AN ANALYSIS OF THE INVENTORY RANGE PROBLEM

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ABSTRACT

In contrast with inventory problems that are concerned with when and how much to order of a given repair part, an analysis is made as to which of the many parts in a complex equipment should be considered for stockage as repair parts. An analytic model is developed that provides inventory range decisions that minimize total repair and inventory support costs at a given maintenance activity. The way in which parts are interrelated in the equipment parts hierarchy is explicitly recognized in the model. As by-products, the model prescribes the kind of repair actions to apply to a failed unit of the equipment and, indirectly, the kind and amount of repair capability that should be provided. The model has potential application to other problems involving the parts hierarchy, such as determining optimal module sizes.

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I. INTRODUCTION

Sometime prior to the entrance of a new equipment into the military inventory, procedures are applied to determine initial requirements for repair parts for the various activities involved in the operation, maintenance, and backup support of the equipment. The focal point of this provisioning operation usually consists of a provisioning conference, attended by military and industry technicians. During the conference, two main functions are accomplished with respect to supply support of the new equipment. One function consists of determining which of the many different parts in the equipment should be stocked as repair parts at the various support activities. The other function consists of determining how much to initially stock of those parts selected as candidates for such stockage.

Viewed as decision processes, the first function may be identified as an "inventory range problem" since it concerns the determination of the range of parts to carry at each activity in support of the equipment. The second function may be identified as an "inventory depth problem" since it involves the determination of desired depth of stockage for those parts selected by the range decision process. In actual practice, these two decision processes are not usually separated from each other and applied sequentially. Instead, parts within the equipment are individually reviewed and decisions are made simultaneously as to whether to provision none or some specified quantity at a particular support activity. In other words, the solution to the range problem is often viewed as a zero answer to the depth problem, despite the fact that there are specific factors bearing upon the range decision that are not relevant to the depth decision. The simultaneous answer to both problems is satisfactory only as long as it is obtained intuitively, where presumably all factors bearing upon both problems are integrated in the minds of the decision makers in reaching the solution.

Almost all of the research in inventory theory has been directed towards obtaining analytic solutions to the inventory depth problem. In general, this work must be viewed as assuming an a priori selection of a part as a candidate for stockage. Even though these solutions to the depth problem may yield zero as an answer for a particular part at a particular activity, this cannot be considered as a proper answer to the range problem for that part, because many factors bearing directly upon whether or not to stock the part are omitted from the analysis. Looking further into why these factors are omitted, it is because the analysis of the depth problem is usually confined to one part at a time and functional relationships between different parts within the equipment are largely ignored. For the range problem, however, these interpart relationships are dominant factors in obtaining proper solutions. Of course, the ideal goal would be to develop a single analytic model incorporating all factors bearing upon both the range and depth problem and which yields desired answers to both problems in an integrated fashion.

Although this paper is basically concerned with an analysis of the inventory range problem, a model is developed which includes both range and depth decision processes. This feature results from the strong influence of depth decisions upon range decisions; in fact, the range problem cannot be considered as an isolated problem in the same sense that the depth problem can be separately analyzed. As a consequence, the model represents a first step towards a generalized decision technique covering both problem areas. However, the depth decision process enters into the model as a self-contained submodel and therefore retains its integrity insofar as application of most extant results on this problem are concerned. In this respect, the depth problem may be suboptimized within the framework of an optimizing range model, but whether or not this results in a general optimizing model for both decision problems is not known. Further research may indicate that a stronger interrelationship between the two decision functions exists and which must be considered before a true integrated optimizing model can be developed.

A. Environmental Complexities

Inventory range decisions must be made within an environment containing four general dimensions of complexity. First, there is the parts hierarchy, wherein all of the parts of an equipment may be ranked in terms of inclusion to form components, assemblies, subassemblies, etc. Second, there is repair capability which at a given repair facility may range from very much to very little. Third, there is a double support echelon structure--one for inventories, which determines for each activity in a supply system a source for resupply, and one for repair, which determines where an assembly may be sent for repair if beyond the repair capability of the activity at which the failure generates. Finally there is time, since repair capabilities, support echelon structures, and even the equipment parts hierarchy can change over time.

In this paper, only the first two dimensions of complexity are explicitly recognized and incorporated in the analysis of the inventory range problem. Thus, although the parts hierarchy and varying repair capabilities are considered, only one activity is assumed to perform repair and supply functions in support of the equipment. Also, a stationary future is assumed whereby no changes in the parts hierarchy and repair capabilities are permitted. Furthermore, it is assumed that range decisions made at the time of provisioning are adhered to throughout the future support of the equipment. Thus, the possibility of changing the mix of parts carried in stock over time is not considered in the analysis. Although it is intended to extend the analysis at a later time to the multi-echelon and dynamic cases, some discussion is presented in this paper of these environmental characteristics in order to further clarify the scope of the present analysis in context with the larger problem.

B. Trade-offs Between Repair and Stockage

Both in practice and in theory, the various dimensions of environmental complexity are intimately interrelated insofar as the inventory range problem is concerned. For example, there is a distinct trade-off between the amount of repair capability provided a support activity and the range of parts provided as spares. As repair capability is reduced, a fewer number of more costly assemblies must be stocked in greater depth in order to maintain support effectiveness. With more repair capability, a wider range of smaller parts will be required to support the repair operations.

Conversely, inventory range decisions directly affects the requirement for and utilization of repair capabilities. If only a few larger assemblies are carried as repair parts, all repair operations involving the removal and replacement of smaller parts are thereby precluded. A large range of repair parts will increase the utilization of repair capabilities that are provided and even encourage the expansion of the capabilities in terms of different kinds of repair that are attempted as well as augmentation of repair personnel and equipment.

Similar trade-offs exist with respect to the supply and repair echelon structures. For example, as repair capability and range of stockage are decreased at lower activities, either the repair capability and range of stockage must be increased at higher activities in the repair echelon structure or the depth of stockage for higher cost assemblies must be increased at higher activities in the supply echelon structure. Thus, range decisions at any activity in the support system affect not only decisions relative to repair capability at that activity but also repair and stockage decisions at other activities in the over-all support system.

It is evident that the inventory range problem is exceedingly difficult when the full complexity of its environment is considered. However, by making certain simplifying assumptions concerning the nature of the environment, practical solutions can be obtained. These assumptions and their implications are presented in further detail during the course of the subsequent analysis.

C. Characteristics of Range Decision Process

Traditionally, range determinations have been made largely by intuition and judgment. In some cases these determinations prove to be satisfactory; in other cases, they do not. However, some of the aspects of the approach taken in making these judgmental decisions may prove instructive in attempting to apply more rigorous and analytic methods to the problem.

The most apparent thing that is noticed when observing technicians making range decisions is that for most parts in an equipment, the decisions are made rapidly with little or no deliberation or discussion. On the other hand, the decisions for a few parts appear difficult and require extensive consideration and discussion. In general, this behavior suggests that the number of factors required to obtain solutions vary from a few for some parts to many for others. In turn, when seeking analytic methods for making these decisions, it may be inferred that relatively simple methods, involving few factors, may be applied to most of the items, with further gradations of complexity required for successively smaller numbers of items. In other words, one may expect that a curve as shown in Exhibit 1, expressing the percent of parts in the equipment as a function of the difficulty of the decision process, may characterize the range determination problem.

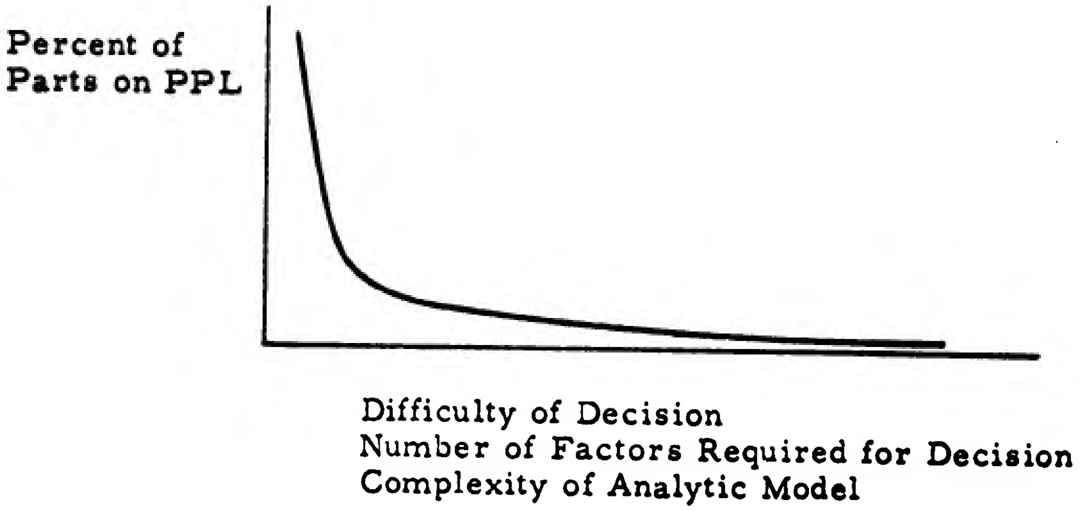

EXHIBIT 1 - PERCENT OF PARTS AS A FUNCTION OF DECISION DIFFICULTY

There also appears to be a correlation between the complexity of the part and the difficulty in making range decisions. In general, the more complex the part is, in terms of number of assemblies, subassemblies, etc., that it contains, the more difficult the decision. This, of course, corresponds with the previous observation on the percent of parts as a function of decision difficulty, since most parts in an equipment are either detail parts or simple assemblies, with only a few parts being complex assemblies. Again, it might be deduced that any analytic technique for solving the range problem might vary in sophistication according to the degree of part complexity.

Whether or not these observations, concerning the way in which range decisions are now made, can serve as guidelines in the development of analytic models will become apparent in the later development. Also, it will be of interest to see whether or not the model that is presented later displays the same characteristics associated with present decision methods.

D. Organization of Paper

The remaining sections of the paper may be placed into three main logical groups. The first group, Sections II through V, establishes basic definitions and factors bearing upon the problem and, in particular, defines and describes the Parts Provisioning List which provides data concerning the parts that are subject to the range decision process. The second group, Sections VI and VII, develops the basic range decision model for simple assemblies that consist only of detail parts. The last group, Sections VIII and IX, extends the range decision model to higher assemblies and to equipments of arbitrary complexity.

II. THE PARTS PROVISIONING LIST

In practice, range determinations are made for items listed on a Parts Provisioning List (PPL). This document, usually prepared by a contractor, contains information concerning the various articles or parts within an equipment. Since most of the subsequent discussion centers around the PPL, a detailed description of it is warranted. In addition, several definitions are established for later use.

A. Definition of Equipment

A given PPL, no matter how obtained or constructed, consists of a list of separately identified parts. These parts are normally assembled together into an article or articles designed to perform prescribed functions. This article or articles will be identified as an "equipment" throughout the subsequent discussion.

This definition of an equipment may or may not correspond with the general meaning of the term. One common definition of an equipment is as follows:

> "A component or components and necessary assemblies, subassemblies, and parts connected or associated together to perform an operational function."

When considering an equipment defined in this way, it is frequently possible to identify portions of it which also satisfy the same definition. Also, there may be a larger system containing the equipment under consideration which in turn satisfies the definition. In general, when considering an array of articles ordered in terms of inclusion when assembled, operated, or associated together, it is frequently ambiguous as to which level in the resulting hierarchy should be identified as an equipment. Similarly, the terms "system," "component," "assembly," "subassembly," etc., as identifying labels for other levels in the hierarchy, become matters of individual judgment. To avoid this ambiguity for working purposes, an equipment is therefore defined by association with a given PPL; the choice of what to include on a given PPL is, of course, a matter of judgment, but once the choice is made, it will define an equipment by association.

B. Definition of Part

A part is defined to be an article or an assemblage of articles that distinguishes it from any other part insofar as designed use is concerned. Two parts are considered the same, despite possible physical variations, if they are completely interchangeable under all operational conditions relating to the use of the parts in the equipment. If a part can be disassembled into several component parts, the collection of component parts is no longer identified as a part; only the assembly itself is considered to be a part. Although the identification of articles as parts is sometimes ambiguous (for example, a reel of wire which is cut into various lengths, each length serving a different use within an equipment), common interpretations are satisfactory in subsequent use of the term. In particular, articles represented on a PPL are identified as parts and numbers are normally assigned (part numbers, stock numbers, drawing numbers, etc.) which uniquely identify one part from another insofar as any subsequent use is concerned.

C. Definition of Item

A particular part referred to by a separate entry on the PPL will be defined as a line item or, for short, an item. Each item is identified by a unique item number. There may be several different line items on a PPL referring to the same part or article. This is caused by the use of the same part in different portions of the equipment; the assignment of different line items enables unique identification of each separate use.

In addition, more than one unit of a part may be represented by an item on the PPL, since multiple applications of the same part at the same place in the equipment parts hierarchy are, by convention, not individually listed. A common example is several identical bolts fastening one part to another. The set of bolts, in this case, is identified as one line item. The same bolt used elsewhere in the equipment would require another line item.

If the same part is included in several line items on the PPL, it is common practice to fully describe the part on its first appearance in the list of items. Abbreviated descriptions are then used in subsequent appearances, with a reference to the first appearance.

D. Range Prejudgments

The choice of which physical articles or parts to include and separately list on the PPL is a matter of judgment. Furthermore, the exercising of this judgment predetermines, to a significant degree, part of the range problem. In fact, it is impossible to construct a PPL without making explicit and implicit range determinations.

The most important decision affecting the range problem concerns the matter of inclusion--deciding which items to list on the PPL and which items to exclude. Articles that are not represented on the PPL will not be purchased and stocked in the supply system under any range determination procedure based upon the PPL, including that which is presented later. Therefore, great care must be exercised in deciding which articles should or should not be identified as items on the PPL.

Common reasons for excluding articles from representation on the PPL are as follows:

1. The article is of such common use that it is presumed to be already available or can easily be obtained at all of the locations where its need might occur. Such articles as solder, common sizes and kinds of wire, common sheet metals and bar stock, and many kinds of common hardware items are examples.

2. The article cannot physically be separately removed and replaced and is therefore not considered to be capable of separate supply and replacement. Examples are parts potted in sealed components or parts fabricated into permanent assemblies.

3. Prejudgments are made concerning the need to remove and replace the article. Even though the article is physically capable of being separately removed and replaced, a judgment is made that the need for such action will never occur or that the chance of such requirement is vanishingly small. Examples might be a bracket which incurs no environmental stress, or a laminated core in a solenoid.

4. Prejudgments are made concerning the practicability and economic advisability of removing and replacing the article. An example might be a chassis in an electronic module, which is possible to remove and replace but would require detaching and reattaching dozens of electronic components. The prejudgment in this example might be that the cost of removing and replacing the chassis would be far greater than the cost of the entire module.

5. Articles may be omitted from representation on the PP through error or oversight. This may occur when originally preparing the PPL from engineering drawings or by inadvertently dropping items from the PPL during subsequent processing.

Instructions for preparing PPL's normally are written to prevent some of the above kinds of exclusions from occurring. Usually, only the second kind of exclusion is permitted: articles which cannot physically be separately removed and replaced. However, in many cases, judgment must be exercised in excluding articles according to even this criterion. These decisions usually depend upon estimates of resources available to accomplish repair operations. As one extreme, the lack of all tools and maintenance personnel would exclude most parts according to this criterion. As the other extreme, the availability of sufficient tools and skilled personnel could enable the removal and replacement of all articles in the equipment, possibly to the point where it becomes difficult to identify articles as items. As an extreme example, an ordinary light bulb is normally considered to be an item such that any part of it cannot be physically removed and replaced; yet, with sufficient resources, the filament could be removed and replaced when it burns out, or the glass could be replaced should it break. In fact, it is difficult to cite examples of articles that cannot be further separated into constituent elements, given sufficient resources.

The main conclusion to be drawn from the above discussion is that any methodology or set of criteria for constructing a PPL represents a partial solution to the theoretical range problem. To use the light bulb example, the failure to list the glass, filament, metal base, etc., as separate items on the PPL automatically eliminates these articles from stockage in the supply system. Furthermore, it is impossible to establish a universal set of criteria, since any PPL, however constructed, can always be further detailed by relaxing implicit assumptions on resources available for remove and replace. The only theoretical limit to this process is a reduction to the common elements--an absurd extreme. Yet, any identification of items short of this extreme carries with it prejudgments as to what "logical" replacement parts are and, correspondingly, an initial solution to the range problem.

For working purposes, then, further discussion of the range problem must be confined to items as listed on a PPL. It must be assumed that the selection of items for inclusion has been as complete as possible, with only those articles excluded that are manifestly, according to technical judgment, not suitable for separate stockage as repair parts.

E. Indenturing

Items on the PPL are normally visually arranged and ordered in an organized fashion. A number may be associated with each item to indicate its position in the hierarchy of parts which constitute the equipment when assembled. This number, defined as an indenture number, represents an ordering of items by inclusion: an item with a given indenture number contains and is largely composed of all immediately subsequent items with the same or larger indenture number. The ordering of items in this way is often called a "top-down breakdown" sequence which has inferences, not always valid, concerning the sequence in which parts must be removed when the equipment is completely disassembled.

A usual way of defining the manner in which the items are sequenced on the PPL and the indenture numbers assigned is shown in Exhibit 2. Another way to visualize this method for assigning indenture numbers is shown in Exhibit 3. Here, the indenture number identifies various levels in the parts hierarchy. From such a schematic, sequential line item numbers are assigned by working from left to right, exhausting all lower parts in a component, assembly, subassembly, etc., at each level before stepping to the next component, assembly, subassembly, etc., at the same level.

