SOME COMPARISONS OF SLOSHING BEHAVIOR IN CYLINDRICAL TANKS WITH FLAT AND CONICAL BOTTOMS

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H. Norman Abramson and Guido E. Ransleben, Jr.

Technical Report No. 4
Contract No. DA-23-072-ORD-1251
SwRI Project 43-768-2

15 May 1959

Department of Engineering Mechanics
Engineering Analysis Section

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APPROVED:

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ABSTRACT

Some results obtained from sloshing experiments with rigid model cylindrical tanks in translation, and having both flat and conical bottoms, are discussed and compared. On the basis of both total force and wall pressure distribution measurements, it is concluded that sloshing behavior in tanks with conical bottoms can be represented quite adequately by an "equivalent" flat bottom (based on equal liquid volumes).

INTRODUCTION

The practically important problem of sloshing behavior in a cylindrical tank of circular cross section and having a conical bottom has recently been studied theoretically (1)*. This analysis is, however, limited to free oscillations, while the designer more frequently desires information applicable to forced oscillations. Because the sloshing characteristics of tanks with <u>flat</u> bottoms undergoing forced oscillations are readily available (2 - 5), the question arises as to the possibility of utilizing theoretical flat bottom sloshing characteristics for tanks with non-flat bottoms. Of course, it is recognized that for extremely shallow liquid depths the bottom shape must govern the fluid motion, and that for relatively great liquid depths the bottom shape will not influence the fluid motion to any appreciable extent. In view of the latter fact, the question then becomes one of investigating sloshing behavior at moderately shallow depths to explore the possibility of utilizing theoretical results for the flat bottom case for problems involving non-flat bottoms, on the basis of an "equivalent" flat bottom tank having an equal liquid volume.

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* Numbers in parenthesis indicate References at end of paper.

With the notation of Fig. 1, we may define a tank with an equivalent flat bottom depth h_{eq} such that the liquid volumes are equal. Thus, assuming the liquid surface to be at the junction of the cylindrical wall and conical bottom,

$$\frac{h_{eq}}{d} = \frac{h_o}{3d} = \frac{h_o}{6r_o} \qquad [1]$$

At other surface levels, the equivalent flat bottom location would shift slightly. On this basis, the natural frequencies for the conical bottom given in (1) are compared with corresponding values for the flat bottom given in (2), in Fig. 2. $d\omega^2/a$ is a dimensionless frequency parameter, where ω is the frequency and a is the acceleration in the direction of the tank longitudinal axis. The difference between the two theories is slight in the first, and generally the most important, mode, but increases somewhat in the higher modes, particularly at very shallow depths.

EXPERIMENTAL RESULTS

The test facility and measurement techniques employed to obtain the data reported here have been described elsewhere (6). The models were mounted vertically and oscillated in the transverse direction.

The results of total sloshing force measurements are shown in Figs. 3 - 6, in terms of amplitude and phase, with ρ being the fluid density and X_o the excitation amplitude. The theoretical curves shown in each of these figures correspond to those for an equivalent flat bottom tank. Note that the liquid depth of h/d = 0.25 (Fig. 6) corresponds to the liquid surface being approximately at the juncture of the conical bottom and the cylindrical wall, and thus represents a rather shallow condition (approximating the condition assumed in Fig. 1).

The results of wall pressure distributions for both flat and conical bottoms at h/d = 0.50 are shown in Figs. 7 and 8, for two values of the frequency parameter*. h' represents the height from the equivalent flat bottom to the point at which the pressure is measured.

The data presented in Figs. 3 - 8 are intended to be only representative examples of a vast amount of data collected as part of a more general research program. However, no data were collected for h/d values less than 0.25, and no reliable pressure distribution data were obtained for h/d less than 0.50. It may be of interest to note that the comparisons between measured total force and integrated pressure distributions almost invariably showed agreement within less than 10%, and usually within only a very few percent.

DISCUSSION

The total force data of Figs. 3 - 6 show generally excellent agreement with the equivalent flat bottom theory, at least through

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* It is planned to publish, at an early date, a more detailed study of pressure distributions during sloshing.

the second mode. Some of the discrepancy in the third mode in Fig. 5 is believed to be due to the fact that the flat bottom was actually "humped" to a significant extent. It is clear that even at the rather shallow depth of $h/d = 0.25$, the agreement between the measured total force and the flat bottom theory is excellent.

