criteria of failure; we need trustworthy rules or procedures for dealing with questions of stability. These questions are the deciding factors in most of the structures we use, but only constructive analyses and empirical formulae for their solution exist.

These, then, are the general and broad requirements which the structural design engineer must face in aeronautical applications. It is around this need that contract DA 44-177-AMC-115(T) was, in essence, formulated. The prime purpose of the research was to generate a substantial understanding of some of the yet unresolved questions in the fields of strength and stability of geometric form. It is a fact, relatively easily established by review of the current state of the art, that in all except the simplest possible cases, our ability to describe precisely and unquestionably the load-carrying capability of a structural system leaves much to be desired. The discovery that analytical processes and procedures of monolithic and built-up columns, frameworks, plates, and beam-type structures were totally inadequate to describe the instability behavior of thin shells in compression posed a problem of the first significance. The general theory of buckling of thin circular cylindrical shells was developed by Lorenz (reference 1), Timoshenko (reference 2), Southwell (reference 3), and Flügge (reference 4), but the predictions of their theories and the observations of the experimentalists were almost completely at variance. The scatter that characterized all experimental research on compressed shells is so great that the value of such experimental data as a point of commencement for analytical discussion is not without question. As a result of this state of affairs, it became accepted in the early 1930's that the linear theory of the classical approach was incapable of representing the basic phenomena and a new nonlinear large deflection theory was required for the solution of the problem. Donnell (reference 5) laid the foundations of such a theory and Kármán and Tsien (reference 6) extended this to discuss possible states of equilibrium of such bodies. In the succeeding years, continued corrections, additions, and refinements to the procedures were made. An excellent resume of the state of the art at the beginning of this decade is given both in the work of Thielemann (reference 7) and also that of Fung and Sechler (reference 8). The gap between experience and prediction was not closed. Nevertheless, today, despite the obvious deficiency of the approach, the whole field of theoretical research on the stability of shell bodies is dominated by the Kármán-Donnell considerations.

It must be admitted, however, that there has been within the past several years a valuable contribution to our possible understanding made by Yoshimura (reference 9). He has demonstrated from the principles of differential geometry that circular cylindrical shells can be transformed into assemblies of plain triangles and that such transformation involves bending only along the sharp ridges which form the boundaries to such shapes. There is no stretching of the middle surface. This primarily inextensional approach has been verified by the studies made by Ponsford (reference 10), there is no improvement in the theoretical description of the actual collapsed load. It is clear that the subject still remains profitable and open. It is equally transparent that its solution is vital. Unless this step can be made for the isotropic structure, there is little hope of success in the more complex problem of the anisotropic shell such as will characterize the laminated and fiber reinforced materials, which are our prime concern.

One of the most profitable applications of the laminated and fiber reinforced materials appears to be their use in sandwich construction. The field of sandwich construction is a good example of the comprehensive process of design integration. It is a concept in which structural functioning and manufacture cannot be separated in any practical sense. As a review of the literature shows, there is need for development in both areas. At the beginning of this research project, there were many analytical treatments of the stability of sandwich plates. Each was unsatisfactory in the sense that it dealt only with a small facet of the overall problem of instability and face wrinkling. There was a clear need for an integrated theory and there still is a clear need for careful experimental study of the behavior of such structures under a variety of load and restraint conditions. There is need for development in manufacturing and inspection procedures.

In the main body of the report, a unified theory for the general instability and face wrinkling of flat sandwich plates and columns subjected to in-plane loading is presented (reference 11). The theory developed takes into account bending stiffness of the faces, continuity of the structure at the interfaces, core normal strains perpendicular to the plane of the plate and in-plane orthotropy of the core shear moduli. The results are presented as general equations with specific examples in graphical form. Of importance, aside from the governing equations, is the presentation of the compatible boundary conditions so that they may be reconciled with the design connections and supports. Buckling criteria are shown to be significantly influenced by the manner in which sandwich elements are supported.

Stability, as we emphasized in our earlier remarks, is but one facet of the question of concern. Criteria of failure are of equal import. In this region there is little by the way of analysis which concerns itself with anything but the purely elastic deformations. The state of the art of the analysis of initial plate buckling, both elastic and inelastic, and the postbuckling behavior of flat plates, as they existed at the time, were discussed by Gerard (reference 12) and Becker and Gerard (reference 13) in 1957. Myriad references were given concerned with the postbuckled load-carrying ability of flat plates. It was reported that all such analyses, with the exception of that of Mayers and Budiansky (reference 14) for square plates, were based on purely elastic considerations and, hence, left the problems of theoretical prediction of the behavior of the postbuckled plate in general and plate maximum strength unsolved. The one inelastic analysis of square plates (reference 14) provides no clearly defined maximum strength criterion and, on the basis of limiting results, overestimates empirically determined maximum strength criteria for long plates which undergo elastic buckling, and form, initially, square wave patterns.

More recently, additional postbuckling analyses applicable to plate elements have been performed by Stein (reference 15) in which, again, only elastic deformations are considered. However, Stein's studies include both theoretical and experimental data that strikingly illustrates the phenomenon of the buckle aspect ratio changing in "jumps" as loading beyond buckling progresses for plates of finite length. Unfortunately, quantitative correlation over the complete postbuckling range between the theoretical and experimental results reported in reference 14 is precluded since the theory is based on elastic analysis. It was conjectured then that the extension of the work of Mayers and Budiansky (reference 14) to include the effect of wavelength change should shed additional light on the problem of the theoretical prediction of the maximum strength of postbuckled flat plates. This subject (reference 16), then, constituted an important part of the research effort discussed in the main body of the report. As is shown, the maximum strength of plates can be predicted with great accuracy by accounting for plasticity and buckle wavelength changes.

The remarks in the previous paragraphs have tended to imply that the research efforts carried out under this contract were confined to specific details, but this is not entirely true. We were concerned with the broad question of engineering research philosophy and the practical aspect of design analysis. It was our purpose to demonstrate that when practical problems of engineering are attacked on a sound scientific basis rather than an ad hoc basis, experimental research plays a more vital role than that of checking theoretical conjecture. We operated under the premise that when valid experimental results are synthesized, sound engineering theories can be developed in areas in which there has previously been much endeavor and little accomplishment. It was recognized that the close coupling of experiment and theory is vital to the development of the science.

DISCUSSION OF RESEARCH CARRIED OUT

We implied in our introductory remarks that the state of knowledge with regard to the stability of shell bodies was most unsatisfactory. Fung and Sechler (reference 8) very clearly summarized the feeling of many when they said: "In a sense the instability of thin elastic shells remains the most challenging of all classical problems in the theory of elasticity. The wide disparity between the theoretical and the experimental results on a problem such as the buckling of cylindrical shells subjected to end compression is well known. In spite of the concreteness of the problem, the simplicity of the geometry, and the ease of qualitative experimental observation, the most elaborate and undaunted calculations on the buckling load for thin shells often leave an unacceptably large "error" between theory and experiments.

We cannot deny the concreteness of the problem, the simplicity of the geometry, the complexity of the calculations, or the magnitude of the error, but we challenge the ease of the qualitative observation. Indeed, a main emphasis of our research centered around the thought that the problem of error prediction and observation arose as much from the inadequacies of the experiments as the deficiencies of the theories.

As a consequence of this philosophy, we reviewed the test procedures of the past and decided that a new approach to the subject of testing cylindrical shells for stability was not only desirable but necessary. We recognized that the perfect cylindrical shell could not be fabricated. We realized that the first step in our understanding was to determine by test the value of the critical load most probable for a perfect body. The procedure adopted is described in detail in reference 17 and the philosophy which underlies it is outlined in reference 18. Briefly, the technique developed was to arrange the test vehicle in such a manner that it was possible under progressively increasing load to completely fill the cylindrical shell with elastic buckles. A shell so filled is shown in Figure 1. Experience proves that under these conditions the distribution of the number of buckles as a function of load can be associated with the logistic curve and that the point of maximum buckle generation can be associated with the classical critical load. The results of this experiment, extracted from reference 17, are as shown in Figures 2 and 3.