EXHIBIT 2 - ASSIGNMENT OF INDENTURE NUMBERS

Indenture Number

1	2	3	4	5	6	7	8
Equipment							
	Component						
	Attaching parts for component						
		Detail parts of component not contained in assemblies or subassemblies					
		Assembly					
		Attaching parts for assembly					
			Detail parts of assembly not contained in subassemblies				
			Subassemblies				
			Attaching parts for subassembly				
				Detail parts of subassembly not contained in sub-subassemblies, etc.			

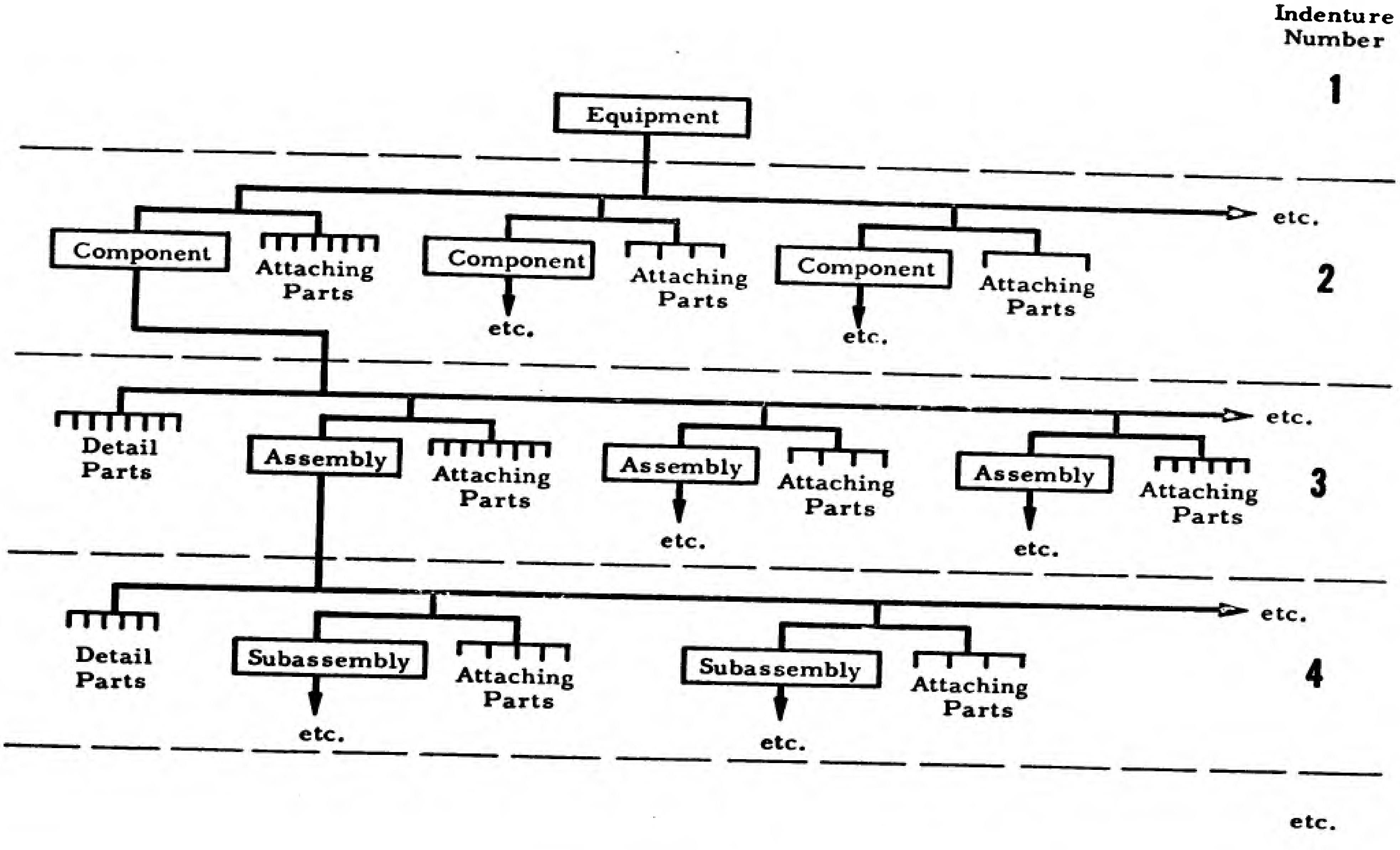

EXHIBIT 3 - SCHEMATIC OF PARTS BREAKDOWN

It should be noticed that neither the item sequence number nor the indenture number assigned in this fashion uniquely identifies the position of a particular part in the parts hierarchy. By position is meant the identification of higher parts in which the given part is included and the extent to which the given part is composed of lower parts. In addition, the labels "component," "assembly," "subassembly," etc., as implicitly defined in Exhibits 2 and 3, cannot be uniquely assigned to each part (although they are unique to line items), because a given part may be at different levels in the parts hierarchy and hence receive different labels. Since the range problem is more concerned with parts than with line items, it will be necessary later to develop a notational method to overcome this ambiguity.

F. Bulk Material

Various conventions are used to include on the PPL bulk materials, such as paint, wire, solder, lubricants, etc., that are used during normal maintenance and servicing of the equipment. One way is to list all such items at the end of the PPL, without particularly identifying particular portions of the equipment to which they apply. Another way is to list them with each logical portion (component, assembly, etc.) in the parts hierarchy to which they apply. In this case, a full description of the items is usually given only on this first appearance on the list, with abbreviated descriptions, referenced to the first appearance, for all subsequent appearances. This latter way of listing these kinds of items on the PPL will be assumed to pertain in the subsequent discussion.

The inclusion or exclusion of items of this nature is subject to the same judgmental determination as previously described. In many cases, this determination is more difficult than for the more specialized items, since it requires some understanding as to what might be required for normal maintenance and servicing. Not only does the degree of maintenance and servicing vary widely from one location to another, but at a given location what constitutes "normal" maintenance and servicing is subject to interpretation and judgment.

G. Special Tools and Instruments

Special tools and test instruments are frequently required to accomplish repair, maintenance, testing, and checkout of the equipment or portions of the equipment. Items of this nature are usually listed at the end of the PPL or on a separate document altogether. In the latter case, they may pertain to equipments on several PPL's. It will be assumed that any such items required for the particular equipment on a PPL will be listed at the end of the PPL.

H. Item Data

For each item listed on a PPL, the following data will be assumed to be available prior to the analysis subsequently discussed:

1. Item Sequence Number - a number uniquely assigned in some systematic manner to each item on the PPL. Although there is a one-to-one correspondence between item and item sequence number, a given kind of part may have several item sequence numbers due to multiple appearances on the list.

2. Item Name - an abbreviated verbal description of the part, usually obtained from Federal Cataloging Handbook H6-1 or assigned by the manufacturer of the equipment.

3. Indenture Number - a number indicating the position of the item in the parts hierarchy as previously described.

4. Stock Number - a number which is associated with the article or part and which uniquely distinguishes the article from other articles having different physical characteristics. The stock number should be a Federal Stock Number, when available, otherwise, a number assigned by the manufacturer of the part or by the equipment contractor.

5. Number of Applications - the number of units of the article included in the item representation. This is normally interpreted as the number of applications of the part in the "next higher assembly."

For subsequent working purposes, the format of the above data is immaterial. However, standard formats for PPL data, which include the data elements listed above, have been developed and are now in use.

I. Matched and Associated Items

Certain kinds of parts may be separately listed on the PPL, but must be always considered together with other parts in all supply and maintenance functions. Such items as gears, pistons, cylinders, bearings, resistors, and vacuum tubes may be matched electronically in systems or circuits, machined to special fits, or lapped as matched sets or assemblies. For the particular equipment under consideration, such matched sets of parts may be considered as one part for all supply and maintenance operations; yet, in other equipment or different portions of the same equipment, a part in such a set may be used independent of the set. Therefore, the convention is usually adopted to separately list the components of matched sets on the PPL and to reference the fact that they are members of a matched set.

A somewhat similar relationship between different parts exists in terms of normal usage. A particular part may be such that if it fails and must be removed and replaced, other designated parts are also normally removed and replaced. A familiar example is a carburetor wherein particular gaskets are always removed and replaced whenever the carburetor is opened up to replace an internal part. This type of relationship between parts is usually one-sided. For example, replacing the float in a carburetor will cause a replacing of gaskets, but if a gasket leaks and needs replacing, the float would not also be replaced. A frequent cause of this kind of association between parts is that particular parts, such as gaskets and seals, must be damaged in order to remove and replace a failed item, and must therefore also be replaced.

To identify these interrelationships between items on the PPL, a referencing system will be assumed. For each item on the PPL which, when removed and replaced, causes other items to normally be removed and replaced at the same time, the item sequence numbers of these other items will be listed. The referenced item sequence numbers will be preceded by an "M" if the parts involved are in a matched set and by an "A" if they are associated through usage. These reference item sequence numbers are another kind of item data assumed to be provided on the initial PPL in addition to those previously listed.

J. Example of PPL

Exhibit 4 shows a portion of a PPL organized in the fashion described above and containing the required item data. It also illustrates, in several places, the kind of range prejudgments that commonly occur. For example, the servo motor (item sequence number 226) is not further broken down into component parts. Therefore, any such component parts are automatically excluded from consideration for stockage in the supply system. Similar observations apply to the potentiometer (item sequence number 222), the synchro (item sequence number 228), and possibly other items.

EXHIBIT 4 - EXAMPLE PORTION OF PPL

Item Sequence Number	Indenture Number	Item Name	Part Number	Reference Numbers	Number of Applications
221	3	Roll Angle Drive Assy	2016272		2
222	4	Potentiometer	2016502-1		1
223	4	Servo Motor Assy	1996844		1
224	5	Plug	2016497-2		1
225	5	Marker	2016489-4		1
226	5	Servo Motor	2016303-1		1
227	4	Synchro Assy	1996845		1
228	5	Synchro	2016500-1		1
229	5	Plug	2016497-2		1
230	5	Marker	2016489-3		1
231	4	Housing	2016261		1
232	4	Clamp	1996819		3
233	4	Clamp	1996678		1
234	4	Stop Clamp Assy	1996836		1
235	5	Clamp	1996834		1
236	5	Spring	1996824		2
237	5	Stop	1996825		2
238	5	Stop	1996833		1
239	5	Screw	MS 35457-1		1
240	5	Screw	MS 35265-2		1
241	5	Screw	MS 35265-3		1
242	5	Washer	MS 35338-39		1
243	5	Washer	MS 35338-40		1
244	4	Bracket Assy	1996832		1
245	5	Bracket	1996830	A246	1
246	5	Gasket	1996831		1
247	4	Dial and Hub Assy	1996835		1
248	5	Dial	1996818		1
249	5	Hub	1996817		1
250	4	Pin	1996823		1
251	4	Gear	2016263	M252	1
252	4	Gear	2016262	M251	1
253	4	Support Block	1996675		1

III. REPAIR AND SUPPLY SUPPORT SYSTEMS

As previously stated in general terms, the range problem is closely interrelated with decisions concerning the repair capability to be provided at each support activity and with the nature of the supply and repair echelon structures. In this section, the repair and supply support systems are described in further detail and definitions of repair, repair capability, and levels of the repair and supply echelons structures are established. In general, the contents of this section establishes further descriptions of the environment within which range decisions must be made and provides an additional foundation for the analysis presented in this paper and in subsequent work on the more general range problem.

A. Definition of Repair

In the subsequent analysis of the range problem, the word "repair" will be frequently used with a rather specific meaning. It is therefore defined as any process or activity applied to a part believed to be inadequate for its intended present or future use because of malfunction, degradation of performance, deterioration, wear, or other cause. Furthermore, the process must directly result in the withdrawal of a part or parts from stock, except as specifically noted in the subsequent analysis.

Examples of repair processes according to this definition are maintenance, repair, overhaul, test, inspection, checkout, calibration, and diagnosis. Units of a part to which these processes are applied will usually be referred to as "failed" or "defective" units, although such terms are not always appropriate. For example, a part may be subject to periodic test or overhaul even though it has neither failed nor is defective. Processes such as lubrication, adjustment, or protective operations which can be performed without disturbing the use of the part are excluded as repair processes.

B. Repair Capabilities

To accomplish repair operations, a wide variety of resources may be used. These include manpower in terms of many kinds of skills and levels of experience, tools, machines, general and specialized test and checkout equipment, repair shops, fixtures, jigs, and so forth. Also, inventories of repair parts and materials should be included. In general, "repair capability" should be defined in terms of these resources, both as to the range of different kinds of resources and the amount of each kind. However, in later work, repair capability will refer to the aggregate of resources, other than spare parts, that is available for use or expenditure during repair processes.

A basic feature of present methods for making range determinations is that explicit and implicit judgments are made concerning the type and degree of repair capability at the various support activities. In most cases, the extent of the repair capability has been predetermined. In fact, often such capabilities already exist with an implicit assumption that supporting the equipment under consideration will not significantly alter these capabilities. In any case, a main problem in making range determinations by present methods is to fully understand just what repair capabilities already exist or are planned to exist, and what their limitations are relative to the equipment under consideration. This is frequently a matter of interpretation and judgment and often constitutes the deciding factor in making particular range decisions.

Since decisions concerning repair capability and range of parts to be stocked are so greatly interrelated, the repair capability should be a decision variable along with inventory range in any analytic formulation of the range problem. This, in fact, is a characteristic of the range decision model presented later, where necessary repair capabilities are determined in consonance with range decisions. Provisions are made to reflect repair capabilities that may already exist or that might exist as a consequence of overriding policies that are imposed above and beyond the model itself. However, in common with present methods for making range decisions, the model requires knowledge of what repair capabilities are needed for different repair processes and what capabilities already exist that may be applied towards the need.

C. Supply Echelon Structure

Since range decisions at one support activity can affect stockage requirements not only at that activity but also others, in particular, those higher up in the supply echelon system, it is necessary that this supply support structure be well defined. In theory, the supply support system should be a policy variable, but, if so, all equipments supported by the system should be included in the analysis. Since this is manifestly impractical and unrealistic under present conditions, it will be assumed that the supply structure relative to the equipment being considered, whether or not it already exists, is well defined and prescribed. Thus, it is assumed to be an input to the analysis rather than a policy variable.

Actually, two main assumptions concerning the supply system are included in the analysis of the range problem. One is that the echelon structure is defined by the identification of a normal source of resupply for each activity. The other is that if a part is stocked at an activity, it is governed by a known and well-defined stockage depth policy. Other than a mild restriction imposed later, there is no particular limitation concerning the depth policy that is used as long as it can be stated analytically. In particular, most of the depth models resulting from past research in inventory theory may be accepted.

D. Repair Echelon Structure

In addition to a supply echelon structure, there may typically be a repair echelon structure. Such a structure usually results when repair capability is significantly different from one activity to another, where it may be desirable to send a failed assembly to an activity other than the one at which it failed from repair. Often, activities may exist whose only function is to repair parts sent from other activities and which have no direct support responsibility for the end equipment.

Like the supply echelon structure, the repair structure theoretically should be a policy variable. Again, however, this would require consideration of all equipment supported by the system. Therefore, it must be assumed that the repair echelon structure for the equipment under consideration is given and well defined. Under this assumption, the structure is defined in terms of identifying, for each repair activity, a normal destination for failed parts if beyond the activity's repair capability. Of course, whether or not to forward a particular failed unit of a part will be an outcome of the inventory range decision process. However, the choice of where to send the unit is assumed given.

E. Source, Maintenance, and Recoverability Codes

As part of the provisioning process, codes are assigned to each part in the equipment which, to a large extent, reflect the results of range and repair decisions and, to a lesser extent, depth decisions. A source code is assigned to first of all identify whether or not the part is procured for stockage and second, to indicate anticipated usage and to prescribe normal methods for resupply. For example, the set of source codes signifying stockage further identify parts as to whether anticipated usage is relatively high or sufficiently low as to be classed as "insurance" items, whether parts are relatively easy to manufacture at principal using activities if necessary, and whether parts are procured in accordance with life expectancies as caused by deterioration or anticipated obsolescence. The set of codes signifying nonstockage further identify parts as to whether they can be manufactured within the maintenance system if needed; whether they are to be procured if demands occur; whether they are to be assembled from other parts if needed; whether they are normally impractical for stocking, maintenance, or manufacture; and so forth.

Maintenance codes are assigned to parts to indicate general kinds of activities at which specified repair processes are authorized. First, support activities are broadly categorized according to their principal function, with implications concerning their position in the repair echelon structure. Examples are shore-based overhaul activity, tender or repair ship, specialized repair facility, contractor repair facility, activity to which equipment is assigned, and so forth. Then, using these activity designators, two maintenance codes are assigned to parts. The first code identifies the lowest authorized maintenance echelon capable of removing and replacing the part and, therefore, identifies the lowest echelon authorized to stock the part for replacement purposes. The second code identifies the lowest maintenance echelon to which lower echelons are authorized to send repairable parts when beyond their repair capability.

The recoverability code reflects the recoverability characteristics of parts removed or replaced in equipments during normal maintenance procedures. It identifies whether it is normally economical and practical to repair the part; otherwise, whether or not the part can be economically salvaged and its component parts reclaimed. If a part is to be neither repaired nor salvaged, it is identified by the code as being expendable or consumable.