The wall pressure distribution data of Figs. 7 and 8 show that the conical bottom data is virtually identical to the flat bottom data, at least to the depth of the lowest pressure cell ($h'/h = 0.620$*). It is unfortunate that no pressure data are available for $h/d < 0.50$; however, on the basis of the total force measurements there is no reason to expect that the agreement would not continue to be excellent, at least down to $h/d = 0.25$.

CONCLUSIONS

It appears from the experimental data presented here that there is ample justification for the use of an equivalent flat bottom theory to describe sloshing behavior in tanks with non-flat bottoms, except for extremely shallow fluid depths ($h/d \ll 0.25$). This conclusion appears to be valid at least through the second mode, and possibly through the third. This knowledge should be of extreme value during the design stage of new configurations.

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* At $h/d = 1.00$, similarly excellent agreement was obtained, the location of the lowest pressure cell being at $h'/h = 0.30$.

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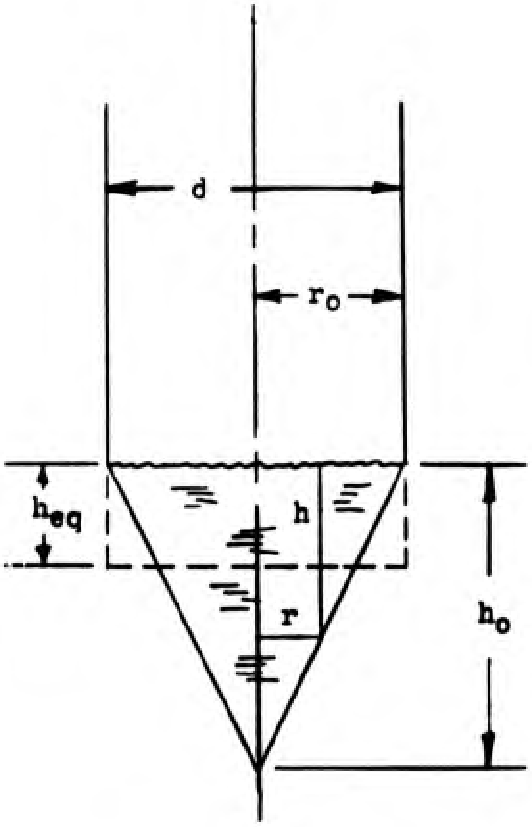
FIGURE CAPTIONS

1. CONICAL BOTTOM AND EQUIVALENT FLAT BOTTOM
2. COMPARISON OF FLAT AND CONICAL BOTTOM NATURAL FREQUENCIES
3. TOTAL FORCE MEASUREMENTS FOR FLAT BOTTOM MODEL WITH $h/d = 0.50$
4. TOTAL FORCE MEASUREMENTS FOR CONICAL BOTTOM MODEL WITH $h/d = 0.50$
5. TOTAL FORCE MEASUREMENTS FOR FLAT BOTTOM MODEL WITH $h/d = 0.25$
6. TOTAL FORCE MEASUREMENTS FOR CONICAL BOTTOM MODEL WITH $h/d = 0.25$
7. COMPARISON OF WALL PRESSURE DISTRIBUTIONS FOR FLAT AND CONICAL BOTTOMS AT $d\omega^2/a \approx 2.5$, $h/d = 0.50$
8. COMPARISON OF WALL PRESSURE DISTRIBUTIONS FOR FLAT AND CONICAL BOTTOMS AT $d\omega^2/a \approx 7.7$, $h/d = 0.50$



