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(Final Report)

CONCENTRATED VORTEX MOTIONS IN FLUIDS

(ABSTRACTS OF PAPERS)

SYMPOSIUM AT ANN ARBOR, JULY 6 - 11, 19..

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LIST OF CONTENTS

INTERNATIONAL UNION OF THEORETICAL AND APPLIED MECHANICS

Abstracts of Papers for the Symposium on

CONCENTRATED VORTEX MOTIONS IN FLUIDS

at

The University of Michigan, Ann Arbor

July 6-11, 1964

'Remember when discoursing about water to adduce first experience and then reason.'

VORTEX MOTIONS

(i) 'So moving water strives to maintain the course pursuant to the power which occasions it, and if it finds an obstacle in its path it completes the span of the course it has commenced, by a circular and revolving movement.'

(ii) 'Of the eddies one is slower at the centre than on the sides, another is swifter at the centre than on the sides; others there are which turn back in the opposite direction to their first movement.'

(iii) 'It often happens that when one wind meets another at an obtuse angle, these same winds circle round together and twine themselves together into the shape of a huge column, and becoming thus condensed the air acquires weight. I once saw such a hollow column assume the shape of a man above the sand of the seashore, where these winds were raging round together and digging stones of a considerable size from this hollow, and carrying sand and seaweed through the air for the space of a mile and dropping them in the water, whirling them round and transforming them to a dense column which formed thick dark clouds at its upper extremity; and beyond the summits of the mountains these clouds were scattered and followed the direct course of the wind when it was no longer impeded by the mountains.

(iv) 'From that time the said eddies with various revolving movements, proceed to consume the impetus that has been begun. And they do not remain in the same positions, but after they have been formed thus, turning, they are borne by the impetus of the water in the same shape, in which they come to make two movements; the one is made in itself by its own revolution, the other as it follows the course of the water which is carrying it along all the time that it is destroying it.'

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LIST OF CONTENTS

LIST OF CONTENTS (CONTD.)

LIST OF CONTENTS (CONTD.)

LIST OF CONTENTS (CONTD.)

LIST OF CONTENTS (CONTD.)

LIST OF CONTENTS (CONTD.)

LIST OF CONTENTS (CONTD.)

ELEMENTARVORGÄNGE TURBULENTER BEWEGUNG

W. Albring, Dresden

Durch O. Reynolds wurden in den Bewegungsgleichungen für turbulentes Strömen, die instationären Glieder durch Mittelwerte über längere Zeiten ersetzt. Obwohl damit exakte Ausdrücke für die Spannungskomponenten hergeleitet sind, konnte die umgestaltete Bewegungsgleichung nicht integriert werden. Man begnügte sich später mit halbempirischen Ansätzen.

Es ist aber lohnend einmal über die nichtstationären Vorgänge nachzudenken, und darüber, wie das Abbremsen der Drehung von Wirbeln zu Stande kommt. Oseen lieferte für das Erlöschen der Zirkulation des Potentialwirbels in zäher Flüssigkeit eine Integration der Navier-Stokesschen Gleichungen. Der Vortragende zeigt, dass für Überlagerungen von Wirbelfeldern tiefer Einblick in die Wirbelbewegung gegeben werden kann an Hand einer Integration der instationären Bewegungsgleichungen. Betrachtungen über die Bilder von vergrösserten länger lebigen Wirbeln aus Einzelwirbeln schliessen an, eine Betrachtungsweise die von Bedeutung ist für den Übergang laminar-turbulent.

EXACT SOLUTIONS FOR THE UNSTEADY
TWO-DIMENSIONAL VORTEX SHEET PROBLEM

Richard C. Alexander
The Johns Hopkins University, Baltimore

The problem is considered in which any number of infinitely long wedges are spaced equally apart in incompressible, frictionless liquid with their tips at the origin. At $t = 0$ the wedges are jerked impulsively at constant velocity q, the velocity vectors making equal angles with each other. Vortex sheets are shed from the wedge tips with their common core at the origin. Exact expressions are found for the velocity potentials for all time after the wedges have receded infinitely far away, i.e. exact solutions for vortex sheets with no obstacles present.

The vortex sheets are logarithmic spirals whose sheet strength decreases toward the core center as r/t. A finite pressure high is found at the core, the fluid being at rest there, and the pressure decreases along the sheet branches as $-(r/t)^2$.

Author indicates a method for including the wedges in the flow field. The total solution with wedges present is still essentially that for logarithmic spirals for $r \ll qt$ where qt is the distance of wedge tips from the core.

The analysis gives insight into the nature of unsteady vortex sheets originating from obstacles in real fluid flows. The single branched vortex sheet is of particular interest since this often occurs in nature behind such sharp-edged obstacles as airfoils and mountain ranges. At sufficiently high Reynolds numbers and low Mach numbers, the perfect fluid solutions are valid beyond a small distance r_0 from the core and outside a thin, unsteady, spiralling boundary layer proceeding from core to obstacle in flow. Under the above conditions, the spiral is logarithmic except near the obstacle. There is thus a "logarithmic spiral region". Viewed from the boundary layer, the impressed pressure is quadratic in r/t and the exterior velocity decreases toward the core center as r/t, so long as $r \gg r_0$

THEORETICAL ANALYSIS OF THE EFFECT OF COMPRESSIBILITY ON THE PRIMARY AND SECONDARY FLOWS IN VORTEX TUBES

Olof Anderson

United Aircraft Corporation Research Laboratories, East Hartford, Connecticut

A theoretical analysis was performed to determine the effects of compressibility on the three-dimensional flow within a vortex tube having turbulent end-wall boundary layers (secondary flow) and laminar primary flow. The secondary flow was analyzed by an integral boundary layer technique and the primary flow was analyzed by solving the general equations for a viscous, compressible, heat-conducting fluid. All solutions obtained include the interaction between the primary and secondary flows.

The results of the analysis indicate that:

(1) Compressibility has little effect on the primary flow streamlines and radial distributions of circulation and secondary mass flow for cases in which the tangential Reynolds number, radial Reynolds number and vortex tube geometry are fixed and in which the maximum local Mach number does not greatly exceed one.

(2) The radial temperature gradients in the primary flow due to thermal conduction and viscous dissipation which have been predicted by Deissler and Perlmutter (Proc. 1958 Heat Transfer and Fluid Mechanics Inst.) for one-dimensional vortex flows with low radial mass flows, also occur for three-dimensional vortex flows in cases with high total radial mass flows because of the passage of a major portion of the total mass flow into the end-wall boundary layers.

(3) The total temperature of the secondary flow tends to decrease relative to the total temperature of the injected flow by the action of viscous stresses produced by the secondary flow.

This investigation was conducted by United Aircraft Corporation Research Laboratories under Contract AF 04(611)-8189 and AF 04(611)-7448 with the Research and Technology Divisions, Edwards Air Force Base, California, Air Force Systems Command, United States Air Force.

THE VELOCITY DISTRIBUTION IN A SINGLE TRAILING VORTEX FAR DOWNSTREAM

G. K. Batchelor, Department of Applied Mathematics and
Theoretical Physics, University of Cambridge

The flow field investigated is steady and axisymmetric, and far from the axis of symmetry the velocity components relative to cylindrical co-ordinates (x,r,φ) are of the form

$$u = U, \quad v = 0, \quad w = C_0/r,$$

where U and C_0 are constants. It is assumed that the axial velocity component u everywhere differs from U by a small amount only, so that, as in the theory of axisymmetric wakes without swirl,

$$u \frac{\partial}{\partial x} + v \frac{\partial}{\partial r} \text{ may be replaced by } U \frac{\partial}{\partial x}$$

in the equations of motion.

The circulation function $C(= rw)$ decays, under the action of viscosity, independently of u, and has the same Gaussian asymptotic dependence on r as for a line vortex dependent on t instead of x. The effect of decay of the swirling motion is to increase the pressure near the axis and consequently to decrease the axial velocity there, in this way the induced drag associated with the generation of the trailing vortex is gradually manifested as an ordinary wake with axial velocity defect. The asymptotic expression for the axial velocity defect is the sum of the two terms, one of the form

$$x^{-1} \times \text{function of } (r^2 U/4\nu x),$$

as for a wake without swirl, and the other of the same form multiplied by log x. The two functions have been determined from the equation of motion.

ON THE RELATION BETWEEN VORTEX MOTION AND THE TRANSPORT OF MATTER IN A KÁRMÁN VORTEX STREET AT LOW REYNOLDS NUMBERS

E. Berger, Berlin

An analysis of the Kármán vortex street by means of the hot-wire technique in the wake of a circular cylinder at low Reynolds numbers seems to indicate that there exists a slip between transport of matter and the motion of the vortices in the wake.

This assumption is based on the discrepancy between the two experimentally determined values of the carrier velocity U_v of the vortices in the street, namely the phase velocity of the vortices $U_{v\varphi}$ and the velocity U_{vm} of the vortex-centre measured directly by the hot-wire.

On the other hand the value of the carrier velocity of the vortices U_{vc}, calculated by potential theory from data of the analysis is in agreement with the phase-velocity $U_{v\varphi}$. For this agreement the length of the vortex-trail and the viscosity are insignificant.

A possible reason for this discrepancy $U_{vc} = U_{v\varphi} < U_{vm}$ is that matter is transported through the vortices.

The application of Friedmann's vortex theorems on incompressible viscous flow yields a criterion for the conservation of vortex-lines, which is always satisfied in plane flow.

Thus, a slip between the transport of matter and the motion of the vortices seems to be possible only if the flow is not plane.

New measurements have shown that at low Reynolds numbers the vortices are shed slantwise and not parallel to the cylinder.

The x-component of the vorticity-vector (caused by the oblique shedding) and the main flow result in a spiraling vortex flow, which is probably responsible for the slip between transport of matter and the motion of the vortices in the wake.

ON THE CURVATURE AND TORSION OF A SINGLE VORTEX FILAMENT

R. Betchov, Aerospace Co, Los Angeles

We consider a very thin vortex filament in an unbounded, incompressible and inviscid fluid. The filament is not necessarily plane. Each portion of the filament moves with a velocity which can be approximated in terms of the local curvature of the filament. This approximation leads to a pair of intrinsic equations giving the curvature and the torsion of the filament, as functions of the time and the arc length along the filament. It is found that helicoidal vortex filaments are elementary solutions, and that they are unstable.

The intrinsic equations also suggest a linear mechanism that tends to produce concentrated torsion and a non-linear mechanism tending to disperse the singularities.

ANNULAR HYDRAULIC JUMPS

A. M. Binnie

Hydraulic jumps are possible in an open horizonal channel if the entering uniform stream is superundal, that is, if its Froude number F exceeds unity. The jumps can be fixed in space by the adjustment of an obstruction, such as a weir, at the channel outlet. With F only slightly above unity, the energy to be dispersed is small, and the jump is undular with waves of cnoidal form. As F is increased, the waves become partially broken and their amplitude is reduced. At large values of F, the energy to be dissipated is great, the jump is fully turbulent and all trace of waves disappears. The same sequence has been found on the free surface of a swirling stream of water discharged from a reservoir down a vertical pipe. An unbroken air core exists, and discontinuities in the radial thickness of the stream are of the three kinds described above. Slides will be shown to illustrate the jumps. The experimental conditions were more difficult than those usually encountered in a horizontal channel, for the jumps moved up and down in an irregular manner over a range of several inches. The purely undular form was much more sensitive than the fully turbulent. Readings of mean wall pressure were obtained together with measurements of core diameter off photographs. With the axial velocity taken as uniform, they support the assumption of uniform angular velocity over a cross-section more closely than the assumption of irrotational motion.