Thus, this work demonstrated that the most probable maximum load which can be sustained by a cylindrical shell in axial compression is consistent with the classical value. It is not, of course, inconsistent with the value derived from the Kármán-Donnell approach, but other factors mitigate these theories as will be discussed later. The procedure adopted in the basic work referred to in reference 17 has been extended and expanded in reference 19. Here we have been able to demonstrate the power of the approach from several viewpoints. We have verified by its use the fact that the failing stress of a cylindrical shell in compression is uninfluenced by the nature of the load distribution which causes the critical condition. At the same time we have been able to show that the statistical process adopted gives the capability of assessing in a broad manner not only the quality of representative shells but also the distribution of quality within the shells themselves. Figures 4 and 5, extracted from reference 19, illustrate clearly the results referred above. The development of the novel procedures to which we have referred brought to the forefront a secondary question of some importance. The question of whether or not plastic flow played any part in the buckling process or whether it was solely confined to the postbuckling behavior. The fact, however, that to maintain a cylindrical shell in the elastic condition under buckling calls for a restriction in the depth to which the buckle is permitted to develop to a dimension of the same order as the thickness is a clear indication that nonlinearities must be included in the Kármán-Donnell type of instability analysis. These conjectures based upon careful experimentation led the way for Mayers and Rehfield to undertake further analytical research on the instability of shell bodies. Their paper (reference 20), "Further Nonlinear Considerations in the Postbuckling of Axially Compressed Circular Cylindrical Shells," was undertaken to establish (1) the limitations on the validity of the Karman-Donnell theory and (2) the effect, relative to results based on application of the Kármán-Donnell theory, of utilizing a relatively more accurate formulation of the extensional strain-displacement relations. Significant limitations of the Kármán-Donnell theory when it is applied to shells of practical interest (say, $100 < R3t < 3000$) are shown to be related to the size of the postbuckling deformations, the magnitude of the elastic strains (or stresses) and, paradoxically, a mathematical sensitivity of the problem analysis to the use of periodic functions for describing the deformation state. The effect of the improved strain-displacement relations is shown to be manifested in the generation of load-shortening curves dependent upon cylinder R/t ratio. The curves are the first to be obtained for infinitely long cylinders which are assumed to buckle into periodic wave forms.

The analysis is carried out on the basis of a variational technique in conjunction with computations performed on a high-speed computing machine. The variational principle utilized is that of Reissner and appears to be the first application of this principle to a postbuckling problem.

On the basis of the results obtained, it is concluded that the Kármán-Donnell elastic theory, number of degrees of freedom assumed in the deformation functions notwithstanding, cannot under any conditions, except perhaps coincidentally for a specific R/t value, describe the postbuckling behavior of axially compressed thin shells in the practical range of structural design interest. In fact, the converged solution of the Kármán-Donnell elastic theory apparently applies to a cylinder of infinite R/t ratio corresponding to a vanishing value of applied stress. Since the R/t dependence of the load-shortening curves obtained, although finite, is small, it is further concluded that little improvement toward a satisfactory solution of the problem can be gained from any theory in which the strain-displacement formulation does not take cognizance of the size of the rotations to be expected in a realistic situation and the physical law between stress and strain does not account for inelastic behavior. Finally, with reference to both the inelastic problem and the fact that practical shells reflect finite length as well as a local rather than overall buckle pattern, it may be stated that the final successful analysis of the axially compressed thin shell problem must include the presence of prebuckling deformations due to either initial imperfections or the nature of the end conditions or a combination of both. The main numerical results of the analysis are included in Figures 6 and 7.

Concurrent with these researches into the stability of shell bodies, an investigation was carried out on the general instability and face wrinkling of flat sandwich plates and columns subjected to in-plane loading (reference 11).

The governing differential equations and associated boundary conditions, applicable to establishing the interrelated buckling modes of both sandwich plates and columns, are developed utilizing a variational procedure. An integral expression, whence the above equations and boundary conditions have been derived, is also presented. For a design problem, the direct variational approach reflecting nonclassical boundary conditions should prove a useful tool for obtaining satisfactory approximate solutions. While other investigations have treated portions of this problem, it is believed that this is the first analysis wherein integral and differential formulations are presented together with a complete statement of boundary conditions in a truly plate-type analysis.

The inclusion of two distinct in-plane core shear moduli in the plate problem reveals that, for structures in which general instability is of primary consideration, equality of the moduli leads to the most efficient design, whereas for face wrinkling, unequal moduli, with the greater one oriented with the loading direction, are preferable. It appears, however, that the slight gains possible by this orthotropy are overshadowed by the significance of the core extensional modulus in a direction perpendicular to the faces. It would appear that there should be less emphasis on off-the-shelf cores and more emphasis on the creation of tailor-made cores to suit the individual design requirement.

The primary intent of this work was to obtain a better understanding of the behavior of sandwich panels rather than just to construct another set of charts of system parameters which, when extended to certain limiting cases, would give the proper agreement with already known classical theories. This intent did lead to the formulation of a representative practical design chart, which can be readily extended to encompass a reasonable range of system parameters. In similar fashion, solutions applicable to other boundary conditions may be obtained and design charts constructed. These will give the designer a direct method to evaluate the necessary trade-offs for a given design problem; however, much more theoretical and experimental work is needed before the potentially attractive composite structure can effectively compete with sheet and stringer construction. It should be remembered that accelerated aircraft development during World War II amplified and justified the need for extensive knowledge of efficient sheet and stringer design. Until items are manufactured to provide the impetus in composite construction design, there will not be the competitive urge to improve fabrication procedures, inspection techniques, handling and storage methods, and, last but not least, reliable test methods for the determination of such items as elastic moduli. This last point is brought out in reference 21 where it is shown that certain test results, which show the greatest consistence and logically the greatest reliability, do so only because they are fundamentally inadequate.

Prior face wrinkling theories for columns have, by neglecting certain degrees of freedom, injected an additional mode termed as shear instability. The unified approach shows this to be an artificial lower bound with, of course, an attendant weight penalty. In addition, the unified theory treats face wrinkling of plates rather than merely columns and should prove useful in further studies which include the effect of initial waviness. This type of imperfection is due to manufacturing limitations; it most certainly is two dimensional, and this is beyond the range of any column analysis.

The plate theory developed herein has been applied to the development of stability criteria for initially flat sandwich plates with boundary conditions corresponding to simply supported faces and fully supported cores. This type of boundary condition is shown to be only one of eight possible types that fit within the designation of "simple support". Attempts to correlate the theoretical buckling criteria with test data (reference 22) obtained from uniaxial compression tests on all "simply supported" fiberglass and fiberglass face - aluminum honeycomb core sandwich plates have been unsuccessful, since (1) the test panels reflected initial deviations from flatness and (2) the test boundary conditions did not correspond to any of the eight possible types of simple support due to the presence of various degrees of rotational restraint on all edges and relative displacement of the plate unloaded edges. Of great import, nevertheless, are the joint findings of reference 22 and the present investigation which indicate that substantial developing and testing are required to achieve and establish realistic sandwich plate behavior for various edge conditions ranging from simply supported to clamped and combinations thereof, and that the present unified theory, when applied to specific boundary condition solutions, can provide the medium for achieving ultimate correlation with valid experiments. Finally, since buckling of plate structures is not necessarily tantamount to failure, the present analysis can be extended to encompass nonlinear effects which lead to satisfactory prediction of postbuckling behavior and failure. A basic approach to the determination of postbuckling behavior and ultimate strength of plates has been developed in reference 23. The main numerical results are presented in Figures 8 and 9.

Although instability is one of the most important questions from the aerospace point of view, the maximum strength of a component is of considerable importance and significance. Analysis of structural elements from this point of view can be used to advantage in preliminary structural design without invoking time-consuming and costly experimental programs. At the same time such analysis is also needed to enable us to project the behavior of newly introduced and attractive structural elements such as beryllium. Mayers, Nelson, and Smith (reference 16) therefore undertook a new analysis of the postbuckling behavior of flat simply supported rectangular plates with straight unloaded edges uniaxially compressed beyond elastic buckling into the plastic range. The incorporation of wavelength effects is seen to lead to clearly defined maximum loads, a phenomenon that could not be established in a previous inelastic postbuckling analysis restricted to square plates. The present analysis yields excellent correlation with the postbuckling behavior and empirically developed maximum strength criteria obtained from the results of various independent investigations conducted with panels of various relatively ductile materials supported between intermediate axial rows of knife edges.

The results of the analysis firmly establish the upper bound to the load-carrying ability of axially compressed, simply supported plates with straight unloaded edges (analog to stiffened-panel plate element) to be that of the square configuration and suggest, for optimum design considerations, that stabilizing elements be considered which both force the square wave pattern (initial buckling mode for long plates) to persist during postbuckling and insure that deflections out of the plate are precluded along nodal lines.