It can be seen from the above descriptions, that the source, maintenance, and recoverability codes describe in a general way results of range decisions, the repair and supply echelon structures, and the repair characteristics for each part. However, they are nonspecific in the sense of being system oriented rather than activity oriented. For example, the codes do not specify range decisions by activity nor do they precisely define the supply and repair echelons pertaining to the part. Of course, they are not intended to precisely define the environment of the part and to prescribe exact range decisions; instead, they serve only as general guides in these directions. As a result, the codes can serve neither to define the environment for an analytic range decision technique nor as a framework for results of range decisions.

On the other hand, these codes can be assigned to each part from environmental data input to the analytic range decision process and from results that are outputs. Since more detail is represented in these inputs and outputs than is included in the present code definitions, a new code structure could be developed to include more detail and perhaps be more meaningful. However, the construction of such codes is considered beyond the scope of this paper.

In summary, the source, maintenance, and recoverability codes, as presently defined and used, will not enter into the subsequent analysis, and even though a more precise code system to serve the same functions can be developed from the range decision model, this will not be accomplished.

IV. THE PARTS HIERARCHY

Since the solution to the range problem is critically dependent upon the way in which the various parts are assembled together to make up the equipment under consideration, it is necessary to precisely define the way in which the parts are so organized. In this section, a notational system is developed to identify the position of each part in the parts hierarchy and definitions are established for different kinds of assemblies.

A. Item Hierarchy Number

In Section II, the present method for listing items on the PPL and assigning indenture numbers was described. Although this method indicates the way in which parts are organized in terms of inclusion, it is not sufficiently precise for analytic purposes. Therefore, it is necessary to define an item numbering system which describes in further detail the position of each item in the parts hierarchy and which portrays the complexity of the part associated with the line item.

The numbering system is developed from a schematic diagram of the parts breakdown, similar to that shown in Exhibit 3. Suppose, then, that the equipment under consideration can be broken down into its constituent parts as shown in Exhibit 5. The only difference between the schematics in Exhibits 3 and 5 is that in Exhibit 5 no special distinction is made between attaching parts and detail parts, and assemblies are not identified as components, assemblies, subassemblies, etc. There is a distinction in Exhibit 5, however, between detail parts represented by circles and assemblies represented by rectangles. Otherwise, the way in which the items are related in terms of inclusion is evident from the schematic.

As before, different levels in the parts breakout are identified; this time, however, by a level number rather than an indenture number (although they are synonymous). The different items are numbered as indicated on the schematic with each item receiving as many numbers,

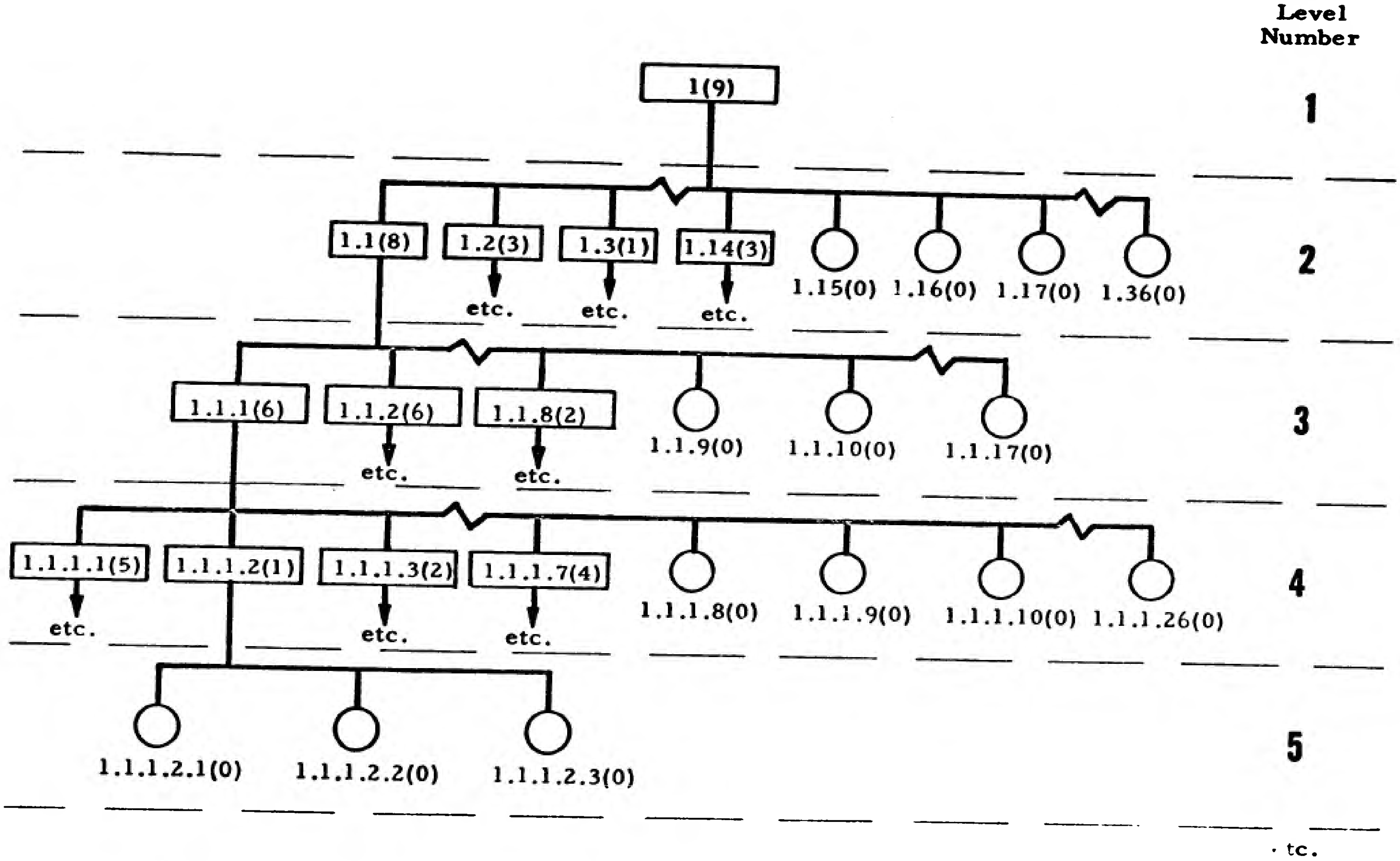

EXHIBIT 5 - EXAMPLE PARTS BREAKDOWN AND HIERARCHY NUMBER ASSIGNMENTS

separated by decimal points, as given by the level number. Thus, an item on level 5 is assigned five numbers separated by decimal points. This string of numbers, including the decimal points, is identified as the item hierarchy number, or H-number.

The portions of the H-number as separated by decimal points identify, by reading from left to right, the highest, next highest, etc., assembly in which the item is contained. The last portion of the H-number is a sequence number to uniquely identify the item in its next higher assembly. Thus, it is readily apparant that the H-number for each item uniquely identifies all higher assemblies containing the item.

There is a one-to-one correspondence between H-numbers and item sequence numbers as defined in Section II. In fact, if the H-number numbers are written down, registered on the left with blank portions represented by zeros, and ordered in strict numeric sequence (employing the usual practice with regards to decimal points), the items in the equipment, will be listed in about the same sequence as described in Section II. Although the H-number could be used as an item sequence number, its length is too cumbersome for most practical purposes.

B. Assembly Order Number

The H-number for each item is appended with a number in parentheses which indicates the number of lower levels that exist for the item's constituent parts. This number is identified as the assembly order number. It identifies, generally, the complexity of the part associated with an item in terms of the extent to which it can be further broken down into lower assemblies.

If the assembly order number is zero, this signifies that the part for the item cannot be further separated into smaller parts. Such parts will generally be referred to as detail parts. If the assembly order number is different from zero, the part will generally be referred to as a Kth order assembly, where K is the assembly order number. Whereas the H-number identifies items, the assembly order number is associated with parts.

C. Subordinate Parts

For later purposes, it is convenient to define, for a given part, the parts on the next level down in the parts hierarchy as being subordinate parts for the given part. Thus, subordinate parts for an assembly consist of those assemblies and detail parts which directly make up the given assembly. Detail parts have no subordinate parts, whereas all subordinate parts for first-order assemblies are detail parts. In general, the assembly order numbers for subordinate parts must be at least one less than the assembly order number for the assembly to which the parts are subordinate.

D. Rule for Multiple Appearances

Since a particular part may be applied in more than one place within the equipment, it may receive more than one H-number. It might happen that a PPL is constructed such that the assembly order number differs from one such H-number to another. This results from deliberately or inadvertently failing to breakdown the part the same way in each application and represents prejudgments concerning item exclusion that affects range decision.

To preclude this situation from occurring, a rule is imposed that a part will have the same assembly order number in all of its appearances in the equipment. The application of the rule may mean an augmentation of the PPL to further breakout a part in some of its appearance so as to be consistent with appearances where it is represented in greater detail.

It should be noted that whereas a PPL is assumed to be constructed according to this rule for analytic purposes, it may for other purposes breakout an assembly only in its first appearance, with appropriate references made in later appearances. Also, even with the rule, a part may have different assembly order numbers for different PPL's.

E. Connecting Parts

Although connecting parts receive no special consideration in the assignment of H-numbers, some further discussion is warranted to serve as a guide as to where they should be located in the parts hierarchy as represented by the schematic of Exhibit 5.

Connecting parts, which can consist of either detail parts or assemblies, are generally of two kinds. One kind of connecting part connects two or more assemblies of the same or different orders and cannot be properly considered as an integral component of any of the relevant assemblies. Thus, if one of the assemblies is removed and replaced, the connecting part is not normally also removed and replaced. A typical example is a cable assembly, connecting one electronic assembly to another, which is not stocked with, or removed and replaced with, either of the two assemblies.

The other kind of connecting part is an integral part of a particular assembly with respect to normal supply and repair operations. An example is a bracket which holds one assembly to another, but which is a part of one of the assemblies in the sense that if the assembly is stocked as a repair part, the bracket is a constituent part of the stocked assembly. Also, whenever the assembly is removed and replaced, the bracket remains an attached part of the removed unit.

A connecting part of the first kind is included in the parts hierarchy at the same level as the parts which it connects. A connecting part of the second kind is included in the parts hierarchy at a level less than that of the assembly of which it is considered an integral part. In all respects, such a connecting part is treated as a detail part or lower order assembly and no special distinction or ordering sequence is needed.

V. REPAIR AND INVENTORY COSTS

Since solutions to the inventory range problem will affect many of the costs associated with the over-all repair and supply support of the equipment, it is necessary to identify and define these costs. The cost factors defined in this section will form the main inputs to the analytic range decision model.

In general, costs affected by range decisions may be divided into two categories: those associated with repair processes and those associated with supply operations. The various costs in each of these categories are further defined and described below in general terms. Some of these costs will be further defined in context with their subsequent application in the range decision model.

A. Repair Costs

Included as repair costs are all direct and indirect costs of services and facilities required to accomplish repair processes. Excluded are costs of parts and materials identified on the PPL are used in the repair processes. The various kinds of repair costs are defined below.

1. Initial Repair Setup Costs

Included as initial repair setup costs are fixed costs incurred to initially establish repair capabilities and which are independent of how much actual repair is subsequently accomplished. Examples ples are costs of technical manuals and repair instructions, establishment of repair records and data, fixed costs of acquiring tools and equipment necessary for repair, fixed costs of personnel, etc.

2. Repair Batch Setup Costs

These costs include all fixed cost incurred in setting up for the repair of a batch of failed assemblies and which are independent of the number of assemblies that are repaired at a time. Examples are paperwork costs (e.g., workorders), costs of setting up machines, tools, test equipment, jigs, fixtures, etc., needed for the repair process, fixed administration costs, costs of dislocating maintenance personnel, etc.

3. Per-Unit Repair Costs

Included as per-unit repair costs are all costs, on a per-unit basis, directly related to the repair of a failed unit of an assembly. Examples are labor costs for diagnosis, remove and replace, inspection, test, etc., prorated operating costs of tools, test equipment, and other equipment and facilities, prorated overhead costs, etc. Excluded are costs of parts that are replaced during the repair process.

B. Inventory Costs

Inventory costs include all direct and indirect costs associated with purchasing parts and carrying them in inventory. All inventory costs are defined on a per-part basis and are futher described below.

1. Initial Inventory Setup Costs

Initial inventory repair costs consists of all fixed or one-time costs incurred in entering a new part into an inventory. Included are initial cataloging costs, costs of establishing initial stock records, costs of establishing warehouse space, paperwork costs associated with initial provisioning, etc.

2. Administration Costs

Inventory administration costs include all costs of administering a part held in inventory which are incurred throughout time but which are independent of the amount carried in stock. Included are costs of maintaining inventory asset and financial records, costs of producing recurring and nonrecurring inventory reports, costs of taking physical inventories and making reconciliations, costs of maintaining catalogs and other inventory data, etc.

3. Holding Costs

Included as inventory holding costs are those costs associated with keeping units of the part in stock and which depend upon the amount held. Examples are cost of warehousing, interest on the investment in the inventory, depreciation, deterioration, obsolescence, etc.

4. Ordering Costs

Ordering costs, consisting of costs incurred in obtaining a resupply of the part, are of two kinds. Fixed ordering costs are independent of the amount ordered and contain such costs as paperwork costs, communication costs, processing costs by the supplier, receiving costs, etc. Variable ordering costs depend upon the amount ordered and include purchase costs of the units ordered, shipping costs, packaging costs, etc.

5. Shortage Costs

Shortage costs result from lack of a replacement part when needed. Included are extra ordering costs for expedited procurement and/or costs associated with loss or degradation of the use of an assembly or equipment due to lack of the part when needed.

C. Other Costs

In addition to the repair and inventory costs described above, there are many kinds of costs that relate directly or indirectly to the support of equipment but which are of a special nature or of lesser importance insofar as the inventory range problem is concerned. Some of these costs are described below. They are only suggestive of the many kinds of costs associated with the support of operational equipments. Some of them are further described as incorporated in the subsequent analysis of the inventory range problem; others may bear upon the range problem but are not included in the present analysis due to the complexity they introduce. Still others are dismissed as being not relevant to the analysis.

1. Salvage Costs

Parts that fail but which are not processed for subsequent reuse may be subject to various kinds of salvage operations. They may be scrapped or junked, or in the case of assemblies, they may be disassembled and serviceable components may be recovered for use as repair parts. More infrequently, they may be reworked and modified to become repair parts that differ from their original specifications and which are identified by new part numbers. In any case, two general kinds of costs may be associated with salvage operations--fixed and variable. The fixed costs include all processing, paperwork, advertising, and other costs which are independent of how many units of the part are salvaged. Variable costs include all inspection, identification, handling, processing, disassembly, and other costs which depend upon the number of units salvaged.

In addition to fixed and variable salvage costs, revenues may accrue from the salvage. If parts are scrapped, the revenues are based upon their scrap value. If parts are disassembled for recovery of components, the revenues are based upon the procurement value of recovered parts. In general, total revenue obtained from salvage is a function of the number of units salvaged.

2. Costs of Failure Reporting

When certain parts fail, procedures may be established for reporting the failure, such reports being used for various supply and maintenance purposes. If failure reports are required, costs are incurred which include costs of preparing the report, transmitting it to the responsible agencies, processing the information on the report, and other actions associated with the initiation and subsequent use of the report.

3. Costs of Automatic Failure Detecting Systems

Some equipment or parts may be provided with automatic devices that monitor the performance of the part and signal actual or impending malfunctions. Such devices incur not only initial procurement and recurring operating costs, but also all repair and inventory costs previously described since they constitute equipments or parts subject to failure.

VI. RANGE DECISION MODEL FOR FIRST-ORDER ASSEMBLIES

As a first step in obtaining a general solution to the inventory range problem, it is convenient to consider a simplified problem whose solution will serve as a basis for extensions to more general problems. Accordingly, a PPL consisting of only one first-order assembly is considered, and only one activity is permitted for repair and supply support. Certain additional assumptions are made to further reduce the problem. For this problem, a cost function is developed which, when minimized, provides least-cost inventory range decisions.

A. Description of Problem

The particular range problem analyzed in this section is characterized and defined by the following assumptions:

1. The PPL under consideration consists of one first-order assembly i, made up from detail parts j, with $j = 1$ to n.
2. Repair and supply support is provided by one activity.
3. Each failure of the assembly resulting in a repair process is caused by one and only one detail part.
4. Failed detail parts and units of the assembly that are not repaired have no salvage value. In particular, failed units of the assembly are not salvaged for usable detail parts.
5. Range decisions are made at the time of initial provisioning and upon the basis that they are not changed in the future.
6. All cost and failure factors remain constant over time. This assumption, together with the previous one, characterizes the problem and subsequent decision model as being stationary time.

The inventory range problem corresponding to these assumptions is to determine, for each part on the PPL (both the assembly itself and its constituent detail parts), whether or not it should be provisioned and carried in stock for replacement purposes.

B. Repair and Stockage Strategies

It is presumed that there is a number of "installed" units of the first-order assembly being operated according to designed use. Each unit of the assembly that fails is brought to the repair facility for some type of corrective or remedial action. Three main kinds of such actions or strategies may be identified and defined as follows:

1. Strategy A--Immediate Repair

The defective detail part that caused the assembly to fail is removed from the assembly and replaced with a serviceable unit. The failed detail part is discarded and considered unavailable for subsequent reuse. The replacement unit of the detail part is drawn from inventory; the availability of replacement units is governed by a prescribed inventory depth policy. The function served by the assembly is not accomplished until use of the assembly is regained after the repair process.

