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ACOUSTIC DETERMINATION OF A BUBBLE SIZE DISTRIBUTION

Subproject SR 011 01 01, Task 0401-11

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ABSTRACT

The purpose of this investigation was to determine the feasibility of measuring bubble size distributions acoustically. The experiment consisted of a comparison of a photographically and an acoustically determined bubble size distribution of an electrolytically generated bubble screen. The approximate range of bubble radii considered was from 15μ to 85μ. Results indicate the acoustic method may be reasonably accurate in determining a relative bubble size distribution for spherical bubbles with radii greater than 32μ, but more information is needed about the damping constants of nonresonant bubbles; and a more precise evaluation of an integral in the acoustical theory is needed before the validity of the acoustical method can be established.

ADMINISTRATIVE INFORMATION

This investigation was made from 12 July to 24 September 1965 under the U. S. Navy Mine Defense Laboratory In-House Independent Research Program, Subproject SR 011 01 01, Task 0401, Subtask 11, authorized by Bureau of Ships letter L1-1/60(315), serial 315-36 of 26 May 1959. The bulk of the experimental work was done by the first author during his employment in the summer of 1965.

The authors wish to acknowledge the help and comments given by J. E. Blue and J. C. Wicke, Jr. throughout this investigation.

APPROVED AND RELEASED 9 DECEMBER 1965

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1. The purpose of this investigation was to determine the feasibility of measuring bubble size distributions acoustically, so that bubble distributions naturally occurring in the sea could be measured and studied. Results are encouraging but inconclusive.

2. This work will be continued at this Laboratory under the U. S. Navy Mine Defense Laboratory In-House Independent Research Program, Subproject SR 011 01 01, Task 0401.

3. This report is made to document the technical information contained herein and to distribute it to interested activities.

R. T. Miller

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INTRODUCTION

During World War II the theory of sound transmission in the sea, especially bubble theory and the effects of bubbles on underwater sound, was studied and vastly improved. This was brought about primarily in an effort to solve the many problems concerning ship cavitation and, more generally, those concerning the wakes of ships. Since that time, this theory has been extended by Carstensen and Foldy (Reference 1). Their work disclosed that bubble distributions greatly affected sound transmission; therefore, a need arose for a method to analyze these distributions in the sea.

This report presents the results of an experiment made to discover whether the theory, and its resulting approximations, could be used to determine acoustically the size distribution of bubbles in a continuous-flow bubble screen.

Acoustic waves, such as those used in sonar, are very much affected upon entering water containing bubbles. The waves are scattered, refracted, reflected, and absorbed by the bubbles more than by the water itself, even if only a small percentage of the volume encountered is composed of bubbles. The amount of scattering, reflection, and absorption is highly dependent, however, on the frequency of the incident sound wave. Sound of some frequencies may undergo almost complete extinction, whereas that of other frequencies is hardly affected while passing through the same bubbly medium. This phenomenon can be explained easily if the equations of motion of the bubbles are analyzed. All bubbles act as small spherical oscillators, capable of expansion and contraction, each bubble having a fundamental resonant frequency of oscillation dependent primarily on its size. The acoustic wave may be considered as the driving force for the bubbles, with each bubble obeying the laws of forced harmonic motion. If the wave frequency is near the resonant bubble frequency, the bubble is highly excited, and the sound wave undergoes extensive scattering and absorption. If the incident wave has a frequency far from the resonant bubble frequency, however, it will pass the bubble virtually unaffected.

By employing a relationship between resonant bubble frequency and bubble radius, coupled with a relationship between sound attenuation and the number of resonant bubbles present, one should be able to derive the actual size distribution of the bubbles, but this has not been established experimentally. By varying the frequency of the incident sound and measuring the attenuation in this experiment, data were gathered for these relationships. The results were compared with a later determination of a similar distribution established photographically, using empirical curves for rate of bubble rise as a function of bubble radius. In the photographic method, a picture of the bubble screen was taken with a one-half second exposure and later extensively analyzed. The lengths of the bubble traces were measured and, using the rate of rise of air bubbles in water as a function of their radii, a bubble size distribution was determined. The relatively close agreement observed between photographic and acoustic results indicates that the theory of sound in a fluid medium containing bubbles (Reference 1) is adequate to correlate the two sets of data after more information concerning the damping constants of nonresonant bubbles is obtained.

ACOUSTIC DETERMINATION OF BUBBLE SIZE DISTRIBUTION

THEORY

As early as 1933, Minnaert (Reference 2) established the relationship between the radius of an air bubble in water (R) and its resonant frequency (f) as

$$f = \frac{1}{2\pi R} \sqrt{3\gamma P/\rho} \tag{1}$$

where P is the hydrostatic pressure, ρ is the fluid density and γ is the ratio of specific heats of the air in the bubble; or, in a more usable form,

$$fR = 330 \tag{2}$$

where f is in cps, R in cm, and P is atmospheric pressure. This equation is valid for a single bubble over the range of bubble sizes considered in the investigation. According to Reference 3, surface tension may not be neglected for bubble radii smaller than 10μ (micron) and Equation 1 must be modified for these smaller bubbles. This resonant frequency corresponds to the fundamental mode of vibration of the bubble, and for practical purposes the harmonic modes may be neglected.

If one begins with the wave equation in a fluid medium and applies the boundary conditions for an actual bubble, as carried out in Reference 3, the following equation is obtained for the extinction cross section for the bubble.

$$\sigma_e = \frac{4\pi R^2\left(\frac{\delta}{\eta}\right)}{\left(\frac{f_r^2}{f^2} - 1\right)^2 + \delta^2} \tag{3}$$

Here, R is the bubble radius; f_r is the resonant frequency of the bubble, f is the frequency of the incident sound; η is the ratio of the bubble circumference to the wavelength of the incident sound in the fluid; and δ has become known as the "damping constant" (not to be confused with the classical damping constant). This extinction cross section σ_e represents the sum of the scattering and absorption cross sections of the bubble. Physically, the sound energy flowing through an area σ_e perpendicular to the incident sound beam is equal to the total energy absorbed and scattered by the bubble.

