

A. 631590

AFFDL-TR-65-65

# A CLOSED-FORM SOLUTION TO LIFTING REENTRY

ROLAND N. BELL

TECHNICAL REPORT AFFDL-TR-65-65

JANUARY 1966

CLEARINGHOUSE FOR FEDERAL SCIENTIFIC AND TECHNICAL INFORMATION			
Hardcopy	Microfiche		
\$ 8.60	0.75	72 pp	A2
EXCESSIVE COPY			

*copy 1*

AIR FORCE FLIGHT DYNAMICS LABORATORY  
 RESEARCH AND TECHNOLOGY DIVISION  
 AIR FORCE SYSTEMS COMMAND  
 WRIGHT-PATTERSON AIR FORCE BASE, OHIO

Distribution of This Document Is Unlimited

## NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Copies of this report should not be returned to the Research and Technology Division unless return is required by security considerations, contractual obligations, or notice on a specific document.

AFFDL-TR-65-65

# **A CLOSED-FORM SOLUTION TO LIFTING REENTRY**

*ROLAND N. BELL*

**Distribution of This Document Is Unlimited**


**BLANK PAGE**

FOREWORD

This report was prepared by Roland N. Bell of the Gasdynamics Branch, Flight Mechanics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. The work was carried out under Project No. 1366, "Aerodynamics and Flight Mechanics," Task No. 136602, "Performance and Trajectory Analysis of Hypervelocity Vehicles," and is part of the continuing Laboratory effort to study the advantages of high L/D reentry systems. The work was administered by the Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command. The research reported in this study was conducted during the period August 1964 through March 1965. The manuscript was released by the author in April 1965 for publication as an RTD Technical Report.

The author is particularly indebted to Dr. Wilbur L. Hankey, Jr., Research Engineer of the Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, for his guidance and suggestions in finding the successful approach to the final solution as well as for helpful suggestions throughout the investigation. Appreciation is also expressed to L. E. Zaytown, Jr., 1/Lt, USAF, Digital Computation Division, Systems Engineering Group, Wright-Patterson Air Force Base, Ohio, for his assistance in programming and running the comparative computer study. Finally, thanks are extended to Miss Robin Hawkins for aid in accomplishing the manuscript.

This technical report has been reviewed and is approved.

  
For PHILIP P. ANTONATOS  
Chief, Flight Mechanics Division  
Air Force Flight Dynamics Laboratory

## ABSTRACT

This report derives closed-form expressions for predicting the longitudinal and lateral range attainable by lifting reentry vehicles. The resultant equations sensitively and accurately define the influence of L/D ratio, bank angle and entry velocity variations over a spectrum of values. To illustrate the usefulness of the method, the derived expressions were used to conduct a parametric reentry study covering a range of L/D ratios from 0.5 to 4.0, bank angles from  $0^\circ$  to  $75^\circ$  and entry velocities from  $0.89V_c$  to  $0.99V_c$ . The results of this study are compared with those obtained from a high speed computer study using the same range of reentry conditions. As an aid to future investigators, a series of curves is presented giving longitudinal and lateral range values for various selected L/D, bank angle and entry velocity values. For those wishing to investigate reentry under conditions not covered by these curves, a detailed "recipe" for utilizing the method is included in an appendix. A comparison of the results of this method with those of more rigorous methods for the same reentry conditions shows that the closed-form solution has sufficient accuracy and sensitivity to be of considerable value to those persons requiring a rapid, preliminary estimate of vehicle performance.

**BLANK PAGE**

TABLE OF CONTENTS

	PAGE
INTRODUCTION . . . . .	1
ANALYSIS	
Assumptions . . . . .	2
Development of Solution . . . . .	3
Comparative Computer Study . . . . .	7
DISCUSSION OF RESULTS . . . . .	9
CONCLUSIONS . . . . .	11
REFERENCES . . . . .	12
APPENDIX UTILIZATION PROCEDURE . . . . .	13

ILLUSTRATIONS

FIGURE	PAGE
1 Vehicle Force Orientation . . . . .	3
2 Reentry Time vs Termination Velocity for Various Reentry Velocities . .	15
3 Total Range vs Termination Velocity for Various Reentry Velocities . . .	16
4 Velocity at $\Delta\psi = 90^\circ$ for Various Entry Velocities and $(L/D) \sin \phi$ Values .	17
5 Longitudinal Range Series Values . . . . .	19
6 Lateral Range Series Values . . . . .	29
7 Local Velocity - Circular Velocity Ratio . . . . .	35
8 Longitudinal Range Variation With Reentry Velocity and Bank Angle, $L/D = 0.5$ . . . . .	36
9 Longitudinal Range Variation With Reentry Velocity and Bank Angle, $L/D = 1.0$ . . . . .	37
10 Longitudinal Range Variation With Reentry Velocity and Bank Angle, $L/D = 2.0$ . . . . .	38
11 Longitudinal Range Variation With Reentry Velocity and Bank Angle, $L/D = 3.0$ . . . . .	39
12 Longitudinal Range Variation With Reentry Velocity and Bank Angle, $L/D = 4.0$ . . . . .	40
13 Lateral Range Variation With Reentry Velocity and Bank Angle, $L/D = 0.5$ . . . . .	41
14 Lateral Range Variation With Reentry Velocity and Bank Angle, $L/D = 1.0$ . . . . .	42
15 Lateral Range Variation With Reentry Velocity and Bank Angle, $L/D = 2.0$ . . . . .	43
16 Lateral Range Variation With Reentry Velocity and Bank Angle, $L/D = 3.0$ . . . . .	44
17 Lateral Range Variation With Reentry Velocity and Bank Angle, $L/D = 4.0$ . . . . .	45
18 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.99$ , $L/D = 0.5$ . . . . .	46
19 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.99$ , $L/D = 1.0$ . . . . .	47

## ILLUSTRATIONS (Cont'd)

FIGURE	PAGE
20 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.99$ , $L/D = 2.0$ . . . . .	48
21 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.99$ , $L/D = 3.0$ . . . . .	49
22 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.99$ , $L/D = 4.0$ . . . . .	50
23 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.94$ , $L/D = 0.5$ . . . . .	51
24 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.94$ , $L/D = 1.0$ . . . . .	52
25 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.94$ , $L/D = 2.0$ . . . . .	53
26 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.94$ , $L/D = 3.0$ . . . . .	54
27 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.94$ , $L/D = 4.0$ . . . . .	55
28 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.89$ , $L/D = 0.5$ . . . . .	56
29 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.89$ , $L/D = 1.0$ . . . . .	56
30 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.89$ , $L/D = 2.0$ . . . . .	57
31 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.89$ , $L/D = 3.0$ . . . . .	58
32 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.89$ , $L/D = 4.0$ . . . . .	59
33 Comparison Between Closed-Form and Computer Maximum Longitudinal Range Values, $V_1/V_c = 0.99$ . . . . .	60

ILLUSTRATIONS (Cont'd)

FIGURE	PAGE
34 Comparison Between Closed-Form and Computer Maximum Longitudinal Range Values, $V_1/V_c = 0.94$ . . . . .	61
35 Comparison Between Closed-Form and Computer Maximum Longitudinal Range Values, $V_1/V_c = 0.89$ . . . . .	62
36 Comparison Between Closed-Form and Computer Maximum Lateral Range Values, $V_1/V_c = 0.99$ . . . . .	63
37 Comparison Between Closed-Form and Computer Maximum Lateral Range Values, $V_1/V_c = 0.94$ . . . . .	64
38 Comparison Between Closed-Form and Computer Maximum Lateral Range Values, $V_1/V_c = 0.89$ . . . . .	65
39 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.983$ , $L/D = 3.0$ . . . . .	66
40 Footprint Comparison Between Closed-Form and Computer Solutions, $V_1/V_c = 0.983$ , $L/D = 4.0$ . . . . .	67

TABLES

TABLE		PAGE
1	Reentry Conditions and Vehicle Characteristics . . . . .	8
2	Longitudinal Range . . . . .	14
3	Lateral Range . . . . .	14

## SYMBOLS

a	$V_1/V_c$
b	$(L/D) \sin \phi$
$C_D$	drag coefficient
$C_L$	lift coefficient
D	drag (lbs)
g	acceleration ( $f/s^2$ ) (32.174)
h	altitude above earth surface (ft)
L	lift (lbs)
m	vehicle mass (slugs)
R	range value (ft)
$R_E$	earth radius (NM) (3437.74)
r	radial distance from earth center to vehicle (ft)
S	ground track range (ft)
T	reentry time (sec)
t	time (sec)
V	velocity (f/s)
x	$V/V_c$
$\gamma$	flight path angle (degrees)
$\Delta$	change in a given quantity
$\sigma$	vehicle heading angle in computer program (degrees)
$\phi$	vehicle bank angle (degrees)
$\psi$	vehicle heading angle (degrees)
$\Omega$	earth rotation rate (rad/sec)

SUBSCRIPTS AND SUPERSCRIPTS

$\dot{(\ )}$	differentiation with respect to time
$(\ )_c$	earth circular value
$(\ )_X$	distance measured along initial heading flight path
$(\ )_Y$	distance measured perpendicular to initial heading flight path
$(\ )_{\Delta\psi = 90^\circ}$	point on trajectory where heading is perpendicular to that at start of reentry
$(\ )_1$	initial reentry conditions
$(\ )_2$	parameter value at some cutoff condition

**BLANK PAGE**

## INTRODUCTION

Because lifting reentry permits significant mission flexibility while subjecting the crew to smaller g loads, it is the object of considerable investigative effort as typified by References 1 through 5. However, it does not readily lend itself to extensive scrutiny through closed-form solutions, although there is a continuing effort to do so as seen by the numerous treatments of the subject. Typical of these efforts are references 6 through 9. However, in most cases, one or more of three drawbacks are found in these treatments which tend to limit the applicability and usefulness of the solution. First, they may be restricted to L/D ratios  $\leq 2.0$ . Second, they may be somewhat cumbersome and difficult to employ because each parameter does not stand alone thus permitting the effect of variations in that single parameter to be easily noted. This second limitation makes parametric studies difficult to undertake and somewhat detracts from the usefulness of the solution. Third, they may not adequately handle lateral range prediction.

The solution proposed in this report is not restricted by these limitations, and retains good accuracy over a wide range of the several variables involved. It is a closed-form integration of the equations of motion governing unpowered, lifting, banked reentry, and is restricted by a minimum of assumptions. The method has been employed to conduct a very thorough, broad, parametric analysis of numerous L/D ratios, bank angles and entry velocities to ascertain the limits of its range of applicability. The method fills an existing need for a rapid, accurate means of parametrically analyzing lifting, maneuvering reentry. The method can be employed with ease and does not require expensive computational equipment.

## ANALYSIS

### ASSUMPTIONS

The initial goal of this investigation was the development of a solution to the general equations of motion governing lifting reentry which would not be hampered by restrictive assumptions. Hence, it would be applicable to virtually any vehicle that could be devised, undergoing any type of possible reentry maneuver. As long as the vehicle could be assumed to generate a given L/D and bank angle while enduring reentry heating (g loads generally being unimportant) no weight, size, configuration or attitude limitations on the solution should exist. However, as the investigation progressed, it became obvious that some assumptions must be made in order to proceed to a meaningful solution. The following assumptions were considered to be the least restrictive that could be employed.

(1) The atmospheric reentry angle is sufficiently small to allow the approximations  $\gamma = 0^\circ$  and  $\cos \gamma = 1.0$ . Experience gained in the X-20 (Dyna-Soar) program indicated that  $|\gamma| \leq 2.0^\circ$  is generally true for lifting reentry, mainly due to heating and structural considerations. Therefore, this assumption is not really restrictive and introduces insignificant error.

(2) The value of the L/D ratio is held constant throughout the reentry. While this is not the ideal trajectory from a minimum heating, maximum maneuverability standpoint, it closely approximates an actual trajectory, generally deviating only in the early stages. As a rule, when L/D modulation does take place (usually for phugoid damping or to limit heating), deviations from a nominal value are not excessive and are often in both directions from the nominal, thus reducing the error incurred by the assumption of constant L/D. In addition, it is possible to account for L/D variation in the solution, but the solution becomes so cumbersome the time loss outweighs any gains in accuracy or simulation which might result and negates an important attribute of the method -- speed.

(3) Local circular velocity is held at a constant value during reentry and the value chosen is assumed unimportant as long as the local gravity and radius vector are consistent with the chosen value. The selected values are those existing at sea level. Since the change in the value from sea level to 300,000 ft altitude, where reentry is assumed to occur, is less than 1 percent, any error incurred is assumed to be insignificant. Comparison with reentry studies beginning at other altitudes also justifies this assumption.

(4) A nonrotating earth is assumed. This is a standard assumption justified by noting that earth rotation may be accounted for by proper timing of retrofire or by taking it into account in the final computer analysis where accuracy is all-important.

(5) The bank angle is held constant throughout reentry, except when the heading change reaches  $90^\circ$ , at which time it becomes  $0^\circ$  instantaneously until the completion of reentry. Again, as in the case of the L/D ratio, this does not result in the optimum reentry maneuver and the ability does exist to vary the bank angle if desired. However, to do so is quite cumbersome and reduces the usefulness of the solution.

(6) Finally, reentry velocities are limited to suborbital values for reasons which are readily apparent from examination of the final expressions obtained. This is perhaps the most restrictive of all the assumptions and is merely accepted as the penalty for obtaining the solution.

## DEVELOPMENT OF SOLUTION

The general equations of motion governing the atmospheric flight of a nonthrusting, lifting, banked vehicle may be written in the form (Figure 1):

$$m\dot{V} = -D - mg \sin \gamma \quad (1)$$

$$mV = \frac{mV^2 \cos \gamma}{r} + L \cos \phi - mg \cos \gamma \quad (2)$$

$$mV\dot{\psi} = \frac{L \sin \phi}{\cos \gamma} \quad (3)$$

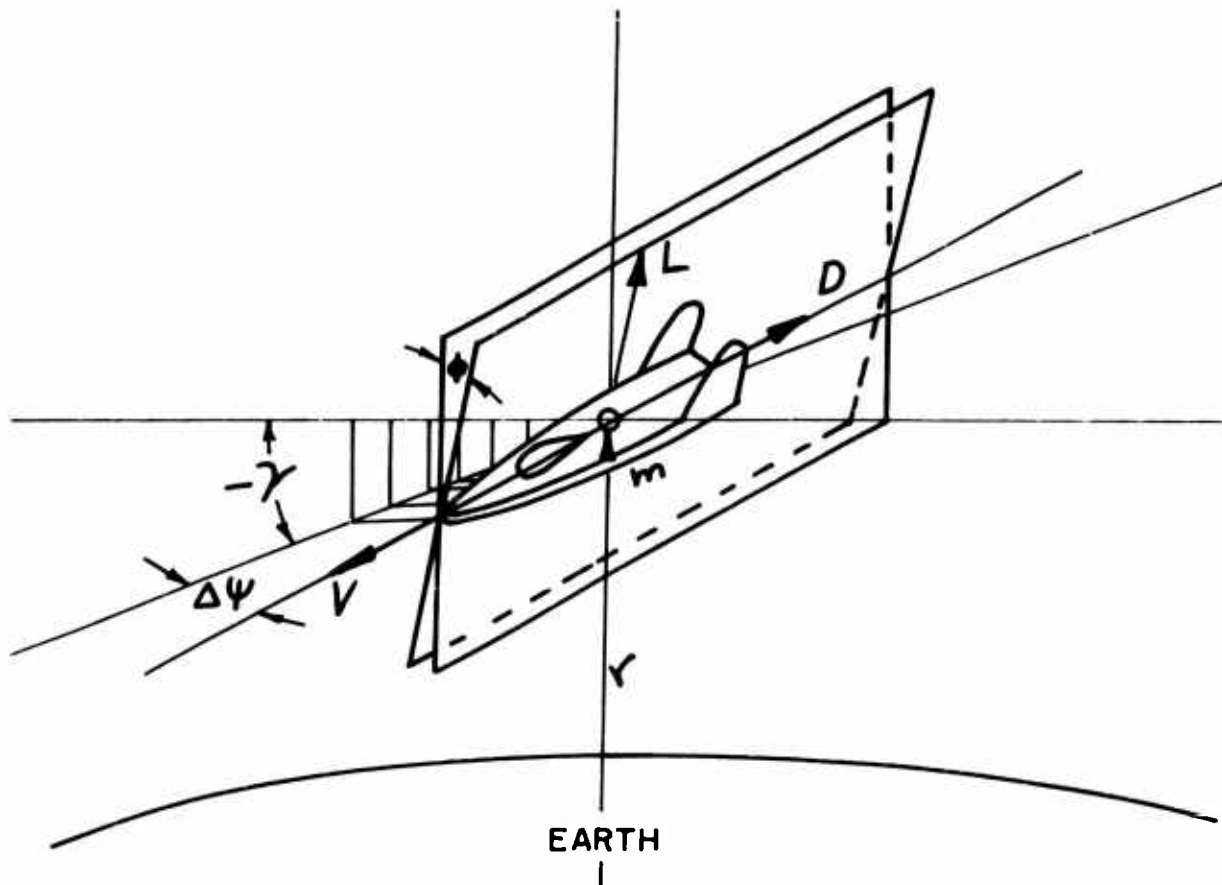


Figure 1. Vehicle Force Orientation

From the previously listed assumptions, these equations may be reduced to:

$$\dot{V} = -\frac{D}{m} \quad (4)$$

$$\frac{V}{r} + \frac{L \cos \phi}{mV} - \frac{g}{V} = \frac{L \cos \phi}{mg} + \left( \frac{V^2}{V_c^2} - 1 \right) = 0 \quad (5)$$

$$\dot{\psi} = \frac{L \sin \phi}{mV} \quad (6)$$

Before proceeding to the ultimate goal of this report, the longitudinal and lateral range prediction equations, several other results of significance can be obtained from Equations 4, 5, and 6. Combining Equations 4 and 5, obtain:

$$\dot{V} = \frac{D}{L} \frac{g}{\cos \phi} \left( \frac{V^2}{V_c^2} - 1 \right) \quad (7)$$

From this we obtain:

$$dt = \frac{L}{D} \frac{\cos \phi}{g} V_c^2 \frac{dV}{V^2 - V_c^2} \quad (8)$$

Assuming the time at the start of reentry is zero, the time (T) required to perform a reentry is given by:

$$T = \frac{L}{D} \frac{\cos \phi}{2} \frac{V_c}{g} \ln \left[ \frac{(1 - \frac{V}{V_c}) (1 + \frac{V_1}{V_c})}{(1 + \frac{V}{V_c}) (1 - \frac{V_1}{V_c})} \right]^* \quad (9)$$

This equation has been evaluated for several reentry and terminal velocities and the results are shown in Figure 2.

It is also possible to calculate the ground-track range from Equations 4 and 5 by eliminating time and installing distance as the independent variable. Thus, obtain:

$$\frac{VdV}{ds} = \frac{D}{L} \frac{g}{\cos \phi} \left( \frac{V^2}{V_c^2} - 1 \right) \quad (10)$$

which may be rewritten in the form:

$$ds = \frac{L}{D} \cos \phi \frac{V_c^2}{g} \frac{dV}{V^2 - V_c^2} \quad (11)$$

Again, by letting the distance at the start of reentry be zero, the ground track range (S) is given by:

$$S = \frac{L}{D} \frac{\cos \phi}{2} \frac{V_c^2}{g} \ln \left[ \frac{1 - \frac{V^2}{V_c^2}}{1 - \frac{V_1^2}{V_c^2}} \right] \quad (12)$$

This equation has also been evaluated for various reentry and terminal velocities and the results are shown in Figure 3.

Now, defining the change in the heading angle,  $\Delta\psi$ , by the expression:

$$\Delta\psi = \int \dot{\psi} dt = \int \frac{\dot{\psi}}{V} dV \quad (13)$$

and by combining Equations 4 and 6 to obtain:

$$\frac{\dot{\psi}}{V} = -\frac{L}{D} \frac{\sin \phi}{V} \quad (14)$$

---

\*Underlined equations are of particular importance.

the change in the heading angle may be written:

$$\Delta \psi = -\frac{L}{D} \sin \phi \int_{V_1}^V \frac{dV}{V} \quad (15)$$

which integrates readily to yield:

$$\Delta \psi = \frac{L}{D} \sin \phi \ln \frac{V_1}{V} \quad (16)$$

This defines the heading change with velocity change during reentry. While Equation 16 is useful in this form, it has even greater utility if rewritten in the form:

$$\frac{V}{\Delta \psi = 90^\circ} = \frac{V_1}{\exp \left( \frac{\pi}{2(L/D) \sin \phi} \right)} \quad (17)$$

since it is necessary to know the vehicle velocity when the heading change reaches  $90^\circ$  in order to employ the closed-form solution. The results of employing Equation 17 for a wide range of vehicle and reentry conditions are plotted in Figure 4 which is used extensively for problem solution.

Having an equation for the heading change, it now is possible to proceed to the final results. Assuming, as is commonly done, that the longitudinal and lateral range can be written in the form:

$$R_X = \int V \cos \Delta \psi dt = \int \frac{V}{\dot{V}} dV \cos \Delta \psi \quad (18)$$

and

$$R_Y = \int V \sin \Delta \psi dt = \int \frac{V}{\dot{V}} dV \sin \Delta \psi \quad (19)$$

respectively; then, by combining Equations 4 and 5 to obtain:

$$\frac{V}{\dot{V}} = -\frac{L}{D} \frac{\cos \phi}{g} \frac{V}{1 - \frac{V^2}{V_c^2}} \quad (20)$$

and using Equation 16 for  $\Delta \psi$ , we obtain:

$$R_X = \frac{L}{D} \frac{\cos \phi}{g} \int_{V_2}^{V_1} \cos \left( \frac{L}{D} \sin \phi \ln \frac{V_1}{V} \right) \frac{V dV}{1 - \frac{V^2}{V_c^2}} \quad (21)$$

and

$$R_Y = \frac{L}{D} \frac{\cos \phi}{g} \int_{V_2}^{V_1} \sin \left( \frac{L}{D} \sin \phi \ln \frac{V_1}{V} \right) \frac{V dV}{1 - \frac{V^2}{V_c^2}} \quad (22)$$

for the longitudinal and lateral ranges, respectively.

In order to integrate, and also to simplify the writing of these expressions, the following substitutions are made:

$$x = \frac{V}{V_c} \quad a = \frac{V_1}{V_c} \quad \frac{V_2}{V_c} = 0 \quad (23)$$

Obviously, the last term in Equation 23 needs some explanation regarding its value. Recall that a major purpose of the method is to obtain general expressions for use over a wide range of conditions. Since altitude effects have been assumed unimportant, reentry could be assumed to occur at any velocity from circular speed down to zero in the extreme or, in other words, the ratio  $V_1/V_c$  could vary from 1 to 0. Thus, the range obtained from reentry to some cutoff value, say at  $\Delta\psi = 90^\circ$ , could be written as the range from reentry to zero velocity minus the range from cutoff to zero velocity with no loss in generality. Extensive simplification is introduced into the analysis by this reasoning. Thus, each problem may be treated as if presented in the form:

$$\int_{V_1}^{V_2} dV = \int_0^{V_2} dV - \int_0^{V_1} dV \quad (24)$$

which is quite proper for a continuous function.

Thus, employing the terms in Equation 23, Equations 21 and 22 become:

$$R_X = \frac{L}{D} \frac{V_c^2 \cos \phi}{g} \int_0^a \cos \left( \frac{L}{D} \sin \phi \ln \frac{a}{x} \right) \frac{x dx}{1-x^2} \quad (25)$$

and

$$R_Y = \frac{L}{D} \frac{V_c^2 \cos \phi}{g} \int_0^a \sin \left( \frac{L}{D} \sin \phi \ln \frac{a}{x} \right) \frac{x dx}{1-x^2} \quad (26)$$

To accomplish the integration, the following substitution is made:

$$x = e^{-\frac{y}{2}} \quad dx = -\frac{1}{2} e^{-\frac{y}{2}} dy \quad b = \frac{L}{D} \sin \phi \quad (27)$$

Then Equations 25 and 26 have the form:

$$R_X = \frac{L}{D} \frac{\cos \phi}{2} \frac{V_c^2}{g} \int_{-2 \ln(a)}^{\infty} \cos \left[ b \left( \ln a + \frac{y}{2} \right) \right] \frac{e^{-y} dy}{1 - e^{-y}} \quad (28)$$

$$R_Y = \frac{L}{D} \frac{\cos \phi}{2} \frac{V_c^2}{g} \int_{-2 \ln(a)}^{\infty} \sin \left[ b \left( \ln a + \frac{y}{2} \right) \right] \frac{e^{-y} dy}{1 - e^{-y}} \quad (29)$$

However, if we write the following relation:

$$\frac{1}{1 - e^{-y}} = \sum_{k=0}^{\infty} e^{-ky} \quad (30)$$

then Equations 28 and 29 may be written:

$$R_X = \frac{L}{D} \frac{\cos \phi}{g} \int_{-2 \ln(a)}^{\infty} \cos \left[ b \left( \ln a + \frac{y}{2} \right) \right] \sum_{n=1}^{\infty} e^{-ny} dy \quad (31)$$

and

$$R_Y = \frac{L}{D} \frac{\cos \phi}{2} \frac{V_C^2}{g} \int_{-2 \ln(a)}^{\infty} \sin \left[ b \left( \ln a + \frac{y}{2} \right) \right] \sum_{n=1}^{\infty} e^{-ny} dy \quad (32)$$

Equations 31 and 32 are readily integrable if one expands the sine and cosine terms and integrates term by term. By proceeding as above, and judiciously combining terms, the final expressions become:

$$R_X = \frac{L}{D} \frac{\cos \phi}{2} \frac{V_C^2}{g} \sum_{n=1}^{\infty} \frac{n \left( \frac{V_1}{V_C} \right)^{2n}}{n^2 + \left( \frac{L}{D} \frac{\sin \phi}{2} \right)^2} \quad (33)$$

$$R_Y = \left( \frac{L}{D} \right)^2 \frac{\sin \phi \cos \phi}{4} \frac{V_C^2}{g} \sum_{n=1}^{\infty} \frac{\left( \frac{V_1}{V_C} \right)^{2n}}{n^2 + \left( \frac{L}{D} \frac{\sin \phi}{2} \right)^2} \quad (34)$$

which define, respectively, the longitudinal and lateral range capabilities of a lifting, banked, nonthrusting reentry vehicle. Note that these expressions are valid for any  $L/D$  value, any bank angle  $< 90^\circ$  and any entry velocity in the range  $0 \leq V_1/V_C < 1$ , since Equation 33 has

the form  $\sum_{n=1}^{\infty} \frac{1}{n}$  for  $V_1/V_C = 1$  and would not converge. For practical reasons, values of  $V_1/V_C > 0.995$  were not considered in this analysis since this is a reasonable maximum value for most suborbital analyses.