$$h = h_o \left(1 - \frac{r}{r_o}\right)$$

Fig. 1. CONICAL BOTTOM AND EQUIVALENT FLAT BOTTOM

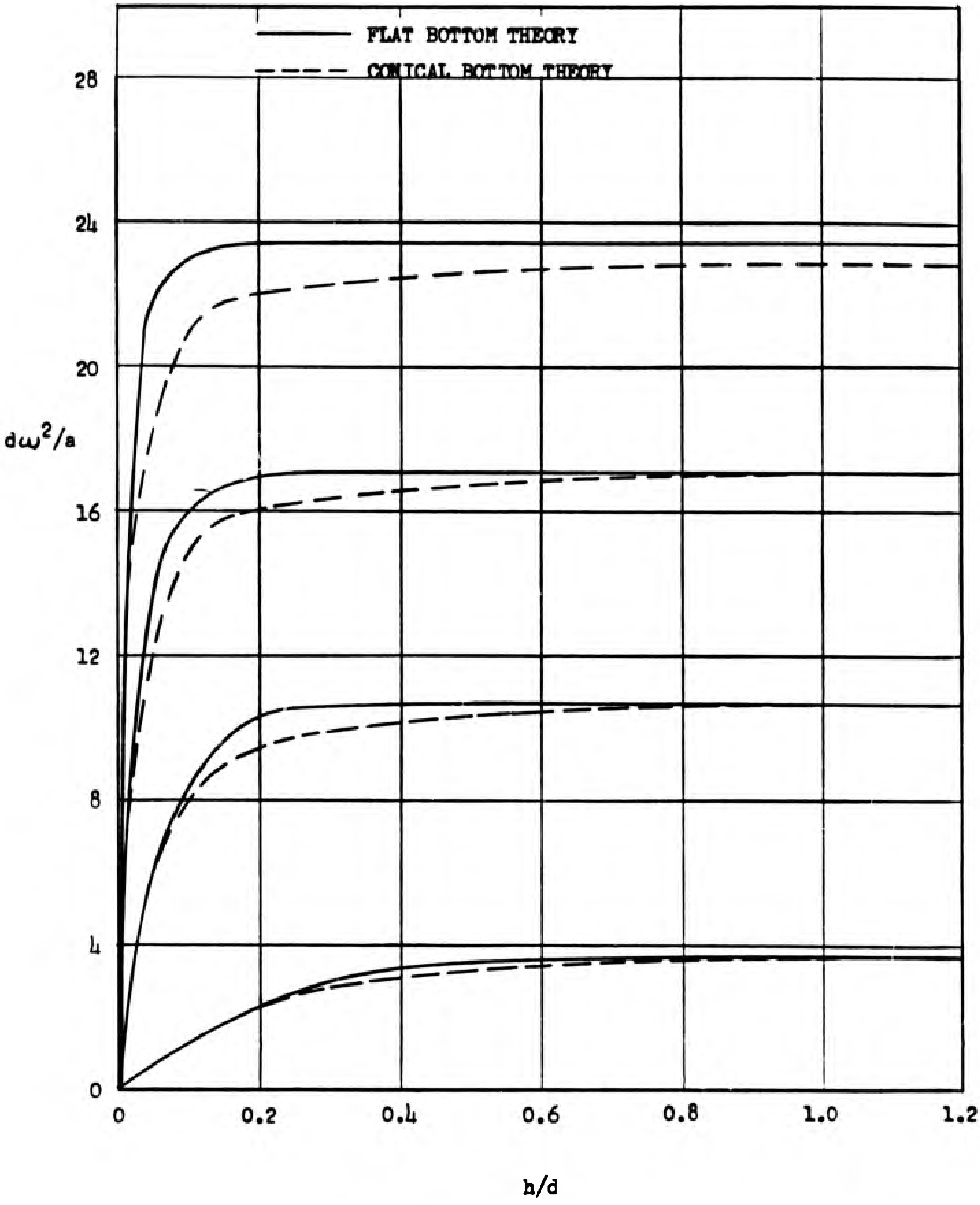


Fig. 2. COMPARISON OF FLAT AND