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A. A. Mathiasen
Editor

Haystack Pointing System: Mathematical Development for Satellites and Belts

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HAYSTACK POINTING SYSTEM: MATHEMATICAL DEVELOPMENT FOR SATELLITES AND BELTS

A. A. MATHIASEN

Editor

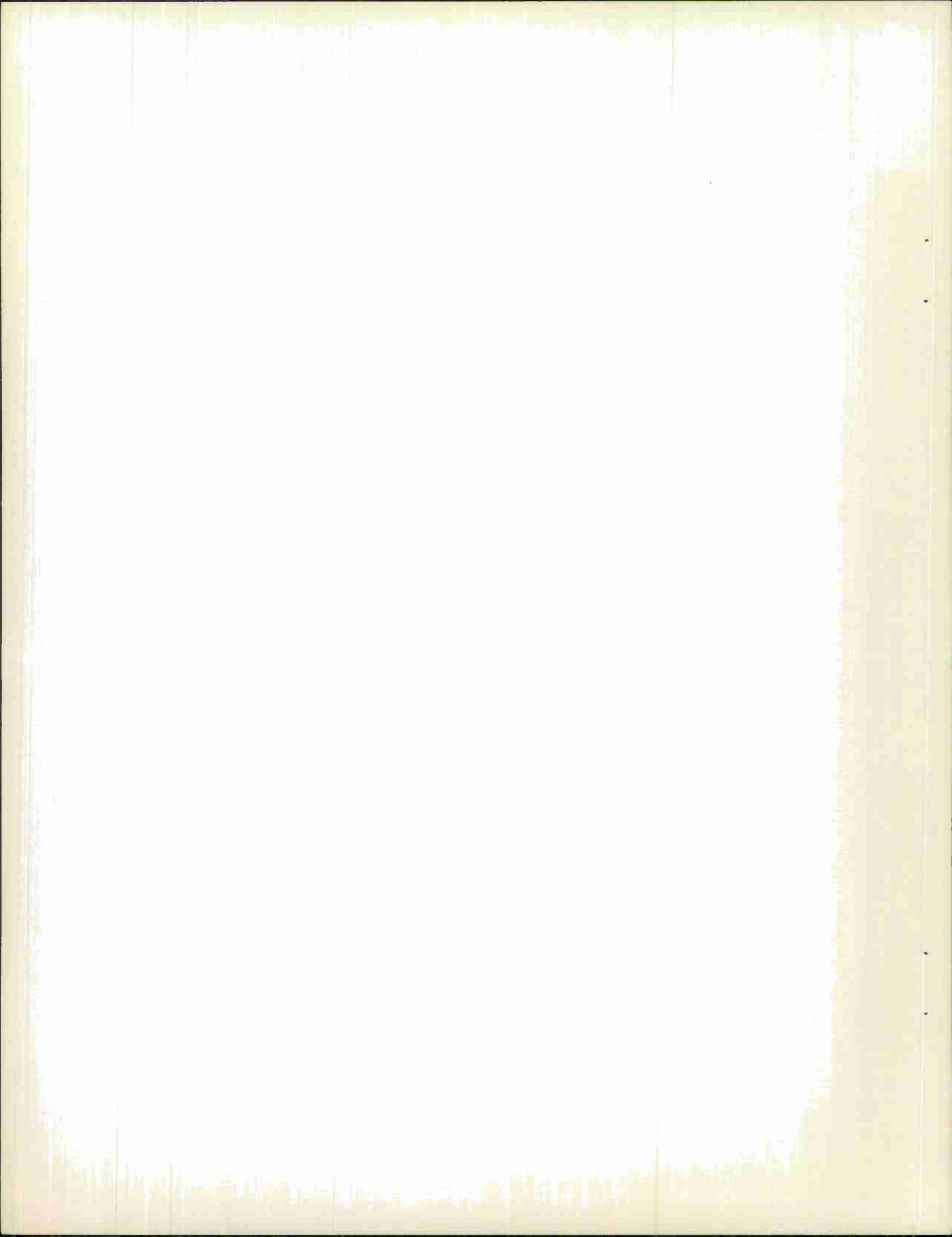*Group 62*

TECHNICAL NOTE 1965-49

23 SEPTEMBER 1965

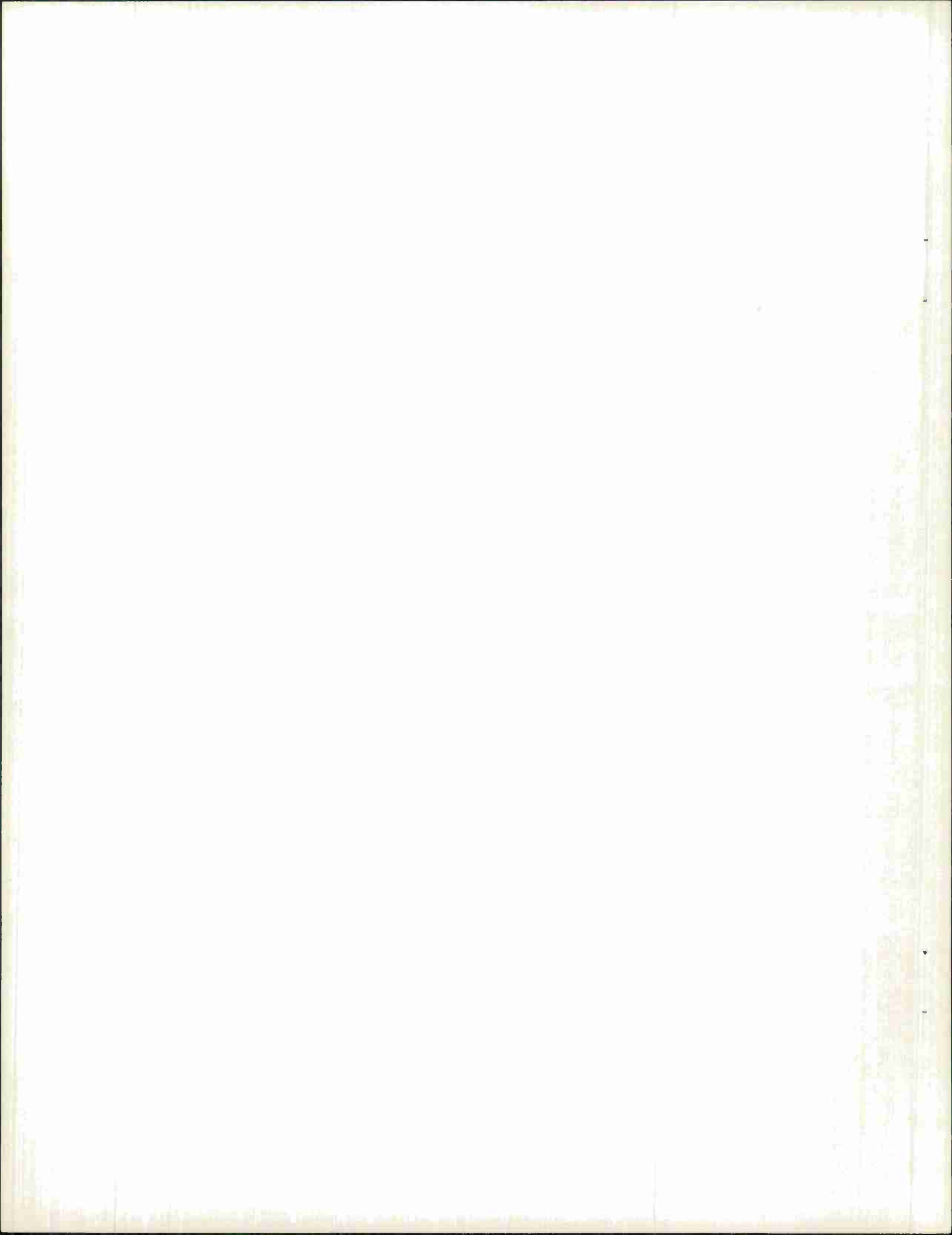LEXINGTON   MASSACHUSETTS

ABSTRACT

Satellites are implicitly completely described by their orbital elements. The conditions for going from the mean orbital elements of a satellite to osculating elements considering perturbations caused by the ellipsoidal earth, and then to celestial coordinates and their rates of change are derived. For belts, it is necessary to fix a point at which it is desired to direct an antenna. This is done by taking the intersection of a right-ascension half-plane, a