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TRANSLATION

THE METHOD OF TRANSVERSE APPROXIMATION IN A HYPERSONIC FLOW

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THE METHOD OF TRANSVERSE APPROXIMATION
IN A HYPERSONIC FLOW

I. I. Aleksenko, R. G. Barantsev
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§ 1.

In the problem of hypersonic flow around blunt bodies the first and main step is the construction of a flow in the transonic region approaching the nose of a body. In spite of the huge amount of work devoted to this theme, it is still difficult to point out a method that is close to optimum in both simplicity and accuracy. In this sense the problem remains unsolved even for an ideal gas when both plane and axial symmetry exist.

The existing approximate analytical methods can be divided into five groups:

1) the Newton formula and its empirical modifications [1 – 4];

2) the model of an incompressible liquid [2, 4, 5 – 7];

3) the expansion according to the parameter $\varepsilon = \frac{x - 1}{x + 1}$, where x is the ratio of the heat capacities [1 – 3];

4) power series in the vicinity of the stagnation point [8 – 10];

5) power series in the vicinity of the shock wave [4, 11 – 13].

Unfortunately, in spite of the rich selection, not one of these methods is free of significant deficiencies: the first is limited only by the surface of the body; the second and fourth are connected to the vicinity of the stagnation point and also cannot capture the whole region of the effect; in the third the applied form of the expansion according to ε does not consider the ordinary irregularities of asymptotic behavior, in view of which the region of applicability of this method remains very limited; in the fifth only the reverse problem is solved and difficulties appear with convergence.

The lack of a successful analytical approach to the solution of this problem, the importance of which needs no explanation, has lead to just this problem being one of the first to be treated by computers. Numerical methods of solving the reverse problem immediately gave birth to three variants [4, 14 – 15]. The central place in the direct problem was occupied by the method of integral correlations of A. A. Dorodnitsyn, developed in the works of O. N. Belotserkovskiy [16] and his followers [17 – 21].

The successes attained with the help of high-speed computers are, no doubt, great. But it is still premature to claim that the basic possibilities have been exhausted and that only insignificant refinements remain. An imperfection of a qualitative character is that a sharp break is formed between accurate, but extremely tedious numerical methods and comparatively simple, but very rough analytical methods. In any problem at any degree of development of computer mathematics it is natural that there is a position in which the possibilities of the analytical apparatus, as the more general, are used to the end, and the complexity of the program is at the level of complexity of the problem. In hypersonic aerodynamics neither one nor the other has yet been attained. However, the accumulated experience in particular that connected with the first and second approximations of Belotserkovskiy, has already given material sufficient for the selection of roots which are not carried away into extremes.

In the search for a suitable approximate method we must, on the one hand, get rid of difficulties that are not connected with the actual problem and, on the other hand, try to retain the difficulties of an intrinsic character; in fact, we must look for them. The specifics of the examined problem are such that an approximate method with any pretense of success evidently should not avoid the following peculiarities: 1) nonlinearity; 2) effects resulting from the mixed nature of the type of equations; 3) implicit dependences.

While these requirements are rather great, the possibilities of simplific tions that retain the stated properties are pretty good. For example, in a number of investigations it has been observed that the change in hydrodynamic magnitudes across the layer between the shock wave and the body can be described by very simple dependences with a great degree of accuracy. Lees [3] even speaks about the linear character of certain functions. It is just this smoothness of behavior along the transverse coordinate that guaranteed the successes of the first and second approximations of Belotserkovskiy.

Thus, it is expedient to search for a reasonable simplification of the problem along the path towards a suitable dependence of the solution upon the transverse variable. This approach is naturally called the method of transverse approximation.

The good results obtained in works [16 – 21] in linear and quadratic approximations speak in favor of this method. However, these works leave a sense of dissatisfaction in connection with the ackwardness of the numerical diagrams, which seems excessive under the assumptions that are made. An impression is created that it makes sense to look for a more economic route to find results of the same level.

Among the analytical investigations work [22] should be noted; in this the longitudinal component of velocity is approximated by a cubic parabola according to a variable of the Dorodnitsyn type in boundary layer theory. Unfortunately, due to the additional suppositions in this work we do not obtain a boundary problem that calculates the reverse effect of the transonic zone.

Thus, in our opinion, the opportunity is ripe to find that variant of the method of transverse approximation which would contain simplicity and accuracy applicable for engineering. This is the goal we have set for ourselves.

In this work we are basically setting forth the results of the first stage of investigation, in which the simplest type of approximation (linearization of the function of flow) and the simplest method of determining the coefficients (from the conditions in a shock wave) are examined. This problem leads to the integration of one non-linear differential equation of the first order with one parameter and two conditions at different points. In the case of bodies that form conic cross sections, this equation is integrated analytically. Planar and axisymmetric bodies with rectilinear generators, spheres, ellipsoids and spherical segments, are examined. For concreteness and simplicity in the calculations the gas is considered ideal, and the Mach number, infinite. The accuracy turned out to be low, but better than was expected from so rough an approach and from such simple apparatus.

The basic value of this work is that in light of the obtained results the last section of the path to the optimum variant of the method becomes more evident.

§ 2. Statement of the Problem

The system of coordinates. Let us examine a symmetric flow around bodies of rotation and the profiles made by a supersonic flow of gas. In this case the whole picture of flow-around can be examined in the half-plane zOr (Fig. 1).

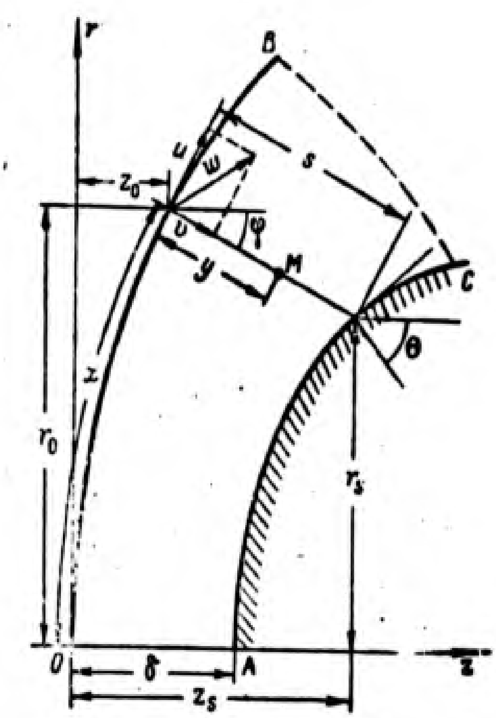

Fig. 1.

