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EFFECT OF ECCENTRICITY OF STIFFENERS ON THE GENERAL INSTABILITY OF STIFFENED CYLINDRICAL SHELLS UNDER TORSION

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Haifa, Israel.

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ERRATA

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Page 2, second line of Eq. (1) should be *

$$+ \Big\{ - \underline{[(1/R^2)M_{x,x}]}_{,x} - (N_\phi/R) - \underline{[(1/R^2)M_{\phi,\phi}]}_{,\phi} + [(1/R^2)M_{x\phi,x\phi}] -$$

Page 2, second line of Eq. (2) should be *

$$+ [M_{x,xx} + M_{\phi,\phi\phi} - \underline{M_{x\phi,x\phi}} + M_{\phi x,x\phi} + RN_\phi + RN_{xo} W_{,xx}$$

Page 4, the last line of Eq. (6) should be *

$$+ \int_0^{2\pi} [N_x \delta u + N_{x\phi}\delta v - M_x \delta (w_{,x}/R)]_{x=o}^{x=(L/R)} \underline{R^2 \, d\phi}$$

* The corrected terms are underlined

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ABSTRACT

A method of analysis of eccentrically stiffened cylindrical shells, developed earlier, is applied to buckling under torsion. In the calculations about 350 shells covering a wide range of shell and stiffener geometries have been considered. It is found that for long cylinders rings are the important stiffeners, and internal rings stiffen more than external rings. For short cylinders the effect is inverted, external rings yielding higher critical loads than internal rings. Stringers have little effect in long cylinders, but in short cylinders they stiffen the shell against buckling in torsion as much as rings. Outside stringers always stiffen the shell more than inside stringers.

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SYMBOLS

A_n	=	coefficient of axial displacement
A_1	=	cross-sectional area of stringer
A_2	=	cross-sectional area of frame (ring)
a	=	distance between frames (rings)
$a_n = (D_{1n}/D_{0n}) = (A_n/C_n)$	=	defined by Eqs.(11)
B_n	=	coefficient of circumferential displacement
b	=	distance between stringers
$b_n = (D_{2n}/D_{0n}) = (B_n/C_n)$	=	defined by Eqs.(11)
C_n	=	coefficient of radial displacement
D	=	$[Eh^3/12(1-\nu^2)]$
D_{0n}, D_{1n}, D_{2n}	=	expressions defined by Eqs.(13)
E, E_1, E_2	=	moduli of elasticity of shell, stringers and frames, respectively
E_n	=	defined by Eqs. (8) and (12)
e_1, e_2	=	distance between centroid of stiffener cross-section and middle surface shell, positive when inside (see Fig.1)
$F(n)$	=	expression defined by Eq.(16)
F_n	=	defined by Eqs. (8) and (12)
G_1, G_2	=	shear moduli of stringers and frames, respectively
h	=	thickness of shell

I_{11}, I_{22}	=	moment of inertia of stiffener cross-section about its centroidal axis
I_{01}, I_{02}	=	moment of inertia of stiffener cross-section about the middle surface of the shell
I_{t1}, I_{t2}	=	torsion constant of stiffener cross-section
L	=	length of shell between bulkheads
$M_x, M_\phi, M_{x\phi}$	=	moment resultants acting on element
m	=	integer
$N_x, N_\phi, N_{x\phi}$	=	membrane force resultants acting on element
n	=	integer
$N_{x0}, N_{\phi 0}, N_{x\phi 0}$	=	prebuckling membrane force resultants
p	=	hydrostatic pressure
R	=	radius of shell
T	=	torque applied on the bulkheads
t	=	number of circumferential waves
u	=	non-dimensional axial displacement ($=u^*/R$)
u^*	=	axial displacement
v	=	non-dimensional circumferential displacement ($=V^*/R$)
v^*	=	circumferential displacement
w	=	non-dimensional radial displacement ($=w^*/R$)
w^*	=	radial displacement

x	= non-dimensional axial co-ordinate ($=x^*/R$)
x^*	= axial co-ordinate
Z	= $(1-\nu^2)^{1/2}\ (L/R)^2\ (R/h)$
z	= non-dimensional radial co-ordinate ($=z^*/R$)
z^*	= radial co-ordinate
β	= $\Pi R/L$
δ_{mn}	= kronecker delta ($\delta_{mn} = 0$ $m \neq n$) ($\delta_{mn} = 1$ $m = n$)
ϵ_x ϵ_ϕ $\gamma_{x\phi}$	= middle surface strains
ζ_1	= $(E_1A_1e_1R/bD)$
ζ_2	= $(E_2A_2e_2R/aD)$
η_{01}	= (E_1I_{01}/bD)
η_{02}	= (E_2I_{02}/aD)
η_{t1}	= (G_1I_{t1}/bD)
η_{t2}	= (G_2I_{t2}/aD)
κ_x κ_ϕ $\kappa_{x\phi}$	= non-dimensional changes of curvature and twist of the middle surface
λ_p	= $(R^3/D)p$
μ_1	= $(1-\nu^2)\,(E_1A_1/Ebh)$
μ_2	= $(1-\nu^2)\,(E_2A_2/Eah)$
μ_t	= $(T/2\pi D)$

ν	=	poisson's ratio
ϕ	=	circumferential co-ordinate
χ_1	=	$(1-\nu^2)\ (E_1A_1e_1/EbhR)$
χ_2	=	$(1-\nu^2)\ (E_2A_2e_2/EahR)$

Subscripts following a comma indicate differentiation

1. INTRODUCTION

In Ref. 1 Van Der Neut showed, for the case of buckling under axial compression, that the eccentricity of stiffeners with respect to the skin has great importance. He found that higher buckling loads are obtained with outside stiffening.

A simpler method of analysis which includes the effect of eccentricity of the stiffeners is given in Ref.2. This method has also been generalized for other types of shells (Ref.3) and has been employed for analysis of the general instability of stiffened circular conical shells under hydrostatic pressure (Refs.4 and 5).

Experimental evidence of the importance of the eccentricity of the stiffeners was first given by tests carried out at the College of Aeronautics, Cranfield (Ref.6) and more recently by the spectacular results of tests at the NASA Langley Research Center (Ref.7). The problem has recently also been studied by other investigators (Refs.8,9,10,11).

In Ref.2, some effects of the eccentricity of the stiffeners upon the critical load were studied, but since only a few cases were calculated, without variation in the shell geometry, some other effects were not noticed. During recent calculations of the general instability of conical shells with non-uniformly spaced stiffeners (Ref.5) the eccentricity effect of the rings was found to be inverted for some short and thick shells. The reason for this inversion was discussed there. Further study and calculations showed that inversion of the eccentricity effect is more general and appears also with other loads. The results of the study for the case of cylindrical shells under hydrostatic pressure are discussed in Ref. 12, and here the case of general instability of cylindrical shells under torsion is considered.

In the case of torsion it is found that for long cylinders rings are the important stiffeners, and internal rings stiffen more than external rings. For short cylinders the effect is inverted, external rings yielding higher critical loads than internal rings. Stringers have little effect in long cylinders, but in short cylinders they stiffen the shell against buckling in torsion as much as rings. Outside stringers always stiffen the shell more than inside stringers.

2. STABILITY EQUATIONS

For a cylindrical shell, Eq. (14) of Ref.3 becomes

$$o = \delta U = \int_0^{2\pi}\int_0^{(L/R)} \Big\{ (-N_{x,x} - N_{x\phi,\phi})\,\delta u + (-N_{\phi,\phi} - N_{x\phi,x})\,\delta v +$$

$$+ \{-[(1/R^2)M_{x,x}] - (N_\phi/R) - [(1/R^2)M_{\phi,\phi}] + [(1/R^2)M_{x\phi,x\phi}] -$$

$$- [(1/R^2)M_{\phi x,x\phi}] - [(1/R)(N_{x0}\,w_{,x})_{,x}] - [(1/R)(N_{\phi 0}\,w_{,\phi})_{,\phi}] -$$

$$- [(1/R)(N_{x\phi 0}\,w_{,\phi})_{,x}] - [(1/R)(N_{x\phi 0}\,w_{,x})_{,\phi}]\}\,R\delta w \Big\}\, R^2 dx d\phi +$$

$$+ \int_0^{2\pi} \{ [N_x R\delta u + N_{x\phi} R\delta v - M_x \delta(w_{,x})] + [(M_{x,x}/R) - (M_{x\phi,\phi}/R) +$$

$$+ (M_{\phi x,\phi}/R) + N_{x0} w_{,x} + N_{x\phi 0} w_{,\phi}]\, R\delta w \}\Big|_{x=0}^{x=(L/R)} R d\phi \qquad (1)$$

With the assumptions that the prebuckling forces N_{x0}, $N_{\phi 0}$ and $N_{x\phi 0}$ are not functions of the coordinates and that the admissible radial displacements vanish at the boundaries, Eq.(1) becomes

$$o = \delta U = -\int_0^{2\pi}\int_0^{(L/R)} \Big\{ (N_{x,x} + N_{x\phi,\phi})\delta u + (N_{\phi,\phi} + N_{x\phi,x})\delta v +$$

$$+ [M_{x,xx} + M_{\phi,\phi\phi} - M_{\phi x,x\phi} + M_{\phi x,x\phi} + RN_\phi + RN_{x0} w_{,xx} +$$

$$+ RN_{\phi 0} w_{,\phi\phi} + 2RN_{x\phi 0} w_{,x\phi}]\,(\delta w/R) \Big\}\, R^2 dx d\phi +$$

$$+ \int_0^{2\pi} [N_x \delta u + N_{x\phi}\delta v - (M_x/R)\,\delta(w_{,x})]\Big|_{x=0}^{x=(L/R)} R^2 d\phi \qquad (2)$$

It is assumed that when the cylindrical shell is loaded by hydrostatic pressure and torques acting on the bulkheads of the shell, the prebuckling stress state is satisfactorily represented by the membrane forces.

$$N_{x0} = -(pR/2)$$

$$N_{\phi 0} = -pR$$

$$N_{x\phi 0} = -(T/2\pi R^2) \qquad (3)$$

For pure torsion N_{x0} and $N_{\phi 0}$ are zero.