When considering a nonhomogeneous size distribution of bubbles such that N(R)dR is the number of bubbles per cubic centimeter with radii between R and R + dR, the total extinction cross section per cubic centimeter, S_e, can be obtained from Equation 3 by integrating overall radii:

$$S_e = \int_0^\infty \sigma_e \; N(R)dR = \int_0^\infty \frac{4\pi R^2 \; N(R)\left(\frac{\delta}{\eta}\right)}{\left(\frac{f_r^2}{f^2} - 1\right)^2 + \delta^2} \; dR \tag{4}$$

To evaluate this integral exactly, δ, which is a function of radius and frequency, must be known completely. However, δ is known only for resonant size bubbles, and thus this integral can be evaluated only by making approximations.

If one assumes that only resonant bubbles contribute appreciably to S_e and that N(R) does not change rapidly for radii near resonance, then S_e can be evaluated approximately as (Reference 3)

$$S_e = \frac{2\pi^2 \; R_r^3 \; N(R_r)}{\eta_r} \tag{5}$$

where R_r and η_r are the resonant values of R and η.

The attenuation by a bubble screen (in db) of sound of frequency f, denoted by K_r, is proportional to S_e and thus

$$K_e = C_1 \, R_r^3 \, N(R_r) \qquad (6)$$

where C_1 is a constant. Thus, if one measures the attenuation of sound through a bubble screen for a given frequency f, then the approximate number of bubbles with radius R_r is given by:

$$N(R_r) = \frac{K_e}{C_1 \, R_r^3} \qquad (7)$$

Thus a relative number distribution is obtained, because the attenuation at each frequency, when properly weighted, should be proportional to the number of resonant size bubbles present for that particular frequency. If the frequency is varied over the entire range of resonant radii, a complete "spectrum" of bubble sizes and numbers should be obtained.

EXPERIMENT

This experiment was performed in a large concrete tank, measuring 4 m by 50 m and filled with tap water to a depth of 1 m. A bubble screen was generated electrolytically, using a 1-m length of No. 12 tinned copper wire as the cathode with a large copper anode positioned about 10 m away. The wire was placed near the bottom of the tank so as to produce a bubble screen measuring 1-m wide, 1-m high, and with a thickness varying from the wire diameter at the bottom of the tank to about 10 cm at the surface (see Figure 1). This varying thickness is caused by the greater diffusion of the smaller bubbles due to their slower rate of rise. A current of 135 ma was supplied to the electrodes by a 10-volt tap from a 12-volt automotive-type battery. This current changed less than ±3 ma during both the acoustical and photographic measurements.

For the acoustical attenuation measurements, the bubble screen was placed midway between, and perpendicular to a line joining, two transducers (see Figure 1). A BM-102 projector and a M-146 hydrophone were positioned facing each other, 60 cm apart, and 46 cm from the bottom of the tank. For the frequencies considered here this separation constituted far-field conditions for the transducers and individual bubbles, but not for the entire bubble screen. The instrumentation used is described in Appendix A.

In the actual experiment a single-frequency continuous-wave signal was projected with no bubble screen, and a voltage level (in db, reference 1 volt) proportional to the received sound pressure level was recorded. This measurement was repeated for frequencies from 20 kc to 110 kc in 5-kc increments, and from 110 kc to 210 kc in 10-kc increments. This procedure was repeated with the bubble screen in place. The attenuation of the sound through the bubble screen is then defined to

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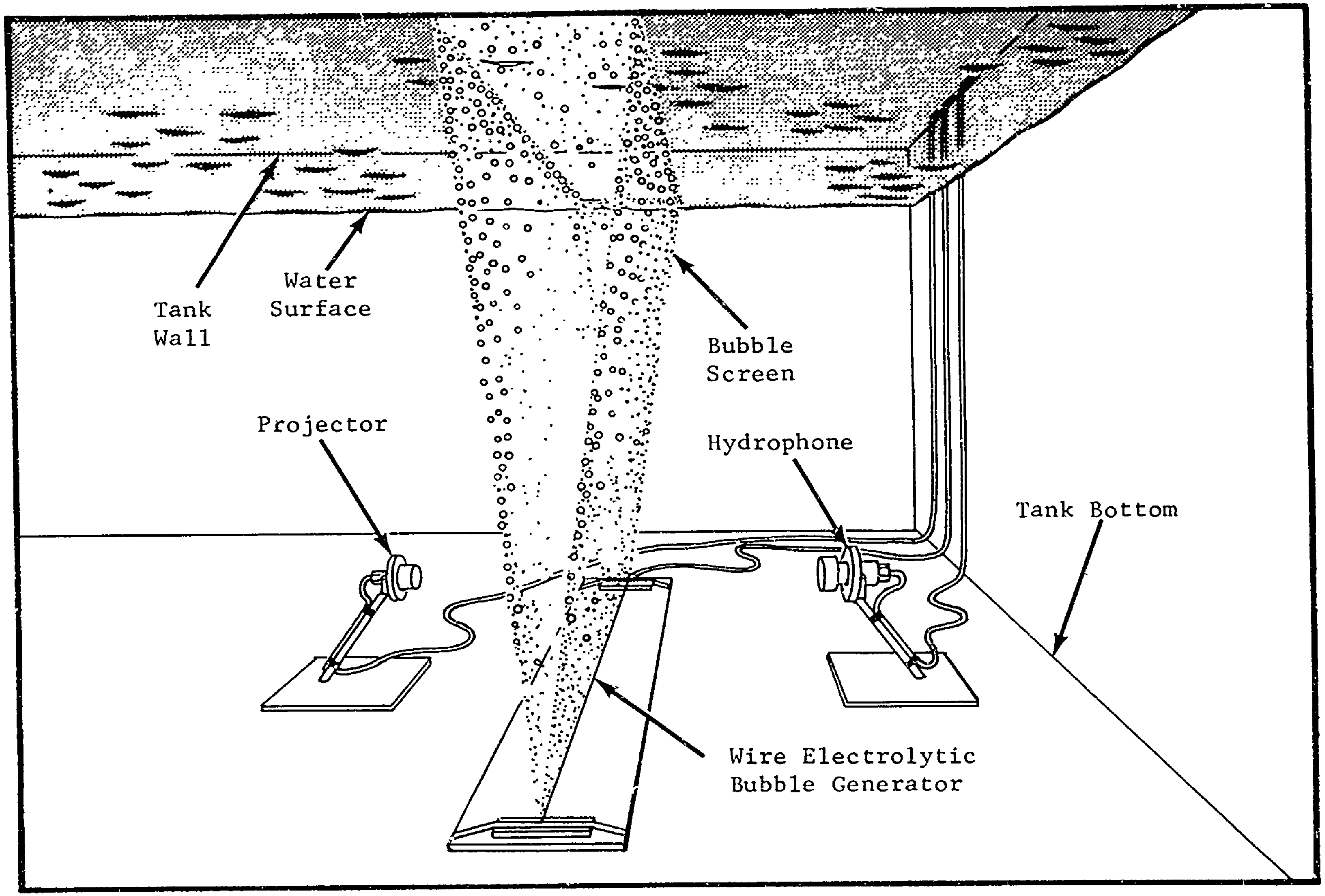