We must determine the flow parameters at least in the minimum region of the effect bounded by the section of the axis OA, the contour of the body AC:

$$z_s = f(r_s) + \delta, \tag{2.1}$$

the shock wave OB:

$$z_0 = l(r_0) \tag{2.2}$$

and the limiting characteristic BC; z_s, r_s are coordinates of the contour of the body in the system zOr; z_0, r_0 are the coordinates of the shock wave; Δ is the distance between the body and the shock wave along the Oz axis, so that f(0) = 0.

In the future we will use a curvilinear system of coordinates connected with the shock wave. For the following reasons it is convenient to take the shock wave as the initial coordinate line:

1) in contrast to the surface of the body, which can have fissures, the shock wave is a smooth surface;

2) a larger number of boundary conditions is given for the shock wave and for the body.

In this the form of the body is considered given, and the form of the shock wave is determined in the process of solving the problem.

The position of point M in the curvilinear system of coordinates is determined by coordinates x and y; x is calculated from point 0 along the shock wave to the intersection with the normal to the shock wave passing through point M, and y according to the normal to the wave.

The equation of the contour of the body is written in the form

$$y = s(x).$$

Angles φ and θ are calculated from the normal to the shock wave or to the contour of the body respectively to the direction of the Oz axis. The connection between coordinates z, r and x, y is given by the formulas:

$$\left.\begin{array}{l} z=z_0+y\cos\varphi; \quad r=r_0-y\sin\varphi, \\ \cos\varphi=\frac{dr_0}{dx}; \quad \sin\varphi=\frac{dz_0}{dx}. \end{array}\right\} \quad (2.3)$$

<u>The equation for the flow function.</u> The gas is considered to be nonviscous in a state of thermodynamic equilibrium. The heat content of a unit mass of stagnated gas is assumed constant in the whole flow. The motion of such a gas is described by the equations:

$$\operatorname{div} \rho\vec{w} = 0, \tag{2.4}$$

$$\operatorname{rot}\vec{w} \times \vec{w} = T\nabla S, \tag{2.5}$$

$$\frac{w^2}{2} + i = i_0, \tag{2.6}$$

$$\nabla i = T\nabla S + \frac{\nabla p}{\rho}. \tag{2.7}$$

$$(\vec{w}\nabla S) = 0. \tag{2.7'}$$

The magnitudes ρ, $\vec{w}$, p, i, i_0, T, S are dimensionless ratios: ρ is the density to ρ_1; $\vec{w}$ is the velocity vector to w_1; p is the pressure to $\rho_1 w_1^2$; i and i_0 are enthalpy and heat content to w_1^2/I; T is the temperature to T_1; S is the entropy to w_1^2/IT_1, where ρ_1, w_1, T_1 are the density, velocity and temperature of the approaching flow; I is the mechanical equivalent of heat.

On the strength of (2.4) let us introduce the function ψ (x, y) according to the formulas

$$\psi_x = r^{\nu-1}\rho v\left(1 - y\frac{d\varphi}{dx}\right); \quad \psi_y = -r^{\nu-1}\rho u. \tag{2.8}$$

The subscripts x and y designate differentiation with respect to the corresponding coordinates; u and v are projections of $\vec{w}$ on the tangent and normal to the shock wave; for planes v = 1 and for axisymmetric cross sections v = 2. Due to the symmetry of the flow rot $\vec{w}$ has only one nonzero component Ω in the direction perpendicular to the plane zOr

$$\Omega = -\frac{v_x}{1 - yd\varphi/dx} + u_y - \frac{d\varphi/dx}{1 - yd\varphi/dx}u.$$

Putting this value of rot $\vec{w}$ in (2.5) and using (2.8), we will obtain

$$\frac{v_x}{1-y\,d\varphi/dx}-u_y+\frac{d\varphi/dx}{1-y\,d\varphi/dx}u=-r^{\nu-1}\rho T\frac{dS}{d\psi}. \qquad (2.9)$$

Let us examine the equation of the state in the form

$$p=p(\rho,\, S), \qquad (2.10)$$

i.e., let us take ρ and S as independent parameters of the state of the gas.

Differentiating (2.8), we find

$$r^{\nu-1}\rho v_x=\frac{\partial}{\partial x}\left(\frac{\psi_x}{1-y\,d\varphi/dx}\right)-\frac{\psi_x}{1-y\,d\varphi/dx}\left(\frac{\nu-1}{r}\frac{\partial r}{\partial x}+\frac{\rho_x}{\rho}\right),$$

$$r^{\nu-1}\rho u_y=\psi_{yy}+\psi_y\left(\frac{\nu-1}{r}\frac{\partial r}{\partial y}+\frac{\rho_y}{\rho}\right).$$

The derivatives ρ_x/ρ and ρ_y/ρ can be obtained from the equality

$$\frac{d\rho}{\rho}=-\frac{1}{1-w^2/a^2}\left[\frac{\frac{\psi_x}{1-y\,d\varphi/dx}\,d\left(\frac{\psi_x}{1-y\,d\varphi/dx}\right)+\psi_y d\psi_y}{(r^{\nu-1}\rho a)^2}-\right.$$
$$\left.-\frac{\nu-1}{r}\frac{w^2}{a^2}dr+\left(T+\frac{1}{\rho}\frac{\partial p}{\partial S}\right)\frac{dS}{a^2}\right],$$

which is a combination of equations (2.6), (2.7) and (2.10) in the differential form and formulas (2.8). The dimensionless velocity of sound $a=\sqrt{dr/d\rho}$. The derivatives $\frac{dr}{dx}$ and $\frac{dr}{dy}$ can be obtained from relationships (2.3). As a result the expressions v_x and u_y depend upon the derivatives of the function ψ, the form of the shock wave and the parameters of the state of the gas ρ, a, T, dr/dS.