As in Ref.2 the internal forces and moments are

$$N_x = [Eh/(1-\nu^2)][u_{,x}(1+\mu_1)+\nu(v_{,\phi}-w)-\chi_1 w_{,xx}]$$

$$N_\phi = [Eh/(1-\nu^2)][(v_{,\phi}-w)(1+\mu_2)+\nu u_{,x}-\chi_2 w_{,\phi\phi}]$$

$$N_{x\phi} = [Eh/2(1+\nu)][u_{,\phi}+v_{,x}] \qquad (4)$$

and

$$M_x = (-D/R)[w_{,xx}(1+\eta_{01})+\nu w_{,\phi\phi}-\zeta_1 u_{,x}]$$

$$M_\phi = (-D/R)[w_{,\phi\phi}(1+\eta_{02})+\nu w_{,xx}-\zeta_2(v_{,\phi}-w)]$$

$$M_{x\phi} = (+D/R)[(1-\nu)+\eta_{t1}]w_{,x\phi}$$

$$M_{\phi x} = (-D/R)[(1-\nu)+\eta_{t2}]w_{,x\phi} \qquad (5)$$

By substitution of Eqs.(4) and (5) into Eq. (2) one obtains

$$\delta U = -\int_0^{2\pi}\int_0^{(L/R)}\Big\{[Eh/(1-\nu^2)]\{u_{,xx}(1+\mu_1)+[(1-\nu)/2]u_{,\phi\phi}+$$

$$+[(1+\nu)/2]v_{,x\phi}-\chi_1 w_{,xxx}-\nu w_{,x}\}\delta u+$$

$$+[Eh/(1-\nu^2)]\{[(1+\nu)/2]u_{,x\phi}+v_{,\phi\phi}(1+\mu_2)+[(1-\nu)/2]v_{,xx}-$$

$$-(1+\mu_2)\,w_{,\phi}-\chi_2\,w_{,\phi\phi\phi}\,]\,\delta v\,-$$

$$-(D/R^2)\,[\,\zeta_1(-u_{,xxx})+\zeta_2(2w_{,\phi\phi}-v_{,\phi\phi\phi})+(1+\eta_{01})\,w_{,xxxx}+$$

$$+(\,2+\eta_{t1}+\eta_{t2})\,w_{,xx\phi\phi}+(1+\eta_{02})\,w_{,\phi\phi\phi\phi}+$$

$$+12\,(R/h)^2\;\;[(1+\mu_2)(w-v_{,\phi})-\nu u_{,x}]+$$

$$+\lambda_p\,[\,(1/2)\,w_{,xx}+w_{,\phi\phi}\,]+2\mu_T\;w_{,x\phi}\,]\,\delta w\Big\}R^2dxd\phi\,+$$

$$+\int_0^{2\pi}[N_x\delta u+N_{x\phi}\delta v-M_x\delta(w_{,x}/R)\,]\,R^2dxd\phi\Big|_{x=0}^{x=(L/R)} \qquad (6)$$

where $\lambda_p=(R^3/D)\,p$

and $\mu_T=(T/2\pi D)$ (7)

3. DISPLACEMENTS, BOUNDARY CONDITIONS and SOLUTION of THE FIRST TWO STABILITY EQUATIONS

Following Ref. 13 the assumed admissible displacements are taken as

$$u=\sin t\phi\sum_{n=1,3,5\ldots}A_n\cos n\beta x+\cos t\phi\sum_{n=2,4,6\ldots}E_n\cos n\beta x$$

$$v=\cos t\phi\sum_{n=1,3,5\ldots}B_n\sin n\beta x+\sin t\phi\sum_{n=2,4,6\ldots}F_n\sin n\beta x$$

$$w=\sin t\phi\sum_{n=1,3,5\ldots}C_n\sin n\beta x+\cos t\phi\sum_{n=2,4,6\ldots}D_n\sin n\beta x \qquad (8)$$

The shell is assumed to be simply supported. The displacements, Eqs. (8), fulfil the following boundary conditions:

a) Geometrical boundary conditions

$$\left.\begin{array}{l} w = 0 \\ v = 0 \end{array}\right\} \text{ at } x = 0,\ (L/R) \tag{9}$$

b) Equilibrium boundary conditions

$$\left.\begin{array}{l} N_x = 0 \\ M_x = 0 \end{array}\right\} \text{ at } x = 0,\ (L/R) \tag{10}$$

Hence, with the displacements given by Eqs. (8), the line integrals at the boundaries which appear in Eq. (7) vanish.

Substitution of the displacements, Eqs. (8),into the first two stability equations yields:

$$A_n = (D_{1n}/D_{0n})\, C_n = a_n C_n$$

$$B_n = (D_{2n}/D_{0n})\, C_n = b_n C_n \tag{11}$$

and

$$E_n = (D_{1n}/D_{0n})\, D_n = a_n D_n$$

$$F_n = -(D_{2n}/D_{0n})\, D_n = -b_n D_n \tag{12}$$

where, as in Ref. 2,

$$D_{0n} = [(1-\nu)/2](1+\mu_2)t^4 + [(1+\mu_1)(1+\mu_2) - \nu]n^2\beta^2t^2 + (1+\mu_1)[(1-\nu)/2]n^4\beta^4$$

$$D_{1n} = -[(1+\nu)/2]\chi_2 n\beta t^4 + (1+\mu_2)\{\chi_1 n^3\beta^3 + [(1-\nu)/2]n\beta\}t^2 +$$

$$+ \chi_1 [(1-\nu)/2]n^5\beta^5 - \nu[(1-\nu)/2]n^3\beta^3$$

$$D_{2n} = [(1-\nu)/2]\chi_2 t^5 + \{(1+\mu_1)\chi_2 n^2\beta^2 - [(1-\nu)/2](1+\mu_2)\}t^3 +$$

$$+ \left\{\{[(1+\nu)/2]\nu - (1+\mu_1)(1+\mu_2)\}n^2\beta^2 - [(1+\nu)/2]\chi_1 n^4\beta^4\right\}t \tag{13}$$

The admissible displacements, Eqs. (8), have now become functions of C_n and D_n only

$$u = \sin t\phi \sum_{n=1,3,5\ldots} a_n \cos n\beta x\, C_n + \cos t\phi \sum_{n=2,4,6\ldots} a_n \cos n\beta x\, D_n$$

$$v = \cos t\phi \sum_{n=1,3,5\ldots} b_n \sin n\beta x\, C_n - \sin t\phi \sum_{n=2,4,6\ldots} b_n \sin n\beta x\, D_n$$

$$w = \sin t\phi \sum_{n=1,3,5\ldots} \sin n\beta x\, C_n + \cos t\phi \sum_{n=1,3,5\ldots} \sin n\beta x\, D_n \qquad (14)$$

4. SOLUTION

Substitution of the displacements, Eqs. (14), into Eq. (7) yields

$$0 = \int_0^{2\pi}\int_0^{(L/R)} \{ \sin t\phi \sum_{n=1,3,5\ldots} F(n) \sin n\beta x\, C_n + \cos t\phi \sum_{n=2,4,6\ldots} F(n) \sin n\beta x\, D_n$$

$$+ \mu_T [t \cos t\phi \sum_{n=1,3,5\ldots} n\beta \cos n\beta x\, C_n - t \sin t\phi \sum_{n=2,4,6\ldots} n\beta \cos n\beta x\, D_n] \}\cdot$$

$$\{\sin t\phi \sum_{m=1,3,5\ldots} \sin m\beta x\, \delta C_m + \cos t\phi \sum_{m=2,4,6\ldots} \sin m\beta x\, \delta D_m \}\, dx d\phi \qquad (15)$$

where

$$F(n) = \zeta_1(-n^3\beta^3 a_n) + \zeta_2(-2t^2 - b_n t^3)$$

$$+ (1+\eta_{01})n^4\beta^4 + (2+\eta_{t1}+\eta_{t2})n^2\beta^2 t^2 + (1+\eta_{02})t^4 +$$

$$+ 12(R/h)^2 [(1+\mu_2)(1+b_n t) + \nu n\beta a_n] - \lambda_p (0.5 n^2\beta^2 + t^2) \qquad (16)$$

Note that, since the line integrals at the boundaries vanish, Eqs. (15) are the Galerkin integrals of the problem.

Integration of Eq.(15), taking into account that δCm and δDm are arbitrary, yields a set of simultaneous algebraic equations:

$$(\pi/2\beta)\sum_{n=1,3,5\ldots} F(n)\,\delta_{nm} C_n - \mu_T 4t \sum_{n=2,4,6\ldots} [nm/(m^2-n^2)]\, D_n = 0 \quad (m=1,3,5\ldots)$$

$$\mu_T\, 4t \sum_{n=1,3,5\ldots} [nm/(m^2-n^2)]\, C_n + (\pi/2\beta)\sum_{n=2,4,6\ldots} F(n)\,\delta_{nm} D_n = 0 \quad (m=2,4,6\ldots) \qquad (17)$$

For any given pressure, the lowest eigenvalue μ_T of the determinant of the coefficients of C_n and D_n of Eqs.(17) yields the critical torque. For any given torque the lowest eigenvalue λ_p of the determinant yields the critical external pressure. For pure torsion $p = 0$ and therefore λ_p vanish. It should be noted that the value of t (the number of circumferential waves into which the shells buckle) which "minimizes" the critical load has to be used.

5. NUMERICAL RESULTS AND DISCUSSION

The critical torques have been calculated for 350 stiffened cylindrical shells covering a wide range of shell and stiffener geometries. The shell and stiffener geometries of 200 ring-stiffened shells considered in the main study are given in Table 1. In order to examine the effect of stiffener geometry (changes in (I_{22}/ah^3), (A_2/ah) and (e_2/h),) on the critical values of torque, and special behaviour of very short shells, 75 additional shells have been considered.

For comparison, another 14 stringer-stiffened shells, 16 combined ring and stringer-stiffened shells and 15 shells with equivalent thickened skin have been investigated.

In the computations, the effect of η_t on the critical values is usually neglected ($\eta_t = 0$ is assumed), but the effect of varying η_t has also been studied on 30 typical ring-stiffened shells.

In all cases, the critical torques for the stiffened shells have been compared with those for unstiffened shells of identical dimensions.

Geometries of the additional shells considered in the supplementary studies are given in Tables 2 to 5 and 7 to 11.

In the present discussion (L/R) ranges from 0.35 for very short shells to 3.0 for long ones. (R/h) varies from 100 to 2000. $(e_2/h) = 1$ is taken for a small eccentricity, 5 for a large eccentricity, as is discussed later. Two values of (I_{22}/ah^3) are used in the computations: 2 and 5, and (A_2/ah) is taken as 0.5 which is a realistic value from a design point of view. Note that the combination $(A_2/ah) = 0.5$, $(e_2/h = 5$ and $(I_{22}/ah^3) = 5$ represents a heavily stiffened shell with a large eccentricity of stiffener.

All computations have been carried out on the Elliot 803 and 503 computers of the Technion's computation center. Results are given in Tables 5 to 11.

In Table 6, the convergence of the solution, computed from Eq.(17), is shown for 24 typical cases. From Table 5 it is found that 193 cases converge within less than one percent, 46 cases converge within one percent up to less than three percent, and for 16 cases convergence was only obtained to within three to six percent. From the present calculations convergence of solution is found to depend upon location of stiffeners. Outside stiffeners always converge much faster than inside ones for which convergence depends very much upon the magnitude of the eccentricity. It also depends to a lesser extent upon the moment of inertia for a small eccentricity, and upon the geometry of the shell in the case of a large eccentricity. For example (See Table 6), for inside stiffeners convergence is fairly rapid for a small value of $(e/h) = 1$ and a small moment of inertia, $(I_{22}/ah^3) = 2$, a 5th order solution yields an accuracy of less than one percent. Note that an n^{th} order solution represents a solution with $(2n - 1)$ terms. For the same eccentricity but larger moment of inertia, $(I_{22}/ah^3) = 5$, convergence is obtained in many cases with a 7th order solution. In some cases, however, accuracy of solution is only within three percent. With high values of eccentricity, $(e/h) = 5$, 7th to 10th order solutions are employed. In some of these cases even 10th order solutions (19 terms) yield an accuracy within more than three percent and the convergence is very slow. For these high values of eccentricity the magnitude of moment of inertia hardly affects the convergence of solution. In this case convergence depends upon the geometry of the shell. It is slow in the following ranges: For short and thin shells $(L/R) = 0.5$ and $(R/h) = 1000, 2000$; for shells with medium values of length and thickness $(L/R) = 1.0, 1.5$ and $(R/h) = 250, 500, 1000$; and for long and thick shells $(L/R) = 2.0, 3.0$ and $(R/h) = 100$ (see table 6).