The series terms in Equations 33 and 34 have been evaluated over the following variable values:  $(L/D) \sin \phi$  from 0 to 4.0 and  $V_1/V_C$  from 0 to 0.995. The results are plotted in Figures 5 and 6. For convenience, Figure 7 is a plot of velocity values ratioed to earth circular values and will aid in the transfer among curves in the report.

#### COMPARATIVE COMPUTER STUDY

To ascertain the accuracy and range of applicability of this closed-form solution, and also to serve as a basis for future use, an inclusive parametric study was undertaken and accomplished employing this solution. The parameters studied and the range of values for each were: vehicle  $L/D$  from 0.5 to 4.0; entry velocity from  $0.89 V_C$  to  $0.99 V_C$ ; and bank angles from  $0^\circ$  to  $75^\circ$ . The results of this study are shown in Figures 8 through 17. To check these results a parametric computer study was also run over this range of parameter values utilizing the Six-Degree-of-Freedom Computer Program (Reference 10) which originated in the Flight Dynamics Laboratory. A comparison between the results obtained from this computer study and those obtained employing the closed-form approximation is given in Figures 18 through 32.

The vehicle characteristics for the computer study are completely arbitrary, but are considered reasonable for early, manned, maneuverable vehicles. Previous studies (Reference 1) indicated that maneuverability and velocity loss were not greatly affected by mass and reference area variations within reasonable limits. The values chosen for the study are listed in Table 1.

TABLE I  
REENTRY CONDITIONS AND VEHICLE CHARACTERISTICS

$h_1 = 300,000 \text{ ft}$ $\gamma_1 = -0.8^\circ$ $\sigma_1 = 90^\circ$		mass = 500 slugs Area = 500 ft <sup>2</sup> $\Omega = 0 \text{ rad/sec}$
L / D	$C_L$	$C_D$
0.5	0.63978	1.27956
1.0	0.41662	0.41662
2.0	0.17248	0.08624
3.0	0.0873	0.0291
4.0	0.05208	0.01302

## DISCUSSION OF RESULTS

An endeavor to analyze each curve generated on a point-by-point basis would yield no useful results. Rather, the general characteristics and trends are briefly considered, with emphasis being placed on the characteristics that are most significant. Before proceeding with this discussion, however, two conditions which were chosen for final parameters in the computer runs should be considered.

The first condition is the reentry angle of  $-0.8^\circ$  on which the computer runs were based. Previous experience indicated that guidance capabilities and vehicle structural limitations resulted in a nominal reentry angle of slightly less than  $-0.8^\circ$  with a tolerance of approximately  $\pm 0.5^\circ$ . This value has been carried over to the present report. Some trial cases were run at other reentry angles from  $0^\circ$  to  $-2.0^\circ$  to test the vehicle sensitivity to reentry angle. It was found to have very little influence except for low L/D, high entry velocity conditions. The  $-0.8^\circ$  reentry angle also somewhat reduced the excessive phugoid motion inherent in vehicles flying at constant L/D and very small incidence angles.

The second condition imposed for all computations was a cutoff velocity of 4000 f/s. This was done for two reasons: (1) It was felt that hypersonic aerodynamics cannot be applied below this value; (2) From the author's previous experience, it was found that the vehicle should be approximately over the intended landing sight by the time the velocity dropped to 4000 f/s in order to make a safe dead stick landing. With these comments in mind, it is possible to consider the comparative figures obtained by solving identical reentry problems employing the closed-form and computer solutions.

Considering Figures 18 through 32 it is evident that the method possesses no gross errors since curve shape is quite similar in all cases. Generally, accuracy improves with increasing L/D and decreasing reentry velocity. The first trend is highly desirable since the lower L/D and ballistic configurations are already well documented in the literature. Disagreement at the higher reentry velocities is rather easily explained from two aspects. First, there is the characteristic phugoid motion which causes the vehicle to skip out and back into the atmosphere several times before beginning its smooth reentry. While doing this skipping, considerable longitudinal range is being obtained, but very little lateral range is achieved until a significant loss in velocity has occurred. Then, the vehicle begins to turn but does not have sufficient velocity remaining to generate the side force necessary to reach the maximum potential lateral range. As reentry velocity decreases, phugoiding is reduced and the two solutions agree more closely in their range predictions. An actual reentry would probably be conducted with a modulated L/D to reduce the phugoid motion, and to increase lateral range. Nevertheless, the closed-form solution does not account for this phugoid motion, thus tending to underpredict longitudinal range slightly while overpredicting lateral range, in some cases to an unacceptable degree. Perhaps some modification to the solution could be used to reduce this error to a tolerable level. As a further point of interest, lateral range was found to be virtually independent ( $< 1\%$  variation) of entry angle ( $\gamma$ ) over the range investigated.

Figures 33 through 38 are of particular value since these show a comparison between the closed-form and computer maximum predicted longitudinal and lateral ranges as a function of L/D and reentry velocity. The values from these figures are those generally quoted when speaking of range capability and return-from-orbit performance. It may be seen that agreement is good except for the high L/D, high velocity lateral range values which, as stated earlier, are unacceptably in error.

Figures 39 and 40 are presented to show the effects of altitude variation and earth rotation on the attainable range (Reference 11). Note that slightly different aerodynamics were employed and a reentry angle of  $0^\circ$  was assumed at 250,000 ft. While this reentry angle is slightly unrealistic from a guidance standpoint, it may be seen that the closed-form expression predicts performance adequately.

There is disagreement between the closed-form and the computer solutions regarding the predicted longitudinal and lateral range coordination. For instance, computer runs predict maximum lateral range for a bank angle ( $\phi$ ) near  $60^\circ$ , while the closed form predicts a value nearer to  $45^\circ$ . Therefore, entire footprints should be mapped out with the closed-form solution in order to decide on approximate vehicle characteristics. Here, the solution speed is readily appreciated as entire footprints are quickly obtained.

F. S. Nyland (Reference 12), in his independent investigation of the same problem, utilized an approach very similar to that developed in this study and obtained identical results. However, the simpler resultant expressions and the graphical presentation of the series expansions make the closed-form solution described in this report a more rapid technique to use over a greater variety of imposed conditions. In addition, this report shows extensive comparisons with computer solutions, illustrating the range of applicability and the regions where significant errors occur.

## CONCLUSIONS

The closed-form solution is not without its limitations. However, its advantages far outweigh these adverse characteristics.

1. The proposed closed-form solution predicts the longitudinal and lateral range capabilities of maneuverable spacecraft with good accuracy over a wide range of reentry conditions and vehicle characteristics.

2. The solution is rapid and simple to employ allowing extensive parametric studies in a minimum of time.

3. Care must be employed in the mechanics of application especially in reading the curves representing the series expansions.

4. Each parameter stands alone and hence may be varied independently. In addition, the built-in sensitivity of the solution permits a rapid study of the effects of small variations in any given parameter.

5. Entire footprints should be mapped out because of some discrepancies in the bank angle-range correspondence. However, the difficulty is minor unless severe bank angle restrictions are likely to be imposed on the final configuration.

6. Overall, the solution fills a need for a rapid, accurate, simple method for predicting the performance of maneuverable reentry vehicles over a wide range of conditions and vehicle characteristics and attitudes.

REFERENCES

1. Bell, R. N. and Hankey, W. L., Jr. Application of Aerodynamic Lift in Accomplishing Orbital Plane Change. ASD TDR 63-693. Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio. September 1963.
2. Wallace, R. A. and Gray, W. A. "Minimum Lift-Drag Ratio Required for Global Landing Coverage." Technical Note in AIAA Journal. Vol I, No. II. November 1963.
3. Knott, P. R. and Johnson, D. T. "Determination of Positions and Orientation Angles for a High L/D Glide Reentry to Hit a Predetermined Earth Target." AFFDL TM 64-36. Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. October 1964.
4. Bruce, R. W. "The Combined Aerodynamic-Propulsive Orbital Plane Change Maneuver." AIAA Paper No. 65-20. AIAA Second Aerospace Sciences Meeting, New York, N. Y. January 1965.
5. Cuadra, E. and Arthur, P. D. "Orbit Plane Change by External Burning Aerocruise." AIAA Paper No. 65-21. AIAA Second Aerospace Sciences Meeting, New York, N. Y. January 1965.
6. Baradell, D. L. and McLellan, C. H. "Lateral Range and Hypersonic Lift-Drag Ratio Requirements for Efficient Ferry Service From a Near-Earth Manned Space Station." Second Manned Space Flight Meeting. April 1963.
7. Slye, R. E. An Analytical Method for Studying the Lateral Motion of Atmosphere Entry Vehicles. NASA TN D-325. September 1960
8. Chapman, D. R. An Approximate Analytical Method for Studying Entry Into Planetary Atmospheres. NASA TR R-11. 1959.
9. Eggers, A. J., Allen, H. J., and Neice, S. E. A Comparative Analysis of the Performance of Long-Range Hypervelocity Vehicles. NACA Report No. 1382. 1958.
10. Air Force Flight Dynamics Laboratory. Six-Degree-of-Freedom Flight Path Study Generalized Computer Program. FDL TDR 64-1. Wright-Patterson Air Force Base, Ohio. October 1964.
11. Benson, B. R. and Nash, R. C. "Constant L/D Reentry Study." FDM TM 63-11. Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. July 1963.
12. Nyland, F. S. Hypersonic Turning With Constant Bank Angle Control. Rand Corporation Memorandum RM-4483-PR. March 1965.

## APPENDIX

## UTILIZATION PROCEDURE

While the closed-form solution is simple and straightforward to use, a step-by-step "recipe" is delineated here to prevent interpretation errors and to insure the maximum benefit from its use. First, the following initial conditions and vehicle information are required: (1) The vehicle L/D values; (2) The range of bank angles ( $\phi$ ) to be considered; and (3) The reentry velocity range to be investigated. With this information at hand the recommended procedure is as follows:

- (1) Calculate all  $(L/D) \sin \phi$  values resulting from the range of values chosen for L/D and  $\phi$ .
- (2) For the reentry velocity values chosen, and the  $(L/D) \sin \phi$  values calculated, read the velocity value from Figure 4 where  $\Delta \psi = 90^\circ$ .
- (3) From Figures 5a through 5e read two values of the longitudinal range series function--one at the reentry velocity chosen and one at the velocity where  $\Delta \psi = 90^\circ$  or at  $V = 4000$  f/s, whichever is greater, but both, naturally, on the same  $(L/D) \sin \phi$  curve previously calculated.
- (4) Subtract the second reading from the first resulting in the net longitudinal series value.
- (5) Multiply the net series value by the quantities in front of the series in Equation 33. The result obtained is the longitudinal range capability of the vehicle under consideration for the specified reentry and attitude conditions.

The procedure for obtaining the lateral range capability parallels that for the longitudinal range in some cases, but in others it may become somewhat more involved; as will be seen. Steps 1 through 4 are essentially the same as those above and are merely repeated for completeness.

- (1) Calculate the applicable  $(L/D) \sin \phi$  values.
- (2) Read the velocity for  $\Delta \psi = 90^\circ$  under the imposed conditions.
- (3) From Figures 6a through 6c read the two values of the lateral range series function--one at the reentry conditions and one at the velocity value for  $\Delta \psi = 90^\circ$  or  $V = 4000$  f/s, whichever is greater, and again, both on the same  $(L/D) \sin \phi$  curve.
- (4) Subtract the second reading from the first resulting in the net lateral range series value. Here the similarity between the two calculations may end--according to this test: If the value of the velocity for  $\Delta \psi = 90^\circ$  as read from Figure 4 is less than 4000 f/s, the net lateral range value calculated above is the total value. If the velocity for  $\Delta \psi = 90^\circ$  is greater than 4000 f/s further computation is necessary and is described in steps 5 through 7. For either case, multiply the net lateral range series value from step 4 by the proper multiplying factor from Equation 34. For the case where the velocity for  $\Delta \psi = 90^\circ$  is less than 4000 f/s, this is the total lateral range for the selected reentry vehicle and imposed conditions and attitude. However, if the velocity for  $\Delta \psi = 90^\circ$  is greater than 4000 f/s, an additional calculation must be made.
- (5) Once the vehicle is normal to the original flight path ( $\Delta \psi = 90^\circ$ ) further turning is detrimental to the mission, generally speaking. However, the vehicle still possesses what may

be a considerable amount of velocity which should be converted to range. To take advantage of this, we assume the vehicle is rolled to a zero bank angle (to preclude further turning and flown in this attitude until the velocity reaches 4000 f/s. To complete the solution then Figures 5 a through 5e for longitudinal range are entered at the velocity for  $\Delta\psi = 90^\circ$  and  $(L/D) \sin \phi = 0$ .

(6) Again, two values for the series function are read--one at the velocity for  $\Delta\psi = 90^\circ$  and one at 4000 f/s.

(7) Subtracting the second reading from the first and multiplying by the proper factor from Equation 33, with  $\phi = 0^\circ$ , results in the final portion of the lateral range value. The sum of the lateral ranges from steps (5) and (7), if any, yields the total lateral range predicted for the reentry vehicle and the conditions and attitude chosen.

As can be seen, the procedure is straightforward, but the steps must be followed with precision to avoid confusion. The underscored items are particularly important and should be carefully noted to prevent obtaining erroneous results. A final note: Extreme care must be exercised in reading the series values from Figures 5 and 6 since subtractions of small or nearly equal quantities are often involved; the smallest error here is multiplied many times in the final result.

As a convenience, the following sample tabulation is extracted from the author's calculation and presented as a method check. A tabulation of this type is repeated as often as necessary to accomplish a complete parametric study for the conditions specified. The initial conditions are  $L/D = 2.0$ ;  $V/V_c = 0.99$ ;  $\phi = 15^\circ$  and  $75^\circ$ .

TABLE II

## LONGITUDINAL RANGE

$\phi$	$(L/D) \sin \phi$	$V_{\Delta\psi = 90^\circ}$	$\Sigma_{\text{entry}}$	$\Sigma_{\Delta\psi = 90^\circ}$ or $(V = 4000 \text{ f/s})$	$\Sigma_{\text{net}}$	$(\frac{L}{D}) \frac{\cos \phi}{2}$	$R_X / R_E$
15°	0.518	1200	3.845	0.27	3.818	0.966	3.685
75°	1.432	11300	3.35	0.17	3.18	0.2588	0.824

TABLE III

## LATERAL RANGE

$\phi$	$\Sigma_{\text{entry}}$	$\Sigma_{\Delta\psi = 90^\circ}$ or $V = 4000 \text{ f/s}$	$\Sigma_{\text{net}}$	$\Sigma_{\Delta\psi = 90^\circ}$	$\Sigma_{V=4000}$	$\Sigma_{\text{net}}$	$(\frac{L^2}{D}) \frac{\sin \phi \cos \phi}{4}$	$(\frac{L}{D}) \frac{\cos \phi}{2}$	$R_Y / R_E$
15°	1.48	0.021	1.459	—	—	—	0.25	—	0.365
75°	1.01	0.107	0.903	0.21	0.025	0.185	0.25	1.0	0.411

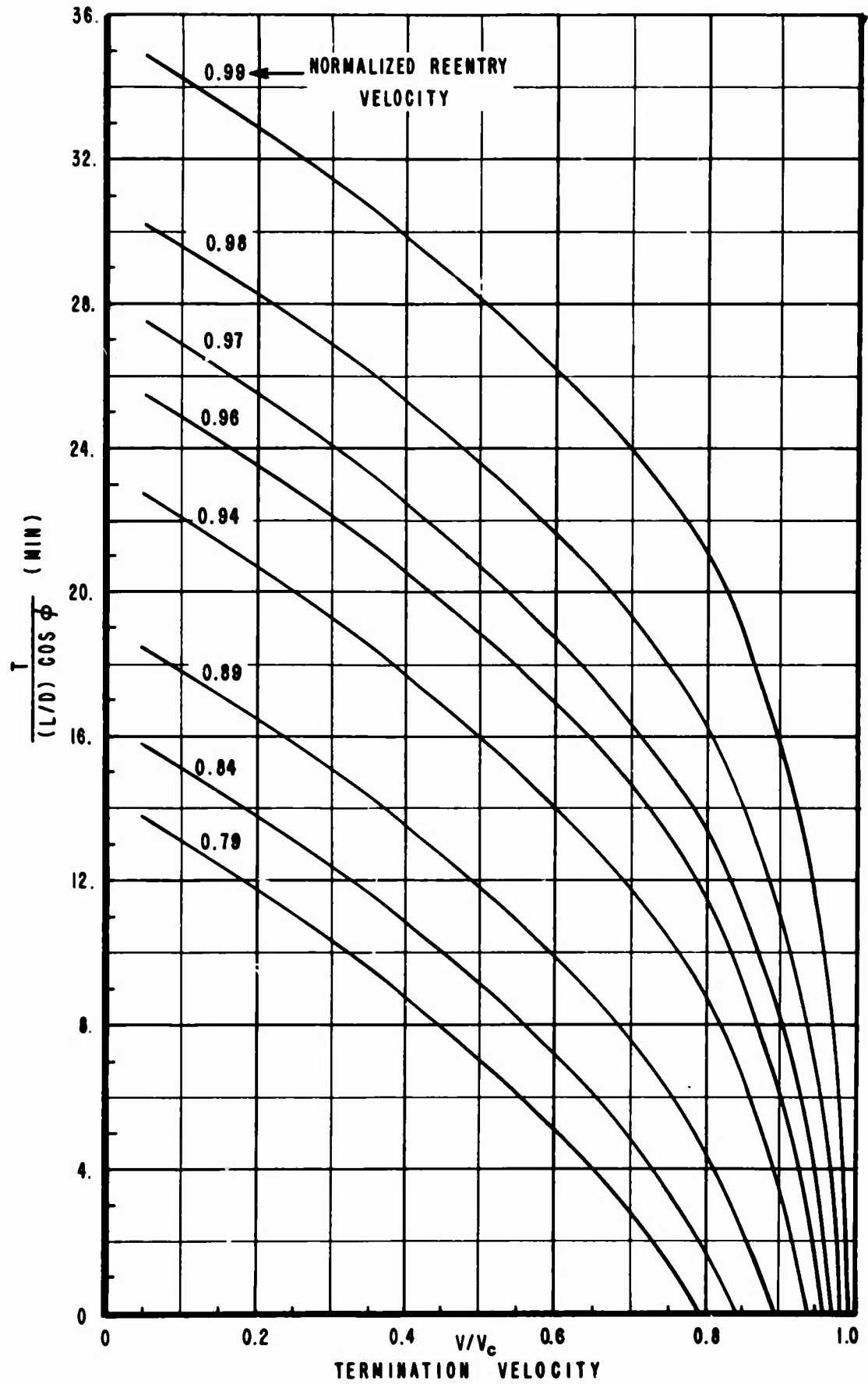


Figure 2. Reentry Time vs Termination Velocity for Various Reentry Velocities

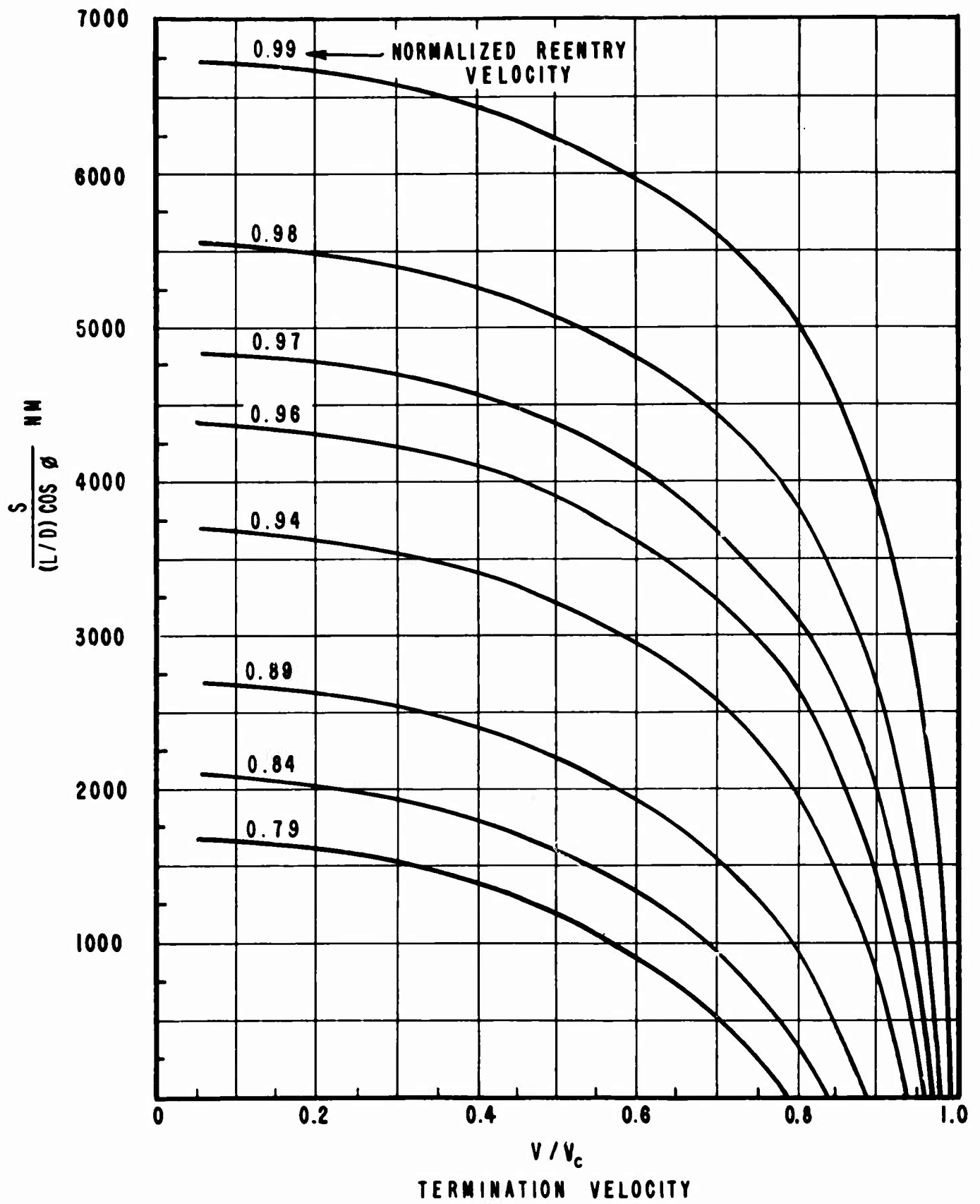
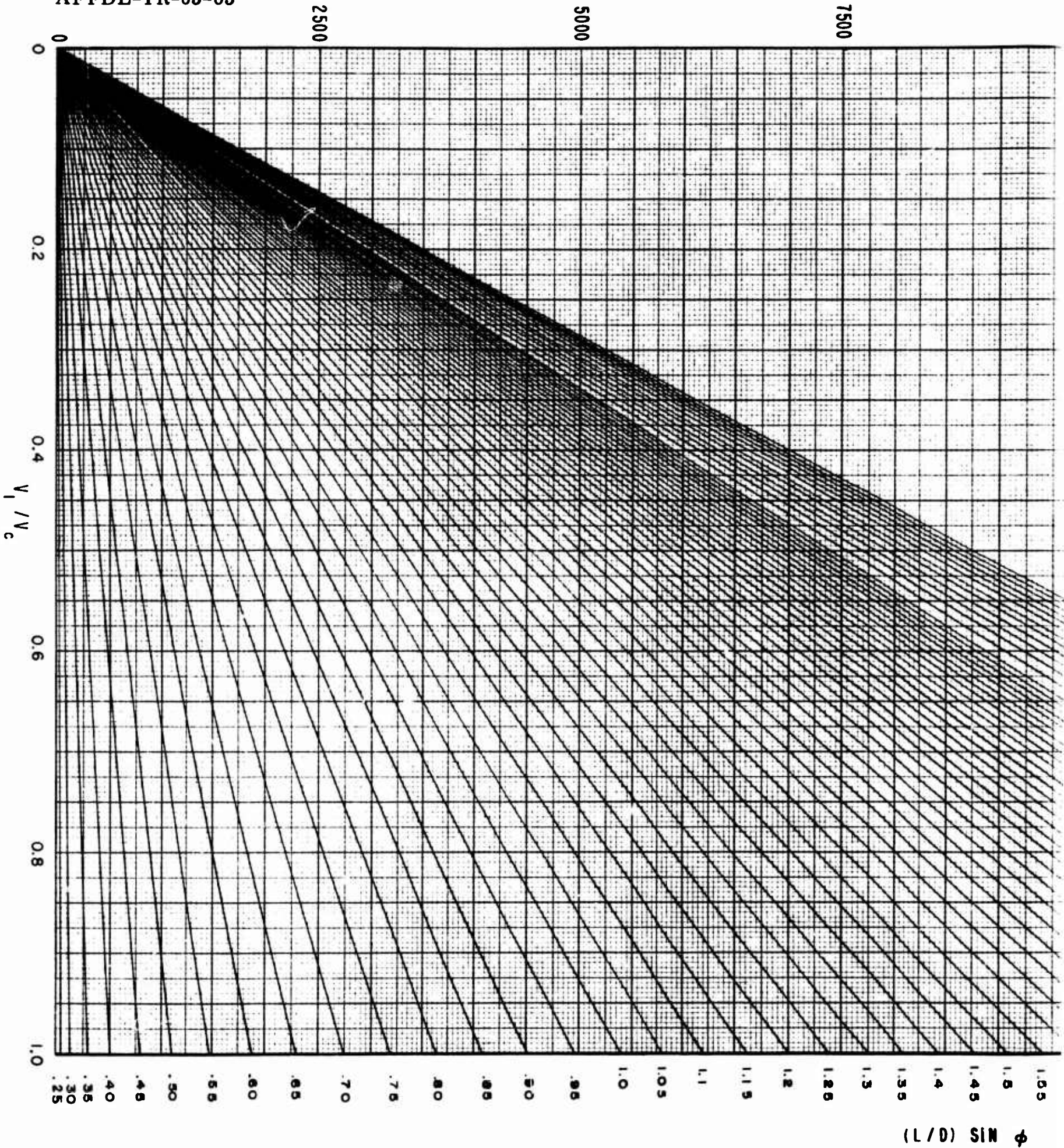