TURBULENT HYPERSONIC WAKES

G. Birkhoff and J. Eckerman
Harvard University, Cambridge

Schlieren photographs, taken at Avco-Wilmington, are shown, of hypersonic wakes behind cones and smooth spheres with diameters d varying from 3 mm to 2 cm for free stream Mach numbers M in the range $5 < M < 20$ and at pressures from 0.05 atm. to atmospheric. An analysis is made of the observed base region geometries, the distance x_{tr} to transition, and the wake radius r_w as a function of the distance x behind the missile. For slender cones, the formula $\xi_{tr} = x_{tr}/d = K(\alpha)M^{1.3}(Re \times 10^{-4})^{-0.5}$ fits the data over the range specified, 2α being the cone vertex angle. Correlations with local Mach and Reynolds numbers are also given. For all shapes, $r_w = Ax^{1/3}$ approximately for $x > 50d$. The effective "turbulence Reynolds number" $Re_T = \Delta(\delta^3)/C_D\Delta\xi$, where $\delta = r_w/d$ and $\xi = x/d$, is estimated. Qualitative discussions are given of observed bifilar and ablation wakes.

DEVELOPMENTS IN THE THEORY OF VORTEX BREAKDOWN

T. Brooke Benjamin

Department of Engineering, University of Cambridge

In a recent paper by the speaker (J. Fluid Mech. 14 (1962), p.593) a theory was developed which appears to explain the vortex breakdown phenomenon satisfactorily. Departing radically from previous theories which supposed the phenomenon to be the outcome of instability of the original flow to small disturbances, the new theory shows it to be in effect a finite transition between two 'dynamically conjugate' states of flow, thus being analogous to the hydraulic jump in open-channel flows. Moreover, several well-known features of hydraulic jumps in practice appears to have counterparts in swirling flows: in particular, a mild vortex breakdown (i.e. one for which the difference between the two conjugate states is small in respect of certain overall parameters) manifests a periodic structure analogous to a weak undular hydraulic jump, whereas a strong breakdown requires considerable dissipation of energy, just as in the turbulent front of a strong hydraulic jump.

The paper to be presented reviews the evidence in favour of the theory and outlines several developments that have been made since the original paper. It is shown how the analogy with open-channel flow can be extended to explain other interesting forms of behaviour in swirling flows, particularly when the cross-sectional structure varies gradually with axial distance, as in leading-edge vortices above delta wings and in swirling flows along gradually diverging or converging ducts. For example, in a swirling flow that is 'supercritical' (according to the definition given in the original paper), the effect of expansion is shown to be generally such as to bring the flow condition towards the critical and hence increase the likelihood of spontaneous vortex breakdown. Brief reference is made to a general theory that has recently been derived for finite-amplitude waves in swirling flows, which complements the vortex breakdown theory in much the same way that the theory of solitary and cnoidal waves bears on the hydraulic jump.

ON COMPRESSIBILITY EFFECTS IN TIP VORTICES

S. N. Brown and K. Stewartson
The University of Durham

The conically symmetric solution of the Eulerian equations of an incompressible fluid obtained by Hall, thought to be descriptive of flow properties in a tip vortex, is generalised to include the effects of compressibility. Fluid properties are assumed to be functions of $r/x = \theta$ where r, x are distances from and along the axis, and conditions are supposed known at $\theta = \theta_2$. The governing equations have an energy integral, and the flow must be homentropic. With a further assumption of slenderness ($\theta \ll 1$) the governing equations take on the form

$$\rho^{\gamma-1}\left[1 - \frac{\gamma - 1}{2}\,\beta\,\frac{d\rho}{du}\right] = \frac{\gamma - 1}{2}\,M_2^2\left(A^2 - u^2\right), \quad -\theta\,\frac{du}{d\theta} = \beta\rho$$

where ρ, u are the reduced density and axial component of velocity and other undefined quantities are constants. For given axial and azimuthal velocities at $\theta = \theta_2$ an acceptable solution is possible for only one value of the radial velocity at $\theta = \theta_2$. This solution extends to $\theta = 0$ if $\gamma < 2$ but if $\gamma > 2$ the density vanishes at a non-zero value of θ. Some comments are made about the solution when the Mach number of the flow at the outside edge of the vortex is small.

DEVELOPMENT AND STABILITY OF STRONG VORTICES IN THE ATMOSPHERE

J. A. Businger, K. Bergman and J. Turner
University of Washington, Seattle

Some simple theoretical models of vortices are analyzed for laminar flow. The simplest case that may occur in the atmosphere is probably formed in a field of constant horizontal convergence. The conservation of angular momentum then leads to the development of strong vortices. It is interesting that the steady state solution of the tangential velocity in this case is independent of the circulation around the vortex. The radius r_m of the maximum tangential velocity is given by $\frac{(\nabla_h \cdot \vec{v}) r_m^2}{2\nu} = -2.51$ where ν is the kinematic viscosity and $\nabla_h \cdot \vec{v}$ is the horizontal divergence of the velocity. Less simple convergence patterns have been analyzed in the same way. The stability of the flow has been analyzed, using the perturbation technique assuming that the flow is stable in the core of the vortex.

A more complete approach involves the temperature field with associated buoyant forces. A steady state model may be constructed using heating at the surface. A complication is encountered by the friction with the surface. In this case the loss of vorticity in the core may be compensated for by twisting horizontal vorticity generated by the mean wind into the vertical. A number of observations made on dustdevils and vortices generated in the laboratory may be described to a satisfactory degree of accuracy. A few observations, however, remain difficult to understand. Finally, some speculations are offered on the generation of a large range of vortices from teacup whirls to the solar system.

ON THE OCCURRENCE OF LONGITUDINAL VORTICES IN THE TRANSITION OF A FREE-CONVECTION BOUNDARY LAYER

P. Čolak-Antić, Freiburg, Germany

An experimental investigation of the occurrence of secondary longitudinal vortices in a free-convection boundary layer along a heated vertical plate has shown two distinct types of concentrated longitudinal vorticity. The boundary layer under consideration was already unstable with respect to primary Tollmien-Schlichting waves but not yet turbulent. The experiments have been conducted in air and in water in order to combine the better visualization possibilities with the more accurate determination of the local perturbation velocities.

In the outer region of the boundary layer discrete longitudinal vortices have been made visible by means of a special orientation property of a suspension of aluminium lamella in water as well as by streamline patterns in different planes. These vortices occur only along a part of the primary Tollmien-Schlichting wave length. The generation, amplification and decay of these vortices can be followed visually. It has been possible to indicate their position with respect to the primary wave.

The second type of longitudinal vorticity occurs in the inner part of the boundary layer bearing a very close resemblance to the three-dimensional behaviour of the Blasius boundary layer in the transition zone.

The continuous shifting in spanwise direction of the arrays of longitudinal vortices has made impossible their detailed investigation by two "X" or "V" hot-wire probes though the longitudinal vorticity nature could clearly be distinguished. Additional types of vortex street stabilizing arrangements are being examined. The hot-wire equipment is being improved so as to enable the measurement of longitudinal vortex formations at the very beginning of their generation. This will make possible a direct comparison with a three-dimensional linearized stability theory which is at present being developed in Freiburg.

ON PROBABILITY STRUCTURE IN A SPECTRAL SAMPLE SPACE AS A MEASURE OF ENTROPY IN ROTATIONAL FLOW

Donald Coles

Havard University and California Institute of Technology

Rotational flows which are not uniquely determined by their ostensible data include stalling and unstalling of airfoils, vortex flow in cavities, baroclinic waves in rotating fluids with heat transfer, and circular Couette flow. For the special case of Couette flow with the outer cylinder at rest, as many as 25 different doubly-periodic laminar states (i.e., combinations of axial and tangential wave number) have been observed at a given speed in a given apparatus with a given fluid. As the speed changes slowly, irreversible but repeatable transitions from one state to another occur at certain critical speeds. In each such transition the number of Taylor vortices or the number of tangential waves changes by an integer. Because the particular state which is observed is determined by the entire operating history of the apparatus starting from rest, it can be argued that a deterministic point of view is not entirely appropriate. Alternatively, the population of accessible states at each speed can be represented by a mean state (at the center of gravity in wave-number space) or by a preferred state (toward which the various transitions tend) or by an expected state (at the center of gravity for a statistically generated ensemble of realizations). In the latter case, an entropy defined as Σ p ln p increases roughly like ln n, where n is the number of distinct states. At higher speeds, where the flow becomes first noisy and then turbulent, the characteristic wave numbers are themselves only statistically defined. However, it is possible that the probability structure of the Fourier components of such a flow can still be taken as a basis for a useful definition of entropy in the sense of uncertainty or disorder.

UNSTEADY VISCOUS VORTEX WITH FLOW TOWARD THE CENTER

Robert G. Deissler
NASA-Lewis Research Center, Cleveland, Ohio

An analysis is made of an unsteady viscous vortex consisting of an annular region in which the flow is radial and tangential and a core region in which an assumed uniform axial flow is also present. The effects of boundary layers, except at the outer boundary, are neglected. In order to carry out the analysis, the Navier-Stokes equation for the tangential flow is solved numerically.

The first case considered is that of an initially fully developed viscous vortex which is subjected suddenly to a zero tangential velocity at an outer boundary. In this case the tangential velocities at all radii decay and approach zero as time increases. The rate of decay is greatest when the ratio of outer radius to core radius is not large, and when the radial flow Reynolds number is small. After an initial period during which the effect of the outer boundary spreads throughout the vortex, the shape of the decaying vortex remains similar.

Next a zero tangential velocity is suddenly applied to a fully developed vortex at the radius where the tangential velocity is a maximum. For radii smaller than that radius, the tangential velocities quickly decay to zero. For greater radii the vortex partially decays and then approaches a new asymptotic shape.

The last case considered is the inverse of the first case. That is, the response of an initially radial and axial flow field without tangential flow to a suddenly applied tangential velocity at an outer boundary is considered. The time required to grow a fully developed vortex is found to be about the same as that required to destroy the vortex considered in the first case. The growth and decay of vortexes resulting from relatively short pulses of vorticity applied at an outer boundary are also studied.

DISCUSSION OF EXPERIMENTS ON MULTICELLED VORTICES

Coleman duP. Donaldson

and

Richard S. Snedeker

An experimental study of the character of vortices in simple cylindrical vortex chambers completely open at one end is presented. The vortex chamber used in these studies was so designed that the radial and tangential velocities could be varied independently and the length-to-diameter ratio of the vortex could be varied throughout the range $1 \leq L/D \leq 5$. The results of detailed static and total pressure distributions both inside and outside the vortex chamber are presented and the general properties of the flow field and velocity distributions in and about the vortex chamber that were deduced from these measurements are discussed. Special attention is given to the problem of transition from one-celled to two-celled vortex structure. It is shown that for turbulent vortices in chambers of this configuration having length-to-diameter ratios in the range $1 < L/D < 5$, the character of the flow is primarily dependent on the ratio of a characteristic tangential velocity to a characteristic radial velocity V/U. Transition from one-celled to two-celled vortex structure started for all vortex configurations tested $(1 < L/D < 5)$ when V/U exceeded approximately 3.

INTERACTION OF STARTING VORTEX AS WELL AS KÁRMÁN VORTEX STREET WITH TRAVELLING SHOCK WAVE *

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Syracuse University, Syracuse, New York

The passage of a plane normal shock front (strength S = 1.8 to 2.0) and the associated drift flow over a two-dimensional NASA 23012 one inch chord airfoil mounted at an angle of attack in a shock tube, generated a columnar spiral starting vortex which interacted with the reflected normal shock front. Spark shadowgraph, schlieren and interferometric records of the shock-vortex interaction revealed that the shock front becomes locally distorted; the transmitted vortex core contracts (becomes elliptical) and within approximately 200 μ sec. regains its circular shape; a cylindrical acoustic wavefront (centered on the transmitted vortex) propagates into the quiescent surrounding medium behind the reflected shock front. These experiments considerably extended earlier schlieren observations of shock-vortex interaction by Hollingsworth and Richards[1]. The alternate compression-rarefraction nature of the wavefront as predicted by linearized analyses by Hollingsworth and Richards[2] and later by Ribner[3] has now been verified experimentally and noteworthy deviations have been discovered. A semi-empirical relationship for the wavefront pressure amplitude distribution (expressed as a linear combination of quadrupole, dipole and monopole acoustic sources) was found to represent closely the experimental results.