2. Strategy B--Exchange Assembly, No Repair

The failed unit of the assembly is exhanged for a serviceable unit. The failed unit is discarded and considered unavailable for subsequent reuse. The detail part causing the failure is not stocked or ordered. The availability of replacement units of the assembly is governed by a prescribed inventory depth policy.

3. Strategy C--Exchange Assembly, Repair Later

The failed unit of the assembly is exchanged for a serviceable unit. The failed unit is subsequently repaired, by removing and replacing the defective detail part, and placed in stock for reuse. The availability of replacement units of the assembly and detail part is governed by prescribed inventory depth policies. Stockages of replacement units of the assembly in this case will subsequently be referred to as a "repair buffer stock."

For each failed unit of the assembly entering the repair facility, only one of the three strategies can apply. Which of the three strategies to choose for a particular failed unit depends upon which of the various detail parts caused the failure. The assignment of a strategy to a particular detail part--to be applied whenever a unit of the assembly fails because of that detail part--constitutes a solution to the range problem for that part. If strategy B is applied, the detail part will not be stocked; if strategy A or C applies, the detail part will be stocked to the extent prescribed by the inventory depth policy that is applied.

The aggregate of strategy assignments to detail parts determines a solution to the range problem for the assembly. If strategy A applies to all detail parts, the assembly is not considered for stockage. If strategy B or C pertains to one or more detail parts, stocks of the assembly may be established according to a prescribed inventory depth policy.

The meaning of the last assumption defining the problem--that range decisions cannot change over time--is further extended to mean that the assignments of repair strategies to detail parts cannot be changed once they are made. Thus, each time a unit of the assembly fails because of a particular detail part, the same strategy is assumed to apply.

Even though assignment of repair strategies may indicate that a part should be stocked, the application of a prescribed inventory depth policy may indicate that no stocks should normally be carried. In this case, it is presumed that some alternative is prescribed by the depth policy, such as ordering only when and as needed. In this regard, the solution to the range problem, as provided by assignments of repair strategies to detail parts, serves only to identify those particular parts to be subjected to inventory depth policies with parts not so identified excluded from any present or future stockage or purchase.

C. Criterion for Strategy Assignment

In order to establish assignments of repair strategies to detail parts, and thereby determine a solution to the range problem, a criterion or objective function must be established. The particular criterion chosen is to make the assignments so as to minimize the total expected costs for supporting the assembly with respect to repair and supply.

The various repair and inventory costs associated with the support of the assembly are incurred at different points in time. Some of the costs are setup costs in the sense that they are incurred before or concurrent with the initial use of the assembly. Other costs are incurred each time a unit of the assembly fails, while still others, such as inventory holding costs, are incurred continuously throughout time. In order to obtain a meaningful solution to the range problem by means of assigning repair and stockage strategies to the various parts, a common measure of costs must be defined. With such a measure, the cost consequences of different strategy assignments may be evaluated and compared so as to obtain a minimum cost solution. The measure that will be used consists of all fixed (setup) costs plus the discounted present value of all future costs, where "present" is defined to be the time at which support of the assembly is to commence.

If N represents the collection of all detail parts in the assembly, three mutually exclusive sets, A , B , and C , may be defined, with $A \cup B \cup C = N$, such that strategy A applies to all parts in set A, strategy B to all parts in set B, and strategy C to all parts in set C. For each possible assignment of detail parts to the three sets, a total cost, T , will result consisting of initial costs plus discounted future costs. The problem will then be to find an allocation of parts to the three sets so as to minimize T .

D. Cost Function

A total cost function T , for an arbitrary assignment of detail parts to the three strategy sets, may be developed by investigating the costs attributable to each set. First, costs directly associated with detail parts will be identified, followed by costs associated with the assembly.

1. Costs Associated With Detail Parts in Set A

Since detail parts in set A are to be stocked according to some inventory depth policy, inventory costs, both initial and recurring, will be incurred. Also, since the detail parts are to be removed and replaced, initial and recurring repair costs can be associated with each detail part. Finally, an "assembly shortage cost" must be identified to reflect the cost consequences of losing the use of the assembly while it is being repaired.

These various kinds of costs associated with detail parts in set A may be summarized in the following expression:

$$C_A = \sum_{j \in A} [B_j + I_j(M_j) + R_j(M_j) + S_j(M_j)] \quad , \qquad (1)$$

where

C_A = total costs attributable to detail parts in set A.
B_j = the sum of initial repair setup costs, initial fixed inventory costs, and present value of future inventory administration costs for the jth detail part.
$I_j(M_j)$ = present value of future inventory costs for the jth detail part.
$R_j(M_j)$ = present value of future per-unit repair costs for the jth detail part.
$S_j(M_j)$ = present value of future assembly shortage costs incurred for each failure of the jth detail part.
M_j = expected annual failure rate for the jth detail part.

In this cost function, the terms, B_j , I_j , and R_j , refer generally to costs described in Section V. The term, B_j , includes all initial repair and inventory costs that are incurred once it is decided to carry the jth detail part in inventory and to use it in repair processes. Cataloging costs and costs of special tools needed to remove and replace the part are obvious examples. Also included in this term are all future administration and bookkeeping costs that will be incurred as long as the part is kept in inventory and which are independent of how many units of the part are stocked.

The term, I_j , includes all future ordering, holding, and shortage costs that result from maintaining an inventory of the jth part. These costs are dependent upon how much is carried in stock over time which is determined by a prescribed inventory depth policy.

The term, R_j , includes all costs directly associated with the repair process that involves the removal and replacement of the jth detail part. In addition to direct labor and repair equipment utilization costs, it may include what was previously described as a repair batch setup cost, where in this case the "batch" is one unit of the assembly. Thus, such costs as paperwork costs and costs of setting up tools and equipment to accomplish the repair may be included.

The term, S_j, represents assembly shortage costs incurred each time a unit of the assembly fails because of the failure of the jth detail part. Since no spare assemblies are stocked for replacement in accordance with strategy A, the failed unit will incur shortage costs during the time required to remove and replace the failed detail part. These shortage costs represent costs associated with loss of designed use of the assembly during the repair process.

The costs, I_j, R_j, and S_j, are expressed as functions of the annual failure rate, M_i. Actually, they may also be functions of other parameters; in particular, the inventory costs will depend upon the lead time, unit cost, ordering costs, and other factors in accordance with the inventory depth policy that will be used. The particular parameter, M_j, is explicitly presented to connote primary dependence of the costs upon failure rates of the detail parts as contrasted with costs, B_i, which do not depend upon failure rates.

It should also be remarked that this notation infers that the annual failure rate remains constant throughout the operational life of the assembly. This is in accordance with the assumption concerning the stationary nature of the problem.

2. Costs Associated With Detail Parts in Set B

Since the detail parts in set B are not governed by an inventory depth policy or used in any repair process, no costs directly attributable to these parts will be incurred.

3. Costs Associated With Detail Parts in Set C

Since detail parts in set C are governed by depth policies and are removed and replaced in repair processes, most of the same costs as for parts in set A will be incurred. The main exclusion is the assembly shortage cost term since a serviceable unit of the assembly is provided for use while the failed unit is being repaired. The cost function for detail parts in this set is therefore expressed as follows:

$$C_C = \sum_{j \in C} [B_j + I_j(M_j) + R_j'(M_j)] \quad . \tag{2}$$

In this expression, C_C is the total cost attributable to detail parts in set C and the terms, B_j and $I_j(M_j)$, are the same as defined for set A. The term, $R_j'(M_j)$, includes the per-unit repair costs described previously for set A but excludes any repair batch setup costs. Since failed units of the assembly, as caused by detail parts in set C, are repaired later, a batch may be permitted to accumulate before repair is scheduled. Since the size of the batch determines the amount of repair buffer stock of the assembly, and the repair batch setup cost is a dominant factor in determining the batch size, this cost is more directly associated with the assembly and will be included later.

4. Costs Associated With Assembly

If one or more detail parts are assigned to set B, set C, or both, then stockage of the assembly is governed by a depth policy and assembly inventory costs will be incurred. These costs are expressed as follows:

$$C_i = B_i \delta + J \left(\sum_{j \in B} M_j, \sum_{j \in C} M_j \right) \quad , \tag{3}$$

where

C_i = total present value of costs attributable to the assembly, i.

B_i = the sum of initial fixed inventory costs and present value of future inventory administration costs for the assembly.

δ = 0 if BUC = 0.

= 1 if BUC $\neq$ 0.

$I_i\left(\sum_{j\in B} M_j, \sum_{j\in C} M_j\right)$ = discounted present value of future inventory costs for the assembly, expressed as a function of the annual failure rates for detail parts in set B and set C.

The assembly inventory costs, which include ordering, holding, and shortage costs, are incurred according to a specified depth policy. The stockage of assemblies actually serves two functions: they are used as replacement units when detail parts in set B fail and result in permanent losses from inventory; they are also used as a repair buffer stock when detail parts in set C fail, where issued units of the assembly are later replaced by repaired units. The particular depth policy used may establish only one stock to serve both functions, explicitly recognizing the two kinds of demands according to the notation above, or the policy (or policies) may establish two separate stocks, one for each function. If the total inventory costs for the assembly can be divided according to the two functions, which is particularly possible when two separate stocks are established according to different depth policies, then the assembly inventory costs can be expressed as follows:

$$I_i\left(\sum_{j\in B} M_j, \sum_{j\in C} M_j\right) = I_i\left(\sum_{j\in B} M_j\right) + I_i'\left(\sum_{j\in C} M_j\right) \qquad (4)$$

To facilitate the subsequent solution to the problem, this expression for the assembly inventory costs will be used, with the assum that the total inventory costs for the assembly can be separated, as indicated, according to the two functions served by the assembly stocks.

5. Total Cost Function

The various portions of the repair and inventory costs associated with the support of the assembly, as given by Equations (1) through (4), may now be combined to form a total cost function which is to be minimized over all possible assignments of detail parts to the three sets, A, B, and C. This function is expressed as follows:

$$T_i = \min_{\pi_{A,B,C}} \left(\sum_{j \in A} [B_j + I_j(M_j) + R_j(M_j) + S_j(M_j)] \right.$$

$$+ [B_i \delta + I_i(\sum_{j \in B} M_j)] \qquad (5)$$

$$\left. + \sum_{j \in C} [B_j + I_j(M_j) + R_j'(M_j)] + I_i'(\sum_{j \in C} M_j) \right)$$

The expression within the large parentheses represents total costs resulting from a given assignment of detail parts to the three sets. The symbol $\min_{\pi_{A,B,C}}$ means that the total cost is to be evaluated for all possible assignments of detail parts to sets and the least of these values is to be selected to determine T_i, the minimum or optimal value of support costs for the assembly.

E. Discussion of Cost Function

Although most repair and supply costs involved in the support of the assembly have been included in the total cost function as given by Equation (5), several costs have been omitted because they have no effect upon the minimizing solution. In general, these costs are incurred regardless of the way in which detail parts are assigned to repair and stockage strategies.

Even though a failed unit of the assembly may be exchanged for a serviceable unit, in accordance with repair strategy B or C, there is an elapsed time during which use of the assembly is lost. This time span starts when the unit is first recognized as having failed and ends when a replacement unit is available for use. A shortage cost, similar to the assembly shortage cost assigned to detail parts in strategy set A, should apply to this time lapse. However, if this time lapse is excluded from that upon which the assembly-shortage cost for parts in set A is based, then the indicated cost will be the same for each failure of the assembly regardless of which repair strategy is applied. Therefore it can be excluded from the minimization problem.

In addition, there are several other costs that can occur each time an assembly fails and which are independent of the strategy assignment. Examples are costs of bringing the failed unit to the repair facility and returning with a replacement unit, paperwork costs associated with reporting the failure, communication costs, etc. Also included are diagnoses costs--costs associated with determining the cause of failure and which of the repair strategies is to apply.

In general, only those costs which can affect the choice of repair strategy, and hence inventory range decisions, are included in Equation (5). There may be many costs associated with support of the assembly, in addition to those mentioned above, which have been excluded for this reason.

F. Marginal Costs

The problem of assigning detail parts to the three sets, A, B, and C, so as to minimize total costs is quite difficult. The only general method for solution is to determine total costs resulting from each possible assignment and to then select the minimum of these values. If there are n detail parts in the assembly, this would involve 3^n computations and comparisons; an impossibility if n is sufficiently large, even with the use of a computer. Therefore, it is necessary to consider other ways to solve the problem.

A desirable way to solve this minimizing problem is to consider the detail parts one by one, in any arbitrary sequence, and determine a repair strategy assignment for each part such that when all parts have been assigned, an over-all cost minimization has been achieved. Each part, as it is considered, will contribute an increment of the total cost and the amount of this increment will depend upon which of the three strategies is selected. Therefore, each of the three possible cost increments, or marginal values of total cost, can be evaluated and the strategy associated with the smallest of the three values can be assigned to the part. If this can be accomplished independently for each part, an over-all cost minimization will result. To apply this procedure, it is necessary to investigate the nature of the respective marginal costs and to obtain analytic expressions for them.

1. Marginal Cost for Set A

The marginal value of total cost resulting from an assignment of the jth detail part to set A is readily obtained and is given by:

$$\Delta_{A,j} = B_j + I_j(M_j) + R_j(M_j) + S_j(M_j) \qquad (6)$$

This expression is obtained directly from Equation (1). It may be evaluated for each detail part j separately, since it does not depend upon how the other parts are assigned.

2. Marginal Cost for Set B

The marginal cost resulting from an assignment of the jth detail part to set B is not easily obtained since it depends upon the total assignment of parts to the set. The costs for set B consist of assembly inventory costs which are a function of the summed failure rates over all parts included in the set; they therefore depend upon how many and which detail parts are so included. However, it is possible to find the smallest and largest marginal cost resulting from an assignment of the jth detail part to set B, regardless of which and how many other parts are also included. This will establish bounds within which the marginal cost for the jth detail part must fall.

In order to obtain simple expressions for these bounds, a restriction must be imposed upon the kind of depth policy used for the assembly. This restriction is that the inventory costs resulting from the depth policy must be a concave function of the demand rate for the assembly. This is not a serious restriction since it is satisfied by most standard inventory depth policies.

With this assumption, the maximum and minimum values for marginal costs contributed by an assignment of the jth detail part to set B are given as follows:

$$\Delta_{B,j}\,(\max) = B_i + I_i(M_j)$$

$$\Delta_{B,j}\,(\min) = I_i(M_i) - I_i(M_i - M_j) \tag{7}$$

where

$$M_i = \sum_{j=1}^{n} M_j \quad .$$

These expressions state that the largest marginal cost contributed by the jth detail part results when it is the only member of set B. The smallest marginal cost is given when all detail parts in the assembly are included in set B. These results and their dependency on the concavity assumption for assembly inventory costs is illustrated in Exhibit 6.

3. Marginal Costs for Set C

The marginal cost contributed by including the jth detail part in set C is again dependent upon which and how many other parts are also included. This is caused by the term representing inventory costs for the repair buffer stock. However, again by imposing the assumption that the inventory costs for the repair buffer stock is a concave function of the demand rate, simple expressions for maximum and minimum marginal cost values may be obtained. These expressions are as follows:

$$\begin{aligned} \Delta_{C,j}(\max) &= B_i + B_j + I_j(M_j) + R_j'(M_j) + I_j'(M_j) \\ \Delta_{C,j}(\min) &= B_j + I_j(M_j) + R_j'(M_j) + [I_i'(M_i) - I_i'(M_i - M_j)] \end{aligned} \tag{8}$$

Analogous to set B, the largest marginal cost results when the jth detail part is the only number of set C and the smallest marginal cost results when all detail parts are included in set C.