Having put these expressions for v_x and u_y into equation (2.9), we will obtain the following equation for ψ*:

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* A similar equation was obtained in work [23] in Cartesian coordinates.

$$L(\psi)=\left[\frac{\psi_{xx}}{(1-yd\varphi/dx)^2}+\psi_x\frac{yd^2\varphi/dx^2}{(1-yd\varphi/dx)^3}\right]\left[1-\left(\frac{\psi_y}{r^{\nu-1}\rho a}\right)^2\right]+$$
$$+\psi_{yy}\left[1-\frac{1}{(1-yd\varphi/dx)^2}\left(\frac{\psi_x}{r^{\nu-1}\rho a}\right)^2\right]+\frac{1}{1-yd\varphi/dx}\frac{\psi_x\psi_y}{(r^{\nu-1}\rho a)^2}\times$$
$$\times\left[\frac{2\psi_{xy}}{1-yd\varphi/dx}+\frac{d\varphi/dx}{(1-yd\varphi/dx)^2}\psi_x\right]+\frac{\nu-1}{r}\left(\psi_y\sin\varphi-\psi_x\frac{\cos\varphi}{1-yd\varphi/dx}\right)+$$
$$+\psi_y\frac{d\varphi/dx}{1-yd\varphi/dx}\left[\frac{1}{(1-yd\varphi/dx)^2}\left(\frac{\psi_x}{r^{\nu-1}\rho a}\right)^2+\left(\frac{\psi_y}{r^{\nu-1}\rho a}\right)^2-1\right]=$$
$$=-(r^{\nu-1}\rho)^2\frac{dS}{d\psi}\left[T+\frac{1}{\rho}\frac{\partial p}{\partial S}\frac{\psi_x^2+\psi_y^2(1-yd\varphi/dx)^2}{(r^{\nu-1}\rho a)^2(1-yd\varphi/dx)^2}\right]. \qquad (2.11)$$

The boundary conditions for equation (2.11) are given for the contour of a body, the shock wave and on the section of the Oz axis between them.

1. The section of the axis OA and the contour of the body are the same line of the flow; therefore we can assume

$$\psi=0 \text{ when } x=0 \text{ and } y=s(x). \qquad (2.12)$$

2. From the relationships of compatibility on the shock wave we obtain u = sin φ, ρ v = cos φ. From this we can find that when y = 0

$$\psi=\frac{r_0^\nu}{\nu};\quad \psi_y=-r_0^{\nu-1}\rho\sin\varphi. \qquad (2.13)$$

In the future we will examine the hypersonic motion of an ideal gas with constant heat capacities c_p and c_v; therefore

$$p=(c_p-c_v)\rho T;\ p=\Theta(S)\rho^\varkappa;\ M=\frac{w_1}{a_1}\to\infty;\ \rho=\frac{\varkappa+1}{\varkappa-1}$$

when y = 0, where $\varkappa=\frac{c_p}{c_v}$.

Then the equation for the flow function can be written in the form

$$L(\psi)=-(r^{\nu-1}\rho)^2\frac{1}{2c_p}\frac{dS}{d\psi}\left[1+\frac{\psi_x^2+\psi_y^2(1-yd\varphi/dx)^2}{(r^{\nu-1}\rho a)^2(1-yd\varphi/dx)^2}\right]. \qquad (2.14)$$

The boundary conditions for this equation are:

$$\psi=0 \text{ when } x=0; \quad y\in[0, \delta]; \qquad (2.15a)$$

$$\psi=0 \text{ when } y=s(x); \qquad (2.15b)$$

$$\psi=\frac{r_0^{\nu}}{\nu}; \quad \psi_y=-\frac{\varkappa+1}{\varkappa-1}r_0^{\nu-1}\sin\varphi \text{ when } y=0. \qquad (2.15c)$$

§ 3. The Concepts of the Method

Let us show the basic features of the method on the example of the quadratic approximation of the flow function

$$\psi(x, y)=\psi_0(x)+\psi_1(x)y+\psi_2(x)y^2. \qquad (3.1)$$

The selection of the approximating function is the first stage of the method; it requires an a priori knowledge of the transverse properties of the solution in the problems for bodies of similar form. For example, instead of (3.1) a cubic parabola, a binomial of the n-th degree, a bilinear function, etc., can turn out to be useful.

The second stage is the statement of the problem for the coefficients of the approximating function that are dependent upon x. Here we must clearly understand that the arbitrary rule allowed in approximation (3.1) (or any other approximation) does not make it possible to accurately satisfy all the conditions of the initial problem. Therefore, different methods of determining the coefficients ψ (x), ψ_1 (x), ψ_2 (x) (and the form of the shock wave s (x)) are possible by partially satisfying equation (2.14) and boundary conditions (2.15a, b, c). In this statement the approximation of the non-uniqueness is again approached by the path of the best solution to the simplicity-accuracy dilemma. The equations for the functions

ψ_0 (x), ψ_1 (x), ψ_2 (x), s (x) can be obtained by writing the conditions of the problem either in separate cross sections (not x = const) or in the form of integrals with respect to y. It is clear that we can write as many of these equations as we want, but only four are needed. The boundary conditions for the shock wave (2.15c) and for the body (2.15b) appear as three equations in two boundary cross sections y = 0 and y = s (x); the remaining infinity of equations with respect to x are posed by equation (2.14) for ψ (x, y). The tendency towards simplicity leads to the following variant of the equations for the functions ψ_0 (x), ψ_1 (x), ψ_2 (x), s (x): three from the boundary conditions and one from (2.14) in the cross section y = 0. Let us write these equations. First of all, from the conditions for the shock wave (2.15c) we have

$$\psi_0(x) = \frac{r_0^{\nu}}{\nu}, \tag{3.2}$$

$$\psi_1(x) = -\frac{\varkappa+1}{\varkappa-1} r_0^{\nu-1} \sin\varphi. \tag{3.3}$$

From equation (2.14) we will obtain

$$\psi_2(x) = \frac{1}{2}\psi_{yy}|_{y=0} = -\frac{r_0^{\nu-1}\sin\varphi}{2}\left[\frac{\nu-1}{r_0}\sin\varphi + 6\frac{\varkappa+1}{(\varkappa-1)^2}\frac{d\varphi/dx}{\cos^2\varphi} - 2\frac{(5\varkappa-1)}{(\varkappa-1)^2}\frac{d\varphi}{dx}\right]. \tag{3.4}$$