The density at the vortex core center decreased abruptly due to interaction with the reflected shock front. For the first 3 milliseconds after interaction the transmitted spiral vortex decayed essentially as a circular incompressible viscous vortex, after which a neighboring high density region appeared and the vortex decayed at a faster rate.

The passage of an incident plane normal shock front and the drift flow over a 1/4 inch diameter circular cylinder generated a Kármán vortex street ($N_{Re} = 10^5$ and N_{ST} = .22 for S = 1.8). Its interaction with the reflected normal shock front was recorded by optical techniques. The observed change in vortex spacing, rapid breakdown of the vortex street and the absence of any detectable acoustic wavefronts will be discussed.

*Work supported by U.S. Army Research Office, Durham:
1. A.R.C. 17985, F.M. 2323 (1955) 2. A.R.C. 18257, F.M. 2371 (1956)
3. UTIA Report No. 61 (1959)

ON SOME PARTICULAR CASES OF THE SOLUTION OF LAPLACE'S EQUATION DESCRIBING THE PRINCIPAL PROPERTIES OF VORTEX FORMATIONS IN FLUIDS

Jiří Dráský, Prague, Czechoslovakia

Vortex formations represent a particular case of discontinuous flow of fluids. Mostly, they are unsteady formations with a pronounced whirling motion. They always contain a core with a marked component of rotational motion and one or two discontinuity surfaces or vortex layers rolling up in the form of spirals. They also exhibit centripetal or centrifugal motions.

In the first approximation, the effect of viscosity may be neglected and the medium considered an incompressible fluid. A solution of plane cases, which describes all the fundamental features of vortex formations in the close vicinity of vortex centers is based on the application of the properties of an integral of Cauchy's type in the neighbourhood of end or internal points of the integration line - the boundary line of the flow. The solution represents a particular case of a boundary value problem with non-stationary density on the flow boundary which is constituted by the discontinuity line inside the fluid. In accord with the behaviour of an integral of Cauchy's type in the vicinity of singular points, the solution is evolved as a suitable combination of logarithmic and algebraic singularities. The solution thus obtained is dependent on time insofar as all conditions of compatibility of the flow must be satisfied at all times in every point (particularly on the discontinuity line). Helmholtz's condition modified for such a particular case defines the time variations of vortex circulation which increases by the inflow of vorticity carried by the simple points of the discontinuity line. The discontinuity line assumes the shape of a logarithmic spiral with angle between the tangent and radius vector $\tau = 60°$, in the vicinity of the end singular points, and that of a symmetrical logarithmic double spiral with angle $\tau = 45°$ in the neighbourhood of internal singular points, according to the solution. Several cases of such plane models of vortex formations are considered and with their aid a model is constructed of a flow field with a vortex pair behind a cylinder starting from rest. For this case a comparison of theoretical and experimental results is presented for two different kinds of start (i.e. with uniform and with uniformly accelerated motions). An agreement as regards the independence of the shape of the discontinuity line of time, the path of the centers of vortices behind the cylinder and the time increase in circulation is shown to exist.

OSCILLATING SLENDER DELTA WINGS

P. T. Fink
Sydney University

A film will be shown of the flow past an oscillating slender delta wing, as visualised by means of smoke in a low-speed wind tunnel. Features of the flow will be discussed.

ON VORTICES BEHIND AN OSCILLATING BLUNT BODY

D. W. Fiszdon
Academy of Sciences, Warsaw

RESULTS OF NUMERICAL STUDIES OF INCOMPRESSIBLE FLOW, USING A DIFFERENCE APPROXIMATION OF THE VORTICITY EQUATION*

Jacob E. Fromm

University of California, Los Alamos Scientific Laboratory
Los Alamos, New Mexico

A review is given of the numerical technique that has been employed to study wake flows at low Reynolds numbers. Results of experiments using the numerical program are examined critically in terms of the vorticity associated with the flow. These include flows which are steady and also those in which a vortex street has developed. The vorticity field is illustrated for the various flows in contour form obtained by a plotting technique developed for such displays. Correlations are made between the vorticity fields and other observables of the complete numerical solution such as streamlines, streaklines and pressure distributions for the flow. An example is discussed in which growth and entrainment of induced vorticity at a boundary may be observed.

Some recent work involving the transfer of heat in wake flow is next discussed. The additional considerations involved in the numerical program are outlined and the analogy between heat and vorticity fields is demonstrated. Included is a short film of a transient solution of the wake flow starting from a potential flow solution. The film first shows the growth of the vortex street in a graphic form through the motion of some selected fluid particles (streaklines) and is then followed by plots of the development of isotherms, contours of vorticity, and streamlines.

A brief examination of term by term properties of the vorticity equation are examined for a steady wake flow and then for an example of oscillating flow.

Finally, if time permits, some recent numerical experiments concerned with the Bénard problem will be included.

* This work performed under the auspices of the United States Atomic Energy Commission.

ON A VORTEX INDUCED IN A ROTATING FLUID BY TILT OF THE ROTATION AXIS

D. Fultz
University of Chicago

In the course of tests for another purpose on a rotating rectangular tank containing liquid with a free surface, we have discovered that an intense vortex can be generated near the axis by tilts of the rotation axis of a few parts per 1000. Instead of attaining an ultimate rigid rotation and smooth paraboloidal free surface, the liquid develops a central vortex without any of the usual mechanisms of suction, boundary layer separation, or thermal buoyancies. The vortex usually pulses due to inertial oscillations but may be very weak and nearly steady if the axis tilt is very small, (< one in 10,000). Preliminary experiments at different depths and rotations and some tentative inferences as to the excitation mechanism are given.

SOME NUMERICAL SOLUTIONS FOR THE VISCOUS CORE OF AN IRROTATIONAL VORTEX

I.S. Gartshore
McGill University, Montreal

A swirling viscous laminar core imbedded in an infinite irrotational vortex has been examined using momentum integral methods. Fourth degree polynomials have been used to approximate the tangential and axial velocity distributions, and numerical solutions have been found for various combinations of free stream circulation and initial core swirl.

When the free stream circulation was very small, some solutions were found which exhibited a short region of reversed flow on the axis. For zero free stream circulation, the governing parameters can be conveniently combined to form a Rossby number, and at the onset of axial flow reversal, this parameter was found to have its established critical value.

For large free stream circulation, an axial flow reversal was found which extended downstream indefinitely.

In all cases the axial velocity on the centre-line finally approached W (the free stream value) or $-\frac{10}{11}$ w, and the viscous region grew parabolically. For some combinations of the free stream circulation and initial core swirl, continuous solutions could not be found. The singularities are probably due to a breakdown of the "boundary layer" approximations used in the analysis.

ON THE FINAL STATE OF TRAILING VORTICITY

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The wake far downstream of a lifting body takes the form of a viscous vortex doublet. This paper examines the distribution of velocity and vorticity within the wake, and also its overall motion. If x is distance along the wake, it is found that, for large x, the radius of the wake grows like $x^{\frac{1}{2}}$, and that its centre is displaced transversely by a distance proportional to log x.

At large distance x, the situation is similar to that for large time t in a diffusing line vortex pair, with no variation in the x-direction. The non-linear convective terms in the equations of motion are of smaller order than the viscous terms, and consequently the first approximation to the solution is found by differentiating the known exact solution for the vorticity in a single line vortex. The solution could also contain an arbitrary transverse velocity v. The unbalanced convective terms resulting from this first-order solution are allowed for by proceeding to a second approximation, and it is found that only for one particular choice of v can a non-singular second-order solution be obtained. Other possible methods of estimating v are also discussed.

A similar analysis may be applied to discuss the ultimate state of a viscous vortex ring. In this case the radius of the ring grows like $t^{\frac{1}{2}}$ and its overall velocity is proportional to $t^{-3/2}$. Thus the vortex ring approaches a final position in space, while the displacement of a line vortex pair or a vortex wake increases without limit.

FLOW DUE TO A ROTATING DISC WITH A SINK AT THE CENTER

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The flow due to a disc rotating in an infinite fluid with a sink located at the center of the disc is studied both numerically and experimentally. For the numerical study, the problem is treated as a time-dependent flow due to an instantaneous introduction of the sink. The initial conditions are taken to be the steady state conditions of the rotating disc problem without the sink as solved by von Kármán. It is assumed that the conditions at infinity (both large z and large r) are the same as those given by von Kármán's solution and that the flow is axially symmetric. The problem is formulated in terms of three simultaneous partial differential equations, viz (1) a time-dependent equation in the circumferential vorticity, (2) a time-dependent equation in the circumferential velocity, and (3) an equation of the elliptic type relating the stream function and vorticity. An implicit alternating-direction technique is employed to advance the circumferential velocity and vorticity fields in time. Using these values of the vorticity, the method of successive over-relaxation is then used to solve for new values of the stream function. Radial and axial velocity fields, as well as the vorticities on the disc, are computed by considering the appropriate Taylor's series expansions for the stream function. The computation was carried out on an IBM 7090. Results are presented for the transient state as well as the steady state.

In the experimental study, a 24 inch diameter disc is rotated in a quiescent atmosphere. The relative motion of the air with respect to the disc is observed from above using a rotoscope. The location and shape of the streamline which divides the flow into the sink from the flow outward are observed for various values of the angular speed of the disc and strength of the sink. The results are plotted in dimensionless form and compared with the numerical results.

THE STRUCTURE OF CONCENTRATED VORTEX CORES

M. G. Hall
R.A.E., Farnborough

A review is given of the characteristic features of, and recent work on, that part of a vortex flow in the neighbourhood of the axis of rotation that may be regarded as rotationally symmetric. Only columnar vortex cores, with straight axes, are considered. The surrounding flow is supposed to be known. In dealing with the characteristic features of such spiralling flows the implications of the appreciable radial pressure gradient are noted; the strong interaction between the tangential and axial components of vorticity, which explains some of the more spectacular phenomena observed, is described; and the implication of the existence of an axis of symmetry, on which the tangential velocity must be zero, is examined: it is observed that there must be a region where diffusion is important and, in consequence, that this admits changes in total temperature of a fluid element in the course of its motion. Most recent work on the structure of concentrated vortex cores concerns either exact solutions of the Navier-Stokes equations, which are described briefly, and practical problems, which may be divided into two classes. The first class concerns some pipe flows and the cores of vortices shed from surfaces, where gradients in the axial direction are small compared with gradients in the radial direction. There are specialized, general but approximate, and numerical methods of solving these problems and these are described in some detail. The second class of problem concerns, for example, the cores of hurricanes and the flows in vortex tubes, where the flow does not exhibit the above simple boundary-layer-like character. A brief discussion of this class is given, in which the nature of the difficulties is examined.

INVESTIGATIONS ON VORTEX BREAKDOWN ON A SHARP-EDGED SLENDER DELTA WING

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When the flow separates from the sharp leading edges of a slender wing, two free vortices are generated over the upper surface of the wing. Under certain circumstances, at high angles of incidence and of yaw for example, these vortices show the phenomenon of vortex breakdown.

The experimental investigations, presented herein, deal with the quantitative determination of the flow characteristics in a vortex upstream and downstream of the breakdown point. A sharp-edged flat plate delta wing of aspect ratio $A = 0.778$ and a tip-chord/center-line-chord ratio of $\lambda = 0.125$ was tested at an angle of incidence of 31° and 0° yaw in incompressible flow. The vortex field was measured in four planes, which were normal to the wing, one plane ahead and three behind the breakdown point. In these planes, the pressure distribution, the velocity distribution and the local flow direction were determined.