FIGURE 1. APPARATUS FOR ACOUSTIC ATTENUATION MEASUREMENTS

be the difference in db between these two sets of data. At no frequency did a recorded level vary more than 1 db, using the high damping mode of the meter. Wall echoes were eliminated by placing the transducers in such a way that the echoes were directed down the length of the tank; the returning signal, having been reflected many times, had a negligible level. Surface and bottom reflections were eliminated in the received signal by the directivity of the transducers.

The measured acoustic attenuation through the bubble screen is plotted in Figure 2. As a check, this entire procedure was repeated on a different day and the deviation was less than 1 db at all points. The data point at 35μ (95 kc) was not recorded due to distortion caused by one of the transducers.

PHOTOGRAPHIC DETERMINATION OF BUBBLE SIZE DISTRIBUTION

The acoustic method of determining bubble size distributions, although fast and convenient, relies on approximations which have not been verified under the conditions considered here. To validate the acoustic approach a photographic technique to determine the bubble size distribution was employed.

The basis of the photographic approach chosen is a measured curve describing the rate of rise of bubbles in a fluid as a function of bubble radius taken from Reference 1. This curve is plotted in Figure 3 (Curve 1). Also plotted in this figure are other curves representing variations in position of the curve, as noted in Figure 3. It can be seen there is close agreement between the line which best correlates acoustic and photographic peaks and the line determined by Carstensen and Foldy (Reference 1).

In the experiment the cathode wire was placed about 10 cm from a plate-glass window in the side of the tank so that the pictures could be taker through the window. A black cloth backdrop was placed about 20 cm behind the bubble screen; lighting was provided by two 500-watt photo-flood lamps positioned just above the surface of the water and approximately over each end of the cathode wire.

A Speed Graphic camera with a shutter speed of 0.5 sec was used with an f-stop of 18. This f-stop was chosen because it was found to give the best depth of field with sufficient contrast. A metric scale was positioned in the bubble screen and appeared at one side of the photograph for calibration. The 4-inch by 5-inch negative (Ansco Super Hy-Pan Film Pack, ASA rating of 500) was enlarged to an 8-inch by 10-inch print, providing an overall magnification of 1.2. The camera was located at the same vertical position as the center of the transducers.

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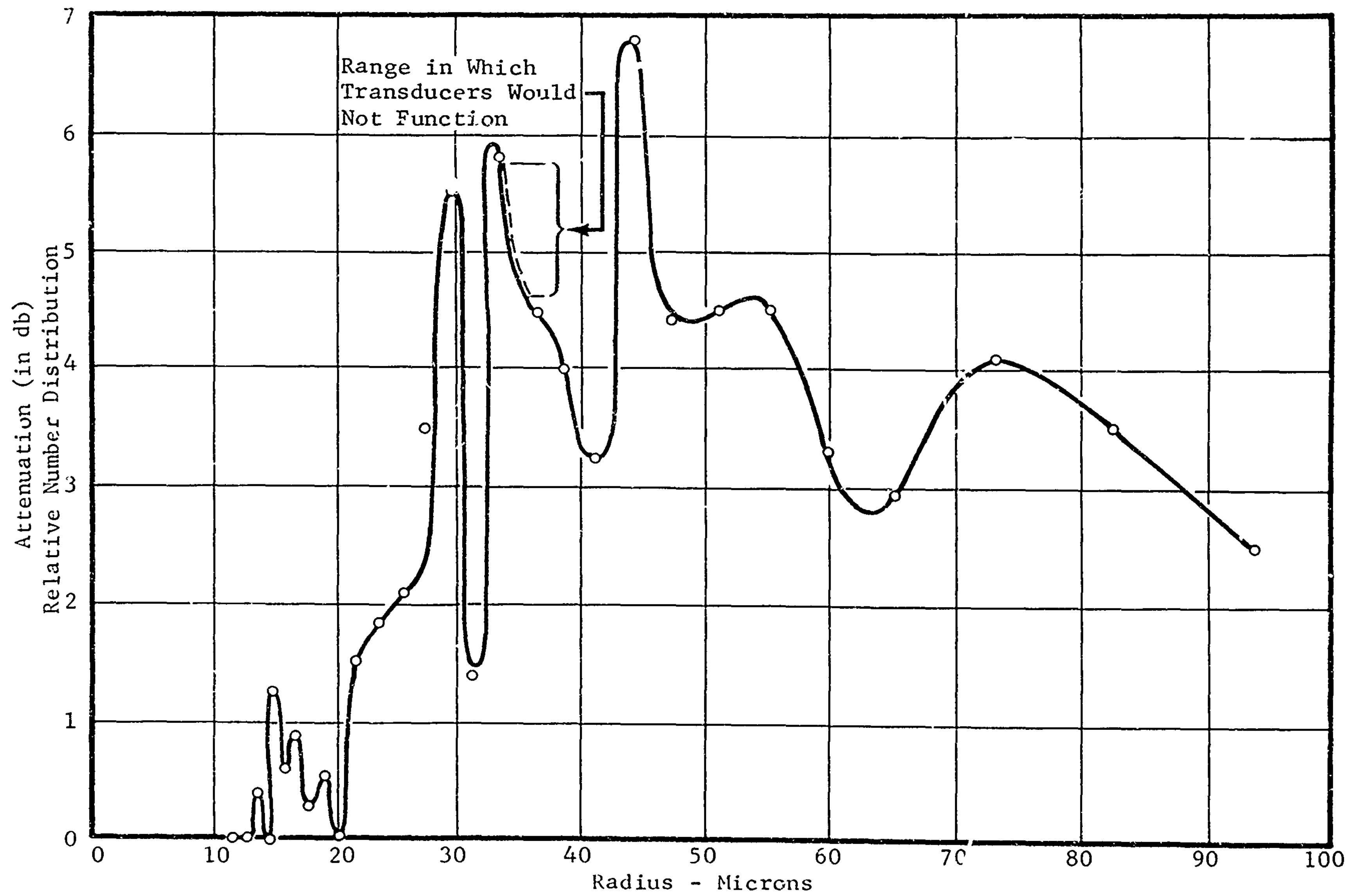