geodetic longitude half-plane, a declination cone, or a time varying central angle vector with the belt. The equations for determining this point from the mean orbital elements and the intersecting surface or ray are derived.

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PREFACE

This document is a collection of notes by W. G. Williams (Parke Mathematical Laboratories, Carlisle, Mass.) that were used by C. W. Adams Associates, 575 Technology Square, Cambridge, Massachusetts in constructing two programs for the Haystack Pointing System under a subcontract with Group 62 of Lincoln Laboratory. The document was prepared by C. W. Adams Associates and is a quite literal transcription of handwritten notes. Included also is an appendix of editorial notes amplifying certain sections of the notes or noting de facto changes. These sections are indicated by an asterisk.
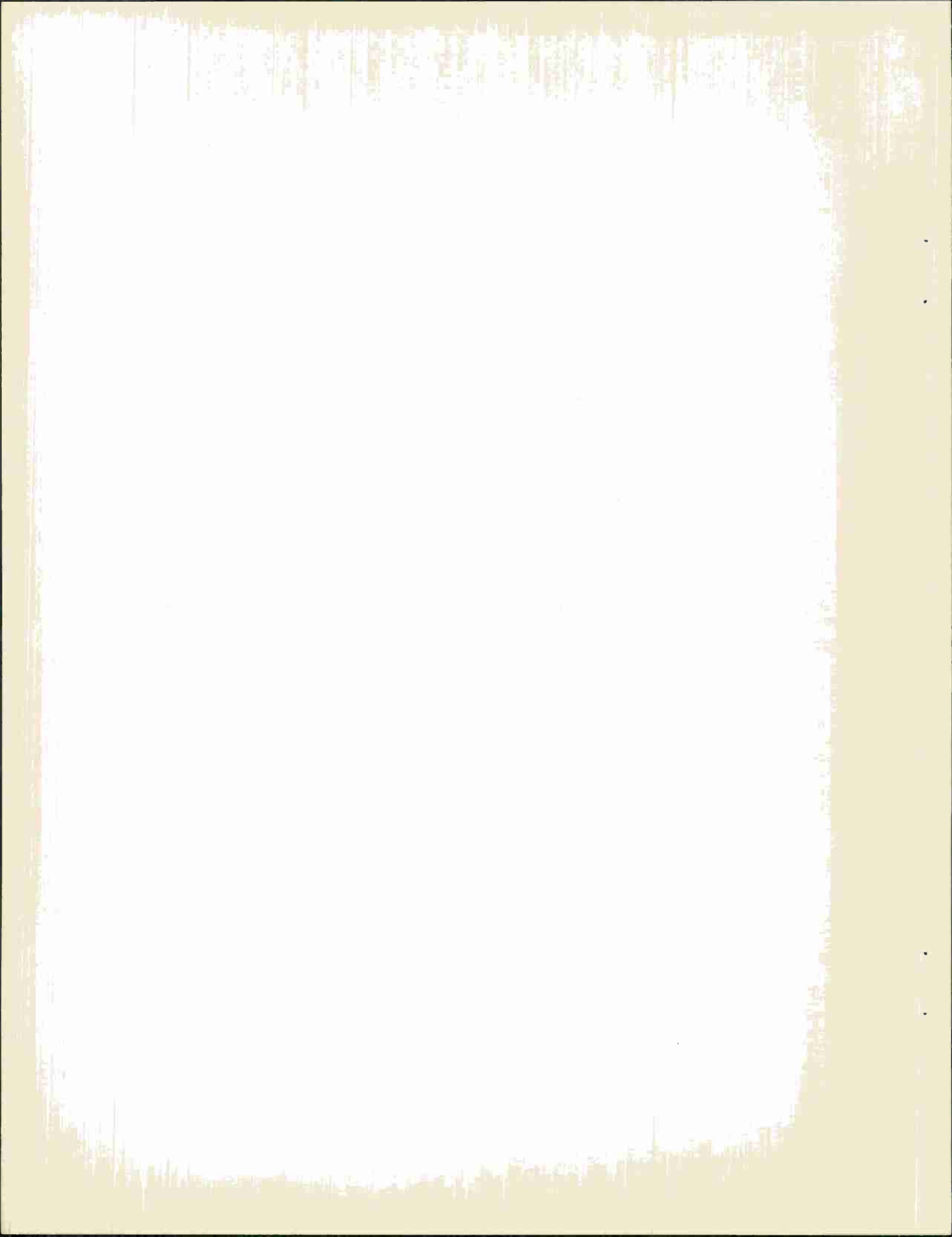
CONTENTS