The results of these investigations are presented. The phenomenon of vortex breakdown is studied with the help of lines of constant dynamic pressure, of constant total pressure and with the local flow direction in the four successive planes. When breakdown occurs, the velocity is considerably reduced in an area near the vortex axis, whose size increases downstream, finally covering the whole region within the rolled up vortex layer. Near the vortex axis backflow was found. Behind the breakdown point a steep increase of static pressure was found.

These measurements are compared with theoretical calculations after H. Ludwieg. The velocity distribution measured in the plane just upstream of the breakdown point agrees very well with the calculated velocity distribution of a conical vortex. The parameters for the velocity distribution at the breakdown point predicted by H. Ludwieg from the theory of stability of spiralling flows are in good agreement with those obtained from the measurements.

ON AN EXPLANATION OF THE FORMATION OF KÁRMÁN VORTEX STREETS

I. Imai
University of Tokyo

The mechanism of the formation of wakes behind bluff bodies at moderate and high Reynolds numbers is considered. The first stage is the shedding of vortex sheets, which can be explained on the basis of the steady solution of the Navier-Stokes equations. For moderate and high Reynolds numbers, we may use, as a good approximation, the asymptotic solution for infinite Reynolds number, which can be obtained by combination of Kirchhoff's dead-water theory and Prandtl's boundary-layer theory. For moderate Reynolds numbers, the laminar wake thus developed will be unstable, leading to the formation of Kármán's vortex street. For high Reynolds numbers, however, the second stage is the very rapid breaking-up of the vortex sheets, which gives rise to the eddy viscosity and hence to a moderate value of the "effective Reynolds number" R^*, about 40 to 50. This Reynolds number is considered to govern the general flow behaviour; the upstream influence is to make the (time-average) flow pattern near the body similar to the steady flow at Reynolds number R^* while the downstream influence is to give the wake the appearance of a quasi-periodic flow analogous to the Kármán vortex street. The formation of secondary and higher Kármán vortex streets further downstream following the breakdown of the primary one, which has been observed by Taneda, may also be explained in a similar fashion.

ON THE EXPLANATION OF VORTEX BREAKDOWN

J. P. Jones
University of Southampton

The explanation of vortex breakdown is still obscure. Although various mechanisms, such as the formation of standing waves, axisymmetric or spiral instabilities and finite transitions from one flow state to another have been proposed, none leads to a situation which can be conclusively tested. There is even controversy over the nature of the flow after breakdown; sometimes there is an axisymmetric burst, at others there is a spiralling flow.

But recent experiments in a water tunnel lend weight to the instability idea. The position of breakdown over a wing at fixed incidence fluctuates with time. When fully established at the most forward position the breakdown is symmetrical in appearance but whenever it travels downstream the spiralling mode appears. It disappears at the furthermost aft position and the breakdown moves upstream, with a more or less symmetrical appearance, to its original position. This suggests that vortex breakdown begins as a spiral instability. Further support comes from flow visualisation on a model delta wing oscillating in the water tunnel. It is found that breakdown begins as a spiral which, with time, grows and distorts into a symmetric "burst". (An important feature, though secondary here perhaps, is that in the oscillatory motion breakdown can occur at comparatively low maximum incidences.)

Some theoretical confirmation has been obtained by examining the stability of an idealised cavity flow i.e. such as occurs along the core of a leading edge vortex in a water tunnel. Vortex breakdown is known to occur in practice in such flows. It is easily shown that instability can arise, in either axisymmetric or spiral modes. The wavelength of the unstable disturbance depends upon the ratio of the peripheral to the axial speed. This work is continuing; it is likely that there will have to be an appeal to turbulent or viscous diffusion to settle some of the issues raised.

HELICAL VORTEX IN INCOMPRESSIBLE SPHERE WAKE*

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A study of the vortex system in the wake of a sphere was made in a low turbulence wind tunnel for Reynolds numbers (Re) between 250 and 40,000 by means of hot wire anemometers.

For Re < 400, periodic fluctuations with a frequency $\cong 0.12\ U_\infty/d$ were observed. Six hot wires around the edge of the wake indicated that the vortex configuration was probably a chain of loops, such as has been observed in a number of visualization experiments reported in the literature. For 400 < Re < 40,000, the wake was turbulent but contained periodic fluctuations with a frequency of about $0.18\ U_\infty/d$.

The case Re = 2000 was the only one studied extensively. Here, the periodic pattern was nearly axisymmetric, being of a single-pitch helix-like form with a preferred rotary direction. This pattern probably prevailed throughout 400 < Re < 40,000. The observed axial fluctuation magnitude and wave-form, and the mean velocity deficit are satisfactorily predicted by a theoretical model which assumes a gaussian velocity deficit extending along a helical path. The observed radial fluctuation shows the presence of a concentrated helical vortex near the wake edge, a feature not accounted for in this model. Attempts to predict the radial and circumferential fluctuations have been unsuccessful. The pitch and radial dimension of the helix were determined from the spatial variation of the phase of the fluctuations. The pitch of the helix just behind the sphere was found to be near in value to that which is theoretically predicted to be most stable. The decay of the periodic fluctuations into turbulence is much slower than for a cylinder wake at a similar Re. The time-average circumferential velocity across a wake diameter indicates that the circulation about the wake axis is small. The sphere thus experiences at most a small torque.

* This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NA7-100, sponsored by the National Aeronautics and Space Administration.

VORTEX FLOW HYDROMAGNETIC STABILIZATION EXPERIMENTS*

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Flow visualization studies have demonstrated the stabilizing influence of a uniform axial magnetic field on the confined, vortex-type flow of an electrolytic conductor generated by impingement of plane and round jets on the interior surface of smooth tubes. In these experiments the flow was introduced along the length of the containing tube (1.1-in. diameter by 6.5-in. length) and removed at the center of one end, so that the velocity field away from the end walls was predominately two dimensional near the concave wall and three dimensional near the center.

With no applied magnetic field, this flow situation is characterized by an apparent oscillatory instability in the boundary layer on the concave interior surface which occurs at values of peripheral tangential Reynolds number as low as 400 (based on tube radius and tangential velocity at a relative radius of 0.8) and which appears to initiate instability (characterized by gross radial convection) throughout the entire flow field.

By employing a saturated aqueous solution of ammonium chloride at 200°F as the working fluid and a 74-kilogauss magnetic field, values of Hartmann number (based on tube radius) of up to 47 were attained, sufficient to increase the peripheral tangential Reynolds number for transition to instability along the concave wall by a factor of from 3 to 4 depending primarily on the jet injection geometry. The transition Reynolds number was observed to vary approximately linearly with the square of the Hartmann number for values of Hartmann number greater than about 25. Still and motion picture photography of dye traces injected through the tube wall enable observation of the instability and the influence of the magnetic field. Significant suppression by the field of the radial components of velocity fluctuations in the interior of the flow is also demonstrated for Reynolds numbers well above the highest transition values corresponding to the maximum attainable Hartmann number.

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ON THE LEADING-EDGE SEPARATION OF A DELTA WING

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The structure of flow near the leading edge of a delta wing at incidence is considered in a qualitative way. The argument is based on low speed wind-tunnel experiments at the AVA, especially on wing-surface oil-film patterns (Anstrichbilder) and on wool-tuft observations.

According to the theory of Mangler and Smith there should be a line of attachment, both at the lower and upper surface, and a line of separation at the leading edge, from which a free vortex sheet emerges. This flow model is, however, disturbed by a secondary separation at the upper surface, which can clearly be recognized from the oil-film patterns (between attachment-line and leading edge). An attempt is made to establish the features of a corresponding, nonviscous, conical flow: the leading edge is no longer a line of separation, but rather a combined line of both separation and attachment.

In spite of the above argument there is reason to maintain the original Mangler-Smith flow model. Wool-tuft observations indicate, that the secondary separation is a phenomenon remaining inside the boundary layer. In one case the field of bound vortices at the upper and lower wing surface has been computed (from the pressure distribution and the flow direction at the outer side of the boundary layer). Within experimental error there is no discontinuity of vorticity at the secondary separation line. A discontinuity of vorticity occurs, of course, at the leading edge, where the bound vortices are continued by free vortices.

PREDICTING EDDY FREQUENCY IN SEPARATED WAKES

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A correlation of the frequency of eddies in the vortex street wake of bluff cylinders of various shapes, given by Roshko,* suggests that this frequency is established principally by the shear velocity of the vortex sheets generated by the body and the asymptotic spacing of these sheets calculated from a "notched hodograph" theory. However, there is nothing in the theory of the growth of disturbances in plane parallel flow, either in the early stages where linearization is valid or in the later non-linear stages examined by Abernathy and Kronauer**) to explain the sharp selectivity of frequency actually observed.

Alternatively, a momentum analysis which equates the drag on the body with the drag of an idealized vortex street wake provides a roundabout means of calculating frequency. A number of ad hoc assumptions are required and the results are sensitive to modest changes in the assumptions.

A kinematic analysis based on inviscid vortex theory is proposed in which disturbances grow in the vortex sheets as they are convected away from the body. This growth culminates in the formation of eddies of fixed strength. The disturbance amplitude and frequency are established by requiring agreement at the separation point between the seed disturbance in the sheet and the velocity induced there by the wake system. Calculations have been carried out for the circular cylinder. The disturbance wave-number is seen to be sharply set. The effects of Reynolds number are studied by delaying disturbance growth for some distance downstream of separation. Many experimental facts are explained.

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* Roshko, A. NACA TN 3169

** Abernathy & Kronauer, J.F.M. 13, pt.1, 1962.

EXACT VORTEX SOLUTIONS OF HYDRODYNAMIC EQUATIONS AND THEIR APPLICATION TO ATMOSPHERIC VORTICES

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Three distinct closed-form exact vortex solutions of the joint system of Navier-Stokes equations of motion and the first law of thermodynamics including the effect of heat conduction are presented. The first solution is thermally active in that the kinetic energy of the vortex motion is derived from a conversion from the latent potential or thermal energy, while the other two solutions represent thermally inactive motions. By putting in reasonable values for the stability factor and the eddy viscosity coefficient, it is found that the thermally active vortex solution can be used to represent the motion- and pressure-fields of the natural atmospheric vortices such as tornadoes and dust storms.

LEAKAGE VORTICES SHED FROM AEROFOILS IN CASCADE

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Experiments have been carried out, first with isolated aerofoils (Reference (1)) and later with aerofoils in cascade, to study the behaviour of leakage vortices, the vortices formed at a gap in the span of an aerofoil or at a gap between the tip of an aerofoil and an adjacent wall. Measurements have been made of total pressure, rotation and angles in the flow, and of pressure distribution and lift and drag forces on the aerofoils.

When the inlet flow is uniform, the effect of the leakage vortices formed at a spanwise gap is to reduce the pressure on the suction and pressure surfaces near the gap. For small gap-chord ratios a slight nett increase in the mean lift may occur, the decrease in pressure on the pressure surface being less than that on the suction surface. The strength of the leakage vortices increases with the size of the gap.

When the inlet flow has a spanwise velocity gradient, secondary vorticity is formed in the channel flow between the blades. Its effect is found to be favourable, since it reduces the strength of the leakage vortices. When a wall is placed within the gap severe separation occurs on this wall. As the gap is increased and leakage vortices form, the extent of the separation is reduced. There is optimum gap-chord ratio for minimum total pressure loss.

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1. Lakshminarayana B. and Horlock J. H.
A.R.C. Report and Memorandum 3316, HMSO London (1963)

THE BREAKDOWN OF CERTAIN TYPES OF VORTEX

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Under some conditions the core of a vortex exhibits a severe disruption at a particular position along its length. The principal result of this is a breakdown from a steady motion, for which the streamlines are spirals about the vortex axis, to an unsteady and turbulent motion.

