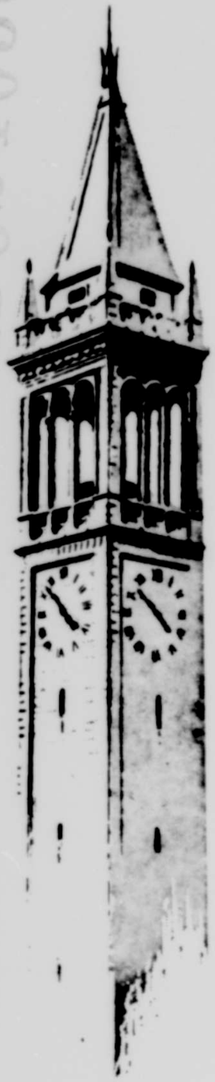

TWO-BEAM DECKHOUSE THEORY WITH SHEAR EFFECTS

by

H. A. Schade

Department of the Navy
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ABSTRACT

The paper deals with the structural response in vertical bending and shear, of the upright ship hull with a deckhouse. The basis for the analysis is that of considering each component as a thin-walled Navier beam and accounting for

a) vertical loading of the deckhouse by a foundation modulus "k" in the usual manner;
b) vertical shear deflection of each component in addition to bending deflection;
c) non-equality of longitudinal strain at deck-edge and deckhouse connection by applying shear lag theory to the deck.

The superstructure is handled as a special case, with only b) above being applicable.

Design charts are presented, based on the solution to the differential equation, for computing (1) longitudinal stresses at midlength, (2) shear flow at bond, and (3) differential deflection of the two components.

It is indicated that the three items (a), (b) and (c) may be of the same order of importance.

A few preliminary experimental results are included.

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Basic Nomenclature

The external loading is applied to the primary member (the hull) identified by the subscript 1; the "following" member (deckhouse or superstructure) is identified by the subscript f.

σ_1 and σ_f	longitudinal stress in sides (x-direction)
$\bar{\sigma}_1$ and $\bar{\sigma}_f$	longitudinal stress in sides at deck (x-direction)
p_1 and p_f	average longitudinal stress (x-direction)
M_1 and M_f	stress moment
S_1 and S_f	section modulus referred to deck
E_1 and E_f	material modulus - normal
G_1 and G_f	material modulus - shear
A_1 and A_f	effective section area including webs and flanges ($A = A_1 + A_f$)
a_1 and a_f	section area (webs only)
I_1 and I_f	section moment of inertia
q_1 and q_f	vertical load/unit length ($q = q_1 + q_f$)
w_1 and w_f	vertical deflection (+ down)
Q_1 and Q_f	vertical shear force ($Q = Q_1 + Q_f$)
e_1 and e_f	distance from bond to individual N.A. ($e = e_1 + e_f$)
z_1 and z_f	vertical co-ordinate distance from individual N.A. (+ down)
2b	beam of hull
y	transverse coordinate distance from ℄
k	spring constant of hull at bond
$\bar{N}$	longitudinal shear flow in deckhouse at bond
Ⓚ	shear deflection factor
M	bending moment generated by external forces

m	constant component of bending moment
a	amplitude of sinusoidal component of bending moment
F_x	longitudinal external force on hull
2λ	length of deckhouse
ρ	effectiveness ratio

Note: The asterisk* indicates modification to symbols to account for difference in material moduli, thus:

$$A_f^* = \frac{E_f}{E_1} A_f$$

$$S_f^* = \frac{E_f}{E_1} S_f$$

$$A^* = A_1 + A_f^*$$

$$I^* = e_1 S_1 + e_f S_f^* + \frac{A_1 A_f^*}{A^*} e^2$$

$$= I_1 + I_f^* + \frac{A_1 A_f^* e^2}{A^*}$$

Parameters Used for Computations

$$B = \frac{A_1 r S_f^*}{(A_1 + r A_f^*) S_1 S_f + A_1 A_f (e_f S_1 + e_1 r S_f^*)}, \qquad \text{(Eq. 7)}$$

$$C = \frac{E_f}{E_1} \frac{A_1 (r e_1 + e_f)}{(A_1 + r A_f^*)(I_1 + I_f^*) + A_1 A_f^* e (r e_1 + e_f)}, \qquad \text{(Eq. 15)}$$

$$\delta = 1 + \frac{\eta}{\omega^2}\left(\frac{\pi}{2\lambda}\right)^2 + \frac{1}{4\omega^4}\left(\frac{\pi}{2\lambda}\right)^4, \qquad \text{(Eq. 17)}$$

$$\theta = 1 + \frac{E_1}{r e_1 + e_f}\left(\frac{I_1 e_f}{a_1 G_1} + \frac{r I_f^* e_1}{a_f G_f}\right)\left(\frac{\pi}{2\lambda}\right)^2 + \frac{E_1}{r e_1 + e_f} \frac{r I_f^* e_1}{k}\left(\frac{\pi}{2\lambda}\right)^4, \qquad \text{(Eq. 17)}$$

$$\omega = \sqrt[4]{\frac{k}{4 r I_f^*} \frac{(r e_1 + e_f)}{E_1 e_1} \frac{B}{C}}, \qquad \text{(Eq. 15)}$$

$$\eta = \frac{1}{2}\left(\frac{1}{a_1 G_1} + \frac{1}{a_f G_f}\right)\sqrt{\frac{k E_1 r I_f^* e_1}{(r e_1 + e_f)} \frac{C}{B}}, \qquad \text{(Eq. 15)}$$

$$r = \frac{1}{2} \frac{1}{\cosh \frac{\pi b}{2\lambda}}\left(\frac{\pi y}{2\lambda} \sinh \frac{\pi y}{2\lambda} + 2 \cosh \frac{\pi y}{2\lambda} - \frac{\pi b}{2\lambda} \tanh \frac{\pi b}{2\lambda} \cosh \frac{\pi y}{2\lambda}\right), \qquad \text{(Eq. 1a)}$$

ψ, φ — Parameters for numerical solution for $p_f \big|_\lambda$ (See Figures 7 and 8)

$(2\lambda\psi'), (2\lambda\varphi')$ — Parameters for numerical solution for $\bar{N}$ (See Figures 9 to 16)

$(4\lambda^2\psi''), (4\lambda^2\varphi'')$ — Parameters for numerical solution for $(w_1 - w_f)$ (See Figures 17 to 24)

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I. Introduction

The treatment of the influence of deckhouses and superstructures upon the bending strength and stiffness of the hull girder for design purposes is far from standardized. Classification Society Rules are not uniform and in part are obscure on this point. Nevertheless, the fact that non-linear distribution of longitudinal primary bending stresses may exist across the total section when a deckhouse-superstructure is present is now well known. When the deckhouse-superstructure is aluminum on a steel hull an additional complication is introduced. The questions of expansion joints and "wobble plates" remain troublesome; both have as objective the suppression of cracking of plates towards the ends of the structure, but these measures are not universally successful.

The problem dealt with here is limited to the primary response of the structure, which is symmetrical about the vertical centerline plane.

For the purpose of avoiding repetitive descriptions, the following (not universally accepted) terminology is used:

a) A deckhouse, with side plating not coplanar with the ship sides so that the vertical deflections of the two are not necessarily the same;

b) a superstructure, where the side plating and main hull sides are coplanar so that identical displacement is forced on both parts at the bond between them.

The word deckhouse is usually used here to describe both where the difference is immaterial. The superstructure may, in fact, be regarded as a special case of the deckhouse.

The two components of the combination of hull and deckhouse or superstructure are treated as if each were a Navier beam. This is thus an extension of the scheme used by Bleich (Ref. 15) and others, but differs from it in the following ways:

a) The boundary conditions at the deckhouse ends differ from those used by Bleich, and seem closer to reality.

b) Shear displacement is accounted for in the sides of both hull and deckhouse. In a superstructure, this is the only influence which distinguishes the composite section behavior from single-beam behavior, since the curvatures of the two components are the same.

c) The treatment of a deckhouse of material different from that of the hull is included in the nomenclature system.

d) The effect of shear lag in the decks between deck-edge and deckhouse side is included.

The focus of interest is on the magnitude and distribution of longitudinal stress in each component, the corresponding longitudinal shear flow at the bond between the two components, and their relative vertical deflection. In particular the question of the effect of the presence of the deckhouse on the longitudinal strength of the hull is important, i.e., the rate of diffusion of longitudinal stress into the deckhouse away from its ends, with consequent modification of the hull longitudinal stresses. This question seems to be most directly studied by making the <u>average</u> longitudinal stress in the deckhouse the primary dependent variable; once this is obtained, the other stresses and deflections can be obtained from it. This has also the advantage of being consistent with shear lag theory, and in fact the same basic nomenclature system is used.

II. Fundamental Assumptions

Assume that the longitudinal connection or bond between deckhouse side and deck is rigid (welded or equivalent), without slip, and assume that longitudinal stresses are related to longitudinal strains there by $\sigma = E\varepsilon$. This latter is equivalent to the assumption that Poisson's ratio μ is zero there, or that longitudinal stresses are unaffected by girth stresses.

The longitudinal stress in the deck at this bond and the longitudinal stress in the deck-edge may differ. Simple beam theory implies they are the same (if camber is neglected); shear lag in the deck plating may modify this.

These assumptions are equivalent to

$$\frac{\bar{\sigma}_f}{E_f} = r\,\frac{\bar{\sigma}_1}{E_1} \tag{1}$$

where r is the ratio of longitudinal deck stress at the bond to the stress at deck-edge.

Precise determination of r would require two-dimensional elastic analysis of the response of all plate elements of the section. For the present purpose assume that r has the same value it would have without the deckhouse, determined by a box-girder analysis of the hull alone, such as that in Ref. 35. According to this analysis, for a sinusoidal bending moment of length 2λ,

$$r = \frac{1}{2}\,\frac{1}{\cosh\frac{\pi b}{2\lambda}}\left[\frac{\pi y}{2\lambda}\sinh\frac{\pi y}{2\lambda} + 2\cosh\frac{\pi y}{2\lambda} - \frac{\pi b}{2\lambda}\tanh\frac{\pi b}{2\lambda}\cosh\frac{\pi y}{2\lambda}\right] \tag{1a}$$

For a bending moment which is constant over the length of the deckhouse r should of course be taken as unity.

For convenience, each of the component longitudinal stresses is decomposed into an average stress and a bending stress:

$$\sigma_1 = p_1 + z_1 \frac{M_1}{I_1}$$
$$\sigma_f = p_f + z_f \frac{M_f}{I_f} \qquad (2a)$$

so that in terms of the component section moduli, the stresses at the deck are

$$\bar{\sigma}_1 = p_1 - \frac{M_1}{S_1}$$
$$\bar{\sigma}_f = p_f + \frac{M_f}{S_f} \qquad (2b)$$

Assume also that the vertical unit load transmitted to the deckhouse through the bond may be represented by

$$q_f = k\,(w_1 - w_f) \qquad (3)$$

where k is not a function of x. It has the dimensions of a stress. Determination of its value is discussed in Appendix III.

III. Equilibrium Relations

Longitudinal force equilibrium across the total section requires

$$A_1 p_1 + A_f p_f = F_x \tag{4}$$

Moment equilibrium across a total section requires (if the resultant of F_x is at the centroid of the hull section)

$$M_1 + M_f - A_f e\, p_f = M \tag{5}$$

From (1), (2b), (4), and (5) eliminate $\bar{\sigma}_1$, $\bar{\sigma}_f$ and p_1 to get the component stress moments as functions of p_f, M and F_x.

$$M_1 = \frac{S_1}{S_1 - rS_f^*}\left[\frac{1}{r}\frac{E_1}{E_f}\left(\frac{(A_1 + rA_f^*)rS_f^*}{A_1} + rA_f^* e\right)p_f + M - \frac{rS_f^*}{A_1}F_x\right] \tag{6a}$$

$$M_f = -\frac{rS_f^*}{S_1 - rS_f^*}\left[\frac{1}{r}\frac{E_1}{E_f}\left(\frac{(A_1 + rA_f^*)S_1}{A_1} + rA_f^* e\right)p_f + M - \frac{S_1}{A_1}F_x\right] \tag{6b}$$

In the further development assume that $F_x = 0$, for the sake of simplicity. The stresses generated by the external longitudinal load F_x are normally very small in comparison with the bending stresses generated by the external bending moment M.

The formula (6b) can also be written in another way:

$$M_f - A_f e_f p_f = -\frac{rS_f^*}{S_1 - rS_f^*}\left(\frac{p_f}{B} + M\right) \tag{7}$$

where

$$B \equiv \frac{A_1 r S_f^*}{(A_1 + rA_f^*) s_1 S_f + A_1 A_f (e_f S_1 + e_1 r S_f^*)} ,$$

and

$$B = \frac{A_1 I_f^* e_1}{A^* I_1 I_f + A_1 A_f (I_1 e_f^2 + I_f^* e_1^2)} \quad \text{for} \quad r = 1 .$$

Consider now the limiting (and structurally unrealistic) case with k = 0; moment equilibrium acrosss a deckhouse section would require

$$M_f - A_f e_f p_f = 0$$

Accordingly, from (7)

$$p_f = - B \cdot M$$

This result is identical with Bleich (Ref. 15, equation 3) if $E_f = E_1$ and $r = 1$, since neither of the other two differences listed in the introduction have been introduced up to this point. This limiting case has no practical importance, but the parameter B is useful in the further development.

Returning to the general case, the deck stress can also be obtained from (1), (2b), (4) and (5) by eliminating M_1 , M_f and p_1

$$\bar{\sigma}_1 = - \frac{1}{s_1 - r S_f^*} \left[\left(\frac{A_f}{A_1} s_1 + S_f + A_f e \right) p_f + M \right] \tag{8}$$

Moment and vertical force equilibrium in the two parts separately and in the total configuration require (see Fig. 1):

$$Q_f = M_f' - A_f e_f p_f' \tag{9a}$$

$$Q_f' = -q_f \tag{9b}$$

$$Q_1' = -q_1 \tag{9c}$$

$$Q_1 + Q_f = M' \tag{9d}$$

Longitudinal force equilibrium of an element of the deckhouse determines the shear flow at the bond with the deck:

$$\bar{N} = -A_f p_f' \tag{10}$$

From (7) and (9a) eliminate M_f to get vertical shear in the deckhouse

$$Q_f = -\frac{r S_f^*}{S_1 - r S_f^*}\left(\frac{p_f'}{B} + M'\right) \tag{11}$$

and this combined with (3) and (9b) yields

$$k(w_1 - w_f) = \frac{r S_f^*}{S_1 - r S_f^*}\left(\frac{p_f''}{B} + M''\right) \tag{12}$$

This equation (12) furnishes the deflection difference as a function of p_f for the case of a deckhouse, where $0 < k < \infty$.

To get the solution for p_f an additional relation is necessary.

IV. Navier Bending

Assume that both parts behave as Navier beams with shear deflection included. The standard equations for this are

$$M_1 = -E_1 I_1 \left(w_1'' + \frac{q_1}{a_1 G_1} \right) \tag{13a}$$

$$M_f = -E_f I_f \left(w_f'' + \frac{q_f}{a_f G_f} \right) \tag{13b}$$

The factor (k) to account for non-uniform vertical shear is included in a_1 and a_f, except where otherwise indicated, in order to simplify the symbols. See Appendix IV for a discussion of its magnitude.

Eliminate M_1, M_f, Q_1, q_1 and q_f from these and from (6), (9) and (11) to get

$$\left(\frac{1}{a_1 G_1} + \frac{1}{a_f G_f}\right) \frac{r S_f^*}{B} p_f'' - \frac{(A_1 + r A_f^*)(I_1 + I_f^*) + A_1 A_f^* e (r e_1 + e_f)}{A_1 E_f e_1 e_f} p_f$$
$$= \frac{1}{E_1} \frac{(r e_1 + e_f)}{e_1 e_f} M - \left(\frac{S_1}{a_1 G_1} + \frac{r S_f^*}{a_f G_f}\right) M'' + (S_1 - S_f^* r)(w_1'' - w_f'') \tag{14}$$

For a <u>superstructure</u> this furnishes the differential equation in p_f directly, since the last term is identally zero. The particular integral is a function not only of M but also of its second derivative. See Appendix II for solutions.

V. Differential Equation for Deckhouse

For the general case ($0 < k < \infty$) of a deckhouse, however, a fourth-order differential equation results from eliminating the deflection difference ($w_1 - w_f$) from (12) and (14):

$$\frac{1}{4\omega^4} p_f^{IV} - \frac{\eta}{\omega^2} p_f'' + p_f = -C\Big[M - \Big(\frac{I_1 e_f}{a_1 G_1} + \frac{r I_f^* e_1}{a_f G_f}\Big)\frac{E_1}{re_1 + e_f} M'' + \frac{r I_f^* e_1}{k}\frac{E_1}{re_1 + e_f} M^{IV}\Big] \quad (15)$$

where

$$C \equiv \frac{E_f}{E_1}\frac{A_1(re_1 + e_f)}{(A_1 + rA_f^*)(I_1 + I_f^*) + A_1 A_f^* e(re_1 + e_f)} ,$$

$$\omega^4 \equiv \frac{k}{4 r I_f^*}\frac{(re_1 + e_f)}{E_1 e_1}\frac{B}{C} ,$$

$$\eta \equiv \frac{k}{4}\Big(\frac{1}{a_1 G_1} + \frac{1}{a_f G_f}\Big)\frac{1}{\omega^2} = \frac{1}{2}\Big(\frac{1}{a_1 G_1} + \frac{1}{a_f G_f}\Big)\sqrt{\frac{k E_1 r I_f^* e_1}{(re_1 + e_f)}\frac{C}{B}} ,$$

$$\frac{\eta}{\omega^2} \equiv \Big(\frac{1}{a_1 G_1} + \frac{1}{a_f G_f}\Big)\frac{E_1 r I_f^* e_1}{(re_1 + e_f)}\frac{C}{B} .$$

If shear lag in the deck is non-existent, or is ignored, so that $r = 1$,

$$C = \frac{E_f}{E_1}\frac{A_1 e}{A^* I^*} \quad (15a)$$

The section modulus of the composite section treated as a single beam, referred to the neutral axis of the house, is the reciprocal of this, so that C is easily calculated by normal naval architecture methods for this case.

Usable solutions can be obtained for many cases by assuming that over the length of the deckhouse the bending moment can be represented by a component constant in x plus a sinusoidal component, i.e., let

$$M = m + a \sin \frac{\pi x}{2\lambda} \quad . \tag{16}$$

If this does not reasonably represent the real bending moment, a finite or an infinite series may be used for the second term.