In the case of the reverse problem, when the function l (r_0) is given, these three equalities fully determine the coefficients ψ_0, ψ_1, ψ_2, and the form of the body is found from condition (2.15b), which leads to a quadratic equation for s (x):

$$\psi_0(x) + \psi_1(x)s(x) + \psi_2(x)s^2(x) = 0. \tag{3.5}$$

In the direct problem that we are solving, l (r_0) is unknown, but the form of the body is given: $z_s = f(r_s) + \delta$, or, according to (2.3),

$$l(r_0) + s(x)\cos\varphi = f[r_0 - s(x)\sin\varphi] + \delta. \qquad (3.6)$$

Thus, we have five equations (3.2 – 3.6) for five functions: $\psi_0(x)$, $\psi_1(x)$, $\psi_2(x)$, $s(x)$, $l(r_0)$; the "extra" connection and the correspondingly "extra" function are the result of using two systems of coordinates. The system of equations (3.2 – 3.6) is not differentiated relative to functions ψ_0, ψ_1, ψ_2, s, and these functions can be easily eliminated. Expressing s through ψ from (3.5) in (3.6) and using (3.2 – 3.4), we will obtain an ordinary differential equation of the second order for $l(r_0)$

$$\Phi(r_0, l, l', l'', \delta) = 0. \qquad (3.7)$$

It remains to clarify the question about the boundary conditions under which we must solve this equation. At point $r_0 = 0$ we have $l = 0$ and $l'(0) = 0$ (when $\delta \neq 0$). These conditions are insufficient for constructing a solution even if point $r_0 = 0$ is not special, since equation (3.7) contains the undetermined parameter δ. Where should we put the additional condition for determining δ? This the fundamental question because here we encounter an interesting peculiarity of the application of the method of curves to equations of the mixed type. As in the case of the equations of the elliptic type, here the nature of the problem is such that there is an interdependence of the solution at all points in the region of the affect. Therefore for the ordinary differential equations appearing in the method of curves a boundary problem or something of this kind, but in no case the Cauchy problem, will be natural. Actually, from the physical meaning of the initial problem (2.14 – 2.15) the solution of equation (3.7) must depend at each point upon the behavior of function f over the whole interval in the region of the effect, which is not obtained in the Cauchy problem.

Thus, together with the conditions at point $r_0 = 0$, there must also be conditions that guarantee the reverse effect of the transonic zone on the stagnation region. However, since, in contrast to elliptical problems, nothing is assigned for the limiting characteristics, it is not immediately evident from where we will select the conditions for the second end of the interval.

After we have used all the boundary conditions of the initial problem, only equation (2.14) itself can be a source of additional conditions. By writing it at any point in the transonic zone, we will obtain the boundary condition for l (r_0). Other variants are also possible; for example, we can integrate equation (2.14) over the whole region or with respect to y in the transonic cross section. One way or another, δ will appear as the only "binding parameter" and it seems insufficient for this purpose. To help it we can bring in another parameter, one that appears in the discovery of the peculiarity at point $r_0 = 0$, for the determination of which we need a second additional condition. We will connect the reverse effect of the transonic zone within the framework of approximation (3.1) through these two parameters. In this relation the same position is held in the second approximation of Belotserkovskiy [16]. But the additional conditions (the finite nature of the velocity gradient at a sonic point) appears to be necessary there for passage through the sonic points, although in principal we can deal with the "free" parameters in another manner.

§ 4. Linear Approximation

To clarify the possibilities of the proposed method it is rational first to select that approximation of the flow function which will allow us to reflect the basic peculiarities of the problem without complicating the mathematical calculating apparatus. The results of works [3], [16], [18 – 21] give a basis to consider it expedient to examine this linear approximation

$$\psi(x, y)=\psi_0(x)+\psi_1(x)y. \tag{4.1}$$

The approximating coefficients ψ_0 and ψ_1 can be determined by various methods. The simplest of all is done by satisfying the boundary conditions for the shock wave, i.e., being given ψ_0 and ψ_1 by formulas (3.2) and (3.3).

Using boundary conditions (3.15b) and formulas (3.2) and (3.3), we find that

$$s(x)=\frac{ar_0}{\sin\varphi}, \tag{4.2}$$

and from (2.3) for r_s we will have

$$r_s=(1-a)r_0, \tag{4.3}$$

where

$$a=\frac{1}{\nu}\frac{\varkappa-1}{\varkappa+1}. \tag{4.4}$$

Equation (3.6), which determines the form of the shock wave, takes on the form

$$\frac{dl}{dr_0}=\frac{ar_0}{f+\delta-l}, \tag{4.5}$$

where $f = f(r_s)$ is the function in equation (2.1) that determines the contour of the body. This is a nonlinear ordinary differential equation of the first order with the initial condition

$$l\ (0) = 0. \qquad (4.6)$$

When $\delta \neq 0$ from (4.5) and (4.6), as a result, we obtain the natural condition $l'\ (0) = 0$.

To determine the flow parameters on the contour of the body we use the expression for ψ_y. When $y = s\ (x)$, on the one hand,

$$\psi_y = -r_s^{\nu-1}\rho u = -r_s^{\nu-1}\rho_T w_T \cos(\theta - \varphi),$$

where ρ_T and w_T are the dimensionless density and velocity on the contour of the body. On the other hand, according to the selected approximation of $\psi\ (x,\ y)$,

$$\psi_y = -\frac{r_0^{\nu-1}}{\nu a}\sin\varphi.$$

Comparing both expressions for ψ_y and considering (4.3), we obtain

$$\rho_T w_T = \frac{1}{\nu a(1-a)^{\nu-1}}\frac{\sin\varphi}{\cos(\theta-\varphi)}. \qquad (4.7)$$

By using (4.7), the Bernoulli equation and the equation of the adiabat, we can find the velocity, pressure and density on the contour of the body depending upon the form of the shock wave.

Let us examine certain cases when equation (4.5), which determines the form of the shock wave, is easily integrated. Introducing the parameter

$$t = \frac{dl/dr_0}{df/dr_s}, \qquad (4.8)$$

by differentiating equation (4.5) we obtain

$$\frac{ar_0}{(df/dr_s)^2}\frac{dt}{dr_0} = t\left[(1-a-t)\,t + \frac{1}{df/dr_s}\frac{d}{dr_s}\left(\frac{r_s}{df/dr_s}\right)\right]. \qquad (4.9)$$

The variables t and r_0 in this equation are separated if

$$\frac{1}{df/dr_s}\frac{d}{dr_s}\left(\frac{r_s}{df/dr_s}\right)=C. \qquad (4.10)$$

We can distinguish three cases here.

1. Condition (4.10) is satisfied if

$$\frac{df}{dr_s}=\operatorname{tg}\theta_0=\text{const}, \text{ i.e., } f=r_s\operatorname{tg}\theta_0.$$

Function f determines a body with rectilinear generators.

2. If C = 0, then condition (4.10) gives

$$f=r_s^2\,\text{const}.$$

Function f determines a parabolic contour of the body.