The phenomenon is examined in some detail for the type of vortex generated by separation of the flow over a highly swept and sharp edge - a leading-edge vortex of a delta wing. Cinematography of low speed flows reveals that at the breakdown the axis of the vortex deforms very abruptly from a straight line. Just downstream of the point of deformation there is a region of periodic flow where the vortex axis itself takes on a spiral form. A little further downstream the spiral form is disrupted to provide the final turbulent state.

An abrupt change can also be observed with swirling flow in a tube; this may have features similar to those for the spiral breakdown of a leading-edge vortex or, as observed by Harvey, may be axisymmetric with the formation of a bubble of stagnant fluid at the axis. Certain points of similarity between the axisymmetric and the spiral form of breakdown can be noted; both involve a severe retardation or reversal of the longitudinal velocity component and an expansion of the stream tubes in the outer regions. Under certain transient conditions the axisymmetric form has been observed to develop initially and then to change to the spiral form.

The sensitivity of the core of a streamwise vortex to an imposed pressure gradient is examined theoretically and an attempt is made to relate the results to certain features of the observations of breakdown.

VORTEX SHEETS FROM LEADING EDGES OF DELTA WINGS

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A discussion on the usefulness of a vortex sheet theory establishes that an exact prevision of the aerodynamics forces is not important. It is better to find a basic scheme as a reference for the interpretation of experiments in viscous fluids.

A review of the international work shows the progress which has been made during recent years and the interest of a better criticism of the hypothesis common to all the theories actually available.

A uniformly valid solution for an incompressible fluid is then presented, or, more exactly, a uniformly valid approximation is established as a first step before a correction near vortex sheets, the sheet axis is near the Mangler and Smith's one, that is to say, rather far from experimental results which are probably significantly affected by viscous effects. The velocity is infinite on the sheet axis which, in the neighbourhood, is asymptotic to a logarithmic spiral.

The extension of the validity of the solution for subsonic compressible fluids presents no difficulty and it is not impossible to find a convenient adaptation for supersonic compressible fluids.

SOME VORTEX SOLUTIONS OF THE NAVIER-STOKES EQUATIONS

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Steady, axially symmetric, viscous vortex flows driven by a radial convection of angular momentum are considered. An attempt to understand such flows is pursued by examining certain special flows for which exact or nearly exact solutions of the incompressible Navier-Stokes equations can be obtained. Exact solutions are considered with the aid of similarity transformations available for the reduction of the partial differential equations of motion to ordinary differential equations. Nearly exact solutions are obtained by a general expansion of the equations of motion for small Rossby number and by linearizing the equations for perturbations about known flows. Particular attention is given to the latter method by employing perturbations about both simple nonrotating flows and strongly rotating flows. Corresponding to these limits, two simple vortex flows which have a radial convection of angular momentum are solved analytically. For large Rossby number the axial decay of swirl superimposed on a stagnation-point flow (with radial inflow) is solved and for small Rossby number the solution for a weak radial flow through a rotating cylindrical chamber is obtained.

It is found that in flows dominated by rotation, the fluid motion is forced to be two-dimensional except for thin shear regions where all necessary adjustment imposed by the boundary conditions are made. The properties of these different shear layers are found to be dependent on the radial gradient of the basic circulation of the flow as well as the Reynolds number.

THE EFFECT OF BLOWING FROM THE LEADING EDGE OF SLENDER WINGS ON THE VORTEX STRUCTURE

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Recent research has shown that with a jet efflux from the leading edge of a slender wing, separated vortex sheets contain the jet spring from the swept edges. It is found that this form of edge blowing produces a suitable flow which is pseudo-conical at subsonic speeds, and the increase in size and strength of the vortices at a fixed incidence result in an increased non-linear lift developed from the wing. The entrainment effect of the jet is shown to eliminate the secondary spring which exists on slender wings without edge blowing.

A theoretical model has been developed which is an extension of that used by Mangler and Smith and is based on the assumptions that the thickness is negligible and its speed is large compared with that in freestream.

With suitable radiation of the jet efflux direction, it is found that not only is the lift augmented, but the loss in thrust is small. The net result is that edge blowing on slender wings increases lift at a fixed incidence and the lift/drag ratio is increased also.

Applications to possible aircraft layouts are briefly discussed.

EXPLANATION OF VORTEX BREAKDOWN BY THE STABILITY THEORY FOR SPIRALLING FLOWS

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From various experimental investigations the breakdown of the free vortices above delta wings is well known. Several theories have been developed in an effort to explain this phenomenon, but they have not been developed far enough to establish which is the right one by comparison with existing experimental results.

In a former paper the author explained breakdown as an instability of the vortex flow which is caused by pressure rise in the flow outside the vortex. This pressure gradient transforms the vortex flow in such a way that the flow becomes unstable in the sense of the stability theory for spiralling flows. This explanation is supported by the fact that vortex breakdown takes place at those velocity profiles of the vortex flow for which it is predicted by the theory. In view of the idealizations of the theory and the inaccuracies of the experiments this is of course no proof of the rightness of this explanation.

In order to obtain more conclusive evidence for the rightness of the given explanation, we studied how the breakdown would develop in detail if it were caused by an instability like that mentioned above, and compared the results with experimental results of D. Hummel. These investigations showed that the first disturbances to be expected in the vortex flow should be helical vortices with a particular pitch angle. This disturbance initiates the process of breakdown while other effects help to produce the observed violent change of flow. An important result of these investigations is that the breakdown itself is not axisymmetric even if one assumes an exact axisymmetric flow before breakdown. A comparison of these results with recent experiments of D. Hummel shows good agreement; one may conclude that at least in this case breakdown is caused by an instability of the kind described above.

ON THE ASYMPTOTIC STRUCTURE OF A CONICAL LEADING-EDGE VORTEX

E. C. Maskell

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In their theory of the flow past a slender delta wing with leading-edge separation, Mangler and Smith (Proc. Roy. Soc. A, 251, pp. 200-217, 1959) conclude that, to the slender-body approximation, the vortex sheet shed from one leading edge makes a trace in any plane x = constant of the form, near its free edge, of a reciprocal spiral, along which the potential difference $\Delta\phi$ is directly proportional to r_1, the radius vector from the free edge. However, their argument is open to the objection that it takes insufficient account of the influence of the more remote parts of the vortex sheet. And in an attempt to remove this objection, the problem is reconsidered in the present paper.

The main conclusion of the present work is that, although the dominant contribution to the circumferential velocity near the centre of the spiral is locally self-induced, the radial velocity is largely contributed by those parts of the vortex sheet remote from the spiral centre. In consequence, the reciprocal spiral of Mangler and Smith requires modification to the form

$$\frac{dr_1}{d\theta_1} = -\frac{C}{\theta_1^{\,2}}\left(1 + \frac{q_1 \sin 2\theta_1 - p_1 \cos 2\theta_1}{1 - \frac{1}{2}KU}\right) + O(\theta_1^{\,-3})$$

where C, p_1, and q_1 are constants that cannot be determined locally, $K = \tan\gamma$, and 2γ is the apex angle of the delta wing. This is a form for which there is some experimental support in the work of Earnshaw (Aeronautical Research Council R. & M. No. 3281, 1961).

The corresponding distribution of $\Delta\phi$ is found to take the form

$$\frac{d\Delta\phi}{d\theta_1} = -\frac{2\pi\, KU\, C^2}{\theta_1^{\,2}} + O(\theta_1^{\,-3})$$

IN-FLIGHT MEASUREMENTS OF THE GENERATION AND DECAY OF A TRAILING VORTEX SYSTEM

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A description is presented of a project which has as its purpose the determination by in-flight measurements of the geometry of a vortex system trailing from the wing of an aircraft. A small, rotating vane system will survey the vorticity close behind the wing. Far downstream, observations will be made by flying past a large tuft grid. Some preliminary measurements obtained with the tuft grid are presented together with data obtained with the vane in wind tunnel model studies.

THE VISCOUS CORE OF AN UNSTEADY, COMPRESSIBLE VORTEX

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Shock tube flow over a vertical wedge produces a vortex sheet at the apex of the wedge. The sheet rolls up to form a growing spiral. For the axisymmetric, inviscid vortex flow a pseudo-stationary theory was given by HOWARD and MATTHEWS assuming that all stream parameters are functions of the pseudostationary coordinate r=R/t only, where R is the radius and t the time. The solutions of this pseudostationary theory are experimentally tested by optical density measurements with a MACH-ZEHNDER-interferometer. The measurements show deviations from the pseudostationary theory in an inner circular region of the vortex; here the density values are greater than predicted by the theory of HOWARD and MATTHEWS. These deviations can be explained by the action of friction. Friction has to be taken into account within an inner viscous vortex core which is growing with the MACH number of the outer flow.

A theory is given for the unsteady, compressible flow in the viscous core of the spiral vortex. Here the conditions for pseudostationarity are not fulfilled, and use must be made of another law of similarity. All the stream parameters are assumed as functions of the similarity coordinate $\eta = R/\sqrt{t}$, in analogy to methods of boundary layer theory. This new coordinate is introduced into the fundamental equations. Because of viscosity, fluid mass is brought from the outer inviscid region into the viscous core; in the equation of continuity, a source term must be taken into account. The initial values for the inner theory are the corresponding pseudostationary solutions by HOWARD and MATTHEWS at the outer edge of the vortex core. The theoretical density distribution in the vortex core is in good agreement with the experimental results. For the velocity distribution, appropriate solutions are found which however cannot be verified experimentally.

ON VORTEX FORMATION IN A FREE BOUNDARY LAYER UNDER THE ASPECT OF STABILITY THEORY

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An attempt is made to explain the formation of vortices in free boundary layers by means of stability theory. The velocity profile of the basic flow investigated is given by $U(y) = 0.5[1 + \tanh y]$. The streamline pattern of the disturbed flow was calculated using the eigen-functions for amplified disturbances which were obtained by a numerical integration of the Rayleigh equation. Here no qualitative distinction between an amplified and the neutral disturbance was found. But the vorticity distribution shows two concentrations of vorticity within one wavelength for amplified disturbances, but only one maximum of vorticity in the neutral case.

Furthermore, the path lines of particles which were initially placed along straight lines parallel to the x-axis were calculated. Lines joining the positions of these particles, called "limit lines", give an impression of the instantaneous shape of the disturbed flow. With increasing time the boundary layer becomes thinner in certain regions and thicker in others. The limit line originally positioned at the critical layer shows in the thicker region a tendency to roll up. Also, a maximum of vorticity exists there.

We know from the nonlinear Helmholtz equation that the limit lines should be identical with lines of constant vorticity and that the maximum value of vorticity cannot be exceeded. The accuracy of the results obtained by the linearized theory is discussed under these aspects. Disagreement was found in the neighbourhood of the critical layer. Using the nonlinear theory of J. T. Stuart up to third order terms, the vorticity shows the tendency expected from the nonlinear equation, but also that the convergence of Stuart's series is not very good in the inviscid case.

VORTEX PLUMES

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Organised vortex columns in the atmosphere occur widely and include such phenomena as dust devils, fire whirls, tornadoes and hurricanes. These flows cover a very wide range of scales, but have the common feature of intense rotation against a background of weak ambient vorticity, and in each case the flow development depends strongly on thermal convection.

The action of thermal convection in producing local amplification of ambient vorticity in a fluid has been investigated in a set of experiments in which turbulent jets of buoyant liquid are released along the axis of a tank of water initially in a state of rigid body rotation. The effect produced is found to depend critically on the condition of the jet. Positively buoyant jets, in which buoyancy acts to increase the jet momentum as in thermals or downwards jets of salt solution in water, generate vortex plumes which rotate faster than their environment; neutral jets produce a negligible net change in the initial distribution of vorticity; and negatively buoyant jets generate columns with reduced mean vorticity.