Figure 3. Total Range vs Termination Velocity for Various Reentry Velocities



**A**

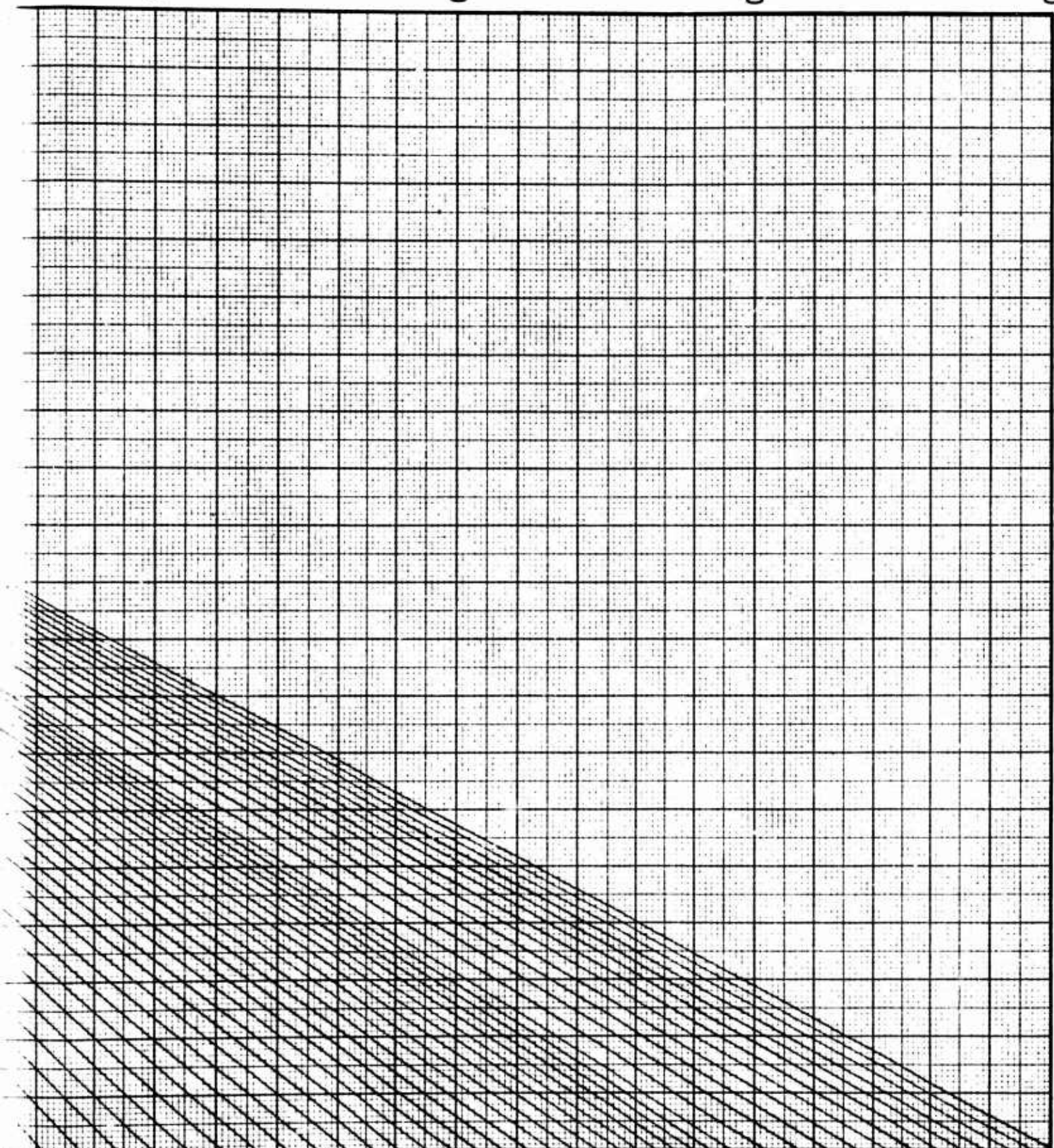
17,500

15,000

12,500

10,000

(f/s)

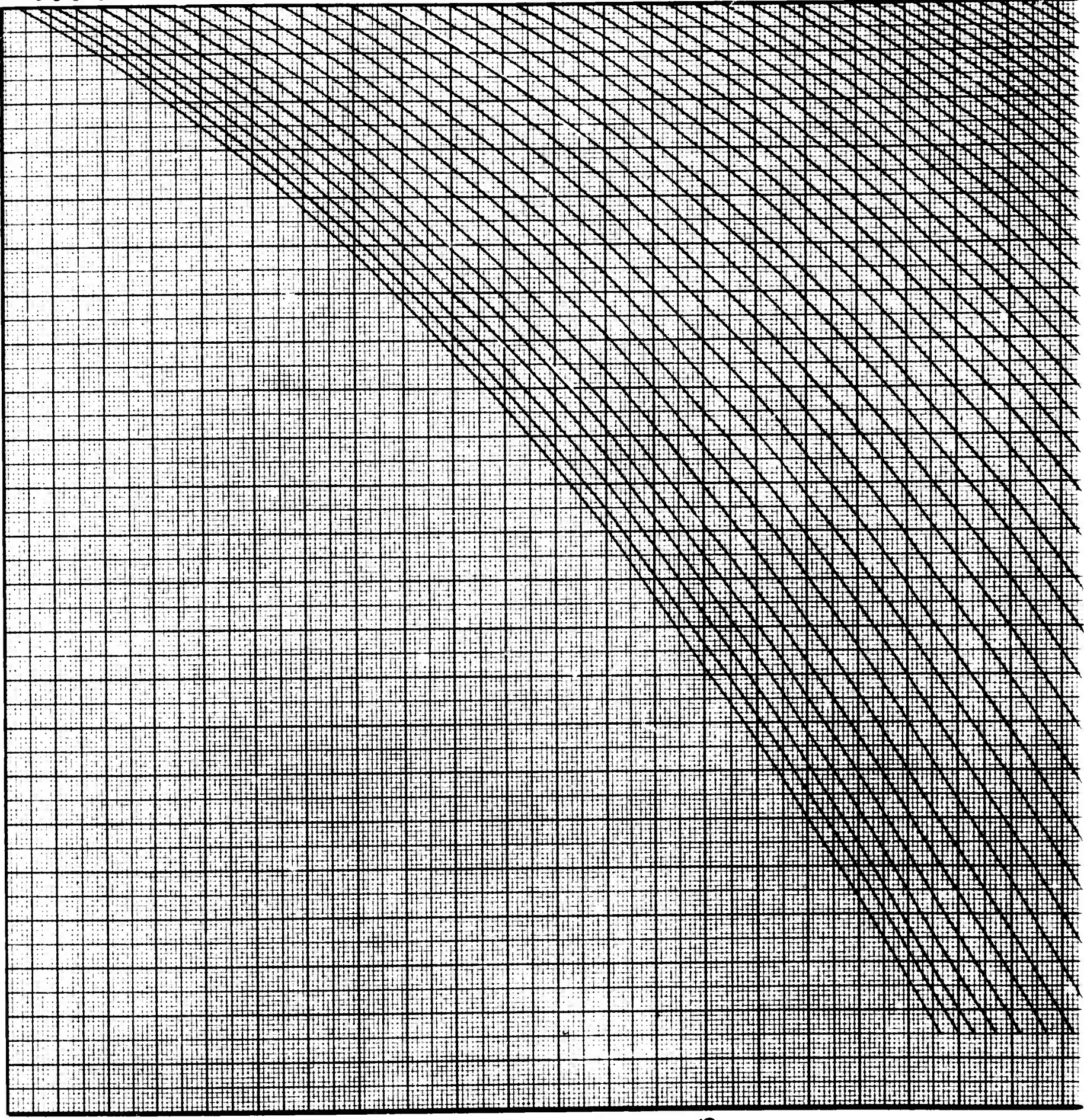


4.0  
3.9  
3.8  
3.7  
3.6  
3.5  
3.4  
3.3  
3.2  
3.1  
3.0  
2.9  
2.8  
2.75  
2.7  
2.65  
2.6  
2.55  
2.5  
2.45  
2.4  
2.35  
2.3  
2.25  
2.2  
2.1  
2.1  
2.1  
2.05  
2.0  
1.95  
1.9  
1.9  
1.85  
1.8  
1.75  
1.7  
1.65  
1.6  
1.55  
1.5

N φ

**B**

0 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3.0 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 4.0



4.5

4.25

4.0

**A**

3.75

$$u^2 (V/N)^2 \left( \phi \frac{D}{L} \sin \phi \right)^2 + 2u^2$$

$8M^2$

Figure 4. Velocity at  $\Delta\psi = 90^\circ$  for Various Entry Velocities and  $(L/D) \sin \phi$  Values

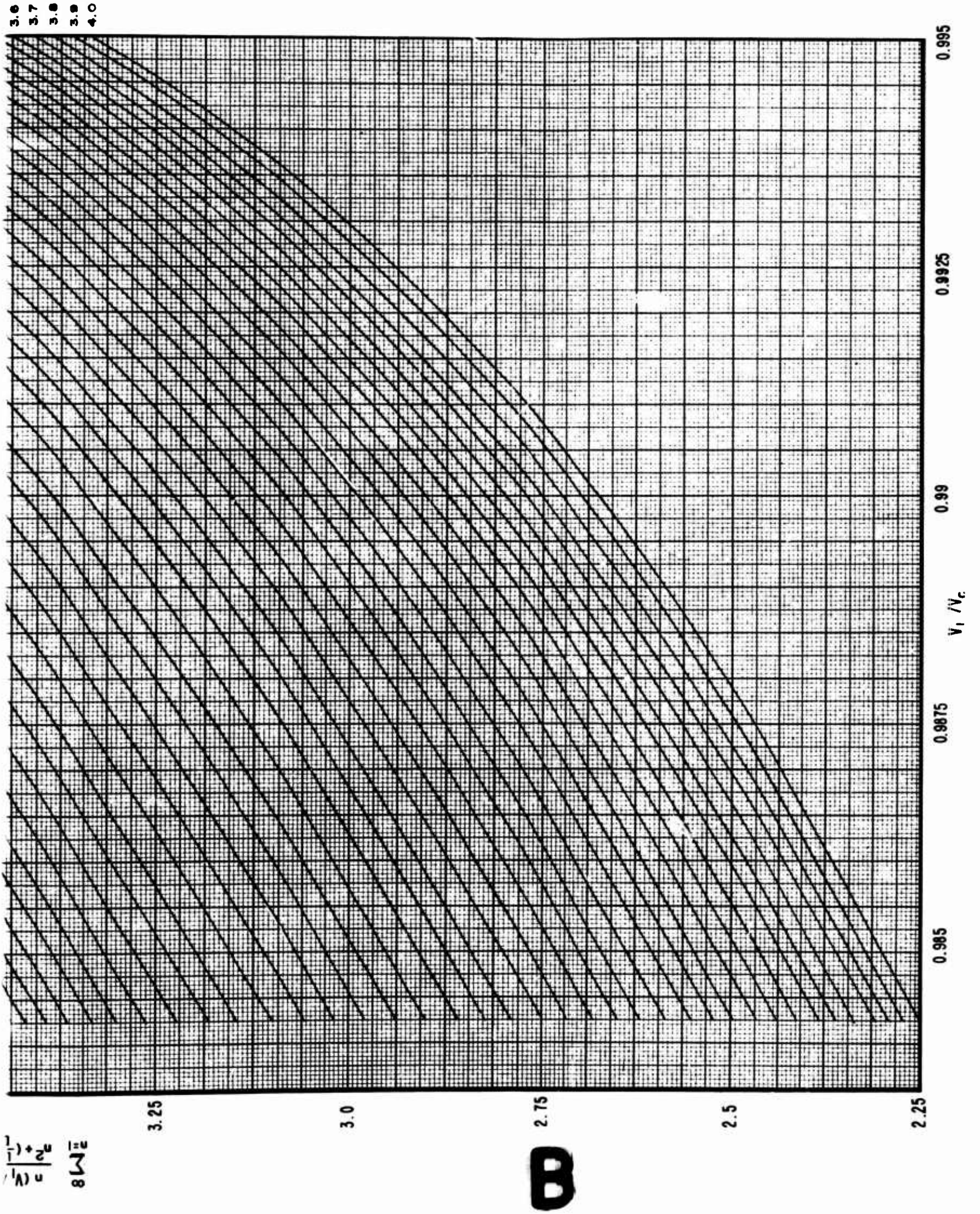
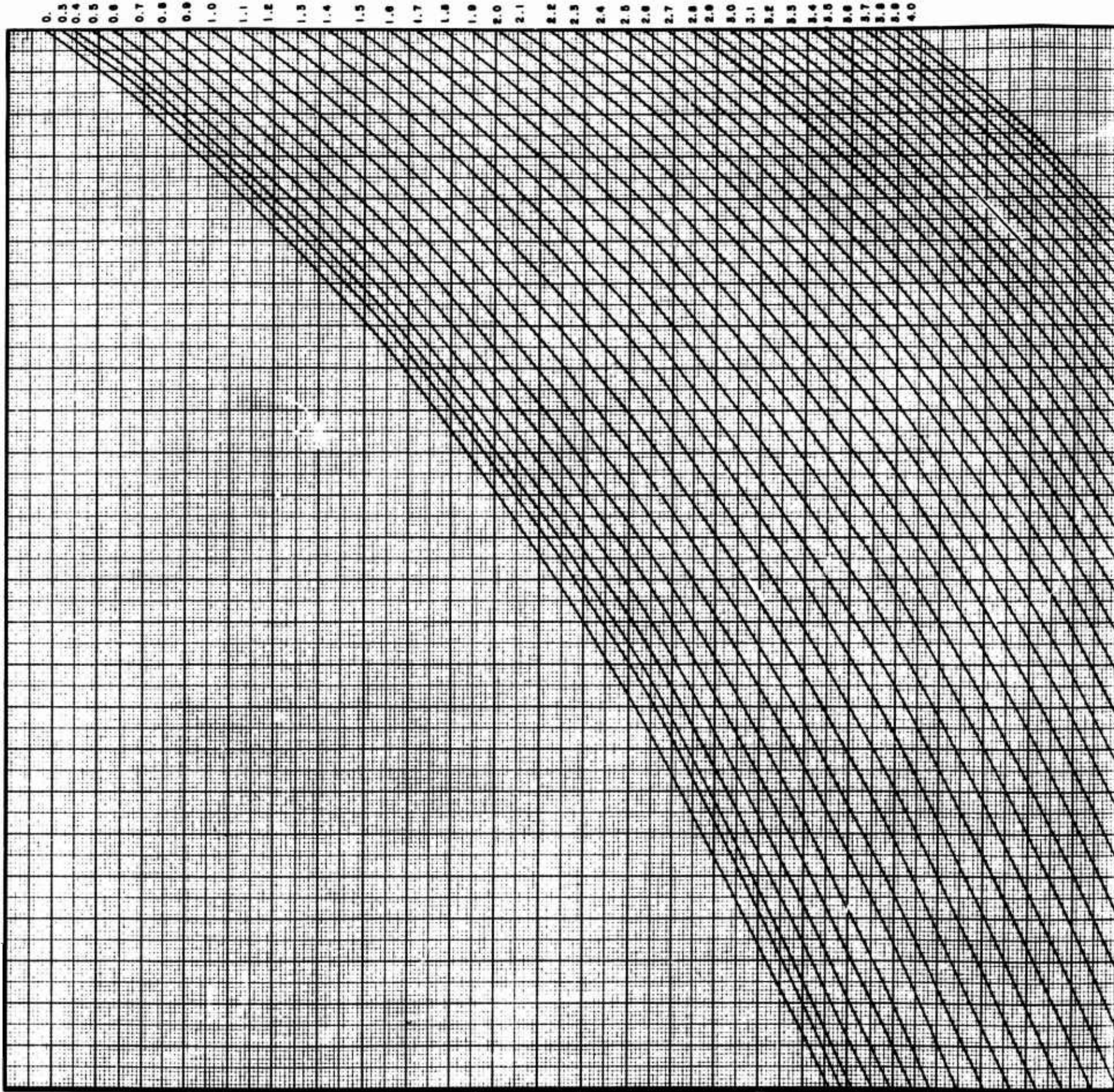


Figure 5a. Longitudinal Range Series Values

$\phi_{NIS} \left( \frac{D}{\lambda} \right)$



$$\frac{z \left( \frac{z}{\phi_{NIS}} \right) + z^u}{u_2 \left( \frac{z}{h} \right) = 8 \mathcal{N} z}$$

**A**

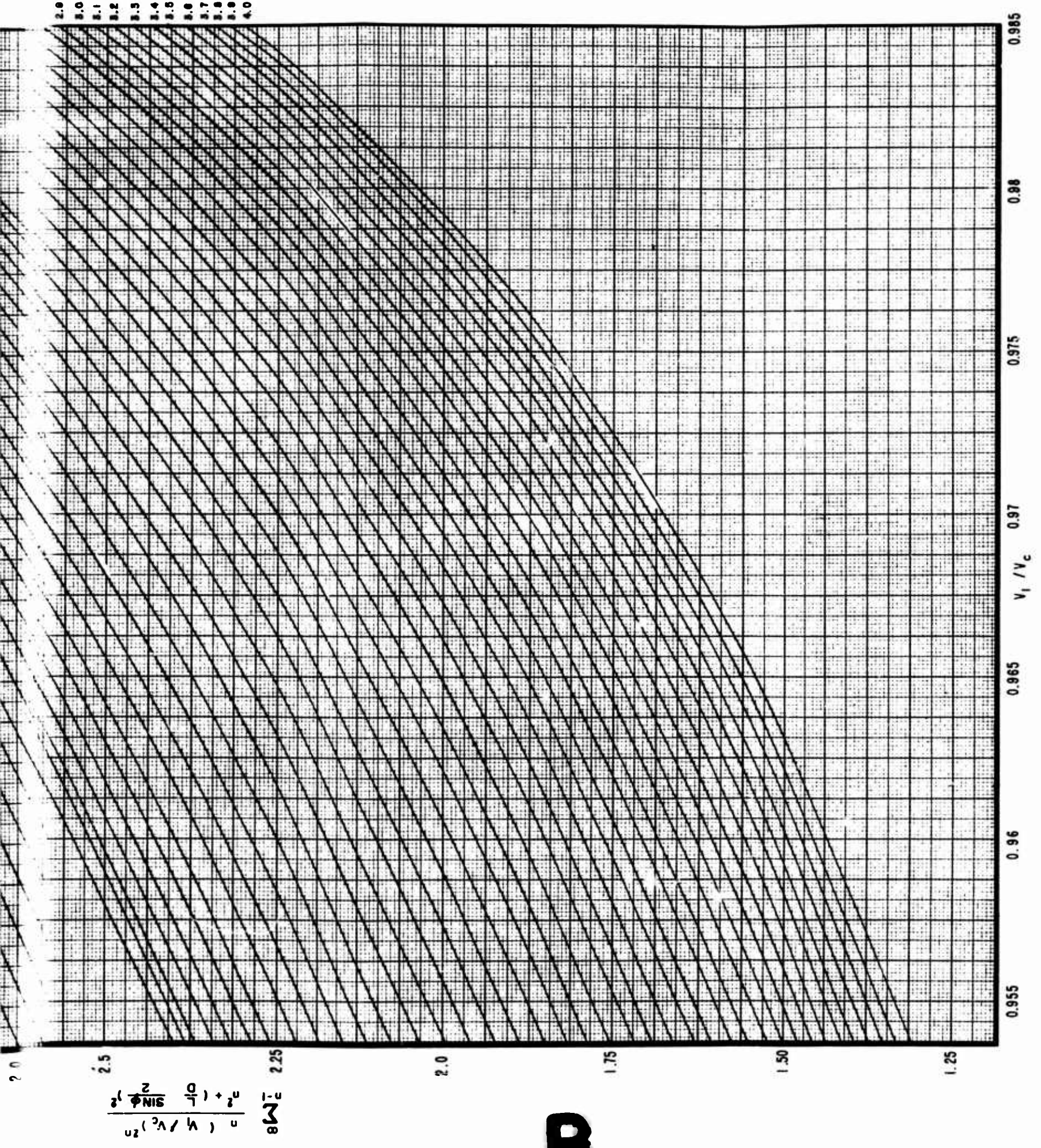
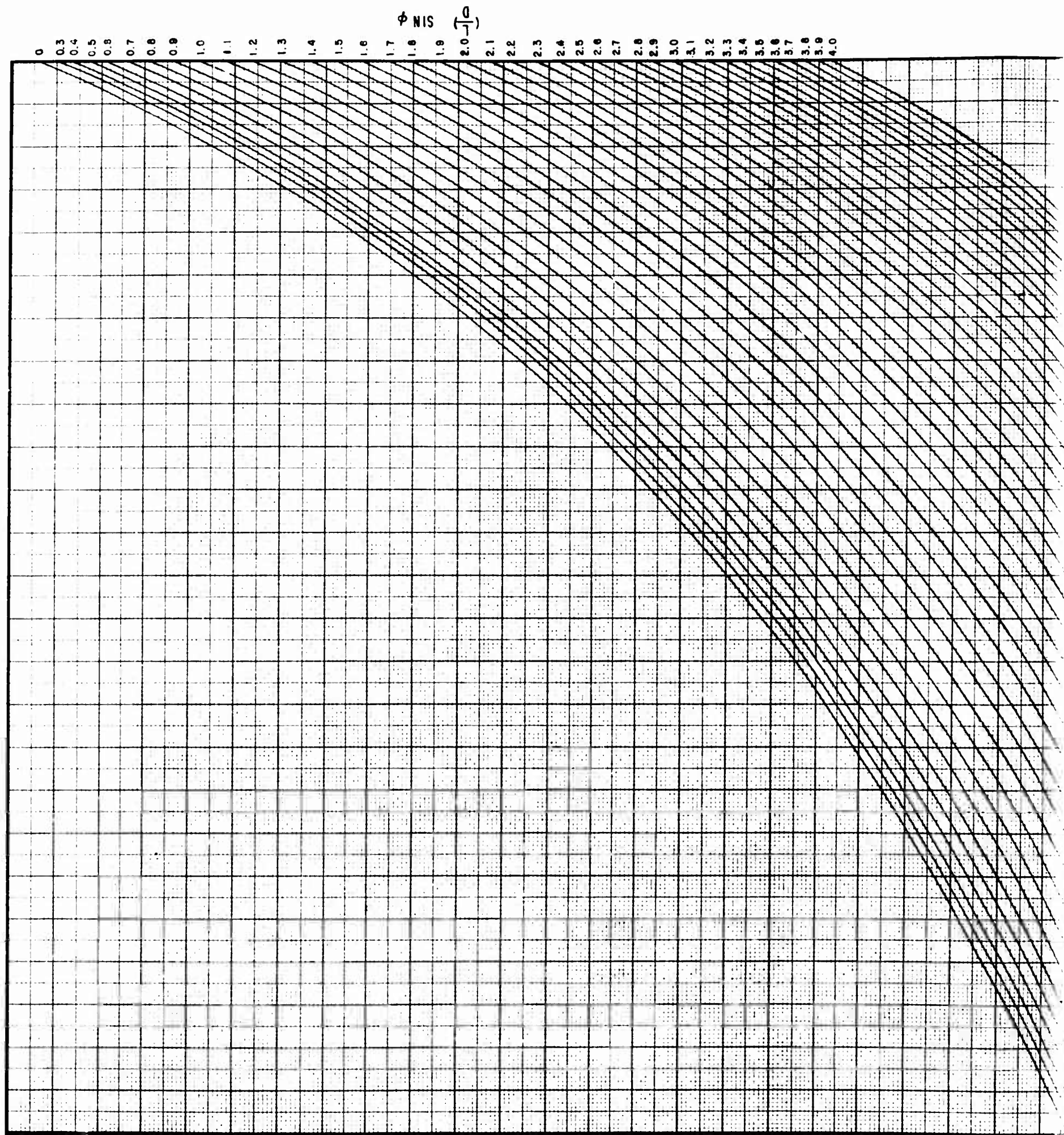


Figure 5b. Longitudinal Range Series Values



2.5

2.25

2.0

1.75

1.5

1.25



$$\sum_{i=1}^n \frac{u_i (\sqrt{v_i})}{2 \phi_{NIS} \left(\frac{1}{g}\right) + 2^u}$$