With this two-term assumed form of bending moment, the differential equation is

$$\frac{1}{4\omega^4} p_f^{\mathrm{IV}} - \frac{\eta}{\omega^2} p_f'' + p_f = -C\left(m + \theta a \sin \frac{\pi x}{2\lambda}\right) \tag{17}$$

where

$$\theta \equiv 1 + \frac{E_1}{re_1 + e_f}\left(\frac{I_1 e_f}{a_1 G_1} + \frac{r I_f^* e_1}{a_f G_f}\right)\left(\frac{\pi}{2\lambda}\right)^2 + \frac{E_1}{re_1 + e_f}\,\frac{r I_f^* e_1}{k}\left(\frac{\pi}{2\lambda}\right)^4 .$$

Its particular integral is

$$-C\left[m + \frac{\theta}{\delta} a \sin \frac{\pi x}{2\lambda}\right]$$

where

$$\delta \equiv 1 + \frac{\eta}{\omega^2}\left(\frac{\pi}{2\lambda}\right)^2 + \frac{1}{4\omega^4}\left(\frac{\pi}{2\lambda}\right)^4$$

VI. Boundary Conditions at Ends of Deckhouse

The deck material between the deckhouse sides is part of the hull, and is included in the hull section, section area A_1. Under these circumstances the ends of the structure which is counted as part of the deckhouse section area A_f are physically free of normal and shear stresses; this requires that

$$\sigma_x \Big|_{0,\,2\lambda} = 0 \qquad (18a)$$

$$\tau \Big|_{0,\,2\lambda} = 0 \qquad (18b)$$

but this is incompatible with the fundamental assumptions at the bond where, because of strain equality in deck and house, σ_x cannot be zero unless M is zero or unless the shear lag distribution produces this result. (See Reference 35.)

As an approximation consider the following, which represent necessary but not sufficient conditions to meet the physical requirement of stress-free end surfaces:

$$p_f \Big|_{0,\,2\lambda} = 0 \qquad (19a)$$

$$Q_f \Big|_{0,\,2\lambda} = 0 \qquad (19b)$$

$$M_f \Big|_{0,\,2\lambda} = 0 \qquad (19c)$$

A solution of the fourth-order differential equation (14) may satisfy only four of these six boundary conditions.

According to (6b), the boundary condition (19a) is equivalent to

$$M_f\Big|_0 = -\frac{r S_f^*}{S_1 - r S_f^*} M\Big|_0 \,, \qquad M_f\Big|_{2\lambda} = -\frac{r S_f^*}{S_1 - r S_f^*} M\Big|_{2\lambda} \qquad (20)$$

which contradicts (19c). An arbitrary choice must be made, and we choose the solution which satisfies (19a) and (19b).

Unless M itself is zero at the ends, such a solution is really the solution for a configuration externally loaded not only by M and Q but also by $M_f\Big|_{0,2\lambda}$ applied externally to the deckhouse ends. Since such moments do not physically exist in the problem under study, they must be cancelled by an internal moment generated by vertical forces operating through the bond near the ends, and this additional internal moment will perturb this solution, but only for a short distance from the ends, according to St. Venant's Principle. (See Figure 2.)

Additional perturbations of the theory in the vicinity of the ends must also occur because of the differences between (18) and (19), but these effects are of still less importance than those discussed in the foregoing; they also attenuate rapidly away from the ends.

The boundary condition of zero vertical shear force across the deckhouse ends, represented by (19b), is equivalent to

$$p_f'\Big|_0 = -B \cdot M'\Big|_0 \,, \qquad p_f'\Big|_{2\lambda} = -B \cdot M'\Big|_{2\lambda} \qquad (21)$$

according to (11). For the assumed bending moment form represented by (16) these become

$$
\begin{aligned}
p_f'\Big|_0 &= -B\frac{\pi}{2\lambda}a\ , \\
p_f'\Big|_{2\lambda} &= B\frac{\pi}{2\lambda}a
\end{aligned}
\qquad (22)
$$

and these, together with (19a)

$$p_f\Big|_{0,\,2\lambda} = 0$$

are the boundary conditions used to determine the solution.

VII. Differential Equation Solution

The complete solution for (17) with these boundary conditions, and for a moment described by (16) is:

$$p_f = -C\psi m + \left[(B\delta - C\theta)\varphi - B \sin\frac{\pi x}{2\lambda} \right] a \tag{23}$$

where

$$\psi = \psi\left(\eta, 2\lambda\omega, \frac{x}{2\lambda}\right),$$

$$\varphi = \varphi\left(\eta, 2\lambda\omega, \frac{x}{2\lambda}\right).$$

The dimensionless functions ψ and φ have three different forms, depending on whether (a) $\eta < 1$, (b) $\eta = 1$, or (c) $\eta > 1$. They are given in Appendix I and their values at mid-length, where $x = \lambda$, are plotted on Figures 7 and 8.

The parameters B, C, δ, and θ are those defined by (7), (15) and (17). The first two, B and C, are parameters of the section geometry; the second two, δ and θ, involve not only the section geometry, but the foundation modulus and deckhouse length also.

The stress distribution across each component is in accordance with Navier distribution:

$$\sigma_1 = \left(1 + \frac{z_1}{e_1}\right) p_1 - \frac{z_1}{e_1} \bar{\sigma}_1 ,$$
$$\sigma_f = \left(1 - \frac{z_f}{e_f}\right) p_f + \frac{z_f}{e_f} \bar{\sigma}_f . \tag{24}$$

The shear flow at the bond, from (10) is $\bar{N} = -A_f P_f'$, and therefore

$$\bar{N} = -\frac{A_f}{2\lambda}\left\{-C(2\lambda\psi')m + \left[(B\delta - C\theta)(2\lambda\varphi') - B\pi\cos\frac{\pi x}{2\lambda}\right]a\right\} \tag{25}$$

The dimensionless functions ($2\lambda\psi'$) and ($2\lambda\varphi'$) are given in Appendix I and are plotted on Figures 9 to 16.

Shear flow in the bond at the deckhouse ends is independent of k and of the length 2λ , because of boundary conditions represented by (21):

$$\bar{N}\Big|_0 = A_f\, B\, Q\Big|_0 ,$$
$$\bar{N}\Big|_{2\lambda} = A_f\, B\, Q\Big|_{2\lambda} . \tag{26}$$

Thus it is a function of section geometry and is proportional to the vertical shear force Q. If Q is zero at the ends, $\bar{N}$ is zero there also.

The difference between the deflections of the two components, from (12), is

$$W_1 - W_f = \frac{1}{k}\,\frac{rS_f^*}{S_1 - rS_f^*}\left[\frac{1}{B}P_f'' + M''\right] \tag{27}$$

and is therefore according to the foregoing

$$W_1 - W_f = \frac{1}{4\lambda^2 k B}\,\frac{rS_f^*}{S_1 - rS_f^*}\left\{-C(4\lambda^2\psi'')m + (B\delta - C\theta)(4\lambda^2\varphi'')a\right\}$$

The dimensionless functions $(4\lambda^2\psi'')$ and $(4\lambda^2\varphi'')$ are given in Appendix I, and are plotted on Figures 17 to 24.

VIII. Summary and Discussion

Computation of stress diffusion into the deckhouse at mid-length, represented by the average deckhouse stress $p_f|_\lambda$, is simple and straightforward, using the formula

$$p_f\Big|_\lambda = -C\psi m + \left[(B\delta - C\theta)\varphi - B\right]a$$

with ψ and φ displayed in Figures 7 and 8 as functions of η and $(2\lambda\omega)$. The other symbols are defined in the nomenclature list. The formula is applicable with a bending moment represented by $M = m + a\sin\frac{\pi x}{2\lambda}$.

For a constant bending moment the stress formula reduces to

$$p_f\Big|_\lambda = -C\psi M$$

and, as noted in connection with (15a) this is simply the stress at the neutral axis of the deckhouse alone computed as if the configuration were a single beam, and modified by the factor ψ picked out from Figure 7. For most conventional (non-constant) bending moment forms this may be an adequate approximation also. This needs further investigation.

Appendix III gives a sample calculation to illustrate the procedure, using the complete bending moment formula. The structure is the model referred to in Reference 35, with a deckhouse added. Here k is computed on the basis of treating the transverse deck beams as clamped at the deck-edges.

Figure 3 shows results of similar computations for a constant bending moment, using a unity value of r and several different values of k , including that for a superstructure. For comparison purposes some preliminary experimental results from a model test run are shown also; the experimental points seem to correspond to a value of k lying between 1.3 x 10^3 and 3 x 10^3 psi, but the experimental points are too preliminary to justify any conclusions about the method of calculating k . They are from the extensive experimental program on deckhouse structural effects now being carried out which will be reported on in detail elsewhere. There were no transverse bulkheads in this model.

The question of the approximate value of k (or equivalent) is of course central in this as in all other theories which recognize differential deflection between hull and deckhouse. However, Figure 3 suggests that its determination may not be difficult, at least with simple structures, and that high precision in its value may not be necessary. The matter may become more difficult when the theory is applied to structures with transverse bulkheads, which will be the subject of a later investigation.

The parameter η , which represents the effect of shear deflection, may be equally or more important, particularly with short deckhouses and stiff deck support (large k). In fact, in the superstructure, which represents the limit of stiffness ($k \to \infty$), shear deflection is the sole factor which distinguishes the response of the structure from that which would be given by treating both components as a single beam. The point is illustrated by results of computations for the same model displayed on Figure 4, with all computations made for $k = 3 \times 10^6$ psi. It is shown on Figure 3 that this value of k gives stress results for the model and loading which are indistinguishable from those for a superstructure. Figure 4 shows that for the full-length house shear deflection has no notable effect, but it exhibits the striking magnification of this effect when the house length is successively shortened, as might be done by using expansion joints. These are calculated results only; experiments of this kind are planned for the model in the future.

Appendix IV shows that uniform vertical shear stress distribution in each component is a safe assumption so that the inclusion of vertical shear deflection in the solution of the deckhouse problem presents no difficulties. There seems to be no reason to ignore this potentially important factor in research or design procedures for short and moderate deckhouses.

In most situations shear deflection of the ship girder is of secondary importance; but in the problem under discussion the loading to which each of the two components responds is coupled with the deflection of that component, so that the importance of deflection becomes enhanced and it is no longer admissible to ignore its shear component when beam theory is used.

A computation according to Bleich (Reference 15) was made for each case identified by $\eta = 0$ on Figure 4, with results which are indistinguishable in each case; the differences between boundary conditions used in the two theories apparently cause significant stress differences only toward the ends. This difference at $x/\lambda = 1/11$ is shown by the comparison made on Figure 5.

The direct deck shear lag effect represented by Γ has importance only when the applied bending moment is not constant. Figure 6 shows results of using three different values of Γ with the same model, loaded with a sinusoidal bending moment and compared with a set of preliminary experimental points. Here a variation in Γ affects the deckedge stresses considerably more than the deckhouse top stresses. The formula (1a) yields $\Gamma = 0.81$, which is considerably greater than that implied by the experimental points, which indicate $\Gamma \approx 0.5$. Application of (1a) is a very crude means of estimating Γ, but a much more elaborate analysis is required to improve it. This matter is under continuing investigation.

For the computation of midlength average longitudinal stress $p_f \big|_\lambda$ only the two design curves displayed on Figures 7 and 8 are needed. The design curves for $(2\lambda\psi')$ and $(2\lambda\varphi')$ are displayed for the purpose of computing the shear flow $\bar{N}$ in accordance with (25) and those for $(4\lambda^2\psi'')$ and $(4\lambda^2\varphi'')$ for the purpose of computing the differential deflection $(w_i - w_f)$ in accordance with (27). The shear flow is a function of the first derivative of p_f, and the differential deflection a function of the second derivative of p_f, so that when the assumed form of bending moment $M = m + a \sin\frac{\pi x}{2\lambda}$ is used as an approximation to a real bending moment to obtain a reasonable approximation of stresses, the corresponding calculated values of shear flow and deflection may not be satisfactory approximations. This matter requires further investigation. However, for design purposes the longitudinal stresses are the design criteria, and for this the moment approximation should be adequate.

The experimental points shown on Figures 3 and 6 and mentioned in the discussion are preliminary in nature, and are used only to help illuminate the calculated results. The large experimental program on deckhouse structural effects currently under way will be reported on in other papers; and it may be expected that it will lead to modifications or extensions of the theory and calculation procedures suggested here.

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Mr. Alaa Monsour and Mr. Hsien-Yun Jan contributed greatly to this study; they made all the numerical calculations, and drafted appendices I and III. Mr. Mansour checked the theory as it was developed, and Mr. Jan checked the final manuscript.

REFERENCES

1. Bruhn, J. "The Stresses at the Discontinuities in a Ship's Structure", TRANSACTIONS, I.N.A., 1899, p. 57.

2. King, J. F. "On Large Deckhouses" TRANSACTIONS, I.N.A. Part 1, 1913, p. 148.

3. Montgomerie, J. "The Scantlings of Light Superstructures", TRANSACTIONS, I.N.A. 1915, p. 52.

4. Taylor, J. L. "The Theory of Longitudinal Bending of Ships", TRANSACTIONS, N.E.C.I.E., vol. 41, 1924-25.

5. Petersen, C. E. Hansen, L. "The Design of Superstructures for Large Passenger Ships", TRANSACTIONS, S.N.A.M.E. 1927.

6. Hovgaard, W. "A New Theory of the Distribution of Shearing Stresses in Riveted and Welded Connections and its Application to Discontinuities in the Structure of a Ship", TRANSACTIONS, I.N.A., 1931, p. 108.

7. Hovgaard, W. "Stress Distribution in Longitudinal Welds and Adjoining Structures", J. Math. Phys., vol. 13, 1934, p. 195.

8. Muckle, W. "Some Considerations in the Application of Light Alloys to Ship Construction", TRANSACTIONS, N.E.C.I., 1943-44.

9. Vasta, J. "Structural Tests on the Liberty Ship S.S. Philip Schuyler", TRANSACTIONS, S.N.A.M.E., vol. 55, 1947, p. 391.

10. Muckle, W. "Experiments on a Light Alloy Model Superstructure", TRANSACTIONS, N.E.C.I., 1948-49.

11. Holt, M. "Structural Test of Models Representing a Steel Ship Hull with Aluminum Alloy and Steel Superstructures", S.N.A.M.E. New England Section, March 1949.

12. Vasta, J. "Structural Tests on the Passenger Ship S.S. President Wilson. Interaction between Superstructure and Main Hull Girder", TRANSACTIONS, S.N.A.M.E., vol.57, 1949, p. 253.

13. Crawford, L. "Theory of Long Ships' Superstructures", TRANSACTIONS, S.N.A.M.E. 1950, vol. 58, p. 693.

14. Muckle, W. "The Scantlings of Long Deckhouses Constructed of Aluminum Alloy", TRANSACTIONS, I.N.A., 1952.

15. Bleich, H. H. "Non-Linear Distribution of Bending Stresses due to Distortion of the Cross-Section", J. of Appl. Mach., vol. 20, 1953, p. 95.

16. Jasper, N. J. "Study of the Strain and Motions of the USCGC Casco at Sea", D.T.M.B. Report No. 781, May 1953.

17. Schade, H. A. "Effective Breadth Concept in Ship-Structure Design", TRANSACTIONS, S.N.A.M.E. vol. 61, 1953.

18. The 11th Research Committee "The Research on the Effect of the Superstructures on the Strength of the Ships". The report of the Ship Research Association of Japan. Tokyo, Japan, No. 5, Sept. 1955.

19. Nissen, H. I. "Deckhuses Medvirken I - Langskibsforbandtet", Nyt Nordisk Forlag. København 1955.

20. Muckle, W. "The Influence of Proportions of the Behavior of Partial Superstructures Constructed of Aluminum Alloy", TRANSACTIONS, I.N.A., 1955, p. 453.

21. Johnson, A. J. "Stresses in Deckhouses and Superstructures", B.S.R.A. Report No. 166, 1955.

22. Crawford, L., & Ruby, W. J. "Final Report (Project SR-121) on Model Tests on Hull-Deckhouses Interaction", (Reed Research, Inc.) for Ship Structure Committee - Serial No. SSC-67, Jan. 1955.

23. Chapman and Sparks "Experiments on Box Girders: Some Contributions to the Theory of Ships Structures, TRANSACTIONS, I.E. & S.S., 1956.

24. Terasawa, I. & Yagi, J. "The Effect of Upper Structure on the Strength of Ship", Part I, Journal of Zosen Kiokai (The Society of Naval Architects of Japan), vol. 100, Feb.1957. Part II, Journal of Zosen Kiokai (The Society of Naval Architects of Japan), vol. 102, Feb. 1958.

25. Caldwell, J. B. "The Effect of Superstructures on the Longitudinal Strength of Ships", TRANSACTIONS, I.N.A., March 1957.

26. Johnson, A. J. "Stresses in Deckhouses and Superstructures", TRANSACTIONS, I.N.A. March 1957.

27. Chapman, J. C. "The Interaction between a Ship's Hull and a Long Superstructure", TRANSACTIONS, I.N.A., March 1957.

28. Lyman, P. T. "A Proposed Method for Obtaining the Deckhouse Foundation Modulus for use with the Bleich's Theory", University of California, Berkeley (Unpublished), 1959.

29. Schokker, J. C. and others "Deckhouse and Hull Interaction" chapter of "The Design of Merchant Ships", 2nd Edition, 1959, Haarlem, Holland.

30. Jaeger, H. E. "The Strength Calculation of Superstructures and Deckhouses", B.S.R.A. Journal, June 1960.

31. Jaeger, H.E. & Woortmen, J.J. "Some New Factors in the Calculation of the Interaction Between a Ship's Hull and a Long Deckhouse", TRANSACTIONS, R.I.N.A., 1961.

32. Chapman, J. C. "Behavior of Long Deckhouses", TRANSACTIONS, I.N.A., 1961.

33. Johnson, A.J. & Ayling, P.W. "Measurements and Predictions of the Influence of Deckhouses on the Strength of Ships", TRANSACTIONS N.E.C.I.E. & S., Jan. 1962.

34. Muckle, W. "The Influence of Large Side Openings on the Efficiencies of Superstructures", TRANSACTIONS, R.I.N.A., 1962.

35. Schade, H.A. "Thin-Walled Box Girder--Theory and Experiment", Schiff und Hafen, Heft 1, 1965.

Appendix I

Solution of (17) to meet the prescribed boundary conditions follows standard procedure and is not repeated here.