3. If C ≠ 0, then according to (4.10) we obtain

$$f=\alpha\left(1-\sqrt{1+\frac{r_s^2}{C\alpha^2}}\right).$$

When C > 0 and $\alpha < 0$ the contour of the body is a hyperbola. When C < 0 and $\alpha > 0$ the contour of the body is an ellipse with semiaxes α and $\beta = \alpha\sqrt{|C|}$.

The conditions $f(r_s) \geqq 0$ and $f(0) = 0$ are considered in integrating equation (4.10).

Analogous dependences for the form of the shock wave that is determined by equation (4.9) will be given below for different specific contours of a body.

§ 5. Conditions for the Determination of δ

In the statement of the problem in the linear approximation (4.1) there remains one undetermined parameter δ, to find which we must assign an additional condition. In the selection of the condition for δ previously we used an arbitrary rule natural for the method. The presentation about the nature of the problem (§ 3) forces us to search for this condition in the transonic zone.

We can use the finite nature of the gradients of the hydrodynamic magnitudes at a sonic point in the shock wave as a general, sufficiently simple, condition. The same result is obtained if we write equation (2.14) at a sonic point in the shock wave, namely

$$r\,l'' = -(\nu-1)\varkappa\left(\frac{\varkappa-1}{\varkappa+1}\right)^{5/2}. \tag{5.1}$$

It is convenient to use this condition in the case of smooth bodies.

For bodies with a corner it is natural to use the condition of achievement of sonic velocity on the body at the corner point [17, 19 – 21]. This condition can be written for $\varphi = \varphi_*$ corresponding to a sonic point in the shock wave by using (4.7) in the form

$$l'(r_0^*) = \operatorname{tg}\varphi_* = \frac{\nu a(1-a)^{\nu-1}\rho_T w_T \cos\theta_*}{1-\nu a(1-a)^{\nu-1}\sin\theta_* \rho_T w_T}. \tag{5.2}$$

We will use formulas (5.1) and (5.2) to determine δ.

§ 6. Flow Around Sharp Bodies with an Attached Shock Wave

In this case $\delta = 0$. The contour of the body can be given by this relationship:

$$z_s = r_s \operatorname{tg}\theta_0 + F(r_s), \tag{6.1}$$

where $F'(0) = 0$ and θ_0 is the angle between the Oz axis and the internal normal to the contour of the body at the nose. According to (4.3)

$$z_s = r_0(1-a)\operatorname{tg}\theta_0 + F[(1-a)r_0].$$

Equation (4.5) takes on the form

$$\frac{dl}{dr_0} = \frac{ar_0}{r_0(1-a)\,\mathrm{tg}\,\theta_0 - l + F[(1-a)r_0]} \qquad (6.2)$$

with the initial condition that $l = 0$ when $r_0 = 0$.

The point $r_0 = 0$, $l = 0$ is special for equation (6.2). Close to it, disregarding the small second order with respect to r_0, this equation can be written in the form

$$\frac{dl}{dr_0} = \frac{ar_0}{r_0(1-a)\,\mathrm{tg}\,\theta_0 - l}. \qquad (6.3)$$

Three cases are possible.

1. If

$$\mathrm{tg}\,\theta_0 > -\frac{2\sqrt{a}}{1-a},$$

then replacement of the variables

$$\xi = -\varepsilon_2 r_0 + l, \qquad \eta = -\varepsilon_1 r_0 + l$$

transforms equation (6.3) into the form

$$\frac{d\eta}{d\xi} = \frac{\varepsilon_1}{\varepsilon_2}\frac{\eta}{\xi},$$

which gives the family of integral curves

$$\eta = C|\xi|^{\varepsilon_1/\varepsilon_2} \text{ and } \xi = 0,$$

where

$$\varepsilon_1 = \frac{1-a}{2}\,\mathrm{tg}\,\theta_0 + \sqrt{\left(\frac{1-a}{2}\,\mathrm{tg}\,\theta_0\right)^2 - a},$$
$$\varepsilon_2 = \frac{1-a}{2}\,\mathrm{tg}\,\theta_0 - \sqrt{\left(\frac{1-a}{2}\,\mathrm{tg}\,\theta_0\right)^2 - a}.$$

When $\theta_0 = \frac{\pi}{2}$ the axis $\eta = 0$ coincides with the axis $r_0 = 0$, the axis $\xi = 0$ coincides with the axis $l = 0$, and the general solution is transformed into the family of curves $\eta = C\xi$ (or $r_0 = Cl$). But as

$\theta_0 \to \frac{\pi}{2}$ the body (or the profile) degenerates into a needle (or a plate) that coincides with the Oz axis. Therefore, the shock wave must approach the contour of the body, i.e., $r_0 \to 0$ at any l, or $\eta \to 0$ at any ξ. And this means that the sought solution to equation (6.3) must correspond to a constant C = 0 in the general solution.

The solution $\xi = 0$ does not fulfill the requirement of approach of the shock wave to the contour of the body as $\theta_0 \to \frac{\pi}{2}$ and therefore it must be rejected. (Actually, if $\theta_0 \to \frac{\pi}{2}$, then $\varepsilon_2 \to 0$, and the solution $\xi = 0$ gives $l \to 0$ at any r_0, i.e., the Or axis).

Thus, close to the special point we have $l = \varepsilon_1 r_0$; therefore the sought solution to equation (6.2) can be presented in the form

$$l = r_0 \operatorname{tg} \varphi_0 + L(r_0), \tag{6.4}$$

where

$$\operatorname{tg} \varphi_0 = \varepsilon_1 = \frac{1-a}{2} \operatorname{tg} \theta_0 + \sqrt{\left(\frac{1-a}{2} \operatorname{tg} \theta_0\right)^2 - a}; \quad L'(0) = 0. \tag{6.5}$$

2. $\operatorname{tg} \theta_0 = \frac{2\sqrt{a}}{1-a}$. Replacement of the variables

$$\eta = l, \quad \xi = r_0 \sqrt{a} - l$$

in (6.3) leads to the equation

$$\frac{d\eta}{d\xi} = \frac{\eta}{\xi} + 1,$$

which gives

$$\eta = \xi \ln \xi + C\xi \text{ and } \xi = 0.$$

All the integral curves, except $\xi = 0$, intersect the contour of the body and therefore cannot determine the shock wave. Consequently, the latter is given by the equation $\xi = 0$ or $l = r_0 \sqrt{a}$ close to the origin of the coordinates. We will obtain the same thing in the first case by a limiting shift

$$\operatorname{tg}\theta_0 \to \frac{2\sqrt{a}}{1-a}.$$

3. When

$$\operatorname{tg}\theta_0 < \frac{2\sqrt{a}}{1-a}$$

all the integral curves at $\delta = 0$ will be logarithmic spirals with a summit at the nose of the body. Therefore they cannot determine the shock wave. In this case we should consider that $\delta \neq 0$; the shock wave is separated from the body.