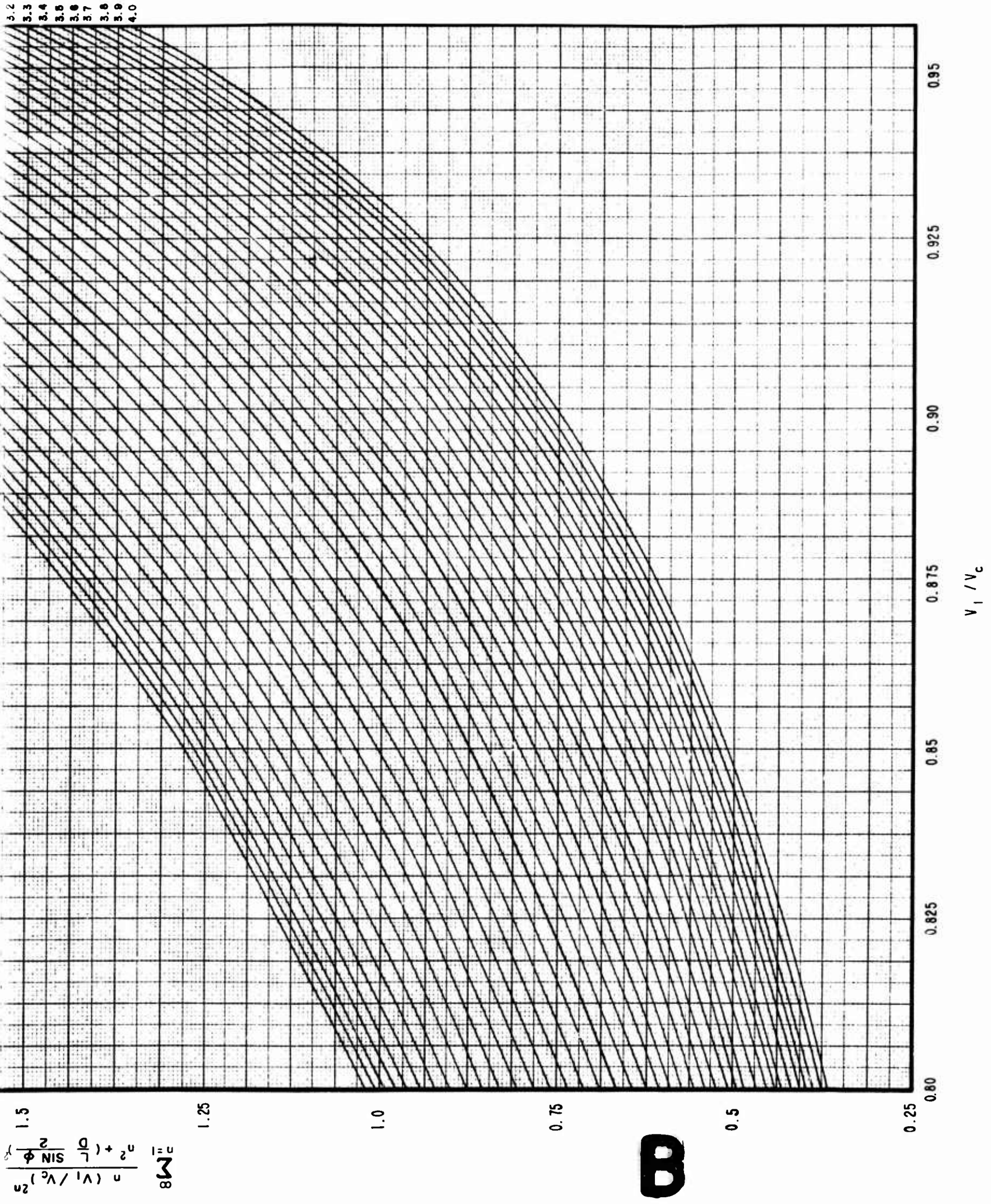


Figure 5c. Longitudinal Range Series Values

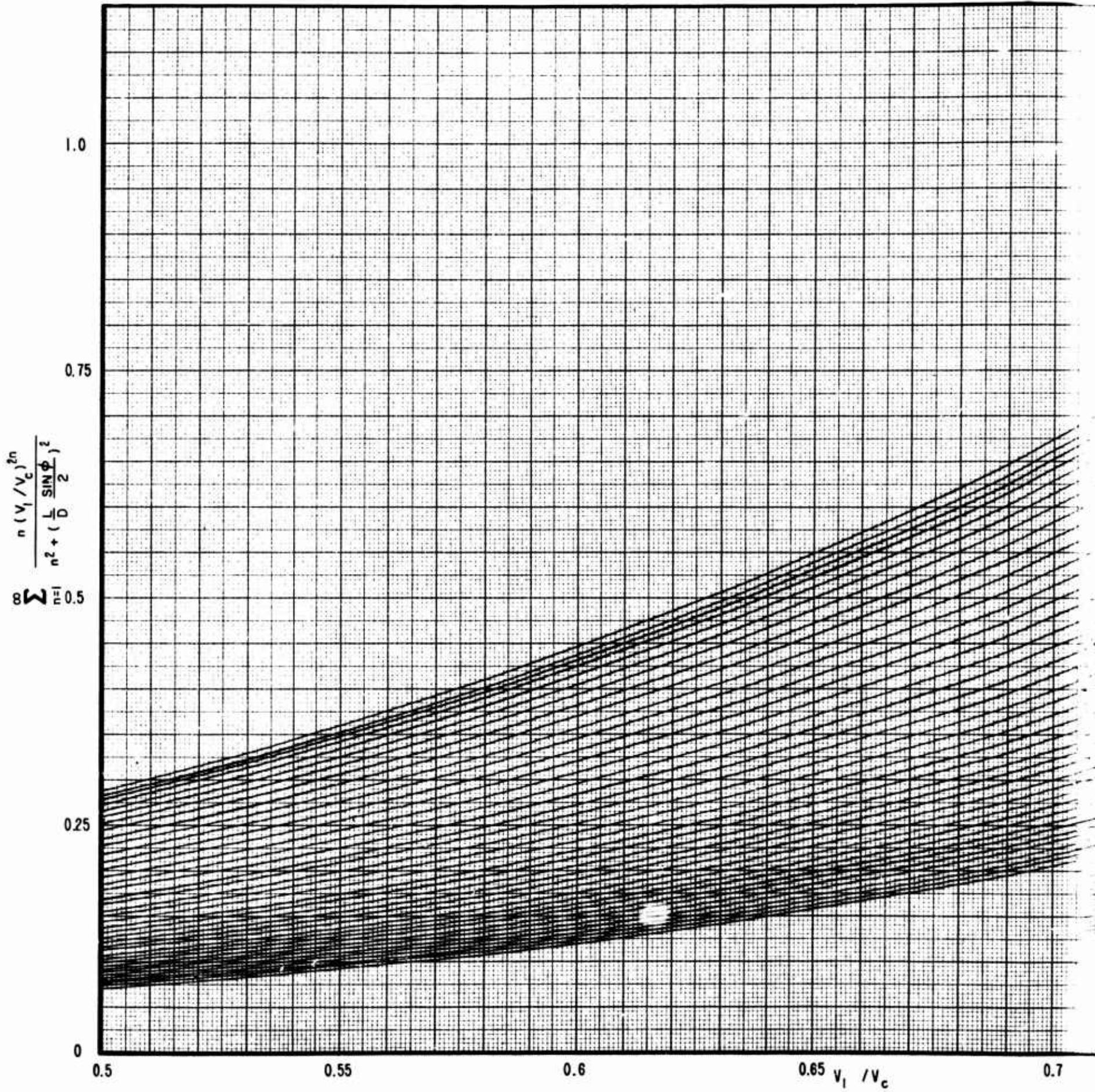
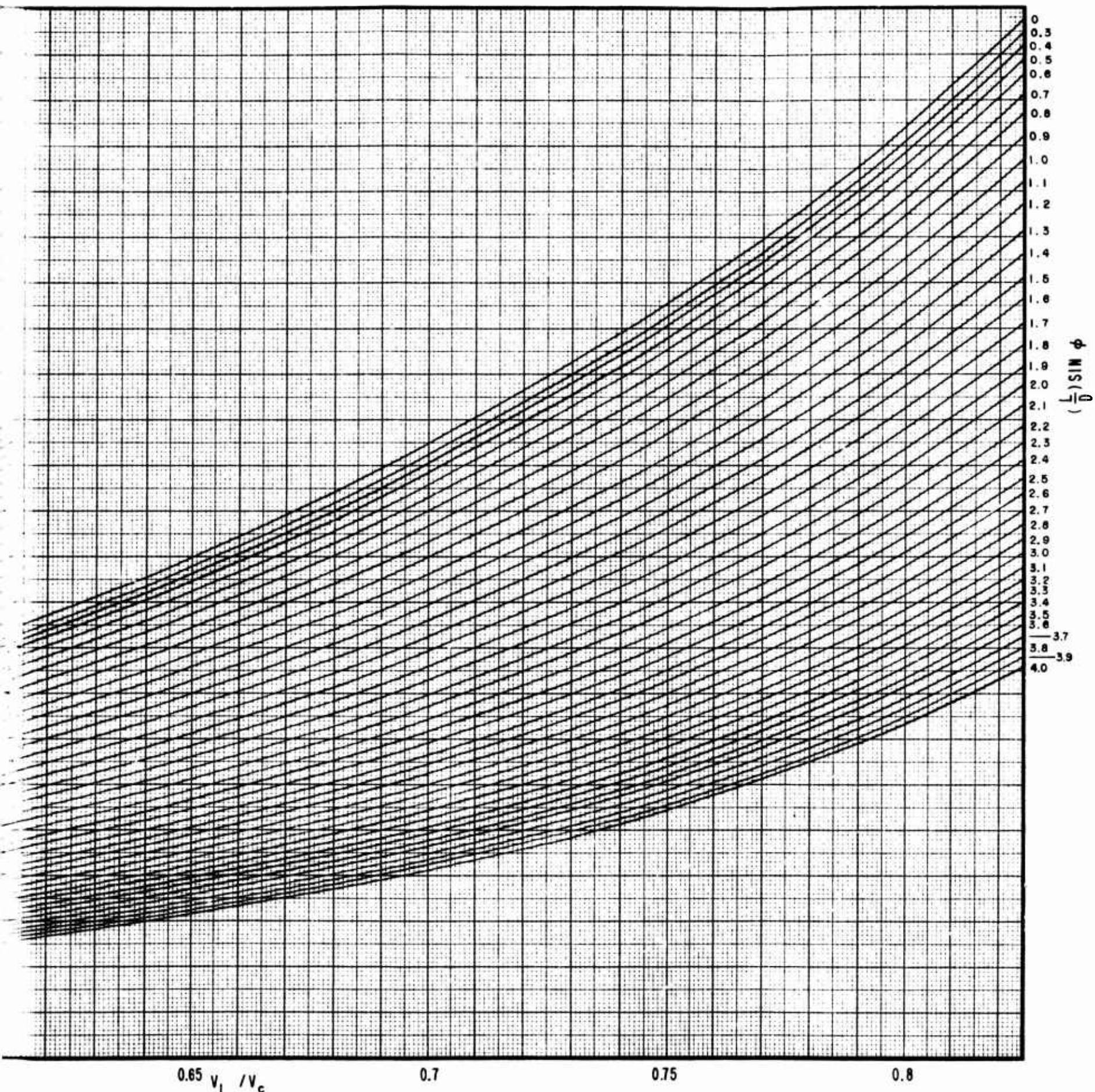


Figure 5d. Longitudinal Range Series Values

**A**



Longitudinal Range Series Values

**B**

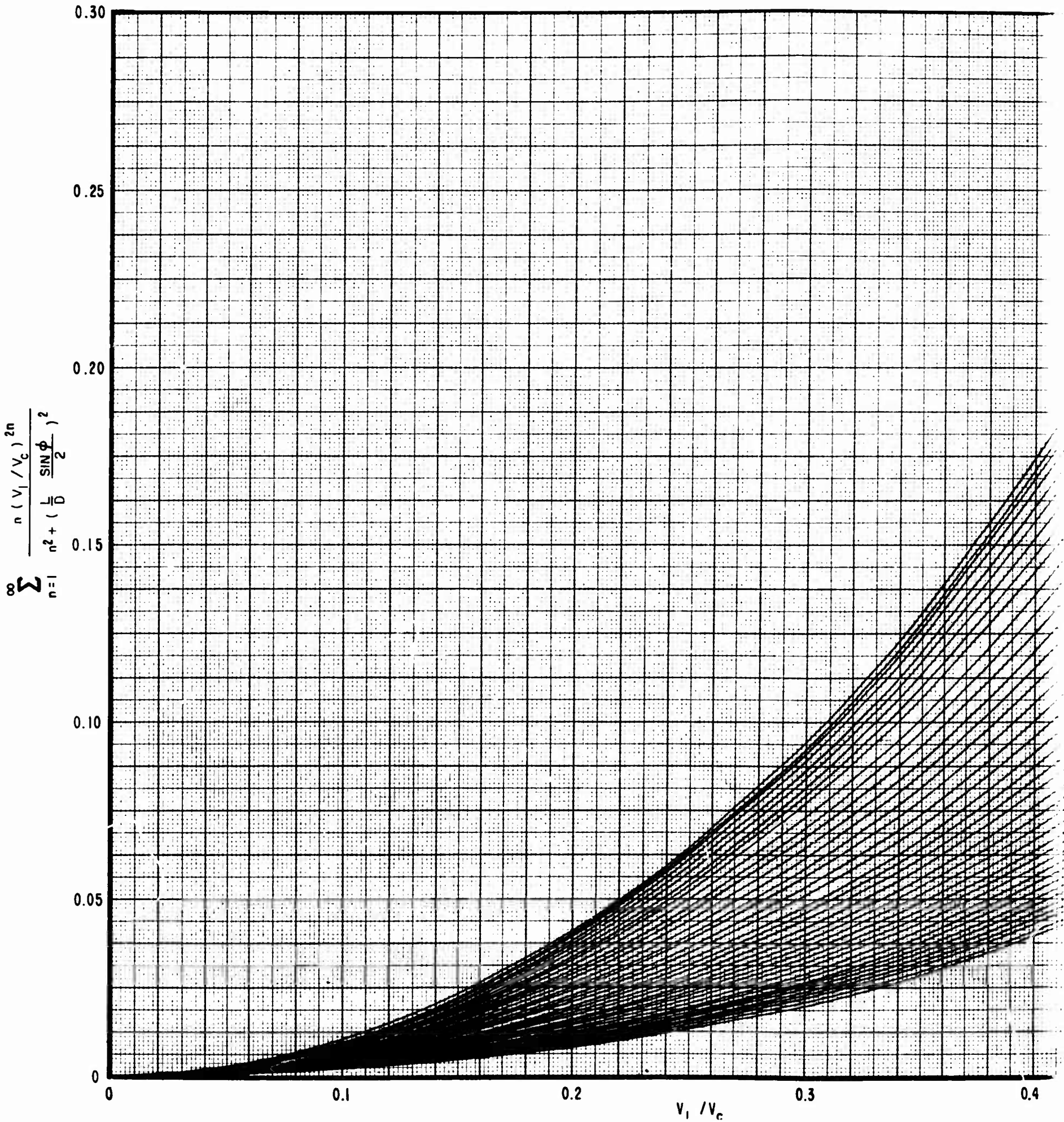


Figure 5e. Longitudinal Range Series Values

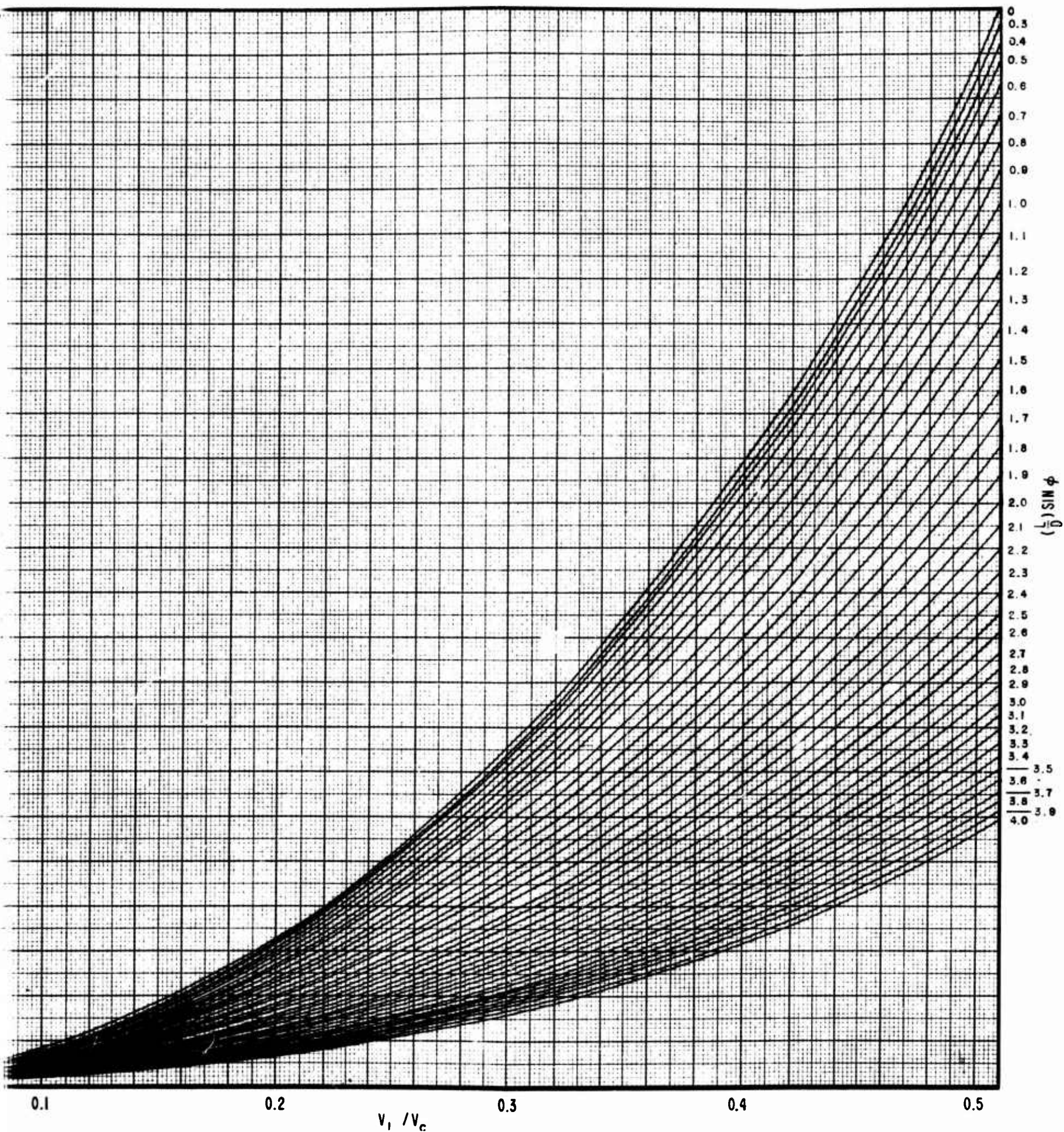


Figure 5e. Longitudinal Range Series Values

**B**

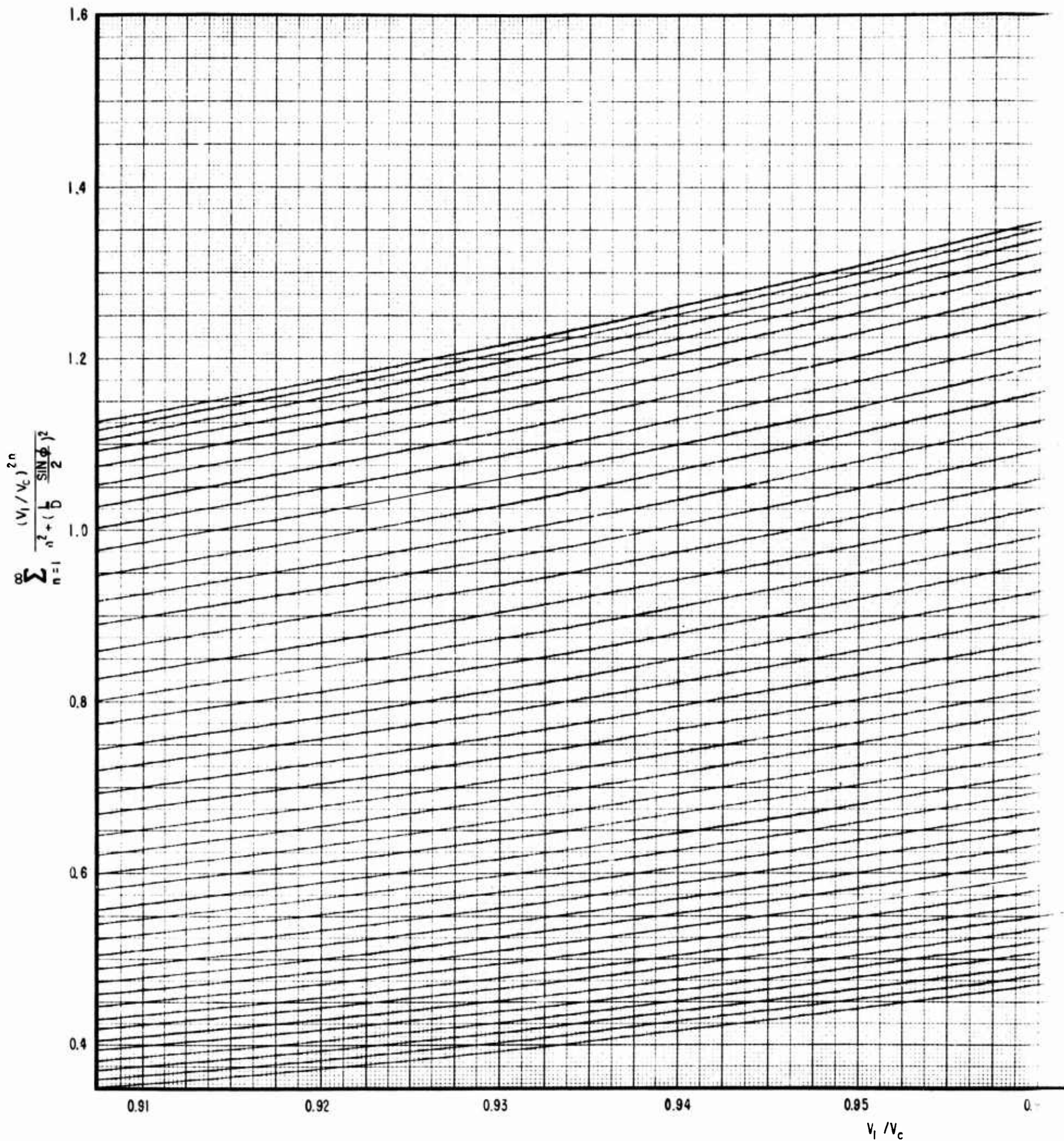


Figure 6a. Lateral Range Series

**A**

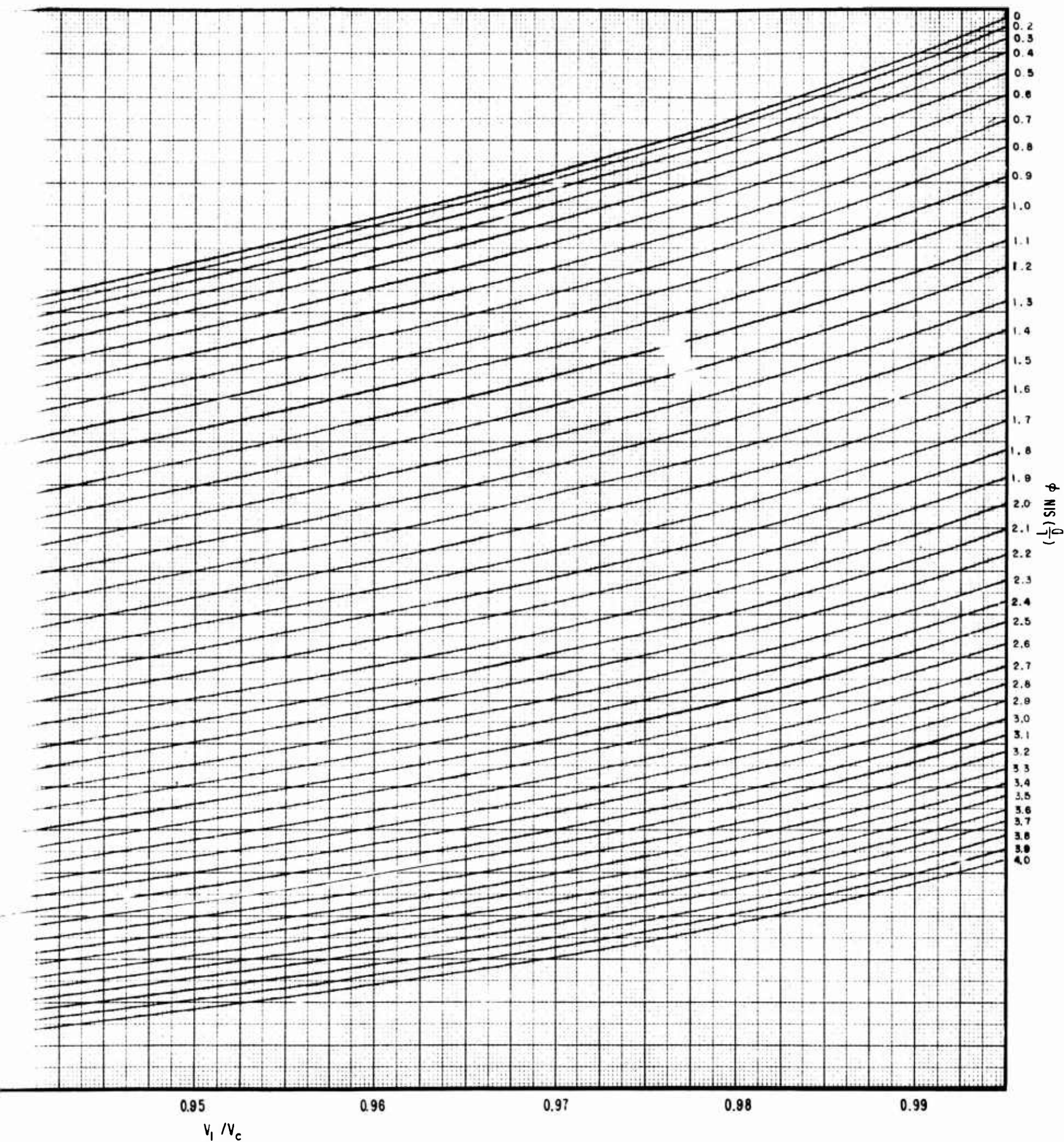
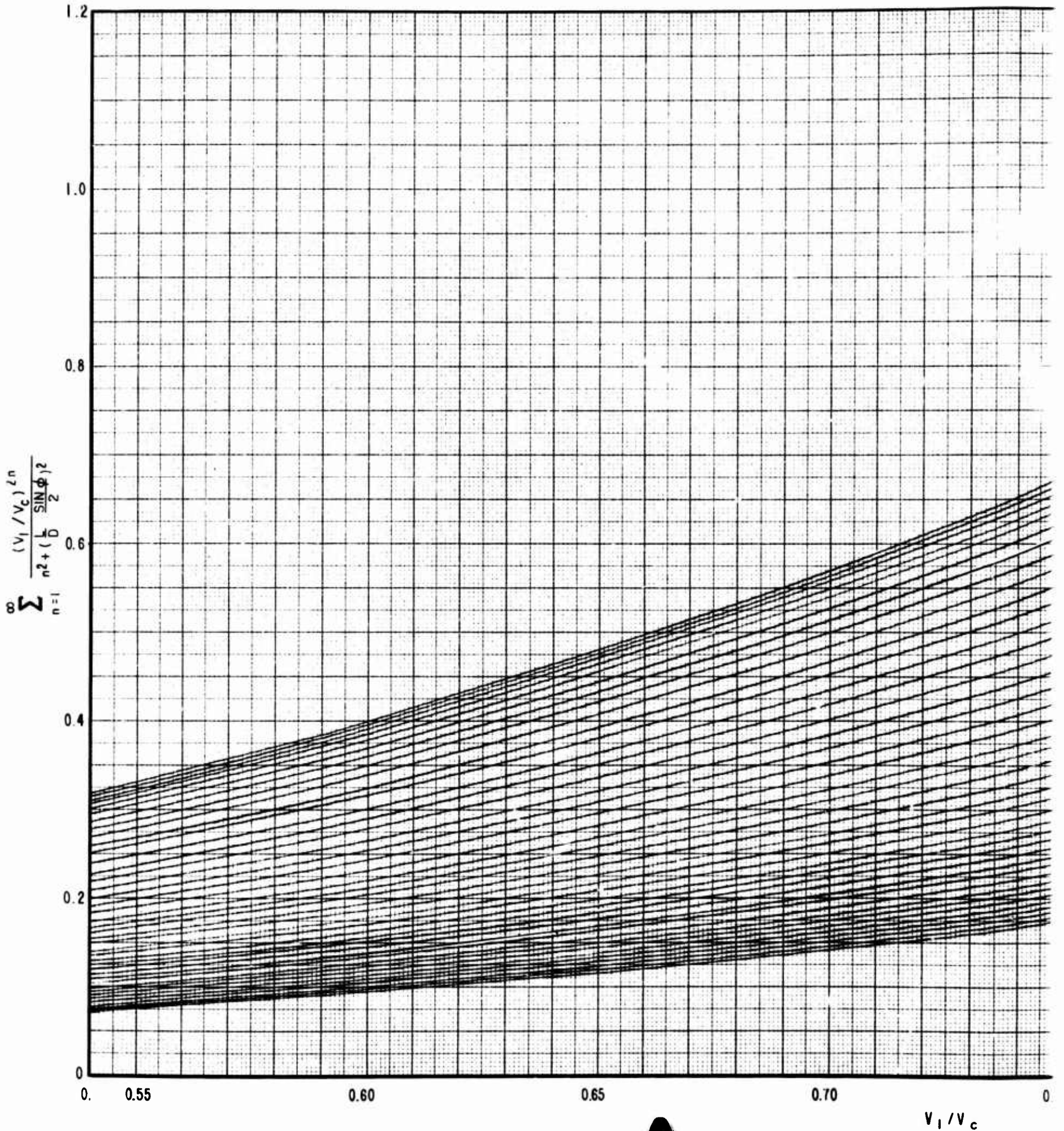