FIGURE 2. MEASURED ACOUSTIC ATTENUATION THROUGH BUBBLE SCREEN

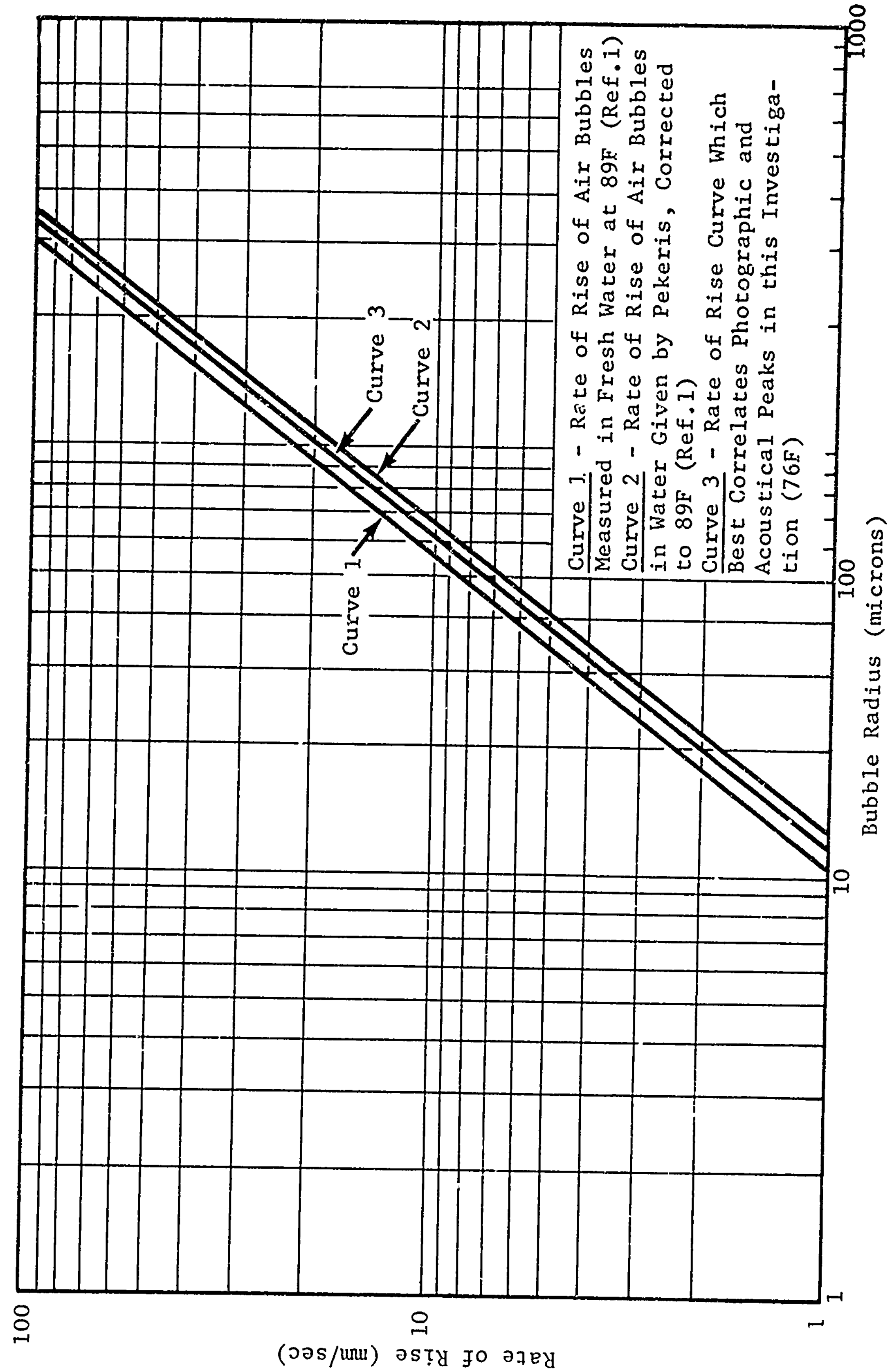

FIGURE 3. RATE OF RISE OF AIR BUBBLES IN WATER

For the size range of bubbles considered, 11μ to 85μ, the 0.5-sec shutter speed constitutes a time exposure, resulting in bubble traces ranging in length from 0.6 mm to 8 mm. A total of 5000 bubble traces were analyzed and tabulated with a 4X comparator. These bubbles were all of the visible bubbles in an area of 138 cm^2 on the photographic print. Of the 5000 bubbles analyzed, less than 20 were greater than 85μ; about 1000 were smaller than 11μ and were not used in this investigation because their size could not be determined accurately. A bubble size distribution calculated from the bubble count data and from Curve 3 of Figure 3 is shown plotted as a dashed curve in Figures 4 through 7. This particular rate-of-rise curve was chosen because it provides the best fit between the two experimental distribution curves; and, within the experimental error involved in measuring this curve, it agrees with the curve by Carstensen and Foldy.