The dimensionless functions of ψ , φ , $(2\lambda\psi')$, $(2\lambda\varphi')$, $(4\lambda^2\psi'')$ and $(4\lambda^2\varphi'')$ have three different forms, depending on whether a) $\eta < 1$, b) $\eta = 1$, or c) $\eta > 1$.

a) $\eta < 1$

$$\psi = 1 - \frac{1}{\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sin 2\lambda\beta}\Big[\sinh\alpha(2\lambda - x)\cos\beta x + \frac{\alpha}{\beta}\cosh\alpha(2\lambda - x)\sin\beta x + \sinh\alpha x\cos\beta(2\lambda - x) + \frac{\alpha}{\beta}\cosh\alpha x\sin\beta(2\lambda - x)\Big] ,$$

$$\varphi = \frac{1}{\delta}\Big[\sin\frac{\pi x}{2\lambda} - \Big(\frac{\pi}{2\lambda}\Big)\frac{\sinh\alpha x\sin\beta(2\lambda - x) + \sin\beta x\sinh\alpha(2\lambda - x)}{\beta(\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sin 2\lambda\beta)}\Big],$$

$$(2\lambda\psi') = -\frac{2\lambda(\beta + \frac{\alpha^2}{\beta})}{\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sin 2\lambda\beta}\Big[\sinh\alpha x\sin\beta(2\lambda - x) - \sinh\alpha(2\lambda - x)\sin\beta x\Big],$$

$$(2\lambda\varphi') = -\frac{\pi}{\delta}\Big\{\frac{1}{\beta(\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sin 2\lambda\beta)}\Big[\beta\Big(\cos\beta x\sinh\alpha(2\lambda - x) - \sinh\alpha x\cos\beta(2\lambda - x)\Big) + \alpha\Big(\cosh\alpha x\sin\beta(2\lambda - x) - \sin\beta x\cosh\alpha(2\lambda - x)\Big)\Big] - \cos\frac{\pi x}{2\lambda}\Big\},$$

$$(4\lambda^2\psi'') = 4\lambda^2 \frac{\alpha^2+\beta^2}{\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sin 2\lambda\beta}\Big[\sinh\alpha(2\lambda-x)\cos\beta x$$

$$+\sinh\alpha x\cos\beta(2\lambda-x) - \frac{\alpha}{\beta}\cosh\alpha(2\lambda-x)\sin\beta x$$

$$-\frac{\alpha}{\beta}\cosh\alpha x\sin\beta(2\lambda-x)\Big] ,$$

$$(4\lambda^2\varphi'') = \frac{\pi}{\delta}\Big\{\frac{2\lambda}{\beta(\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sin 2\lambda\beta)}\Big[2\alpha\beta\cosh\alpha x\cos\beta(2\lambda-x)$$

$$+2\alpha\beta\cos\beta x\cosh\alpha(2\lambda-x) - (\alpha^2-\beta^2)\sinh\alpha x\sin\beta(2\lambda-x)$$

$$-(\alpha^2-\beta^2)\sin\beta x\sinh\alpha(2\lambda-x)\Big] - \pi\sin\frac{\pi x}{2\lambda}\Big\} ,$$

where $\alpha^2 \equiv \omega^2(1+\eta)$, $\beta^2 \equiv \omega^2(1-\eta)$.

b) $\eta = 1$

$$\psi = 1 - \frac{1}{\sinh 2\lambda\gamma + 2\lambda\gamma}\Big[(\sinh 2\lambda\gamma + 2\lambda\gamma)\cosh\gamma x$$

$$-\gamma x(1-\cosh 2\lambda\gamma)\cosh\gamma x - (1-\cosh 2\lambda\gamma)\sinh\gamma x$$

$$-\gamma x\sinh 2\lambda\gamma\sinh\gamma x\Big] ,$$

$$\varphi = \frac{1}{\delta}\Big[\sin\frac{\pi x}{2\lambda} - \frac{\pi}{2\lambda}\frac{(2\lambda-x)\sinh\gamma x + x\sinh\gamma(2\lambda-x)}{\sinh 2\lambda\gamma + 2\lambda\gamma}\Big] ,$$

$$(2\lambda\psi') = -\frac{2\lambda\gamma^2}{\sinh 2\lambda\gamma + 2\lambda\gamma}\Big[(2\lambda-x)\sinh\gamma x - x\sinh\gamma(2\lambda-x)\Big] ,$$

$$(2\lambda\varphi') = -\frac{\pi}{\delta}\left\{\frac{1}{\sinh 2\lambda\gamma + 2\lambda\gamma}\left[\sinh\gamma(2\lambda - x) - \sinh\gamma x\right.\right.$$

$$\left.\left. + (2\lambda - x)\gamma\cosh\gamma x - \gamma x\cosh\gamma(2\lambda - x)\right] - \cos\frac{\pi x}{2\lambda}\right\},$$

$$(4\lambda^2\psi'') = \frac{4\lambda^2\gamma^2}{\sinh 2\lambda\gamma + 2\lambda\gamma}\left[\sinh\gamma(2\lambda - x) - (2\lambda - x)\gamma\cosh\gamma x\right.$$

$$\left. + \sinh\gamma x - \gamma x\cosh\gamma(2\lambda - x)\right],$$

$$(4\lambda^2\varphi'') = \frac{\pi}{\delta}\left\{\frac{2\lambda\gamma}{\sinh 2\lambda\gamma + 2\lambda\gamma}\left[2\cosh\gamma x + 2\cosh\gamma(2\lambda - x)\right.\right.$$

$$\left.\left. -(2\lambda - x)\gamma\sinh\gamma x - \gamma x\sinh\gamma(2\lambda - x)\right] - \pi\sin\frac{\pi x}{2\lambda}\right\},$$

where $\gamma^2 = 2\omega^2$.

c) $\eta > 1$

$$\psi = 1 - \frac{1}{\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sinh 2\lambda\beta}\left[\sinh\alpha(2\lambda - x)\cosh\beta x\right.$$

$$+ \frac{\alpha}{\beta}\cosh\alpha(2\lambda - x)\sinh\beta x + \sinh\alpha x\cosh\beta(2\lambda - x)$$

$$\left. + \frac{\alpha}{\beta}\cosh\alpha x\sinh\beta(2\lambda - x)\right],$$

$$\varphi = \frac{1}{\delta}\left\{\sin\frac{\pi x}{2\lambda} - \frac{\pi}{2\lambda}\,\frac{\sinh\beta x\sinh\alpha(2\lambda - x) + \sinh\alpha x\sinh\beta(2\lambda - x)}{\beta(\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sinh 2\lambda\beta)}\right\},$$

$$(2\lambda\psi') = -\frac{2\lambda(\beta - \frac{\alpha^2}{\beta})}{\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sinh 2\lambda\beta}\left[\sinh\alpha(2\lambda - x)\sinh\beta x\right.$$

$$\left. - \sinh\alpha x\sinh\beta(2\lambda - x)\right],$$

$$(2\lambda\varphi') = -\frac{\pi}{\delta}\left\{\frac{1}{\beta(\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sinh 2\lambda\beta)}\Big[\beta\big(\cosh \beta x \sinh \alpha(2\lambda - x) - \sinh \alpha x \cosh \beta(2\lambda - x)\big) + \alpha\big(\cosh \alpha x \sinh \beta(2\lambda - x) - \sinh \beta x \cosh \alpha(2\lambda - x)\big)\Big] - \cos\frac{\pi x}{2\lambda}\right\},$$

$$(4\lambda^2\psi'') = 4\lambda^2\frac{\alpha^2 - \beta^2}{\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sinh 2\lambda\beta}\Big[\sinh \alpha(2\lambda - x)\cosh \beta x + \sinh \alpha x \cosh \beta(2\lambda - x) - \frac{\alpha}{\beta}\cosh \alpha(2\lambda - x)\sinh \beta x - \frac{\alpha}{\beta}\cosh \alpha x \sinh \beta(2\lambda - x)\Big],$$

$$(4\lambda^2\varphi'') = \frac{\pi}{\delta}\left\{\frac{2\lambda}{\beta(\sinh 2\lambda\alpha + \frac{\alpha}{\beta}\sinh 2\lambda\beta)}\Big[2\alpha\beta \cosh \alpha x \cosh \beta(2\lambda - x) + 2\alpha\beta\cosh \beta x \cosh \alpha(2\lambda - x) - (\alpha^2 + \beta^2)\sinh \alpha x \sinh \beta(2\lambda - x) - (\alpha^2 + \beta^2)\sinh \beta x \sinh \alpha(2\lambda - x)\Big] - \pi \sin\frac{\pi x}{2\lambda}\right\},$$

where $\alpha^2 \equiv \omega^2(\eta + 1)$, $\beta^2 \equiv \omega^2(\eta - 1)$.

These six solutions parameters are plotted on Figures 10 to 27. Only the mid-length ($x = \lambda$) values of ψ and φ are plotted, because the longitudinal stress values reach their maximum values there for the forms of bending moment used. Values of the first and second derivatives, however, are shown over the deckhouse length, because the shear flow and deflection difference may be of interest anywhere along the length.

Appendix II

For a superstructure the last term in (14) vanishes since the curvatures of the two parts are the same ($w_1'' = w_f''$).

For a deckhouse there is an alternative phantom, which was referred to in Reference 17. The vertical forces exerted through the bond because of the foundation modulus k of the structure on which the deckhouse rests, if operating alone, would give the deckhouse a curvature which has the same sign as the curvature of the hull. On the other hand the shear forces transmitted to the deckhouse through the bond, if operating alone, would give the deckhouse a curvature opposite in sign to the curvature of the hull. One plausible hypothesis therefore assumes that these two curvatures cancel, and that the deckhouse does not bend at all. A corresponding differential equation in p_f can be easily obtained.

From (6b),(9b), (11), and (13b) (with $w_f'' = 0$) eliminate M_f, q_f, and Q_f

$$-\left(\frac{1}{a_f G_f}\right)\frac{r S_f^*}{B} p_f'' + \frac{S_1(A_1 + rA_f^*) + A_1 r A_f^* e}{A_1 E_f e_f} p_f = -\frac{r}{E_1 e_f} M + \frac{r S_f^*}{a_f G_f} M'' .$$

This and the superstructure equation (14) (with $w_1'' = w_f''$) have the same form; with the same assumed form of bending moment

$$M = m + a \sin \frac{\pi x}{2\lambda} ,$$

the differential equation covering both situations can be written

$$-\frac{1}{K^2} P_f'' + P_f = -C\left[m + \theta a \sin\frac{\pi x}{2\lambda}\right] ,$$

where for a superstructure

$$K^2 \equiv \frac{B[(A_1 + A_f^* r)(I_1 + I_f^*) + A_1 A_f^* e(e_f + re_1)]}{E_f\left(\frac{1}{a_1 G_1} + \frac{1}{a_f G_f}\right) e_1 A_1 r I_f^*} ,$$

$$C \equiv \frac{E_f}{E_1} \frac{A_1(re_1 + e_f)}{(A_1 + rA_f^*)(I_1 + I_f^*) + A_1 A_f^* e(re_1 + e_f)} ,$$

$$\theta \equiv 1 + \frac{E_1}{re_1 + e_f}\left(\frac{I_1 e_f}{a_1 G_1} + \frac{r I_f^* e_1}{a_f G_f}\right)\left(\frac{\pi}{2\lambda}\right)^2 ,$$

and for a zero-curvature deckhouse

$$K^2 \equiv \frac{[(A_1 + rA_f^*)S_1 + A_1 A_f^* re] B}{E_f\left(\frac{1}{a_f G_f}\right) r I_f^* A_1} ,$$

$$C \equiv \frac{E_f}{E_1} \frac{rA_1}{[(A_1 + rA_f^*)S_1 + A_1 A_f^* re]} ,$$

$$\theta \equiv 1 + \frac{I_f}{a_f G_f} E_1 \left(\frac{\pi}{2\lambda}\right)^2 .$$

The particular integral is

$$-C\left[m + \frac{\theta a}{1 + \frac{1}{K^2}\left(\frac{\pi}{2\lambda}\right)^2} \sin\frac{\pi x}{2\lambda}\right] .$$

With the boundary condition from (19a)

$$p_f\Big|_{0,2\lambda} = 0$$

the solutions are

$$p_f = -C\left[m\left(1 - \frac{\cosh K(x-\lambda)}{\cosh K\lambda}\right) + \theta a \frac{1}{1 + \frac{1}{K^2}\left(\frac{\pi}{2\lambda}\right)^2} \sin\frac{\pi x}{2\lambda}\right] .$$

Only two (not four) boundary conditions can be met for the superstructure, and for the "no-bending" approximation, since these cases are represented by second-order differential equations. Thus their solutions are really solutions for a configuration loaded not only by M and Q , but also by $M_f\Big|_{0,2\lambda}$ and $Q_f\Big|_{0,2\lambda}$ as well. Both these external loadings must be cancelled by internal vertical forces operating through the bond. The resulting perturbations to the solution must be stronger than for the deckhouse solution, owing to the non-zero values of $Q_f\Big|_{0,2\lambda}$ here.

Appendix III

In the following a sample calculation of the average longitudinal stress p_f is given for the 42-foot model referred to in Reference 35, with a deckhouse added. Suppose the bending moment curve over the length of the deckhouse of length 2λ is represented by

$$M = m + a \sin \frac{\pi x}{2\lambda}$$

with

$$m = 2.51 \times 10^6 \text{ lb.-in.}$$

$$a = 0.925 \times 10^6 \text{ lb.-in.}$$

The deckhouse length is

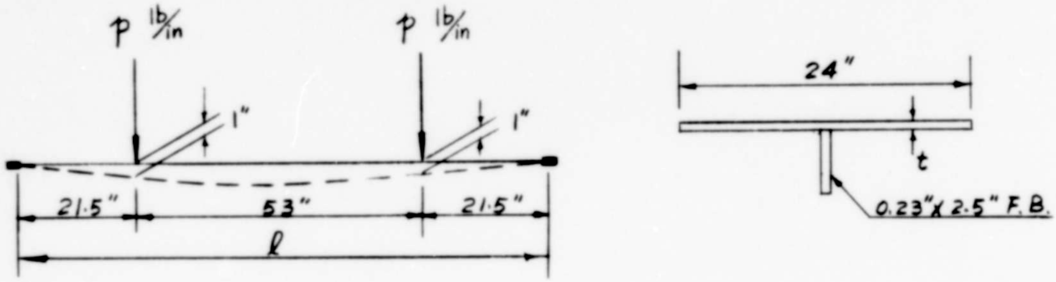
$$2\lambda = 264 \text{ in.}$$

The material properties are

$$E_1 = E_f = 30 \times 10^6 \text{ psi},$$

$$G_1 = G_f = 11.5 \times 10^6 \text{ psi}.$$

The value of k, without the presence of transverse bulkheads, may be estimated by means of the simple beam theory. Consider the deck beam together with the effective deck plating as clamped at the hull sides and loaded with two identical line loads p lb./in. at the deckhouse sides, which produce a unit deflection at the deckhouse sides. Then $k = 2p$ by definition.



Thickness of deck plating (longitudinals diffused) $t = 0.156$ in.

Effectiveness ratio $\rho = 0.58$

Moment of inertia $I = 1.14 \text{ in.}^4$

According to the energy method, p is determined by

$$2 \times \frac{1}{2} \times (24\,p) \times 1 = \frac{1}{2}\int_0^{\ell} \frac{M^2}{EI}\,dx$$

This yields

$$p = 650 \text{ lb./in.}$$

and $k = 2\,p = 1.3 \times 10^3$ psi.

The effective geometric properties and the computation parameters are as follows for constant moment and for sinusoidal moment respectively. The actual deck, bottom, and housetop section areas are modified by application of the shear-lag "effectiveness" ratio ρ to obtain the "effective" geometrical properties. The appropriate values of ρ are obtained from Reference 17. For evaluating ρ, M_1 is assumed to have the same form as that of the imposed moment (either constant or sinusoidal), M_f is assumed to have a sinusoidal form. Consequently, $\rho_f = 0.93$ in both the cases.

	For Constant Moment	For Sinusoidal Moment	
Effective Geometric Properties:			
r	1.0	0.81	
ρ_1	1.0	0.75	
A_1, in.^{2}	50.35	42.12	
a_1, in.^{2}	17.56	17.56	
e_1, in.	29.49	29.19	
I_1, in.^{4}	28,450	23,156	
S_1, in.^{3}	965.4	792.0	
$A_f(A_f^*)$, in.^{2}	15.41	15.41	
a_f, in.^{2}	4.31	4.31	
e_f, in.	9.6	9.6	
$I_f(I_f^*)$, in.^{4}	139.3	139.3	
$S_f(S_f^*)$, in.^{3}	14.5	14.5	
A^*, in.^{2}	65.76	57.53	
e, in.	39.09	38.79	
Calculation parameters:			
B, in.$^{-3}$	0.866×10^{-4}	0.854×10^{-4}	(Eq. 7)
C, in.$^{-3}$	0.641×10^{-3}	0.662×10^{-3}	(Eq.15)
$2\lambda\omega$	2.86	2.87	(Eq.15)
η	0.0695	0.0688	(Eq.15)
δ		1.442	(Eq.17)
θ		1.196	(eq.17)

Therefore the stresses at the midlength of the deckhouse are

a) for constant moment alone

$\psi = 0.435$ (from Figure 7)

$p_f = -C\psi m = -700$ psi.

$p_1 = 214$ psi

$\bar{\sigma}_1 = -1965$ psi

$\bar{\sigma}_f = -1965$ psi

b) for sinusoidal moment alone

$\varphi = 0.380$ (from Figure 8)

$p_f = [(B\delta - C\theta)\varphi - B]a = -314$ psi

$p_1 = 116$ psi

$\bar{\sigma}_1 = -822$ psi

$\bar{\sigma}_f = -666$ psi

Superimposing b) and a) we obtain the total stresses for the combination bending moment

$p_f = -1014$ psi

$p_1 = 330$ psi

$\bar{\sigma}_1 = -2787$ psi

$\bar{\sigma}_f = -2631$ psi

Appendix IV

The Shear Deflection Constant ⓚ

An approximation for this constant applicable to thin-walled rectangular girders can be obtained by integrating the square of the shear flow in vertical plating and dividing by the square of the average:

$$\textcircled{k} = \frac{1}{(depth)\, N_{av.}^2} \int_{depth} N^2\, dz \quad . \tag{a}$$

Applying this to the deckhouse, the shear flow is related to the longitudinal stress in the sides of thickness t by the equilibrium equation

$$\frac{\partial N_f}{\partial z} = - \frac{\partial \sigma_x}{\partial x} t \, .$$

Using (2a) for σ_x , this becomes

$$N_f = - \frac{M_f'}{I_f} \int z t\, dz - p_f' \int t\, dz \, ,$$

and for a single-tiered house with sides of constant thickness and depth 2h

$$N_f = \frac{3}{2h^2(4A_f - 3a_f)} \left[(A_f - a_f)h + \frac{A_f}{4}\frac{1}{h}\left((2h-e_f)^2 - z_f^2\right)\right] M_f'$$

$$- \left[\frac{A_f - a_f}{2} + \frac{a_f}{4}\frac{1}{h}(2h - e_f + z_f)\right] p_f' \, .$$

(Here N_f at the bond is $\frac{1}{2}\overline{N}$, since $\overline{N}$ is the sum of the shears contributed by both deckhouse sides.)

This can be put in terms of p_f' and M' by using (6b) for M_f. Then ⓚ can be obtained as a function of p_f' and M' by carrying out the integrations of formula (a). For the case of a constant bending moment $M' = 0$, and p_f' cancels, so that ⓚ is a function of the geometry only.