Figure 6a. Lateral Range Series Values

**B**



**A**

Figure 6b. Lateral Bar

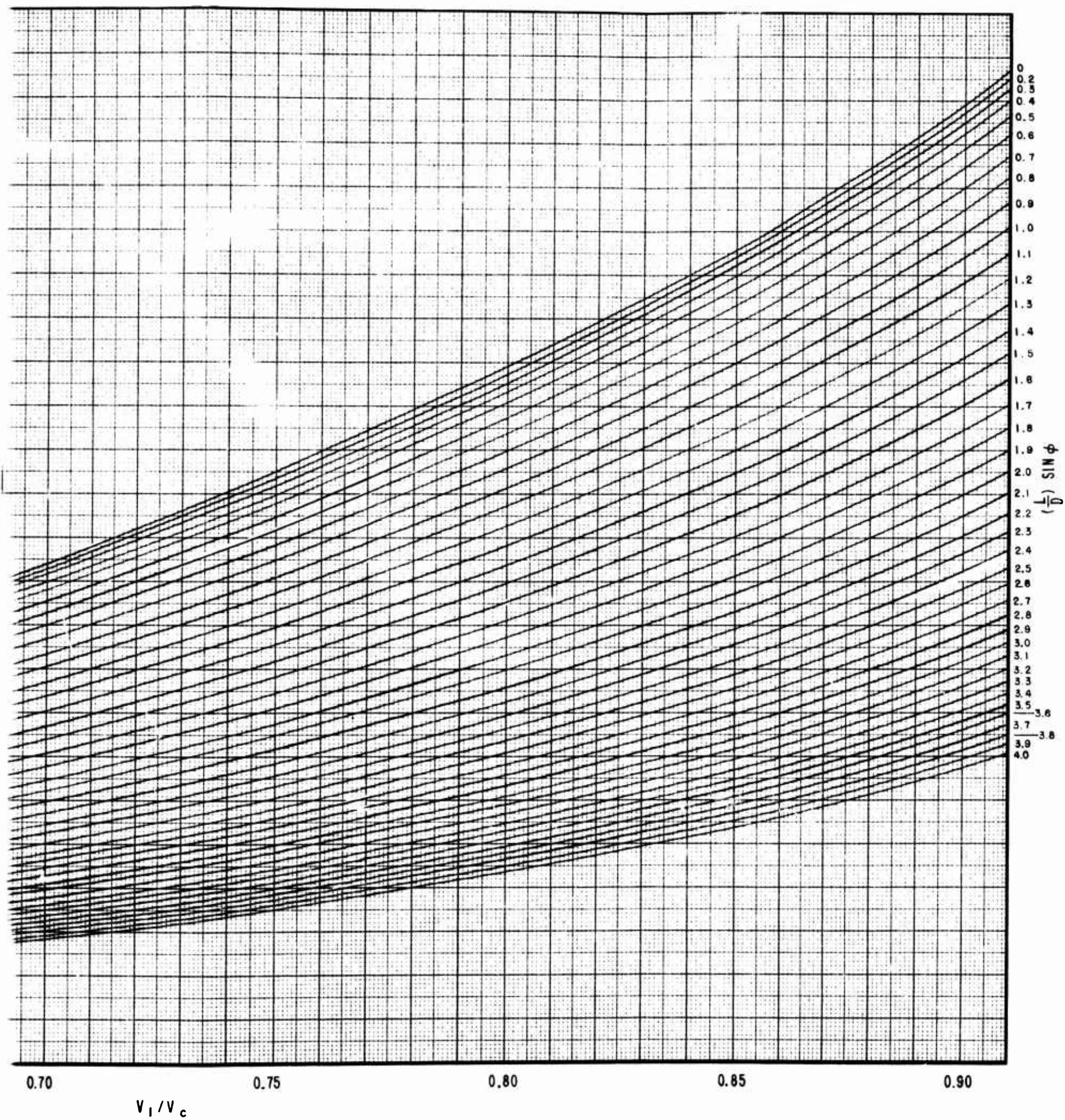


Figure 6b. Lateral Range Series Values

**B**

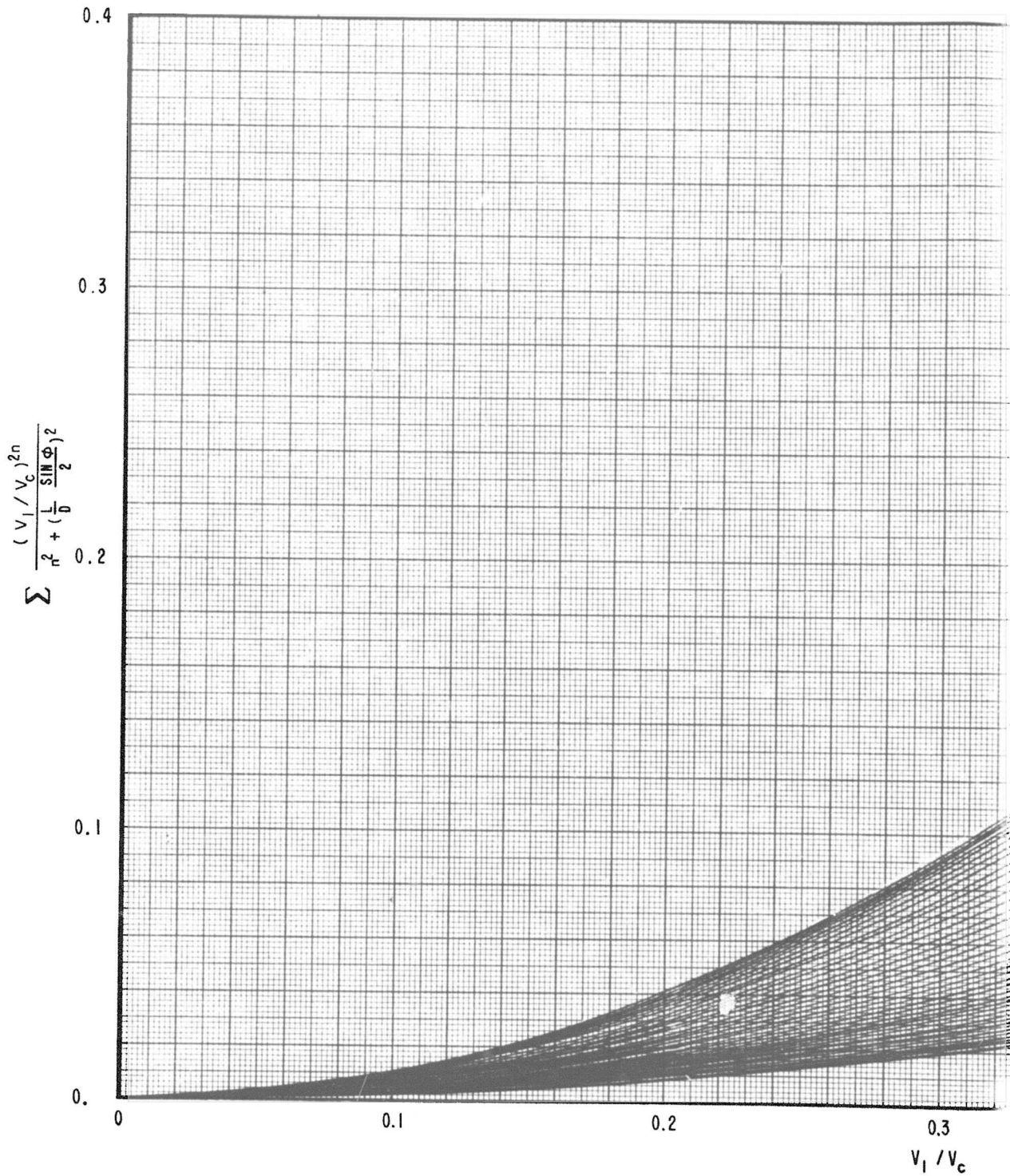
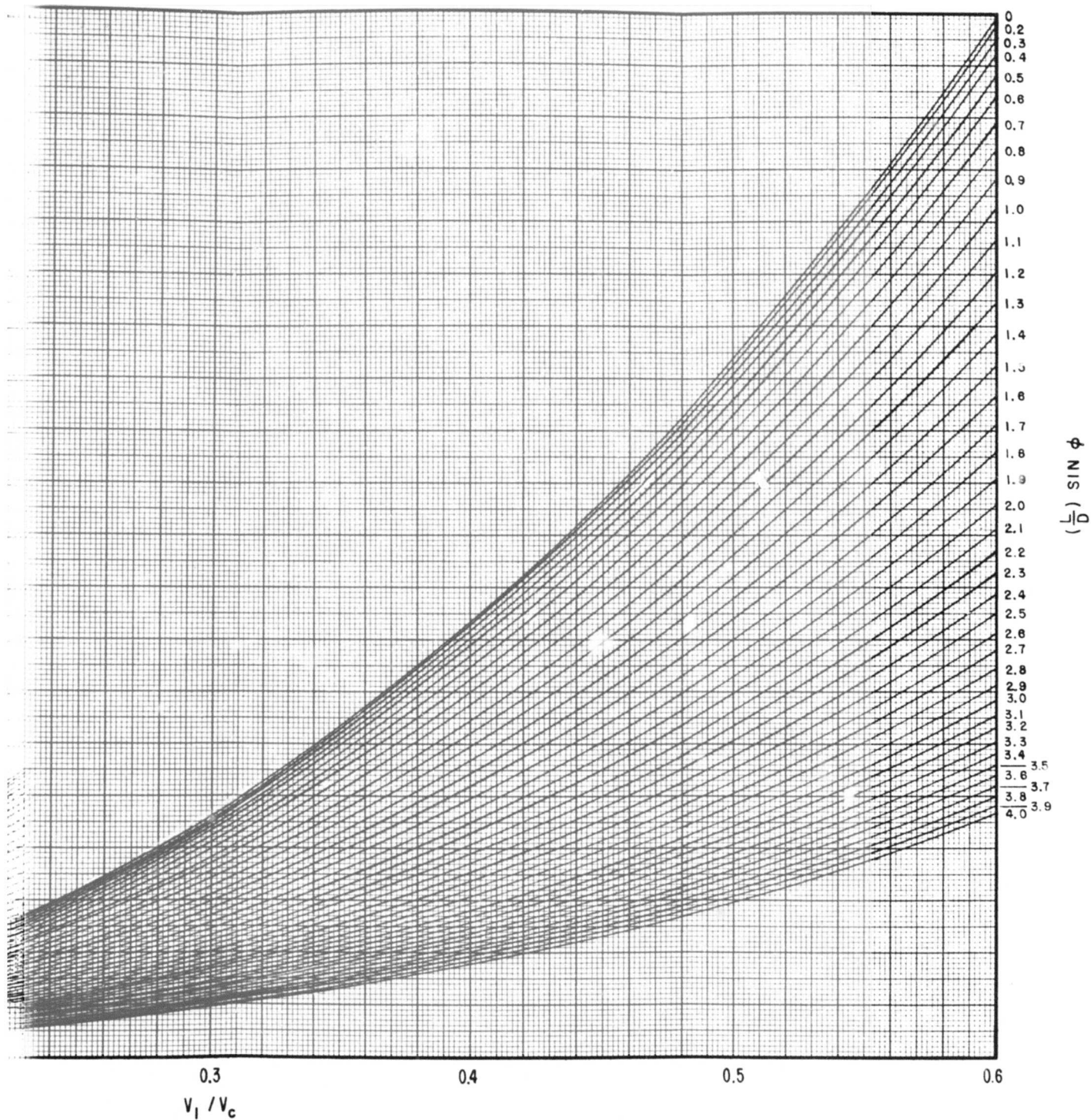


Figure 6c. Lateral Range Series Va



Lateral Range Series Values

**B**

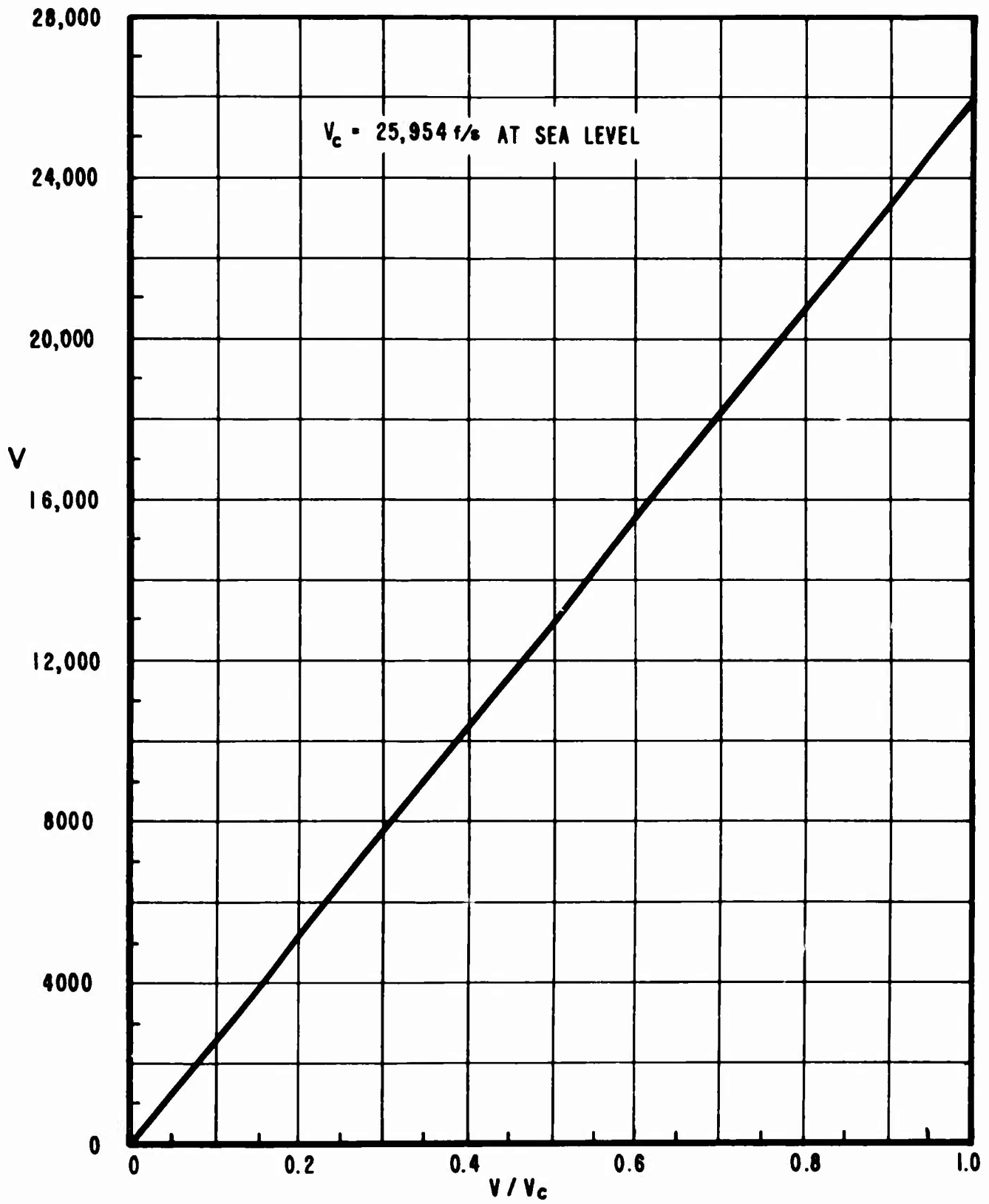


Figure 7. Local Velocity - Circular Velocity Ratio

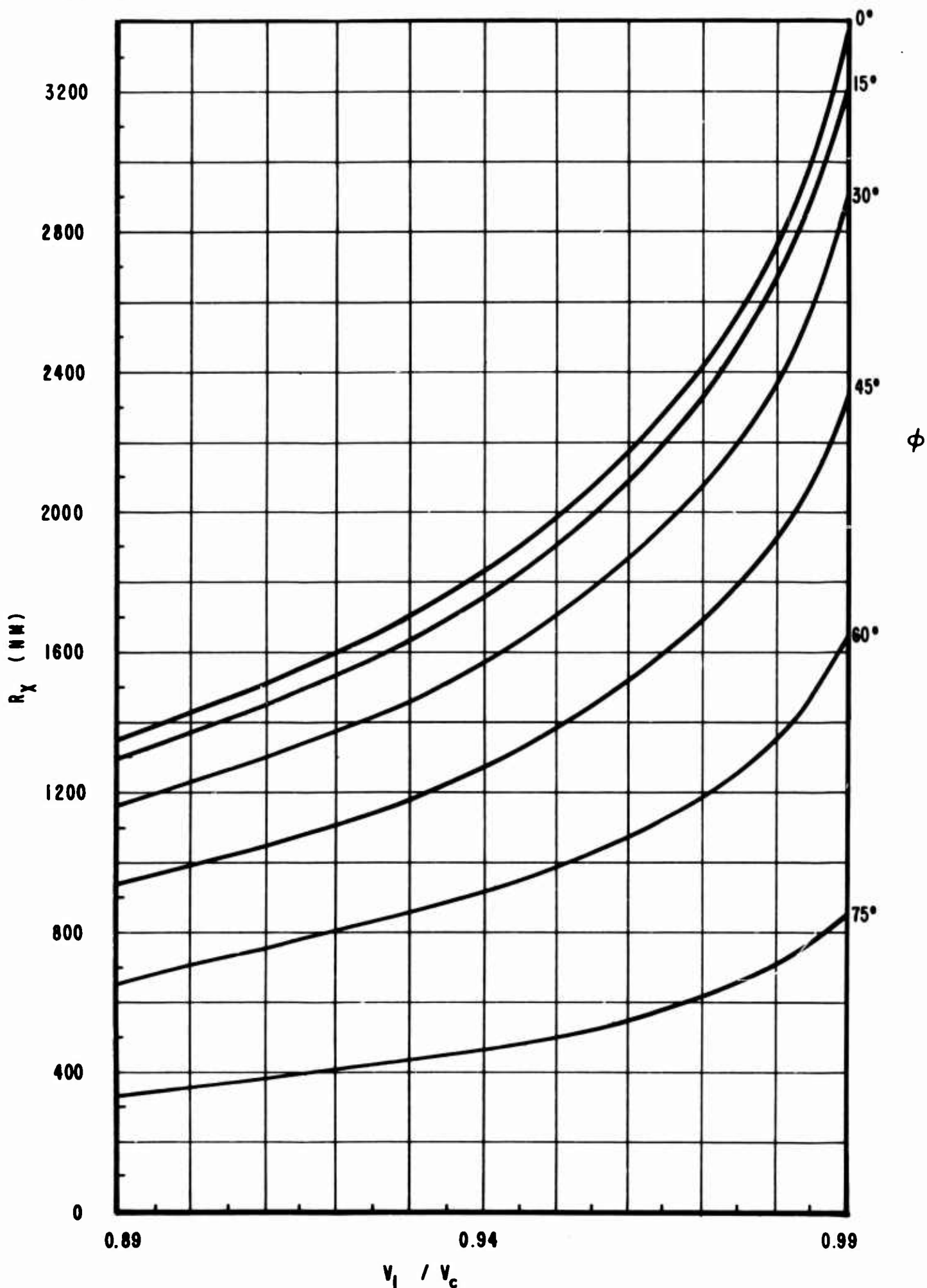


Figure 8. Longitudinal Range Variation With Reentry Velocity and Bank Angle,  $L/D = 0.5$

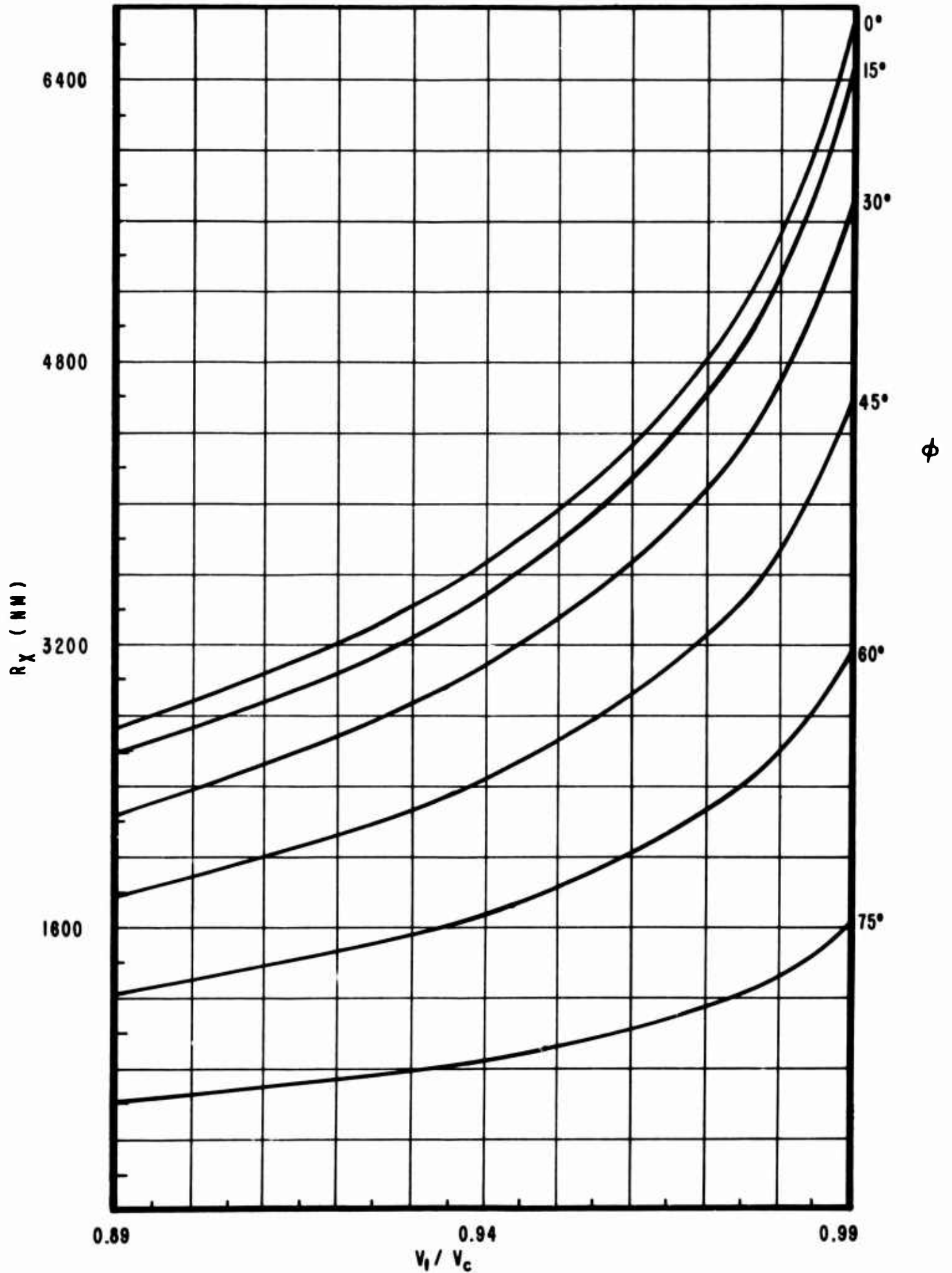


Figure 9. Longitudinal Range Variation With Reentry Velocity and Bank Angle,  $L/D = 1.0$

