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ABSTRACT

An algorithm is developed for solving a special structured linear program. The particular structure studied has a large number of blocks coupled together by a relatively few connecting equations. The method proposed is an extension of [2] and, from the basis, defines a working basis which is much smaller in size than the original. Two methods of updating the working basis are proposed.

INTRODUCTION

In this paper we propose a device for solving a large scale linear programming problem of a special configuration. The method proposed is an extension of "Generalized Upper Bounded Techniques - I" [2] to a larger class of problems.

The special configuration is the following:

$$\left\{\begin{array}{lcl} \boxed{A'_o x_o + A'_1 x_1 + \qquad\qquad A'_L x_L} & = & b_o \\ \quad \boxed{A_1 x_1} & = & b_1 \\ \qquad\qquad\qquad \boxed{A_L x_L} & = & b_L \\ \gamma_o x_o + \gamma_1 x_1 + \qquad\qquad \gamma_L x_L & = & z(\min) \end{array}\right. \tag{1}$$

x is a vector of N components which can be written $x = (x_o, x_1, \ldots, x_L)$ where x_i has n_i components and $N = n_o + n_1 + \ldots + n_L$.

A'_i is an $m_o \times n_i$ matrix;

A_i is an $m_i \times n_i$ matrix .

$M = m_o + m_1 + \ldots + m_L$ is the number of equations of the system which we will assume to be nonredundant and which can be written for short:

$$\left\{\begin{array}{l} \underline{A}x = \underline{b} \\ \gamma x = z(\min) \end{array}\right. \quad \text{where } \underline{A} \text{ is an } M \times N \text{ matrix} \tag{1'}$$

$$\boxed{\text{The basic assumption is that } L \gg m_o .}$$

Although we can save an important computational work with respect to the simplex method by applying the decomposition principle [1] to Problem (1') we will, rather, taking advantage of the very special condition $L \gg m_o$, extend the device given by Dantzig and Van Slyke in [2]: From the basis of the system a "working basis" is derived which is considerably smaller in size. Computations are done, as much as possible, on this working basis which result in a substantial savings in computation and storage. A way is given to find the next working basis from iteration to iteration.

In a first part we give definitions and theorems; in a second part we give the general method; and in a third part we give two different ways of updating.

1. SOME DEFINITIONS AND THEOREMS - S_i will refer (depending on context) either to the set of components of x_i or to the set of corresponding columns of the matrix $\underline{A}$.

Suppose we are at some stage of the process and that $\underline{B}$ is a basis for the system. Let

$$\begin{bmatrix} B_i' \\ 0 \\ B_i \\ 0 \end{bmatrix}$$

be the contribution of the set S_i to the basis $\underline{B}$.

THEOREM 1: At least m_i variables from each set S_i $(i = 1,\ldots,L)$ are basic.

Proof: Suppose S_p has less than m_p basic columns. Then there exists a linear combination of rows of B_p which vanishes: (i) $\lambda_p B_p = 0$. Now we can define $\lambda = (0,\ldots,0, \lambda_p, 0,\ldots,0)$ such that $\lambda\underline{B} = \lambda_p B_p = 0$, which contradicts the fact that $\underline{B}$ is nonsingular.

Remark: It follows from (i) that if B_p has exactly m_p basic columns, it is nonsingular.

THEOREM 2: The number of sets S_i containing more than m_i basic variables is at most m_o .

Proof: By the assumption of full rank, each basis has exactly M vectors. By Theorem 1, each set S_i contains at least m_i basic variables; this leaves m_o to make up sets of more than m_i basic variables.

<u>Definition</u>: A set S_i will be said to be essential if it has more than m_i basic variables; the set S_o is always put in the essential class. The other sets will be called inessential.

$$\xi = \{i \mid S_i \text{ is essential}\}$$

$$\bar{\xi} = \{i \mid S_i \text{ is inessential}\} \quad .$$

II. <u>THE GENERAL METHOD</u> - The reduced system is defined by deleting the subsystems $A_i x_i = b_i$ where S_i is inessential. The set of essential basic columns restricted to the reduced system is our working basis B .

The value of inessential basic variables is determined from:

$$B_i x_i' = b_i \tag{1}$$

where B_i is square and nonsingular. These smaller systems are readily solved.

<u>THEOREM 3</u>: The working basis is a basis for the reduced system.

Proof: The number of equations in the reduced system equals the number of variables in the working basis since for each inessential set we remove exactly m_i equations and m_i basic columns from the square basis $\underline{B}$. The columns of the working basis are linearly independent since they differ from the columns of the basis $\underline{B}$ by a bunch of zeros in the inessential part, and the columns of $\underline{B}$ are linearly independent.