Mr. Robert S. Johnson investigated ⓚ for a range of possible geometries of hull and deckhouse, and in an unpublished paper showed that

$$1.0 < ⓚ < 1.01 \quad .$$

Assuming that this result for constant M would govern also for a more general form of M, it seems clear that for design purposes at least

$$ⓚ \approx 1.0$$

and the actual value of a_f can be used in the formulas, without modification for ⓚ. Since the length-depth ratio of the hull is usually greater than that of the deckhouse, the same conclusion holds for a_1 .

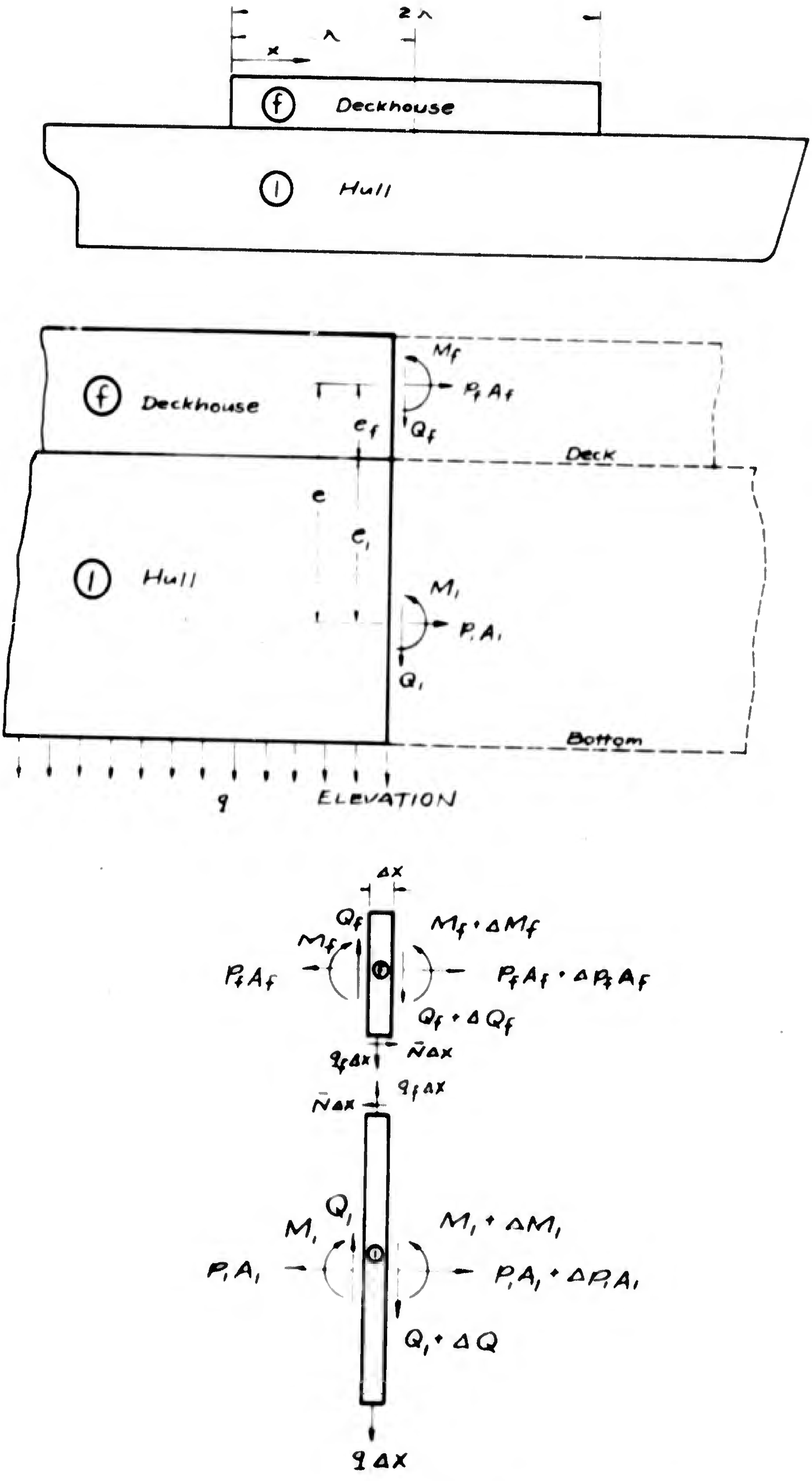


FIG. 1 IDENTIFICATION SKETCHES

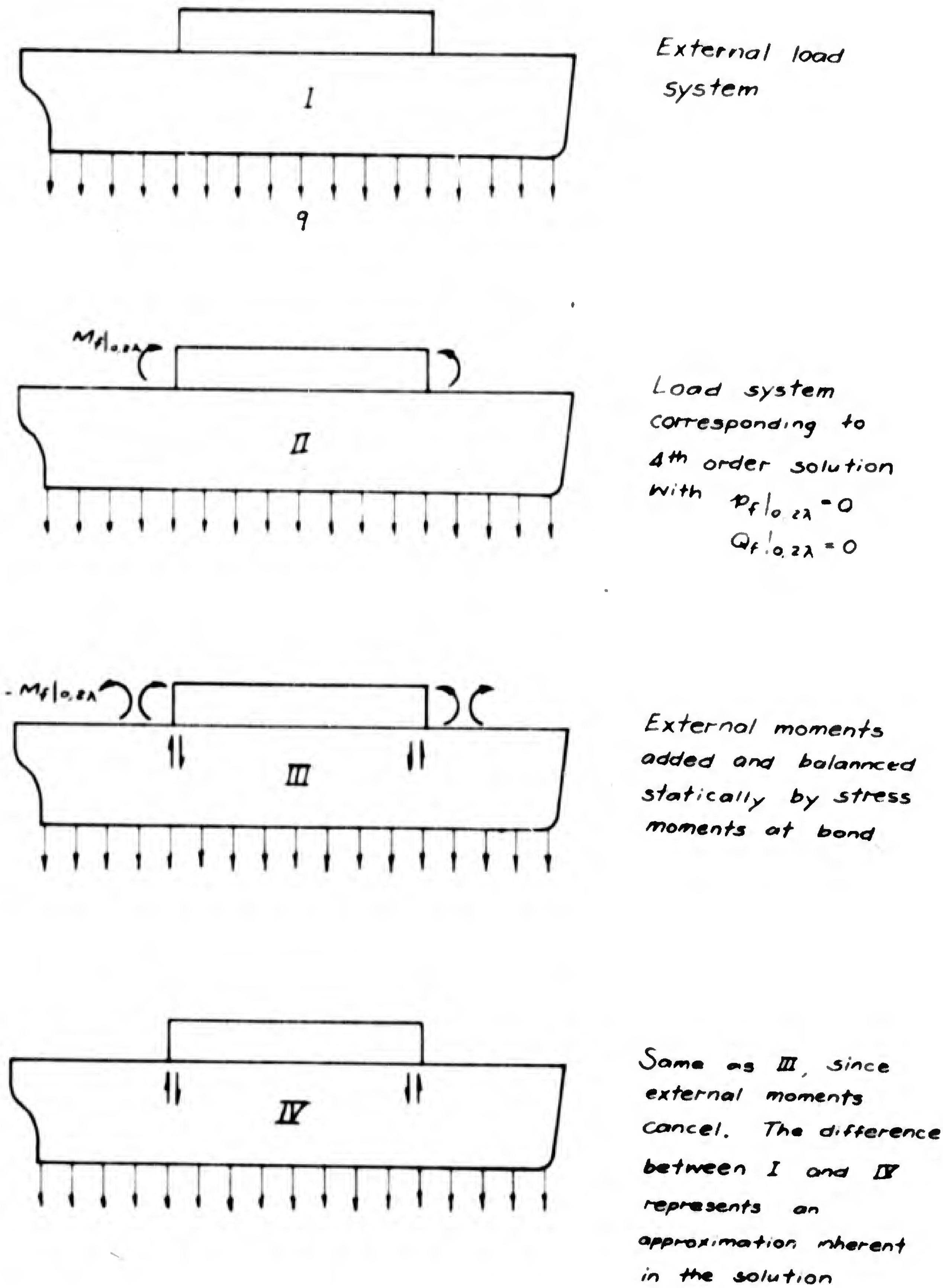


FIG.2 LOAD SYSTEM SKETCHES

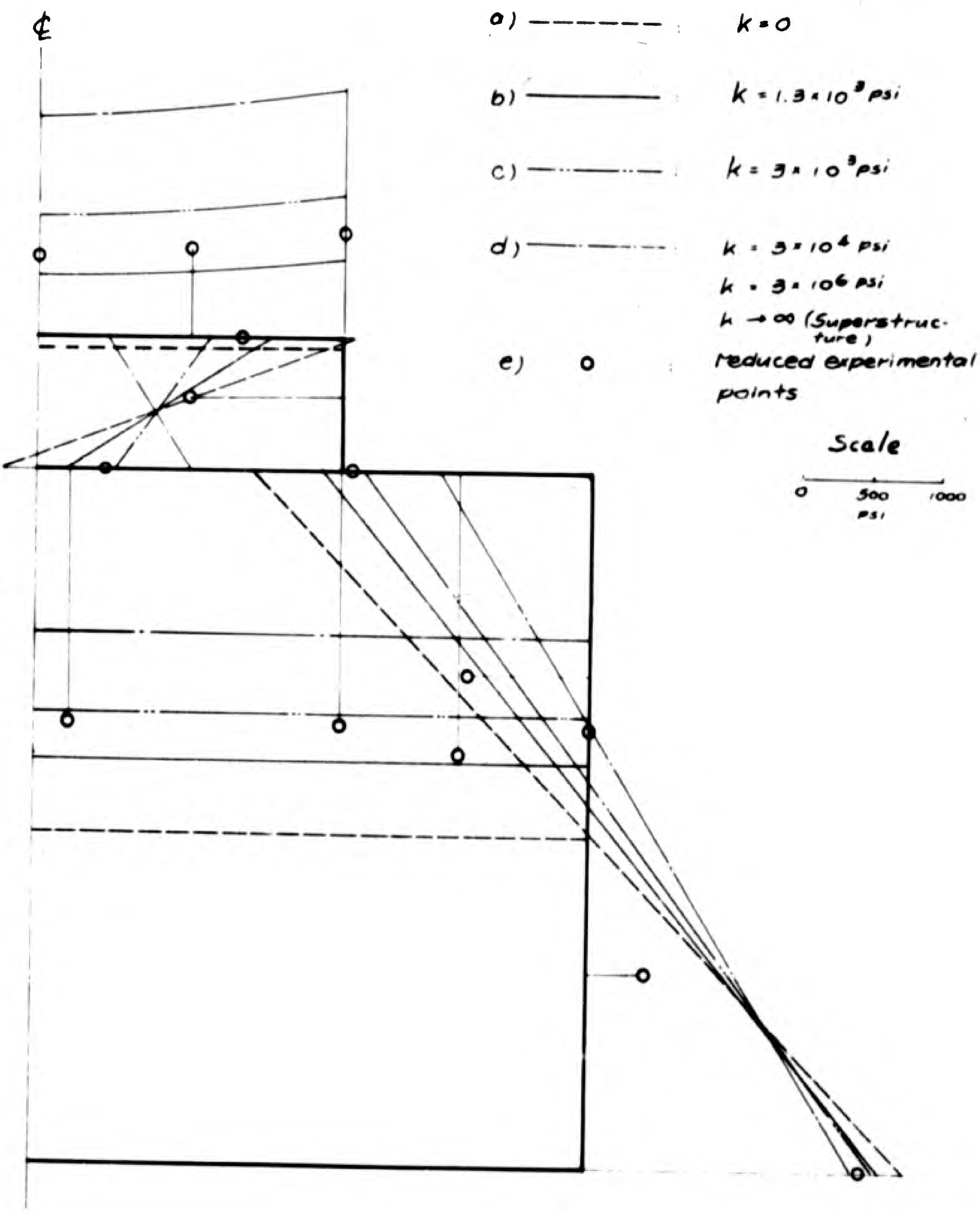


FIG. 3 LONGITUDINAL STRESS AT MIDLENGTH (λ) AS A FUNCTION OF k FOR STRUCTURAL MODEL WITH LONG DECKHOUSE (2λ = 264") — CONSTANT M

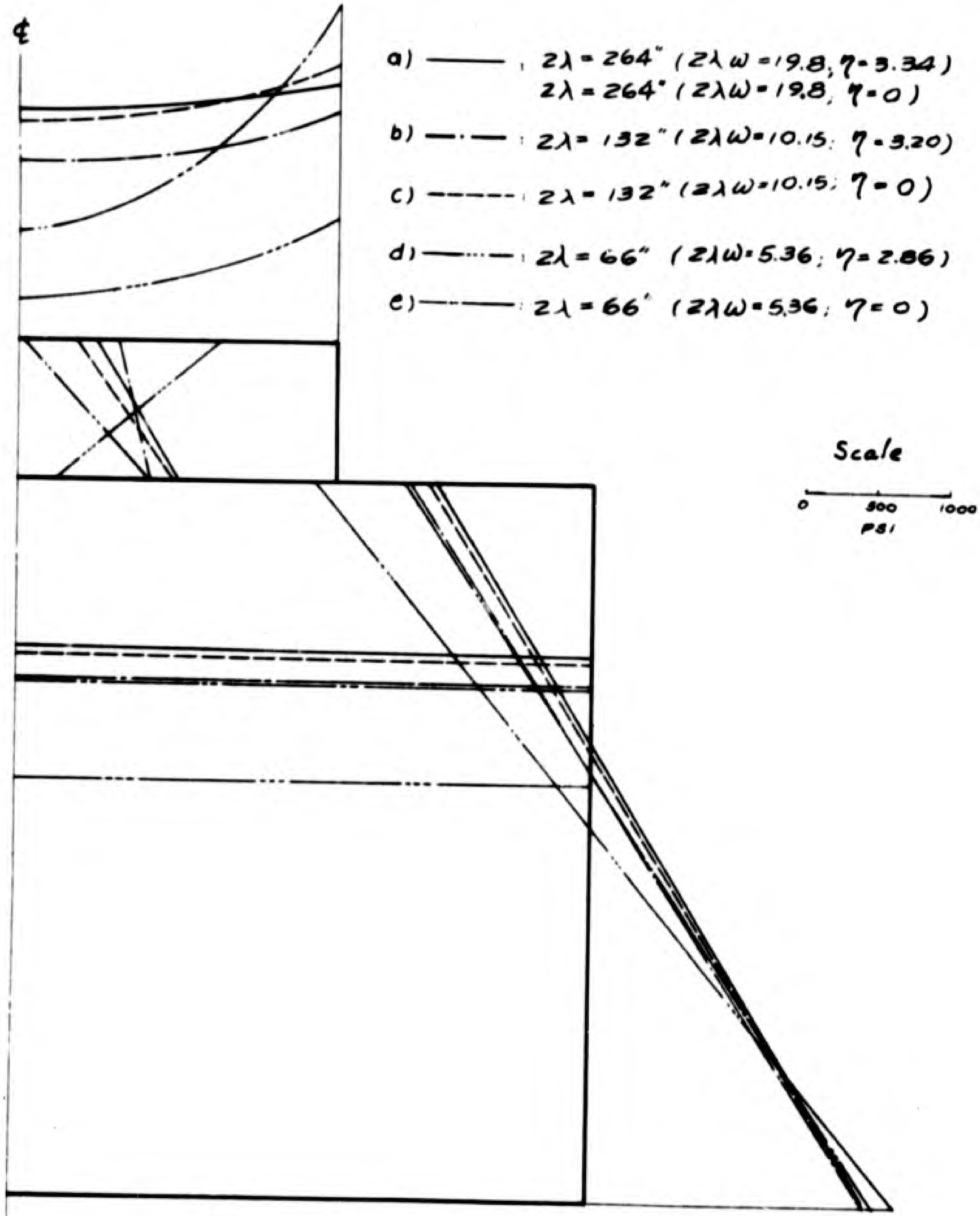


FIG. 4 LONGITUDUINAL STRESS AT MIDLENGTH (λ) AS A FUNCTION OF DECKHOUSE LENGTH (2λ) FOR STRUCTURAL MODEL WITH STIFF FOUNDATION MODULUS (k = 3×10⁶ PSI) — CONSTANT M

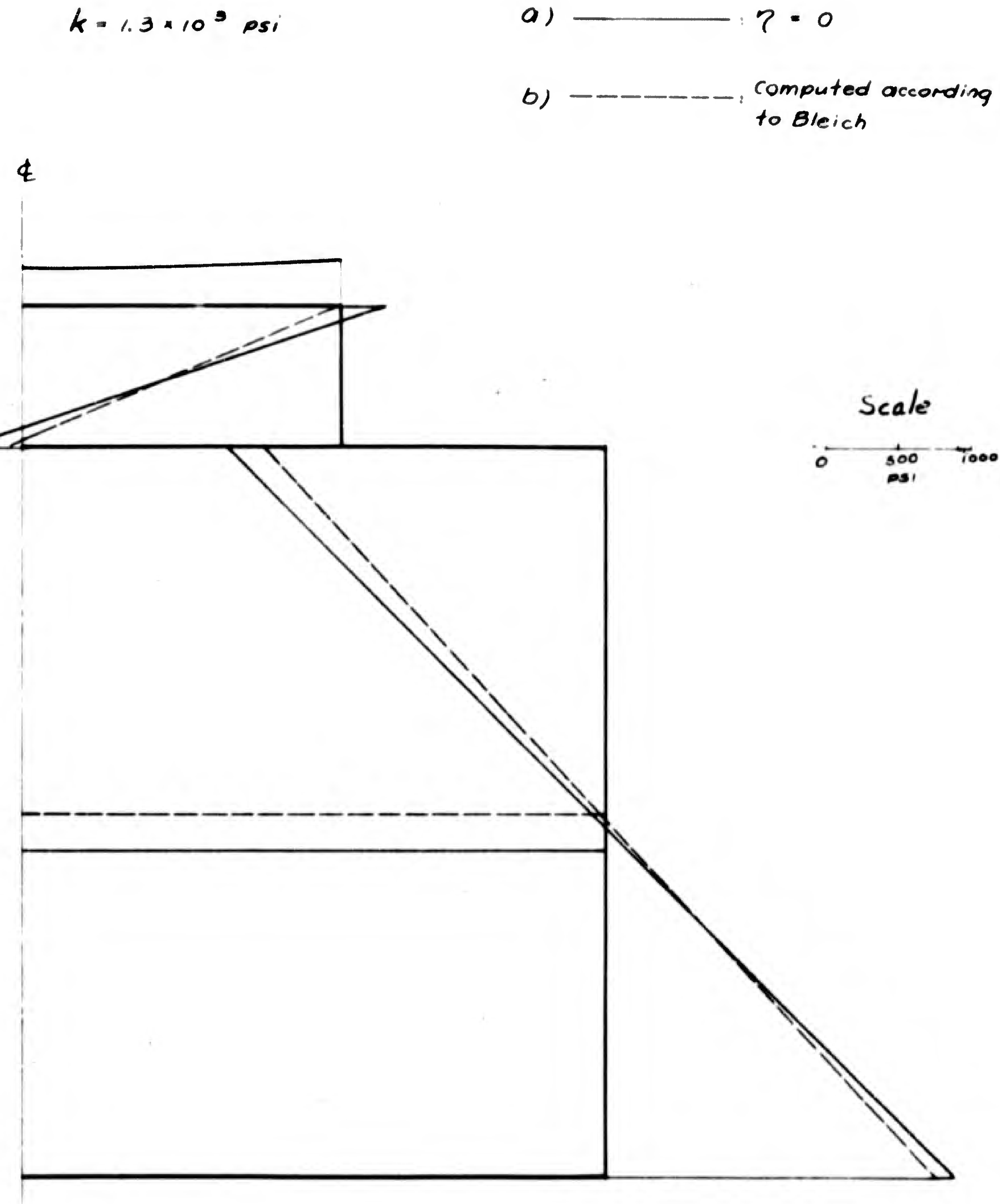


FIG.5 LONGITUDINAL STRESS AT $X/\lambda = 1/11$ WITH TWO DIFFERENT SETS OF BOUNDARY CONDITIONS FOR STRUCTURAL MODEL WITH LONG DECKHOUSE ($2\lambda = 264"$) — CONSTANT M