The essential geometry of an unstiffened cylindrical shell can be defined by the Batdorf parameter Ref. 14

$$Z = (1-\nu^2)^{1/2} (L/R)^2 (R/h).$$

In Ref. 12 the Batdorf parameter was found to be the parameter governing the inversion of the eccentricity effect for ring-stiffened shells under external pressure. The variation of the eccentricity effect with Z is therefore studied in the case of torsion. In Fig.3A the ratio of critical torque with outside rings to that with inside rings is plotted versus Z. Note that the poor convergence of some cases discussed above shows itself as scatter in Fig.3A. One may see from Table 5 that each point of the curves in Fig.3A represents many shells with various shell geometries having identical values of Z and the same configuration of stiffener. As in Ref.12, an inversion of the effect of eccentricity is found for rings, and the inversion is again determined by the shell geometry. A transition range of Z is found at $Z = 1000$ to $Z = 4000$ approximately, beyond which inside rings are always more effective than outside ones. In this high range of Z, beyond inversion, the ratio of critical torque for outside rings to that of inside rings is almost the same for all configurations of stiffeners. On the other hand, below the transition range of Z, outside rings are more effective than inside ones, and the ratio of critical torque for outside rings to that for inside rings depends very much upon the eccentricity (e/h). The ratio increases with eccentricity, and even exceeds the value of 2 for very large eccentricities. From Fig.3A it is found that this ratio has a maximum at a certain value of Z. This value of Z increases with increasing values of moment of inertia; $Z \cong 150$ for $(I_{22}/ah^3) = 2$ and $Z \cong 250$ for $(I_{22}/ah^3) = 5$. In Fig.3B the transition range of Fig.3A is shown enlarged.

Fig.4 is a study of the effect of stiffener area, or rather the parameter (A_2/ah) on the critical torque. It is found that initially the increase in area raises the critical torque appreciably, but for larger areas the curve flattens and there is almost no gain in critical torque.

$(A_2/ah) = 0.5$ used in the present calculations is found to be a reasonable value from this point of view.

The effect of moment of inertia (I_{22}/ah^3) on the critical torque is shown in Fig.5.

Fig.6 shows the effect of eccentricity (e/h). The values of critical torque are found to grow very rapidly with increasing eccentricity. It is seen that values of $(e/h) = 1$ and $(e/h) = 5$ are fair representations of variation of eccentricity.

Fig.7 examines the relative effect of ring and stringers on cylindrical shells under torsion. The rings and stringers presented have identical geometries. It is found that for short shells, $Z < 60$, stringers are more efficient than rings. This figure also shows the effect of equivalent thickening of the skin, which is thickening the skin instead of stiffening it with the same total amount of material, or in other words, with the weight being kept constant. It is found that for long cylinders equivalent thickening yields better results than stringer stiffened shells, but obviously ring stiffened shells are much more efficient. For very short shells equivalent thickening yields higher critical torques than rings with small eccentricity $(e/h) = 1$, but stringers should be more efficient in this range.

In order to study the effect of equivalent thickening of skin in detail, a supplementary study has been carried out. In Table 9 a comparison of the relative effect of equivalent thickening of skin to that of ring stiffening is presented. It is again found from this table that for very short cylinders, or more precisely for cylinders having a small value of Z $(Z < 15)$, equivalent thickening is more efficient. In Table 10 a comparison is made for stringer stiffening and the conclusion, that for long cylinders, or rather cylinders having a large value of Z $(Z > 850)$, equivalent thickening yields higher critical torques, is verified.

In Table 8 the effect of combined rings and stringers is studied and compared with separate rings and stringers. It is concluded from this table and Fig.3 that also for combined stiffening the highest critical torques are obtained in all cases for outside stringers as for other kinds of load, while the optimum location of rings depends upon the parameter Z. From the present computations it is also found that highest values obtained for a combination of rings and stringers are always larger than the sum of the critical values for each kind of stiffener separately.

Finally, the increase in torsional rigidity represented by η_{t2} is considered for the case of ring stiffeners. The critical torques obtained are given in Table 11. In Fig.8 these values are compared with those yielded for identical shells, but with $\eta_{t2} = 0$. It is seen from this figure that when the shell buckles in many circumferential waves, or more precisely when $t \geq 6$ (cases – 176,186), there is almost no gain in critical torque due to η_{t2}, while for cases with a small wave number, $t \leq 5$ (cases – 158,173,183), the critical torque is affected by the torsional rigidity when η_{t2} is large. In Fig.2 three different shapes of cross sections are shown which have the same total stiffener weight and identical stiffener parameters $(A_2/ah) = 0.5$, $(I_{22}/ah^3) = 5$ and $(e_2/h) = 5$. The torsional rigidity has been varied to examine possible realistic values of η_{t2}. It is seen that values of η_{t2} in excess of about 50 are hardly feasible, and values of η_{t2} above 10 can be achieved only with closed cross sections. Hence η_{t2} need not be taken into account for stiffeners of open cross section usually used in practice. Only when a shell is stiffened with stiffeners of closed cross section and buckles with less than 5 circumferential waves the torsional rigidity has to be included in the analysis. It is also found from Table 11 that when η_{t2} is taken into account, the critical torque occurs with the same or a smaller wave number than when $\eta_{t2} = 0$ is assumed.

6. CONCLUSIONS

The analysis shows that for buckling under torsion the eccentricity of the stiffeners is an important factor. For long shells rings are more efficient stiffeners than stringers and internal rings stiffen more than external ones. There is an inversion of the effect of the eccentricity of rings in the range of $1000 < Z < 4000$, and for short cylinders external rings yield considerably higher critical loads than internal ones. Stringers have little effect in long cylinders, but in short cylinders they stiffen the shell against buckling in torsion as much as rings. Outside stringers always stiffen the shell more than inside ones.

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TABLE 1:

GEOMETRIES CONSIDERED IN MAIN STUDY*

L/R	0.5	1.0	1.5	2.0	3.0
R/h	100	250	500	1000	2000
$\pm(e_2/h)$	1	5			
A_2/ah	0.5				
I_{22}/ah^3	2	5			

(*) 200 possible variations of geometry have been chosen.

TABLE 2:

VARIATIONS OF STIFFENER GEOMETRY STUDIED ON RING STIFFENED SHELLS

L/R	1.0	3.0				
R/h	1000					
$\pm(e_2/h)$	1	3	5	8		for $A_2/ah = 0.5$; $I_{22}/ah^3 = 5$
A_2/ah	0.1	0.5	1.0	1.5		for $\pm(e_2/h) = 5$; $I_{22}/ah^3 = 5$
I_{22}/ah^3	1	2	3	5	8	for $\pm(e_2/h) = 5$; $A_2/ah = 0.5$

TABLE 3:

SHELL AND STIFFENER GEOMETRIES FOR STRINGER STIFFENED SHELLS

$\pm(e_2/h)$	5	R/h	100	L/R	0.5	0.7	1.5	3.0
A_2/bh	0.5							
I_{22}/bh^3	5	R/h	1000	L/R	1.0	3.0		

TABLE 4:

SHELL AND STIFFENER GEOMETRIES FOR SHELLS STIFFENED WITH RINGS AND STRINGERS

L/R	1.0 3.0	$\pm(e_1/h)$	$\pm(e_2/h)$	I_{11}/bh^3	I_{22}/ah^3	A_1/bh	A_2/ah
R/h	100 1000	5	5	5	5	0.5	0.5

TABLE 5 :

CRITICAL TORQUES AND TORQUE RATIOS OF RING STIFFENED SHELLS

TORCY CASE No.	GEOMETRY OF SHELL L/R	R/h	Z	GEOMETRY OF STIFFENER e_2/h	A_2/ah	I_{22}/ah^3	n order of solution	(μ_T) INSIDE	t	n order of solution	(μ_T) OUTSIDE	t	(μ_T) UNSTIFF.	t	T OUTSIDE / T INSIDE	T INSIDE / T UNSTIFF	T OUTSIDE / T UNSTIFF
1 and 28	0.5	100	23.85	1	0.5	2	5	1093	7	5	1336	7	438.7	11	1.223	2.491	3.046
2 and 29		250	59.63				5	2361	10	5	2905	10	764.9	15	1.231	3.087	3.798
3 and 30		500	119.3				5	4519**	14	5	5475	13	1234	20	1.212	3.663	4.438
4 and 31		1000	238.5				7	8810	19	5	10320	17	2041	25	1.171	4.316	5.055
5 and 32		2000	477.0				5	17590**	23	5	19140	22	3418	31	1.089	5.145	5.601
6 and 33	1.0	100	95.40				5	914.8*	6	5	1114	6	263.5	9	1.218	3.472	4.228
7 and 34		250	238.5				7	2203	10	5	2583	9	511.7	13	1.172	4.306	5.048
8 and 35		500	477.0				5	4412*	11	5	4786	11	854.7	16	1.085	5.162	5.600
9 and 36		1000	954.0				5	8441	14	5	8695	13	1437	19	1.030	5.872	6.049
10 and 37		2000	1908				5	15690	17	5	15500	16	2419	24	0.987	6.487	6.406
13 and 38	1.5	100	214.7				5	896.1***	6	5	1050	6	209.9	8	1.171	4.268	5.000
14 and 39		250	536.6				5	2188 **	8	5	2370	7	414.7	11	1.083	5.276	5.715
15 and 40		500	1073				5	4182	10	5	4268	9	698.7	13	1.020	5.986	6.109
16 and 41		1000	2147				5	7738	11	5	7587	11	1176	16	0.980	6.583	6.454
17 and 42		2000	4293				5	13920	14	5	13310	13	1979	20	0.956	7.032	6.720
18 and 43	2.0	100	381.6				5	893.0	5	5	982.7	5	181.6	7	1.100	4.916	5.410
19 and 44		250	954.0				5	2110 *	7	5	2182	7	359.7	10	1.034	5.867	6.066
20 and 45		500	1908				5	3931	8	5	3874	8	604.8	12	0.986	6.499	6.406
21 and 46		1000	3816				5	7088	10	5	6817	10	1020	14	0.962	6.949	6.683
22 and 47		2000	7632				5	12550	12	5	11840	12	1717	17	0.943	7.310	6.894
23 and 48	3.0	100	858.6				5	864.0	5	5	888.5	4	148.1	6	1.028	5.836	6.001
24 and 49		250	2147				5	1935	6	5	1914	6	293.9	8	0.989	6.584	6.512
25 and 50		500	4293				5	3479	7	5	3337	7	494.8	10	0.959	7.032	6.744
26 and 51		1000	8586				5	6133	8	5	5764	8	833.3	12	0.940	7.361	6.918
27 and 52		2000	17170				5	10700	10	5	9941	10	1404	14	0.929	7.620	7.079
53 and 78	0.5	100	23.85	5	0.5	2	5	1400	5	5	2656	5	438.7	11	1.898	3.190	6.054
54 and 79		250	59.63				7	3100	8	5	6233 *	7	764.9	15	2.011	4.052	8.149
55 and 80		500	119.3				7	5912	11	5	12420 *	10	1234	20	2.100	4.792	10.07
56 and 81		1000	238.5				9	11550	15	5	24190	13	2041	25	2.094	5.659	11.85
57 and 82		2000	477.0				10	23000 *	21	5	45130	17	3418	31	1.962	6.729	13.21
58 and 83	1.0	100	95.40				7	1199	5	7	2477	5	263.5	9	2.065	4.552	9.401
59 and 84		250	238.5				8	2921 *	8	5	6058 *	7	511.7	13	2.074	5.709	11.84
60 and 85		500	477.0				9	5913 **	10	5	11330	8	854.7	16	1.916	6.918	13.25
61 and 86		1000	954.0				10	12460 ***	14	5	20510	10	1437	19	1.647	8.665	14.27
62 and 87		2000	1908				8	30120 **	18	5	36330	13	2419	24	1.206	12.450	15.02
63 and 88	1.5	100	214.7				9	1163	5	5	2451	4	209.9	8	2.108	5.538	11.68
64 and 89		250	536.6				10	2934 *	7	5	5563	6	414.7	11	1.896	7.074	13.42
65 and 90		500	1073				10	6339***	10	5	10050	7	698.7	13	1.585	9.073	14.38
66 and 91		1000	2147				7	15760 **	12	5	17780	9	1176	16	1.128	13.41	15.12
67 and 92		2000	4293				5	32050	12	5	31080	11	1979	20	0.970	16.20	15.70
68 and 93	2.0	100	381.6				10	1150	5	5	2317	4	181.6	7	2.013	6.330	12.75
69 and 94		250	954.0				10	3114***	7	5	5128	5	359.7	10	1.647	8.658	14.26
70 and 95		500	1908				7	7867 *	8	5	9152	6	604.8	12	1.163	13.01	15.13
71 and 96		1000	3816				5	16180	9	5	15910	8	1020	14	0.983	15.87	15.60
72 and 97		2000	7632				5	30220	10	5	27670	10	1717	17	0.916	17.60	16.12
73 and 98	3.0	100	858.6				9	1336 **	4	5	2125	3	148.1	6	1.591	9.021	14.35
74 and 99		250	2147				9	3740 *	6	5	4524	4	293.9	8	1.210	12.73	15.40
75 and 100		500	4293				5	8014 **	6	5	7847	5	494.8	10	0.979	16.20	15.86
76 and 101		1000	8586				5	14880	7	5	13530	7	833.3	12	0.909	17.86	16.24
77 and 102		2000	17170				5	26740	8	5	23090	8	1404	14	0.86[illegible]	19.04	16.44