INITIAL TRACKING PROGRAM CALCULATIONS
(June 25, 1963)

Preliminary Remarks

A.   The coordinate system which Smithsonian Astronomical
      Observatory (SAO) employs is the customary sidereal
      system referred to the equinox of 1950 and the equator
      as of the date. The coordinate system employed by
      Lincoln Laboratory is the customary sidereal system
      but referred to the equinox of date and the equator of
      date. In the SAO system the measurement of the right
      ascension of the ascending node is from the equinox of
      1950 along the fixed equator to the ascending node of
      the moving equator and thence along this equator to the
      ascending node of the satellite (Kozai, A.J., 65, 621,
      1960). No problems should be encountered by making
      only a simple correction to the ascending node of the
      satellite to take into account the precession of the
      equinox alone (that is, negating nutational effects),
      since, to within high precision, both equators (SAO
      and LL) will coincide after the precession term has
      been built in.

B.   The SAO mean elements will be given as 'a function of
      time, generally having the form

$$b = b_0 + b_1 t + b_2 t^2 + \ldots + \sum A_i \sin(B_i + C_i t)$$

      and occasionally a term of the form $D \sin \omega$ or $E \cos \omega$
      to take into account the effect of the third harmonic of
      the terrestial gravitational field.

      Time t in this expression is given by

$$t = T - T_0$$

      and is in days, where T_0 is the reference epoch (that is,
      a certain Julian Day and fraction thereof) in days.

C. The outline of the problem is as follows:

1. Once having been given the date and the particular satellite to be followed, evaluate the mean elements (whose form has been cited in B) for an initial time T.

2. From this evaluation calculate the short period perturbations (which will be given) and add to these mean elements to get the instantaneous osculating elements at the time T.

3. From these osculating elements calculate the right ascension and declination and radius of satellite.

4. Proceed to the time T+δT and repeat this process.

Note: It may turn out that it will be sufficient to calculate the osculating elements only once during the given pass. If so, then item 2 above will be calculated this once and then we will assume that b_0 in the general expression of $\underline{B}$ is this osculating element and that $b = b(t)$ will be the time change to a sufficient precision for the pass.

D. The elements employed by SAO are the usual elliptic set:

ω = argument of perigee

Ω = longitude (right ascension) of ascending node

i = inclination

e = eccentricity

M = mean anomaly

n = anomalistic (mean) motion = $\dot{M}$

a = semi-major axis

Units are: degrees for angular quantities
            mega-meters (10^6 meters) for linear quantities
            revolutions for M and derivatives of M

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SAO employs a Modified Julian Date which is related to the Julian Date as follows: MJD = JD - 2,400,000.5

Calculation of Satellite Right Ascension (α), Declination (δ) and Radius (ρ).

Given the SAO mean elements (as described above) and the initial time of the calculation (i.e., T which is a J.D. and some fraction) and the reference epoch (which may be an M.J.D. and some fraction) to:

1. Calculate MJD of T and then calculate $t = T - T_0$.

2. Evaluate the orbital elements:

   $\omega = \omega(t)$

   $\Omega = \Omega(t)$

   $i = i(t)$

   $e = e(t)$

   $M = M(t)$

   $n = \dot{M}(t)$

3. Correct Ω for the precession of the equinox by adding to Ω the term 3.508×10^{-5} (MJD-33281), i.e.,

   $\Omega = \Omega + 3.508 \times 10^{-5}$ (MJD-33281)

4. Evaluate:

   $$p = \left(\frac{GM}{n^2}\right)^{\frac{1}{3}} (1-e^2)$$

   (GM is a constant which is to be specified)

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5. Evaluate:

$$a = \left(\frac{GM}{n^2}\right)^{\frac{1}{3}} \left\{ 1 - \frac{A_e}{p^2} \left(1 - \frac{3}{2} \sin^2 i\right) \sqrt{1-e^2} \right\}^{\frac{1}{3}}$$

which can be approximated as:

$$a = \left(\frac{GM}{n^2}\right)^{\frac{1}{3}} \left\{ 1 - \frac{1}{3} \frac{A_e}{p^2} \left(1 - \frac{3}{2} \sin^2 i\right) \sqrt{1-e^2} \right\}$$

(A_e is a constant to be specified)

* 6. Calculate the eccentric anomaly E from the Kepler equation:

$E = M + e \sin E$

This will be done by an iterative process; as an initial trial value use the expression:

$$E = M + \left(e - \frac{e^3}{8}\right)\sin M + \frac{1}{2}e^2 \sin 2M + \frac{3}{8}e^3 \sin 3M + 0(e^4)$$

7. Calculate the true anomaly v from the relations

$$\sin v = \frac{\sqrt{1-e^2}}{1-e \cos E} \sin E; \quad \cos v = \frac{\cos E - e}{1-e \cos E}$$

8. Calculate argument of latitude u from the expression

$u = \omega + v$

9. Evaluate $\rho = a (1-e \cos E)$

10. Calculate short period perturbations from the expressions

$$d\Omega = - \frac{A_e}{p^2} \cos i \left\{ v - M + e \sin v - \tfrac{1}{2}\sin 2(v+\omega) - \right.$$

$$\left. \frac{e}{2} \sin (v+2\omega) - \frac{e}{6} \sin (3v + 2\omega) \right\}$$

$$di = \frac{1}{4} \frac{A_e}{p^2} \sin 2i \left\{ \cos 2(v+\omega) + e \cos (v+2\omega) + \right.$$

$$\left. \frac{e}{3} \cos (3v+2\omega) \right\}$$

$$du = \frac{A_e}{p^2} \left[(2 - \frac{5}{2} \sin^2 i)(v - M + e \sin v) + (1 - \frac{3}{2} \sin^2 i) \right.$$

$$\left\{ \frac{2}{3e} (1 - \frac{e^2}{2} - \sqrt{1-e^2}) \sin v + \frac{1}{6} (1 - \sqrt{1-e^2} \sin 2v \right\}$$

$$- (\frac{1}{2} - \frac{5}{6} \sin^2 i) e \sin (v+2\omega) - (\frac{1}{2} - \frac{7}{12} \sin^2 i)$$

$$\left. \sin 2(v+\omega) - \frac{e}{6} \cos^2 i \sin (3v+2\omega) \right]$$

$$d\rho = \frac{1}{3} \frac{A_e}{p} (1 - \frac{3}{2} \sin^2 i) \left\{ -1 - \frac{1}{e} (1 - \sqrt{1-e^2}) \cos v + \right.$$

$$\left. + \frac{\rho}{a} \sqrt{\frac{1}{1-e^2}} \right\} + \frac{1}{6} \frac{A_e}{p} \sin^2 i \cos 2(v+\omega)$$