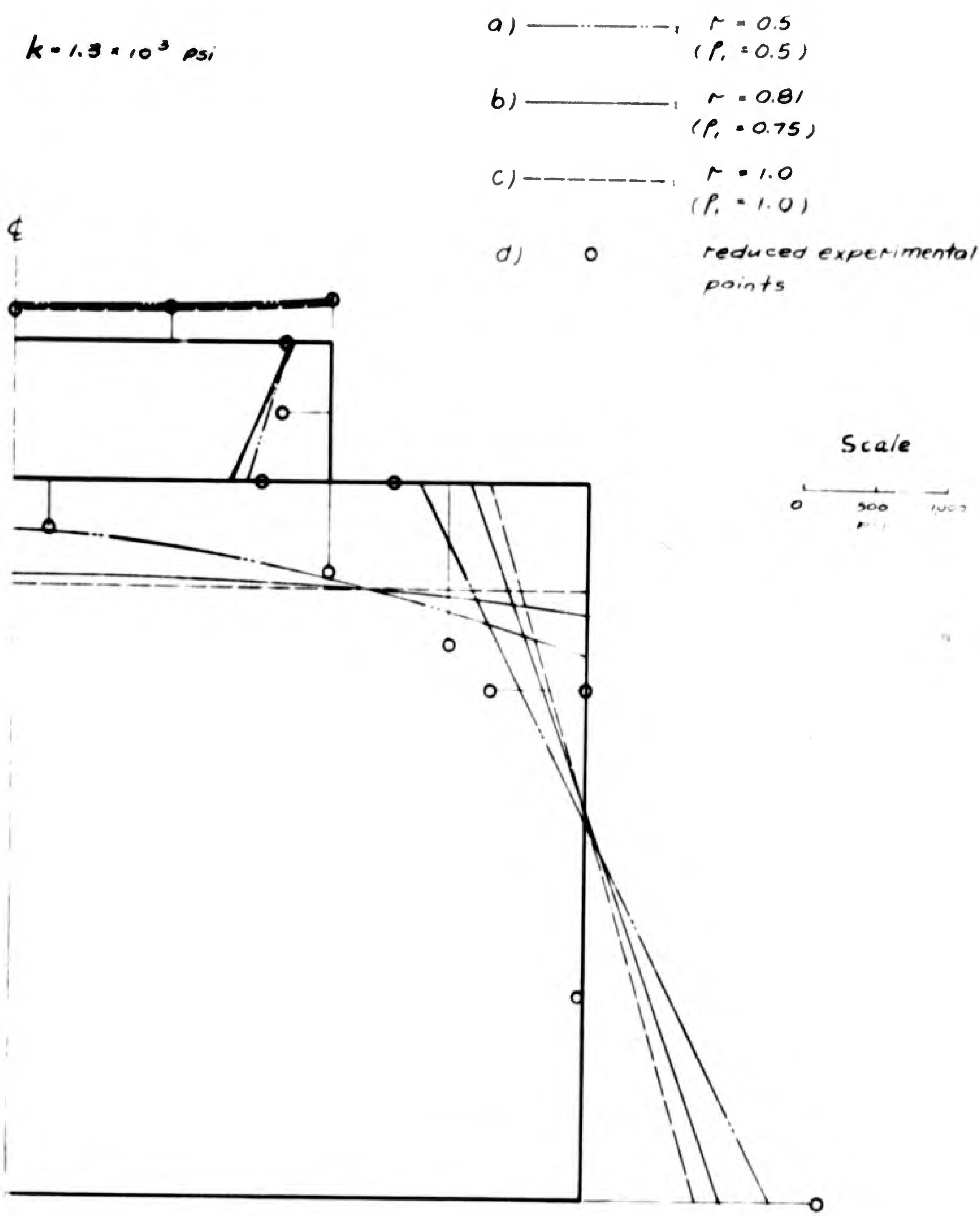


FIG. 6 LONGITUDINAL STRESS AT MIDLENGTH (λ) FOR THREE DEGREES OF DECK SHEAR LAG (r) FOR STRUCTURAL MODEL WITH LONG DECKHOUSE (2λ = 264") — SINUSOIDAL M