ANALYSIS OF RESULTS

Four different comparisons are made between the acoustically and photographically determined curves in Figures 4 through 7. The photographic curves (dashed curves) are the same in all figures, whereas each of the four acoustic curves correspond to a different integral value of the power to which R is raised in Equation 7. This was done because of the uncertainty introduced by the approximations involved in obtaining Equation 7 from Equation 4, and is essentially a test of the validity of these approximations. The curve in Figure 2 corresponds to the zero power relation in which a distribution simply proportional to the attenuation is assumed. The solid curve in Figure 4 corresponds to the inverse first power; in Figure 5 it corresponds to the inverse second power, in Figure 6 to the inverse third power, and in Figure 7 to the inverse fourth power. Of these five curves, the inverse third power relation, shown in Figure 6, was shown by regression analysis to provide the best fit; it also is the one predicted by the approximated theory. A regression analysis of these four curves for the range of radii from 32μ to 75μ shows, however, that the inverse third power curve is only 2 percent better than the inverse second power curve.

The range of best fit of the radius parameter in these curves is from 32μ to 60μ. For radii greater than 60μ the numbers of bubbles counted for each radius increment is less than 30, and the statistics are no longer as meaningful. For radii less than approximately 32μ, Figure 6 shows that the acoustic and photographic curves no longer agree. This disagreement has been attributed to various phenomena affecting the acoustic properties of the bubble:

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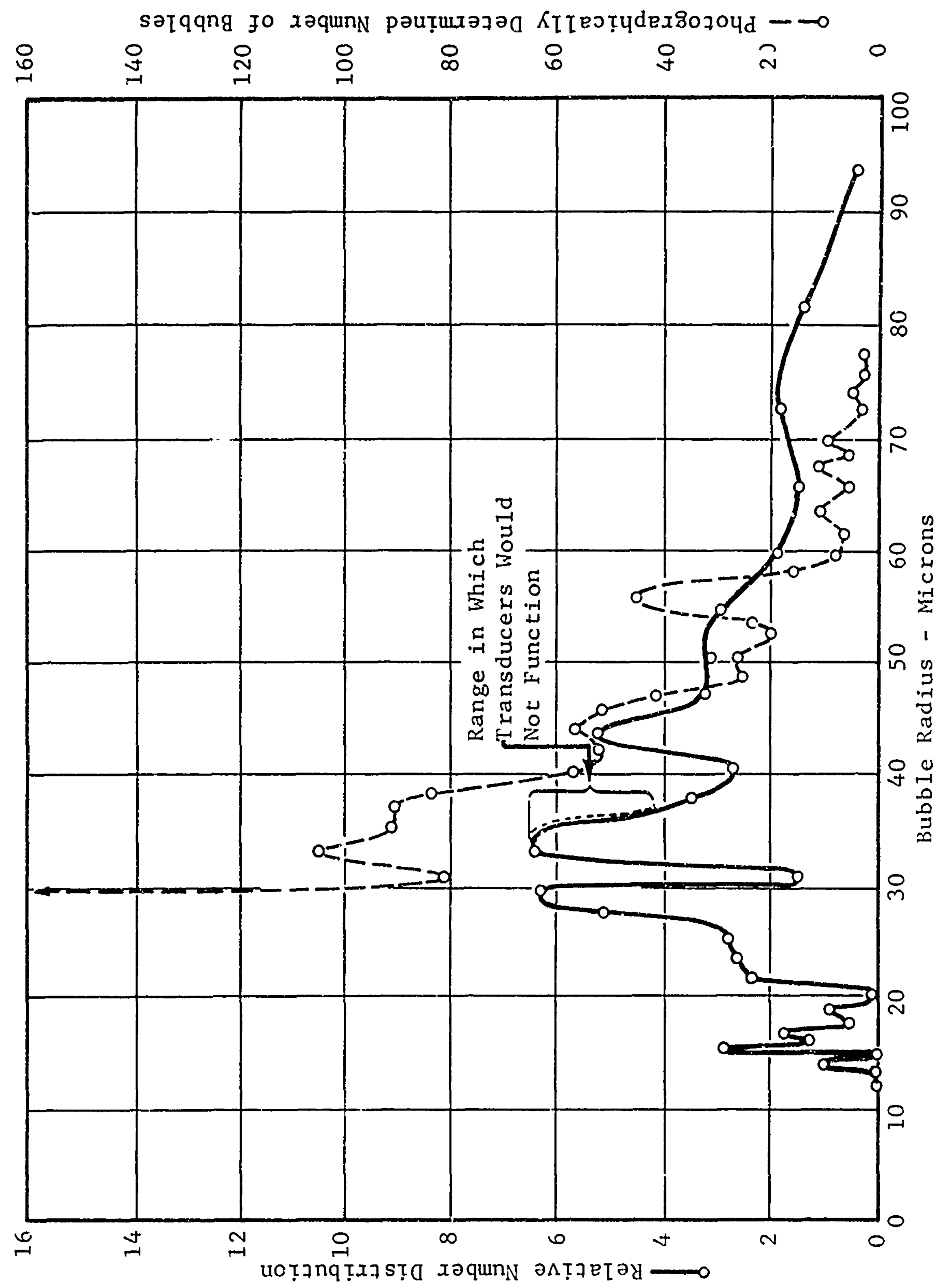