The plate analysis, when applied in conjunction with an appropriate stress-strain curve for beryllium, offers a plausible explanation for the somewhat anomalous results obtained with several beryllium plates considered to be simply supported. This particular instance points out the need for the development of adequate theory and analysis techniques applicable to the prediction of failure criteria in basic structural elements and the concomitant establishment of experimental results for such structures constructed of new and attractive materials which together can lead to early utilization of such materials in efficient designs. The accepted design criteria for plates and shells (both solid and sandwich types) in compression, for example, are well known to be significantly conservative with respect to the potential performance of these structures as established by analysis and experiment which meticulously account for the effects of, for example, boundary conditions, imperfections, and finite length. Some of the more important results of this study are presented graphically in Figures 10, 11, and 12.

As a prelude to further developments in shell theory and the behavior of sandwich-type construction, research efforts were also made on the buckling of clamped conical shells under external pressure (reference 23) and on the "Combined Influence of Higher-Order Linear Effects and Nonlinear Effects on the Lateral Vibration Behavior of Solid and Sandwich Beams" (reference 24).

The first of these papers is based on solution of modified Donnell-type stability equations in the presence of slightly relaxed boundary conditions for the middle surface displacements. It is then shown that the elastic restraints implied by the proposed solution closely approximate the boundary conditions of the perfectly clamped shell. Numerical results covering a wide range of geometries are obtained for uniform hydrostatic pressure loading and are compared with those for corresponding simply supported shells. Correlation with equivalent cylindrical shells then yields a simple approximate analysis.

The second paper presents the results of a study of the combined influence of higher order linear effects and nonlinear effects on the lateral vibration behavior of solid and sandwich beams in the presence of axial load. The higher order linear effects considered are transverse shear and rotatory inertia. The nonlinear effects considered are those which arise due to stretching of the longitudinal fibers of the beam during vibration. This stretching results from the practical consideration of variable axial boundary restraint on an internal member in a structural assembly. The predicted vibration behavior provides new design information for solid and sandwich beams of either metal or plastic construction and correlates excellently in various limiting cases with the theoretical and experimental results obtained by previous investigators.

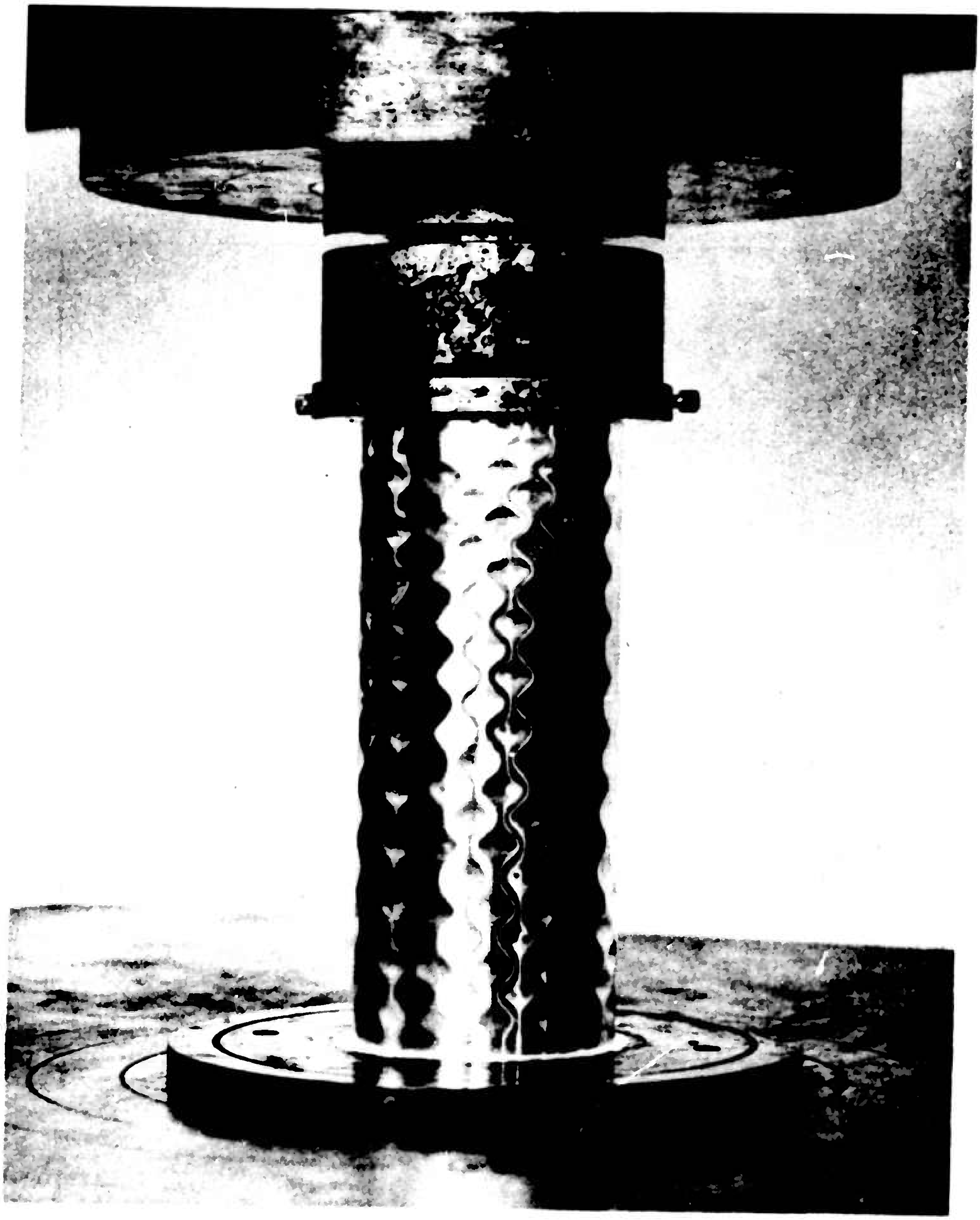


Figure 1. Photograph of Cylinder With Completely Developed Buckle Pattern.