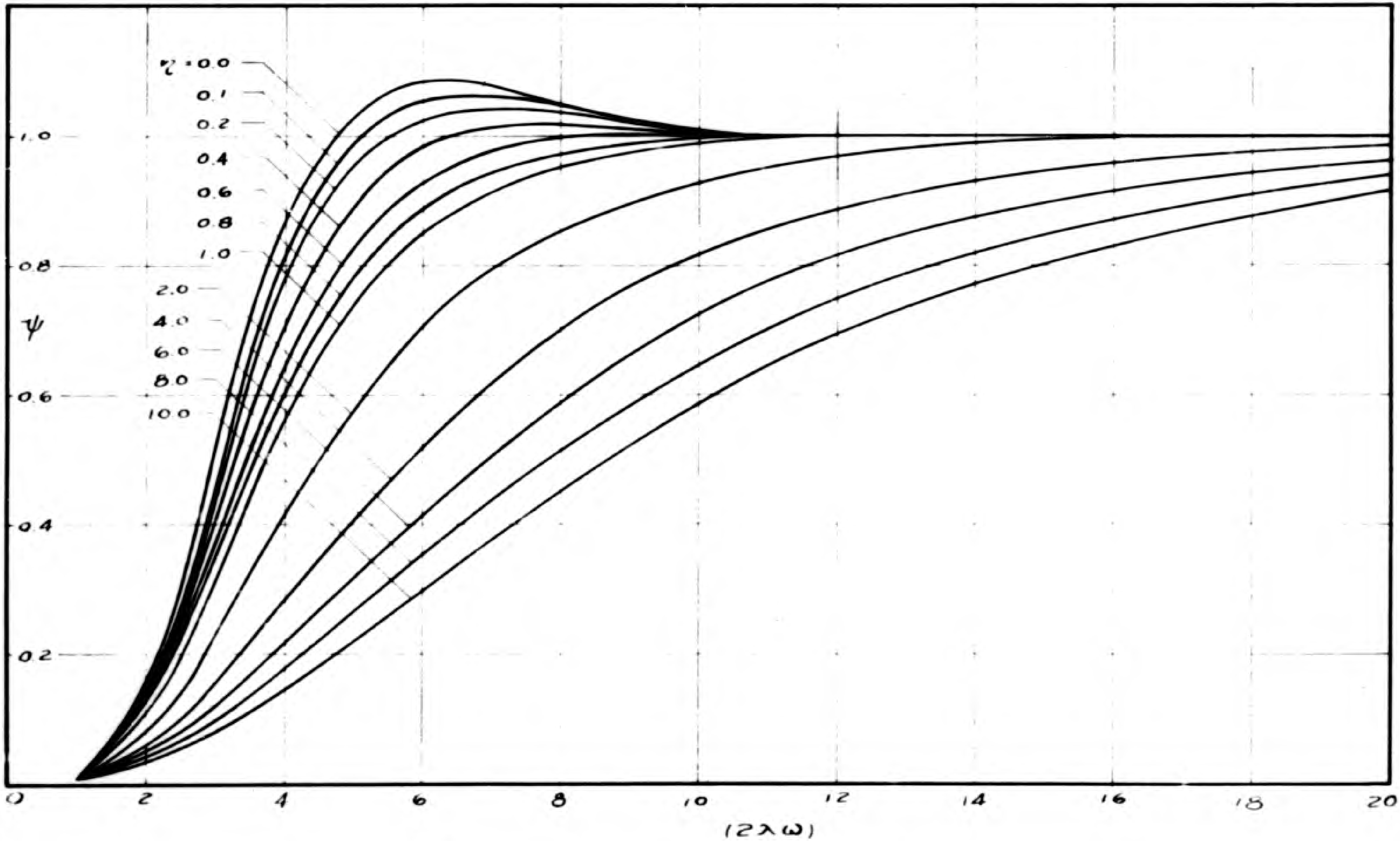


FIG. 7 DESIGN CURVES FOR ψ AT $X = \lambda$

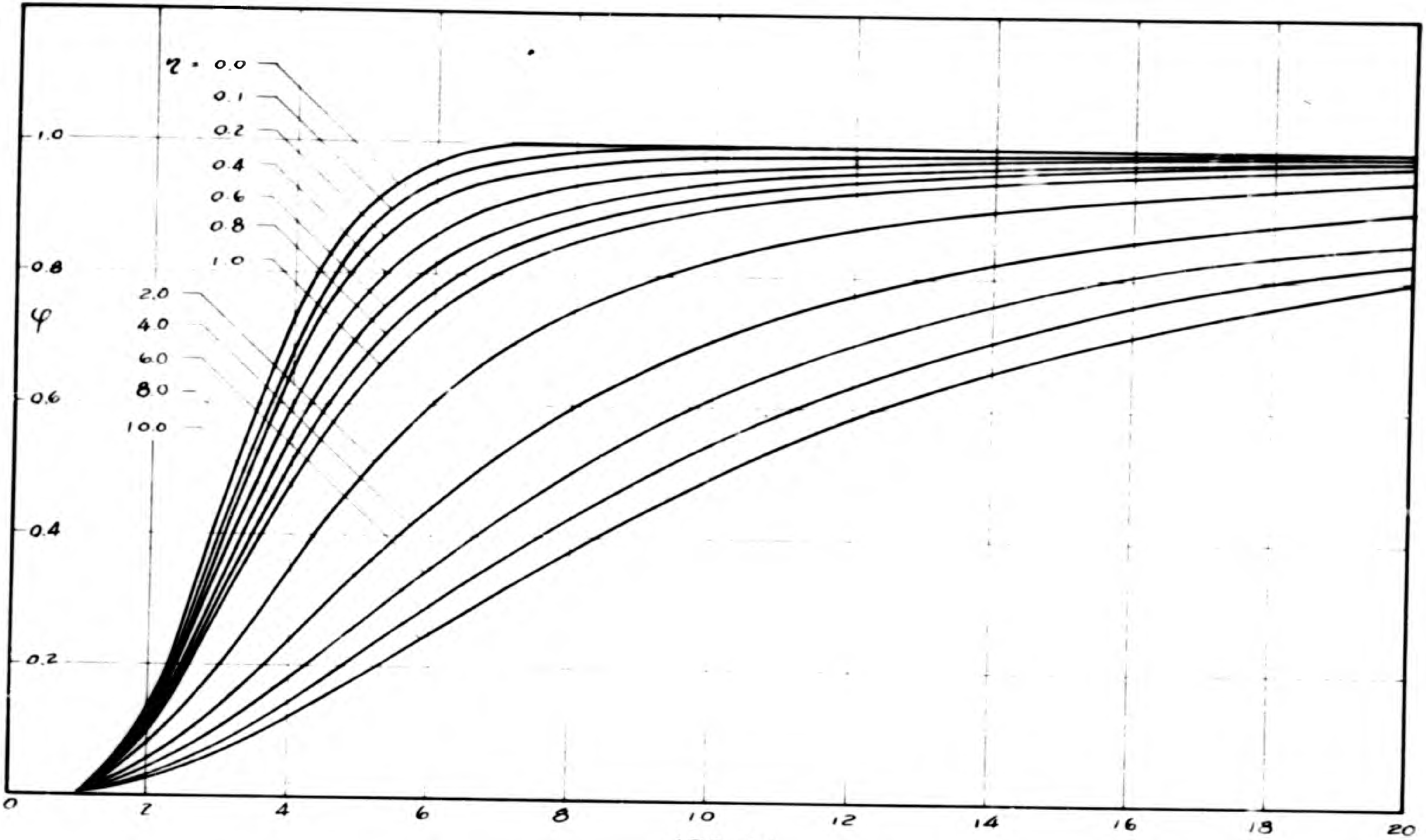


FIG. 8 DESIGN CURVES FOR φ AT $x = \lambda$

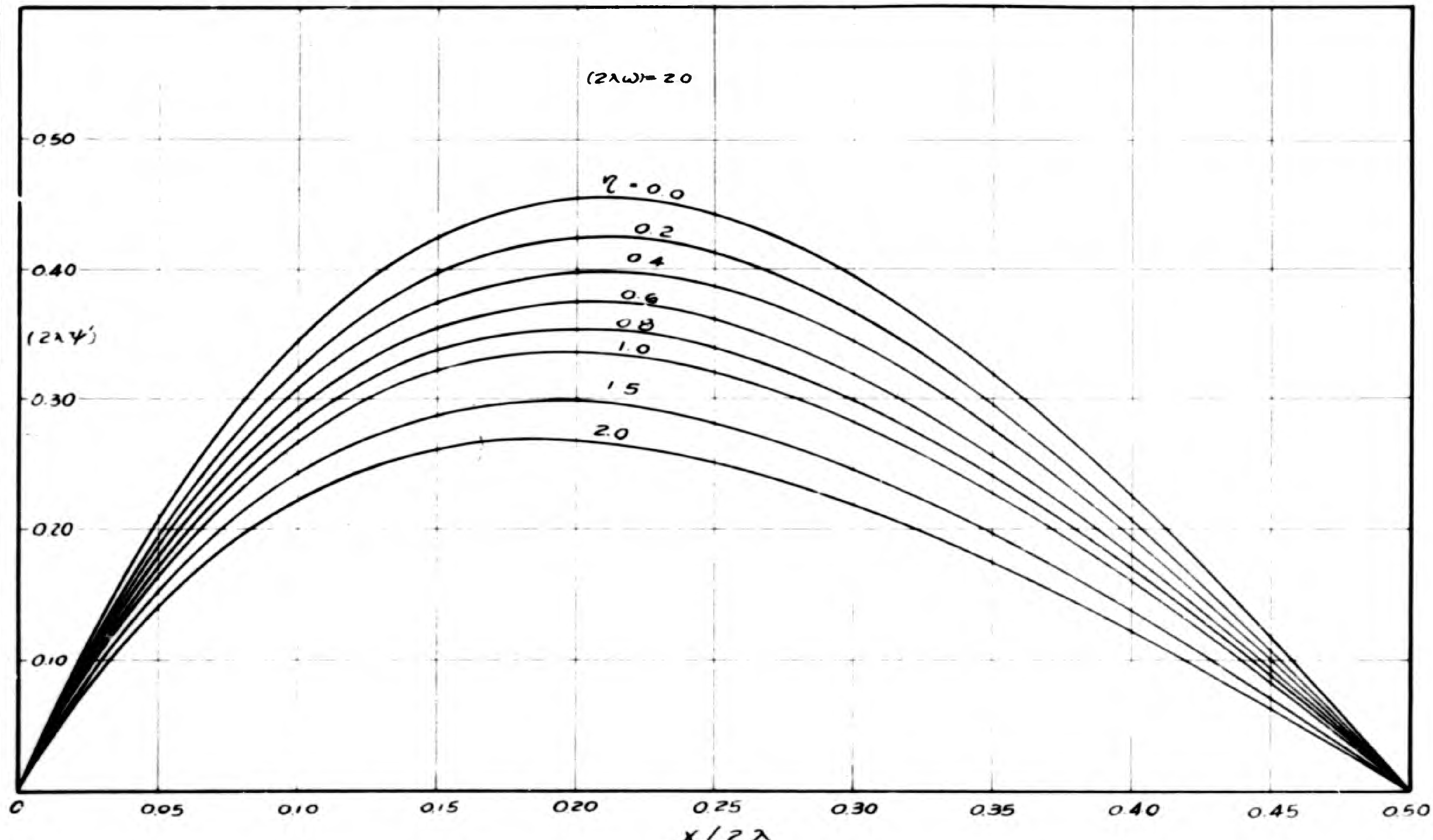


FIG. 9 DESIGN CURVES FOR $(2\lambda\psi')$ AT $(2\lambda\omega) = 2.0$

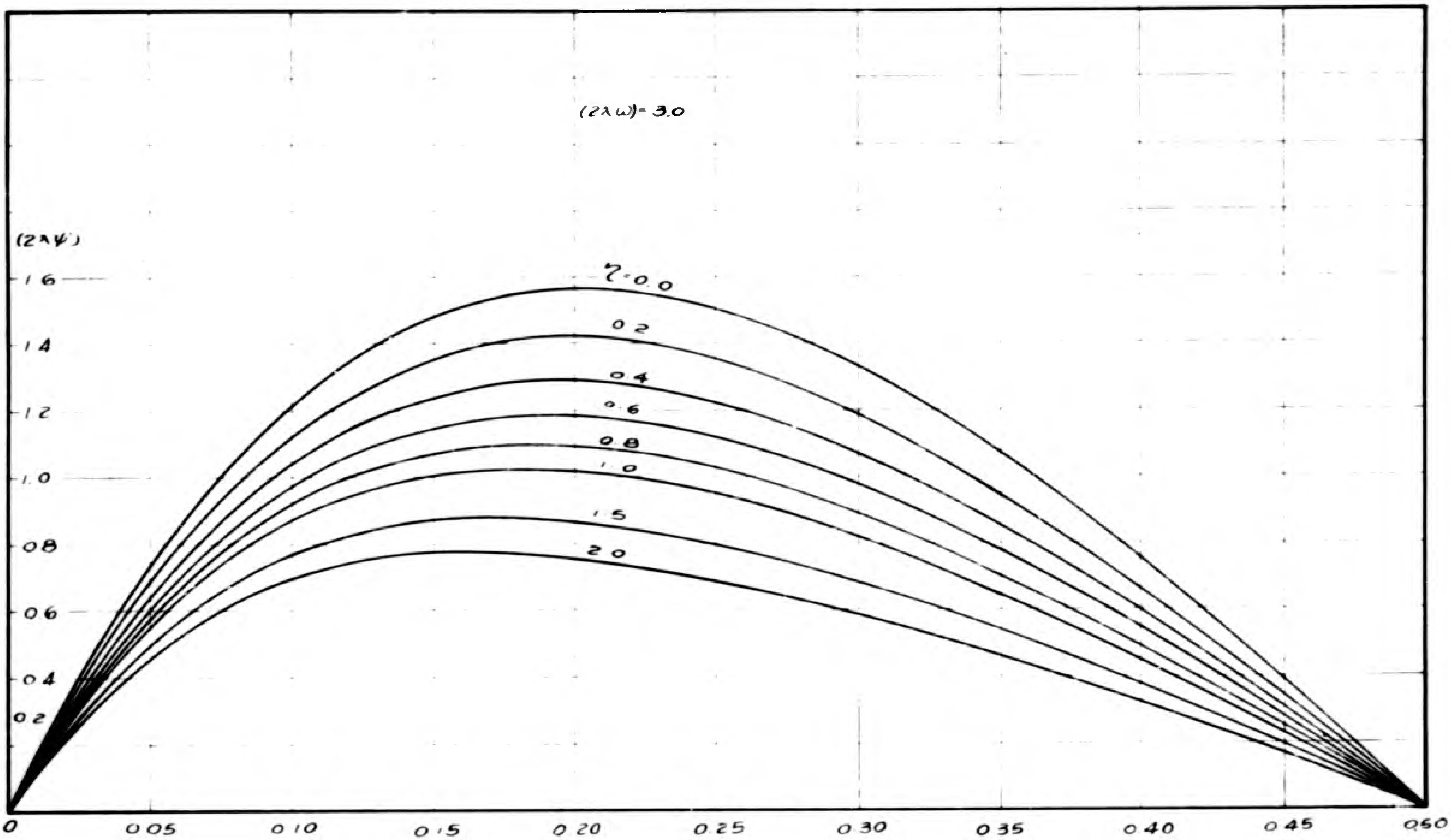


FIG. 10 DESIGN CURVES FOR $(2\lambda\psi')$ AT $(2\lambda\omega) = 3.0$

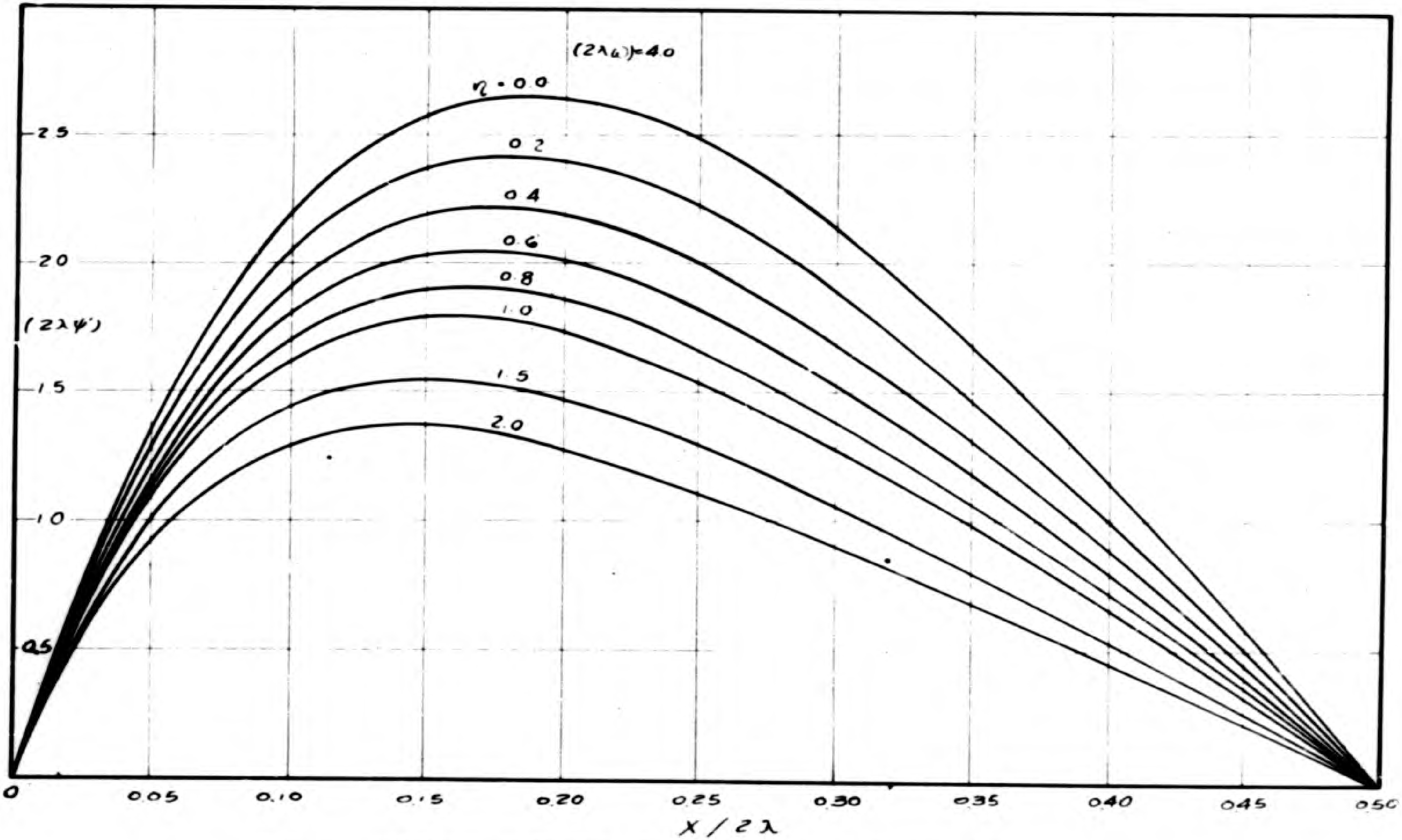


FIG. 11 DESIGN CURVES FOR $(2\lambda\psi')$ AT $(2\lambda\omega) = 4.0$

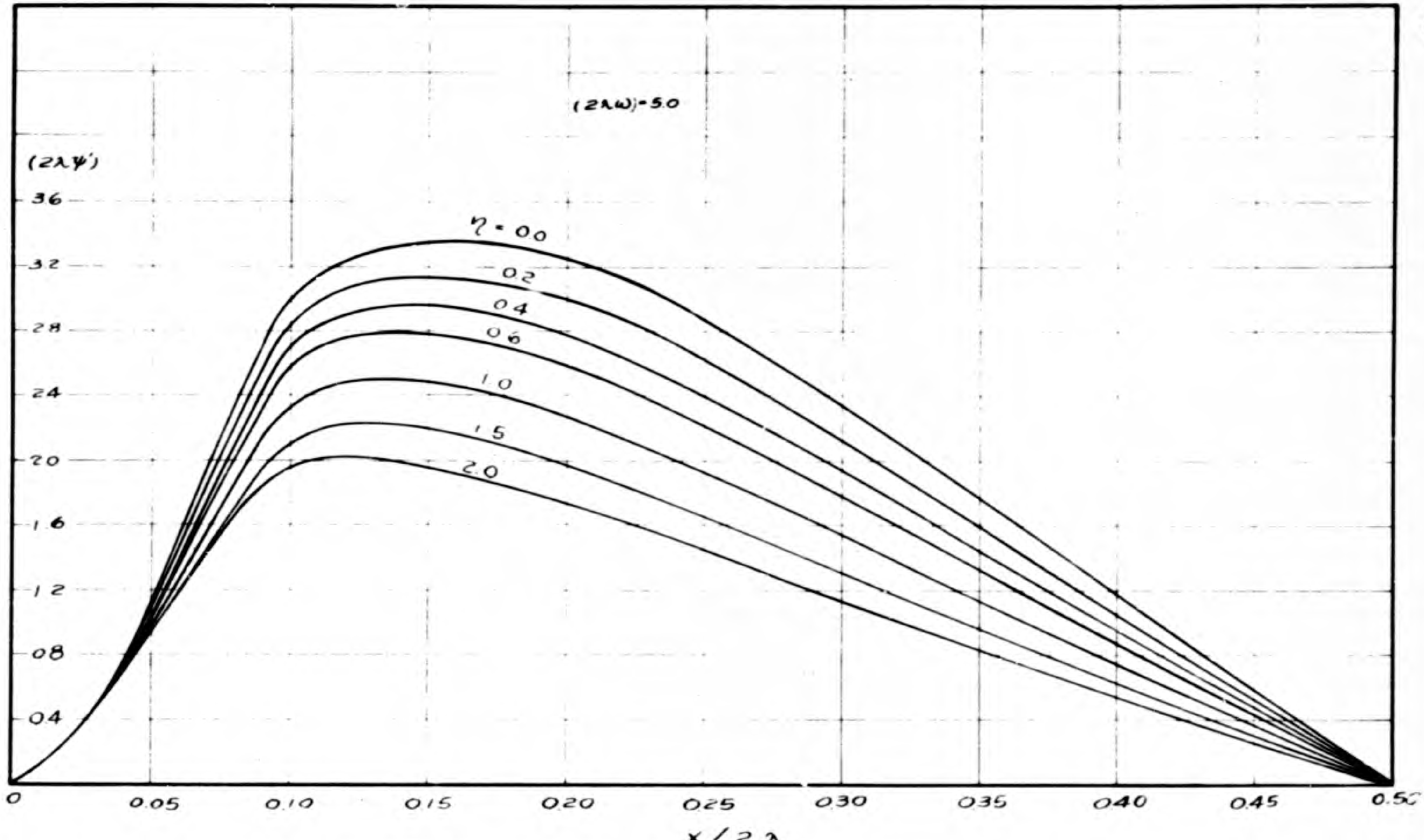


FIG. 12 DESIGN CURVES FOR $(2\lambda\psi')$ **AT** $(2\lambda\omega) = 5.0$

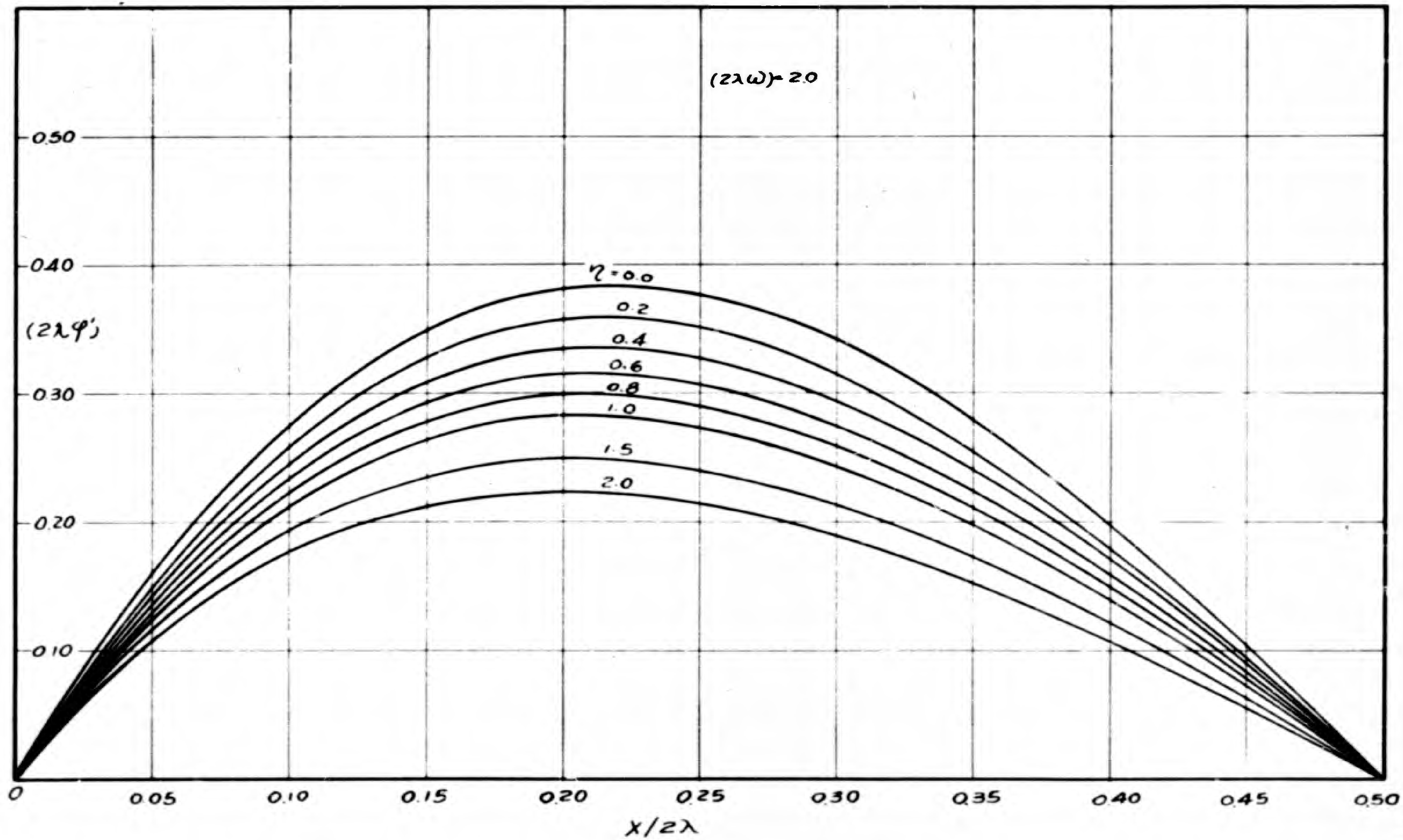