Thus, an attached shock wave in front of a sharp body exists if

$$\operatorname{tg}\theta_0 > \operatorname{tg}\theta_0^* = \frac{2\sqrt{a}}{1-a}, \qquad (6:6)$$

and is determined by equation (6.4). The function $L(r_0)$ can be determined by numerical integration of equation (6.2) depending upon function $F(r_s)$ in equation (6.1). The flow parameters on the contour of the body are determined with the help of equation (4.7) depending upon ν, x, $\frac{dl}{dr_0}$ and $\frac{dz_s}{dr_s}$.

Flow around a cone ($\nu = 2$) and a wedge ($\nu = 1$). In this case $F(r_s) \equiv 0$, $L(r_0) \equiv 0$, $\theta = \theta_0 = \text{const}$ and $\varphi = \varphi_0 = \text{const}$.

According to equation (6.6) the limiting angles θ_0^* at which the shock wave separates from the body are equal to $\theta_0^* = 44°15'$ for a wedge and $\theta_0^* = 33°12'$ for a cone (if $\varkappa = 1.4$). The exact values of these angles are 44°25' and 32°28' ($M \to \infty$) respectively. The error does not exceed 0.8%.

Table 1 introduces the results of calculating φ_0 according to formula (6.5) and w on the surface of cones according formula (4.7). This table also gives the values of φ_0 and w according to the calculations of Kopal [27].

Table 1

θ_0^0	φ_0^0 from (6.5)	φ_0^0 from [27]	w for a cone from (46)	w for a cone from [27]
77,5	76,36	76,30	0,971	0,972
50	45,28	45,11	0,707	0,714
45	39,20	38,94	0,631	0,635

The obtained results testify to the sufficient accuracy of the linear approximation of ψ for determining the picture of flow around cones by a hypersonic flow with an attached shock wave.

In conclusion let us introduce the equation of the flow lines between the attached shock wave and cone, according to the linear approximation of ψ:

$$r \operatorname{tg} \theta_0 - z \frac{C}{r \operatorname{tg} \varphi_0 + z} = 0.$$

For a wedge these lines are curves parallel to the surface.

§ 7. A Body with Rectilinear Generators. A Separated Shock Wave

Let us consider flow around a cone (or a wedge) in the presence of a fissure at the point $r_s = R$, considering that $\theta_0 < \theta_0^*$, i.e., when the shock wave is separated from the body as is shown in § 6.

The simplest case is $\theta_0 = 0$ (a flat cut). The contour of the body in the nose section when

$$r_s \in [0; R]$$

is given by the relationship $z_s = \delta$. The equation for determining $l\ (r_0)$ takes on the form

$$\frac{dl}{dr_0} = \frac{ar_0}{\delta - l}. \tag{7.1}$$

At the initial condition $l(0) = 0$ it determines an ellipse

$$ar_0^2 + (l - \delta)^2 = \delta^2, \quad r_0 \in \left[0, \frac{R}{1-a}\right] \tag{7.2}$$

with the semimajor axis parallel to the Or axis.

As was shown in § 5, the unknown magnitude δ can be determined from the condition of achieving sonic velocity at the corner point $r_* = R$. Then relationship (5.2) will determine l/dr_0 at this point, namely

$$\left(\frac{dl}{dr_0}\right)_* = \nu a (1-a)^{\nu-1} \rho_T w_T. \tag{7.3}$$

Placing the magnitude of dl_0/dr_0 from (7.3) and the magnitude $\delta - l$ from (7.2) into (7.1) and considering (4.3), we will obtain

$$\frac{\delta}{R} = \sqrt{\left(\frac{1}{\nu(1-a)^\nu \rho_T w_T}\right)^2 + \frac{a}{(1-a)^2}}. \tag{7.4}$$

When $\varkappa = 1.4$ for the axisymmetric case ($\nu = 2$) we have $\frac{\delta}{R} = 0.476$, and for plane ($\nu = 1$) we have $\frac{\delta}{R} = 0.871$.

To determine the flow parameters on a flat cut from (4.7) we can obtain the formula

$$\rho_T w_T = \frac{1}{\nu a (1-a)^{\nu-1}} \cdot \frac{a r_s}{\sqrt{(1-a)^2 l^2 - a r_s^2}}; \quad r_s \in [0; R]. \tag{7.5}$$

On Fig. 2 for the plane ($\nu = 1$) and axisymmetric ($\nu = 2$) cases we have constructed shock waves obtained from equation (7.2) ($\varkappa = 1.4$; $M = \infty$). For comparison we have also shown the shock waves calculated by Chubb [17] ($\nu = 1$; $\varkappa = 1.4$; $M = \infty$) and Holt [21] ($\nu = 2$; $\varkappa = 1.4$; $M = 5.8$) according to the method of Belotserkovskiy. On Fig. 3 we have introduced the distribution of pressure over a flat cut at $\nu = 2$, one obtained in the linear approximation of ψ and one found by Holt [21] at M = 5.8. From an examination of the figures we can conclude that the linear approximation of ψ allows us to obtain a basically reliable picture of the distribution of pressure in a flat cut and the form and position of the shock wave, although the distance δ is obtained somewhat increased for $\nu = 1$.

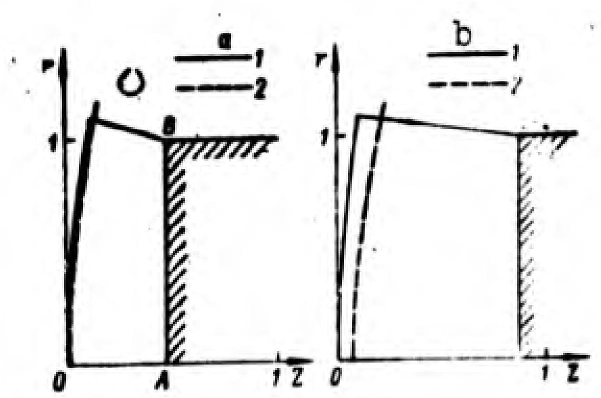

Fig. 2. The shock wave for a cylinder and a plate. a) 1, from (7.2); 2, Holt [21]; b) 1, from (7.2); 2, Chubb [17].