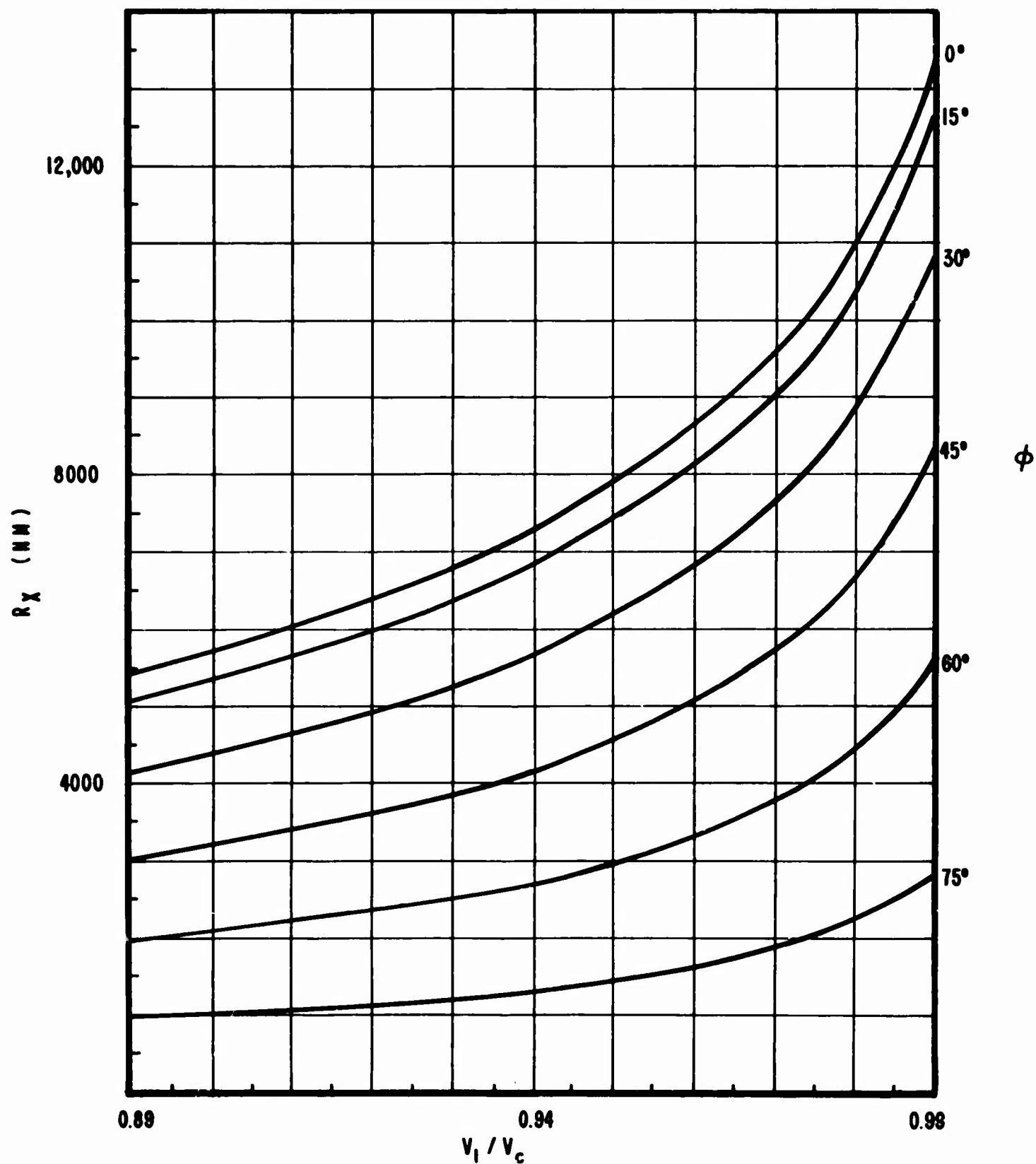


Figure 10. Longitudinal Range Variation With Reentry Velocity and Bank Angle,  $L/D = 2.0$

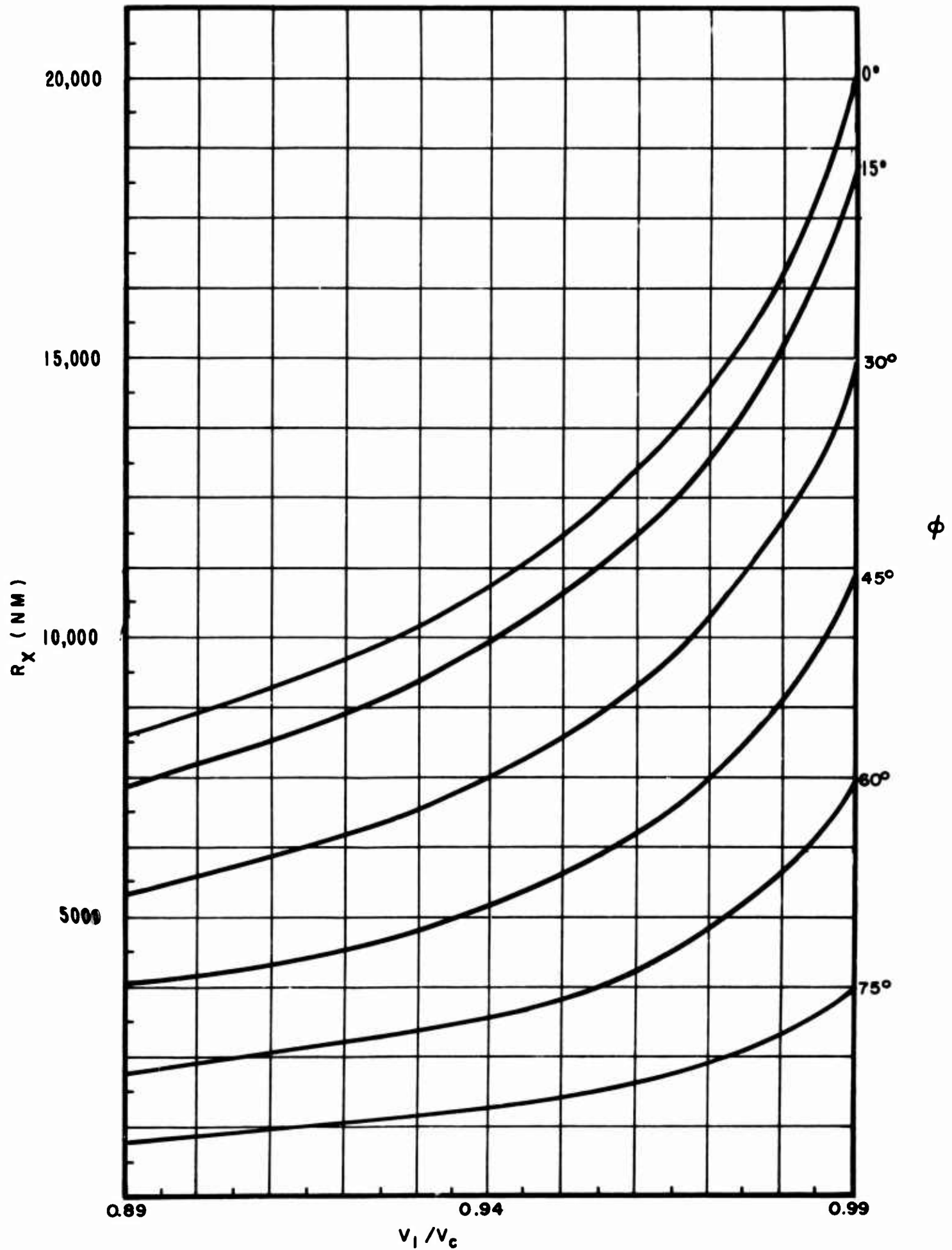


Figure 11. Longitudinal Range Variation With Reentry Velocity and Bank Angle,  $L/D = 3.0$

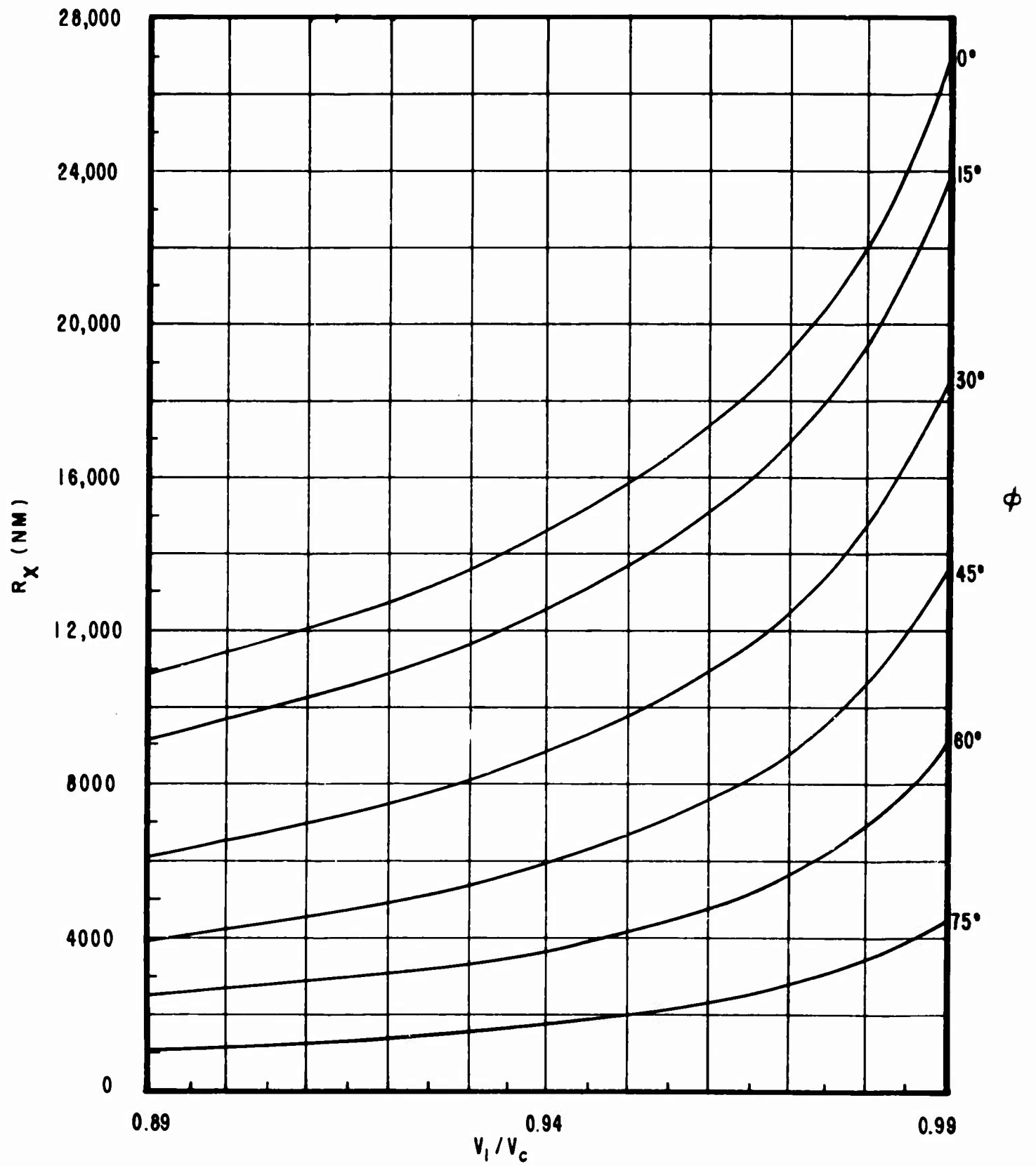


Figure 12. Longitudinal Range Variation With Reentry Velocity and Bank Angle,  $L/D = 4.0$

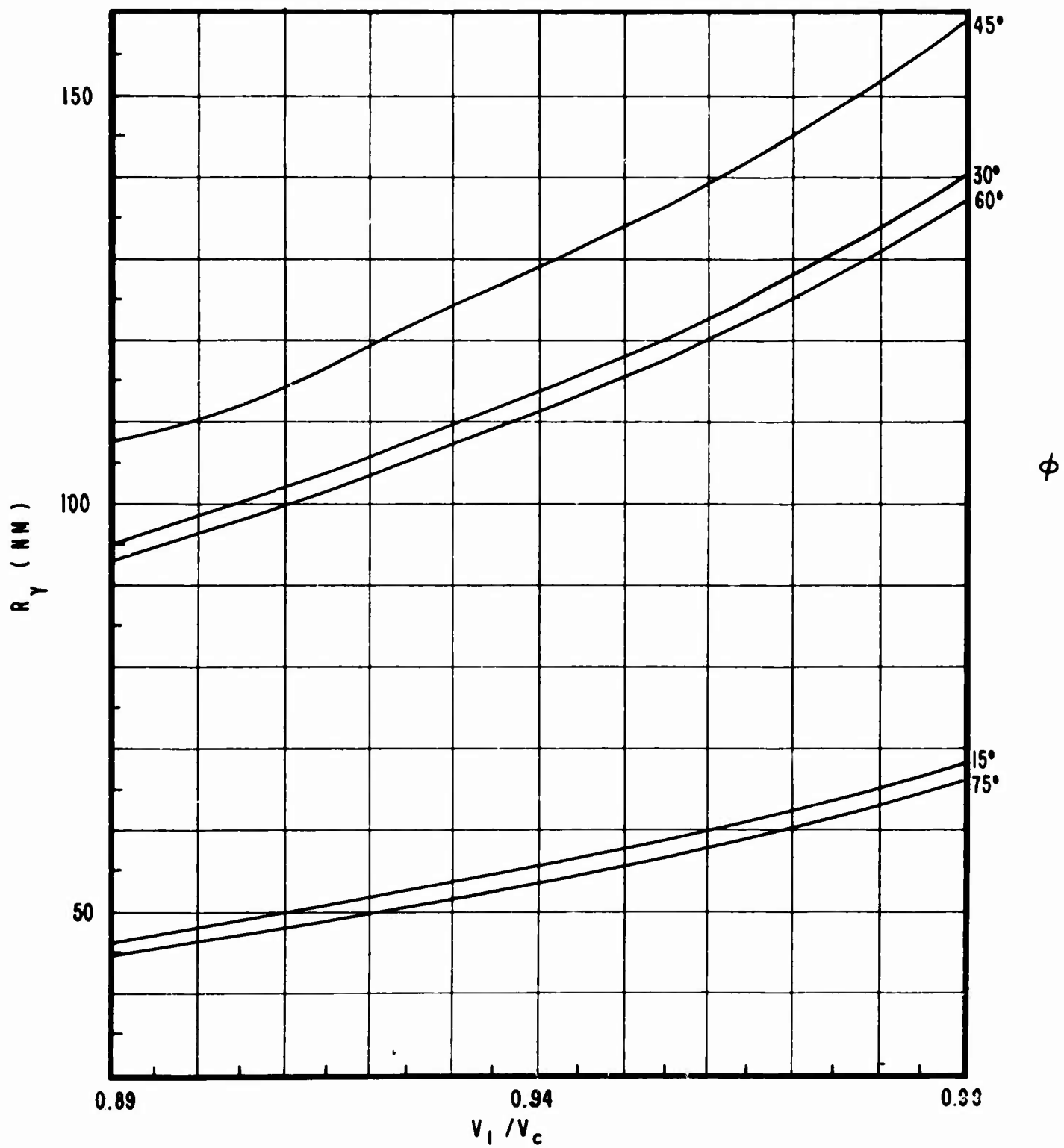


Figure 13. Lateral Range Variation With Reentry Velocity and Bank Angle,  $L/D = 0.5$

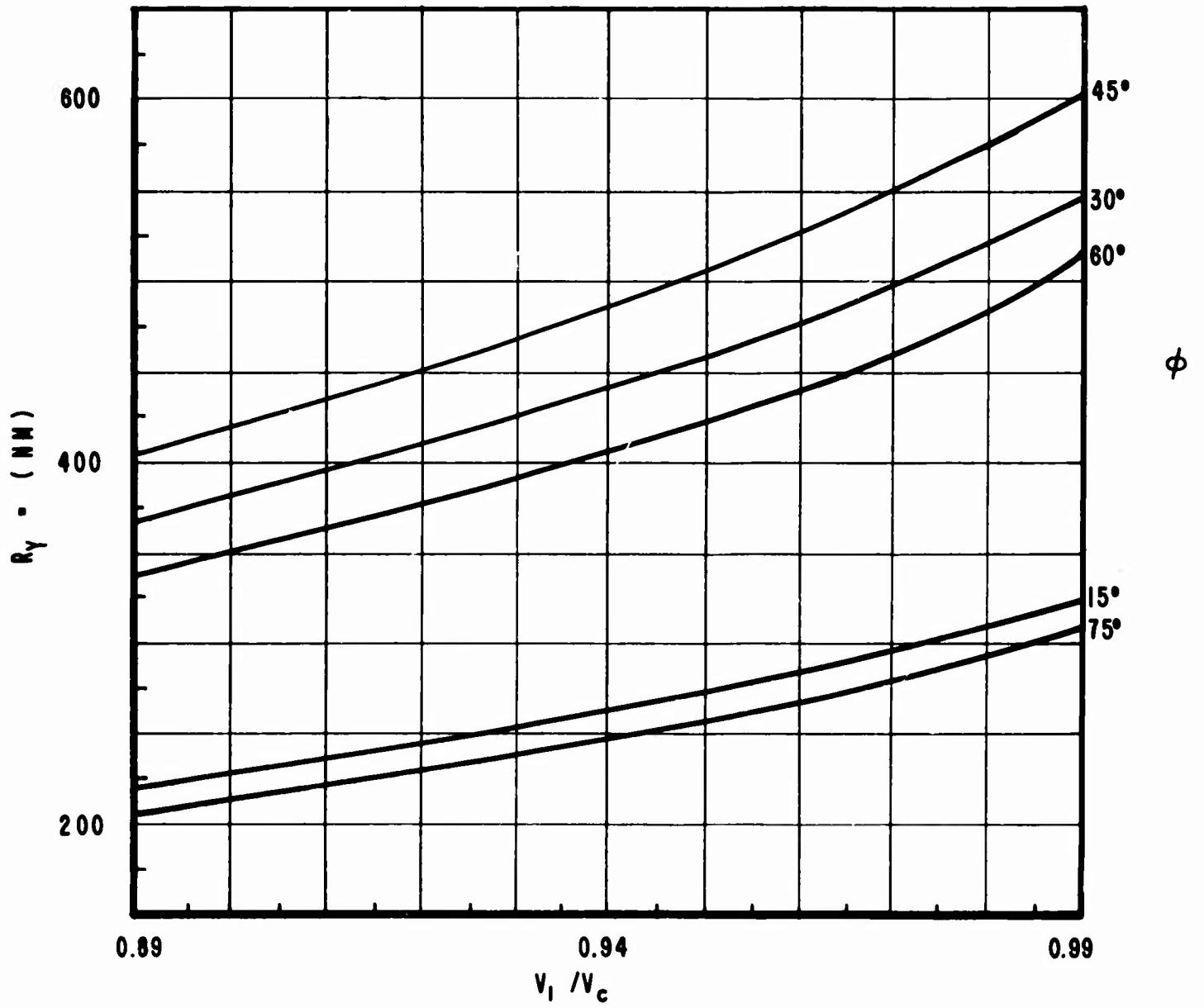


Figure 14. Lateral Range Variation With Reentry Velocity and Bank Angle,  $L/D = 1.0$

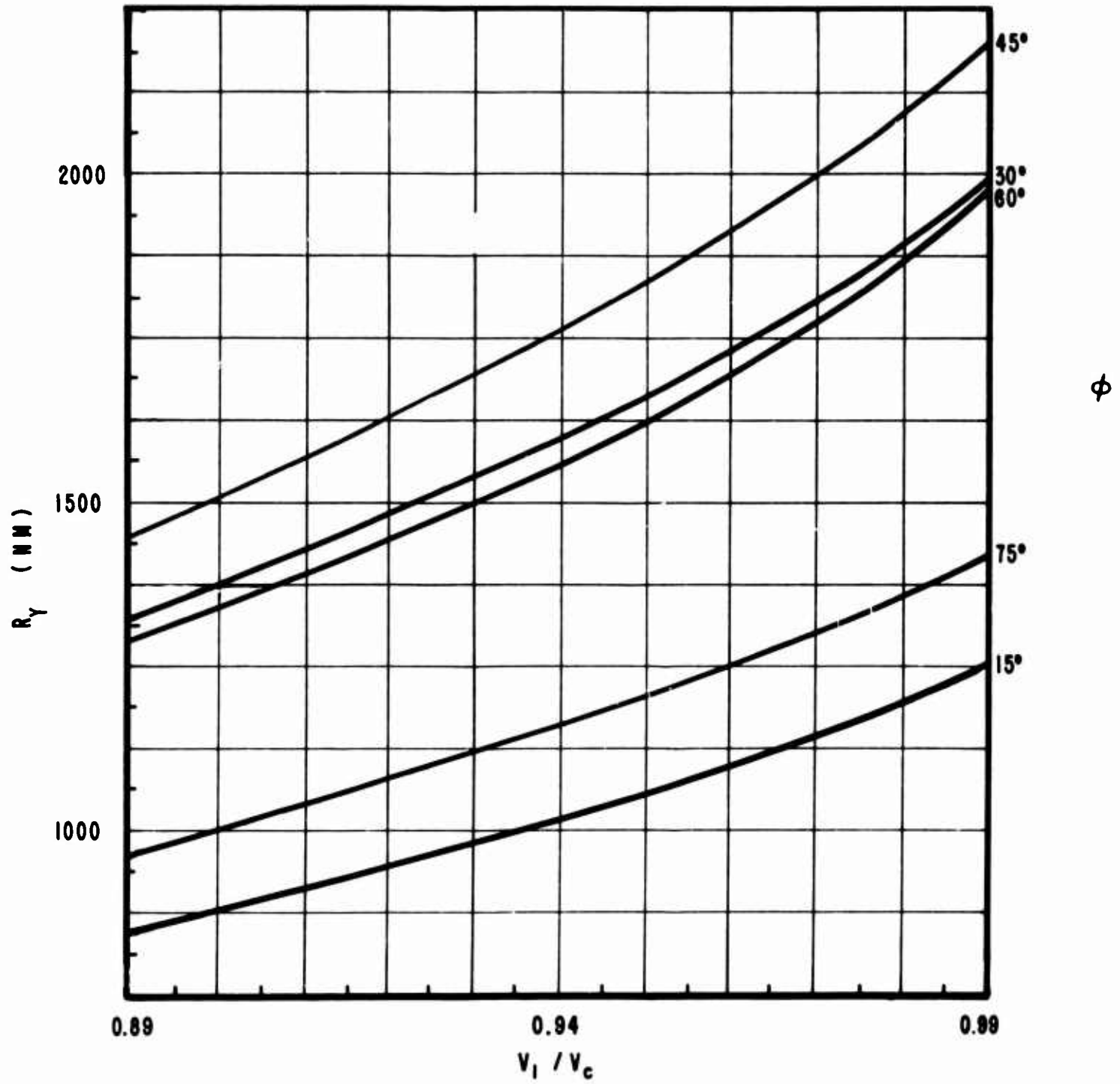


Figure 15. Lateral Range Variation With Reentry Velocity and Bank Angle,  $L/D = 2.0$

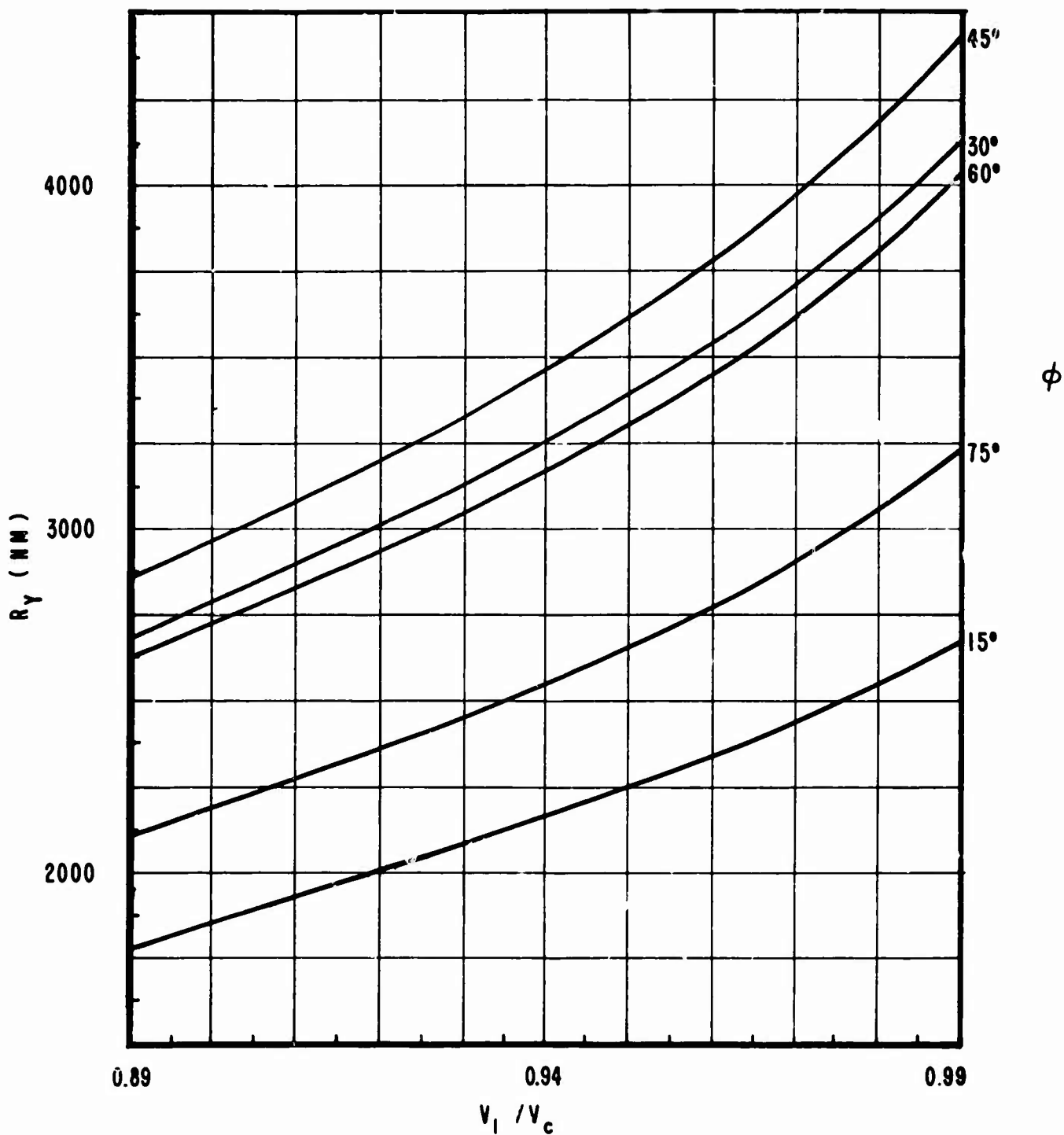


Figure 16. Lateral Range Variation With Reentry Velocity and Bank Angle,  $L/D = 3.0$