FIG. 13 DESIGN CURVES FOR $(2\lambda\varphi')$ AT $(2\lambda\omega) = 2.0$

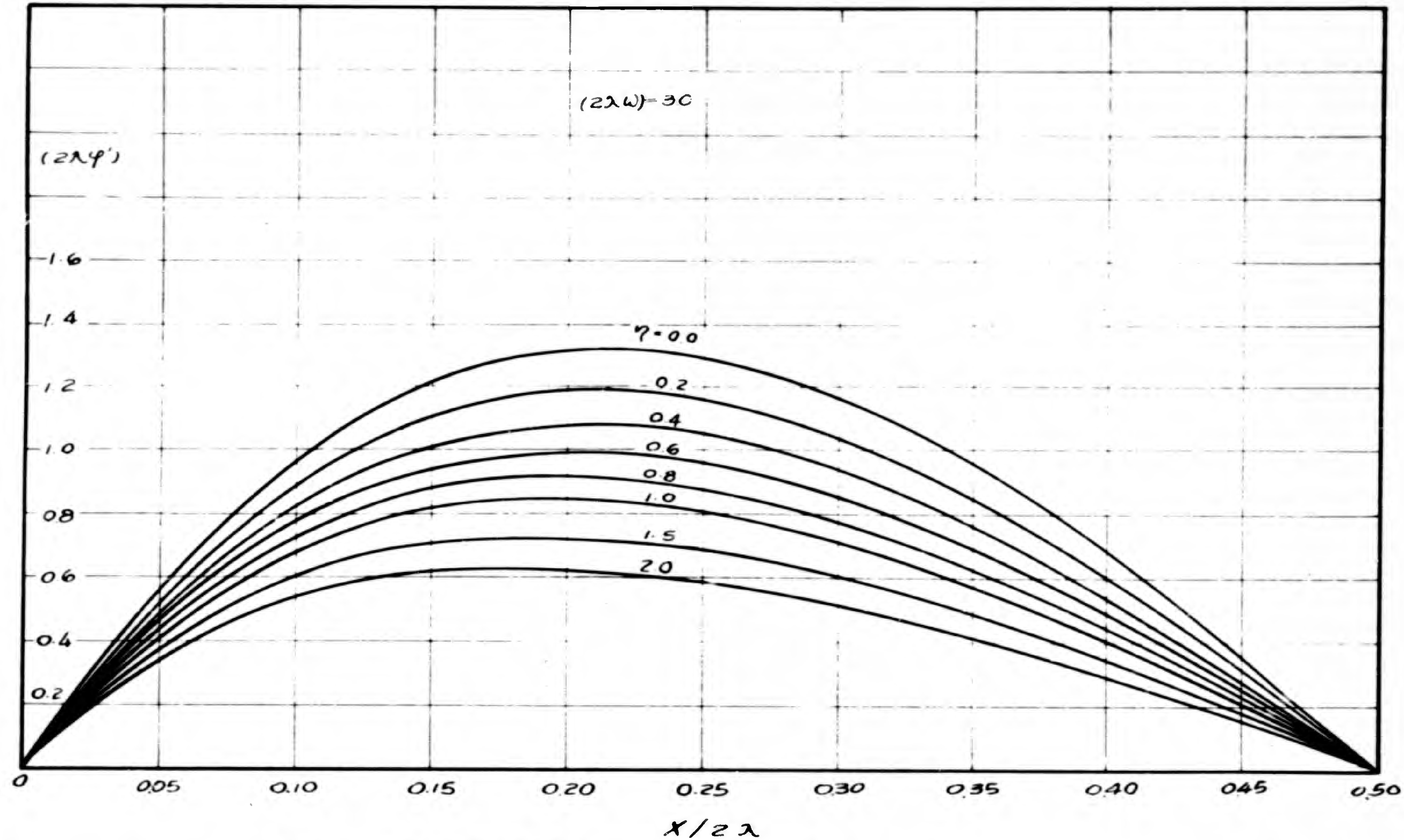


FIG. 14 DESIGN CURVES FOR $(2\lambda\varphi')$ **AT** $(2\lambda\omega) = 3.0$

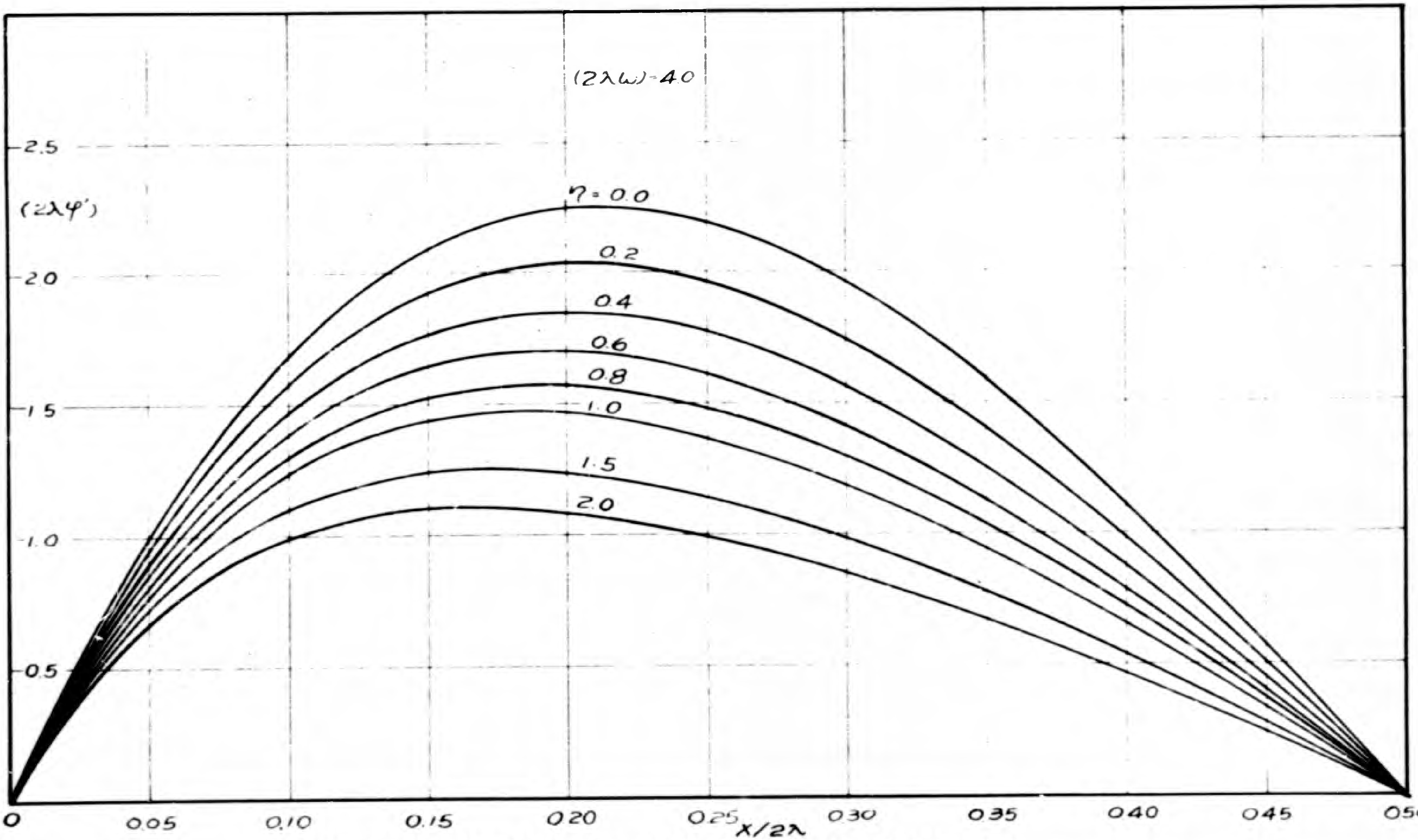


FIG. 15 DESIGN CURVES FOR $(2\lambda\varphi')$ AT $(2\lambda\omega) = 4.0$

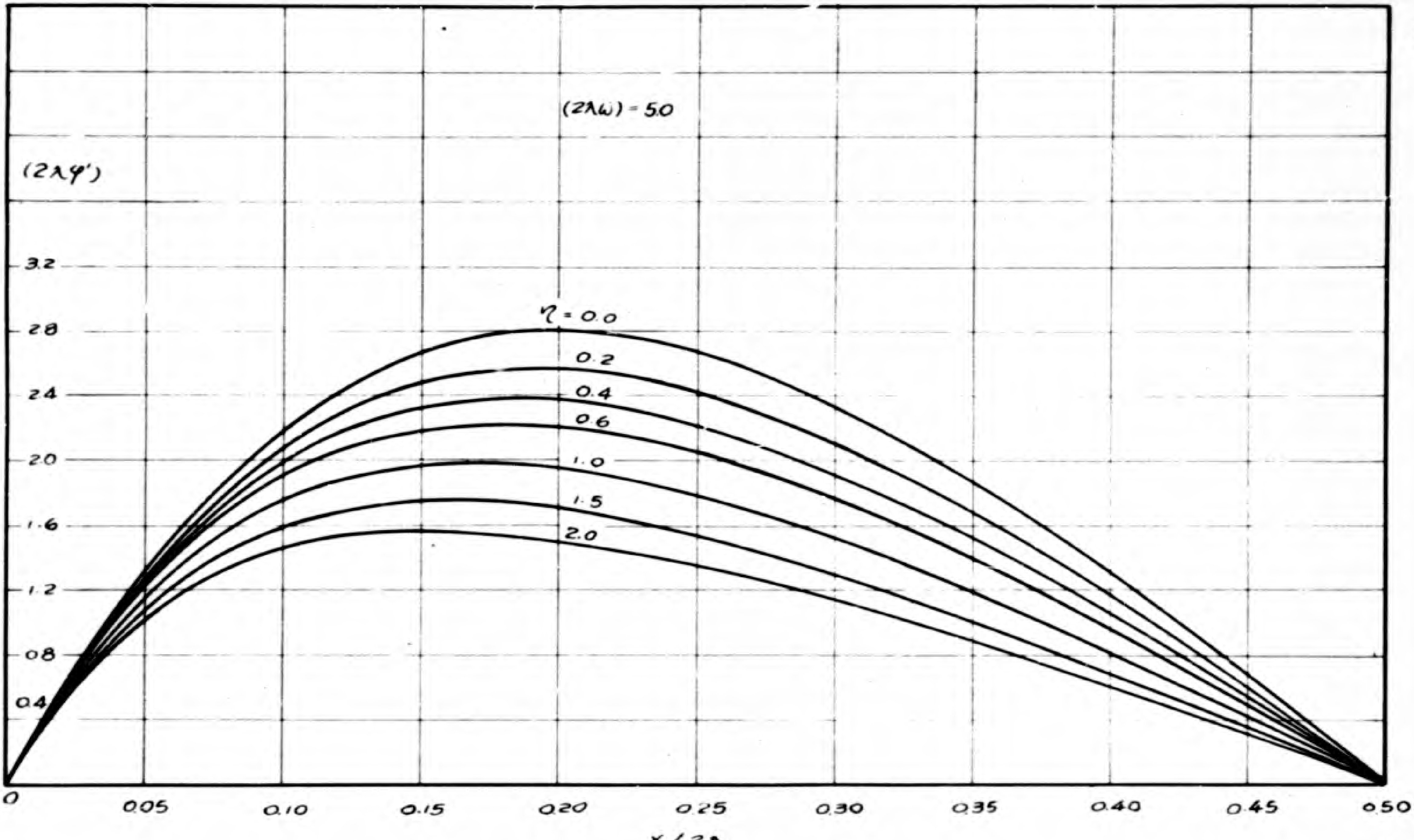


FIG. 16 DESIGN CURVES FOR $(2\lambda\varphi')$ AT $(2\lambda\omega) = 5.0$

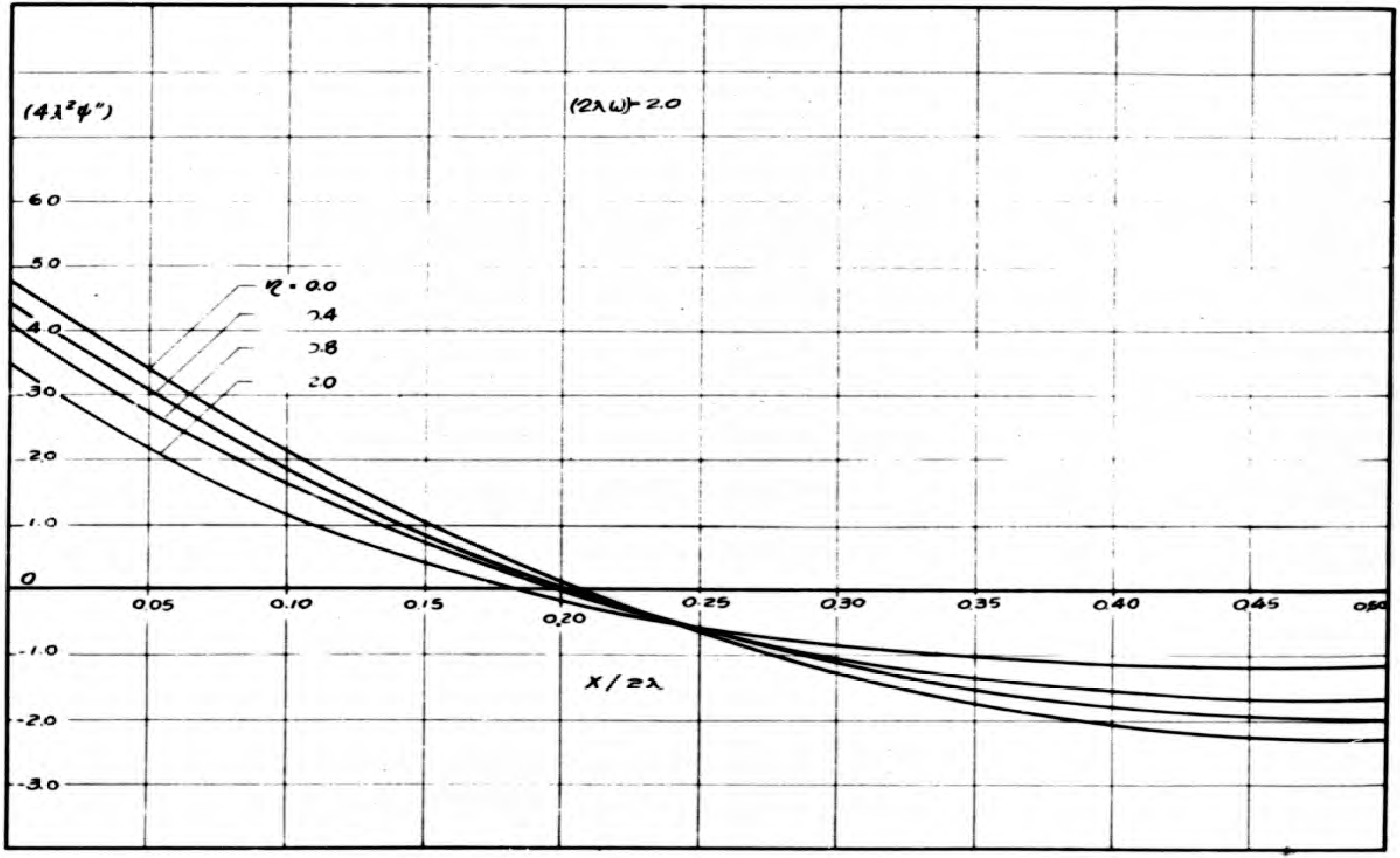


FIG. 17 DESIGN CURVES FOR $(4\lambda^2\psi'')$ AT $(2\lambda\omega) = 2.0$

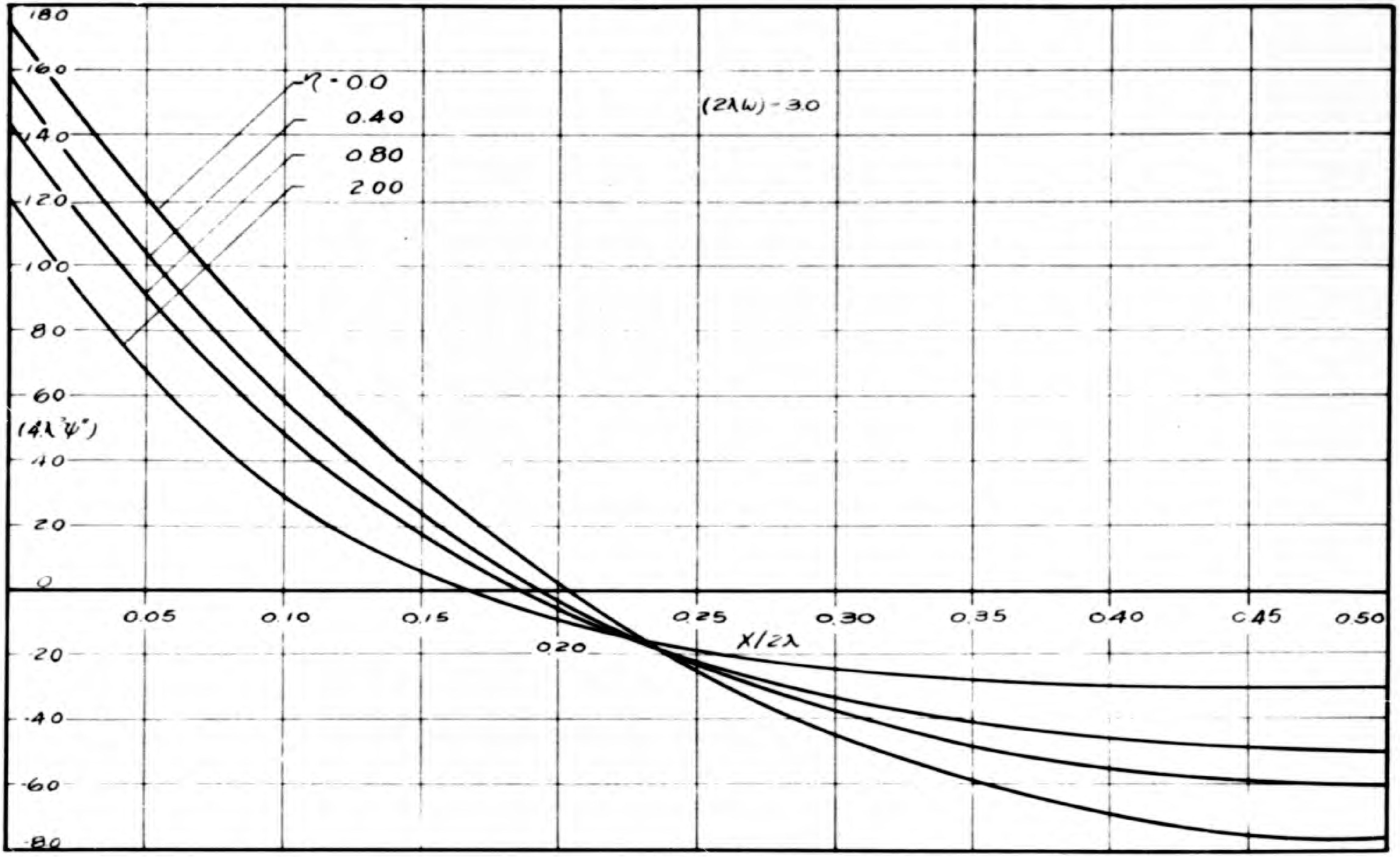


FIG. 18 DESIGN CURVES FOR $(4\lambda^2\psi'')$ AT $(2\lambda\omega) = 3.0$

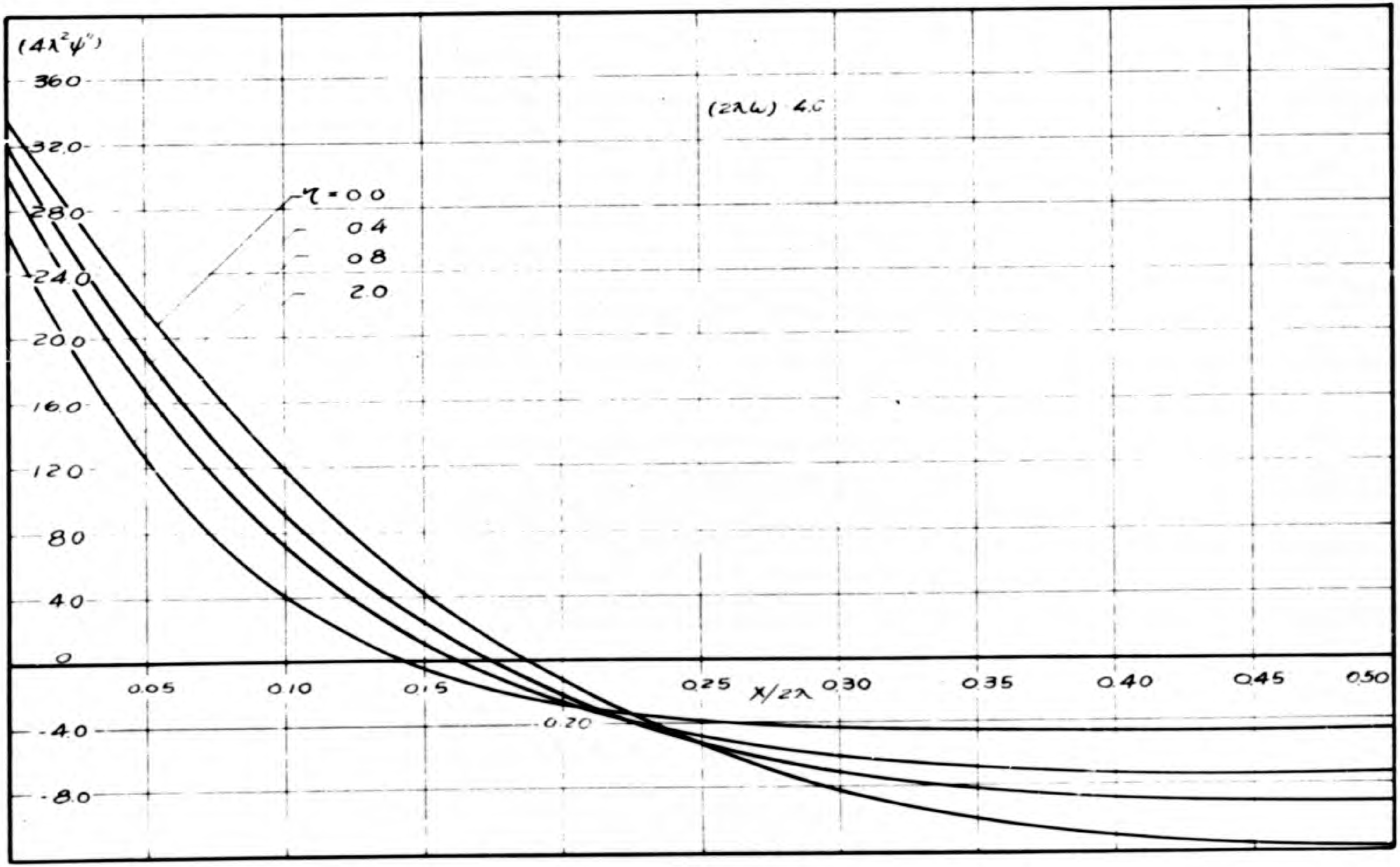