TABLE 5 (contd.) CRITICAL TORQUES AND TORQUE RATIOS OF RING STIFFENED SHELLS

TORCY CASE No.	GEOMETRY OF SHELL L/R	R/h	Z	GEOMETRY OF STIFFENER e_2/h	A_2/eh	I_{22}/eh^3	order of solution n	(μ_T) INSIDE	t	order of solution n	(μ_T) OUTSIDE	t	(μ_T) UNSTIFF.	t	T OUTSIDE / T INSIDE	T INSIDE / T UNSTIFF	T OUTSIDE / T UNSTIFF
103 and 128	0.5	100	23.85	1	0.5	5	5	1444	6	5	1699	6	438.7	11	1.176	3.292	3.873
104 and 129		250	59.63				5	3217 **	9	5	3835 *	9	764.9	15	1.192	4.206	5.014
105 and 130		500	119.3				7	6236	12	7	7458	12	1234	20	1.197	5.050	6.045
106 and 131		1000	238.5				7	12350*	17	5	15070	17	2041	25	1.220	6.050	7.384
107 and 132		2000	477.0				7	25310*	23	5	28800	20	3418	31	1.138	7.404	8.426
108 and 133	1.0	100	95.40				7	1268	6	5	1522 *	5	263.5	9	1.200	4.814	5.776
109 and 134		250	238.5				7	3096 *	9	5	3720 **	8	511.7	13	1.202	6.050	7.269
110 and 135		500	477.0				7	6357	12	5	7199	10	854.7	16	1.132	7.438	8.423
111 and 136		1000	954.0				5	12940 *	13	5	13510	12	1437	19	1.044	9.005	9.400
112 and 137		2000	1908				5	24450	15	5	24670	15	2419	24	1.009	10.11	10.20
113 and 138	1.5	100	214.7				7	1254	6	5	1490 **	5	209.9	8	1.188	5.975	7.098
114 and 139		250	536.6				6	3256	8	5	3571 *	7	414.7	11	1.124	7.663	8.612
115 and 140		500	1073				5	6430	9	5	6674	8	698.7	13	1.038	9.208	9.553
116 and 141		1000	2147				5	12090	10	5	12100	10	1176	16	1.001	10.28	10.29
117 and 142		2000	4293				5	21980	12	5	21530	12	1979	20	0.980	11.11	10.98
118 and 143	2.0	100	381.6				7	1263	5	5	1478 *	5	181.6	7	1.170	6.952	8.134
119 and 144		250	954.0				5	3246 *	6	5	3378	6	359.7	10	1.041	9.025	9.391
120 and 145		500	1908				5	6131	8	5	6171	7	604.9	12	1.007	10.14	10.20
121 and 146		1000	3816				5	11180	9	5	11010	9	1020	14	0.984	10.96	10.79
122 and 147		2000	7632				5	19990	11	5	19370	11	1717	17	0.969	11.64	11.28
123 and 148	3.0	100	858.6				5	1301 **	4	5	1368	4	148.1	6	1.051	8.785	9.236
124 and 149		250	2147				5	3022	5	5	3025	5	293.9	8	1.001	10.28	10.30
125 and 150		500	4293				5	5495	6	5	5382	6	494.8	10	0.980	11.11	10.88
126 and 151		1000	8586				5	9791	7	5	9445	7	833.3	12	0.965	11.75	11.34
127 and 152		2000	17170				5	17140	9	5	16390	9	1404	14	0.956	12.20	11.67
153 and 178	0.5	100	23.85	5	0.5	5	7	1568	5	7	2808	5	438.7	11	1.791	3.573	6.400
154 and 179		250	59.63				7	3441	7	7	6522	7	764.9	15	1.895	4.498	8.526
155 and 180		500	119.3				7	6614	10	7	12890 *	10	1234	20	1.949	5.364	10.45
156 and 181		1000	238.5				8	13010 *	14	6	26680	13	2041	25	2.051	6.373	13.07
157 and 182		2000	477.0				10	25880 **	20	7	50670	16	3418	31	1.958	7.571	14.83
158 and 183	1.0	100	95.40				7	1361	5	7	2613	4	263.5	9	1.920	5.167	9.919
159 and 184		250	238.5				9	3245	7	6	6682	7	511.7	13	2.061	6.337	13.06
160 and 185		500	477.0				10	6469 **	10	7	12670	8	854.7	16	1.958	7.569	14.82
161 and 186		1000	954.0				10	14180 ***	14	7	23290	10	1437	19	1.642	9.866	16.20
162 and 187		2000	1908				9	33200 **	18	7	41890	13	2419	24	1.262	13.72	17.30
163 and 188	1.5	100	214.7				9	1321	5	7	2601	4	209.9	8	1.968	6.295	12.39
164 and 189		250	536.6				10	3264 **	7	7	6286	6	414.7	11	1.922	7.870	15.16
165 and 190		500	1073				10	7280 ***	10	7	11450	7	698.7	13	1.572	10.42	16.38
166 and 191		1000	2147				10	16280 **	13	7	20540	9	1176	16	1.261	13.85	17.47
167 and 192		2000	4293				7	36630	12	7	36180	11	1979	20	0.987	18.51	18.28
168 and 193	2.0	100	381.6				9	1347 **	5	5	2605 *	4	181.6	7	1.934	7.416	14.34
169 and 194		250	954.0				10	3545 ***	7	7	5823	5	359.7	10	1.642	9.857	16.19
170 and 195		500	1908				6	9304 *	8	7	10470	6	604.9	12	1.174	14.75	17.32
171 and 196		1000	3816				7	18370	10	7	18510	8	1020	14	1.008	18.01	18.14
172 and 197		2000	7632				7	34550	10	7	32190	9	1717	17	0.932	20.12	18.75
173 and 198	3.0	100	858.6				10	1423 **	4	7	2390	3	148.1	6	1.679	9.614	16.14
174 and 199		250	2147				10	4098 **	6	7	5166	4	293.9	8	1.260	13.95	17.58
175 and 200		500	4293				7	9158	6	7	9051	5	494.8	10	0.988	18.51	18.29
176 and 201		1000	8586				7	17040	7	7	15780	6	833.3	12	0.926	20.45	18.94
177 and 202		2000	17170				7	30610	8	7	27130	8	1404	14	0.886	21.80	19.32

TABLE 5.(contd.) **CRITICAL TORQUES AND TORQUE RATIOS OF RING STIFFENED SHELLS**

TORCY CASE No.	GEOMETRY OF SHELL L/R	R/h	Z	GEOMETRY OF STIFFENER e_2/h	A_2/eh	I_{22}/eh^3	order of solution	(μ_T) INSIDE	t	order of solution	(μ_T) OUTSIDE	t	(μ_T) UNSTIFF.	t	T OUTSIDE / T INSIDE	T INSIDE / T UNSTIFF	T OUTSIDE / T UNSTIFF
203 and 221	1.0	1000	954.0	3	0.5	5	8	13650 **	14	5	17330	11	1437	19	1.229	9.494	12.05
204 and 222	3.0		8586				5	12650	7	5	11730	7	833.3	12	0.927	15.18	14.08
205 and 223	1.0	1000	954.0	8	0.5	5	10	16820 ***	12	5	34750	9	1437	19	2.065	11.70	24.18
206 and 224	3.0		8586				5	25050	6	5	23630	6	833.3	12	0.943	30.07	28.36
207 and 225	1.0	1000	954.0	5	0.1	5	10	13090 **	15	5	15890	12	1437	19	1.214	9.104	11.06
208 and 226					1.0		10	14550 ***	13	5	28250	10	1437	19	1.941	10.12	19.66
209 and 227					1.5		10	14750 ***	13	5	31030	9	1437	19	2.103	10.26	21.60
210 and 228	3.0	1000	8586	5	0.1	5	5	11710	7	5	11280	7	833.3	12	0.963	14.05	13.53
211 and 229					1.0		5	20110	7	5	18360	6	833.3	12	0.913	24.14	22.03
212 and 230					1.5		5	21740	7	5	19810	6	833.3	12	0.911	26.10	23.77
213 and 231	1.0	1000	954.0	5	0.5	1	10	11850 ***	15	5	19400	11	1437	19	1.637	8.246	13.50
214 and 232						3	10	13030 ***	14	5	21520	10	1437	19	1.651	9.066	14.98
215 and 233						8	10	15640 ***	13	5	26020	10	1437	19	1.664	10.88	18.11
216 and 234						10	10	16560 ***	13	5	27620	10	1437	19	1.668	11.52	19.23
217 and 235	3.0	1000	8586	5	0.5	1	5	14090	7	5	12670	7	833.3	12	0.899	16.91	15.21
218 and 236						3	5	15640	7	5	14360	7	833.3	12	0.918	18.77	17.24
219 and 237						8	5	19140	7	5	17860	6	833.3	12	0.933	22,96	21.43
220 and 238						10	5	20400	6	5	19150	6	833.3	12	0.939	24.49	22.98
239 and 242	1.0	1000	954.0	3	1.0	5	10	12870	15	5	19550	11	1437	19	1.519	8.952	13.61
240				5			10	14550 ***	13	–		–	1437	19	–	10.12	–
241 and 245				5	1.5	3	10	13840 ***	13	5	29390	9	1437	19	2.124	9.627	20.46
244					–	5	–		–	5	31030	9	1437	19	–	–	21.60
246 and 252	0.4	100	15.26	5	0.5	2	5	1562	5	5	2858	5	551.8	12	1.830	2.831	5.179
247 and 253	0.35	100	11.69				5	1710	5	5	3047	5	642.7	12	1.782	2.661	4.742
248 and 254	0.4		15.26			5	5	1737	5	5	3010	5	551.8	12	1.733	3.147	5.454
249 and 255	0.35		11.69				5	1887	5	5	3196	5	642.7	12	1.694	2.936	4.973
250 and 256	0.4		15.26	1		2	5	1226	7	5	1490	7	551.8	12	1.216	2.221	2.699
251 and 257						5	5	1595	6	5	1863	6	551.8	13	1.168	2.890	3.375

No asterix - accuracy of convergence within less than one percent.
* accuracy of convergence within one to two percent.
** " " " " two to three "
*** " " " " three to six "