G. Solution Algorithm

A computational procedure or algorithm, based upon the marginal costs developed above, may now be established for accomplishing the

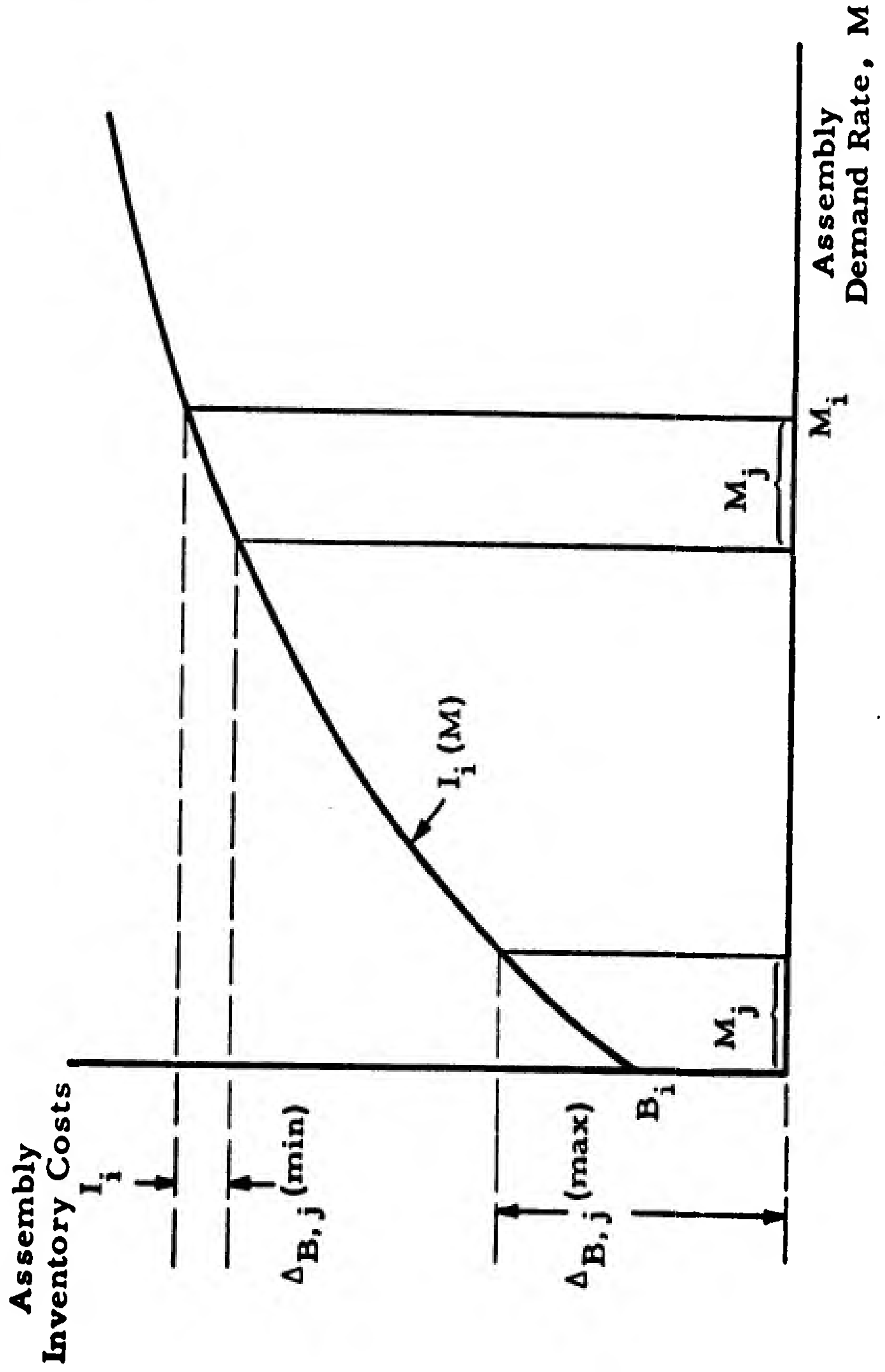

EXHIBIT 6 - MARGINAL COST BOUNDS FOR ASSIGNMENT TO SET B

minimization indicated in Equation (5). This procedure is recursive in nature and is described as follows:

1. Using Equations (6), (7), and (8), compute the following for each detail part j:

$$\Delta^1_{A,j} = \Delta_{A,j}$$

$$\Delta^1_{B,j}(\max) = \Delta_{B,j}(\max) - B_j$$

$$\Delta^1_{B,j}(\min) = \Delta_{B,j}(\min)$$

$$\Delta^1_{C,j}(\max) = \Delta_{C,j}(\max) - B_i$$

$$\Delta^1_{C,j}(\min) = \Delta_{C,j}(\min)$$

2. Let A^1 = set of all detail parts for which

$$\Delta^1_{A,j} \leq \Delta^1_{B,j}(\min)$$

and

$$\Delta^1_{A,j} \leq \Delta^1_{C,j}(\min) .$$

Let B^1 = set of all detail parts for which

$$\Delta^1_{B,j}(\max) \leq \Delta^1_{A,j}$$

and

$$\Delta^1_{B,j}(\max) \leq \Delta^1_{C,j}(\min) .$$

Let C^1 = set of all detail parts for which

$$\Delta^1_{C,j}(\max) \le \Delta^1_{A,j}$$

and

$$\Delta^1_{C,j}(\max) \le \Delta^1_{B,j}(\min) \ .$$

3. Define and compute the following for detail parts j, not in A^1, B^1, or C^1:

$$\Delta^2_{A,j} = \Delta_{A,j}$$

$$\Delta^2_{B,j}(\max) = I_i(M_{B^1} + M_j) - I_i(M_{B^1})$$

where

$$M_{B^1} = \sum_{j \epsilon B^1} M_j$$

$$\Delta^2_{B,j}(\min) = I_i(\hat{M}_{B^1}) - I_i\ (\hat{M}_{B^1} - M_j)$$

where

$$\hat{M}_{B^1} = M_i - \sum_{j \epsilon A^1} M_j - \sum_{j \epsilon C^1} M_j$$

$$\Delta^2_{C,j}(\max) = B_j + I_j(M_j) + R'_j(M_j) + [I'_i(M_{C^1} + M_j) - I'_i(M_{C^1})]$$

where

$$M_{C^1} = \sum_{j \epsilon C^1} M_j$$

$$\Delta^2_{C,j}(\min) = B_j + I_j(M_j) + R'_j(M_j) + [\, I'_i(\hat{M}_C1) - I_i(\hat{M}_C1)]\, M_j$$

where

$$\hat{M}_C1 = M_i - \sum_{j \in A^1} M_j - \sum_{j \in B^1} M_j$$

4. Let $A^2 = A^1$ + all detail parts for which

$$\Delta^2_{A,j} \le \Delta^2_{B,j}(\min)$$

and

$$\Delta^2_{A,j} \le \Delta^2_{C,j}(\min) .$$

Let $B^2 = B^1$ + all detail parts for which

$$\Delta^2_{B,j}(\max) \le \Delta^2_{A,j}$$

and

$$\Delta^2_{B,j}(\max) \le \Delta^2_{C,j}(\min) .$$

Let $C^2 = C^1$ + all detail parts for which

$$\Delta^2_{C,j}(\max) \le \Delta^2_{A,j}$$

and

$$\Delta^2_{C,j}(\max) \le \Delta^2_{B,j}(\min) .$$

5. Repeat step 3 with all superscripts increased by one.

6. Continue the iteration until an ℓ is obtained for which

$$A^{\ell+1} = A^{\ell}$$

$$B^{\ell+1} = B^{\ell}$$

$$C^{\ell+1} = C^{\ell}$$

Assign all detail parts not in A^{ℓ}, B^{ℓ}, or C^{ℓ} to either set B or set C according to the least of $\Delta^{\ell}_{B,j}(\min)$ and $\Delta^{\ell}_{C,j}(\min)$. Each detail part in the assembly is now assigned to one of three sets A, B, or C.

7. For this assignment of detail parts to sets, calculate

$$C_C + C_i - \sum_{i \in B \cup C} \Delta_{A,i}$$

using Equations (2), (4), and (6). If this is greater than B_i, retain the parts assignments obtained above. If less than B_i, assign all detail parts to set A.

H. Discussion of Solution Algorithm

The basic principle in the solution algorithm is that although only ranges can be expressed for marginal costs of parts in sets B and C, these ranges for practical problems are generally relatively small. Thus, the first pass through the recursive procedure will usually select some of the parts for definite assignment to a particular set according to least marginal cost. Having made such assignments, the marginal cost ranges can then be narrowed for application to the remaining unassigned parts. Each subsequent iteration of the procedure further narrows the ranges, enabling further parts to be definitely assigned. When the point is reached where no further parts can be assigned to particular sets, the ranges should be so narrow that it does not particularly matter, with respect to cost, to which set the residue of parts is assigned.

Although the algorithm may not, in particular cases, provide an absolute minimum solution, due to the relatively arbitrary assignment of parts in the terminating iteration, it is expected to be nearly optimal for most problems. The problem for which the procedure is least adequate is where no parts are selected for assignment in the first iteration. However, this is expected to be a rare occurrence; in fact, it is expected that for many problems, all parts in the assembly will be selected in the first pass.

It should be noticed that the procedure is based upon comparing marginal costs excluding B_i, the initial and recurring inventory administration costs for the assembly. In the last step of the procedure, it is determined whether or not the savings afforded by placing parts in sets B and C rather than A will offset the cost, B_i. If so, the assignments provided by the procedure are valid; if not, the least-cost solution is to place all parts in set A.

Although the solution algorithm represents a simplified way to solve the minimization problem at the expense of a rather mild restriction on the kind of inventory depth policy used, a further simplification is possible if the inventory cost is a linear function of the failure rate. In this case, specific values of marginal costs can be obtained for assignments to set B and set C rather than ranges of values, and only the first pass through the solution algorithm is necessary. This is illustrated as follows: If

$$I_i\left(\sum_{j\epsilon B} M_j\right) = I \sum_{j\epsilon B} M_j$$

and

$$I_i'\Big(\sum_{j\in C} M_j\Big) = I'\sum_{j\in C} M_j \quad ,$$

where I and I' are constants, then

$$\Delta^1_{B,j}(\max) = \Delta^1_{B,j}(\min) = IM_j$$

and

$$\Delta^1_{C,j}(\max) = \Delta^1_{C,j}(\min) = B_j + I_j(M_j) + R_j'(M_j) + I'M_j \; .$$

Unfortunately, however, this linearity condition seldom holds for realistic depth policies and this simplification is not normally applicable.

I. Sample Problem

To illustrate the procedure for solving the range problem for first-order assemblies, particular functions for inventory and repair costs may be assumed. The inventory depth policy that will be used for this purpose consists of an economic lot-size policy with no safety stock (case of demand certainty). The annual cost, as a function of the lot size, for this policy is as follows:

$$C(Q) = HU\frac{Q}{2} + K\frac{M}{Q} + UM \quad ,$$

where

H = Annual holding cost rate
U = Unit price
Q = Lot size
K = Fixed order cost
M = Annual demand rate

The value of Q which minimizes C(Q) is given by:

$$Q = \sqrt{\frac{2KM}{HU}}$$

Replacing this value for Q in C(Q), the result is as follows:

$$C(Q) = \sqrt{2KMHU} + UM$$

Assuming an infinite program and discrete annual compound discounting with an annual interest rate, r, the present value of future inventory costs is given by:

$$I(M) = \alpha C(Q) = \alpha[\sqrt{2KMHU} + UM] , \qquad (9)$$

where

$$\alpha = \frac{1 + r}{r} = \text{discount factor} .$$

Equation (9) will be used to represent inventory costs for all detail parts and for the assembly excluding the assembly repair buffer stock. For the repair buffer stock, an analogous depth policy will be used which is defined as follows:

$$C'(B) = HUB + K' \frac{M}{B} \quad ,$$

where

$C'(B)$ = Annual cost as a function of the repair batch size
B = Repair batch size
K' = Repair batch setup cost

If the optimizing value of B is substituted the present value of future inventory costs for the repair buffer stock is found to be:

$$\begin{aligned} I'(M) &= \alpha C'(B) + UB \\ &= 2\alpha\sqrt{K'MHU} + \frac{K'MU}{H} \quad , \end{aligned} \tag{10}$$

where the term, UB, represents the initial cost of establishing the buffer stock, B.

The cost functions for per-unit repair costs and assembly shortage costs will be assumed as follows:

$$\begin{aligned} R(M) &= \alpha RM \\ S(M) &= \alpha SM \quad , \end{aligned} \tag{11}$$

where

R = Per-unit repair cost
S = Per-unit assembly shortage cost

Substituting Equations (9), (10), and (11) in Equation (5), the following total cost function is obtained:

$$
\begin{aligned}
T_i = \min_{\pi_{A,B,C}} \Bigg\{ & \sum_{j\in A} \left[B_j + \alpha\sqrt{2KHU_jM_j} + \alpha M_j(U_j + R_j + S_j)\right] \\
& + B_i\delta + \alpha\sqrt{2KHU_i\sum_{j\in B} M_j} + \alpha U_i \sum_{j\in B} M_j \\
& + \sum_{j\in C} \left[B_j + \alpha\sqrt{2KHU_jM_j} + \alpha M_j(U_j + R'_j)\right] \\
& + 2\alpha\sqrt{K'HU_i}\sum_{j\in C} M_j + \sqrt{\frac{K'U_i}{H}\sum_{j\in C} M_j} \Bigg\}
\end{aligned}
\qquad (12)
$$

In this function, the subscript i refers to the assembly and the subscript j refers to the jth detail part (j = 1 to n). The parameters α, K, and H have been assumed the same for all parts and, hence, are not subscripted.

To solve this minimization problem by the marginal cost procedure, it is necessary to verify that inventory costs form a concave function of demand rate. This will be established if the second derivative of the cost function, with respect to demand rate, is negative for all positive values of demand rate. This is demonstrated as follows:

$$\frac{\partial^2 I(M)}{\partial M^2} = \frac{\partial^2}{\partial M^2} \left[a\sqrt{2KMHU} + UM\right]$$

$$= - \frac{a}{4M} \sqrt{\frac{2KHU}{M}}$$

Similarly, it can be shown that $\dfrac{\partial^2 I'(M)}{\partial M^2} \leq 0.$

To facilitate the computation, the following expressions may be derived for marginal costs:

$$\Delta^1_{A,j} = B_j + a[\sqrt{2KHU_jM_j} + M_j(U_j + R_j + S_j)]$$

$$\Delta^1_{B,j}(\max) = a[\sqrt{2KHU_iM_j} + U_iM_j]$$

$$\Delta^1_{B,j}(\min) = a[\sqrt{2KHU_i}\ (\sqrt{M_i} - \sqrt{M_i - M_j}) + U_iM_j] \tag{13}$$

$$\Delta^1_{C,j}(\max) = B_j + a[\sqrt{2KHU_jM_j} + M_j(U_j + R_j') + \sqrt{K'HU_iM_j}\ (2 + \frac{1}{aH})]$$

$$\Delta^1_{C,j}(\min) = B_j + a[\sqrt{2KHU_jM_j} + M_j(U_j + R_j') + \sqrt{K'HU_i}\ (2 + \frac{1}{aH})\ (\sqrt{M_i} - \sqrt{M_i - M_j})$$

J. Numeric Examples

To further illustrate the computation procedure, two numeric examples will be used, based upon the particular cost functions developed above and expressed by Equation (12). The first example considers a first-order assembly containing only two detail parts. The second example is more complex and better illustrates the iterative nature of the computation procedure.

1. Example I

For the first numeric example, the Dial and Hub Assembly in Exhibit 2 will be used. This is a first-order assembly consisting of only two detail parts. Values for parameters pertaining to the various parts are assumed to be as shown in Exhibit 7.

EXHIBIT 7 - ASSUMED PARAMETER VALUES FOR EXAMPLE I

Part	M_j	U_j	$R_j = R_j'$	S_j
Dial and Hub Assembly	\$25	\$16		
Dial	16	4	\$1	\$1
Hub	9	9	5	1

In addition to parameter values given in Exhibit 7, the following values for parameters common to all parts will be assumed:

$\alpha = 5$

$H = .20$

$K = \$10$

$K' = \$20$

$B_i = B_j = \$50$

With these parameter values, Equations (13) may be used to obtain the values for marginal costs shown in Exhibit 8.

EXHIBIT 8 - MARGINAL COSTS FOR EXAMPLE I

Part	$\Delta_{A,j}$	$\Delta_{B,j}$(min)	$\Delta_{B,j}$(max)	$\Delta_{C,j}$(min)	$\Delta_{C,j}$(max)
Dial	$610	$1,320	$1,400	$770	$1,010
Hub	815	760	840	890	1,130

From the results in Exhibit 8, it is seen that the Dial should be assigned to set A since $\Delta_{A,1}$ = $610 is less than both $\Delta_{B,1}$ (min) = $1,320 and $\Delta_{C,1}$(min) = $770. Although it is evident that the Hub should be assigned either to set A or B, it is ambiguous from the data as to which of the two choices should be made. In this particular instance, however, an assignment of the Hub to set B would incur $\Delta_{B,2}$(max) = $840 as the marginal cost, since the Dial has already been assigned to set A. Since this is larger than $\Delta_{A,2}$ = $815, the Hub should be assigned to set A. This conclusion can also be reached by applying the second iteration as prescribed by the solution algorithm. The final solution for this example, then, is that both detail parts are assigned to repair strategy A whereby they are carried in inventory and the assembly is repaired each time it fails by immediate removal and replacement of the defective part.

As a partial check of this result, the present value of total expected costs, assuming detail parts only are stocked, may be computed and compared against those incurred if just units of the assembly were stocked. These results are as follows:

$$C_A = \sum_{j \epsilon A} \Delta_{A,j} = \$610 + \$815 = \$1,425$$

EXHIBIT 10 - MARGINAL COSTS FOR EXAMPLE II

Part	$\Delta^1_{A,j}$	$\Delta^1_{B,j}$(min)	$\Delta^1_{B,j}$(max)	$\Delta^1_{C,j}$(min)	$\Delta^1_{C,j}$(max)
Housing	68.66	45.75	60.00	45.91	88.66
Housing Cover	53.63	45.75	60.00	30.88	73.63
Impeller Shaft	72.07	45.75	60.00	49.32	92.07
Impeller	163.66	91.50	111.21	118.16	177.29
Bushing	3418.54	4152.57	4192.31	1476.25	1595.47
Pulley	47.07	91.50	111.21	41.57	100.70
Nipples	53.63	137.25	160.97	45.43	116.59

Inspection of the marginal costs in Exhibit 10 reveals that only two parts--the Impeller and Bushing--can be assigned unequivocably to strategy sets, the Impeller being assigned to set B and the Bushing to Set C. For the remaining parts, it is ambiguous as to which assignment will result in least cost. Therefore, a second application of the computation procedure is necessary, based on the fact that assignments for two of the parts are now known. The results of this computation are shown in Exhibit 11.