FIG. 19 DESIGN CURVES FOR $(4\lambda^2\psi'')$ AT $(2\lambda\omega) = 4.0$

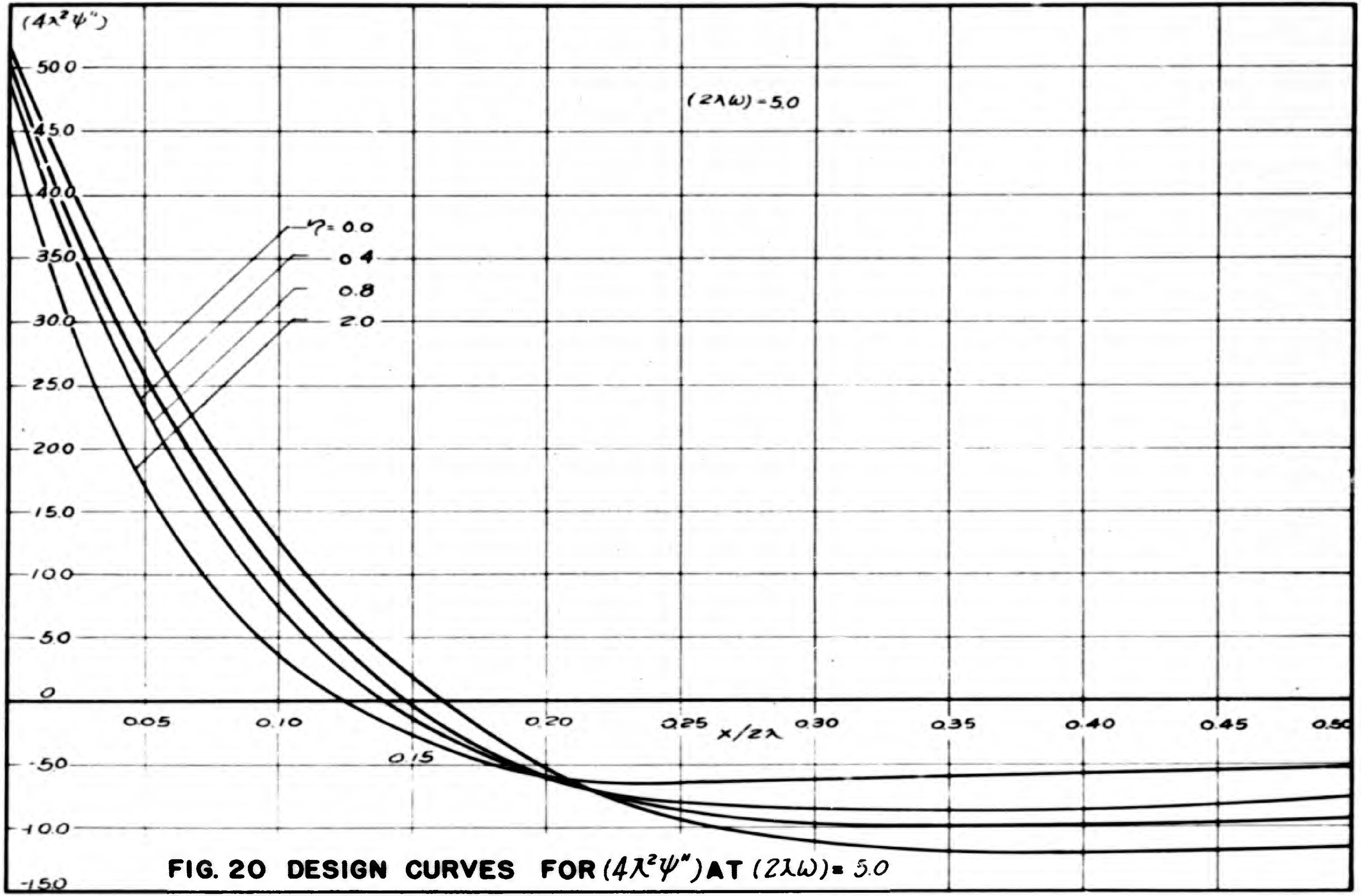


FIG. 20 DESIGN CURVES FOR $(4\lambda^2\psi'')$ AT $(2\lambda\omega) = 5.0$

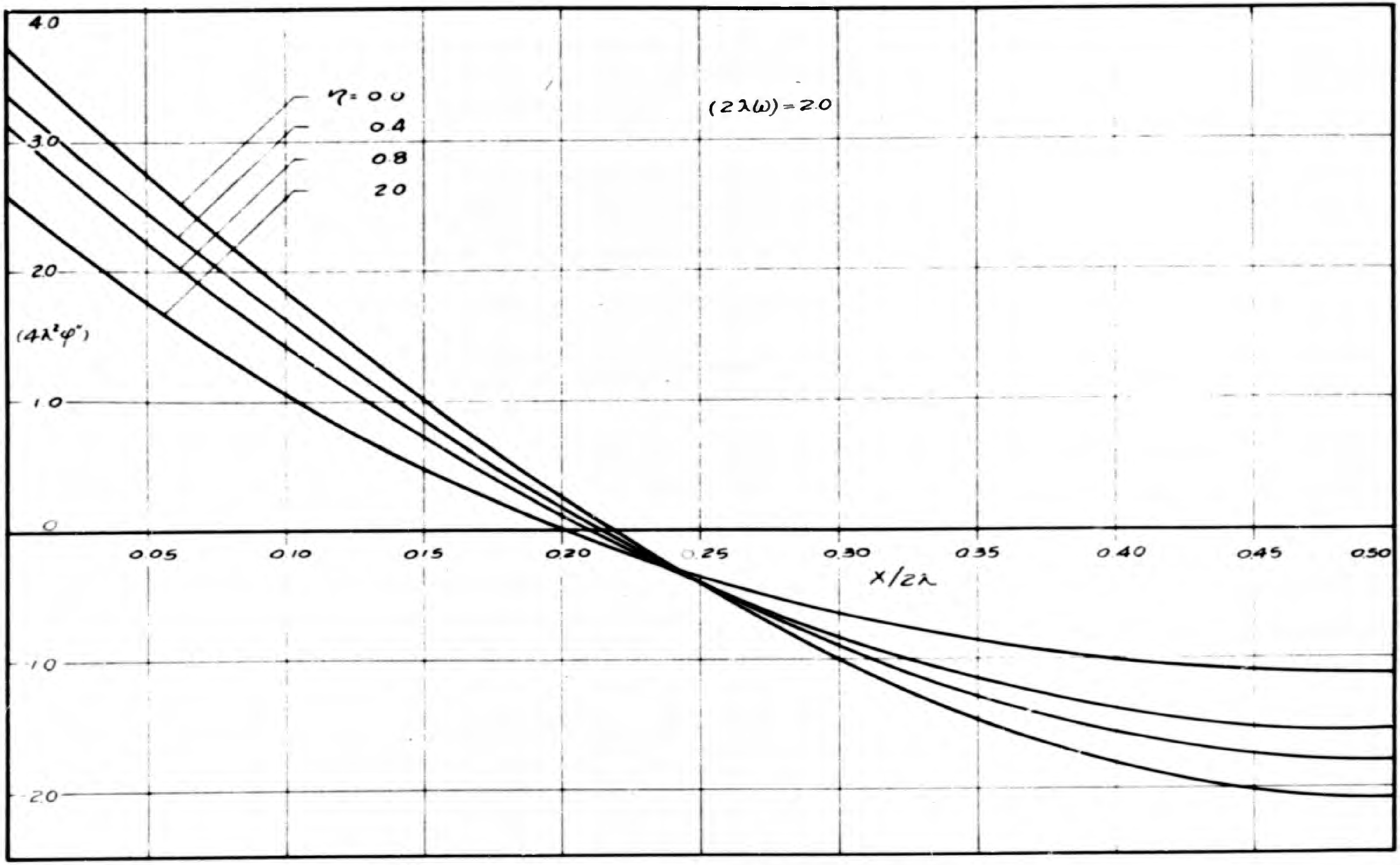


FIG. 21 DESIGN CURVES FOR $(4\lambda^2\varphi'')$ AT $(2\lambda\omega) = 2.0$

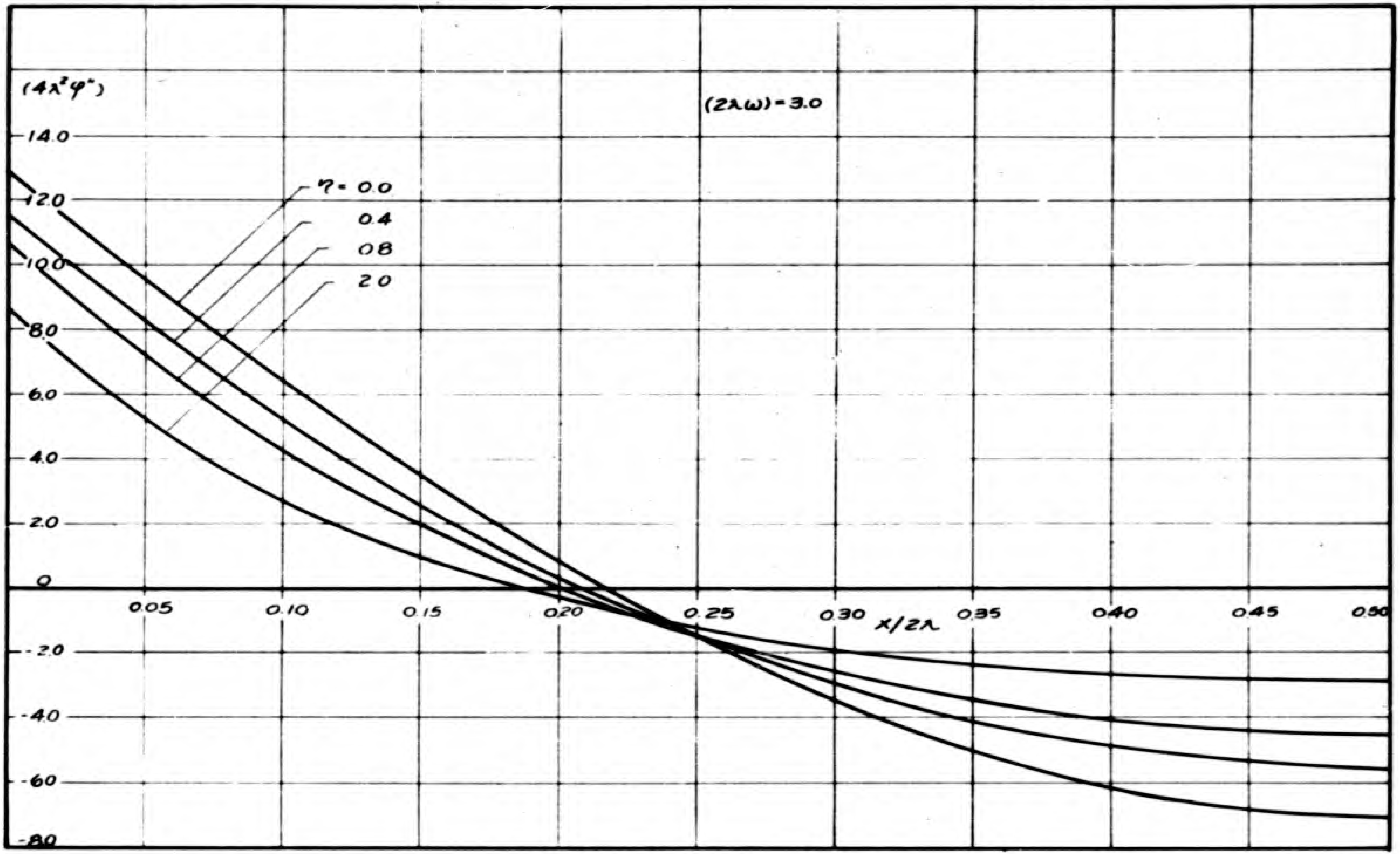


FIG. 22 DESIGN CURVES FOR $(4\lambda^2\varphi'')$ AT $(2\lambda\omega) = 3.0$

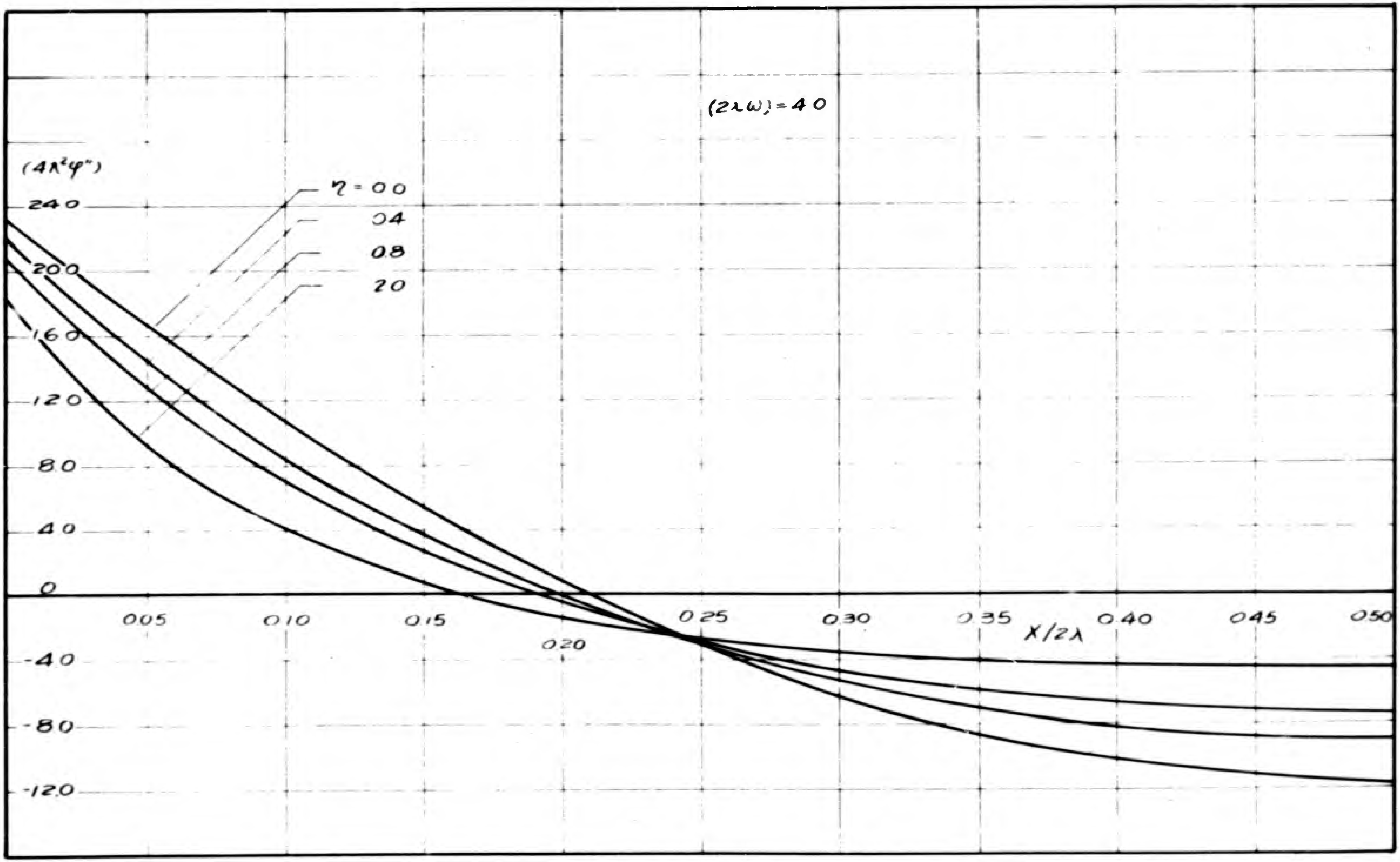


FIG. 23 DESIGN CURVES FOR $(4\lambda^2\varphi'')$ AT $(2\lambda\omega) = 4.0$

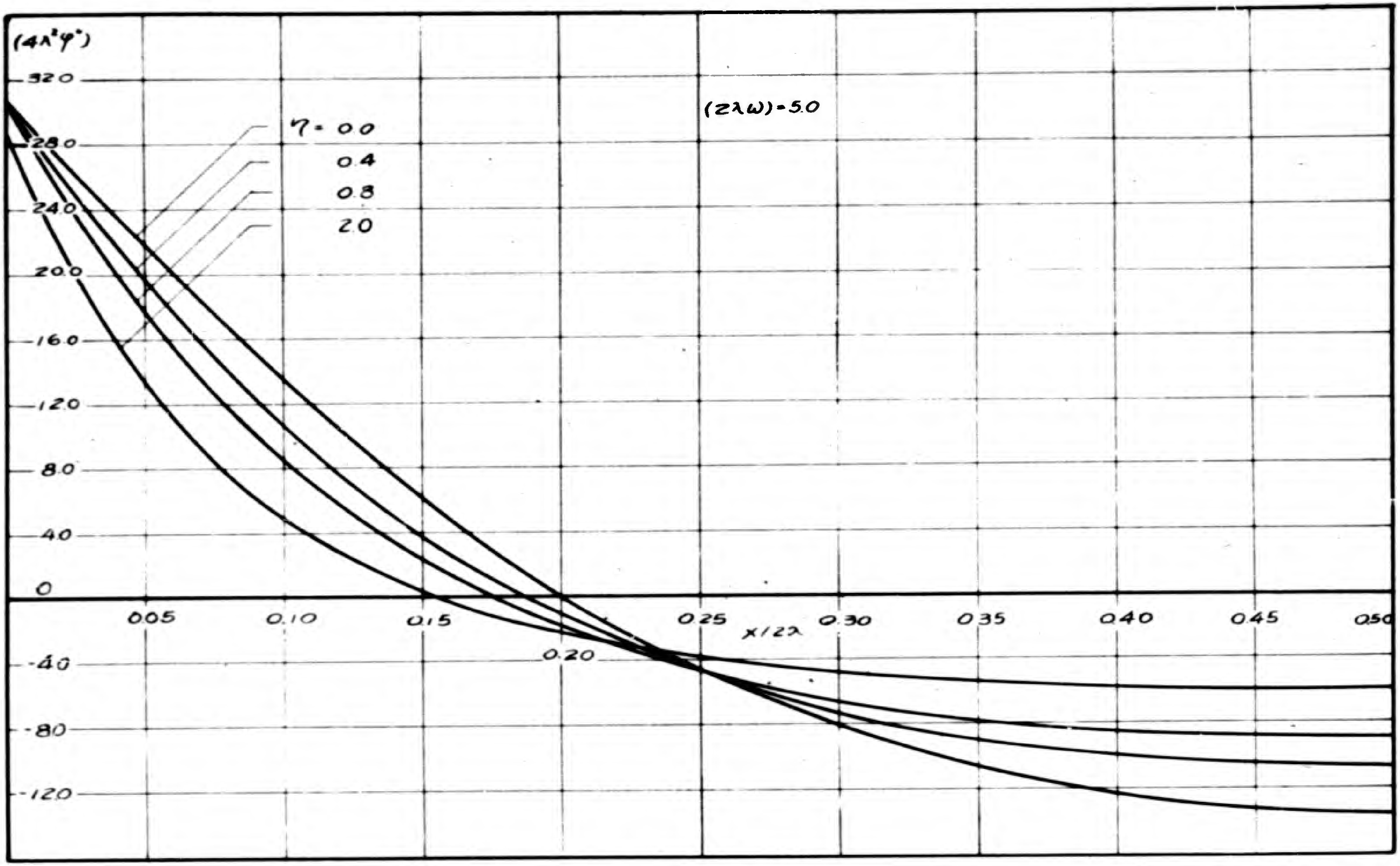


FIG.24 DESIGN CURVES FOR $(4\lambda^2\varphi'')$ AT $(2\lambda\omega) = 5.0$