11. Apply perturbation in range (from center of earth)

$$\rho = a(1 - e \cos E) + d\rho$$

12. Apply perturbation to node.

$$\Omega \rightarrow \Omega + d\Omega$$

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13. Apply perturbation to inclination in the form

$$\left.\begin{array}{l}\sin i \to \sin i + \cos i \ di \\[6pt] \cos i \to \cos i - \sin i \ di\end{array}\right\} \text{where trig functions on right are unperturbed.}$$

14. Apply perturbation to argument of latitude in the form

$$\left.\begin{array}{l}\sin u \to \sin u + \cos u \ du \\[6pt] \cos u \to \cos u - \sin u \ du\end{array}\right\} \text{where trig functions on right are unperturbed}$$

15. Calculate the right ascension (α) and the declination (δ) from the relations

$$\sin \delta = \sin i \ \sin u \qquad\qquad\qquad\qquad = A_3$$

$$\cos \alpha \cos \delta = \cos \Omega \cos u - \cos i \sin \Omega \sin u = A_1$$

$$\sin \alpha \cos \Omega + \sin \Omega \cos u + \cos i \cos \Omega \sin u = A_2$$

Calculation of Pass Visibility at This Time T

1. Calculate the right ascension of Greenwich at the time T from the Newcomb Formula

$$\alpha_G = \left[\overbrace{0.2769193981 + 0.002737909263 \, (T-2,415,020)}^{\text{round off to 10 places}} + \right.$$
$$\left. + 0.0000010752 \ (\frac{T-2,415,020}{3.6525 \times 10^4})^2 \right] \ 2\pi \ (\text{in radians})$$

* 2. Calculate right ascension of radar α_R at time T from

$$\alpha_R = \alpha_G + \Lambda_R$$

where Λ_R = longitude of radar (West is negative; East is positive)

* 3. Calculate declination of radar from the relation

$$\delta_R = \ell - m \sin 2\ell + \frac{m^2}{2} \sin 4\ell$$

where ℓ = geographic latitude (North taken positive)

$m = 3.359076852 \times 10^{-3}$

4. Calculate

$$B_1 = \cos \delta_R \cos \alpha_R$$

$$B_2 = \cos \delta_R \sin \alpha_R$$

$$B_3 = \sin \delta_R$$

5. Calculate Γ, the angle between satellite direction (from earth center) and radar direction (from earth center) from

$$\cos \Gamma = \sum_{i=1}^{3} A_i B_i$$

6. Convert geographic minimum "look" angle β at radar to geocentric "look" angle β' by means of

$$\beta' = \beta + m \sin 2\ell - \frac{m^2}{2} \sin 4\ell$$

* 7. Calculate geocentric distance of radar from the relation

$$R = R_p [1 + (\mathcal{K} \cos^2 \ell) + \frac{3}{2} (\mathcal{K} \cos^2 \ell)^2] + h$$

where:

R_p = polar radius of earth

h = height of radar above sea level

$\mathcal{K} = 3.34783 \times 10^{-3}$

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* 8. Calculate visibility zone half angle G from the relation

$$\cos G = (\frac{R}{r}) \cos^2 \beta' + \sin \beta' \sqrt{1 - (\frac{R}{r})^2 \cos^2 \beta'}$$

where r is the geocentric radius of satellite.

* 9. Satellite is visible when

$$\Gamma \leq G$$

INITIAL BELT TRACKING PROGRAM
(September 9, 1963)

The following theoretical treatment is based upon the
fact that the same tracking accuracies will ultimately be
required of the belt program as are required of the satel-
lite program; and, further, that the orbit parameters will
in effect be given in terms of SAO mean elements. If the
second assumption is unfounded, the analysis will be simpli-
fied considerably in an obvious way since a large share of
the analysis consists of going from mean to "instantaneous"
osculating elements in a straightforward way. If the first
assumption can be realized, the number of iterations re-
quired is obviously less.

* Determination of First and Last Belt Visibility Times

Oth Approximation

1. Calculate cos G (given in satellite Initial Track-
   ing Program, which shall be referred to as ITP)
   from equation 8 of the Visibility section with
   r=a where a is gotten at 5 of the first section
   of ITP. Convert to sin G, noting that
   $0 \leq G \leq \pi/2$.

2. Calculate the mean elements of the belt at the be-
   ginning of the J.D. (Julian Day) of interest.

3. Determine the first and last visibility right as-
   censions of the radar, α_R, from the equalities of
   the following relation(in which δ_R is the declina-
   tion of the radar calculated in 3 of the visibility
   section of ITP):

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$-\sin G \leq [\sin i \cos \delta_R \sin (\Omega - \alpha_R) + \cos i \sin \delta_R]$

$\leq \sin G$.

Note that there will be, in general, two such sets of contacts per sidereal day because of the rotation of the earth; thus when the two α_R's have been found, there will be two more derived by adding $\Omega + \frac{\pi}{2} - \alpha_R$ to $\Omega + \frac{\pi}{2}$ in the 0th approximation.

4. The universal times for these contacts can be derived from these α_R's by solving for α_G in ITP visibility 2, and then inverting the Newcomb Formula of ITP visibility 1.

This completes the 0th approximation

5. To start the 1st approximation, solve the following expression for the argument of the latitude, u, of the belt at these contact α_R's from the expression:

$\cos \delta_R \cos (\alpha_R - \Omega) \cos u + [\cos \delta_R \cos i \sin (\alpha_R - \Omega)$

$+ \sin \delta_R \sin i] \sin u = \cos G$

1st Approximation Series

6. From the T's in 4 update all the mean elements to the appropriate times.

7. At each of these times use the mean elements updated in 6 and the u's calculated in 5 and calculate the geocentric distance of the belt, ρ, from the formula:

$$\rho = \frac{a(1-e^2)}{1+e\,\cos(u-\omega)}$$

8. With these values of ρ recalculate cos G (and, hence, sin G) by replacing a in the 0th order expression by ρ.

9. Repeat steps 3, 4 and 5 deriving new contact right ascensions, times and arguments of latitudes.

10. Repeat the cycle from 6 through 9 until consistency is attained.

2nd Approximation

11. From the mean elements finally settled upon in 10 and from the relation that $v = u - \omega$ where u is gotten from 10 and from the relations that $\rho = a(1-e\,\cos E)$ and $M = E - e\,\sin E$, calculate the values of v and M (which are needed in $\delta\Omega$ and δu) and calculate the short period perturbations of ITP (namely, $d\Omega$, di, du and $d\rho$). Apply (i.e., add as in ITP) these mean elements settled on in 10 thereby determining the osculating elements.