CONICAL BOTTOM NATURAL FREQUENCIES

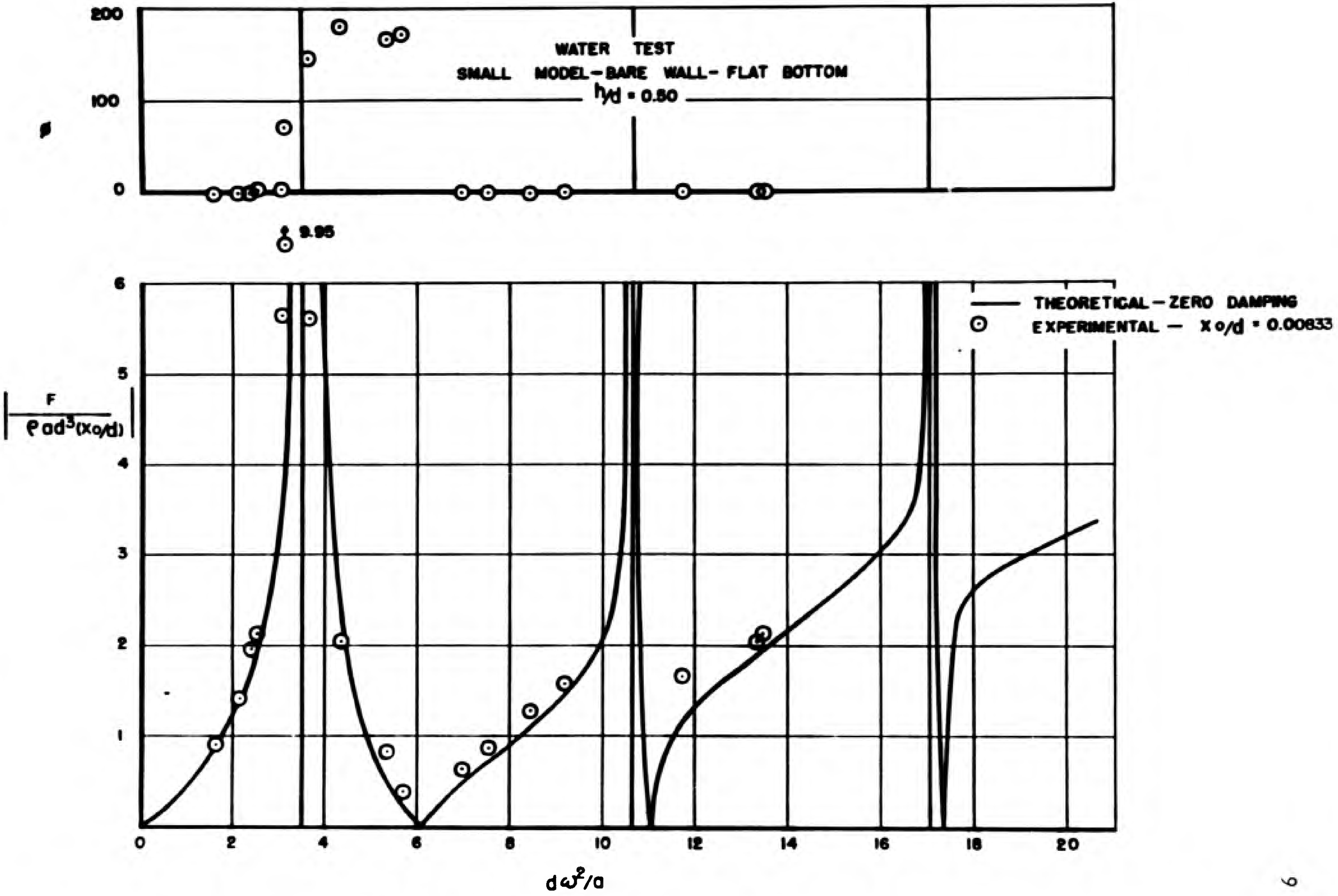


Fig. 3. TOTAL FORCE MEASUREMENTS FOR FLAT BOTTOM MODEL WITH h/d = 0.50