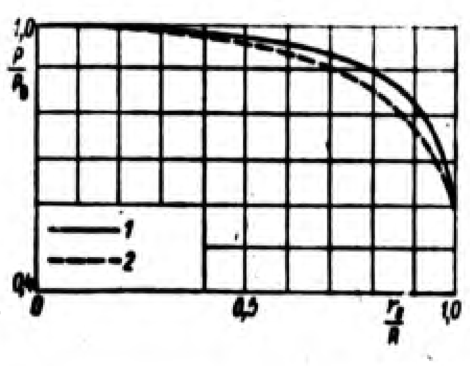

Fig. 3. 1 – Holt [21]; 2 – (7.5).

If $0 < \theta_0 < \theta_0^{*}$, then the form of the body is determined by equation

$$z_s = r_0(1-a)\,\mathrm{tg}\,\theta_0 + \delta;\quad r_0 \in \left[0;\ \frac{R}{1-a}\right]. \qquad (7.6)$$

and to determine the form and position of the shock wave $z_0 = l\,(r_0)$ we have

$$\frac{dl}{dr_0}=\frac{ar_0}{r_0(1-a)\,\mathrm{tg}\,\theta_0+\delta-l}. \qquad (7.7)$$

Introducing as an independent variable

$$t_1=\frac{dl}{dr_0},$$

by differentiating equation (7.7) we can obtain equation

$$\frac{1}{r_0}\frac{dr_0}{dt_1}=\frac{a}{t_1^3-t_1^2(1-a)\,\mathrm{tg}\,\theta_0+at_1},$$

which gives

$$r_0=C|t_1|\exp\left[-\int_0^{t_1}\frac{t_1-(1-a)\,\mathrm{tg}\,\theta_0}{t_1^2-t_1(1-a)\,\mathrm{tg}\,\theta_0+a}\,dt_1\right].$$

Let us determine the constant C by considering that, because of (7.7),

$$t_1=\frac{ar_0}{r_0(1-a)\,\mathrm{tg}\,\theta_0+\delta-l}, \qquad (7.8)$$

and assuming that $r_0 = 0$. As a result $C = \delta$.

Since

$$\theta_0<\theta_0^*=\mathrm{arctg}\,\frac{2\sqrt{a}}{1-a},$$

then the denominator under the integral sign does not have real roots and it is easy to obtain

$$r_0=\frac{\delta}{\sqrt{a}}\,\frac{|t_1|}{\sqrt{t_1^2-t_1(1-a)\,\mathrm{tg}\,\theta_0+a}}\times$$
$$\times\exp\left[\frac{(1-a)\,\mathrm{tg}\,\theta_0}{\sqrt{4a-(1-a)^2\,\mathrm{tg}^2\,\theta_0}}\,\mathrm{arctg}\,\frac{t_1\sqrt{4a-(1-a)^2\,\mathrm{tg}^2\,\theta_0}}{2a-t_1(1-a)\,\mathrm{tg}\,\theta_0}\right]. \qquad (7.9)$$

If we consider, as in the case of the flat cut, that a sonic velocity is attained at the point of the fissure $r_* = R$, then the magnitude δ is determined by the formula

$$\frac{\delta}{R}=\frac{\sqrt{a}\sqrt{t_*^2-t_*(1-a)\operatorname{tg}\theta_0+a}}{(1-a)|t_*|}\times$$
$$\times\exp\left[-\frac{(1-a)\operatorname{tg}\theta_0}{\sqrt{4a-(1-a)^2\operatorname{tg}^2\theta_0}}\operatorname{arctg}\frac{t_*\sqrt{4a-(1-a)^2\operatorname{tg}^2\theta_0}}{t_*(1-a)\operatorname{tg}\theta_0-2a}\right],$$

we can find t_* from the relationship

$$t_*=\frac{\nu a(1-a)^{\nu-1}\rho_T w_T\cos\theta_0}{1-\nu a(1-a)^{\nu-1}\rho_T w_T\sin\theta_0}.$$

Thus, equations (7.8) and (7.9) determine the form and position of the shock wave at .

Table 2 introduces the values of δ/R when $\varkappa$ = 1.4 for different angles θ_0 of the generators of a cone (ν = 2) and a wedge (ν = 1).

Table 2

θ_0		0°	5°	10°	15°	20°	25°	30°	35°	40°
$\frac{\delta}{R}$	ν = 1	0,871	0,775	0.686	0,600	0,515	0,426	0,338	0,248	0,140
	ν = 2	0,476	0,382	0,308	0,222	0,148	0,074	0,012	—	—

§ 8. Flow Around Spheres and Ellipsoids

Let us consider flow around spheres and ellipsoids that are given in the system z, r by the equation

$$f=\alpha\left[1-\sqrt{1-\frac{r_s^2}{\beta^2}}\right]=\alpha-\Delta\sqrt{\beta^2-r_s^2}, \qquad (8.1)$$

where α and β are semiaxes and $\delta=\frac{\alpha}{\beta}=\frac{1}{\varepsilon}$; when α = β = 1 we will obtain a sphere.

To determine the form of the shock wave we have the equation

$$\frac{dl}{dr_0}=\frac{ar_0}{\alpha-\Delta\sqrt{\beta^2-(1-a)^2r_0^2}+\delta-l} \qquad (8.2)$$

with the initial condition $l\ (0) = 0$.

Let us introduce the parameter

$$t=\frac{dl/dr_0}{df/dr_0}=\frac{ar_0}{f+\delta-l}\,\frac{\sqrt{\beta^2-(1-a)^2r_0^2}}{\Delta(1-a)r_0}. \qquad (8.3)$$

Then equation (8.2) leads to an equation with separating variable

$$\frac{dt}{t\left[t^2-(1-a)t-t^2a\right]}+\frac{(1-a)^2r_0dr_0}{a\iota^2\left[\beta^2-(1-a)^2r_0^2\right]}=0 \qquad (8.4)$$

with the initial condition

$$t(0)=\frac{a\beta}{\delta\Delta(1-a)}. \qquad (8.5)$$

Integrating (8.4), we obtain the implicit equation

$$k_0\ln\frac{(\alpha+\delta-l)-\Delta(1+k_1)\sqrt{\beta^2-(1-a)^2r_0^2}}{(\alpha+\delta-l)-\Delta(1-k_2)\sqrt{\beta^2-(1-a)^2r_0^2}}\cdot\frac{\delta+\Delta k_2\beta}{\delta-\Delta k_1\beta}=$$
$$=\ln\frac{\frac{a}{(1-a)^2}\left[\beta^2-(1-a)^2r_0^2\right]+\Delta(\alpha+\delta-l)\sqrt{\beta^2-(1+a)^2r_0^2}-(\alpha+\delta-l)^2}{\frac{a\beta^2}{(1-a)^2}-\Delta\beta\delta-\delta^2}, \qquad (8.6)$$

where

$$k_0=\frac{\Delta(1-a)}{\sqrt{\Delta^2(1-a)^2+4a}},$$
$$k_{1,2}=\frac{2a}{\Delta(1-a)\left[\sqrt{\Delta^2(1+a)^2+4a}\pm\Delta(1-a)\right]}.$$