This report will give a survey of recent experimental results relating the local amplification of vorticity with the strength of the source of buoyancy, and will also outline related theoretical work based on both turbulent and laminar plume models in an environment with distributed vorticity.

ON THE STABILITY OF VORTEX ROTATIONAL FLUID FLOW

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SOME OBSERVATIONS OF VORTICES FORMED AT A DUCT ENTRY NEAR A PLANE SURFACE

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A pipe having an internal diameter of 2.75 inches was mounted with its entry close to a plane surface and air was drawn through it at speeds up to 160 ft/sec. The pipe could be mounted with its axis parallel or perpendicular to the plane, and in both configurations the distance of the entry from the surface could be varied. Experiments were carried out both with a plain pipe entry and with a fairing fitted to suppress separation at the lip of the pipe.

The vortex formations were observed using a Preston smoke generator. In addition a vane type vorticity 'meter' was placed in the pipe and could be traversed across the pipe section.

With the fairing in position and the pipe axis horizontal in its lowest position a pair of vortices having opposing vorticity formed on the plane just in front of the pipe entry. They were symmetrically disposed about the vertical centre plane of the pipe and their cores passed along the pipe near the bottom of the section. This was the most stable flow regime observed.

With the fairing removed the twin vortex regime became less stable and gave way occasionally to a single vortex which might be in either sense. The flow changed from one regime to the other in apparently random fashion.

This type of flow also occurred at greater heights of the pipe axis, even with the fairing in position.

When the pipe axis was placed perpendicular to the plane a pair of opposed vortices formed near the stagnation point.

AN EXPERIMENTAL INVESTIGATION OF THE LAMINAR BOUNDARY LAYER ON ELLIPTIC CONES AT INCIDENCE

W. Rainbird

N.A.E. Ottawa

LAMINAR FLOW SEPARATION ABOUT ELLIPTIC CONES AT INCIDENCE

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Flow visualisation measurements, using fluorescent dye dilaments, have been made on a series of slender elliptic cones, at incidences up to about 40°, in the N.A.E. 10 x 13 inch water tunnel. All tests were performed at a length Reynolds number of about 2.7×10^{4} thus ensuring that the flow within boundary layers, separated vortex sheets and concentrated vortex cores was laminar. Three models with axis ratios of 1:1, 2:1 and 4:1, and longitudinal area distributions equivalent to a $7\frac{1}{2}^{\circ}$ semi-nose angle circular cone, and two further models with axis ratios of 1:1 and 1.5:1 and area distribution equivalent to a $12\frac{1}{2}^{\circ}$ circular cone, were tested.

Details of the flow at small, moderate and large incidence (relative to equivalent semi-apex angle) have been elucidated and comparisons have been made with some existing wind tunnel results. The positions of all separation and attachment lines and the directions of the "surface" streamlines have been measured and, in the case of the two circular cones, the results have been compared with calculations of the three dimensional laminar boundary layer.

The following are some significant observations concerning the development of the vortices:

(1) The vortices develop conically (for example the vortex axes produced forward pass through the cone apex) except at large incidence where the unsteady effects of the cone base propogate forward.

(2) Vortices of significant strength are not formed immediately following primary boundary layer separation. For example, on the circular cones, primary separation first occurs at a relative incidence of about 0.6 but vortices of significant strength are not observed until the relative incidence is increased to about 1.2.

(3) At large relative incidence secondary separation of the leeward surface boundary layers produces a pair of weak tertiary vortices of opposite sign to the primary vortices. At the same time the vortex sheets shed from the primary separation lines split and roll up into two pairs of vortices, which we have called primary and secondary.

(4) No steady asymmetric vortex positions were observed.

(5) The distribution of streamline helix angle within the primary vortex core is independent of relative incidence.

BACKFLOWS IN ROTATING FLUIDS

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If tangential velocities are larger than axial velocities, rotating fluids moving axially through expanding cross sections can develop secondary, axisymmetric backflows. A rotated free jet has embedded in it a stationary toroidal vortex which expands in size on increase of tangential velocity. The backflow is stabilized at a well-defined stagnation point in front of the jet by control of the radial distribution of impact pressure at some cross-section upstream. With an open swirl tube, an egg-shaped vortex sits in the end of the tube.

Near critical conditions, backflows appear to be somewhat oscillatory. Fully developed systems show back velocities with values as high as one-half the values of initial forward velocities. Flows are highly turbulent, and turbulence conditions affect critical parameters and the appearance of the velocity field.

Size and shape of the secondary flow region and velocity and pressure were measured for a number of systems. Conditions for existence and characteristics of the backflows will be described.

HYDRAULICS OF A ROTATING FLUID

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Observations of typical hydraulic phenomena, including jumps, subcritical and supercritical flows, in a liquid swirling over the inside of a cylinder. The "whirling core" phenomenon.

VORTICES IN THE ATMOSPHERE

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Both the television and infrared sensors aboard the meteorological satellite, TIROS, have observed a wide range of turbulent and vortex motions in the atmosphere as revealed by the clouds. The scale has ranged from clouds associated with tornadoes which were not directly observed to cyclonic circulations several thousand miles in diameter. Eddy motions from the flow around islands and hurricanes have been striking examples of vortices formed in the atmosphere.

Brief descriptions and illustrations of the types and scales of circulations observed will be presented.

BOUNDARY-LAYER INDUCED SECONDARY FLOWS IN JET-DRIVEN VORTEX TUBES

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Flow visualization experiments have been conducted in both a single vortex tube and in a multiple vortex array (vortex matrix) which demonstrate the unusual secondary flow patterns induced by interactions of the primary vortex flow with the end-wall boundary layers. In the simplest geometry, that with planar end-walls, boundary-layer mass ejection produces a number of concentric zones near the axis, in which the axial velocities alternate in sign. Modifications of this flow pattern, produced by geometrical perturbations and by tangential fluid injection in the end walls, have also been investigated. Tangential injection results in a weakening of the axial counter-flows and, if strong enough, can cause cancellation or even reversal of the secondary flows. Alterations of the end-wall geometry have a lesser influence on the secondary-flow structure except in the cases of highly-curved and stepped end walls which produce cylindrical regions where the radial and axial velocities apparently vanish simultaneously. In both cases zones of alternating axial velocities are observed within the "stationary" cylinder.

A theory has been developed which treats the three-dimensional vortex-boundary layer interaction problem. Numerical solutions for the circulation profiles and boundary-layer mass flow distributions have been obtained by an iteration technique. Analytical solutions have been obtained in the limit of large Reynolds number based on the radial flow. In addition to the Reynolds number the solutions depend primarily upon a boundary-layer interaction parameter, which is a measure of the fraction of the total mass flow diverted to the boundary layers.

AN EXPERIMENTAL STUDY OF TURBULENCE LEVELS IN JET-DRIVEN VORTEX CHAMBERS

D. H. Ross

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A vortex flow was produced in a right-circular cylindrical container by means of fine tangential jets in the outer wall; the exit flow passed through a pivoted porous-metal tube which extracted the angular momentum from the flow and was instrumented to act as a sort of angular momentum dynamometer. The torque acting on the tube, together with detailed measurements of the end-wall radial static pressure distribution, was used to calculate the angular momentum balance within the chamber, and thus infer a radial distribution of turbulent shear stress. An "effective turbulent-to-laminar shear ratio", τ_t/τ_ℓ, was calculated on the basis of both two-dimensional and approximate three-dimensional flow models; the latter was obtained by calculating the mass flow diverted into the end-wall boundary layers, based upon the measured circulation distribution and an approximate turbulent boundary-layer theory. Rotating end walls were used to measure the end-wall shear stress.

The latter results were carried out for a variety of mass flows at chamber L/D ratios from 1/5 to 3. The porous tube was deployed both inside and outside the chamber, and the injection geometry was varied. The quantity τ_t/τ_ℓ at a given radius is correlated with the local tangential Reynolds number and other characteristic parameters of the flow; these results are compared with those of other investigators. The results indicate that appreciable turbulence is present in the vortex, even when allowance is made for mass flow diverted into the end-wall boundary layers; corrected τ_t/τ_ℓ ranging from 10 to 10^3 were obtained and could be approximated by the relation $\tau_t/\tau_\ell \big|_m = 1.53 \times 10^{-6} \, Re_{t_m}^{1.44}$.

LAMINAR AND TURBULENT BOUNDARY LAYER DEVELOPMENT ON THE END WALLS OF A VORTEX CHAMBER

Nicholas Rott
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Recent work by Mack, Anderson, and King, which has led to simplifications in the treatment of laminar boundary layer development on the end walls of a vortex chamber by the momentum integral method, is reviewed.

The momentum integral method is applied to the problem of turbulent boundary layers. Radically simplified assumptions on the turbulent shear are made so that an iterative analytical procedure can be found for the solution. The sensitivity of the calculations to different limiting assumptions on the turbulent shear is investigated. In particular, it is found that the total mass carried by the radial flow in the turbulent boundary layer shows very little sensitivity to the assumed shear law, so that this result can be considered valid with some confidence. As expected, the analytical method closely agrees with numerical results obtained previously.

The main advantage of the analytical approximation is the simplification resulting for the treatment of the strong interaction problem between the boundary layer and the external vortex flow.

ON THE ROLLING-UP OF THE CONICAL VORTEX SHEETS FROM A DELTA WING

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This paper continues the sequence, beginning in 1952, in which the author elaborates his conception of the conical rolled-up vortex sheets ('nappes en cornet') above a delta wing at incidence; a conception which is firmly based on the systematic experiments in flow visualisation effected in those days under his leadership at O.N.E.R.A.

The particular problem considered is that of a delta wing in the form of a flat plate extending to infinity downstream at a moderate incidence to a steady, uniform, inviscid, incompressible flow. The flow is assumed to be conical in the sense that the velocity is constant in magnitude and direction along the half-lines through the apex of the wing. The flow is also assumed to be irrotational, the vortex sheets springing from the leading edges being represented by surfaces across which the direction of the velocity changes discontinuously. From this three-dimensional conical flow a two-dimensional pseudo-flow in a transverse plane is constructed. This can be regarded as another incompressible flow, but because the velocity on the transverse plane is not solenoidal, a distribution of sources and sinks must be introduced. In the first place those singularities which lie along the discontinuity surfaces are considered. Boundary conditions derived from the tangency of the velocity to the sheet on both sides of it and from the continuity of its magnitude across the sheet relate the intensities of the vortex and source distributions to the shape of the cross-section of the sheet.

Since it is impossible to imagine that the vortex sheet rolls up indefinitely, at least in the immediate neighbourhood of the wing apex, the presence of a free edge to the sheet must be postulated. The velocity induced at this edge must take an appropriate value and an isolated singularity must occur there. This is shown to be a vortex-sink, a characteristic which explains the impressive stability of the vortex system of the delta wing. The appropriate distribution of the compensating sources is indicated.

It is hoped that these considerations suggest a schematization of the shape of the vortex sheet and the forms of the required singularity distributions. The choice of this would be much assisted by the outstanding theoretical research of Robert Legendre.

ON THE FORMATION OF VORTEX STREETS FROM INSTABILITY WAVES IN THE WAKE OF A THIN FLAT PLATE

Hiroshi Sato
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The two-dimensional wake of a thin flat plate placed parallel to a uniform flow was investigated. The wake is classified into four regions depending on the nature of velocity fluctuations. The first region just behind the plate is the laminar region in which no velocity fluctuation is present. A small-amplitude velocity fluctuation with a sinusoidal wave-form appears in the second, linear region. The observed wave length, the amplification rate, the propagation velocity, and the amplitude and phase distributions of the velocity fluctuation are all in good agreement with predictions of the linearized stability theory. The third is the non-linear region which is followed by the turbulent region. The non-linear region can be divided into two sub-regions. The first sub-region is of transient nature. Distributions of the mean velocity and the velocity fluctuation change in the flow direction. Measurements in the second sub-region indicate that it is in a kind of equilibrium state. The velocity fluctuation consists of a sinusoidal fluctuation of the same frequency as that in the linear region and a fluctuation of doubled frequency. This fact together with visual observations in a water tank indicates the existence of a double row of vortices in the wake. In other words, the vorticity of the instability waves is concentrated and a double row of vortices is formed in the non-linear region.