TABLE 6

CONVERGENCE OF SOLUTION FOR TYPICAL CASES

TORCY CASE No	GEOMETRY OF SHELL L/R	R/h	Z	GEOMETRY OF RING STIFFENER $\lvert e_2/h \rvert$	A_2/ah	I_{22}/ah^3	order of solution n	(μ_T) INSIDE	t	(μ_T) OUTSIDE	t
4 and 31	0.5	1000	238.5	1	0.5	2	2	13885.584	14	14455.283	14
							3	10175.613	15	11031.350	16
							4	9237.418	17	10374.173	17
							5	8925.034	18	10318.594	17
							6	8817.422	19		
							7	8810.110	19		
106 and 131	0.5	1000	238.5	1	0.5	5	2	21245.686	12	22261.464	12
							3	15329.357	13	16513.851	14
							4	13687.228	15	15219.152	15
							5	12987.105	16	15072.520	17
							6	12584.813	17		
							7	12349.506*	17		
18 and 43	2.0	100	381.6	1	0.5	2	2	1333.731	4	1354.542	4
							3	1028.196	5	1072.825	5
							4	910.603	5	986.478	5
							5	892.965*	5	982.675	5
118 and 143	2.0	100	381.6	1	0.5	5	2	2171.429	4	2207.252	4
							3	1509.468	4	1589.450	4
							4	1395.396	4	1502.783	4
							5	1328.203	5	1477.541*	5
							6	1281.326	5		
							7	1262.754*	5		
127 and 152	3.0	2000	17170	1	0.5	5	2	22189.147	7	21054.263	7
							3	17682.251	8	16862.922	8
							4	17139.333	9	16387.590	9
							5	17135.498	9	16385.877	9
157 and 182	0.5	2000	477.0	5	0.5	5	2	62930.321	13	70513.140	14
							3	44049.868	15	53934.786	15
							4	37879.232	16	51118.802	16
							5	34375.456	17	50977.493	16
							6	31598.410	18	50779.774	16
							7	29355.778	19	50673.130	16
							8	27756.476	20		
							9	26543.978	20		
							10	25877.563**	20		

TABLE 6. (contd.) **CONVERGENCE OF SOLUTION FOR TYPICAL CASES**

TORCY CASE No.	GEOMETRY OF SHELL L/R	R/h	Z	GEOMETRY OF RING STIFFENER $\|e_2/h\|$	A_2/ah	$I_{22}ah^3$	order of solution n	(μ_T) INSIDE	t	(μ_T) OUTSIDE	t
62 and 87	1.0	1000	954.0	5	0.5	2	2	27133.305	9	27273.537	9
							3	19538.728	10	21380.982	10
							4	17356.853	11	20521.371	10
							5	16223.040	12	20513.145	10
							6	15333.477	12		
							7	14412.637	13		
							8	13694.476	14		
							9	12984.294	14		
							10	12456.019***	14		
161 and 186	1.0	1000	954.0	5	0.5	5	2	30710.071	8	31648.882	8
							3	22209.343	9	24574.601	9
							4	19782.383	10	23425.357	10
							5	18495.009	11	23417.592	10
							6	17442.266	12	23297.047	10
							7	16550.173	13	23290.437	10
							8	15575.990	13		
							9	14835.947	13		
							10	14181.691***	14		
66 and 91	1.5	1000	2146.5	5	0.5	2	2	24947.730	7	23439.257	7
							3	18623.536	8	18478.991	8
							4	17107.301	9	17779.491	9
							5	16547.980	10	17779.314	9
							6	16162.120	11		
							7	15764.498**	12		
166 and 191	1.5	1000	2146.5	5	0.5	5	2	28451.525	7	27098.510	7
							3	21318.774	8	21401.045	8
							4	19628.826	9	20671.415	9
							5	19020.502	10	20633.969	9
							6	18598.587	10	20543.254	9
							7	18025.013	10	20535.603	9
							8	17448.193	12		
							9	16856.095	12		
							10	16278.921**	13		

No asterix - accuracy of convergence within less than one percent.
* accuracy of convergence within one to two percent.
** " " " " two to three "
*** " " " " three to six "

TABLE 8:

CRITICAL TORQUES FOR COMBINED RING AND STRINGER STIFFENED SHELLS OPTIMUM LOCATION OF STIFFENERS

For all the cases : $A_1/bh = A_2/ah = 0.5$: $I_{11}/bh^3 = I_{22}/ah^3 = 5$

TORCYR+S CASE No	R/h	L/R	Z	e_1/h	e_2/h	$(\mu_T)_S$ Stringers only	t	$(\mu_T)_R$ Rings Only	t	$(\mu_T)_{R+S}$ * Combined	t
1	100	1.0	95.4	5	5	1470	10	1361	5	5980	4
2				–5	–5	1782	12	2613	4	<u>8072</u>	4
3				–5	5	1782	12	1361	5	6281	4
4				5	–5	1470	10	2613	4	6446	4
5	100	3.0	858.6	5	5	222.7	5	1423	4	2352	3
6				–5	–5	346.8	6	2390	3	<u>3280</u>	3
7				–5	5	346.8	6	1423	4	2756	3
8				5	–5	222.7	5	2390	3	2578	3
9	1000	1.0	954	5	5	2049	17	14180	14	23170	9
10				–5	–5	3255	19	23290	10	<u>32060</u>	9
11				–5	5	3255	19	14180	14	26990	10
12				5	–5	2049	17	23290	10	25180	10
13	1000	3.0	8586	5	5	872.8	12	17040	7	17550	7
14				–5	–5	1204	12	15780	6	19230	6
15				–5	5	872.8	12	17040	7	<u>19600</u>	7
16				5	–5	1204	12	15780	6	16940	7

(*) Underlined Values Represent the Highest Critical Torque.

TABLE 7:

CRITICAL TORQUES FOR STRINGER STIFFENED SHELLS

TORCYS CASE No.	L/R	R/h	Z	$\lvert e_1/h \rvert$	A_1/bh	I_{11}/bh^3	order of solution n	(μ_T) INSIDE	t	(μ_T) OUTSIDE	t	μ_T UNSTIFF.	$\frac{\text{T INSIDE}}{\text{T UNSTIFF}}$	$\frac{\text{T OUTSIDE}}{\text{T UNSTIFF.}}$
1 and 2	1.0	100	95.4	5	0.5	5	5	1470	10	1782	12	263.4	5.579	6.765
3 and 4		1000	954				5	2049	17	3255	19	1437	1.426	2.265
5 and 6	3.0	100	858.6				5	222.8	5	346.8	6	148.1	1.506	2.343
7 and 8		1000	8586				5	872.8	12	1204	12	833.2	1.047	1.445
9 and 10	1.5	100	238.5				5	650.3	7	898.4	9	209.9	3.097	4.279
11 and 12	0.7		46.74				5	3102	14	3435	16	332.7	9.327	10.33
13 and 14	0.5		23.85				5	6215	20	6556	22	438.7	14.17	14.95

TABLE 9:

COMPARISON OF EQUIVALENT THICKENING OF SKIN WITH RING STIFFENING

TORCY CASE No	L/R	R/h	R/h*	Z	$\|e_2/h\|$	A_2/ah	I_{22}/ah^3	(μ_T) INSIDE	(μ_T) OUTSIDE	T EQUIV. THICK.	T INSIDE / T EQU.TH.	T OUTSIDE / T EQ.THI.
258 and 259	0.35	100	66.67	11.69	1	0.5	2	1346	1632	560.4	1.408	1.162
260 and 261	0.35	100	66.67	11.69	1	0.5	5	1734	2018	560.4	1.091	0.937
247 and 253	0.35	100	66.67	11.69	5	0.5	2	1710	3047	560.4	1.106	0.650
250 and 256	0.40	100	66.67	15.26	1	0.5	2	1226	1490	467.2	1.468	1.200
251 and 257	0.40	100	66.67	15.26	1	0.5	5	1595	1863	467.2	0.988	0.846
246 and 252	0.40	100	66.67	15.26	5	0.5	2	1562	2858	467.2	1.007	0.587
1 and 28	0.50	100	66.67	23.85	1	0.5	2	1093	1336	359.9	1.112	0.903
262 and 263	0.35	250	166.7	30.14	1	0.5	2	2606	3214	807.4	1.046	0.848
264	0.40	250	166.7	30.14	1	0.5	2	2491		709.7	0.962	
29	0.50	250	166.7	59.63	1	0.5	2	2905		589.3	0.684	
53	0.50	250	166.7	59.63	5	0.5	2	1400		359.6	0.867	
103	0.50	250	166.7	59.63	1	0.5	5	1444		359.6	0.840	
153	0.50	250	166.7	59.63	5	0.5	5	1571		359.6	0.772	

(*) R/h for Equivalent Thickening of Skin.

TABLE 10:

COMPARISON OF EQUIVALENT THICKENING OF SKIN WITH STRINGER STIFFENING

TORCYS CASE No	L/R	R/h	R/h*	Z	$\|e_1/h\|$	A_1/ah	I_{11}/ah^3	(μ_T) INSIDE	(μ_T) OUTSIDE	(μ_T) EQU. THICK.	T INSIDE / T EQU.THI.	T OUTSIDE / T EQU. THICK.
1 and 2	1.0	100	66.67	95.40	5	0.5	5	1470	1782	199.5	0.549	0.378
5 and 6	3.0	100	66.67	858.6				222.8	346.8	110.2	1.671	1.073
3 and 4	1.0	1000	666.7	954.0				2049	3255	1060	1.745	1.100
7 and 8	3.0	1000	666.7	8586				872.8	1204	615.1	2.379	1.725

(*) R/h for Equivalent Thickening of Skin.

TABLE 11:

RELATIVE EFFECT OF TORSIONAL RIGIDITY OF STIFFENER ON THE CRITICAL TORQUE

TØRCYT CASE No	L/R	R/h	Z	e_2/h	A_2/ah	I_{22}/ah^3	η_{t2}	order of solution n	$(\mu_T)_{\eta_{t2}}$	r	$(\mu_T)_{\eta_{t2}=0}$	r	$T_{\eta_{t2}} / T_{\eta_{t2}=0}$
158/0	1.0	100	95.4	5	0.5	5	2.283	10	1437	5	1361	5	1.056
158/1							22.83	10	1887	4			1.386
158/2							42.00	10	2231	4			1.639
158/3							61.20	10	2546	4			1.871
158/4							100.0	10	3133	4			2.302
158/5							228.3	10	4577	3			3.363
173/0	3.0	100	858.6	5	0.5	5	2.283	10	1464	4	1423	4	1.029
173/6							10.00	10	1598	4			1.123
173/1							22.83	10	1814	4			1.275
173/3							42.00	5	2173	3			1.526
173/4							61.20	5	2253	3			1.583
173/5							100.0	5	2409	3			1.693
173/2							228.3	5	2902	3			2.039
176/0	3.0	1000	8586	5	0.5	5	2.283	5	17120	7	17090	7	1.002
176/1							22.83	5	17340	7			1.015
176/3							42.00	5	17550	7			1.027
176/4							61.20	5	17750	6			1.040
176/5							100.0	5	18030	6			1.055
176/2							228.3	5	18960	6			1.110
183/0	1.0	100	95.4	–5	0.5	5	2.283	5	2731	4	2613	4	1.045
183/1							22.83	5	3106	4			1.189
183/3							42.00	5	3415	4			1.307
183/4							61.20	5	3704	4			1.417
183/5							100.0	5	4255	4			1.628
183/2							228.3	5	5839	3			2.235
186/2	1.0	1000	954	–5	0.5	5	2.283	5	23510	10	23420	10	1.002
186/1							22.83	5	24290	10			1.035
186/3							42.00	5	25020	10			1.070
186/4							61.20	5	25740	10			1.098
186/5							100.0	5	26970	9			1.150
186/0							228.3	5	30830	10			1.317