12. Repeat steps 3, 4 and 5, thereby determining the contact right ascensions of radar, the times of contact (these are the crucial times) and the arguments of latitude of belt at contact.

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13. Using these osculating elements of 11, determine
    the contact right ascension-declination sets of
    the belt at contact from 15 of ITP.

Determination of Right Ascension-Declination Values of Belt inside Visibility Times

For a given visibility interval the procedure is as
follows:

14. Jump time from 1st contact by Δt and update mean
    elements to this time.

15. Solve 5 for "upper" and "lower" argument of lati-
    tude of belt using the value of ρ in cos G derived
    from 11.

16. Apply short-period perturbations to these "upper"
    and "lower" points to get osculating elements.

17. Using these values of the elements, again solve 5
    for the "end point u's".

18. Using these u's solve for the end point right
    ascensions and declinations by means of 15 from ITP.

19. Using "appropriately" averaged elements (i.e., the
    mean of the end point osculating elements) calculate
    the right ascension and declination of the belt
    (and radar range if desired) for a range of u's

intermediate to these end point u's from 15 of
ITP (and from 7 above).

20. Jump time from 1st contact by $2\Delta t$ and cycle from
    15 through 19.

21. Jump to $3\Delta t$, $4\Delta t$....etc., until end of visibility
    interval.

Remarks

a) The right ascension-declination values derived in
    19, 20 and 21 are sufficient to construct a pre-
    cise schedule for the

    1) fixed latitude (declination) schedule
    2) belt scan schedule
    3) fixed right ascension schedule
    4) fixed longitude schedule

    For example, a precise plot of the values in a form
    such as α vs. δ with T as a parameter contains all
    the information necessary.

b) The above treatment has been done in complete
    generality. For certain "degenerate" situations
    and/or with reduced accuracy requirements, it can
    be considerably simplified with little effort if
    desired.

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ADDITIONS TO INITIAL BELT TRACKING
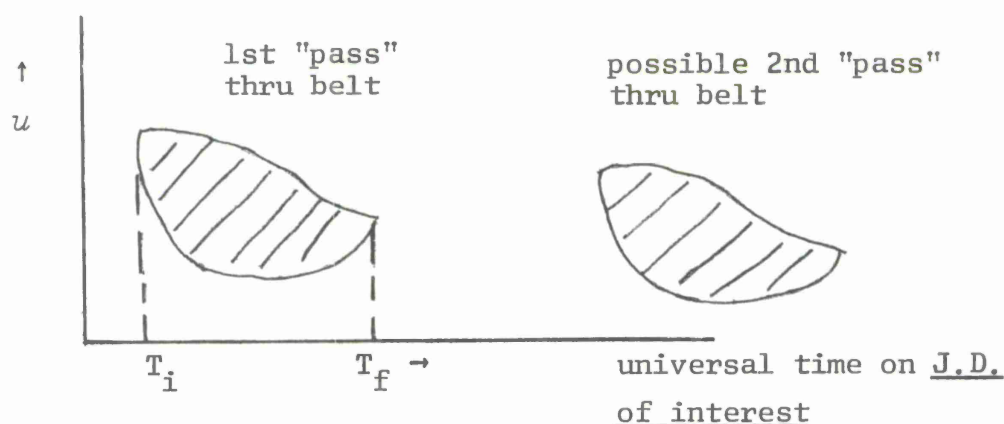## PROGRAM AND MISCELLANEOUS ADDENDA
(September 10, 1963)

Initial Belt Tracking Program

* 1. If instantaneous osculating elements are employed as
inputs rather than SAO mean elements, the calculations
are considerably simplified in that the process is
ended at 10 at the termination of what has been desig-
nated the 1st approximation series. Further, in steps
1-10 replace "mean elements" with "osculating elements"
mentioned, and use "osculating" for "mean" in 14 and
delete 16 and 17 which are rendered unnecessary.

2. In terms of the desired schedules the calculation of 18
is superfluous. All that is required is the calculation
of the "end point u's". Thus the output of the program
will be as follows:

	T	u	
Initial contact time	T_1	u_1	Initial argument of latitude
Intermediate times	$T_1 = T_1 + \Delta T$ $T_2 = T_1 + 2\Delta T$	u_1^u, u_1^ℓ u_2^u, u_2^ℓ	Upper and lower $(u + 1)$ u's at in- termediate times.
	. . .	. . .	
Final contact times	T_f	u_f	Final argument of latitude.
	. . .		

The interval ΔT should be chosen such that a smooth and precise interpolation scheme can be worked out for times and u's in between these calculated. Thus, schematically we may represent the output in the following graphical form:

↑
u

1st "pass" thru belt   possible 2nd "pass" thru belt

T_i T_f →   universal time on <u>J.D.</u> <u>of interest</u>

From these visibility "footprints" or, more precisely, the appropriately interpolated u vs. T outline of the "prints", the belt schedules can be worked out as follows:

A.   <u>Fixed Latitude (Geodetic) or Declination Schedule</u>

Given: δ_B to remain fixed

a)   from $\sin\delta = \sin i \, \sin u$ ◄——————————(I)
     solve for u_B corresponding to $\delta = \delta_B$

b)   From "foot print" solve for visibility times, i.e.,

↑
u

u_B →

T →

visibility true for δ_B constant

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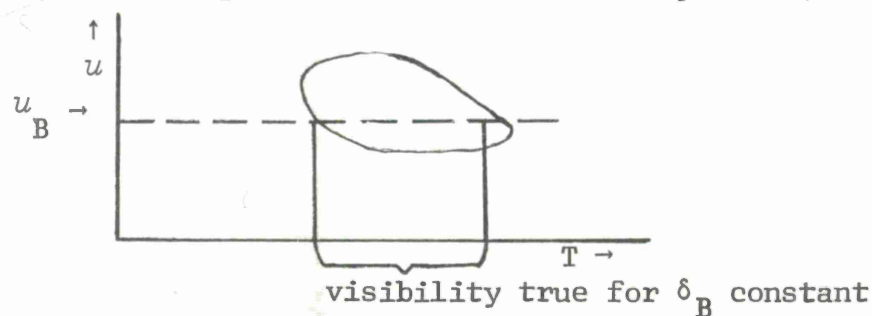
c) From either

$$\cos\alpha \; \cos\delta = \cos\Omega \; \cos u - \cos i \; \sin\Omega \; \sin u \quad \longleftarrow \underline{\quad\quad} \text{(II)}$$

or $\sin\alpha \; \cos\delta = \sin\Omega \; \cos u - \cos i \; \cos\Omega \; \sin u \quad \longleftarrow \underline{\quad\quad} \text{(III)}$

with $\delta = \delta_B$ and $u = u_B$

solve for α_B.

d) From

$$\rho = \frac{a(1-e^2)}{1+e\cos(u-\omega)} \quad\quad\quad\quad \longleftarrow \underline{\quad\quad} \text{(IV)}$$

with $u = u_B$ solve for ρ_B

* B. <u>Constant Belt Scan Schedule</u>

Given: $\frac{dv}{dt} = k$ to remain fixed where v = true anomaly

(Note: $\frac{du}{dt} = \frac{d}{dt} (v+\omega) = k + \dot\omega$ where $\dot\omega$ = perigeal pre-cession; thus $\dot u$ can be calculated with $k + \dot\omega$ given as they are. However, what is probably meant by belt scan is that $\frac{du}{dt} = k$. At any rate, one can get to $\dot u$ from k in a straight forward fashion.)

a) At $T = T_1$ we have u_1 from which $(\alpha_1, \delta_1, \rho_1)$ can be gotten by means of (I), (II) and/or (III) and (IV).

b) At $T = T_1 + \Delta T$ we have $u = u_1 + (\frac{du}{dt}) \Delta t$ from which the next (α, δ, ρ) set can be calculated.

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* c) Repeat (b) until belt is no longer visible at horizon

* d) Reverse sign of $\frac{du}{dt}$ and, repeating (b) scan "backwards" to other horizon

   e) Continue c) and d) until time has run out.

   (This process is pictured below)
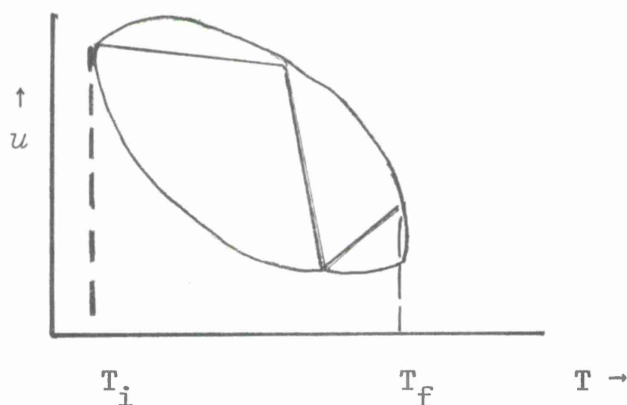
C. Fixed Right Ascension Schedule

Given $\alpha_B \leftarrow$ to remain fixed

(Note: When $\alpha_B = 0$, $\pi/2$, π, $3\pi/2$, u assumes certain degenerate values; this situation must be handled separately, obviously)

a) Set $\alpha \rightarrow \alpha_B$ and from the following (III divided by II)

$$\tan \alpha = \frac{\sin\Omega \ \cos u - \cos i \ \cos\Omega \ \sin u}{\cos\Omega \ \cos u - \cos i \ \sin\Omega \ \sin u} \quad\longleftarrow\quad\text{(V)}$$

calculate $u \rightarrow u_B$

b) Solve for visibility times as in A.b) above

c) Solve for δ_B and ρ_B from (I) and (IV)

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D. Fixed Longitude Schedule

Given $\alpha_B = \alpha_G + \Lambda \leftarrow$ Fixed longitude (W = - ; E = +)

$\uparrow$

Greenwich

which varies as a function of time, α_G being

given by the Newcomb Formula

At each time step repeat a), b) and c) of C, the
fixed right ascension schedule.

General Notes

I) Let χ be angle between orbit plane (osculating) and
meridian

then

$$\cos \chi = \sin i \cos (\alpha_M - \Omega)$$

where α_M = right ascension of meridian of interest (such
as the α_B's of above).

II) It should be emphasized that the Belt Tracking Program
provides a beautifully simple way of calculating satellite
visibility times as follows:

a) Calculate the visibility "footprint" for the orbit
of the satellite thought of as a belt, in precisely
the same way (including short period perturbations)
as previously given.

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b) Now, noting that $u = \text{mod } 2\pi$, calculate u <u>as a</u> <u>function</u> of time from the start of the J.D. of interest to end by means of the formulas of satellite tracking program.

c) Compare these with "footprints", when u falls inside satellite is visible, when outside invisible. This can be represented schematically on next page.

Notes:
1. Radar assumed in Northern hemisphere
2. Orbit assumed unperturbed
3. B is mirror image of A
   w.r.t. J.D. + $\frac{1}{2}$ true
4. Perigee assumed at $\omega = \frac{\pi}{3}$
5. 8 hr. orbit assumed
6. A and B not drawn to scale