A theoretical analysis was made of the non-linear region retaining non-linear terms in the equation of motion and considering the deformation of mean-velocity distribution due to the Reynolds stress. Distributions of mean-velocity and velocity fluctuation calculated show a fairly good agreement with experimental results.

CONCENTRATION OF VORTICITY IN NEW HURRICANES

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Theories of hurricanes have dealt fairly satisfactorily with all aspects of their mechanics except their initiation. Normally one thinks of rotation relative to the earth as being intensified by horizontal convergence and a stretching of the vortex lines representing the vertical component of vorticity; but for this to happen in the troposphere there must be a simultaneous divergence aloft with the establishment of anticyclonic vorticity there. Surface heating produces convection clouds rather than a larger scale convergence-divergence system of the order of 200 km. wide and 12 km. deep. The new theory now put forward suggests the following mechanism. If thermal convection is vigorous in an area, because the atmosphere is, on the whole stably stratified except in the convection clouds, there will be a large amount of horizontal spreading out of thermals at stable layers. These diverging parcels of air effect a forcible stirring of a fluid which is initially in a stable configuration of rotation, that is one in which the vorticity and the direction of rotation round the centre have the same sense. A completely stirred body of fluid possessing rotation must have zero vorticity otherwise parcels of it are not interchangeable except by the application of finite forces. Therefore the stirred fluid will tend to acquire zero absolute vorticity and a distribution of tangential velocity like $v = k/r$. The consequence of this is that the air mass stirred by convection has its vorticity concentrated near the centre and some of the energy of the convection is converted into energy of the mean motion. The vortex has rotation relative to the earth of the same sign throughout the troposphere. The low pressure is achieved not by horizontal divergence in the upper troposphere but by subsidence of the lower stratosphere and the tropopause takes the shape of the surface of a bath plug vortex. Once the low pressure centre is established with rotation it becomes a heat source because the air temperature at sea level is lowered and a steady state baroclinic vortex with friction, which we know as a hurricane, can become established.

THE IMPROVED CALCULATION OF THE MANGLER-SMITH MODEL OF LEADING-EDGE SEPARATION FROM A SLENDER DELTA WING AT INCIDENCE

J. H. B. Smith

R.A.E., Farnborough

The calculation by Mangler and Smith (Proc. Roy. Soc.A 251 200-217, 1959) of the flow past a low-aspect-ratio flat-plate delta wing at incidence agree imperfectly with the available observations. In a region of substantially conical flow, the lift is predicted correctly at incidences up to one-half the apex angle, but the position of the core of the spiral vortex and the distribution of pressure over the surface of the wing do not agree satisfactorily with observation.

There are a number of possible explanations for this: the vortex sheet model of the shear layer shed from the leading edge may be inadequate; the secondary separation of the flow streaming outboard under the vortex may exert a significant influence; slender-body theory may not be appropriate to the study of the separated flow at the Mach numbers and aspect ratios of the tests; the account of the interaction between the core of the vortex and the outer part of the spiral may be seriously over-simplified; and, finally, it may be inadequate to apply the boundary conditions satisfied along the length of the vortex sheet at so small a number of isolated points. Arguments based on experiments tend to discount all but the last two of these explanations. Mr. Maskell's paper here is aimed at the penultimate and this paper reports on attempts made with the help of an electronic computer to improve on the accuracy with which the original model was calculated.

Two obvious improvements suggest themselves. First, the boundary conditions that the vortex sheet is a stream surface and that the pressure is continuous across it must be applied at enough points along the sheet to make the possible discrepancies between these points negligible. Second, the extent of the finite part of the sheet so treated should be increased so as to reduce the effect of the simplified treatment of the core. Although the governing differential equation is linear, the determination of the shape and strength of the vortex sheet from the boundary conditions requires the solution of highly non-linear equations by iterative methods.

The limited success achieved so far is described in the paper.

CORNER VORTICES ASSOCIATED WITH SEPARATION
UPSTREAM OF AN OBSTACLE ON A WALL

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In the steady flow past an obstacle of finite width projecting from a wall, one or more vortices may be observed, looped round the front of the obstacle in the corner which it forms with the wall and trailing downstream from its sides. (See for example the Frontispiece and Fig. XII·35 of Thwaites' "Incompressible Aerodynamics"). More correctly described, these vortices are coiled vortex sheets or shear layers, produced by separation of the wall boundary layer upstream of the obstacle. Flow along their cores, draining away the fluid which enters them, is an essential feature which they have in common with vortices produced in other three-dimensional flows with separation, such as the leading-edge vortices of slender wings.

Previous accounts of such corner vortices are first reviewed, and attention is drawn to other work in which observations reported contain evidence of them. Some further experimental observations of corner vortices at cylindrical obstacles are presented and discussed.

The nature of corner vortices is then considered further from various points of view. The realignment of the vorticity of the wall boundary layer by the corner vortices is qualitatively as predicted by secondary-flow calculations for small perturbations of an inviscid shear flow; the validity of the secondary-flow approach is discussed. When the flow is entirely laminar the separated shear layer sometimes rolls up into an array of vortices; this process is compared with the rolling-up of a two-dimensional shear layer into a moving array of vortices.

Finally, the significance of the vortices in practical situations is reviewed, with reference to examples from several fields of fluid dynamics.

EXPERIMENTAL INVESTIGATION OF VORTEX STREETS

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Vortex streets behind circular cylinders and flat plates were investigated experimentally in a water tank. Photography and hot-wire techniques were used to examine the flow of vortex streets.

The main results obtained are as follows.

(1) The minimum critical Reynolds number at which the stable Kármán vortex street is formed artificially in the wake of a circular cylinder is about 1.0, while the critical Reynolds number at which the Kármán vortex street appears spontaneously is about 30.

(2) Walls increase the stability of wakes. For example, the critical Reynolds number at which the Kármán vortex street appears spontaneously is about 120 for $\delta/d = 1.5$, while it is about 30 for $\delta/d = 10$, where d is the diameter of the cylinder and δ is the distance between the two walls.

(3) Not only laminar wakes but also turbulent wakes show a strong tendency to arrange themselves into the configuration of the Kármán vortex street. A Kármán vortex street continues to exist for some distance, then it always breaks down. After destruction, however, a new Kármán vortex street is constructed again further downstream. This repeats over and over again until the wake decays by viscous diffusion.

(4) It has been believed that the vortex street of symmetrical arrangement cannot exist stably. If, however, an appropriate periodic disturbance is given to the wake, we can produce the stable symmetrical vortex street. The values of λ/d and h/λ of symmetrical vortex streets are quite different from those of ordinary Kármán vortex streets, where λ is the distance between successive vortices and h is the distance between two rows. It should be noted that the strength of vortex filaments of the symmetrical vortex street is different in two rows.

(5) The vortex filaments behind tapered cylinders and whirling cylinders are straight but inclined to the cylinder axis.

(6) Two Kármán vortex streets interact each other when they are approached. Either the symmetrical arrangement or the antisymmetrical arrangement is

ON THE PERIODIC SHEDDING OF VORTICES FROM A CIRCULAR CYLINDER AT HIGH REYNOLDS NUMBERS

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The communication presents the results of experiments by the author's co-workers, K. Yamamoto and M. Iuchi, on the flow past a circular cylinder at Reynolds numbers from 3×10^4 to 10^6. A pair of trip wires was attached on the cylinder at ±50° from the stagnation point with a view to simulating the flow at higher Reynolds numbers. The experiments bring out the following features.

The first transition from high to low values of drag coefficient (sub- to supercritical transition) is of critical nature, being much more discontinuous than previously reported. Introduction of trip wires makes the transition less discontinuous.

The second transition from low to high values of drag coefficient (super- to transcritical transition) is more or less gradual. At lower supercritical Reynolds numbers no periodic shedding of vortices is observed. At higher supercritical Reynolds numbers vortex shedding occurs intermittently, the fraction of the time of occurrence increasing as the Reynolds number is increased. At transcritical Reynolds numbers vortex shedding is of the same nature as at subcritical values, but with Strouhal number increased by about 40 per cent. The indication is that at supercritical Reynolds numbers there exist two types of flow, the one with and the other without periodic vortex shedding, the former occurring more frequently with increasing Reynolds number.

The flow at supercritical Reynolds numbers is characterized by the existence of a laminar separation bubble followed by a relatively short dead-air region, which makes the wake markedly constricted. At transcritical Reynolds numbers the bubble disappears and a prolonged dead-air region develops behind the cylinder. The wake is wider than in the supercritical flow, but slightly narrower than in the subcritical flow.

These results seem to give answers to some of the questions raised by A. Roshko (1961) in his measurements at very high Reynolds numbers.

CONCENTRATION OF VORTICITY NEAR THE TIPS
OF WINGS WITH ENDPLATES AND TIP-TANKS

J. L. Taylor
N. T. H. Trondheim

The experimental and theoretical work summarized was started in Trondheim in 1962, and continued in 1962-3 during the author's sabbatical year in England.

Although the original approach was slightly different, and concerned the possibility of regenerating tip-vortex energy, it can be described as an attempt to optimize wing performance (L/D) by reducing the effective induced drag, for given span, by means of wing-tip modifications. End-plates reduce the induced drag, but have little effect on L/D due to the increased surface area and profile drag. Ordinary tip-tanks should theoretically give a net increase in L/D, after allowing for the increase in surface area; though not strictly the theoretical optimum, they are probably not far from it.

The difficulty in achieving the predicted increase in L/D in actual practice seems never to have been properly elucidated, but it can easily be traced to interaction between the tip vortex and the tapering rear part of the tip body, creating a sort of induced form drag. Experiment fully confirms this.

In terms of the vortex wake, this means a reduction in the core diameter of the tip vortex and hence an increase in the vortex energy. Having traced the source of the performance loss, it is a fairly simple matter to eliminate it, and this again is confirmed by experiment.

Without necessarily decrying the classical use of the idealized vortex wake, consisting, in the simple case of a plain wing, of a plane vortex sheet, it is suggested that attention should in general at least equally be given to the actual (rolled-up) vortex wake, consisting of the tip vortices with finite cores, whose diameter is critical

ON THE CHARACTERISTICS OF THE CORES OF DYING AND GROWING VORTICES

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There are two different possibilities in the action of viscosity on plane or quasi plane vortices.

In the first case - e.g. in a Kármán vortex street - the whole vorticity of the system is concentrated in the vortices. The vortex strength contained in every single vortex can neither increase nor decrease, except for the overlapping of vortices of different sign. The vorticity, however, will diffuse due to the action of viscosity.

This behaviour can be shown by photographs of the water surface marked by aluminium powder. The strengths of the individual vortices were calculated by the method due to Hooker.

In the second case - e.g. a shear layer - not all the vorticity of the system is concentrated in vortices at the beginning, and the amount of vorticity in the vortices increases downstream until the vortices become turbulent and explode.

In order to verify the growth of vortices experimentally measurements in the shear layers of a jet were made using a CT-hot-wire anemometer with a coupled linearisation stage. The hot-wire signals were interpreted with the aid of a very simple mathematical model.

According to theoretical investigations by Schade & Michalke and according to these measurements one can prove that in a certain range of Reynolds numbers the viscosity is of no importance as far as the generation and rise of vortex strength is concerned. And even instability and transition to turbulence are not - as formerly assumed by the author - governed by the Reynolds number of the individual vortex but depend on the relation of the vorticity concentrated in the vortex to the vorticity remaining in the shear layer.