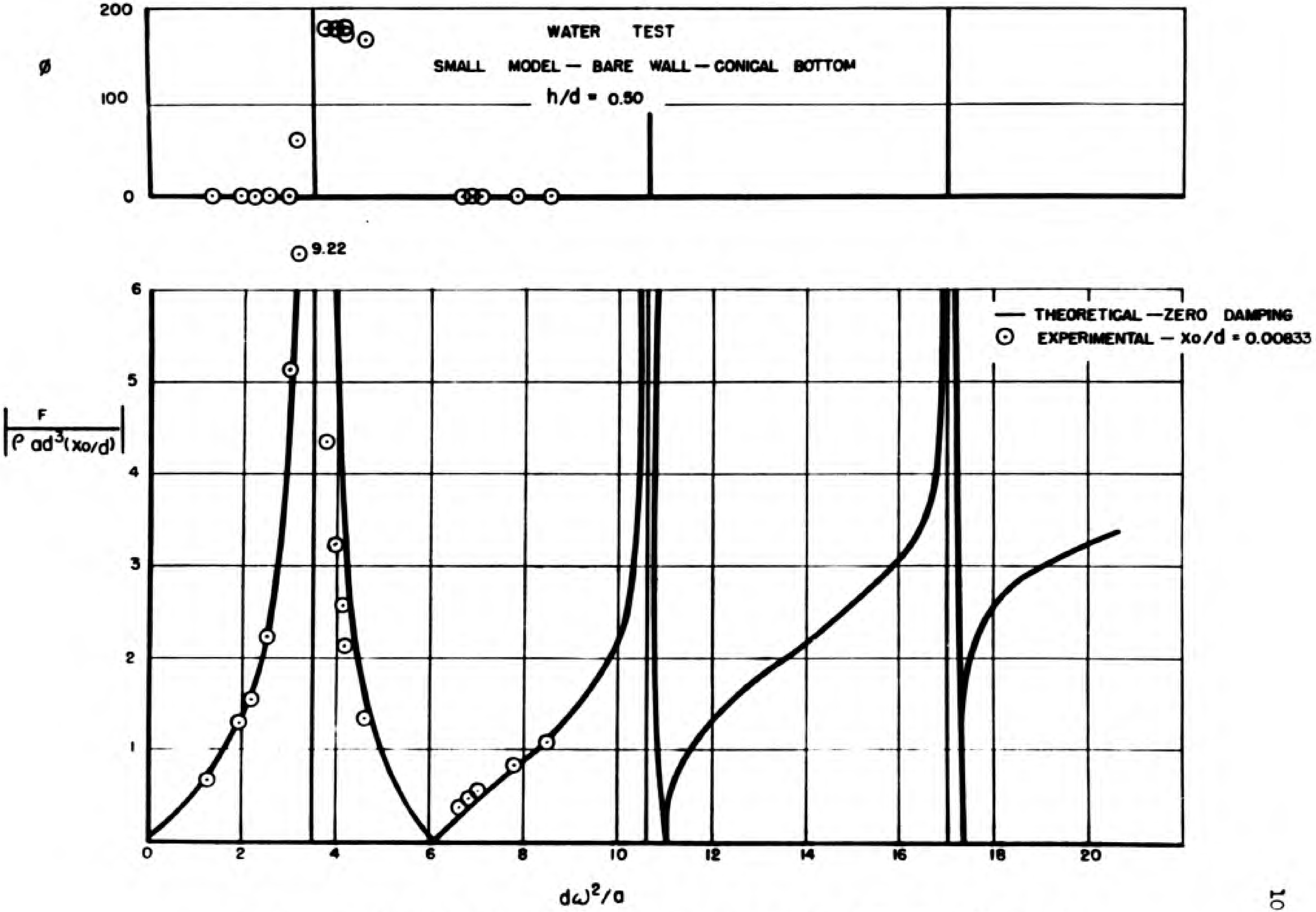


Fig. 4. TOTAL FORCE MEASUREMENTS FOR CONICAL BOTTOM MODEL WITH h/d = 0.50

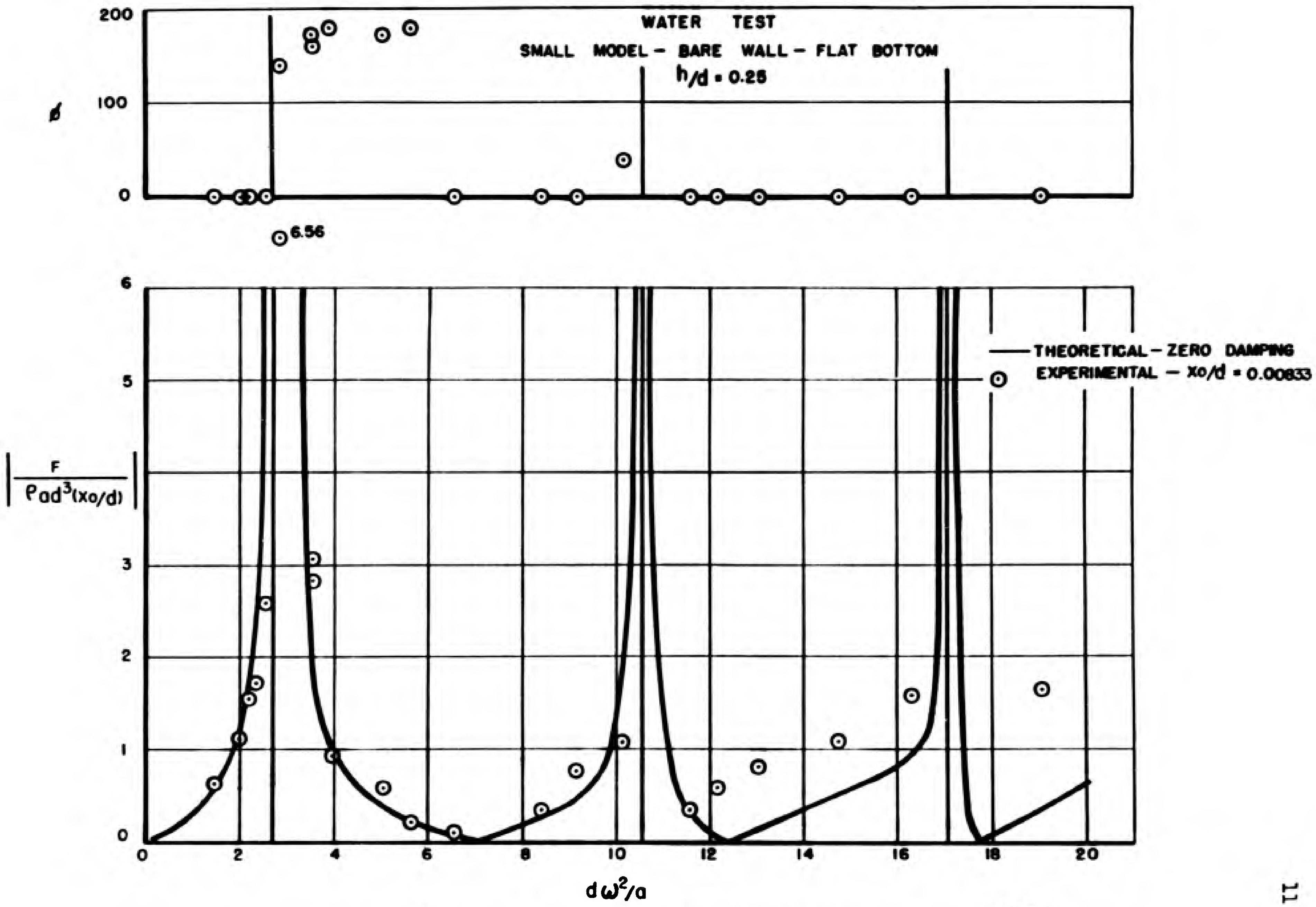