SATELLITE VISIBILITY
PERIODS
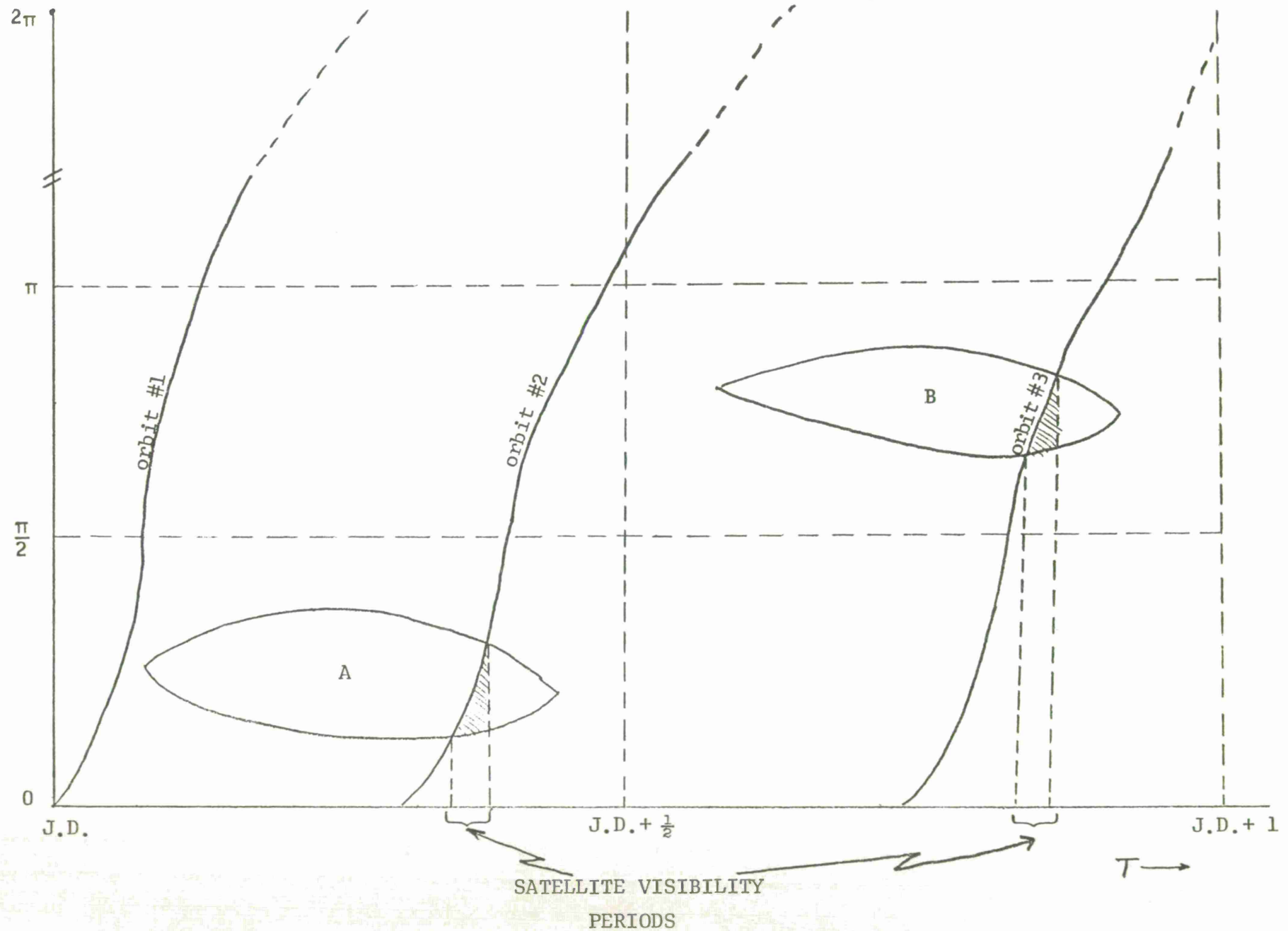
PRE-DOPPLER CALCULATIONS
(February 6, 1964)

Satellite/Belt Doppler Effect

Assumptions

We will neglect ionospheric, tropospheric and relativistic considerations:

a) For further discussion of these see:
    Mass and Vassy; Chapter **1** of Advances in Space
    Science and Technology, 4, **1**-38 (1962).

b) For a more detailed analysis of the first and
    second items see:

    1) Grier and Weiffenback, Proc. I.R.E. 48,
        507-516 (1960)

    2) von Handel and Hohndorf, I.R.E. Trans on Mil.
        Elec. 3, 162-172 (1959)

c) Note that neglect of relativistic effects is
    $\sim(\frac{V}{C})^2$ in relative frequency shift and hence will
    not become "appreciable" ($\sim$ **1** cps) until fre-
    quencies of **1**-10GC (KMc) are employed with the ve-
    locities ($\sim$ 10km/sec) "normally" encountered in
    satellite technology.

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Theory

It can be shown that the 1st order Doppler shift is given by the following expression:

$$-\left(\frac{\Delta f}{f}\right) c = \dot{\rho} \tag{1}$$

in which

Δf .=. Doppler shift in frequency

f .=. Frequency employed by transmitter

Note: It is assumed here that there is an active transmitter on board the satellite. If we are dealing with a passive satellite or with belts, i.e., bouncing e-m radiation off the satellite/belt, then,

$$-(\Delta f/_f) c = \dot{\rho}_1 + \dot{\rho}_2 \quad \text{where } \rho_1 \text{ .=. transmitter/belt range}$$
$$\rho_2 \text{ .=. belt/receiver range}$$

c .=. velocity of light (vacuum)

ρ .=. range (from transmitter to receiver if transmitter on satellite or (see note above) range from radar to belt or from belt to receiver)

$\dot{\rho}$.=. range rate in which a dot denotes time differentiation.

Now, if

$\vec{r}$.=. vector pointing from center of earth to satellite (or to some specific position on belt)

$\vec{R}$.=. vector pointing from center of earth to radar transmitter (or receiver)

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Γ .=. angle between these two vectors (Note: this
is the same angle Γ specified earlier)

then we see that with

$$\cos\Gamma = \sin\delta\sin\delta_R + \cos\delta\cos\delta_R\cos(\alpha-\alpha_R) \tag{2}$$

where the δ's and α's have the usual meaning of declinations
and right ascensions, respectively; that

$$\rho^2 = r^2 + R^2 - 2rR\cos\Gamma \tag{3}$$

where $\cos\Gamma$ is given by (2).

Performing, now, the indicated time derivatives we see that

$$\dot{\rho} = \frac{1}{\rho}\left\{ [r-R\cos\Gamma]\ \dot{r} - rR\ \left[[\sin\delta_R\ \cos\delta-\cos\delta_R\ \sin\delta\ \cos(\alpha-\alpha_R)]\dot{\delta} - \right.\right.$$

$$\left.\left. [\cos\delta_R\ \cos\delta\ \sin(\alpha-\alpha_R)]\ (\dot{\alpha}-\dot{\alpha}_R)\right]\right\} \tag{4}$$

in which we have noted that:

$$\dot{R} = \dot{\delta}_R \equiv 0$$

i e , the "radar" remains on a small circle R away from the
center of the earth at declination δ moving only due to the
earth's rotation $\dot{\alpha}_R$.