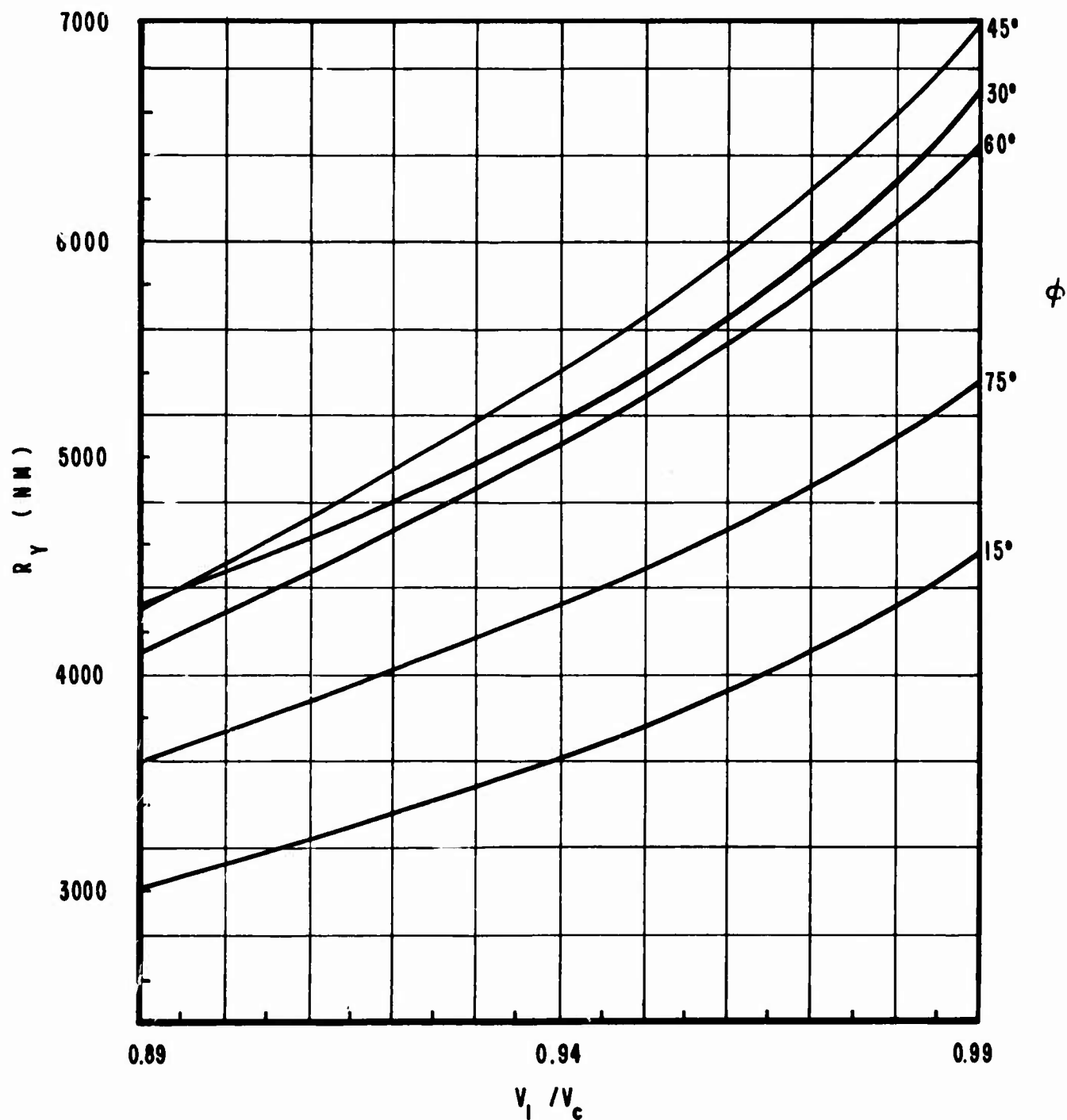


Figure 17. Lateral Range Variation With Reentry Velocity and Bank Angle,  $L/D = 4.0$

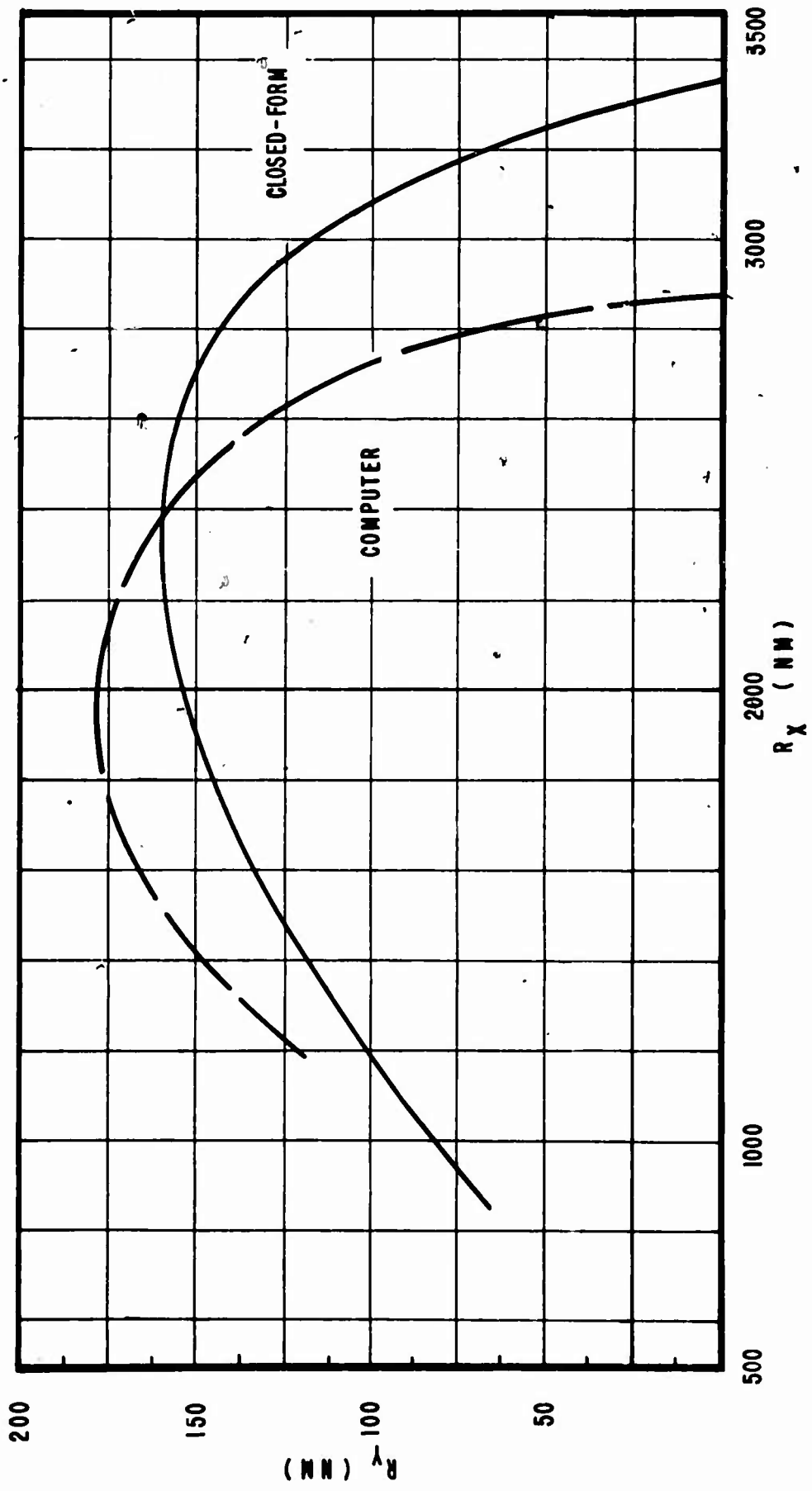


Figure 18. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.99$ ,  $L/D = 0.5$

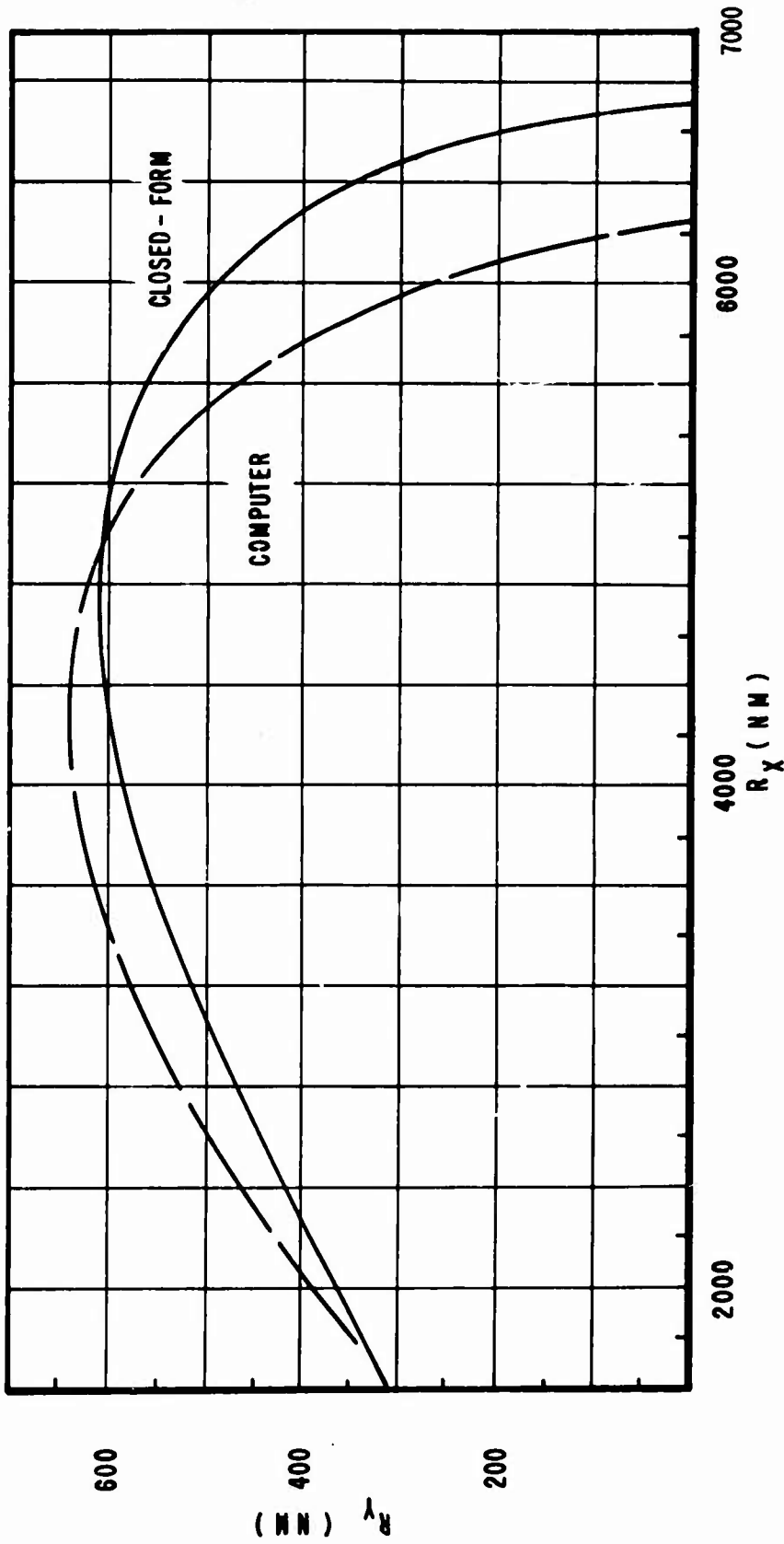


Figure 19. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.99$ ,  $L/D = 1.0$

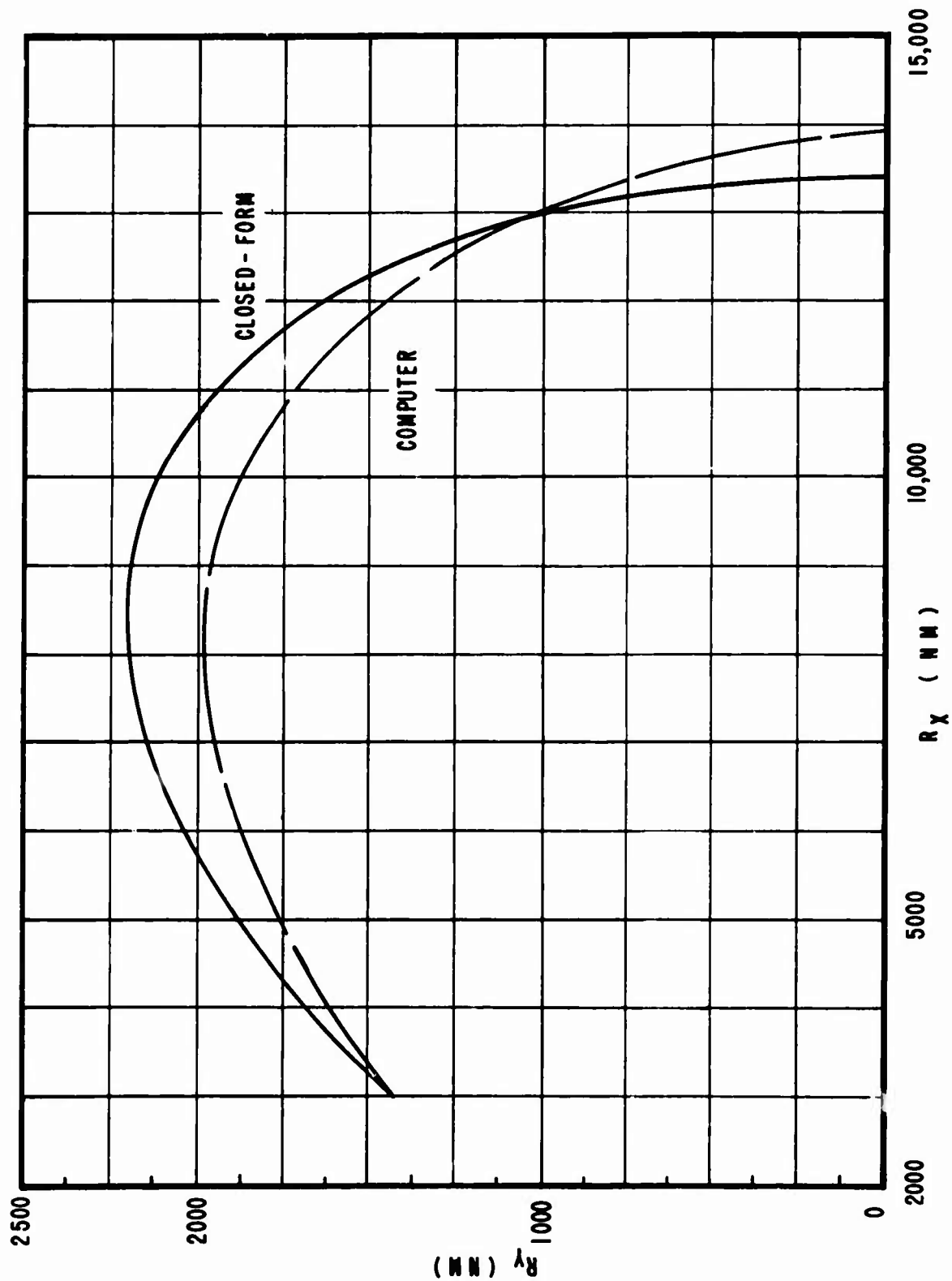


Figure 20. Footprint Comparison Between Closed-Form and Computer Solutions,  
 $V_1/V_C = 0.99$ ,  $L/D = 2.0$

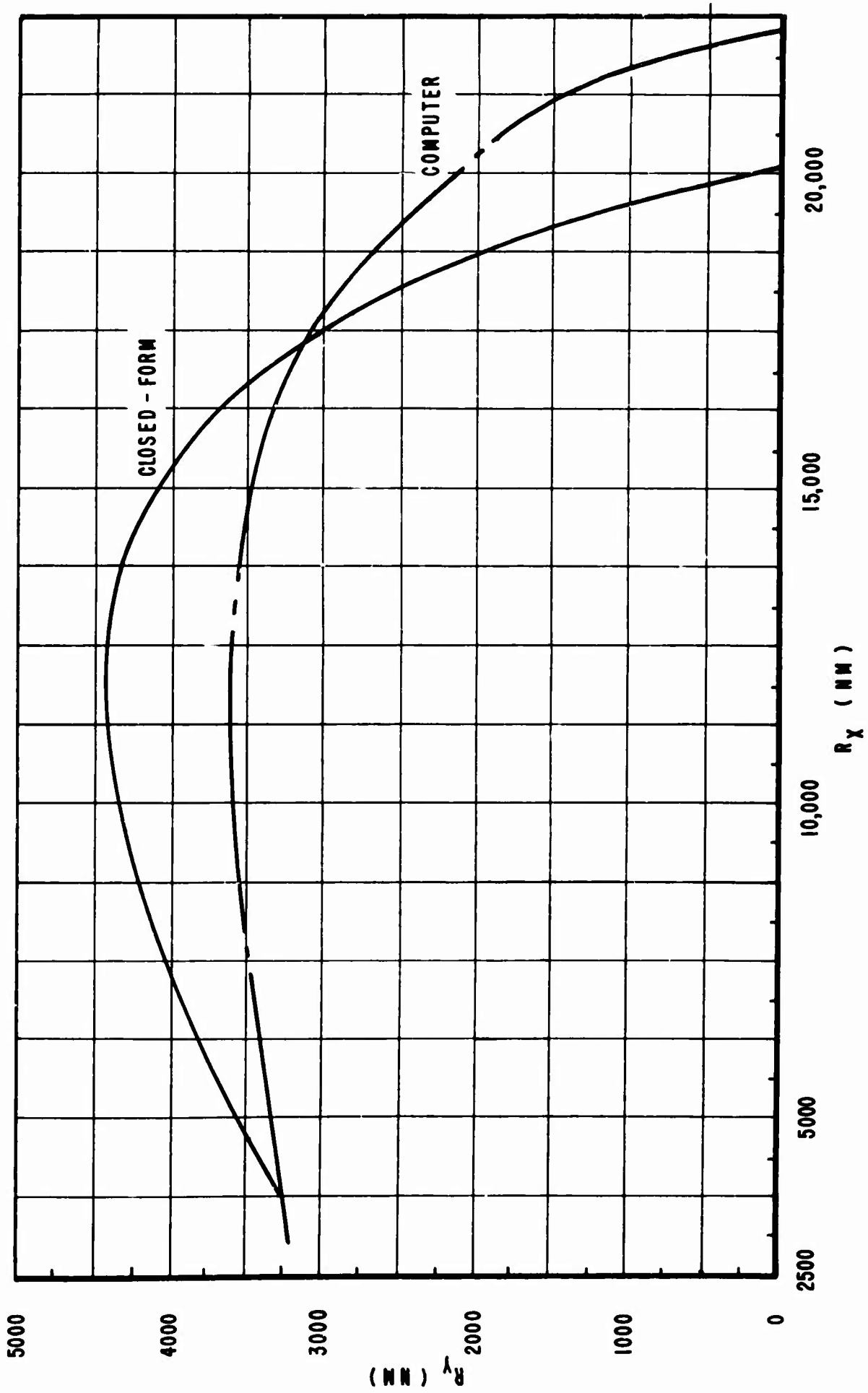


Figure 21. Footprint Comparison Between Closed-Form and Computer Solutions,  
 $V_1/V_c = 0.99, L/D = 3.0$

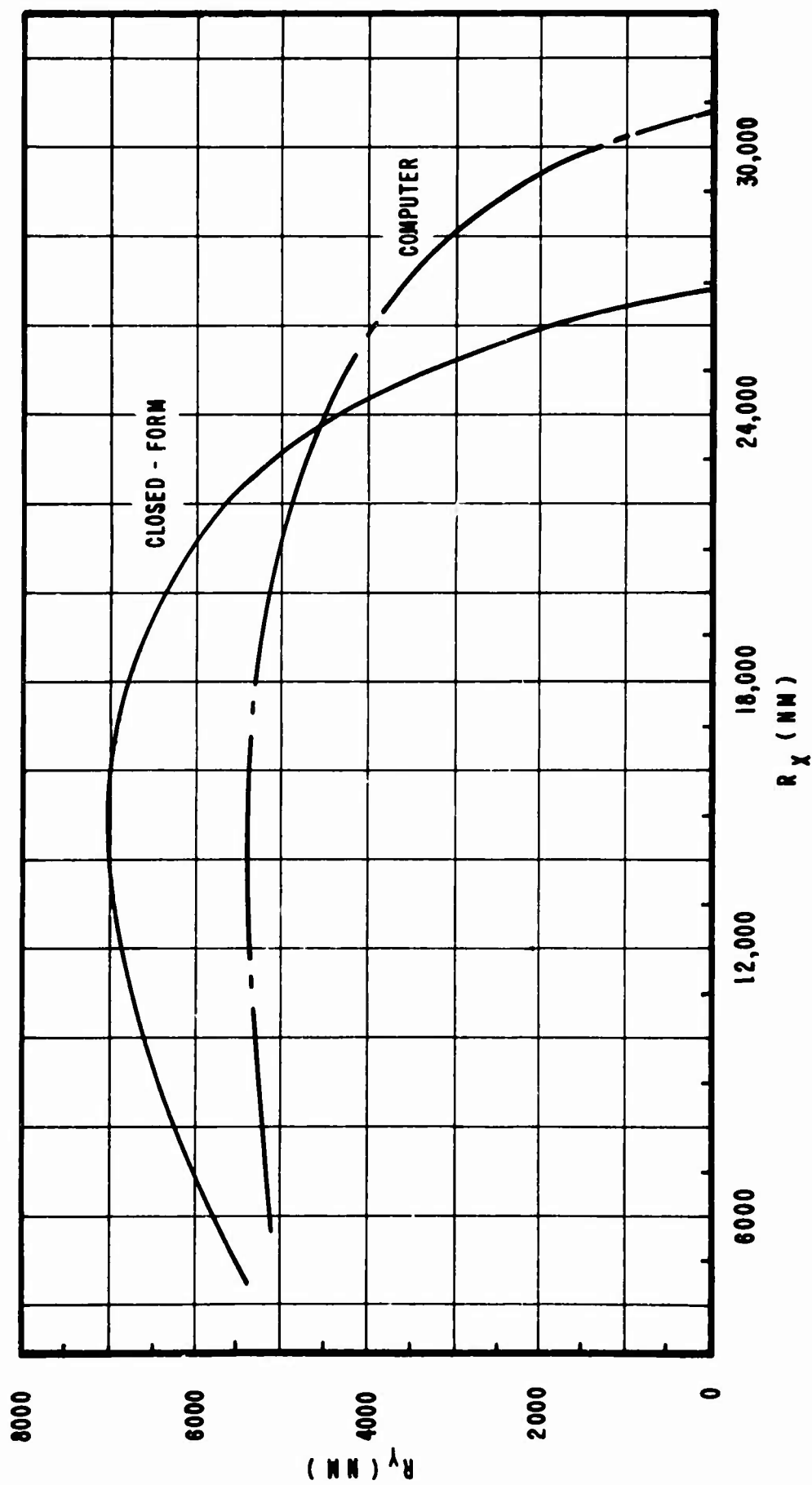


Figure 22. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.99$ ,  $L/D = 4.0$

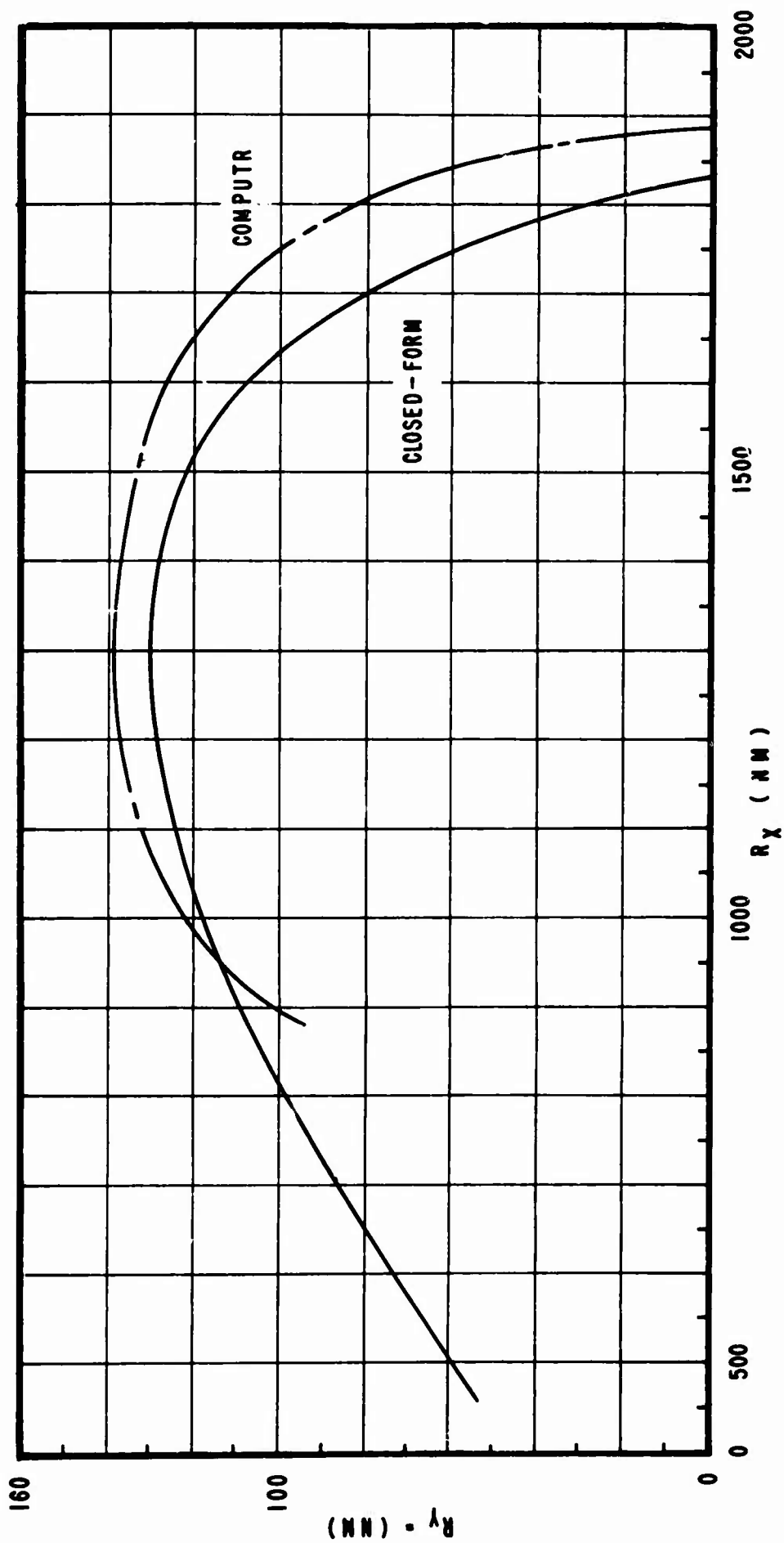


Figure 23. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.94$ ,  $L/D = 0.5$