Fig. 5. TOTAL FORCE MEASUREMENTS FOR FLAT BOTTOM MODEL WITH h/d = 0.25

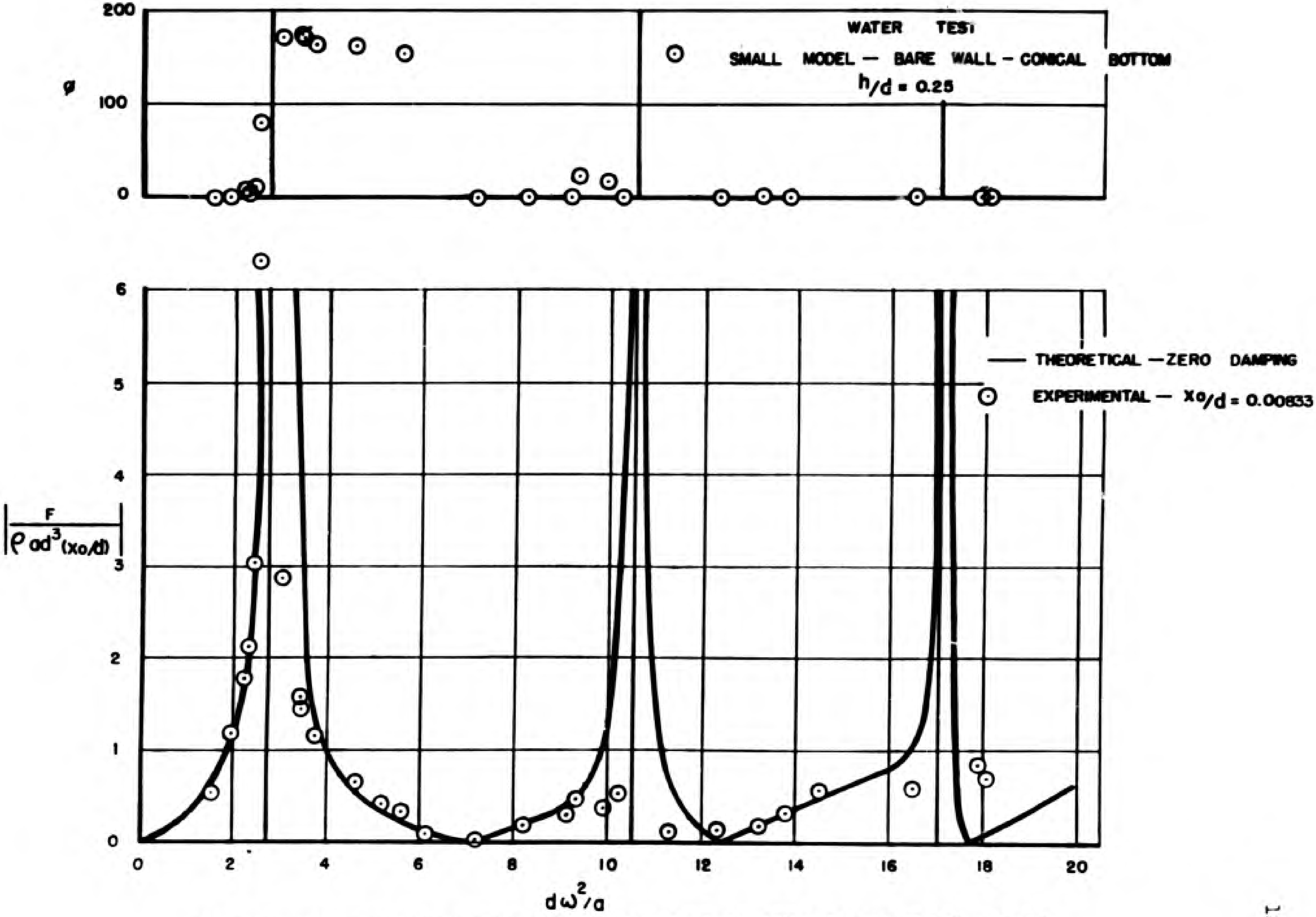


Fig. 6. TOTAL FORCE MEASUREMENTS FOR CONICAL BOTTOM MODEL WITH h/d = 0.25