FIGURE 4. PHOTOGRAPHIC DETERMINATION AND ACOUSTIC DETERMINATION ASSUMING INVERSE FIRST POWER RELATIONSHIP

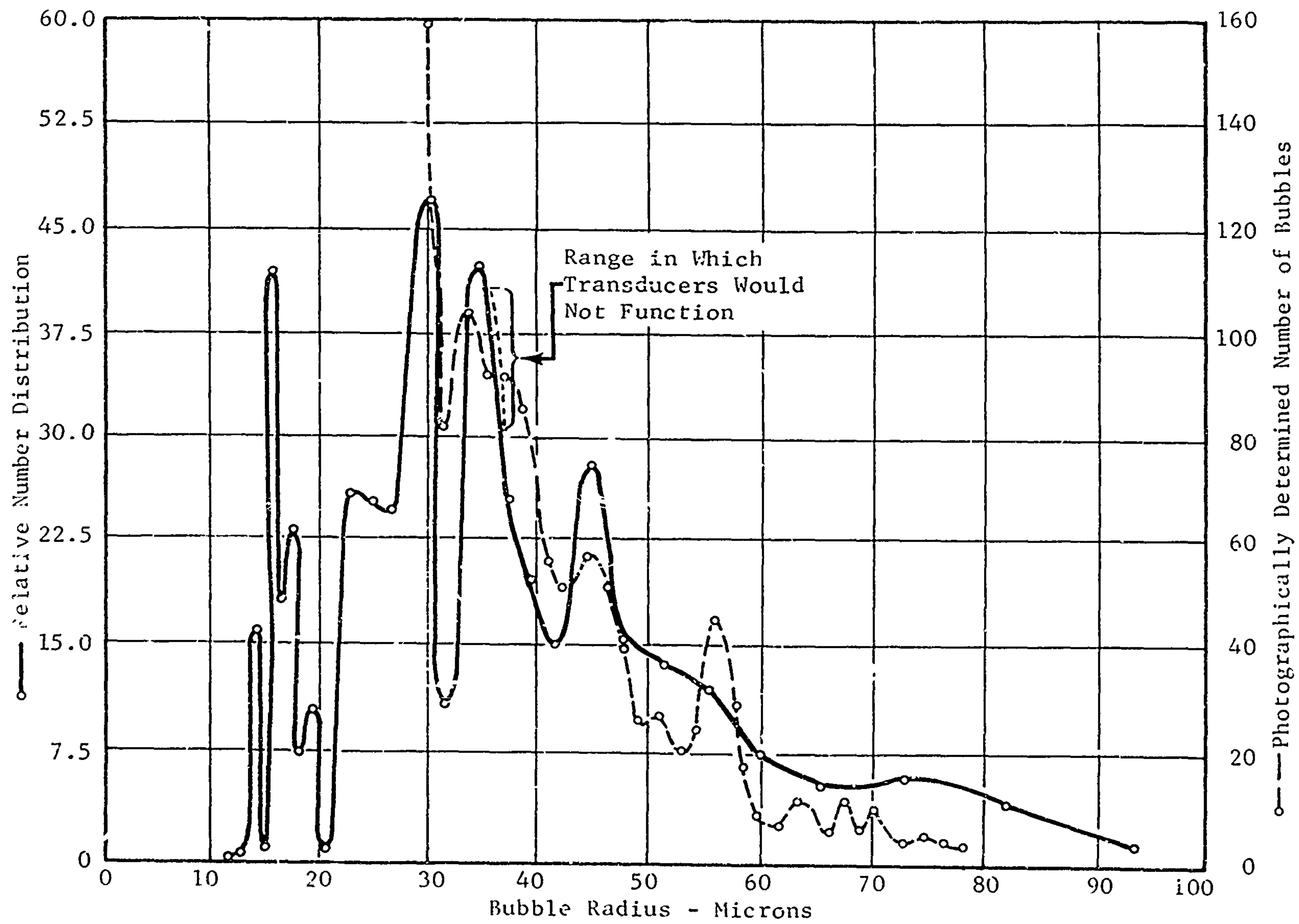

FIGURE 5. PHOTOGRAPHIC DETERMINATION AND ACOUSTIC DETERMINATION ASSUMING INVERSE SECOND POWER RELATIONSHIP

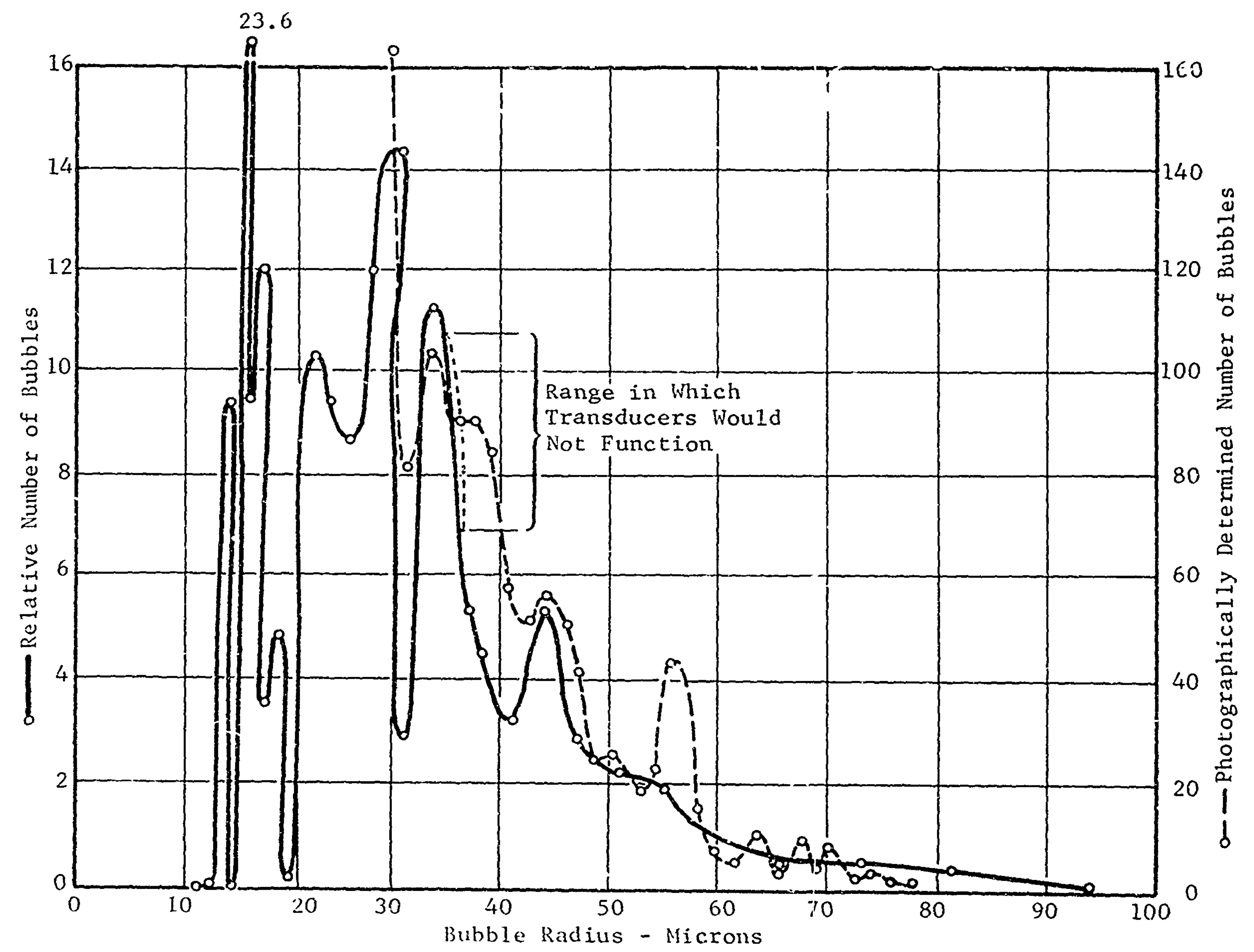

FIGURE 6. PHOTOGRAPHIC DETERMINATION AND ACOUSTIC DETERMINATION ASSUMING INVERSE THIRD POWER RELATIONSHIP

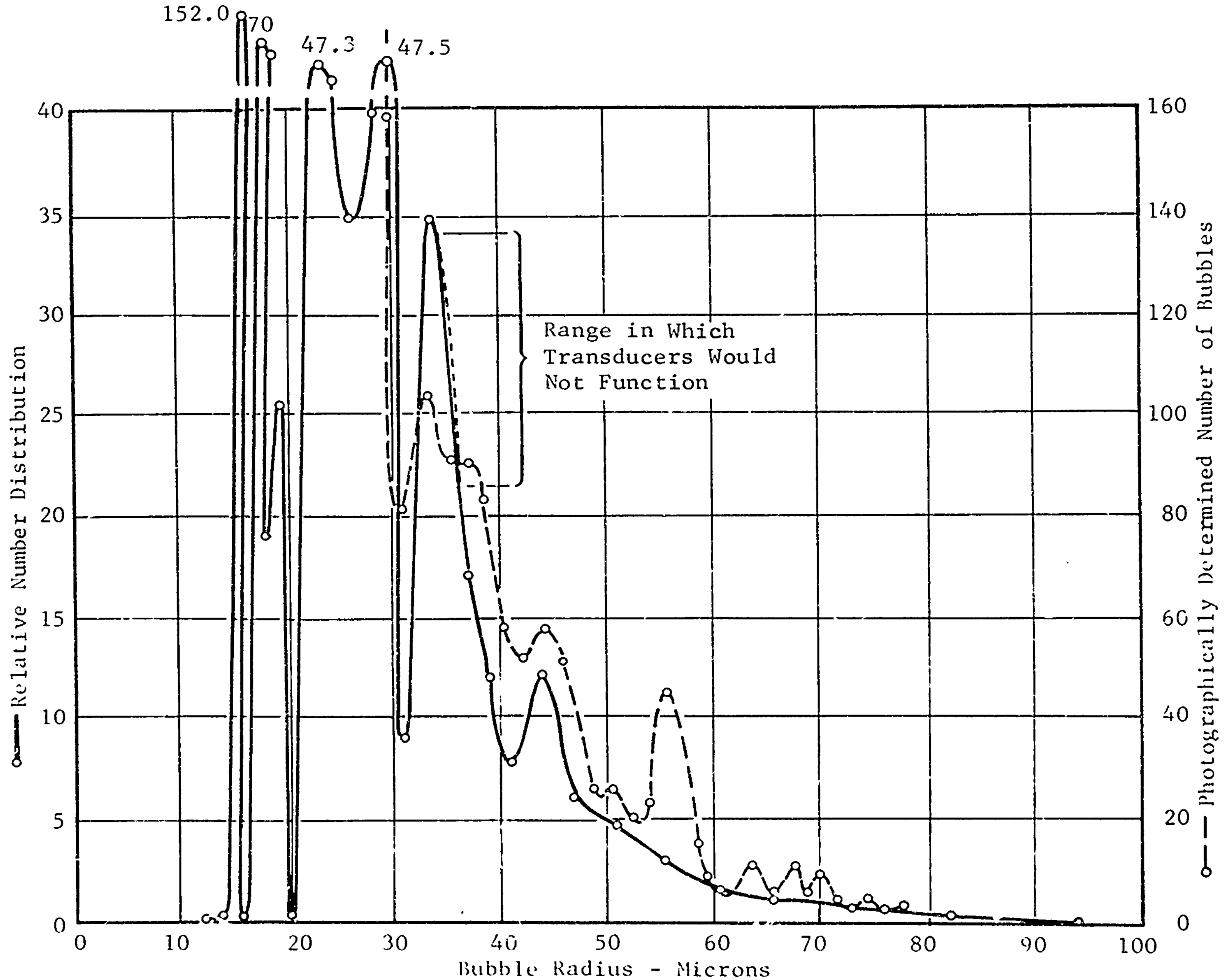

FIGURE 7. PHOTOGRAPHIC DETERMINATION AND ACOUSTIC DETERMINATION ASSUMING INVERSE FOURTH POWER RELATIONSHIP

1. Due to surface tension and acoustic dissipation by heat conduction between the gas and liquid and by viscous friction of the liquid taking part in the oscillation, the acoustic theory given previously is not valid for radii less than about 30μ. Reference 4 gives this limit as 50μ and References 3 and 5 as 32μ. The present investigation agrees with the 32μ limit.