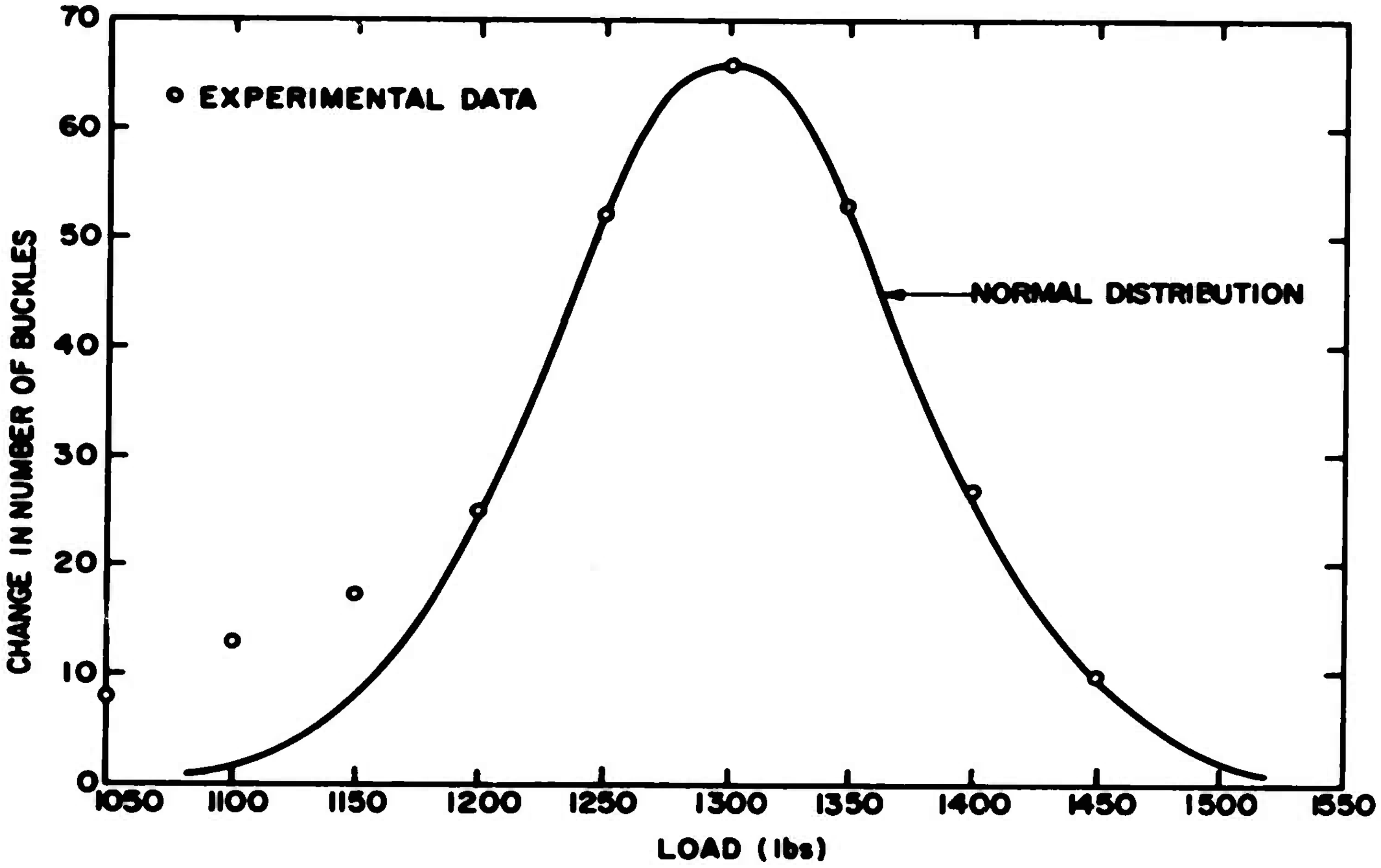


Figure 2. Comparison of Buckle Distribution With Normal Distribution.

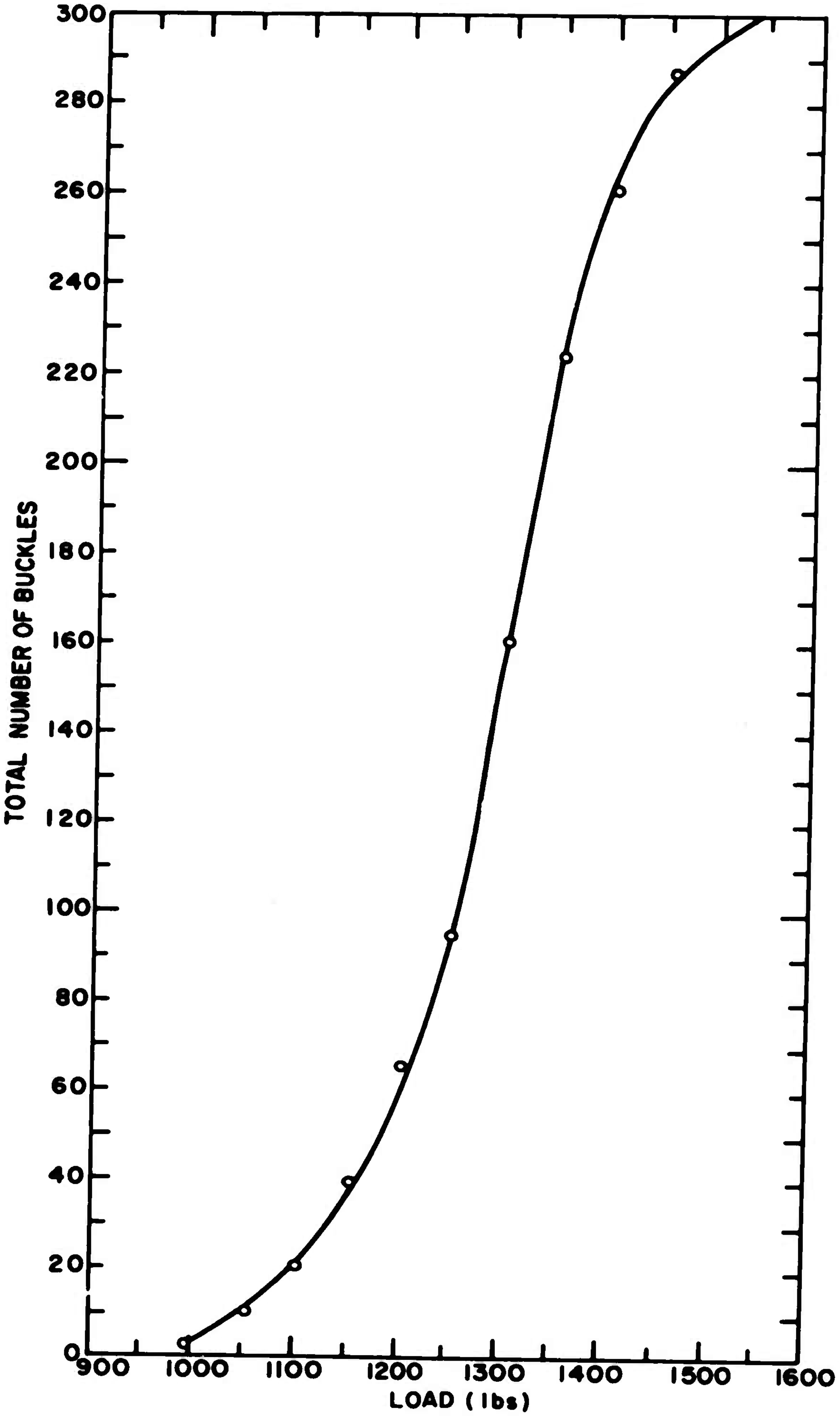


Figure 3. Plot of Total Number of Buckles Versus Load.