The size of the working basis B is $m_o + \sum_{i \in \xi} m_i$, considerably smaller than that of $\underline{B}$. Note that the size of B changes from step to step, and that even the number of blocks in B (which is bounded by m_o) changes from step to step.

Suppose we have a basis $\underline{B}$, the columns of which will be denoted $\underline{B}^j$ (j=1,..

Let $B = [B^{h_i}, i = 1,\dots,|B|]$ be the corresponding working basis.

The problem is:

a) To determine which column enters the basis;

b) To determine which column is dropped from the basis.

1) Let

$\underline{\Pi} = (\pi_o, \pi_i, \dots, \pi_L)$ be the price vector corresponding to $\underline{B}$;

$\pi = (\pi_o, \pi_1 \mid i \epsilon \xi)$ be the price vector corresponding to B ;

$\gamma = (\gamma_o, \gamma_1, \dots, \gamma_L)$ be the cost vector;

$\underline{c} = (c_o, c_1, \dots, c_L)$ be the cost vector associated with $\underline{B}$;

$c = (c_i \mid i \epsilon \xi)$ be the cost vector associated with B .

The equation

(2) $$\underline{\pi}\,\underline{B} = \underline{c}$$

defines the price vector. The point is that this equation can be written

(2') $$\pi B = c \quad ;$$

(2'') $$\pi_o B_i^{\prime} + \pi_i B_i = c_i \qquad i \epsilon \bar{\xi} \quad .$$

We will later give different ways of solving (2'); (2') being solved, substituting π_o in (2'') gives π_i in an easy way.

Let A_σ^s and A'^s_σ denote the s^{th} column of matrices A_σ and A'_σ respectively.

$$\underline{A}_\sigma^s = \begin{pmatrix} A'^s_\sigma \\ 0 \\ A_\sigma^s \\ 0 \end{pmatrix}$$ is the corresponding column of $\underline{A}$.

The column to enter the basis will be given by the usual criterion:

$$\bar{\gamma}^s_\sigma = \gamma^s_\sigma - \underline{\pi}\,\underline{A}^s_\sigma = \min_{\substack{0 \leq \tau \leq L \\ 1 \leq j \leq n_\tau}} (\gamma^j_\tau - \underline{\pi}\,\underline{A}^j_\tau) \quad .$$

If $\bar{\gamma}^s_\sigma \geq 0$, we have found an optimal basic solution; if not, we bring $\underline{A}^s_\sigma$ into the basis.

2) $\underline{A}^s_\sigma \in S_\sigma$. If S_σ is essential we have not to worry about the inessential part of the problem which remains unchanged. We simply pivot in the reduced system with the usual criterion for picking the column to drop. We then have to update the right-hand side, and the entering column is the reduced system. That is, to solve

(3) $$B\,\tilde{A}^s_\sigma = A^s_\sigma \quad ;$$

(4) $$B\,\tilde{b} = b \quad .$$

We postpone to a later part the discussion of how (3) and (4) are actually solved. Note that in this case, since B is assumed to be of a reasonable size, the updating is simple.

If S_σ is not essential we have to solve, instead of (3) and (4), the more complicated systems:

(3') $$\underline{B}\,\underline{\tilde{A}}^s_\sigma = \underline{A}^s_\sigma \quad ;$$

(4') $$\underline{B}\,\underline{\tilde{b}} = \underline{b} \quad .$$

We will now prove that solving (3') and (4') is, in fact, equivalent to solving (3) and (4).

Let $\tilde{A}'^s_\sigma$, $\tilde{b}$ denote the part of the updated columns which correspond to the working basis.

$$\begin{bmatrix} B & B'_k & & B'_\sigma & & B'_L \\ & B_k & & & & \\ & & \ddots & & & \\ & & & B_\sigma & & \\ & & & & \ddots & \\ & & & & & B_L \end{bmatrix} \begin{bmatrix} \tilde{A}'^s_\sigma \\ \tilde{d}_k \\ \vdots \\ \tilde{A}^s_\sigma \\ \vdots \\ \tilde{d}_L \end{bmatrix}, \begin{bmatrix} \tilde{b} \\ \tilde{b}_k \\ \vdots \\ \tilde{b}_\sigma \\ \vdots \\ \tilde{b}_L \end{bmatrix} = \begin{bmatrix} A'^s_\sigma \\ 0 \\ 0 \\ A^s_\sigma \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} b \\ b_k \\ \vdots \\ b_\sigma \\ \vdots \\ b_L \end{bmatrix}$$