VORTEX STREET BEHIND A HEATED CYLINDER

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A detailed experimental study of formation and structure of vortex street behind two-dimensional heated cylinder is underway. It is found that the critical Reynolds Number increases and the corresponding Strouhal Number decreases with increasing cylinder temperature. At a cylinder temperature of 725°C and air at 22°C the critical Reynolds Number is 3.4 times and the corresponding Strouhal Number is 10% lower than that when the cylinder and air temperature are the same.

ON THE FORMATION OF COHERENT VORTEX SHEETS IN STEADY FLOWS (PROPELLERS AND WINGS TIP VORTEX)

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The shape and structure of tip vortices shed from airscrew blades and wings (Aerofoils of finite span) have been investigated by the Author on models in wind tunnels (2 to 80 ms), by means of smoke streams, continuous or intermittent.

Smoke streams when illuminated stroboscopically are particularly convenient for revealing the structure of concentrated vortices as well as the fluid motion around and inside them.

The tip vortex of an airscrew blade can be seen shedding from the tip of the blade. Its mean line can be described as an helix drawn on a cylinder whose diameter is smaller than the diameter of the airscrew and whose pitch and circulation (around it) increase first rapidly immediately behind the airscrew disc, and then very slowly, both of them tending toward a limit. Experiments show that the average pitch 0.5 D from the disc, is a linear function of $\frac{V}{\Omega R}$

$$\frac{h}{\pi D} = a \frac{V}{\Omega R} + (1 - a) \ \mathrm{tg}\, \theta$$

with the slope of the zero lift line of a section of the blade close to the tip. It can be shown by the vortex theory that a has the approximate expression:

$$a = \frac{1}{1 + \frac{n s t}{4 K \sin \theta}}$$

The tip vortex of a wing appears roughly as a cylindrical vortex, outside of which the flow is irrotational.

Data are given of extensive measurements concerning the core diameter, the pressure inside (total and static) the circulation, and the influence of incidence of the wing and of the distance from the origin. Comparisons are made with the theory. The longitudinal component of the speed along the vortex axis is strongly influenced by the shape of the ends of the wing (distribution of the circulation along the span) and by the incidence.

Applications of both investigations are described.

A NUMERICAL INVESTIGATION ON THE ROLLING-UP OF VORTEX SHEETS

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The problem of the Helmholtz instability of a vortex sheet has been considered recently by G. Birkhoff (Vol.XIII of the Proc. of Symposia in Appl. Math., p.55-76, 1962). Already Rayleigh had shown that a vortex sheet is unstable in an approximation which is linearized in amplitude if viscosity and surface tension are neglected. Whether in a non-linearized theory some well-defined final position (limiting curve) of the vortex sheet arises, is not known. There exists the general idea that vortex sheets tend to roll up smoothly into periodic spirals. On the other hand, Birkhoff supposes that the problem is mathematically not well-set in the sense of Hadamard, which would mean that no well-defined final situation is approached when time increases.

In order to decide what occurs actually, a numerical investigation on the motion of a vortex sheet has been performed. One possibility is to replace the vortex sheet by a number of point-vortices. Such calculations have been performed before by Rosenhead (1931) and by Fisher (1959), but the present calculations, performed on a high-speed electronic computer, are based upon a much larger number of vortices. If the vortices are connected in a way which suitably represents the vortex sheet, it is shown very clearly that the vortex sheet begins to roll up at a certain point and that the process of rolling-up continues with time. The point at which the rolling-up begins depends upon the form and amplitude of the initial disturbance.

When the vortex sheet is not replaced by point vortices, the velocity of a certain point of the sheet is determined by the Cauchy principal value of an integration along the vortex sheet. Numerical evaluation of this integral leads to the same calculation as for the point vortices except that an additional velocity contribution is due to the immediate neighbourhood of the point considered. This contribution is roughly proportional to the curvature of the vortex sheet and increases the instability. It is very difficult to take into account this term accurately since it requires numerical differentiations.

The effect of viscosity is to reduce this last term. Moreover, viscosity will transform the spiral of the rolled-up vortex sheet into a region of vorticity, which at some distance has the same influence as a concentrated vortex. Therefore, when seen on a not too small scale, the model of a rolled-up vortex seems certainly to be admissible.

MEASUREMENTS OF THE CORRELATION BETWEEN VELOCITY AND PRESSURE FLUCTUATIONS IN A MOVING VORTEX SYSTEM

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In the past, a lot of hot-wire measurements have been made in the transition region of a boundary layer and attempts have been made to calculate the resulting pressure distribution by assuming a certain kind of flow model. But the question whether the fluctuations are created by a wave or vortex motion cannot be decided by hot-wire measurements alone.

Therefore, an investigation has been made in a wake behind a cylinder where the fluctuations are known to be created by a two row vortex street (Kármán vortex street). The measurements of the velocity and the pressure fluctuations have been made with a combined hot-wire and pressure probe in the Reynolds number range between 40 ... 120. In this range, the fluctuations are very stable and have a discrete frequency.

In order to separate the velocity fluctuations into the two components u' and v', a special hot-wire in the form of a ribbon, has been developed. This ribbon is connected to a Constant Temperature Hot-wire anemometer and the characteristics of this measuring device have been investigated.

The evaluation of the velocity and pressure signals shows that both are very well correlated in the sense of a moving vortex system. An attempt has been made to calculate the vortex strength from these measurements and to find a criteria for a vortex-like motion.

By using this criteria, the velocity fluctuations in the free boundary layer of a jet and in the boundary layer of a flat plate are to be investigated to decide about the type of the occurring fluctuations.

OCCURRENCE OF DUAL REGIMES OF FLOW IN THE CORE OF A VORTEX SINK

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The solution of a differential equation describing possible modes of motion in the core of a vortex sink reveals that, subject to requisite boundary conditions, two different regimes of flow may occur. It is found that the velocity of propagation of pressure waves along the vortex tube plays an important role in differentiating these two regimes.

Experimental investigations have confirmed the physical existence of the wave velocity and its role as a basic parameter. One of the regimes of flow theoretically predicted was found to exhibit properties closely analogous to those predicted theoretically. In place of the other regime predicted there occurs what is known as the two-cell vortex pattern. The experimental investigation of this pattern furnishes new information particularly in regard to its functions in originating motions of higher order within the core region of the vortex sink.

KÁRMÁN VORTEX STREETS

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The Kármán vortex street in its many aspects is taken as a typical example of an unsteady flow and transient motion. From the point of view of wake-flow the Kármán street may be a special case, but as periodic vortex shedding remains detectable up to high Reynolds and Mach numbers many phenomena of wake- and free boundary layer flow may be approached by starting with the Kármán vortices.

Since the review by the author (Adv. Appl. Mech. 6, 1960) it appears that no further treatment of hydrodynamic stability of the point-vortex constellation has been published. Domm (ZAMM 36, 1956) had proved that with non-linear theory all double-row vortex streets in ideal fluids are unstable. From other fields of stability theory we learn that in cases of "inductional stability" fluid friction tends to exercise a stabilizing effect.

Mathematical theory nowadays helps us to understand in a more general way the building-up, and motion, of regions of concentrated vorticity in sheets of discontinuity. Thirty years ago Rosenhead (Proc. Roy. Soc. A, 134, 1931) wrote a pioneer paper on this subject. Nowadays the papers of Hama (Phys. Fluids 5, 1962), Hama and Burke (Univ. Maryland T.N. BN.220, 1960), Abernathy and Kronauer (J.F.M. 13, 1962) and of Michalke (Ing. Arch. 1964) are representative of this new approach. The importance lies in the fact that flow-photographs and oscilloscope-signals of hot-wire techniques can be interpreted.

New facts about the spatial and temporal development of vortex trails and of wakes have been discovered by three experimenters. Tritton (J.F.M. 6, 1959) observed two modifications of the Kármán street at Re ~ 90 and some time later Berger (Z.Flugwiss, 12, 1964) added a third one for 125 < Re < 160. The "flow modes" differ in frequency and in amplitude, which Tritton explained by their different origin. Taneda (J. Phys. Soc. Japan 18, 1963) has since 1952 worked on problems of vortex trails and wakes in general. Of the many enlightening results one may be mentioned: Taneda (Phys. Soc. Japan 14, 1959) discovered the phenomenon of shape-changing and rearranging of the vortices at great distances downstream. Whether these results are related to Tritton's and Berger's findings has still to be clarified.

Birkhoff and Zarantonello (Acad. Press. N.Y. 1957) comprehensively treated jets and wakes, and on periodic wakes we owe to them many theoretical clarifications of general topics. The special problem of the determination of concentrated vorticity e.g. in a single vortex of a vortex street is a most important problem in basic fluid mechanics, and the regular pattern of the Kármán vortex street is obviously the best object of study. But since, as Timme (DVL Ber. 77, 1959) proved, no significant hot-wire signal can be obtained for the centre of a vortex, the difficulties in comparing theory and measurement are great. Berger (Z. Flugwiss, 12 1964) did overcome these difficulties by using the amplitude distribution across the street and, without assumptions regarding the "vortex-age" and the "spacing-ratio" which had been used by Schaefer and Eskinazi (J.F.M. 6, 1959), Berger could evaluate at least the "initial circulation" of a vortex. New elucidation on this subject will be presented during this symposium, but it should be remembered that more than 35 years ago Fage and Johansen (R & M 1104, 1927) attacked this problem. In another flow-field, the jet boundary layer, Fabian (DVL Ber.122, 1960) provided data on concentration of vorticity towards a critical state.

In the related field of body drag Tritton (J.F.M. 6, 1959) provided results for low Reynolds numbers, and Fung (IAS Paper No.60 - 6) for high Reynolds numbers. In Fung's paper the connection with aeroelasticity is given as well. Berger on the other hand took up the phenomenon of "vibration" with regard to damping or stimulating of turbulence downstream. This will be discussed during the symposium.

STEADY AND UNSTEADY MOTIONS AND WAKES OF FREELY FALLING DISKS

W. W. Willmarth, N. E. Hawk, & R. L. Harvey
University of Michigan, Ann Arbor

The motions and wakes of freely falling disks were studied and it was found that the diverse motions of the disks exhibit a systematic dependence on the Reynolds number Re, and the dimensionless moment of inertia I^*. The relation between I^* and Re along the boundary separating stable and unstable pitching oscillations of the disk was determined. The Reynolds number for stable motion of a disk with large I^* is 100, in agreement with the Reynolds number for stability of the wake of a fixed disk. Slightly unstable disks of large I^* were stabilized by reducing the moment of inertia. Results of measurements of the damping moment of disks forced to oscillate about a fixed axis in a wind tunnel suggest that the disk stabilization is caused by an increase in average damping moment when the dimensionless frequency of oscillation, nd/U, increases as a result of the reduction of I^*. The Reynolds number for stable disk motion passes through a maximum, Re = 172, and then decreases with further reduction of I^*. At higher Reynolds numbers the disks exhibited periodic pitching and translational oscillations. The laminar wake behind certain of the oscillating disks consisted of a staggered arrangement of two rows of regularly spaced vortex rings similar to the wake observed behind liquid drops by Margarvey and Bishop. The dependence of the dimensionless frequency of oscillation on I^* and Re was determined along the boundary for stable motion and at higher Reynolds numbers when the wake was turbulent. Tumbling motions of the disks were observed when the Reynolds number was large. Re > 2000, and I^* was greater than a certain value, $I^* = 10^{-2}$.

The frontispiece is from a drawing by Leonardo da Vinci (1452-1519) in the Royal Library at Windsor. The inscription has been rendered as

'Watch the movement of the surface of water, how like it is to that of hair, which has two movements, one following the undulation of the surpace, the other the lines of the curves: thus water forms whirling eddies, part following the impetus of the chief current, part the rising and falling movement.'

The quotations inside the front cover and at the foot of this page are rendered from manuscripts of Leonardo. The originals in the order given here, are located as follows :

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