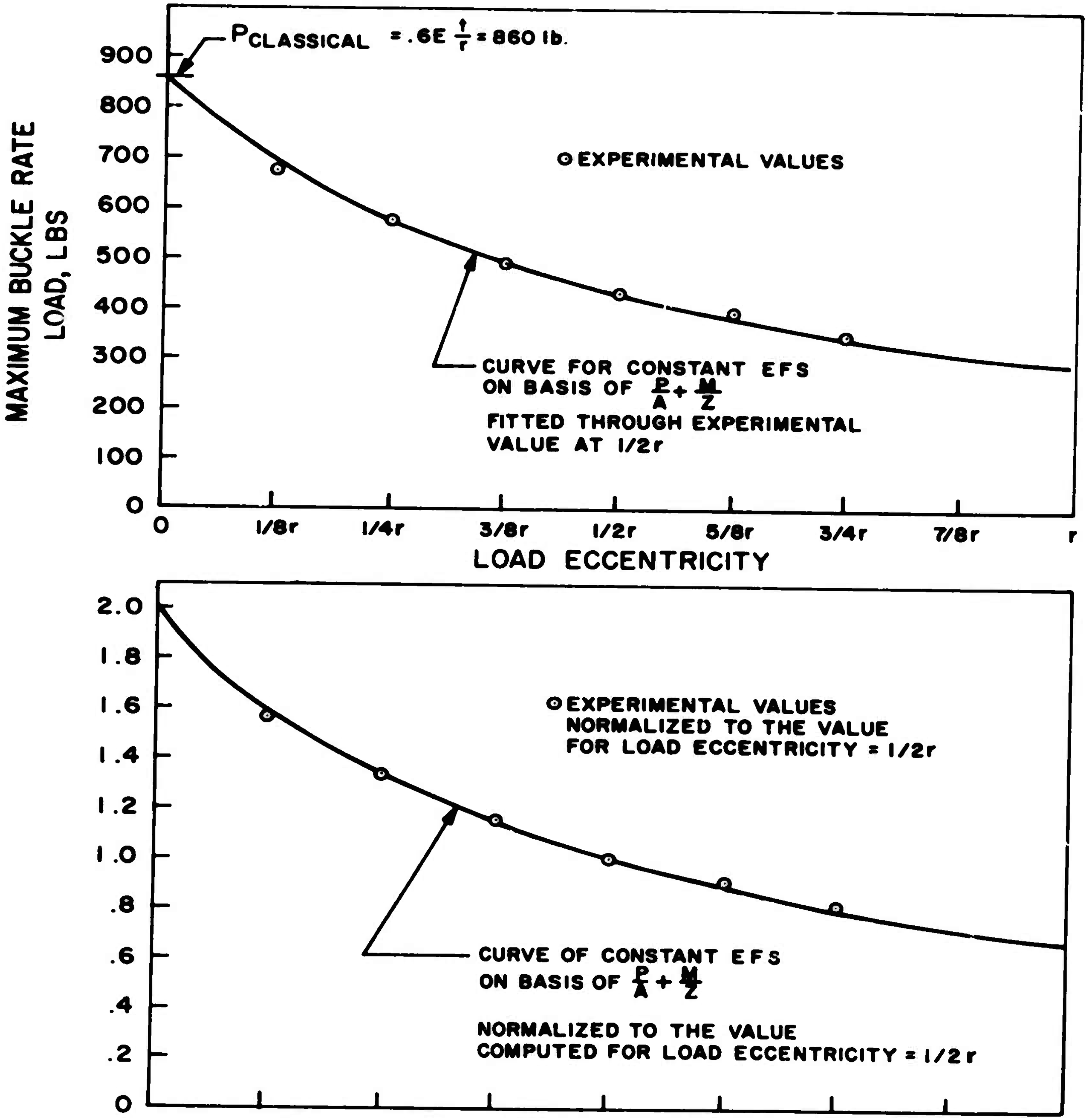
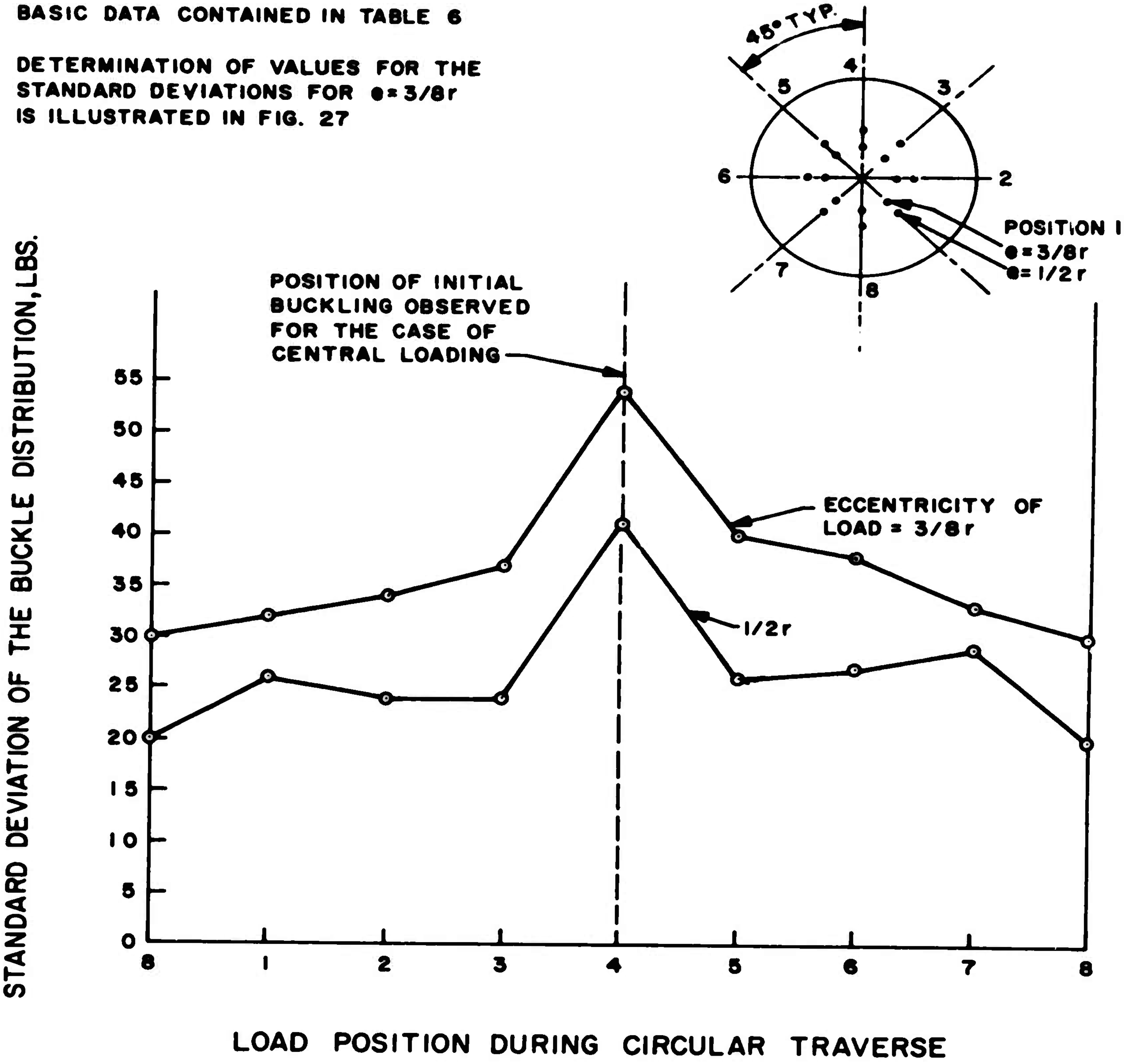


Figure 4. Maximum Buckle Rate Load Versus Load Eccentricity for Shell.



BASIC DATA CONTAINED IN TABLE 6

DETERMINATION OF VALUES FOR THE STANDARD DEVIATIONS FOR e = 3/8 r IS ILLUSTRATED IN FIG. 27

Figure 5. Initial Buckling Region of Shell Specimen as Revealed by Circular Traverses of Loading.

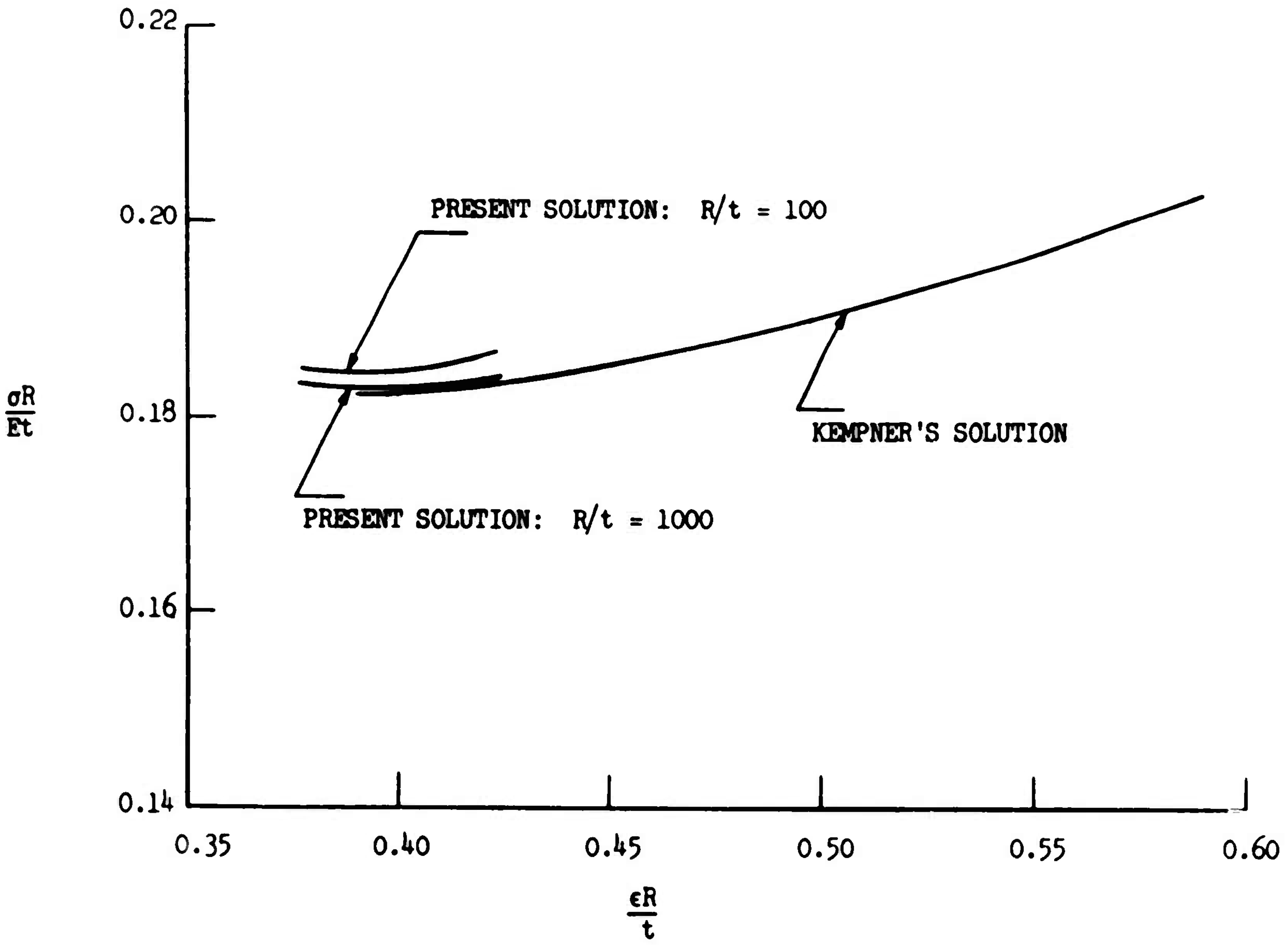


Figure 6. Minimum Postbuckling Stresses Obtained From Present Analysis for Two Values of R/t Compared With Result of Kempner.