EXHIBIT 11 - RECOMPUTED VALUES FOR MARGINAL COSTS

Part	$\Delta^2_{A,j}$	$\Delta^2_{B,j}$(min)	$\Delta^2_{B,j}$(max)	$\Delta^2_{C,j}$(min)	$\Delta^2_{C,j}$(max)
Housing	68.66	47.43	49.77	45.91	46.00
Housing Cover	53.63	47.43	49.77	30.88	30.97
Impeller Shaft	72.07	47.43	49.77	49.32	49.41
Pulley	47.07	95.01	98.79	41.57	41.80
Nipples	53.63	142.74	147.33	45.38	45.70

The ranges of marginal costs for sets B and C are now so narrow that all but one of the parts can be assigned to set C. For the Impeller

Shaft, however, it is still ambiguous as to whether it should be assigned to set B or C. However, if it were assigned to set B, it would incur the high end of the range--\$49.77--whereas if it were assigned to set C, the low end of the range--\$49.32--would pertain. Therefore, this part should be assigned to set C. This result can also be obtained by a third application of the computation algorithm.

Having assigned all detail parts to strategy sets, it must finally be determined whether or not savings afforded by stocking the assembly in accordance with strategies B and C, rather than only detail parts, will offset the initial inventory and administrative costs for the assembly. From Exhibits 10 and 11, it is readily apparent that several detail parts by themselves will warrant the incurrence of this cost. In particular, savings afforded by assigning the Bushing to set C rather than set A will pay the cost, B_i, many times over.

In summary, all parts except the Impeller should be assigned to repair strategy C--each failed unit of the Water Pump should be exchanged for a serviceable unit, with the failed unit being repaired later. If a unit fails because of the Impellor, a new Water Pump should be provided and the failed unit discarded. This results mainly from the high unit repair cost assumed for this kind of failure.

VII. EXTENSIONS OF THE FIRST-ORDER ASSEMBLY MODEL

The range decision model developed in the previous section is generally applicable to any first-order assembly as long as the established assumptions are satisfied. In this section, some of these assumptions are examined and methods are developed by which they may be relaxed, thereby increasing the scope of the model. Also, several special cases of the problem are considered which require minor modification or special interpretations of the previous results.

A. Multiple Failures

A basic assumption in the previous development was that each failure of the assembly is caused by only one detail part. This is an unrealistic assumption since, in fact, several detail parts may contribute to the failure. Whenever this occurs, errors in several of the assumed cost factors in Equation (5) will result. In particular, the total unit repair costs, applicable to parts in set A, will be less than as stated in Equation (5). The total repair cost, when several detail parts are removed and replaced at the same time, will generally be less than the sum of unit repair costs assuming they are removed and replaced at different times. Also, only one assembly shortage cost will be incurred for all the detail parts involved rather than as many as there are detail parts which is assumed in Equation (5).

The most serious error that will result, however, is that the failure rate of the assembly can no longer be represented as the sum of the failure rates of the detail parts. Thus, inventory costs for the assembly are overstated in the total cost function to the extent that multiple failures of detail parts can occur.

There is no simple theoretical way to include in the model the possibility of multiple failures nor to correctly compensate for the errors incurred by application of the model as formulated. However, revisions may be made in the model to incorporate this situation to within an approximation. The use of the revised model will generally provide results with accuracies commensurate with those with which the input data can be estimated.

If independent estimates of failure rates are made for each detail part and for the assembly itself, the failure rate for the assembly may be less than the sum of failure rates for the detail parts. The amount of the difference reflects the degree to which it is believed multiple failures can occur. From these estimated failure rates, normalized failure rates may be defined as follows:

$$M_j' = \frac{M_j M_i}{\sum_{i=1}^{n} M_j} ,$$

where

M_j' = normalized failure rate for the jth detail part

M_j = independent estimate of the failure rate for the jth detail part

M_i = independent estimate of the failure rate for the assembly.

The normalized failure rate should be used in each appropriate cost term of Equation (5) except the term, $I_j(M_j)$. This represents a factoring of these costs to within an approximation of what they would be if multiple failures were properly considered. However, the normalized failure rates should not be used to determine inventory costs for detail parts, since these costs depend upon the actual failure rates and are independent of whether or not multiple failures occur.

In addition, the per-unit repair and assembly shortage costs must be expressed in terms that are not parts-specific. Thus, these costs can no longer be assigned differently for each detail part but must represent an average per failure of the assembly. Finally, it is possible that the portion of the term, B_j, representing initial repair setup costs, cannot be specified differently for each detail part due to use of the same repair capability for removing and replacing the several detail parts causing the assembly failure. However, the remainder of this term, initial inventory setup costs and future administration costs, can be expressed on a per-part basis.

The incorporation of these changes in Equation (5) produces the following revised cost function:

$$
\begin{aligned}
T_i = \min_{\pi_{A,B,C}} \Bigg\{ & \sum_{j\in A} [\, B_j + I_j(M_j)] + R_i\Big(\sum_{j\in A} M_j'\Big) + S_i\Big(\sum_{j\in A} M_j'\Big) \\
& + V_i\delta_1 + B_i\,\delta_2 + I_i\Big(\sum_{j\in B} M_j'\Big) \qquad\qquad (14) \\
& + \sum_{j\in C} [B_j + I_j(M_j)] + R_i'\Big(\sum_{j\in C} M_j'\Big) + I_i\Big(\sum_{j\in C} M_j'\Big) \Bigg\} \quad ,
\end{aligned}
$$

where

B_j = inventory setup costs plus present value of future inventory administration costs for the jth detail part

V_i = initial repair setup costs for repair of assembly i

δ_1 = 1 if AUC $\neq$ 0

= 0 if AUC = 0

δ_2 = 1 if BUC $\neq$ 0

= 0 if BUC = 0

Other terms in the function are as previously defined; the functional forms of the per-unit repair and assembly shortage costs are in accordance with the discussion above.

The minimization in Equation (14) can be accomplished by essentially the same procedure as described in the previous section. If the costs, R_i, R'_i, and S_i, are linear functions of the indicated failure rates, then their marginal values are easily specified. Otherwise, a concavity assumption may be applied in order to obtain simple expressions for their bounds. In this case, the previous algorithm can be extended in an obvious way to allow for a range of values for $\Delta_{A,j}$ as well as for $\Delta_{B,j}$ and $\Delta_{C,j}$. In general, however, it is reasonable to assume that the costs, R_i, R'_i, and S_i, can be estimated only as linear functions in which case the previous algorithm can apply without change.

Theoretically, the value for the term, V_i, depends upon which parts are assigned to sets A and C as well as whether or not one or more part is so assigned. However, this term is included in the computation algorithm in the same fashion as the term, B_i; that is, assignments of detail parts to sets are made by implicitly assuming a zero value for the term, V_i, and then it is determined whether or not savings afforded by assignments to sets A and C, rather than set B, will offset this cost. If it is possible to estimate this cost on a per-part basis despite the complexity imposed by the possibility of multiple failures, the individual costs for parts assigned to sets A and C can be summed to determine a value for V_i to be used in making the test. If the savings are greater than this value for V_i, the assignments can be retained; otherwise, the particular part or parts contributing the most to the cost, V_i, can be reassigned to set B until a balance is reached between savings obtained by assignment to sets A and C and costs contributed to V_i. However, this complexity will probably seldom occur, particularly since it is usually the case that once the repair capability is provided to repair the assembly for a given kind of failure, the marginal cost of increasing the capability to include any other kind of failure is small. This is particularly true when, for each detail part, the chance of another part failing at the same time is significantly large. Of course, if it is believed that, for each detail part the chance of another detail part failing at the same time is quite small, then the results of the previous section would be more appropriate, where the repair capability setup costs can be easily incorporated on a per-part basis.

Having applied Equation (14) to obtain least-cost assignments of detail parts to strategy sets, the question then arises as to which action should apply to a unit of the assembly that has failed because of several detail parts, where the failed detail parts belong to different strategy sets. As an extreme, all three strategy sets may be represented by the failed detail parts. In this situation, the appropriate action to take is unambiguously specified by the results of the range decision. For example, if there is at least one detail part among those causing the assembly failure which has been assigned to strategy B, then there is no replacement for this part in stock and therefore, a replacement unit of the assembly must be provided. This action, consequently, overrides actions prescribed for the other detail parts involved in the failure which have been assigned to other strategy sets. Similarly, if only strategies A and C are represented by the failed detail parts, strategy C is the overriding strategy.

B. Salvage Processes

In the previous section, an assumption was imposed to the effect that no salvage considerations were included in the analysis. This assumption prevented several complexities from entering the problem; in particular, the possibility of obtaining serviceable detail parts through salvage of failed units of the assembly. Although it is not possible to extend the basic model to include all kinds of salvage processes in a manner that is theoretically correct, approximate methods may be included which will probably suffice for practical purposes.

Theoretically, salvage processes can have a dominant effect upon inventory range decisions. Even the case where a net revenue is obtainable from scrapping a failed part can influence the decision as to whether or not to carry the part in inventory. Of more importance is the case where failed units of the assembly are disassembled and serviceable detail parts are recovered and returned to stock. If these two cases can be included in the range decision model, most other kinds of salvage processes will thereby be included. For example, the case where extra units of the assembly are procured for the sole purpose of being cannibalized for parts is implicitly included in the second of the two cases.

The salvage problem is relatively simple for the case where parts (failed detail parts and/or failed units of the assembly) are subject to salvage processes which do not result in serviceable parts being returned to stock. Presumably, this kind of salvage would be accomplished only under conditions where revenue from the salvage (e.g., scrap value) is greater than the cost of the salvage process. Generally, this kind of salvage process is subject to some salvage policy analogous to inventory depth policies. For example, batches of failed units of a part may be permitted to accumulate before being salvaged, or all outstanding parts subject to salvage may be processed at designated times. For a prescribed salvage policy, a net revenue function, N(M), may be developed, expressed as a function of M, the annual failure rate for the part. A typical form for this function is illustrated in Exhibit 12.

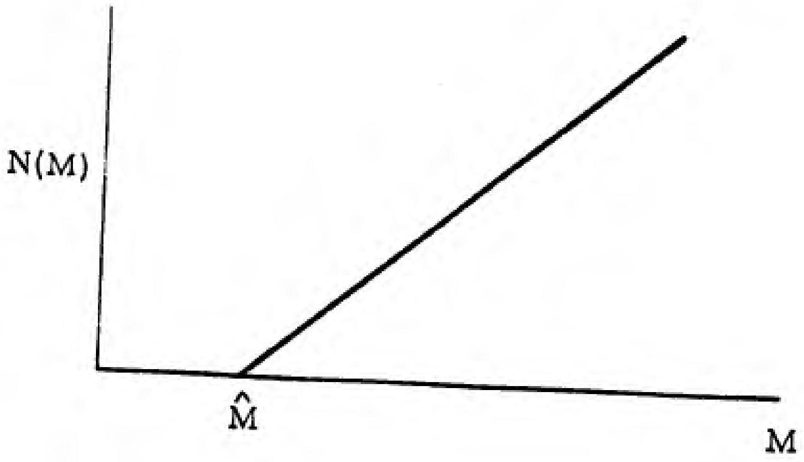

EXHIBIT 12 - TYPICAL SALVAGE NET REVENUE FUNCTION

In the function shown in Exhibit 12, net revenue as a function of M is linear for $M > \hat{M}$ and zero for $M \leq \hat{M}$. Generally, $\hat{M}$ will be greater than zero whenever there is a setup cost associated with the salvage process, where annual failure rates less than $\hat{M}$ will fail to produce enough units for which gross revenue from salvage will fail to offset the fixed salvage costs.

Although a salvage net revenue term may be incorporated in the total cost function, Equation (14), for any part, the inclusion of the term for the assembly itself will be discussed further for illustrative purposes. Here, the term is associated with repair strategy B, where failed units of the assembly are now salvaged instead of being discarded as previously assumed. The additive term is given by:

$$- N_i\left(\sum_{j \in B} M_j'\right) \quad ,$$

where the minus sign indicates that the revenue is a negative cost. If this cost function has the form shown in Exhibit 10, or, more generally, is a <u>convex</u> function of annual failure rate, the same solution algorithm described in the previous section may be applied to obtain least-cost assignments of detail parts to strategy sets. For example, assuming the form shown in Exhibit 10, terms given as $-\max[0, (M_j' - \hat{M})N_i]$ and $-\max[M_j' N_i, (M_j' - \hat{M})N_i]$ are added, respectively, to $\Delta^1_{B,j}(\max)$ and $\Delta^1_{B,j}(\min)$; where N_i is the net revenue per unit of the assembly. These particular marginal costs for N_i result from the fact that $\hat{M}$ may be less than M_j or greater than $M_i - M_j'$. Of course, if $\hat{M} \geq M_i$, the salvage consideration should be omitted from the problem. It should also be noticed that the inclusion of the salvage term in this way acts to increase the range of marginal values for assignments to set B.

The other kind of salvage process is where failed units of the assembly associated with strategy set B are disassembled and serviceable detail parts are returned to stock. This case is quite complex since the utility gained by the reclamation induces assignments of detail parts to set B, whereas the reclaimed detail parts reduce inventory costs for assignments to sets A and C, thereby increasing the attractiveness of assignments to sets A and C rather than set B. Theoretically, of course, there must be some best balance of these contradictory effects.

A cost expression analogous to Equation (14) may be developed incorporating terms reflecting this kind of salvage. This expression is as follows

$$
\begin{aligned}
T_i = \min_{\pi_{A,B,C}} \Bigg\{ & \sum_{j\epsilon A} [B_j + I_j(M_j, \sum_{k\epsilon B} M_k')] + R_i(\sum_{j\epsilon A} M_j') + S_i\,(\sum_{j\epsilon A} M_j') \\
& + V_i\delta_i + B_i\delta_2 + W_i(\sum_{j\epsilon B} M_j') + I_i\,(\sum_{j\epsilon B} M_j') \qquad (15) \\
& + \sum_{j\epsilon C} [B_j + I_j(M_j, \sum_{k\epsilon B} M_k')] + R_j'(\sum_{j\epsilon C} M_j') + I_i'(\sum_{j\epsilon C} M_j') \Bigg\}
\end{aligned}
$$

with the constraint that

$$
\sum_{j\epsilon A\cup C} [I_j(M_j, 0) - I_j(M_j, \sum_{k\epsilon B} M_k')] > W_i(\sum_{j\epsilon B} M_j') \quad .
$$

In this expression, the inventory cost for assignment of the jth detail part to set A or C is now a function not only of the failure rate of the jth detail part but also a function of the failure rate of the assembly as contributed by assignment of detail parts to set B. Also, a term, W_i, has been added to reflect the cost of salvaging failed assemblies in accordance with a prescribed salvage policy. Finally, a constraint has been added to assure that total reduction in inventory costs, as obtained from recovered parts, is greater than the cost of the recovery processes.

As approximations, the following substitutions in Equation (15) may be considered

$$I_j(M_j, \sum_{k\epsilon B} M'_k) = I_j(M_j) - \alpha U_j \sum_{k\epsilon B} M'_k$$

$$W_i(\sum_{j\epsilon B} M'_j) = \alpha\omega_i \sum_{j\epsilon B} M'_j \quad ,$$

where

α = discount factor
U_j = unit purchase price of jth detail part
ω_i = per-unit salvage cost for assembly i .

Here, only the savings in variable procurement costs, as provided by recovery of detail parts through salvage, are represented, while additional savings in other kinds of inventory costs have been omitted. On the other hand, certain salvage costs have also been omitted; in particular, all fixed costs that would induce accumulating batches for salvage. Since these omissions tend to balance out, the approximation is probably reasonable for practical purposes.

When the substitutions are made in Equation (15), the result will differ from Equation (14) only by the term

$$\min \left[0, -\alpha\left(\sum_{j \notin B} U_j - W_i\right) \sum_{j \in B} M_j'\right] ,$$

which also reflects the constraint in Equation (15) that salvage will be accomplished only under assignments for which savings are greater than costs.

Unfortunately, the inclusion of this term complicates the minimization problem, with respect to detail part assignments to strategy sets, and the algorithm previously developed can no longer apply. Since no simple solution technique is apparent, this problem must be considered unsolved at the present time.

C. Predetermined Repair Capability

In the previous development, it was implicitly assumed possible to remove and replace any detail part in the assembly at the incurrence of various costs reflecting not only direct repair costs but also costs of providing the necessary repair capability. However, it may happen that a priori or overriding policies have been established with regard to providing repair capability which would prevent the removal and replacement of some of the detail parts. Detail parts which cannot or are not to be physically removed and replaced at the location, due to lack of repair capability or any other overriding policy, can be automatically assigned to strategy set B. This is because parts assigned to sets A and C require the capability to remove and replace.

Any such preassignments of detail parts to set B, or to any or all of the three sets according to overriding policies or judgments, can be considered as the first pass through the computation recursion procedure. The procedure can then be continued, as previously described, for the remaining uncommitted parts. Thus, a least-cost solution will be obtained to the extent permitted by the overriding policies.

D. Prior Stockage

It may happen that particular detail parts in the assembly or the assembly itself may already be stocked at the location for other purposes. The only effect of such prior stockages is that revised expressions for inventory costs must be used for these parts.

If a detail part is already stocked, the term, $I_j(M_j)$, plus the portion of B_j that represents inventory costs (initial setup and future administrative costs) must be replaced, in Equation (14), by $I_j(M_P + M_j) - I_j(M_P)$, where M_P is the demand for the part for purposes other than in support of the assembly under consideration. This substitution expresses inventory costs for the part in its new application in terms of the marginal increase of such costs over those incurred by previous applications.