We need, therefore, $\dot{r}$, $\dot{\delta}$, $\dot{\alpha}$ and $\dot{\alpha}_R$ in addition to the param-
eters, all of which have been calculated before except
for ρ which is calculated from (3)

We note that

* $$\dot{r} = \frac{a(1-e^2)e\sin v}{(1+e\cos v)^2}\ \dot{v} \tag{5a}$$

where

$$\dot{v} = \frac{a^2\sqrt{1-e^2}}{r^2} \quad \dot{M} \tag{5b}$$

and

$$\dot{\delta} = \frac{\sin i \; \cos u}{\cos\delta} \quad \dot{u} \tag{6a}$$

where

$$\dot{u} = \frac{d}{dt} \quad (\omega + v) = \dot{\omega} + \dot{v} \tag{6b}$$

and

$$* \qquad \dot{\alpha} = \dot{\Omega} + \frac{\cos i}{\cos^2\delta} \quad \dot{u} \tag{7}$$

which can be gotten from Teoste's expression after very tedious algebra or by noting that:

$$\alpha = \Omega + \cos^{-1} \quad \frac{\cos L}{\cos\delta} \tag{8}$$

and finally,

$$\overset{\circ}{\alpha}_R = 1.002737909263 \times 2\pi \; \text{radians/day} \tag{9}$$
$$(\text{i.e., } 7.29211585 \times 10^{-5} \text{radians/sec})$$

NOTE: Both the Newcomb Formula and, hence, the following
        relation in the paper of June 25, 1963 are in error.
        See the accompanying correction.

****

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Hence, it is seen that $\dot{\rho}$ can be calculated after some effort by using that which we knew before, for the most part.

CALCULATION PROCEDURE FOR DOPPLER SHIFT AT TIME T

I   Extract everything from satellite right ascension and declination and radius portion of Initial Tracking Program (June 25, 1963); extract all but 6, 8 and 9 of Pass Visibility, at the time T (using corrected 1 and 2, of course).

II* Decide (in case of belt) what mode to be in:

a)   Fixed declination:
$$\dot{\delta}_B = \dot{u}_B \equiv 0$$

b)   Constant belt scan:
$$\dot{u} = R + \dot{\omega} \text{ or } = K$$

c)   Fixed R.A. and/or Fixed Longitude: both determine δ_B and u_B as a function of time.

* III   Calculate $\dot{r}$, $\dot{\delta}$, $\dot{\alpha}$ and $\dot{\alpha}_R$ from (5), (6), (7) and (9)

*   IV   Calculate ρ from (3)

*   V   Calculate $\dot{\rho}$ from (4)

*   VI   Calculate Δf or $\Delta f/f$ from (1) (for active satellite)

* VII   a)   Double results of VI if transmitter and receiver are at same site (Passive satellites or belts)

b)   Do an additional calculation for other link.

*   Recall note on p.22.

APPENDIX

Editor's Notes

p. 4 Section 6

The iterative process

$$E = M + e \sin E$$

has been replaced by the highly convergent procedure

$$M_i = E_i - e \sin E_i$$

$$\Delta E_i = \frac{M - M_i}{1 - e \cos E_i}$$

$$E_{i+1} = E_i + \Delta E_i$$

The initial value E_0 is found either by the series expansion of E as shown or by a linear prediction based upon the last two calculated values of E.

See "Celestial Mechanics" by Forest Ray Moulton, the MacMillan Co., New York (1959) p. 164.

p. 6 Section 2

The right ascension of the site at time T is found by the coordinate conversion program using stored tables of apparent sidereal times. This value is available to other programs of the pointing system, including the belt and satellite programs.

p. 7 Section 3, 7

The coordinate conversion program computes during system initialization, the declination (geocentric latitude) and the distance from the geocenter to the radar plane.

p. 8 Section 8, 9

The satellite program does not calculate a visibility zone. Whenever called, it computes the celestial coordinates of the satellite for the time given the program.

p. 9

The method of determining the visible portion of a belt given here is not used. Rather, when there are two possible points on a belt (as may happen in the fixed declination and the scan schedules), that point is chosen whose geocentric radius vector makes the smaller angle with the geocentric radius vector to the site.

p. 14 Section 1

SAO mean elements are not used by the belt program. Another set of mean elements determined by special purpose programs at Lincoln Laboratory is used. Only ω and Ω in this set have first derivatives, i.e.

$$\omega = \omega_o + \dot\omega\, t$$

$$\Omega = \Omega_o + \dot\Omega\, t \quad .$$

The other elements are fixed (over a run).

p. 16 Section B

Indeed, belt scan is $\dfrac{du}{dt} = K$

p. 17 Section B-c and B-d

The belt program looks at the radar elevation of the point of the belt as computed by the coordinate conversion program. Whenever the elevation goes from positive to negative, the sign of $\dfrac{du}{dt}$ is changed.

p. 23 Equation 5a

This equation has been replaced by

$$\dot r = \frac{a\, e\, \sin v}{\sqrt{1 - e^2}}\ \dot M$$

which can be gotten from William's expression after very tedious algebra or by differentiating

$$r = a(1 - e \cos E)$$

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p. 24 Equation 7

Because of scaling difficulties, this has been replaced in the program by

$$\dot{\alpha} \cos \delta = \dot{\Omega} \cos \delta + \frac{\cos i}{\cos \delta} \dot{u}$$

The original expression was

$$\dot{\alpha} = \frac{\cos \Omega \, \sin u + \cos i \sin \Omega \, \cos u}{\sin \alpha \, \cos \delta} \dot{u} - \cot \alpha \, \tan \delta \, \dot{\delta} + \dot{\Omega}$$

p. 25 Sections III, IV, V, VI, VII

The calculation of range and range rate is performed by the coordinate conversion program. Doppler is calculated by the interpolation program.

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13. ABSTRACT

Satellites are implicitly completely described by their orbital elements. The conditions for going from the mean orbital elements of a satellite to osculating elements considering perturbations caused by the ellipsoidal earth, and then to celestial coordinates and their rates of change are derived. For belts, it is necessary to fix a point at which it is desired to direct an antenna. This is done by taking the intersection of a right-ascension half-plane, a geodetic longitude half-plane, a declination cone, or a time varying central angle vector with the belt. The equations for determining this point from the mean orbital elements and the intersecting surface or ray are derived.

14. KEY WORDS

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