This can be written:

$$(5)\quad \left\{\begin{array}{lll} B_k & \tilde{d}_k & = 0 \\ \vdots & & = 0 \\ B_\sigma & \tilde{A}^s_\sigma & = A^s_\sigma \\ \vdots & & = 0 \\ B_L & \tilde{d}_L & = 0 \end{array}\right. \qquad (6)\quad \left\{\begin{array}{lll} B_k & \tilde{b}_k & = b_k \\ \vdots & & \\ B_\sigma & \tilde{b}_\sigma & = b_\sigma \\ \vdots & & \\ B_L & \tilde{b}_L & = b_L \end{array}\right. .$$

$$(3'')\quad B\tilde{A}'^s_\sigma = \begin{bmatrix} A'^s_\sigma \\ 0 \end{bmatrix} - \begin{bmatrix} B'_\sigma \\ 0 \end{bmatrix} \tilde{A}^s_\sigma \quad ; \qquad (4'')\quad B\tilde{b} = b - \sum_{i\epsilon\xi} \begin{bmatrix} B'_i \\ 0 \end{bmatrix} \tilde{b}_i \quad .$$

And we see that if we suppose the systems in (5) and (6) are of very low order and that their solution is readily at hand, we have only to solve (3'') and (4''), which are exactly of the same type as (3) and (4).

By the usual simplex criterion we now know which column is to be dropped. Let us suppose that the column to be dropped belongs to the set S_τ. Two cases can occur:

a) S_τ is inessential, so S_τ can only be S_σ. Then the pivoting is only done in the inessential sub-basis B_σ, and the working basis does not change.

b) S_τ is essential, so the method of pivoting is as follows:

<u>Step I</u> - Introduce S_σ in the essential set (the size of the working basis is increased by m_σ) .

<u>Step II</u> - Pivot in the new reduced system.

<u>Step III</u> - If the number of variables in S_τ is now m_τ,† make S_τ inessential (the size of the working basis is decreased by m_τ) .

The algebraic work involved in Equations (2'), (3) and (4) (or (3'') and (4'')) can be done in essentially two ways. The first is the Revised Simplex Method; the justification of using it is that the working basis will remain of a reasonable size. The second is to solve the system, instead of inverting the matrix B , by a triangularization process which is inspired of [3].

III. <u>UPDATING PROCESS</u> -

A. <u>Revised Simplex Method</u>: Suppose we have an inverse of the working basis B^{-1}; we want to find B^{*-1} where B^* is the basis for the next iteration. If S_σ is essential, we pivot in the reduced system and simply apply the revised simplex algorithm in the reduced system. If S_σ and S_τ are inessential, the working basis does not change. If S_σ is inessential and S_τ is essential, we will describe the three steps outlined above.

† If the essential set S_τ had more than $m_{\tau+1}$ basic columns beforehand, this will not be the case.

Step 1. Define

$$\tilde{B} = \left[\begin{array}{c|c} B & B'_\sigma \\ \cline{2-2} & \\ \hline 0 & B_\sigma \end{array}\right] \qquad Q = \left[\begin{array}{c} B'_\sigma \\ \hline \\ \end{array}\right] \;;$$

then it is easy to see that:

$$\tilde{B}^{-1} = \left[\begin{array}{c|c} B^{-1} & Q' \\ \hline 0 & B_\sigma^{-1} \end{array}\right] \quad \text{with} \quad Q' = B^{-1} Q B_\sigma^{-1} \; .$$

Step 2. Now we are as in the first case and we use the modified simplex method to get $\tilde{B}^{*-1}$ (where $\tilde{B}^{*}$ is obtained from $\tilde{B}$ by pivoting in the reduced system).

Step 3. If B_τ is now square, we can rearrange the equations and variables in $\tilde{B}^{*}$ so that it looks like this:

$$\tilde{B}^{*} = \left[\begin{array}{c|c} B^{*} & B'_\tau \\ \cline{2-2} & \\ \hline & B_\tau \end{array}\right] \qquad \tilde{B}^{*-1} = \left[\begin{array}{c|c} B^{*-1} & Q'_\tau \\ \hline & B_\tau^{-1} \end{array}\right] \; .$$

B^{*} and B^{*-1} are now obtained by dropping the column and rows corresponding to B_τ .

B. <u>Compact Basis Triangularization</u>: Any equations of the type $Bx = b$ or $\pi B = c$ where B is an $m \times m$ nonsingular matrix can be solved by triangularizing B (see [3]). It has been pointed out in [3] that triangularizing may be an efficient method in cases where B has a special structure (such as block angular).

Triangularizing is equivalent to pre-multiplying B by a set of elementary matrices E_i

$$T = E_1, E_2, \ldots, E_m B$$

where, in the notation of [3],

T is the triangularized form of B ;

$E_1, \ldots, E_m$ is the compact E-structure.