The same type of substitution should be made if the assembly is already stocked at the location. The terms, $B_i\delta_2 + I_i(\sum_{j\in B} M_j')$ and $I_i'(\sum_{j\in C} M_j')$, in Equation (14) should be replaced, respectively, by $I_i(M_a + \sum_{j\in B} M_j') - I_i(M_a)$ and $I_i'(M_a' + \sum_{j\in C} M_j') - I_i'(M_a')$, where M_a represents demand for the assembly resulting in permanent losses from inventory in accordance with previous uses, and M_a' represents such demands resulting in returns to stock through repair.

With the above substitutions for inventory costs, the solution algorithm may then be applied as previously described. The general result obtained for parts already stocked is that their use for the new application will be encouraged, since the marginal cost of increasing the depth of stock will be much smaller than if no stocks already existed. However, it will not always follow that the depth should so be increased;

cheaper alternatives may pertain for the new application. If so, they will be derived by the solution algorithm.

The existance of prior stockage is only a special case of the general problem of prior application of parts in other equipments. Another case is where it may be desirable to disestablish the stock of a part when prior applications are considered in conjunction with the new application. Still another case is where a particular part has not been stocked in accordance with previous range decisions, but which may now be stocked when considered together with the new applications. These possibilities are discussed further in a later section, where a more general treatment of the problem of prior application is presented.

E. Multiple Application

If a particular detail part appears in the assembly more than once, several of the input factors may have different values according to the application. In particular, the failure rate, per-unit repair cost, repair cycle, and assembly shortage cost may vary from one application to another. This case may be included in the previous procedure, to within a reasonable approximation, by taking as the failure rate for the part the sum of the failure rates over the various applications and by using weighted averages of the other parameters. The averages over the various applications should be weighted by the respective failure rates.

If more than one unit of the assembly is to be supported, care must be taken to express relevant failure rates in terms of the total number of units to be supported. For example, if failure rates for detail parts are expressed in terms of one unit of the assembly, they must be multiplied by the number of units to be supported before being used in the computational procedure.

F. Matched and Associated Parts

If there are matched sets of parts in the assembly, the only special treatment required is that each set be considered as one part, insofar as assignment of parameter values and all computations are concerned. Thus, the failure rate represents the failure rate for the set; the unit price used is the price of the set, and so forth.

Associated parts, however, present a more difficult problem. Consider, for example, two parts, A and B, such that whenever part A is removed and replaced, so is part B. In addition, part B may have a "natural" failure rate independent of part A. The main problem is to ensure that assignments to repair strategy sets are consistent for the two parts; i.e., if part A is to be stocked, then so must part B. On the other hand, if part A is not stocked, then part B may or may not be stocked according to its natural failure rate.

As an approximate solution, the two parts may be treated as a matched set to the extent that part A fails. Thus, the unit price of part A may be increased by the unit price of part B, the unit repair cost becomes the cost of removing and replacing both parts, and so forth. The matched set should be associated with part A in the computations, using the failure rate of part A.

If the natural failure rate of part B is zero or very small, no separate consideration of part B need be made. In this case, all inventory costs for part B are included in costs for part A, considered as a matched set. However, if part B has a significant natural failure rate, it should be identified as a separate part to the extent of its natural failure rate. In this case, inventory costs for part B entered as a separate part should be based upon the rule for prior stockage, where its use in conjunction with part A represents the prior stockage. This results from the fact that part A (as a separate part) can be considered for stockage only if part B is also considered; therefore, part A, considered as a matched set, should bear the burden of the first increment of inventory costs for part B. Part B, stocked against its natural failure rate, should bear only marginal inventory costs if it is also stocked against its failure rate associated with that of part A.

The only difficulty in this approach is that after the computation procedure is applied, part A (considered as a matched set) may be assigned to strategy set B and part B may be assigned to set A or C. In this case, the computation should be repeated, this time with part B assuming all inventory costs to the extent of its natural failure rate. This may increase the cost of assigning part B to set A or C sufficiently as to cause a reassignment to set B.

This process may be extended to the case where more than two parts are included in an associated set. In each such set, a particular part is dominant in that each time it fails, the other parts in the set are considered as having failed. This part is treated like a matched set in the computations, while the remaining parts are separately identified using their natural failure rates and marginal inventory costs, assuming prior stockage in accordance with their inclusion in the matched set. The computation procedure is then applied as described above.

G. Preventive Maintenance

The assembly under consideration may be subject to a preventive maintenance policy such as a periodic inspection or overhaul. Maintenance processes of this type can be incorporated in the range decision model in several ways according to the actions taken during the processes.

Normally, preventive maintenance is accomplished on equipment which has not failed but which is more likely to fail in the future if the preventive maintenance operations are not accomplished. It is possible for detail parts to be removed and replaced or the assembly itself to be replaced during such operations. If so, it is as a result of judgment and experience as to the likelihood of such parts failing before the next scheduled preventive maintenance. By correcting incipient failures in this way, unscheduled failures, with associated deleterious effects, are reduced. Presumably, estimates may be made as to the rates by which each part is subject to removal and replacement. In this case, they can be added to estimates of natural failure rates, with the sum being used as the failure rate in the range decision model.

On the other hand, the preventive maintenance operation may not involve the removal and replacement of parts. Instead, only labor and/ or common materials may be expended as, for example, in lubrication and cleaning functions. In this case, no further consideration need be made insofar as application of the range decision model is concerned, since there is no effect upon parts stockage.

In the preventive maintenance operation, there may be an overriding policy that an exchange unit of the assembly be provided while the maintenance functions are being performed. This policy may pertain regardless of whether or not parts are removed and replaced during the preventive maintenance. If units of the assembly subject to preventive maintenance are thought of as failures, this represents a situation where two classes of failures occur, one class being subject to an overriding maintenance strategy and the other class (natural or unscheduled failures) being subject to maintenance strategies in accordance with the decision model.

With this overriding policy, it is clear that no detail part subject to removal and replacement during preventive maintenance can be assigned to repair strategy A. However, such a part can be assigned to strategy B, since it may be more economic to provide a new unit of the assembly rather than to repair the "failed" unit. In this case, the temporary use of a unit of the assembly, out of a repair buffer stock, becomes a permanent use, and the buffer stock must be replenished. Finally, of course, the detail part may be assigned to strategy C which would be the expected assignment under the overriding policy. In summary, then, a particular detail part, subject to removal and replacement during preventive maintenance where exchange units of the assembly are automatically provided, can be assigned to repair strategies B or C; for the remaining parts, a free choice exists for assignment to the three strategy groups. In addition, regardless of assignments of detail parts to strategy sets, a repair buffer stock must be established to the extent that the assembly is subject to preventive maintenance.

These considerations can be incorporated in a modification of Equation (14) as follows:

$$
\begin{aligned}
T_i = B_i + \min_{\pi_{A,B,C}} \Bigg\{ & \sum_{j\epsilon\hat{A}} [B_j + I_j(M_j)] + R_i \sum_{j\epsilon\hat{A}} M_j') + S_i(\sum_{j\epsilon\hat{A}} M_j') \\
& + V_i\delta_1 + I_i(\sum_{j\epsilon B} M_j') + \sum_{j\epsilon C} [B_j + I_j(M_j)] \qquad (16) \\
& + R_j'(\sum_{j\epsilon C} M_j') + I_i'(M_P + \sum_{j\epsilon C} \hat{M}_j') - I_i'(M_P) \Bigg\}
\end{aligned}
$$

In this cost function, M_P is the annual rate at which the assembly is subject to preventive maintenance, and $\hat{M}_j'$ represents the rate (normalized to reflect multiple failures) at which unscheduled failures occur for the jth detail part. Also, the set, $\hat{A}$, consists of detail parts not subject to removal and replacement during preventive maintenance. Finally, the term B_i is removed from the minimization, since it is automatically incurred as a consequence of the overriding policy concerning exchange of assemblies. The indicated minimization can be accomplished in the usual way.

It should be noticed that all of the preceding discussion assumes a given preventive maintenance policy. Theoretically, there exists interrelationships between such a policy and decisions provided by the range decision model. This is particularly apparent in the tradeoff between scheduled and unscheduled failures, where unscheduled failures are generally reduced as the time between scheduled or preventive maintenance is reduced. Although no attempt has been made to optimize the preventive maintenance policy, some of the ingredients necessary for such an optimization have been suggested.

VIII. RANGE DECISION MODEL FOR HIGHER-ORDER ASSEMBLIES

In the previous sections, a model was developed for solving the inventory range problem for a PPL containing only one first-order assembly and supported by one repair and supply facility. In this section, the model is extended to the case of a general PPL representing an equipment defined as an assembly of specified higher order. However, the assumption that the support system contains only one activity is retained.

A. Basic Range Decision Model for an Assembly of Arbitrary Order

If a complex equipment is considered in terms of its parts hierarchy, organized as illustrated in Exhibit 5, a particular assembly within the hierarchical structure may be arbitrarily selected for a detailed analysis. For illustrative purposes, a particular assembly of order greater than two will be selected. If this is identified as assembly i, it is composed of subordinate parts j, $j = 1$ to n.

If assembly i fails, it is due to the failure of one or more of its subordinate parts. To simplify the analysis, however, an initial assumption is imposed that in each instance the failure is caused by one and only one subordinate part.

To render the failed unit of the assembly again available for use, three main kinds of repair strategies may be identified which are analogous to those defined for first-order assemblies. These strategies are described as follows:

1. Strategy A--Immediate Repair

Some action is applied to the failed subordinate part itself, thereby making assembly i serviceable and again available for its designed use. Although the action taken may be complex, it will generally involve the removal and replacement of the subordinate part or one or more lower parts if the subordinate part is an assembly. However, for this strategy, the assembly i is not itself replaced but is considered unavailable for use until corrective action is taken upon the subordinate part causing the failure.

2. Strategy B--Exchange Assembly, No Repair

The failed unit of assembly i is exchanged (removed and replaced) for a serviceable unit. The failed unit is discarded or otherwise disposed of and is considered unavailable for subsequent reuse. The subordinate part (or any lower part) is not stocked. The availability of replacement units of the assembly is governed by a prescribed inventory depth policy.

3. Strategy C--Exchange Assembly, Repair Later

The failed unit of assembly i is exchanged (removed and replaced) for a serviceable unit. The failed unit is subsequently repaired and returned to stock for reuse. The availability of replacement units of the assembly is governed by a prescribed depth policy.

The main difference between these strategies and those defined for first-order assemblies consists of the repair actions taken according to strategies A and C. Whereas before, the action consisted only of removing and replacing detail parts, the actions now are more complex since the failed subordinate part may itself be an assembly. If it is an assembly, it is subject to the same three strategies described above and these strategies may be compounded down through the parts hierarchy. Despite this complexity, each failed unit of the given assembly i is subject to only one strategy as defined and the choice of strategy for a given failure depends upon which subordinate part caused the failure.

It might be argued that strategies other than the three described above are possible if portions of the parts hierarchy above the given assembly are considered. For example, a failure of assembly i may be alleviated by replacing some higher assembly. However, the principle applied in the subsequent analysis is to decide what actions to take with respect to repair and supply assuming that assembly i itself is to be acted upon. Whether or not to apply such actions to assembly i is considered as part of similar decisions associated with assemblies higher up in the parts hierarchy. Thus, the actions taken for assembly i, as associated with the three strategies defined above, are contingent upon an a priori decision to compensate for the failure of assembly i, considered independent of its higher assemblies.

B. Cost Functions

As in the case of the first-order assembly, various costs may be associated with the three repair and stockage strategies applied to rectify a failure of assembly i. Again, these costs may be included in a cost function as follows:

$$T_i = D_i(M_i) + \min_{\pi_{A,B,C}} \left\{ \sum_{j \in A} [B_j + R_j(M_j) + S_j(M_j) + F_j] + B_i\delta + I_i\left(\sum_{j \in B} M_j\right) + \sum_{j \in C} [B_j + R_j'(M_j) + F_j] + I_i'\left(\sum_{j \in C} M_j\right) \right\} \tag{17}$$

where

T_i = Total future repair and inventory costs for assembly i.

D_i = Per unit diagnosis cost for assembly i.

B_j = Marginal repair capability setup cost for subordinate part j, plus inventory setup and administration costs if j is a detail part.

R_j = Marginal per unit remove and replace cost for subordinate part j.

S_j = Marginal per unit assembly shortage cost for subordinate part j.

F_j = Future repair and inventory costs for subordinate part j.

B_i = Initial inventory setup and future inventory administrative costs for assembly i.

I_i = Future inventory costs for assembly i.

I_i = Future repair buffer stock inventory costs for assembly i.

M_j = Annual failure rate of subordinate part j.

M_i = Annual failure rate of assembly i.

Although not explicitly stated above, all costs in the cost function are in terms of present discounted value. Most of the cost terms are of the same kind as for the case of a first-order assembly, but with somewhat different interpretations in context with the case of a higher-order assembly. Therefore, additional comment is necessary to describe the differences.

The term, $D_i(M_i)$, is a new one occasioned by the fact that assembly i may be a part of a higher-order assembly. Although it may incorporate all costs associated with assembly i that are independent of the repair strategy that is chosen, its main ingredient is the cost of diagnosing the failed unit of assembly i to determine which subordinate part, j, caused the failure. However, it does not include any cost involved in isolating the fault below the subordinate part level. Although this cost is represented as a function of the failure rate for assembly i, it may contain costs that are independent of the failure rate, such as an initial cost for providing test and checkout equipment for fault isolation purposes. Such fixed costs, however, must be incorporated on a marginal basis; that is, the additional cost of fault isolation to the subordinate part level of assembly i over and above the cost of fault isolation to the assembly i level.

The term, B_j, contains initial repair capability setup costs on a marginal basis; that is, the cost of providing additional capability to remove and replace subordinate part j over and above such cost for removing and replacing assembly i. It does not include such costs for removing and replacing any lower parts if subordinate part j is an assembly. In addition, if subordinate part j is a detail part, the term, B_j, includes initial and future administrative costs associated with carrying part j in stock.

Generally, the term, $R_j(M_j)$, represents the marginal per unit repair costs associated with part j but not with lower parts if part j is an assembly. Usually, it consists of the extra costs incurred in removing and replacing part j from assembly i. However, in some instances it may have a zero value as, for example, when a lower part (assuming part j to be an assembly) is removed and replaced "in place," without involving the explicit removal of part j. The assumption of a zero value for this term depends upon which lower part of part j caused the failure. However, even though such lower parts may be identified, it is difficult to explicitly incorporate this consideration in the cost function. Therefore, for practical purposes, the term, $R_j(M_j)$, may be estimated purely upon the basis of marginal remove and replace costs, with the estimation tempered as necessary to reflect the possibility that it may occasionally take on a zero value.

In addition to direct per-unit remove and replace costs, the term, $R_j(M_j)$, contains any fixed costs incurred each time such an action takes place. These costs were previously identified as repair batch setup costs, where the batch in this case is one unit. The term, $R_j'(M_j)$, differs from $R_j(M_j)$ only in this respect; that is, $R_j'(M_j)$ excludes any repair batch setup costs since they are included in the term, $I_j'(M_j)$, for the reasons given in the discussion for first-order assemblies.

The term, $S_j(M_j)$, represents the extra costs that are incurred by repairing (in some degree) assembly i, rather than immediately providing a serviceable substitute. In general, such costs reflect the consequences of a marginal increase in the time before which a unit of assembly i is available for use. This incremental increase in time is of two types. For one type, the length of time the equipment is unavailable for use is increased. For the other type, the repair time for some higher assembly is increased, resulting in an increase in the time before which the higher assembly is returned to stock and available for reissue. For a given failure of assembly i, the kind of time increase incurred and, for the second type, the particular higher assembly affected, depends upon repair strategy choices for assemblies higher in the parts hierarchy than assembly i. Since the value for $S_j(M_j)$ depends upon which type of time increment is involved, it depends upon strategy choices for assemblies of higher order than assembly i. However, these decisions are unknown when assembly i is considered and therefore an approximation must be made for the value of $S_j(M_j)$. The most practical approximation is one based upon the first type of time increment: the marginal increase in time before the equipment is available for use due to repair of assembly i rather than an immediate exchange for a serviceable unit. Although this assumption will in many instances overstate the value of $S_j(M_j)$, the net effect is on the conservative side with respect to reducing the time the equipment is inoperable.

The term, F_j, represents the present value of all initial and future repair and inventory costs associated with subordinate part j and all parts within part j if it is an assembly. If part j is a detail part, $F_j = I_j(M_j)$, the inventory costs as defined in the discussion on first-order assemblies. If part j is an assembly, the value for $F_j(M_j)$ depends upon the mix of repair and inventory decisions applied to part j and all of its lower assemblies and detail parts. For the present discussion, it is assumed that this mix of decisions is prescribed and that the corresponding value for F_j is known.

The terms, B_i, I_i, and I'_i, represent initial and future inventory costs for assembly i and are the same as defined for a first-order assembly. No special interpretation is required in the present context.

Since the cost function given in Equation (17) is of the same form as the one for first-order assemblies, as given in Equation (5), the indicated minimization can be accomplished by the computation procedure previously defined. No special considerations are required for the application of this procedure in the current context.