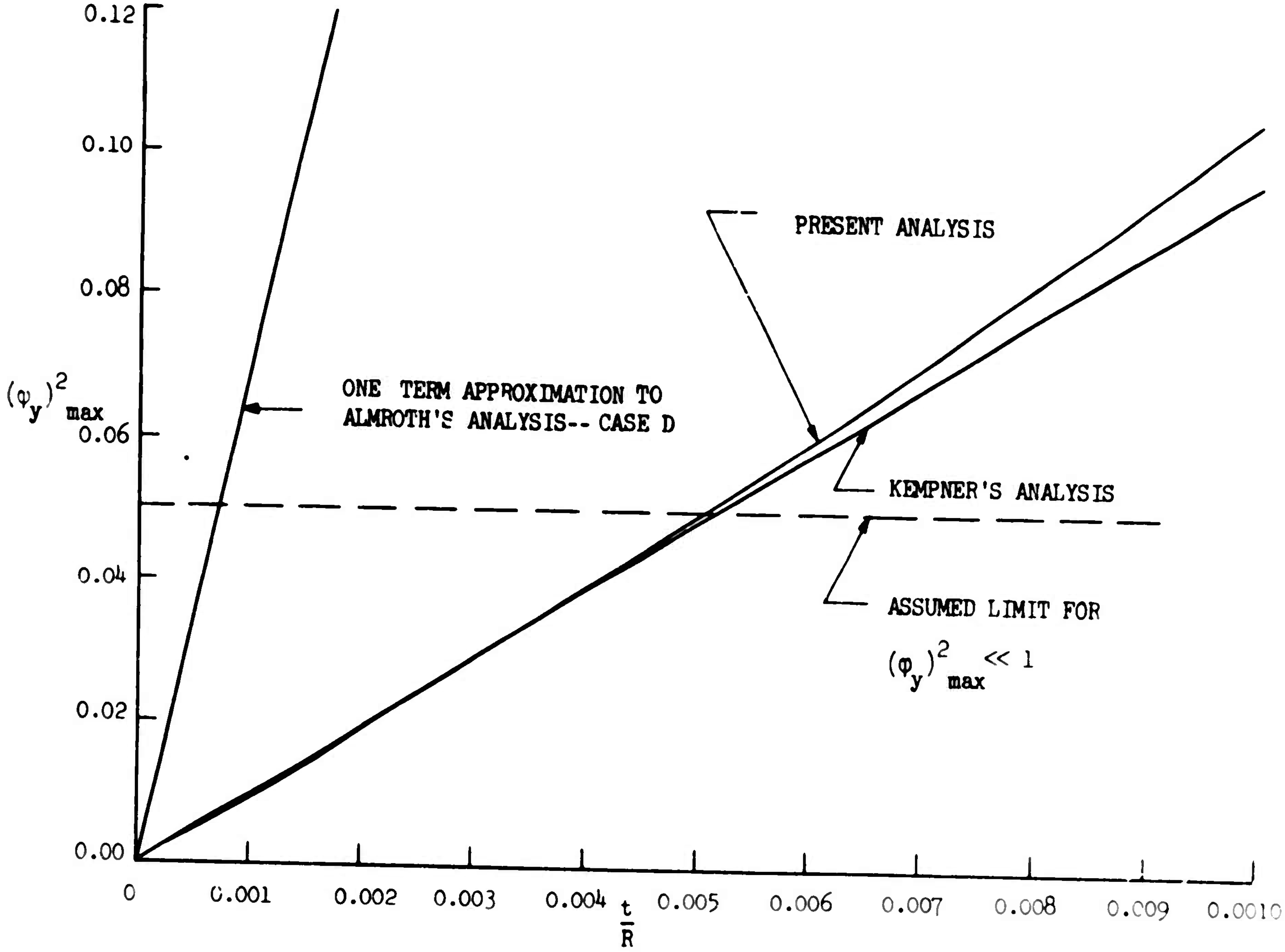


Figure 7. Variation of Maximum Rotation Magnitudes With t/R at $\epsilon R/t = 0.4$ for the Kempner, Almroth and Present Analyses.

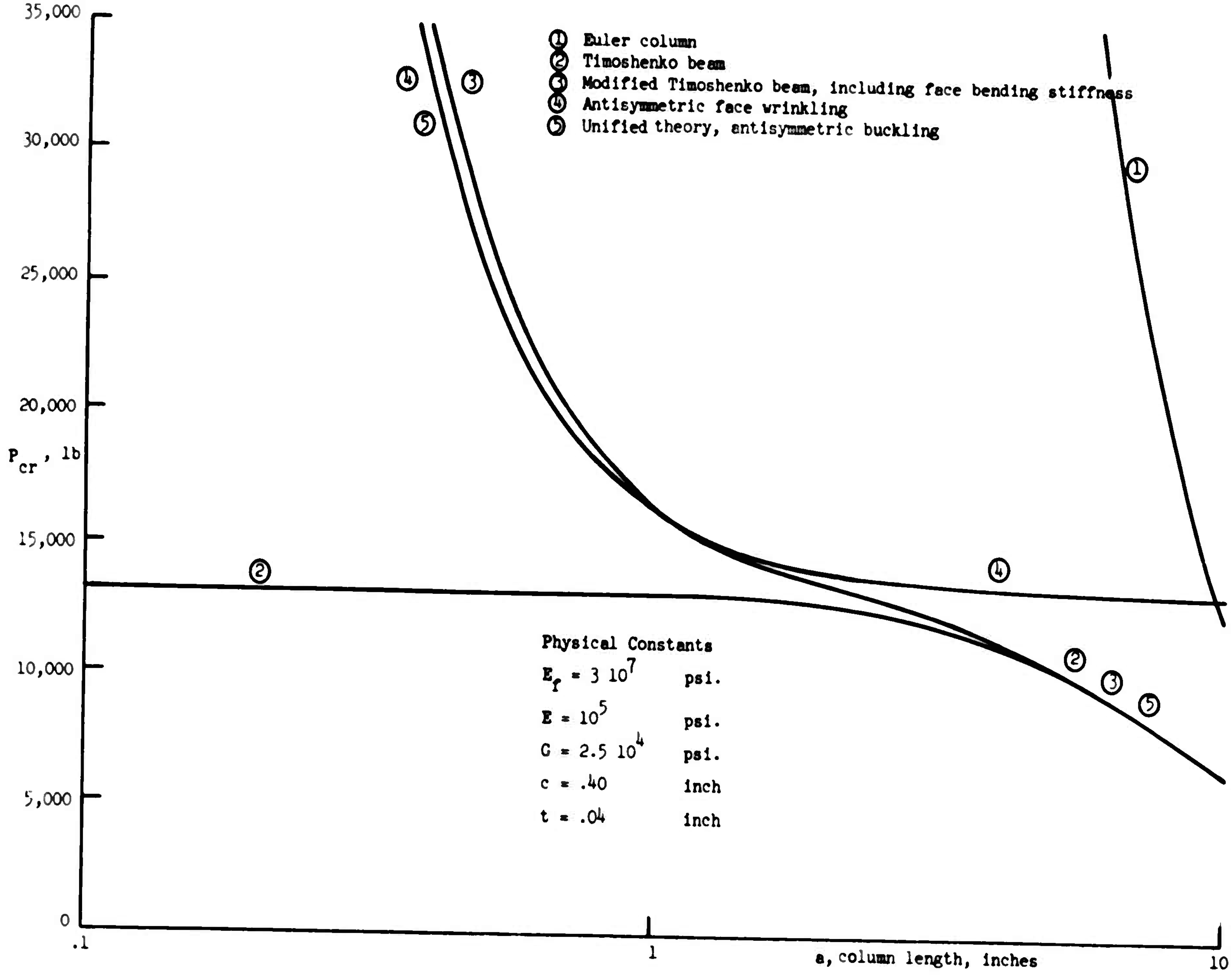


Figure 8. Buckling of a Column; Critical Load Versus Length for Simply Supported Faces and Fully Supported Core; Comparison With Other Buckling Theories.