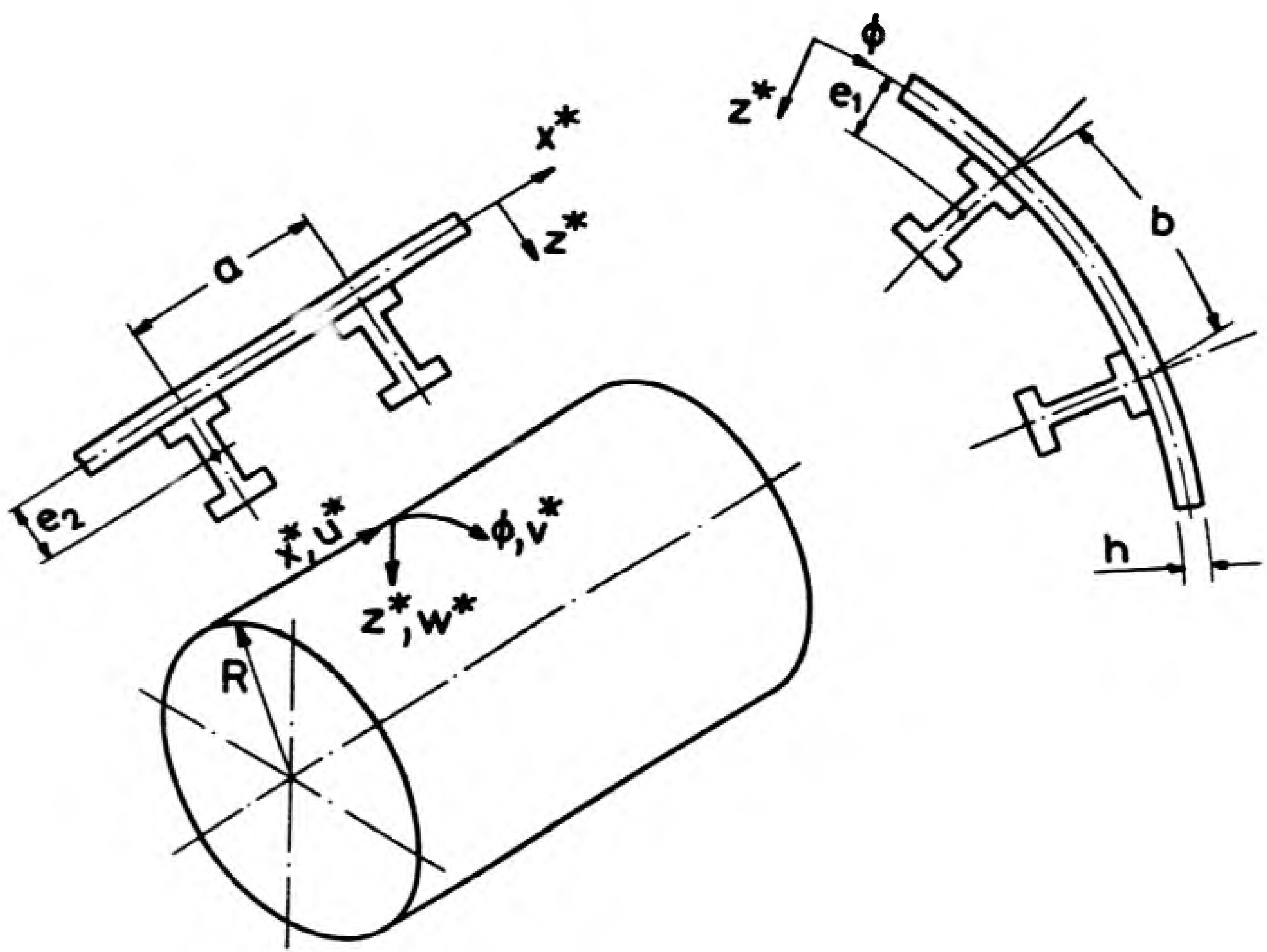


FIG. 1 NOTATION

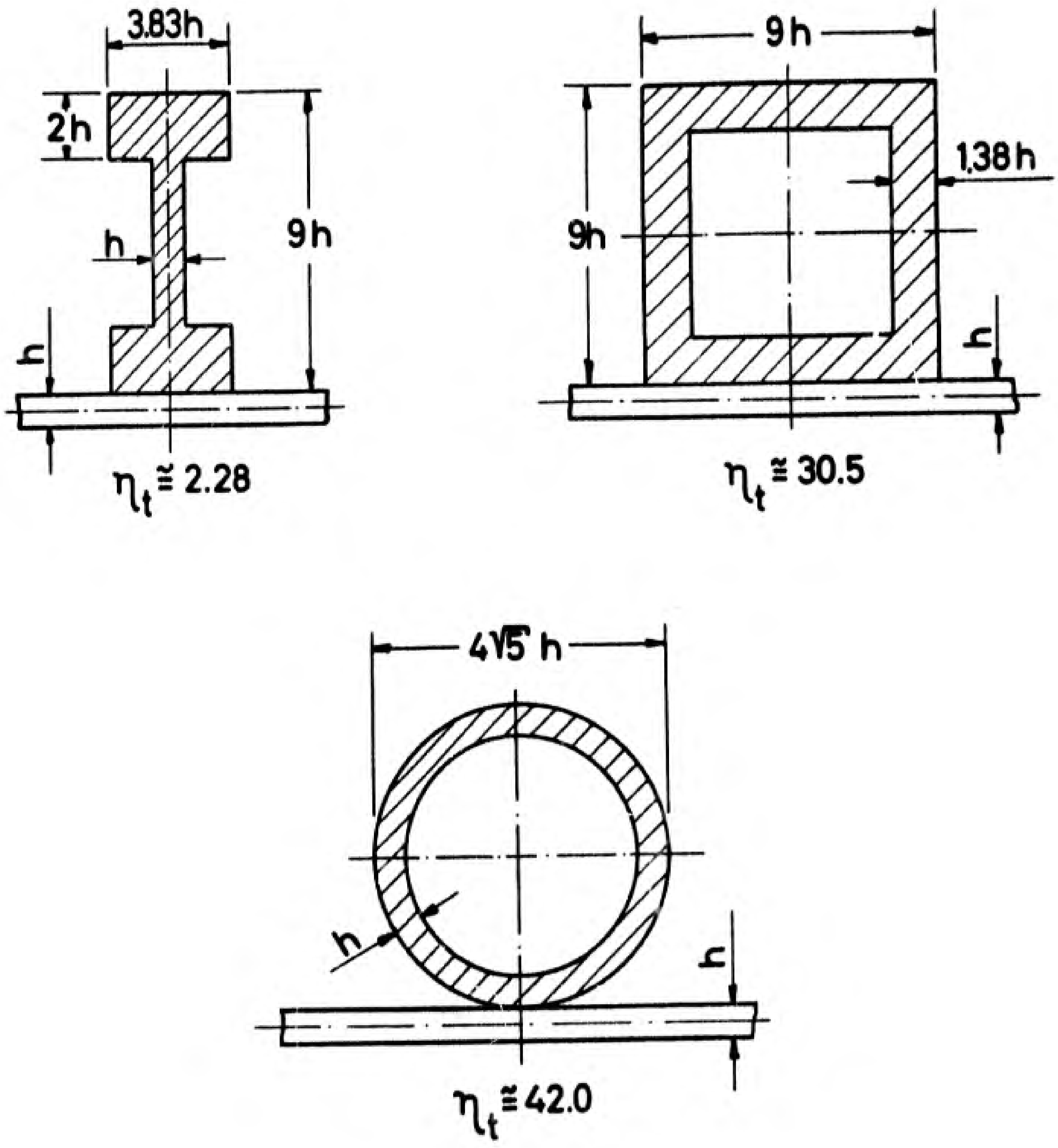


FIG. 2 POSSIBLE CROSS SECTIONS OF STIFFENERS FOR CONSTANT A_2/ah; I_{22}/ah^3; e_2/h [FOR THE SAME TOTAL WEIGHT]

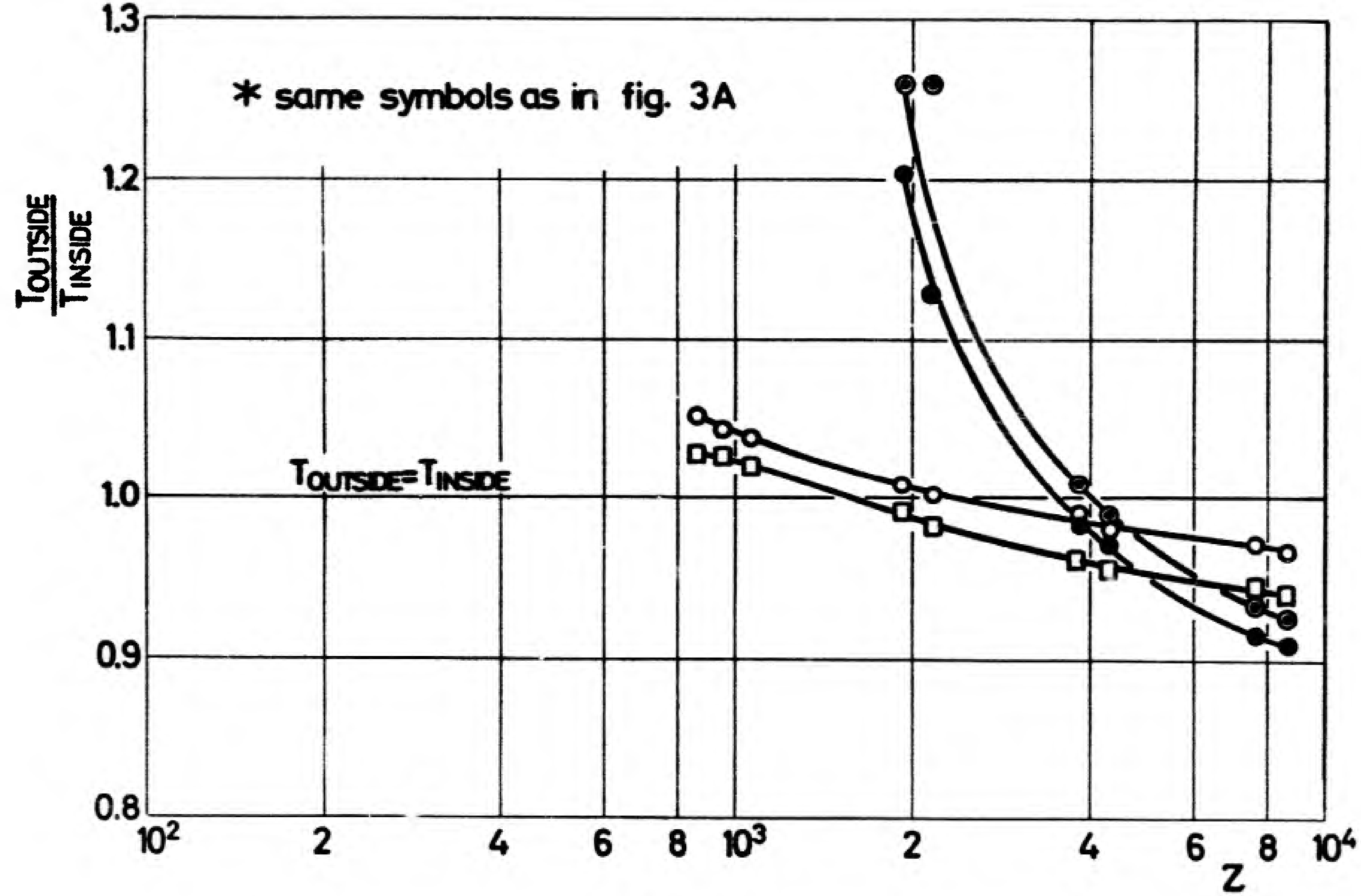
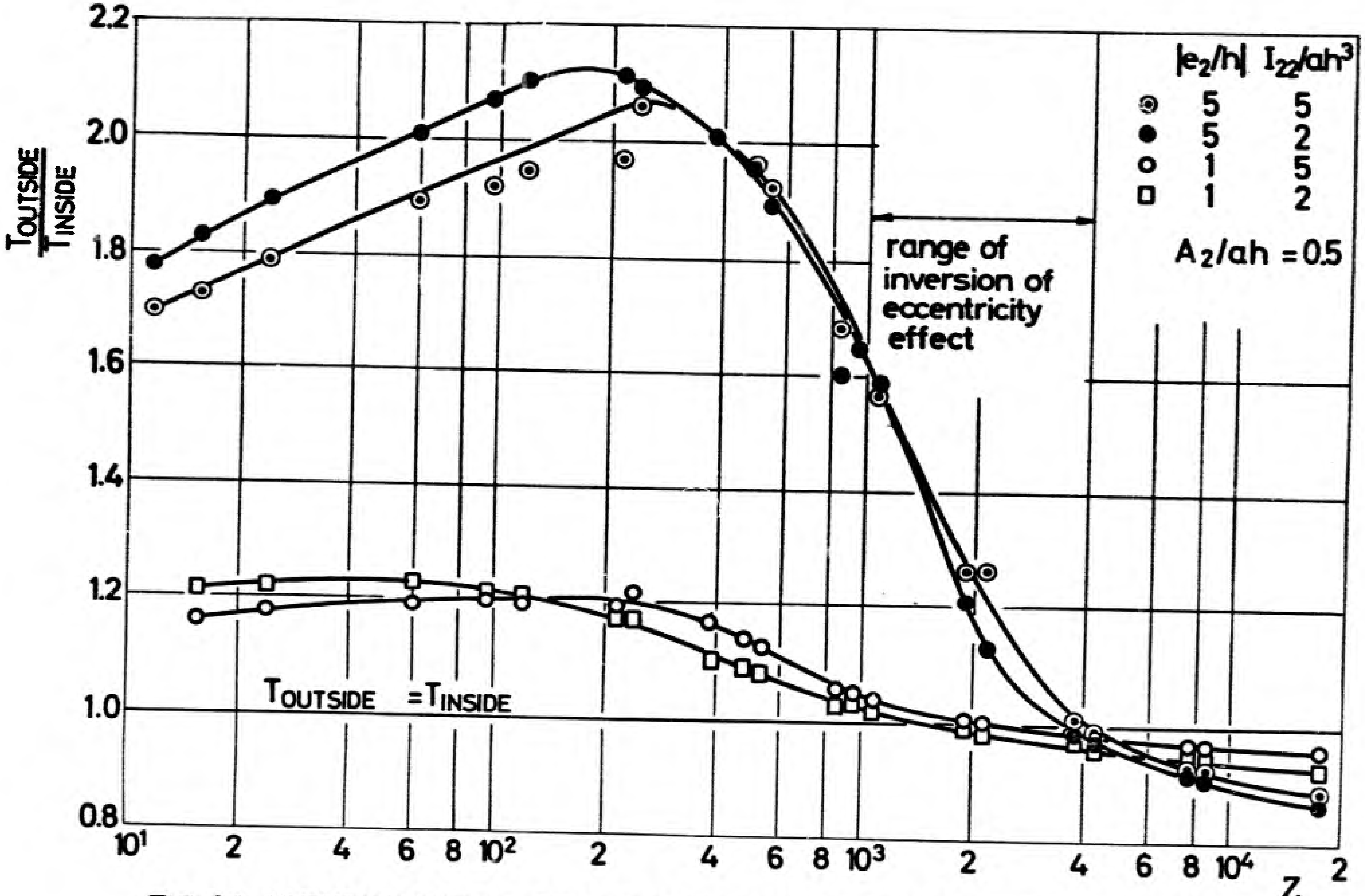