2. Due to "anomalous" bubbles (Reference 5), caused by dust particles adhering to the bubble surface, the functional relationship between attenuation and bubble size may be affected. The percentage of these bubbles, as compared to the total number of bubbles present, increases with decreasing radii. Below 30μ there is evidence to indicate that a majority of the bubbles are of this anomalous type (Reference 5).

Also, due to the inverse third power relationship assumed in the conversion of attenuation to size distribution, increasingly larger numbers of bubbles are required to produce significant attenuation as one considers decreasingly smaller radii. In this investigation the measured attenuations, for bubbles less than 20μ in radius, were less than 1 db. Since the measurement of such small attenuations exceeds the precision capabilities of the acoustic instrumentation, it also provides a likely source for some of the disagreement observed in the curves.

It is unfortunate that the bubble size distribution generated in this experiment apparently peaked below 30μ.

It is to be noted that the ordinate scale for the photographic curves in Figures 4 through 7 is the actual number of bubbles counted in a 96-cm^2 area of the bubble screen, whereas the ordinate scales for the acoustic curves are arbitrary and represent the relative number of bubbles of the various sizes rather than the absolute values.

SUMMARY AND CONCLUSIONS

By comparison of photographically and acoustically determined bubble size distributions, this investigation has attempted to verify that by acoustically measuring attenuation as a function of frequency, a reasonably accurate relative bubble size distribution can be determined. For a heterogeneous mixture of bubbles with radii between 32μ and 85μ, corresponding to resonant frequencies from 39 kc to 110 kc, rather crude agreement in shape was found. To obtain an absolute bubble size distribution acoustically, more information is needed about the "damping constant" for nonresonant bubbles. A better fit between the acoustical and photographic curves possibly could be obtained if the integration involved in the acoustical method could be done by numerical techniques, thus avoiding the errors introduced by approximating the solution of this integral as an expression inversely proportional to the third power of the radius.

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APPENDIX A

INSTRUMENTATION

The block diagram of the acoustic instrumentation used in this experiment is shown in Figure A1. The equipment used may be further described as follows:

1. Test Oscillator: Hewlett-Packard Model 650A

2. Frequency Counter: Hewlett-Packard Model 5238

3. Power Amplifier: McIntosh Model Mc60

4. Preamplifier: Tektronix Model 1121

5. Voltmeter: Ballantine Model 801-58 (High Damping Mode).

Not shown in the diagram, but also used were:

1. Oscilloscope: Tektronix Model 555

2. Amplifier: Keithly Model 102B-R (in series with counter).

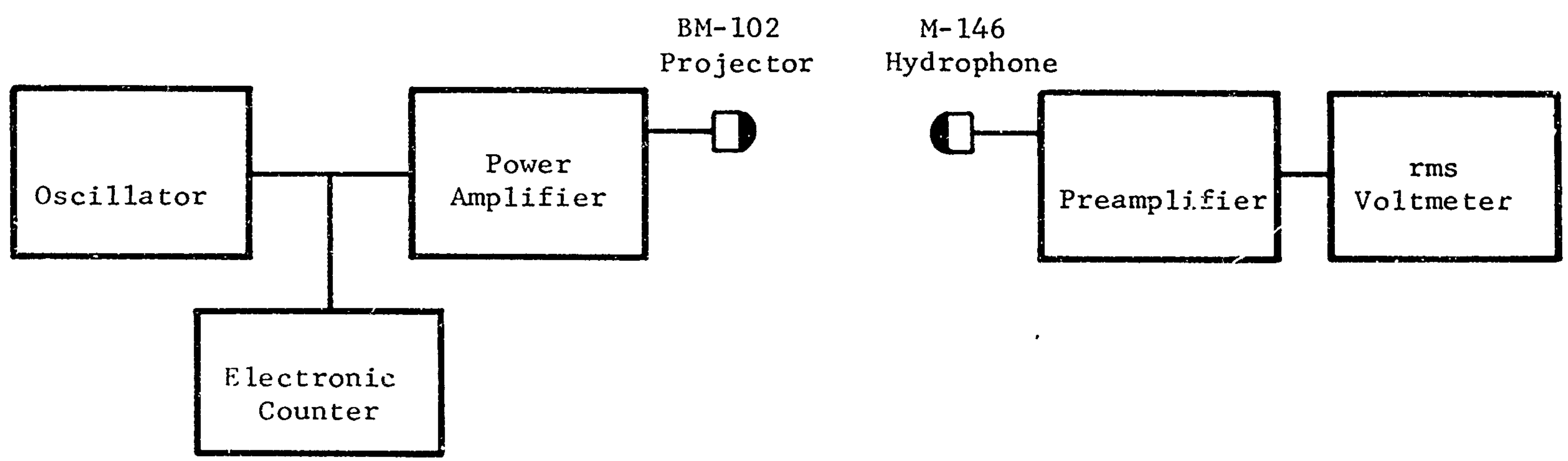

FIGURE A1. BLOCK DIAGRAM OF ACOUSTIC INSTRUMENTATION

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13 ABSTRACT

The purpose of this investigation was to determine the feasibility of measuring bubble size distributions acoustically. The experiment consisted of a comparison of a photographically and an acoustically determined bubble size distribution of an electrolytically generated bubble screen. The approximate range of bubble radii considered was from 15μ to 85μ. Results indicate the acoustic method may be reasonably accurate in determining a relative bubble size distribution for spherical bubbles with radii greater than 32μ, but more information is needed about the damping constants of nonresonant bubbles; and a more precise evaluation of an integral in the acoustical theory is needed before the validity of the acoustical method can be established.

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