COMPARISON OF WALL PRESSURE DISTRIBUTIONS FOR FLAT AND CONICAL BOTTOMS AT $d\omega^2/a \approx 2.5$
$h/d = 0.50$

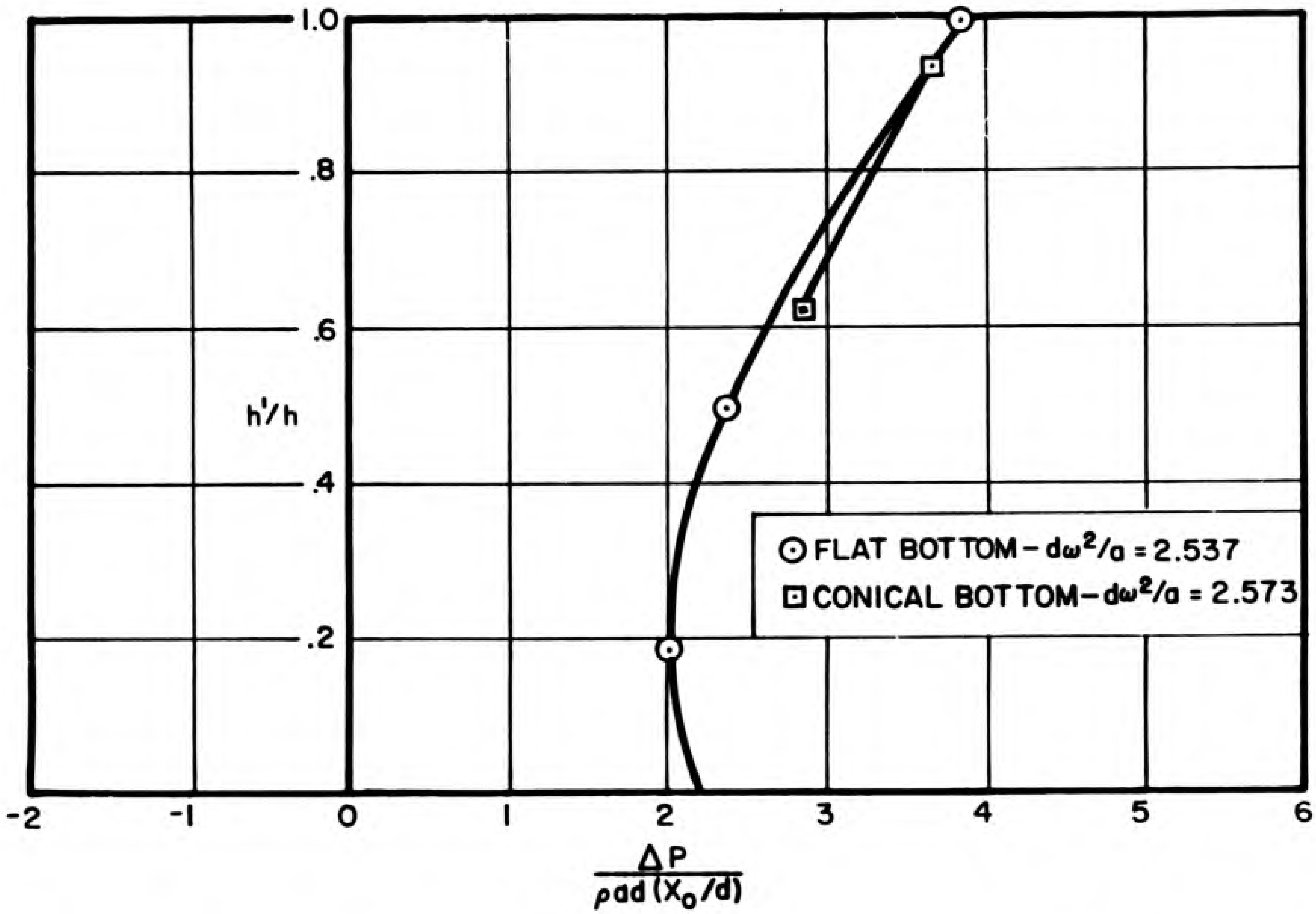


Fig. 7.