FIG. 3B MAGNIFIED INVERSION RANGE OF FIG. 3A*



<u>FIG. 3A</u> VARIATION WITH Z OF RATIO OF CRITICAL TORQUE WITH OUTSIDE RINGS TO THAT WITH INSIDE RINGS

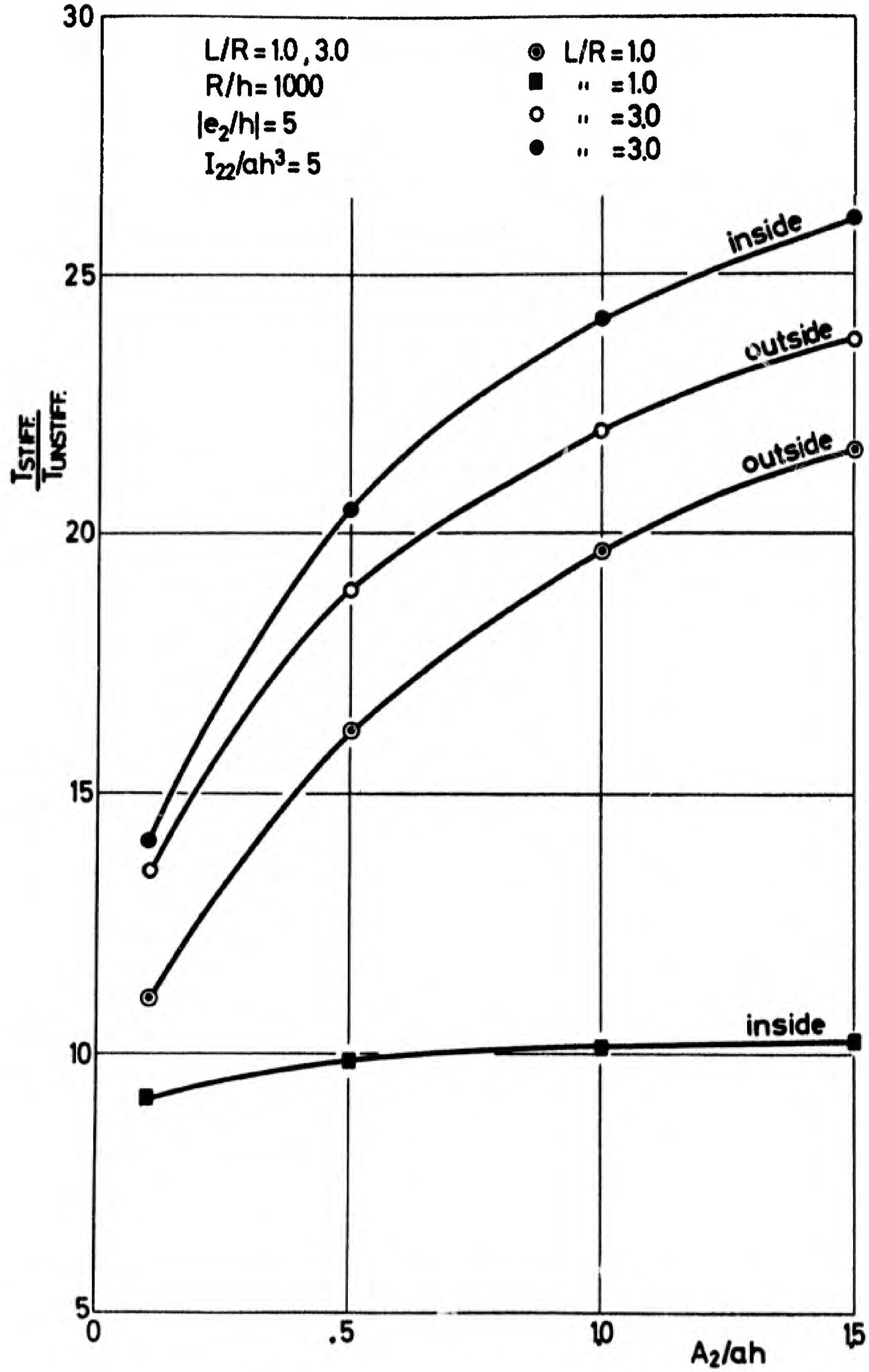


FIG. 4 EFFECT OF RING AREA ON THE CRITICAL TORQUE

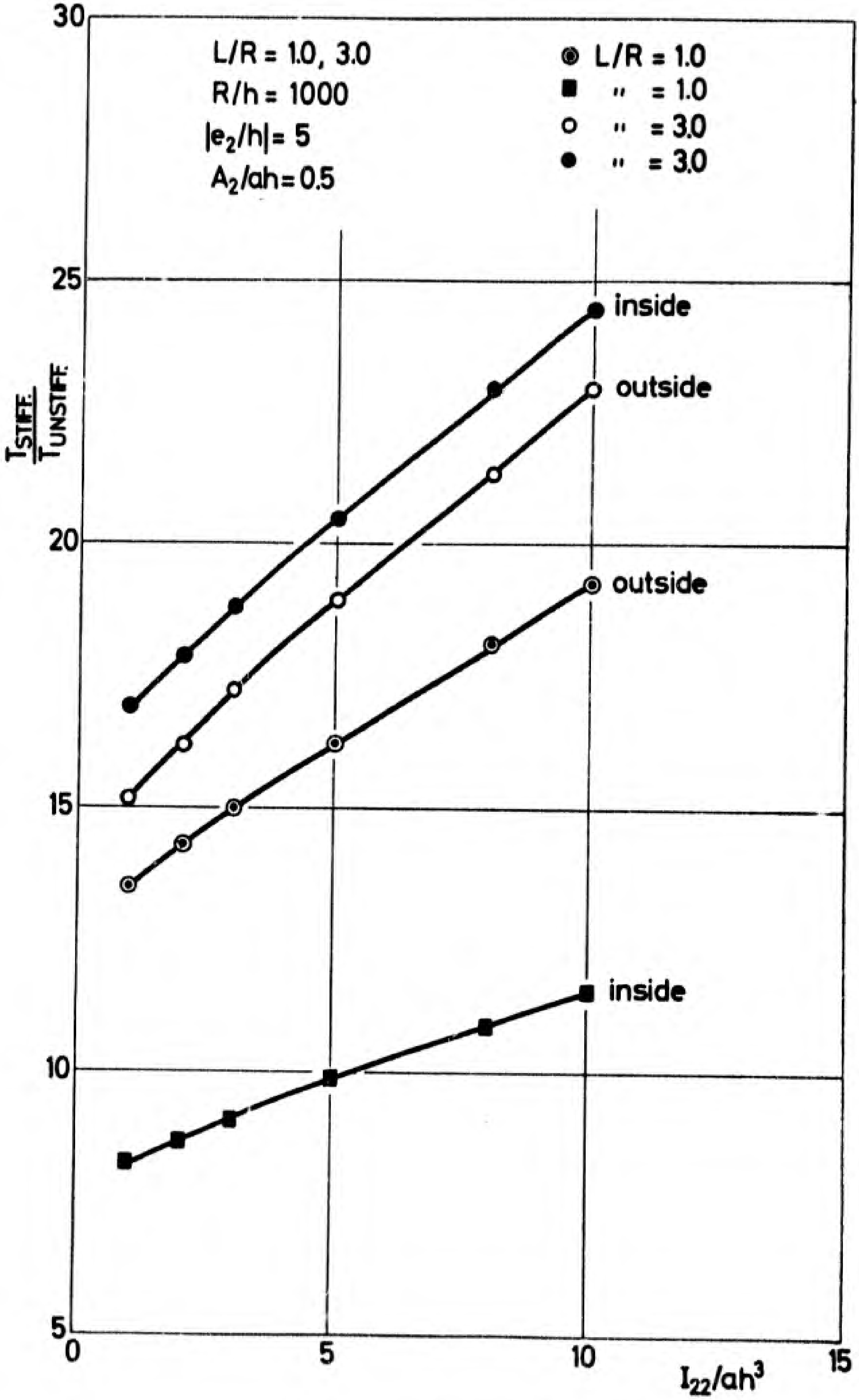


FIG. 5 EFFECT OF MOMENT OF INERTIA OF RING CROSS SECTION ON THE CRITICAL TORQUE

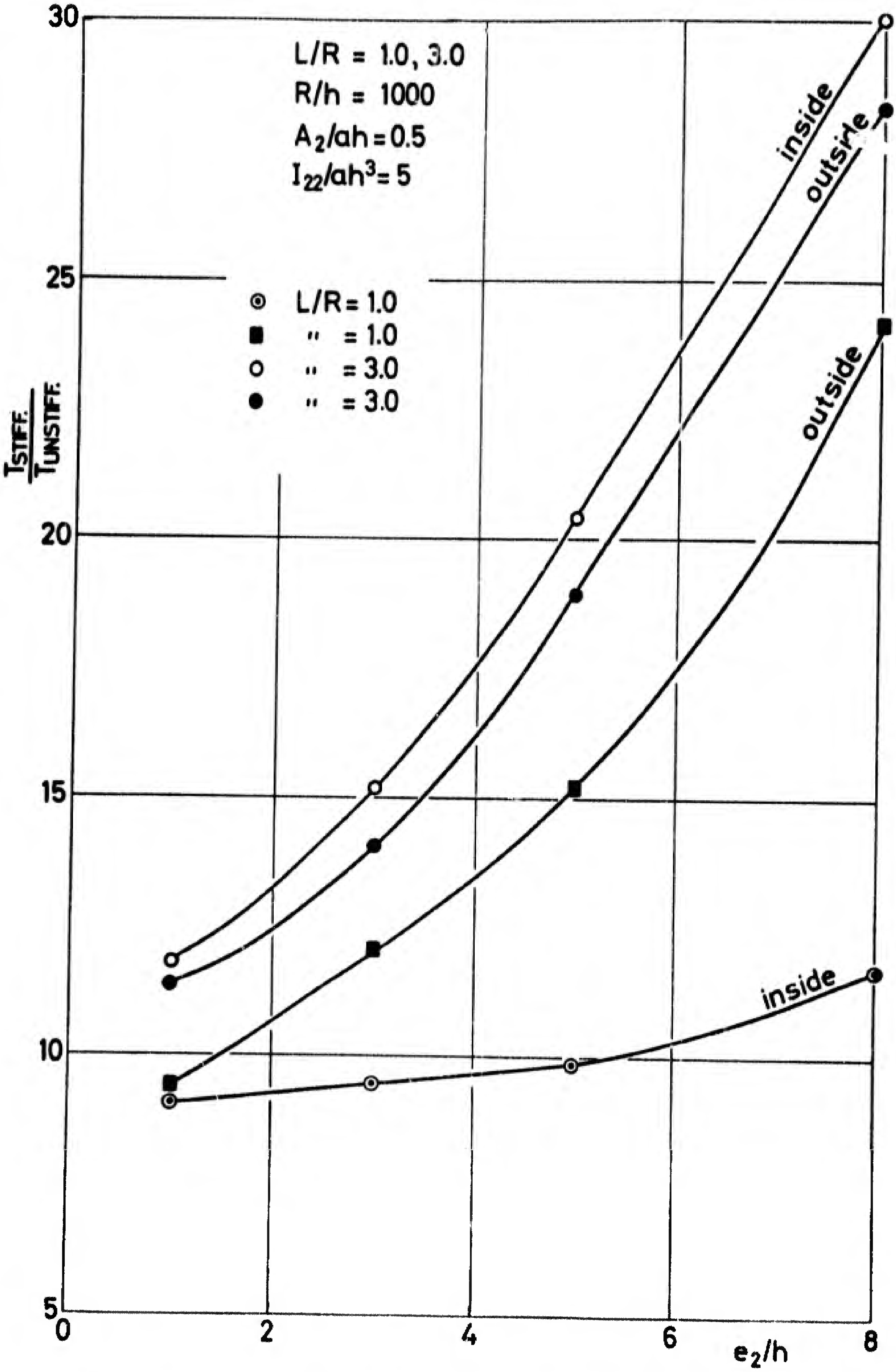