To determine δ let us use equation (5.1) at a sonic point in the shock wave:

$$\mathring{r}_0 l''(\mathring{r}_0)=-(\nu-1)\varkappa\left(\frac{\varkappa-1}{\varkappa+1}\right)^{5/2}.$$

Differentiating equation (8.2), we will find $l''(\mathring{r}_0)$:

$$r_0^{\bullet} l'' (r_0^{\bullet}) = \frac{l_*'}{a} \left[a + l_*'^2 - \frac{\Delta (1-a)^2 r_0^{\bullet}}{\sqrt{\beta^2 - (1-a)^2 r_0^{\bullet 2}}} l_*' \right].$$

Bearing in mind that at a sonic point in the shock wave

$$l_*'^2 = \frac{\varkappa - 1}{\varkappa + 1},$$

from these two relationships we can find $r_0^{\bullet}$ and, writing (8.6) at a sonic point, we will obtain an equation for determining δ:

$$k_0 \ln \frac{\delta - \Delta k_1 \beta}{\delta + \Delta k_2 \beta} - \ln \left[\delta^2 + \Delta \beta \delta - \frac{a \beta^2}{(1-a)^2} \right] = k_0 \ln \frac{a r_0^{\bullet} - l_*' \Delta k_1 \sqrt{\beta^2 - (1-a)^2 r_0^{\bullet 2}}}{a r_0^{\bullet} + l_*' \Delta k_2 \sqrt{\beta^2 - (1-a)^2 r_0^{\bullet 2}}} - \ln \left[\frac{a}{l_*'^2} r_0^{\bullet 2} + \frac{a r_0^{\bullet}}{l_*'} \Delta \sqrt{\beta^2 - (1-a)^2 r_0^{\bullet 2}} - \frac{a}{(1-a)^2} \left(\beta^2 - (1-a)^2 r_0^{\bullet 2} \right) \right]. \quad (8.7)$$

Shock waves constructed from formulas (8.6) and (8.7) for a sphere and ellipsoids (ε = 0.5 and ε = 1.5) are shown on Fig. 4 together with accurate curves taken from [25]. Comparable values of δ are introduced in Table 3.

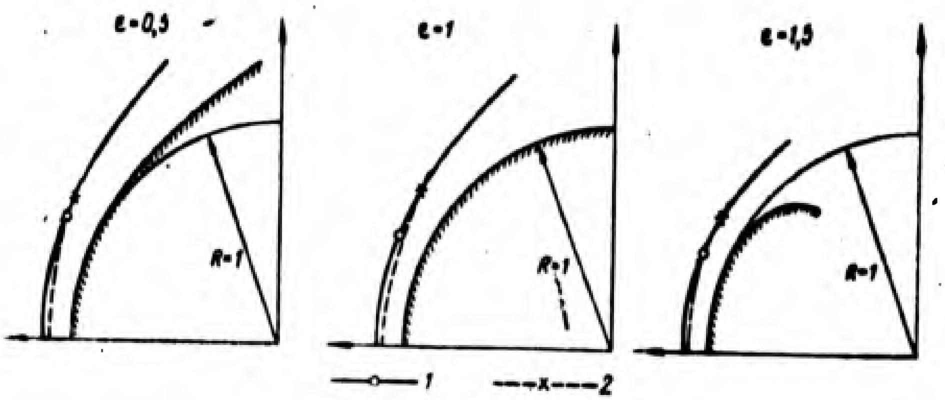

Fig. 4. The shock wave for a sphere and ellipsoids (O, X are sonic points: 1) [25]; 2) (8.6 – 8.7)).

Table 3

a		0,5	1	1,5
δ	from (8,6–8,7)	0,104	0,099	0,094
	from [25]	0,136	0,128	0,119

Let us examine another spherical segment with a semiaperature angle θ^{*}. In this case the shock wave is determined by equation (8.6) with $\alpha = \beta = 1$. To determine δ let us use the condition (5.2) of achieving sonic velocity at the corner point of the segment $r_0^{*} = \frac{\sin\theta_*}{1-a}$. As a result we will obtain the implicit equation

$$\left[\frac{\sin\theta_* - \frac{1}{a}\cos\theta_*\,\mathrm{tg}\,\varphi_*}{\sin\theta_* + \cos\theta_*\,\mathrm{tg}\,\varphi_*}\,\frac{\delta(1-a)+1}{\delta(1-a)-a}\right]^{\frac{1-a}{1+a}} = \frac{\frac{a}{1-a}\sin\theta_*\cos\theta_*\,\mathrm{ctg}\,\varphi_* + \frac{a^2}{(1-a)^2}\sin^2\theta_*\,\mathrm{ctg}^2\varphi_* - \frac{a}{(1-a)^2}\cos^2\theta_*}{\delta^2 - \delta - \frac{a}{(1-a)^2}}.$$

The dependence $\delta = \delta(\theta_*)$ is given in Table 4.

Table 4

θ_*	5°	10°	15°	20°	25°	30°	35°	40°	45°
δ	0,0395	0,0698	0,0885	0,105	0,121	0,128	0,132	0,134	0,138

Conclusion

1. The investigated simplest variant of the method of transverse approximation is very simple, but it is still insufficiently accurate. Nevertheless, in certain cases (§ 6) even in this form it gives a very good result.

2. The best combination of simplicity and accuracy can evidently be attained at the level of the quadratic approximation by writing, for a determination of the coefficients, two equations for the shock wave and two for the body (one from (2.14)).

3. The expediency and the prospect of generalizing this method for real gases, nonequilibrium flows, finite Mach numbers and a non-symmetric flow-around are evident.

Summary

The direct problem of hypersonic flow about blunt-nose bodies is solved by the method of apriori Ansatz of stream function dependence upon the transversal coordinate counted along the normal to the shock wave. The case of the linear approximation is considered in detail.

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