COMPARISON OF WALL PRESSURE DISTRIBUTIONS FOR FLAT AND CONICAL BOTTOMS AT $d\omega^2/a \approx 7.7$
$h/d = 0.50$

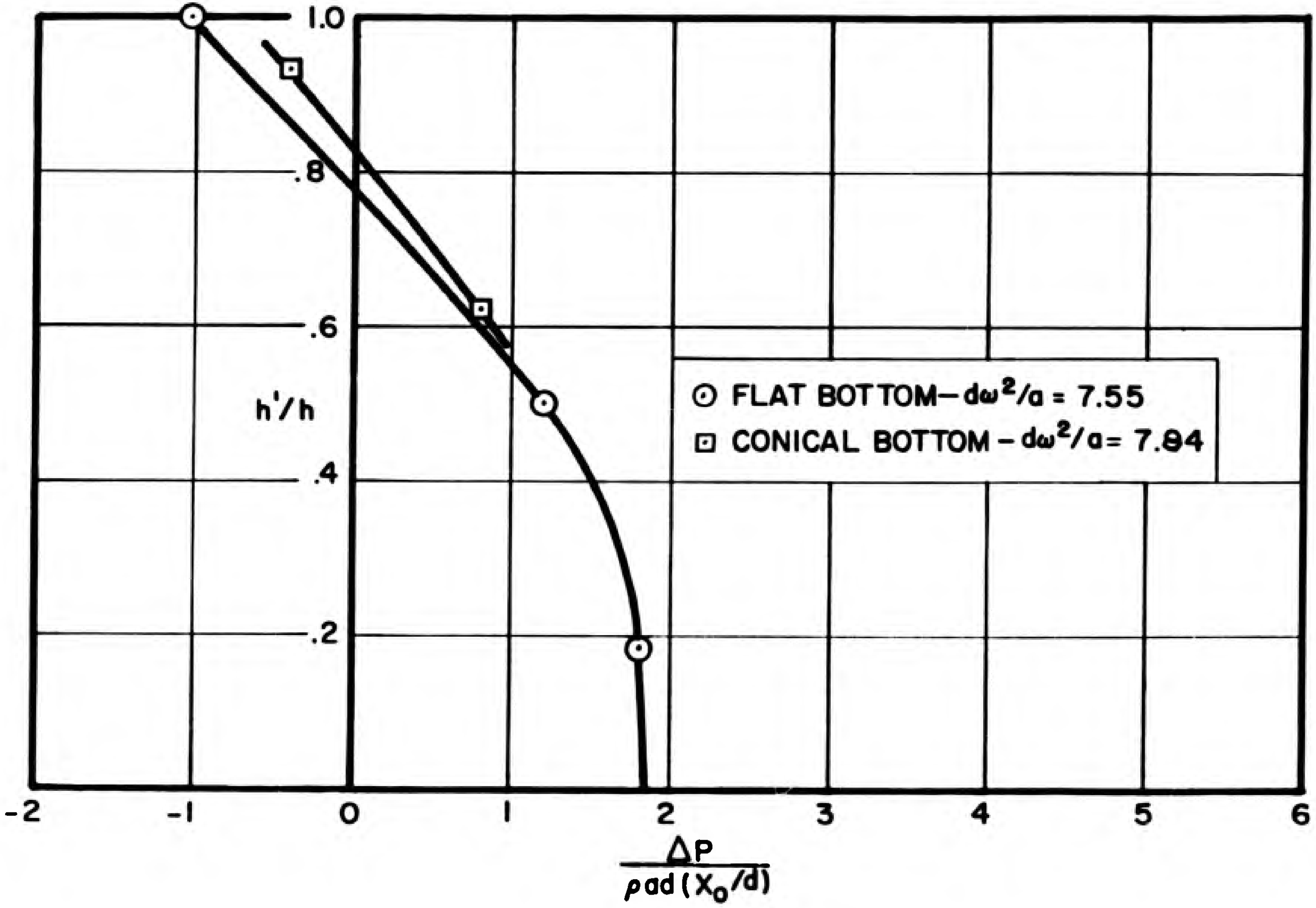


Fig. 8.