To solve $\pi B = c$ (Equations (2') and (2'')), we solve

$$\pi^* T = c$$

and

$$\pi = E_1, E_2, \ldots, E_m \pi^* \quad .$$

To solve $Bx = b$ (Equations (3),(4),(5),(6)), we find

$$b^* = E_1, E_2, \ldots, E_m b$$

and solve

$$Tx = b^* \quad .$$

Having determined S_σ and S_τ , we now show the steps in pivoting.

If S_σ is essential, we pivot in the reduced working basis. If S_σ and S_τ are inessential, we pivot in the smaller basis B_σ . If S_σ is inessential and S_τ essential, then we show the three steps.

Let T_σ be the triangularized form of B_σ , E_σ its compact E-structure, and J the permutation matrix used to triangularize B . Let T be the triangularized form of B, and E its compact E-structure.

<u>Step 1</u>. Introducing S_σ into the essential class, we modify the working basis by introducing B_σ into it. We get†

$$\text{New } T = \left[\begin{array}{c|c} T_\sigma & 0 \\ \hline EJQ & T \end{array}\right] \qquad Q = \left[\begin{array}{c} B'_\sigma \\ \hline 0 \end{array}\right]$$

$$\text{New } E = \left[\begin{array}{c|c} E_\sigma & 0 \\ \hline 0 & E \end{array}\right]$$

<u>Step 2</u>. Apply the pivot operation in the new B.

<u>Step 3</u>. Assume ξ_τ has become inessential; to remove this block from the working basis, assume the columns of S_τ in B lie between i^{th} and $i+m_\tau$ th column. Define

$$T = \begin{array}{l} \begin{array}{ccc} \quad\quad\quad & i \quad & i+m_\tau \end{array} \\ \left[\begin{array}{c|c|c} T_1 & & 0 \\ \hline & T_2 & \\ \hline T_4 & & T_3 \end{array}\right] \begin{array}{l} \\ i \\ i+m_\tau \end{array} \end{array} \qquad E = \begin{array}{l} \begin{array}{ccc} \quad\quad\quad & i \quad & i+m_\tau \end{array} \\ \left[\begin{array}{c|c|c} E_1 & & E_4 \\ \hline & E_2 & \\ \hline & & E_3 \end{array}\right] \begin{array}{l} \\ i \\ i+m_\tau \end{array} \end{array}$$

Then

---

† Note: If we introduce the inessential set S_σ at the top LHS of the T and E, we always obtain the new E structure in this manner.

triangularized form of B, and E its compact E-structure.

Step 1. Introducing S_σ into the essential class, we modify the working basis by introducing B_σ into it. We get†

$$\text{New } T = \left[\begin{array}{c|c} T_\sigma & 0 \\ \hline EJQ & T \end{array}\right] \qquad Q = \left[\begin{array}{c} B'_\sigma \\ \hline 0 \end{array}\right]$$

$$\text{New } E = \left[\begin{array}{c|c} E_\sigma & 0 \\ \hline 0 & E \end{array}\right]$$

Step 2. Apply the pivot operation in the new B.

Step 3. Assume ξ_τ has become inessential; to remove this block from the working basis, assume the columns of S_τ in B lie between i^{th} and $i{+}m_\tau{}^{th}$ column. Define

$$T = \begin{array}{c} \quad\quad\quad\quad i \quad\; i{+}m_\tau \\ \left[\begin{array}{c|c|c} T_1 & & 0 \\ \hline & T_2 & \\ \hline T_4 & & T_3 \end{array}\right] \begin{array}{l} i \\ i{+}m_\tau \\ \end{array} \end{array} \qquad E = \begin{array}{c} \quad\quad\quad\quad i \quad\; i{+}m_\tau \\ \left[\begin{array}{c|c|c} E_1 & & E_4 \\ \hline & E_2 & \\ \hline & & E_3 \end{array}\right] \begin{array}{l} i \\ i{+}m_\tau \\ \end{array} \end{array}.$$

Then

---

† Note: If we introduce the inessential set S_σ at the top LHS of the T and E, we always obtain the new E structure in this manner.

$$\text{new } T = \begin{array}{|c|c|}\hline T_1 & 0 \\ \hline T_4 & T_3 \\ \hline \end{array} \qquad \text{new } E = \begin{array}{|c|c|}\hline E_1 & E_4 \\ \hline & E_3 \\ \hline \end{array}$$

and $$T_T = T_2 \qquad E_T = E_2 \quad .$$

In conclusion, it seems that Method A may be preferred if the working basis is of small size and if the process of the revised simplex algorithm on it is not too long, as Method B will be preferred if the working basis itself is of large dimension.

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13. ABSTRACT

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