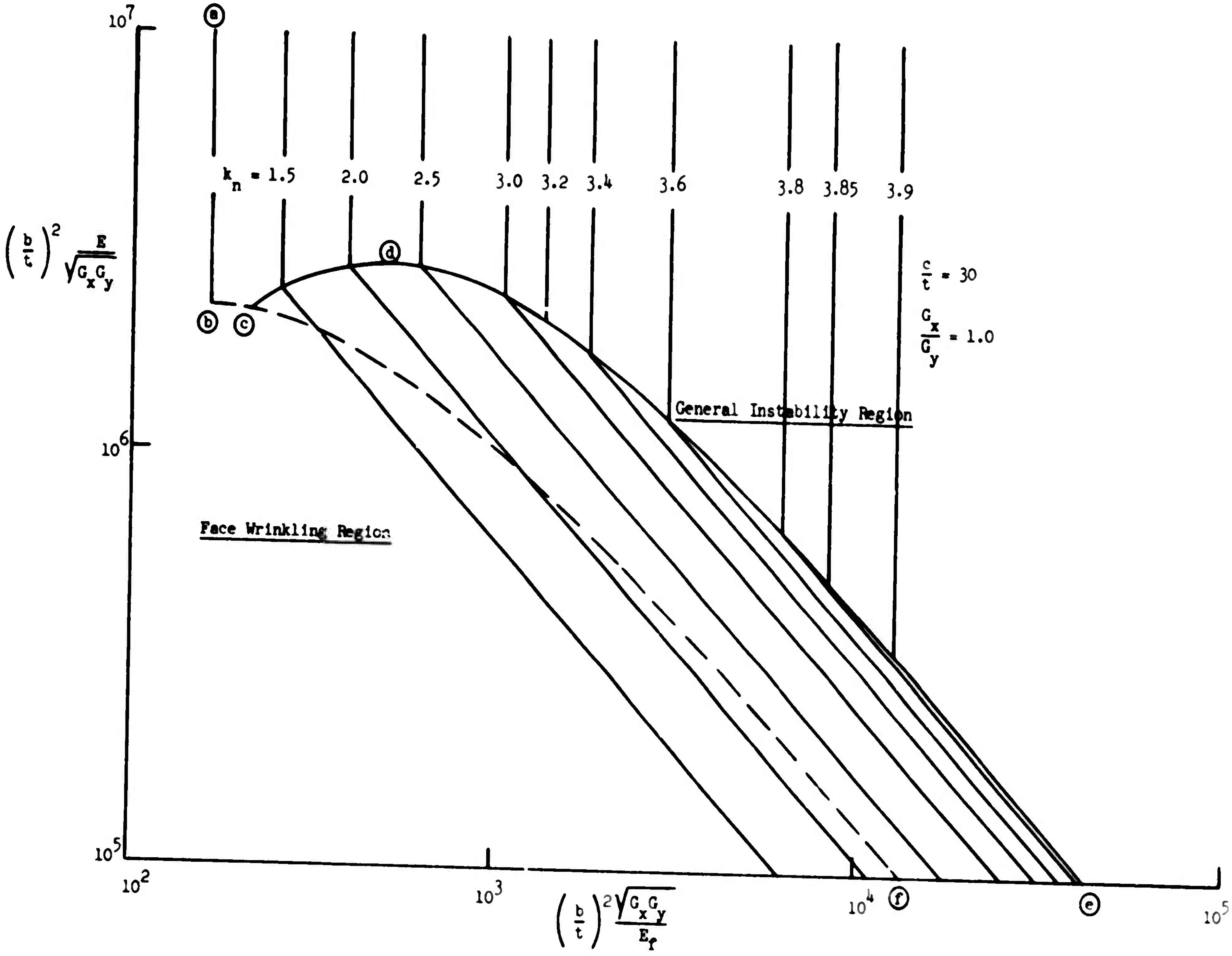


Figure 9. Stability Boundary, ⓐ-ⓑ-ⓒ-ⓓ-ⓔ, Between the Face Wrinkling Modes and the General Instability Mode of a Plate for Simply Supported Faces and Fully Supported Core.

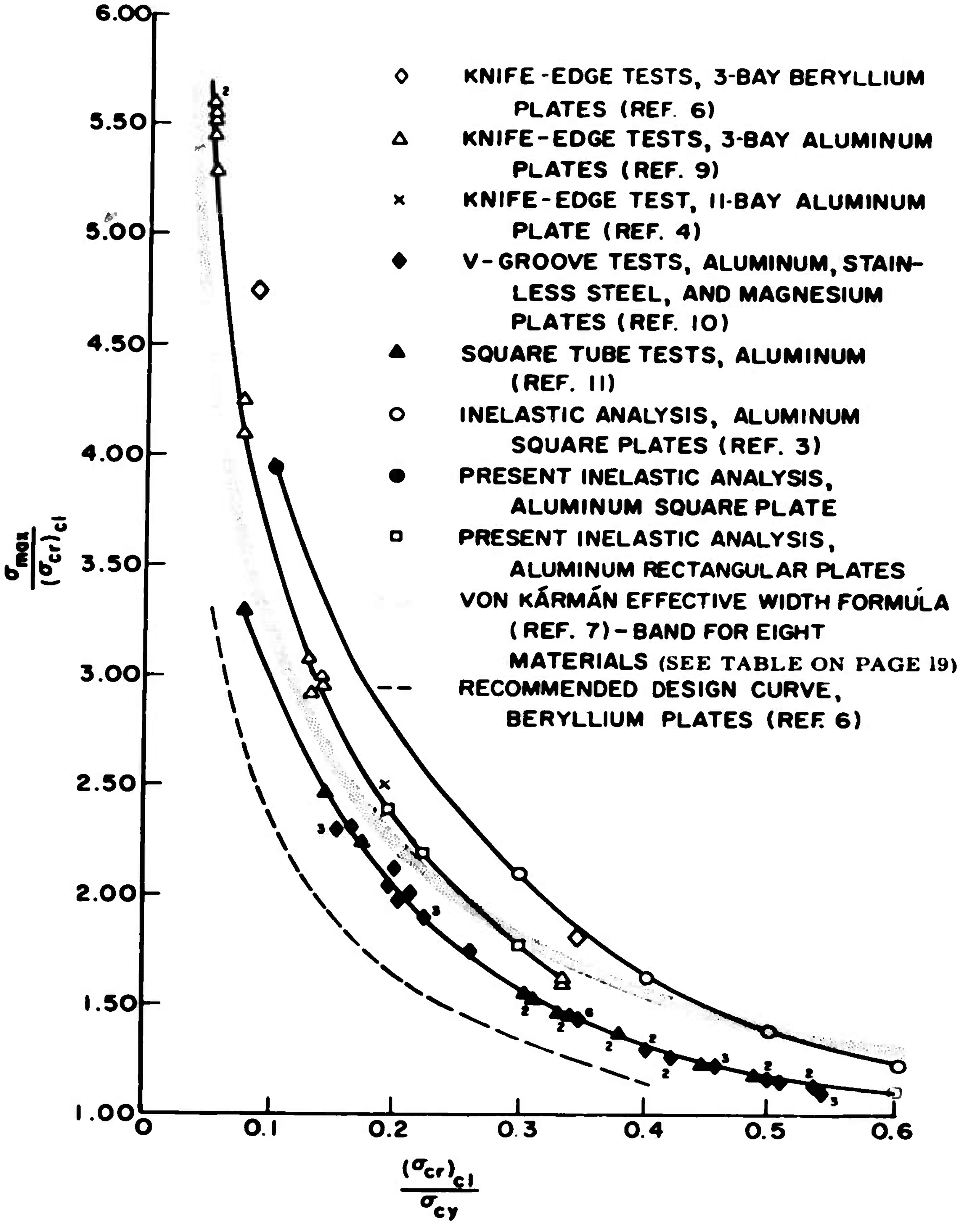


Figure 10. Theoretical and Empirical Maximum Strength Criteria for Simply Supported Rectangular Plates.