C. Recursion Principle

In Equation (17), the term, F_j, represents the present value of all initial and future repair and inventory costs associated with subordinate part j and its lower parts if part j is an assembly. As mentioned previously, the value for this term depends on repair and inventory decisions that are applied. Although, before, such decisions or policies were assumed to be of any arbitrary kind as long as the corresponding costs could be assessed, a particular choice for the set of decisions and their cost consequences may now be considered. In particular, that set of decisions which minimizes F_j over all possible sets of such decisions may be specified. If it is assumed that F_j is the minimum cost for subordinate part j, then T_j, as given by Equation (17), will be a minimum value for repair and inventory costs for assembly i . The problem now is to determine a minimum value for F_j in order to obtain a true minimum for T_i .

The cost function in Equation (17), however, was defined for an assembly of arbitrary order. Therefore, it can apply equally well to subordinate part j if it is an assembly. Thus, if F_j is set equal to T_j , where T_j is given by an equation similar to Equation (17), then F_j will be minimized. In general, this defines an inductive type process where stage k (associated with assembly i) has been minimized given that stage k-1 (associated with subordinate parts to assembly i) has been minimized. Stage 1 in the induction is associated with the lowest first-order assemblies in the parts hierarchy, which has been previously minimized. Thus, the induction is complete and a minimum value for T_i has been determined for each assembly in the equipment, including the equipment itself considered as a higher-order assembly.

The computational procedure is applied recursively in accordance with the induction, starting with all first-order assemblies and then proceeding to all second-order assemblies, third-order assemblies, and so forth up to the equipment itself. Each application of the minimizing procedure provides inputs to the cost functions for assemblies of the next higher assemblies. Since each application of the minimizing procedure establishes repair and inventory range decisions for an assembly, the final computation for the equipment itself completes such decisions for all parts within the equipment. This results in the identification of parts to be stocked and, conceptually, an exact prescription as to what repair actions to take for any given failure of the equipment such that if these actions are taken, over-all support costs for the equipment will be minimized.

D. Numeric Example

To illustrate the entire procedure for a higher-order assembly, a numeric example is developed for a fourth-order assembly. The fourth-order assembly contains two first-order assemblies and one third-order assembly. The inventory depth policies assumed for this example are the same as defined in Section VI.

The PPL for this example is shown in Exhibit 13 which also contains assumed values for item parameters. In addition to these values, the following factors and their values will be used for all items:

$\alpha = 5$

$H = .20$

$K = \$10$

$K' = \$20$

$B = \$50$

$D = 0$

These values have been chosen more to simplify the computation for this example rather than as realistic values for an actual PPL.

Results of the computation for the three first-order assemblies are shown in Exhibit 14. These results show that it does not pay to repair the Trigger Assembly but it does pay to repair the other two assemblies since, for each detail part, $\Delta_{A,i}$ is less than $\Delta_{B,i}$ (min) and $\Delta_{C,i}$ (min).

Exhibit 15 shows the results of the computation for the second- and third-order assemblies. For each assembly, repair is indicated with subordinate parts only being stocked. In the Chassis Assembly, the Trigger Assembly is stocked but the detail parts therein are not stocked in accordance with the previous result. The three detail parts in the Chassis Assembly are not stocked since repair strategy B is cheapest for these parts. In the Wired Chassis Assembly, the Chassis Assembly as a package is indicated for stockage; however, from the previous result, only parts within the chassis are stocked and not the Chassis Assembly itself.

Finally, Exhibit 16 shows results for the Commutator Chassis, a fourth-order assembly. Again, subordinate parts only are indicated for stockage. Although the lower-order assemblies are shown as being stocked, they are considered as "packages" in this respect; whether or not the assemblies themselves are stocked is governed by previous results. In this case, all three lower-order assemblies are not stocked as assemblies but parts within them are stocked for repair.

The final solution to the range problem for the example PPL is summarized in the right-hand column of Exhibit 13.

In this example, direct repair by removing and replacing failed detail parts is favored rather than remove and replace assemblies.

EXHIBIT 13 - PARAMETER VALUES FOR EXAMPLE PPL

Item Seq. No.	Ind. No.	Item Name	Part No.	No. of Applic.	Assy. Order No.	U_i	M_i	$R_i = R_i'$	S_i	Result of Range Dec.
1	1	Commutator Chassis	LD419481	1	4	500.00	100.0			NS
2	2	Screw	MS35266-68	6	D	.05	.1	3	5.00	S
3	2	Washer	AN960-10	6	D	.05	.1	3	5.00	S
4	2	Spacer	1996668	2	D	1.50	.8	10	5.00	S
5	2	Chopper	2016325-1	1	D	42.50	4.0	10	5.00	S
6	2	Wired Chassis Assy.	2023669	1	3	250.00	20.0	20	5.00	NS
7	3	Channel	2016739-1	2	D	2.50	.2	10	2.50	S
8	3	Connector	2016302-3	4	D	3.08	.8	5	2.50	S
9	3	Bi-Stable Magnetic	2016849	1	D	40.00	4.0	10	2.50	S
10	3	Transformer	2016281	2	D	45.00	3.0	10	2.50	S
11	3	Screw	MS35266-68	12	D	.05	.1	5	2.50	S
12	3	Nut	MS20365-1032	12	D	.02	.1	5	2.50	S
13	3	Switch	2016764-1	2	D	6.10	1.0	10	2.50	S
14	3	Resistor	2016321	4	D	1.00	3.0	10	2.50	S
15	3	Var. Resistor	2016781-5	1	D	6.95	6.5	10	2.50	S
16	3	Chassis	2137641	1	2	60.00	1.3	100	2.50	S
17	4	Panel	2023668	1	D	15.00	.1	10	.60	NS
18	4	Support	2009267	1	D	15.00	.1	5	.60	NS
19	4	Bracket	2016819	1	D	10.00	.1	5	.60	NS
20	4	Trigger Assy.	2009159	2	1	5.00	1.0	5	.60	S
21	5	Trigger	2009158	1	D	2.00	.8	3	.05	NS
22	5	Pin	1996679	1	D	2.00	.2	3	.05	NS
23	2	Commutator Relay Driver	2023708	1	1	125.00	40.0	5	5.00	NS
24	3	Transistor	2016338-1	6	D	14.75	15.0	20	1.25	S
25	3	Capacitor	2016487-7	2	D	3.15	4.0	20	1.25	S
26	3	Resistor	RC07GF202J	6	D	.15	20.0	20	1.25	S
27	3	Connector	2016494-1	1	D	1.29	.8	10	1.25	S
28	3	Frame	2016258-1	1	D	3.51	.2	50	1.25	S
29	2	Water Velocity Amplifier	2023712	1	1	100.00	35.0	5	5.00	NS
30	3	Transistor	575R680H04	3	D	30.60	22.0	20	1.00	S
31	3	Capacitor	575R921H54	1	D	1.90	2.0	20	1.00	S
32	3	Resistor	RC07GF202J	3	D	.15	10.0	20	1.00	S
33	3	Connector	2016494-1	1	D	1.29	.8	10	1.00	S
34	3	Frame	2016258-1	1	D	3.51	.2	50	1.00	S

EXHIBIT 14 - RESULTS OF COMPUTATION FOR FIRST-ORDER ASSEMELIES

Item Number	Item Name	$\Delta_{A,i}$	$\Delta_{B,i}(\text{min})$	$\Delta_{B,i}(\text{max})$	$\Delta_{C,i}(\text{min})$	$\Delta_{C,i}(\text{max})$	Range Decision
20	TRIGGER ASSEMBLY						S
21	Trigger	82.85	32.37	40.00	119.75	142.65	NS
22	Pin	61.37	7.37	15.00	68.43	91.32	NS
23	COMMUTATOR RELAY DRIVER						NS
24	Transistor	2,898.66	9,523.15	9,808.00	3,249.36	4,103.91	S
25	Capacitor	573.50	2,536.34	2,723.61	657.52	1,219.33	S
26	Resistor	2,207.32	12,707.19	13,000.00	2,703.88	3,582.32	S
27	Connector	110.31	507.16	600.00	126.78	405.31	S
28	Frame	113.14	126.79	175.00	117.26	261.89	S
29	WATER VELOCITY AMPLIFIER						NS
30	Transistor	5,985.42	11,231.00	11,469.04	6,568.42	7,282.54	S
31	Capacitor	298.49	1,017.10	1,141.42	339.79	712.75	S
32	Resistor	1,119.75	5,091.60	5,316.23	1,344.55	2,018.44	S
33	Connector	109.31	406.80	489.44	125.71	373.63	S
34	Frame	112.89	101.70	144.72	116.99	246.05	S

EXHIBIT 15 - RESULTS OF COMPUTATION FOR SECOND- AND THIRD-ORDER ASSEMBLIES

Item Number	Item Name	$\Delta_{A,i}$	$\Delta_{B,i}(\text{min})$	$\Delta_{B,i}(\text{max})$	$\Delta_{C,i}(\text{min})$	$\Delta_{C,i}(\text{max})$	Range Decision
16	CHASSIS						S
17	Panel	75.05	33.49	54.50	85.21	148.25	NS
18	Support	72.55	33.49	54.50	82.71	145.75	NS
19	Bracket	67.80	33.49	54.50	77.96	141.00	NS
20	Trigger Assembly	125.36	345.86	377.46	259.93	354.74	S
6	WIRED CHASSIS ASSEMBLY						NS
7	Channel	72.07	253.48	320.71	80.00	281.70	S
8	Connector	108.00	1,014.23	1,141.42	140.69	522.25	S
9	Bi-Stable Magnetic	1,226.49	5,074.63	5,316.23	1,400.38	2,125.18	S
10	Transformer	978.69	3,805.18	4,023.86	1,156.73	1,812.77	S
11	Screw	54.49	126.74	175.00	58.46	203.24	S
12	Nut	54.21	126.74	175.00	58.18	202.96	S
13	Switch	167.70	1,267.87	1,408.11	208.80	629.53	S
14	Resistor	269.82	3,805.18	4,023.86	397.86	1,053.90	S
15.	Var. Resistor	749.36	8,251.17	8,528.00	1,046.62	1,877.11	S
16.	Chassis	335.79	1,648.40	1,805.28	1,027.99	1,498.63	S

EXHIBIT 16 - RESULTS OF COMPUTATION FOR FOURTH-ORDER ASSEMBLY

Item Number	Item Name	$\Delta_{A,i}$	$\Delta_{B,i}(\min)$	$\Delta_{B,i}(\max)$	$\Delta_{C,i}(\min)$	$\Delta_{C,i}(\max)$	Range Decision
1	COMMUTATOR CHASSIS						NS
2	Screw	54.74	251.12	320.71	55.60	264.37	S
3	Washer	54.74	251.12	320.71	55.60	264.37	S
4	Spacer	126.95	2,008.95	2,200.00	133.79	706.95	S
5	Chopper	1,330.38	10,045.17	10,447.21	1,365.89	2,572.01	S
6	Wired Chassis Assembly	6,516.62	50,223.61	51,000.00	6,725.09	9,016.62	S
23	Commutator Relay Drive	7,902.93	100,504.02	101,414.20	8,414.98	11,145.56	S
29	Water Velocity Amplifier	9,275.86	87,933.36	88,822.90	9,800.93	12,469.50	S

This results, of course, from the particular values of the parameters that were chosen. More specifically, the prior existence of a complete repair capability was implicitly assumed by the assignment of a relatively low value for B_i, the initial repair and inventory cost. The value chosen is probably less than a reasonable value for inventory costs (initial setup and future administrative) might be; therefore, there is little or no reflection of initial costs for providing repair capabilities. Stockages of assemblies in the form of repair buffer stocks are not indicated basically because a low value was placed on assembly shortage costs. In general, it is less expensive to have the equipment out of commission while being repaired rather than to carry stocks of spare assemblies. If the values for assembly shortage costs were doubled, some of the assemblies would be stocked in the form of repair buffer stocks.

IX. EXTENSIONS OF THE HIGHER-ORDER ASSEMBLY MODEL

As in the case of the first-order assembly model, the higher-order assembly model may be altered so as to relax some of the assumptions incorporated in its development and to incorporate special cases that can occur. Most of the extensions discussed in Section VII for the first-order assembly model can be applied in basically the same fashion to the higher-order assembly model and need not be further discussed. However, the consideration of higher-order assemblies introduces an additional problem of sufficient importance as to require further investigation. This problem occurs when a given part, whether a detail part or an assembly, has multiple applications within a particular higher-order assembly (or equipment), often at various levels in the parts hierarchy. In this section, the higher-order assembly model is extended to consider this case of multiple parts application.

A. Multiple Application

The development of the higher-order assembly model in the preceding section implicitly assumed a one-to-one correspondence between parts and items on a PPL. Therefore, the analysis pertains to items on a PPL as well as to parts since under the assumption they are, in effect, synonymous. A particular item representing more than one unit of a part can still be considered as only one appearance of the part, insofar as computations are concerned, wherein the remarks on multiple applications in Section VII. E apply. However, when the same part is associated with more than one item on the PPL, the problem becomes more complex and must be considered further.

As a simple example of this problem, a second-order assembly may consist of two first-order assemblies, each having a particular detail part in common. In this case, the model previously developed would assume, for each first-order assembly, that the detail part is a unique appearance and does not exist in the other assembly. Thus, for each first-order assembly computation, an initial inventory cost is assumed for the detail part, and future inventory costs are computed by neglecting the possibility of stocking the detail part in support of the other assembly. After independently computing minimizing costs for the two assemblies on this basis, they are passed up to become part of the cost function for the second-order assembly. Not only are costs for the detail part thereby overstated, they can indirectly cause errors in range decisions associated with the second-order assembly. It is clear that some way to allow for this situation is required.

Considering the case of an assembly of arbitrary order and referring to Equation (17), suppose that assembly i appears in more than one line-item on the PPL. As discussed previously, once it is decided to compensate for a failure of assembly i, the actions taken are independent of decisions relating to its higher assemblies. Therefore, repair and supply actions applied to a given failure of assembly i do not depend upon the position of the failed unit in the parts hierarchy. Furthermore, by an assumption imposed in Section IV, assembly i is broken down into its constituent parts the same way for each of its appearances on the PPL and, in particular, has the same subordinate parts for each application. Consequently, the fact that assembly i has multiple applications within the equipment merely means that the installed population of the part is greater than one insofar as range decisions associated with assembly i are concerned. Thus, Equation (17) may be applied directly by interpreting all failure rates, both for the assembly and its subordinate parts, as being total rates over all applications of assembly i.

Suppose next that subordinate part j in assembly i is itself an assembly with multiple application, perhaps with some of its applications in assemblies other than assembly i. By the same reasoning as above, Equation (17) may be applied to subordinate part j, with failure rates based upon the total population of part j in the equipment. However, when considering decisions relating to assembly i, the term, F_j, should include only some portion of the costs, T_j, depending generally upon the extent of the application of subordinate part j in assembly i. Since T_j is not normally a linear function of the failure rate for part j (over all its applications), the particular portion to be substituted for F_j will, in general, depend upon repair strategy decisions for higher assemblies containing part j in its other applications. In theory, this would require a simultaneous consideration of all assemblies containing part j as a subordinate part. However, since this is difficult and impractical, a first-order approximation may be made by assuming T_j to be a linear function of the failure rate for part j and by taking as F_j, the value, T_j, prorated according to the extent of the application of subordinate part j in assembly i. Thus, if $\tilde{M}_j$ represents the total failure rate of part j, over all its applications, and M_j represents the failure rate of subordinate part j for all its applications in assembly i, then F_j may be set equal to

$$\frac{M_j}{\tilde{M}_j} T_j \quad .$$

A similar approximation may be made if subordinate part j is a detail part with multiple applications. In this case, the following substitution is made in Equation (17):

$$F_j = \frac{M_j}{\widetilde{M}_j} I_j(M_j) \quad .$$

Whether subordinate part j is an assembly or a detail part, the term, B_j, in Equation (17) presents an additional problem. Rather than prorating these fixed costs as was done for costs depending upon the failure rate, a better method is to omit them from the initial computation. Then, each higher assembly containing part j as a subordinate part that chose strategy A or C for part j may be inspected, either individually or collectively, to determine whether savings afforded by strategies A or C rather than strategy B will offset the cost, B_j. If so, the term, B_j, may then be added to T_i for the assembly i that contains the smallest number of applications of part j for which the introduces the cost, B_j, at the lowest justifiable level in the parts hierarchy, so as to be properly included in decisions for higher levels. Of course, it may happen that savings afforded by assignment to strategy A or C in some or even all of the applications of part j will not offset the cost, B_j. In this case, strategy B will pertain to all appearances.

B. Prior Application

A given part may not only have multiple application within the equipment under consideration, but may also have applications on equipments already being supported. According to prior range decisions for these equipments, the part may be already stocked or, if not, may now be stocked when the new applications are considered together with previous ones. Also, it is at least theoretically possible that prior stockage of the part should be disestablished, in favor of some higher assembly, when the new application is considered in conjunction with prior ones.

In principle, each time a new equipment is introduced that contains parts common to equipment already being supported, the aggregate of all such equipments should be recomputed using the method above regarding multiple application. Of course, any costs such as initial repair and inventory costs that have already been incurred should be omitted from the computation. However, this approach is usually impractical and the new equipment must be considered separately. In this case, the only recognition of prior application that can be made is if a part is already stocked for the prior application. Here, marginal inventory costs may be used for the part in its new applications in accordance with the method described in Section VII. D. If, in addition to prior stockage, the part has multiple application in the new equipment, the approximate method described above becomes more accurate, since the marginal inventory costs for most depth policies become more nearly linear as a function of failure rate.