FIG. 6 EFFECT OF ECCENTRICITY OF RINGS ON THE CRITICAL TORQUE

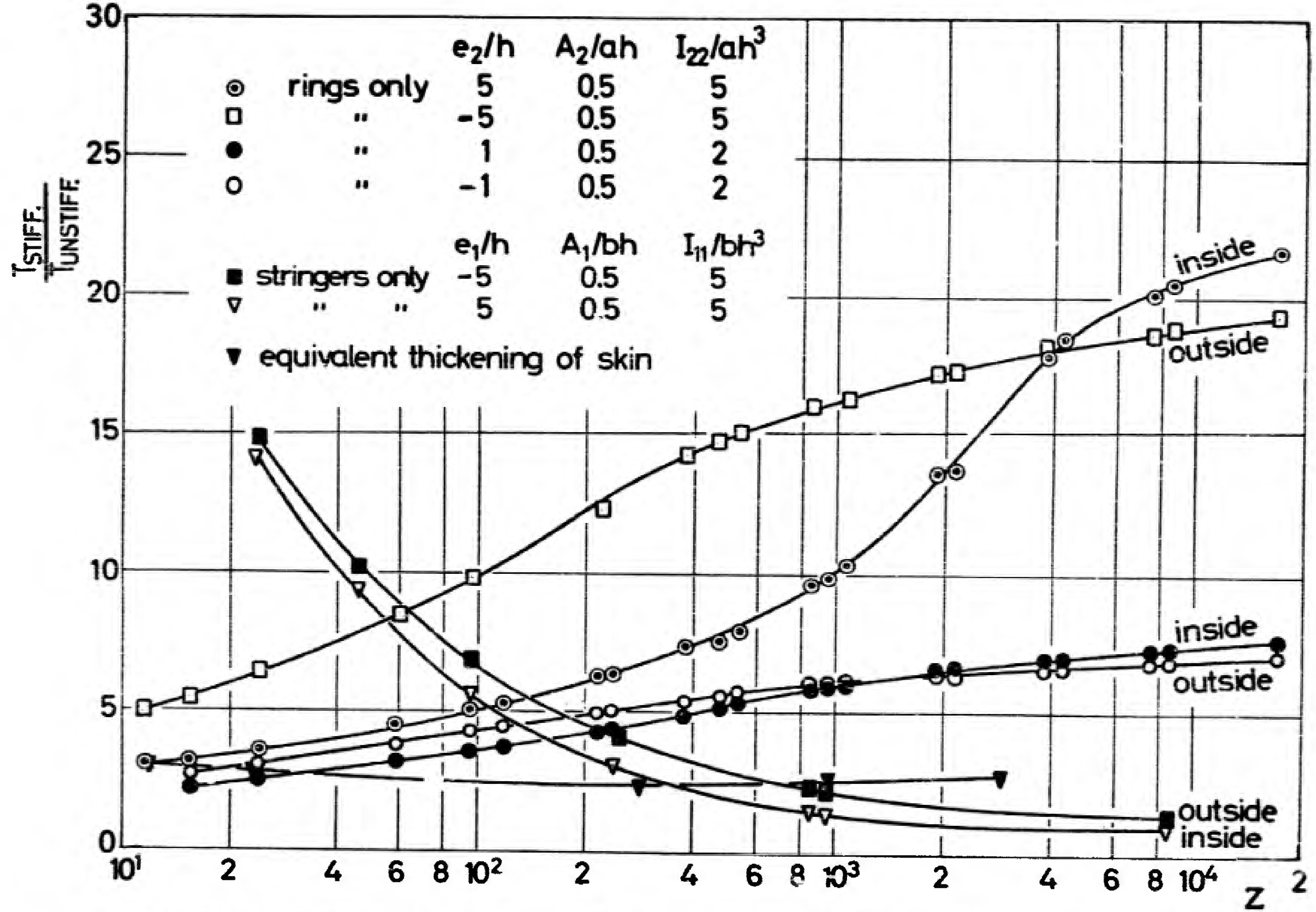
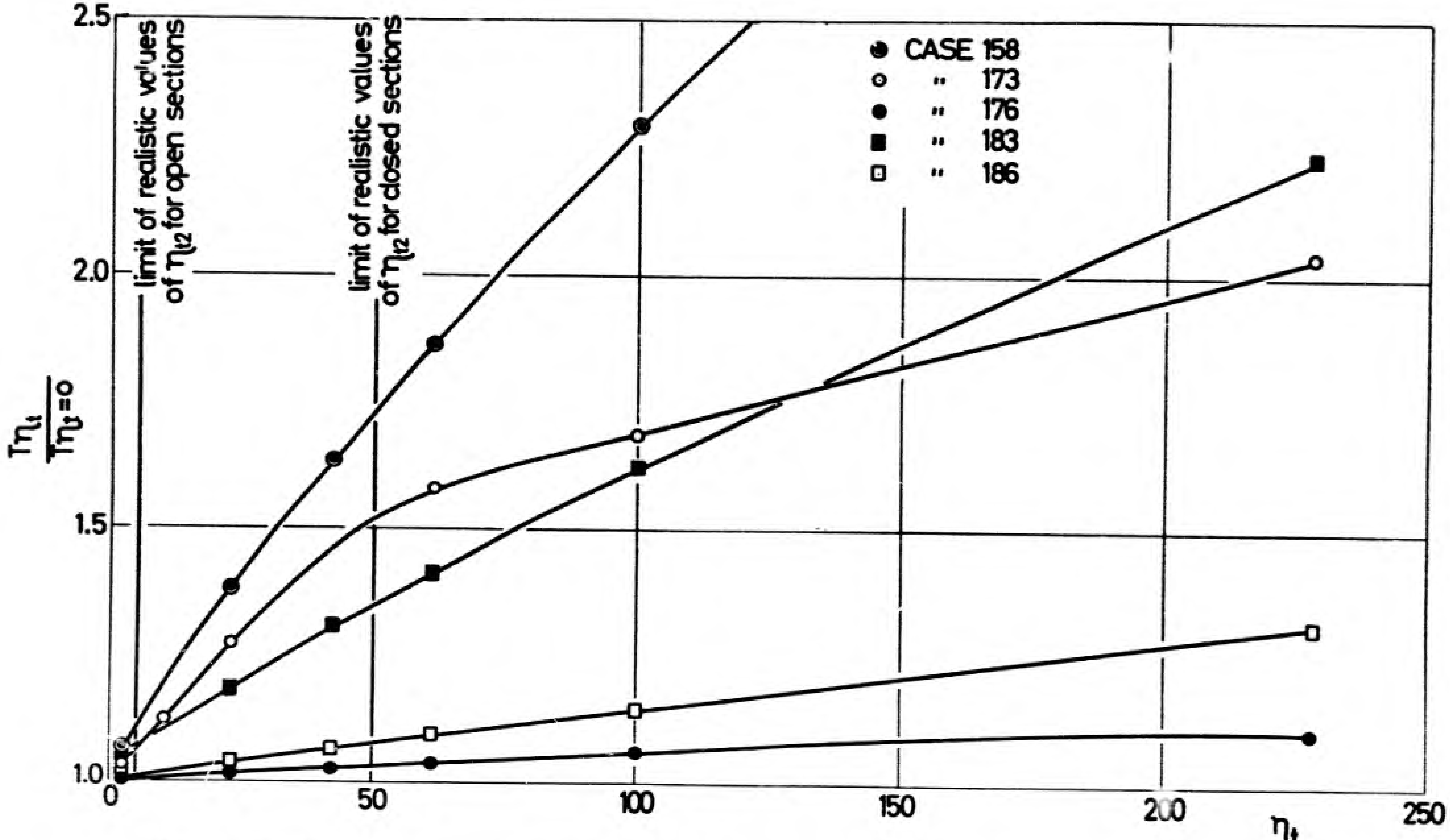


FIG. 7 RELATIVE EFFECT OF STRINGERS, RINGS AND EQUIVALENT THICKENING OF SKIN ON THE CRITICAL TORQUE



<u>FIG. 8</u> RELATIVE EFFECT OF TORSIONAL RIGIDITY OF STIFFENER ON THE CRITICAL TORQUE

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