TABLE

MATERIAL PROPERTIES

Material	E (10^6 psi)	σ_{cy} (10^3 psi)
Aluminum, 2014-T6	10.7	61.3
Aluminum, 2024-T3	10.7	48.0
Aluminum, 2024-T4	10.7	43.6
Aluminum, 7075-T6	10.5	72.6
Magnesium, FS-1H	6.5	26.7
Beryllium, 1.75% BeO	44.0	60.0
Steel, Stainless W	30.0	195.0
Steel, Stainless 18-8-3/4H	28.8	105.0 (With grain) 162.0 (Cross grain)

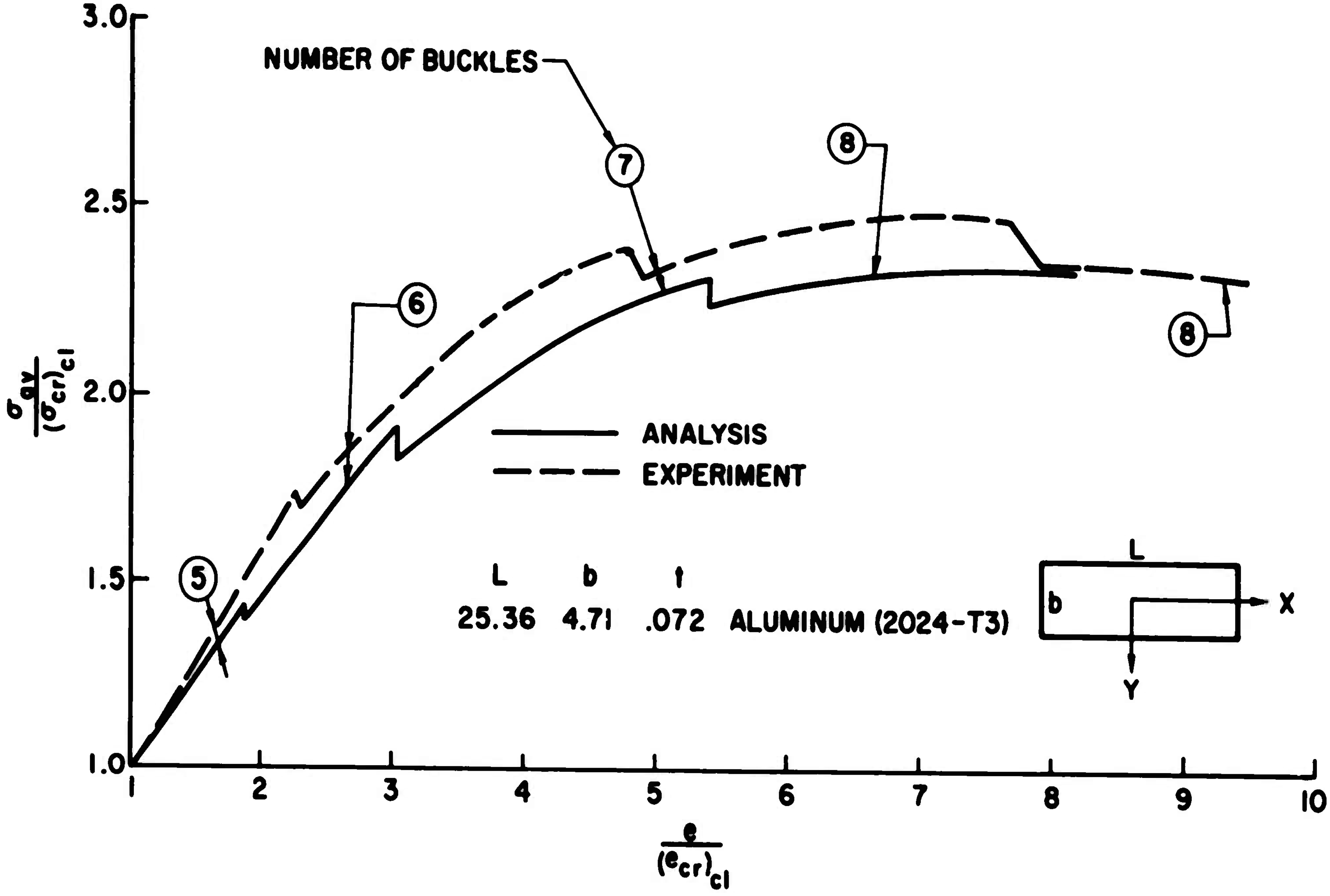


Figure 11. Comparison of Load-Shortening Curves for a Rectangular Plate Based on Prediction of Present Theory and Experimental Result of Reference 27.

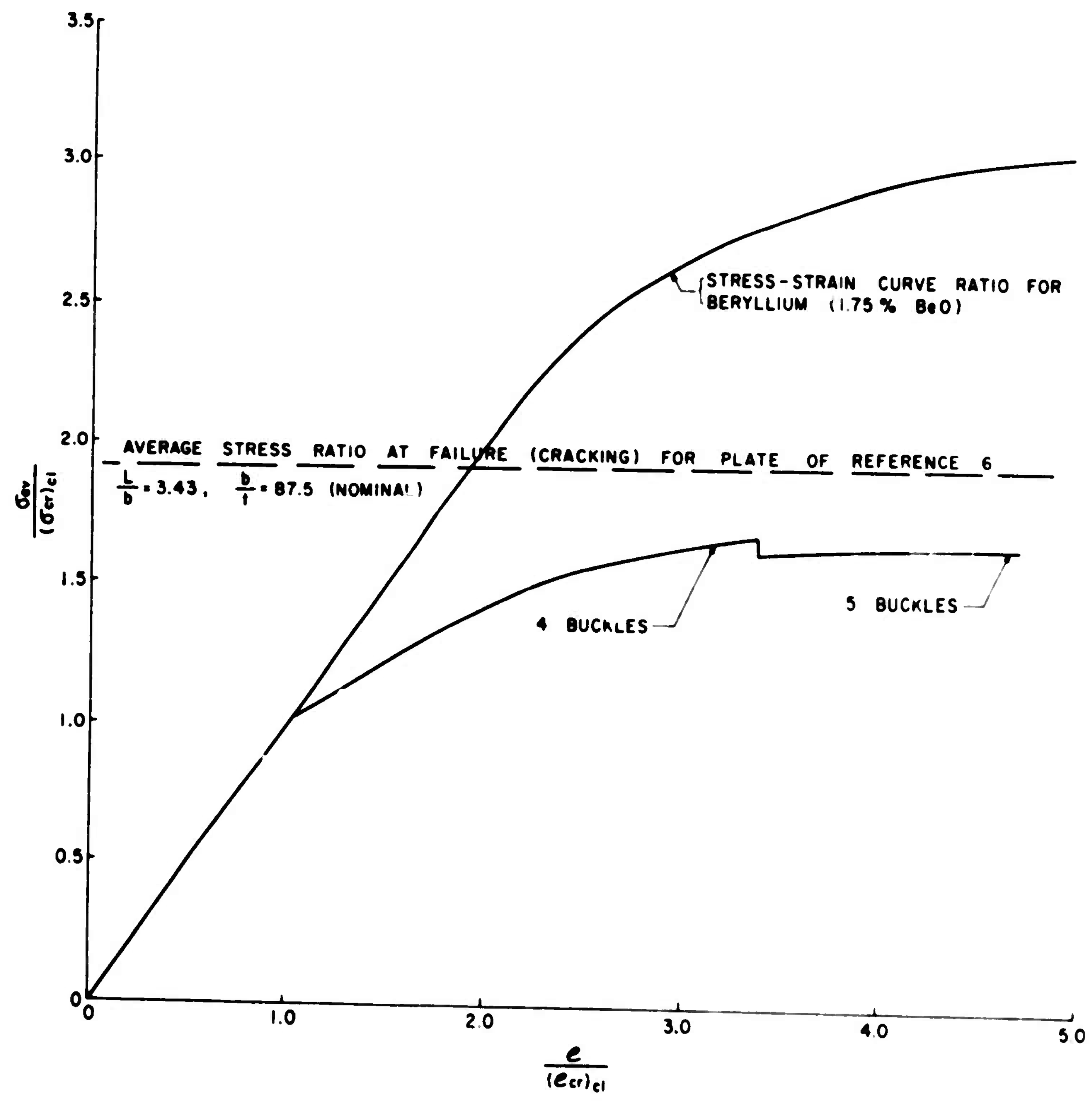


Figure 12. Predicted Load-Shortening Curve for a Rectangular Plate Based on Present Analysis Compared to Experimental Result of Reference 25.

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13 ABSTRACT

This paper presents a condensation of a wide range of research efforts carried out at Stanford University during the period November 1963 - February 1965 on contract DA 44-177-AMC-115(T). In so doing, it outlines new approaches to the experimental study of the instability of shell bodies and provides fresh analyses in this field. It abstracts from recent extensions to the maximum strength analysis for flat plates and gives an outline of a generalization in the theory of buckling of sandwich plates. It refers to research efforts on the stability of conical shells and indicates progress in studies of the influence of higher order linear and nonlinear effects on the lateral vibrations of solid and sandwich beams.

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