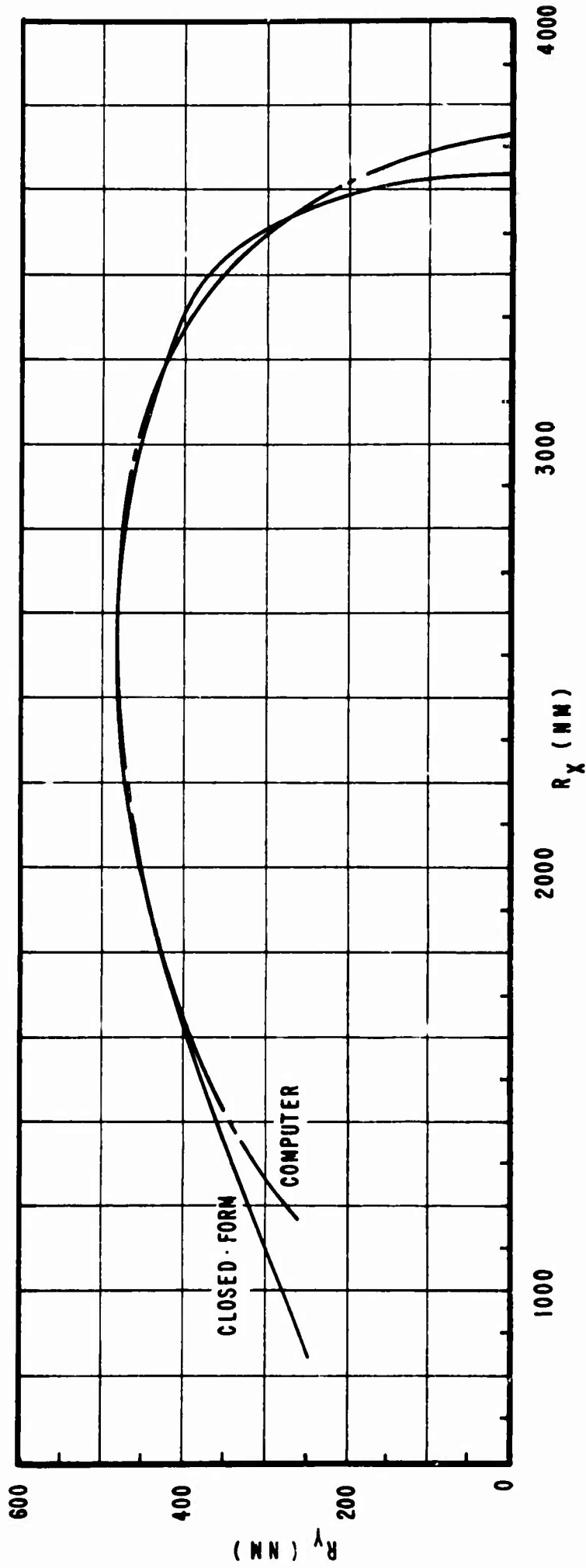


Figure 24. Footprint Comparison Between Closed-Form and Computer Solutions,  
 $V_1/V_C = 0.94$ ,  $L/D = 1.0$

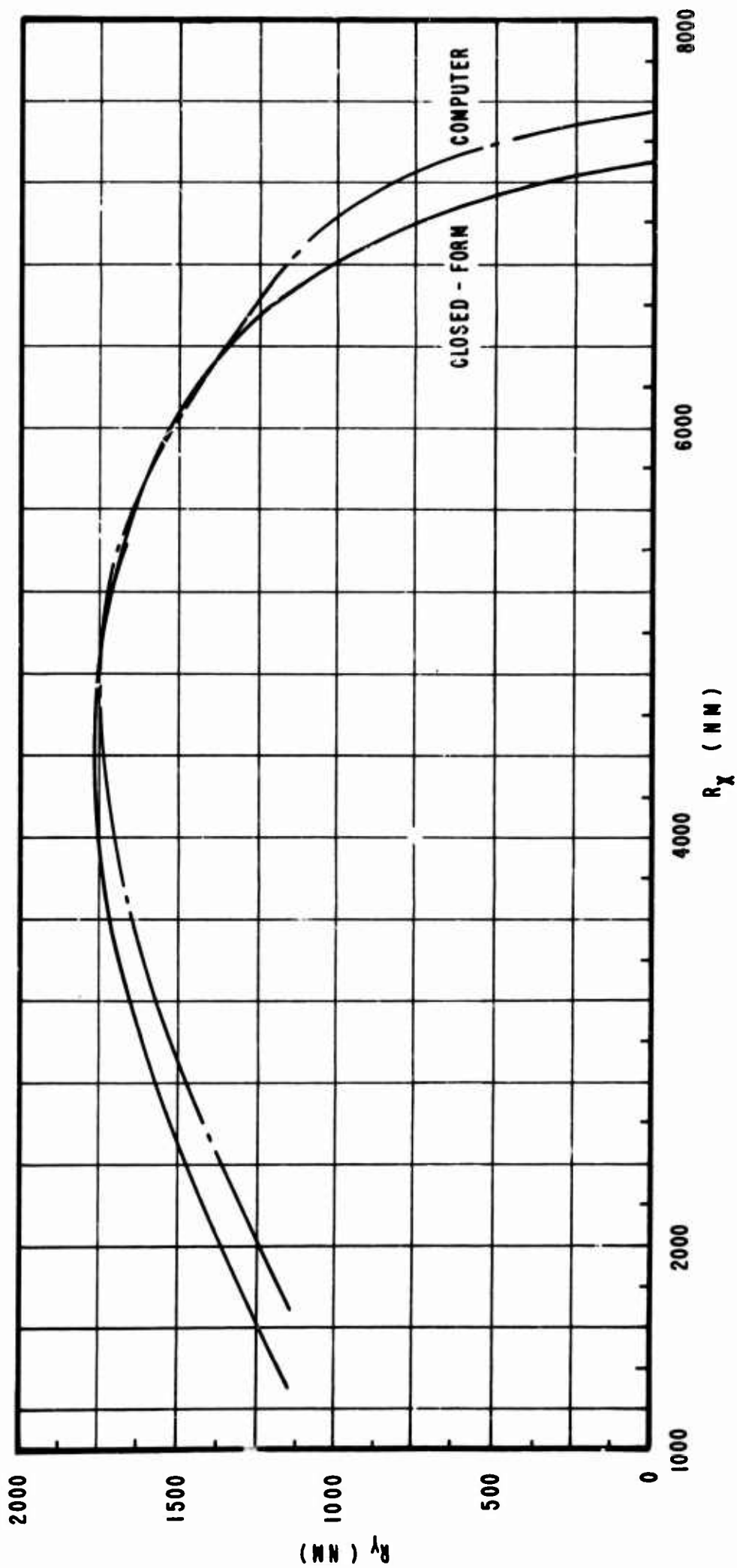


Figure 25. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.94$ ,  $L/D = 2.0$

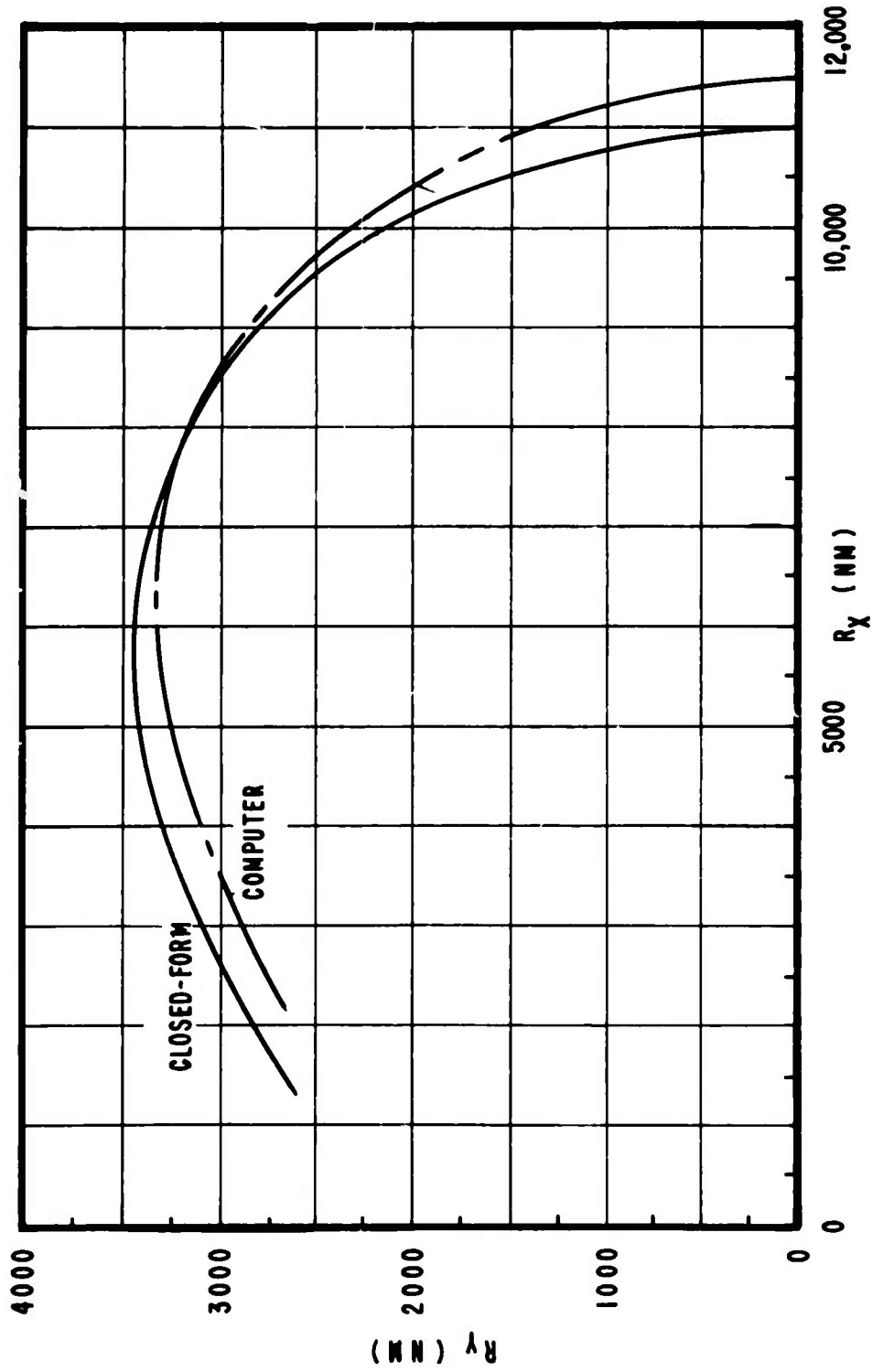


Figure 26. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_C = 0.94$ ,  $L/D = 3.0$

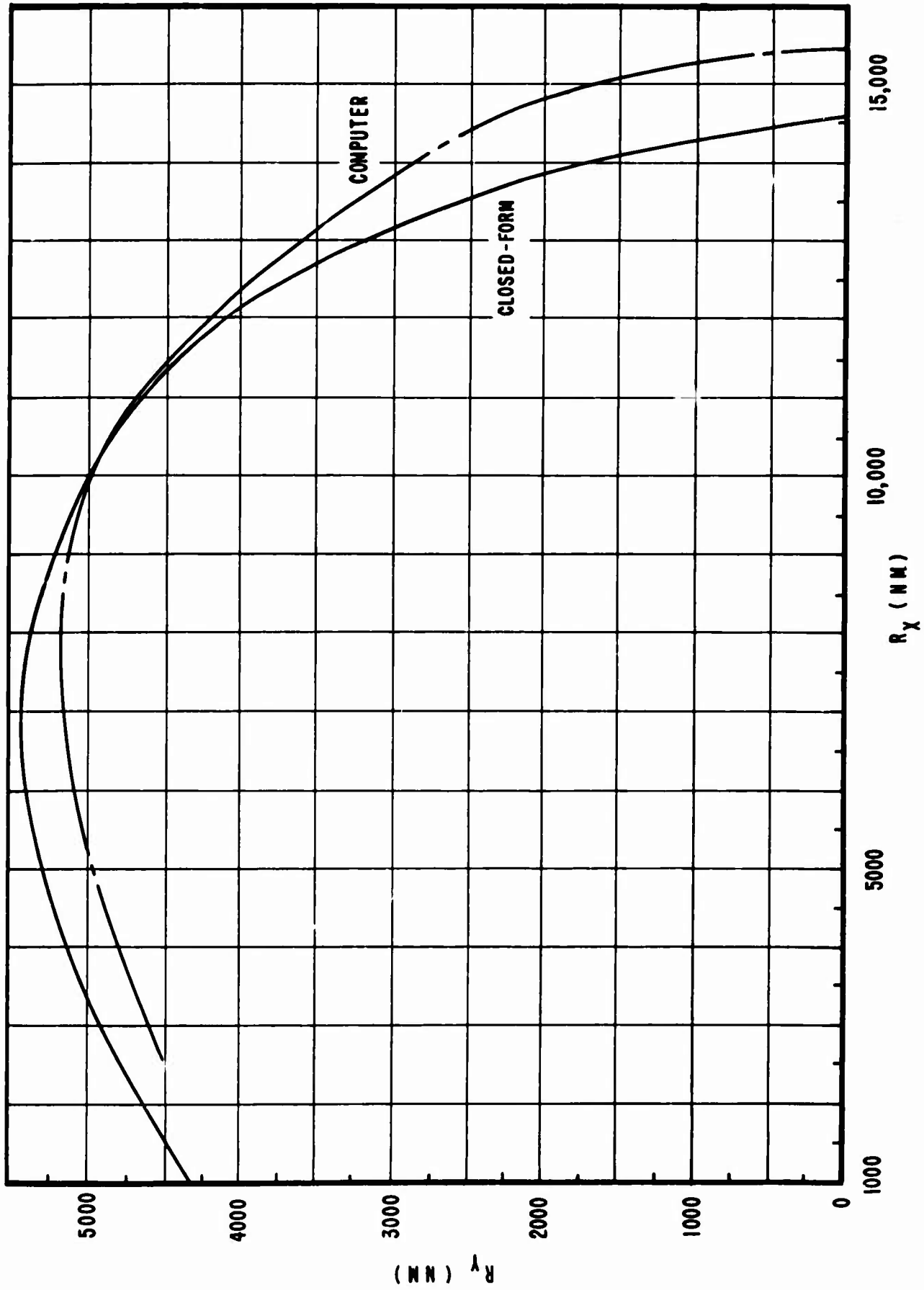


Figure 27. Footprint Comparison Between Closed-Form and Computer Solutions.  
 $V_1/V_c = 0.94, L/D = 4.0$

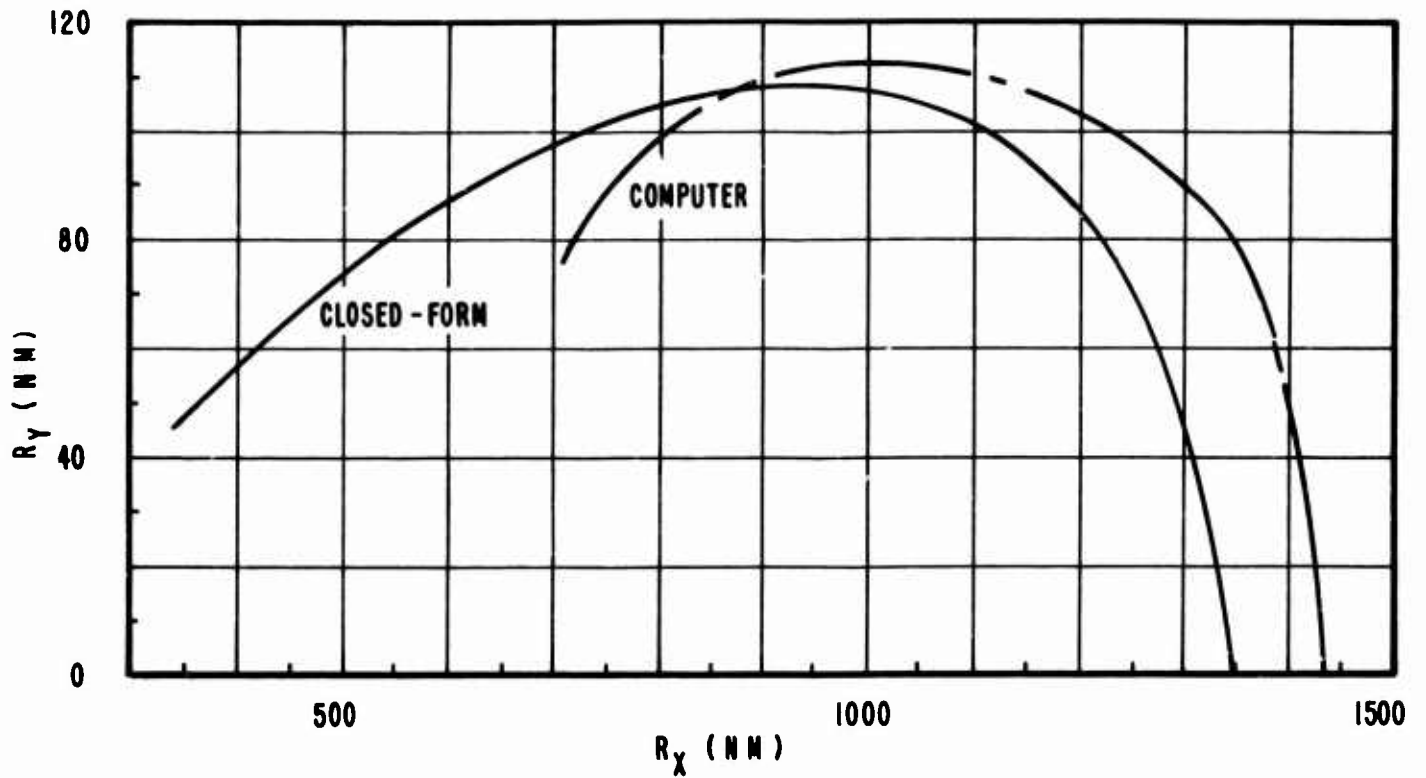


Figure 28. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.89$ ,  $L/D = 0.5$

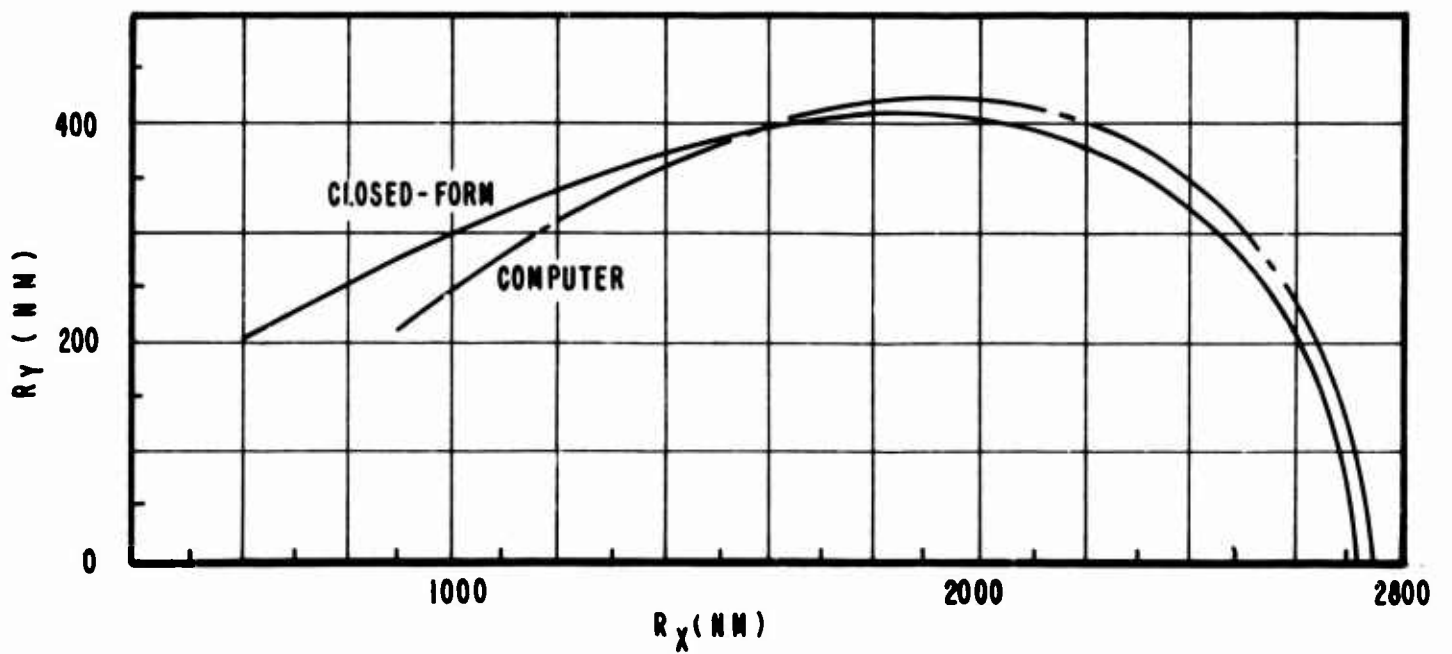


Figure 29. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.89$ ,  $L/D = 1.0$

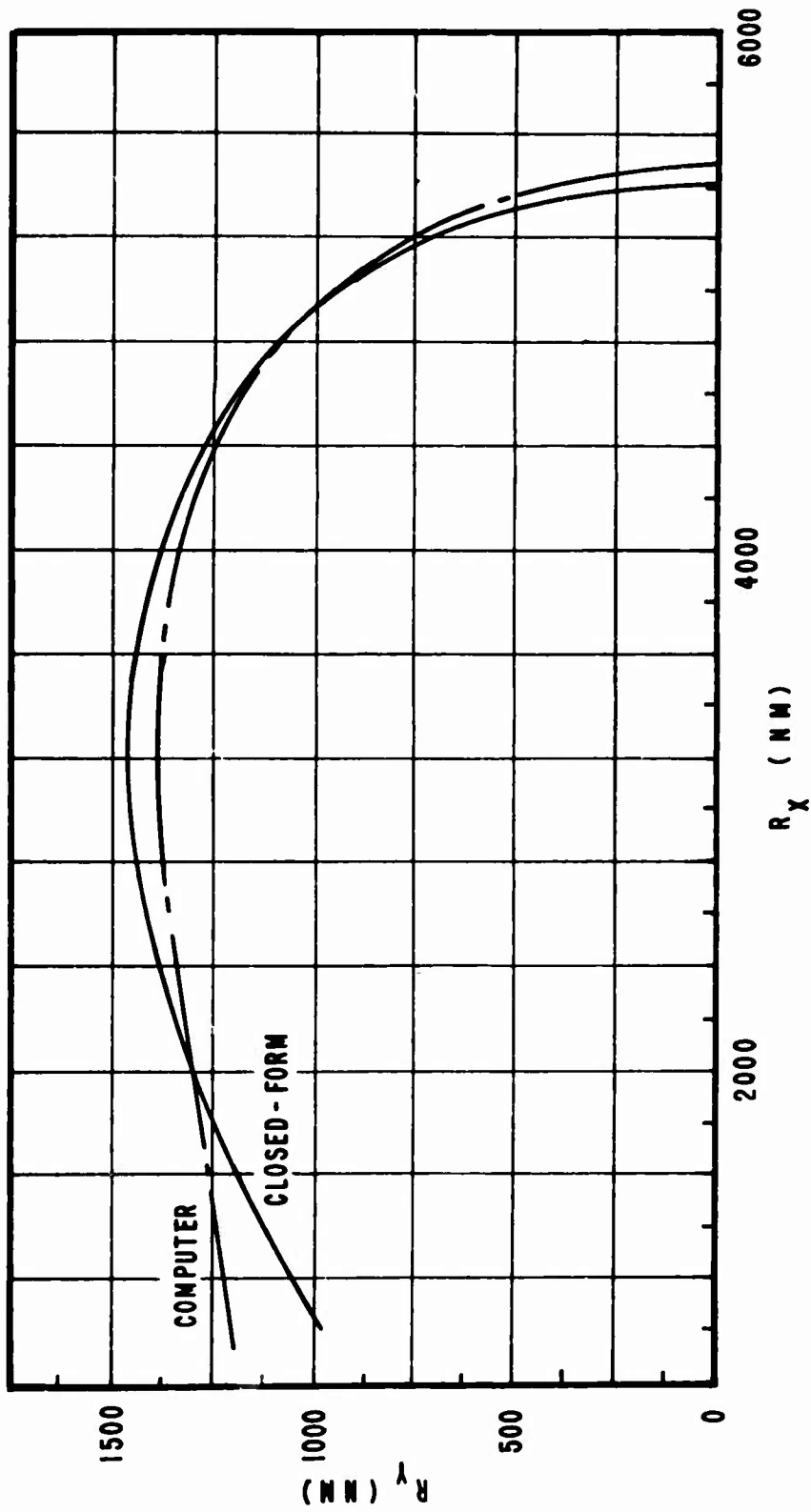


Figure 30. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_C = 0.89$ ,  $L/D = 2.0$

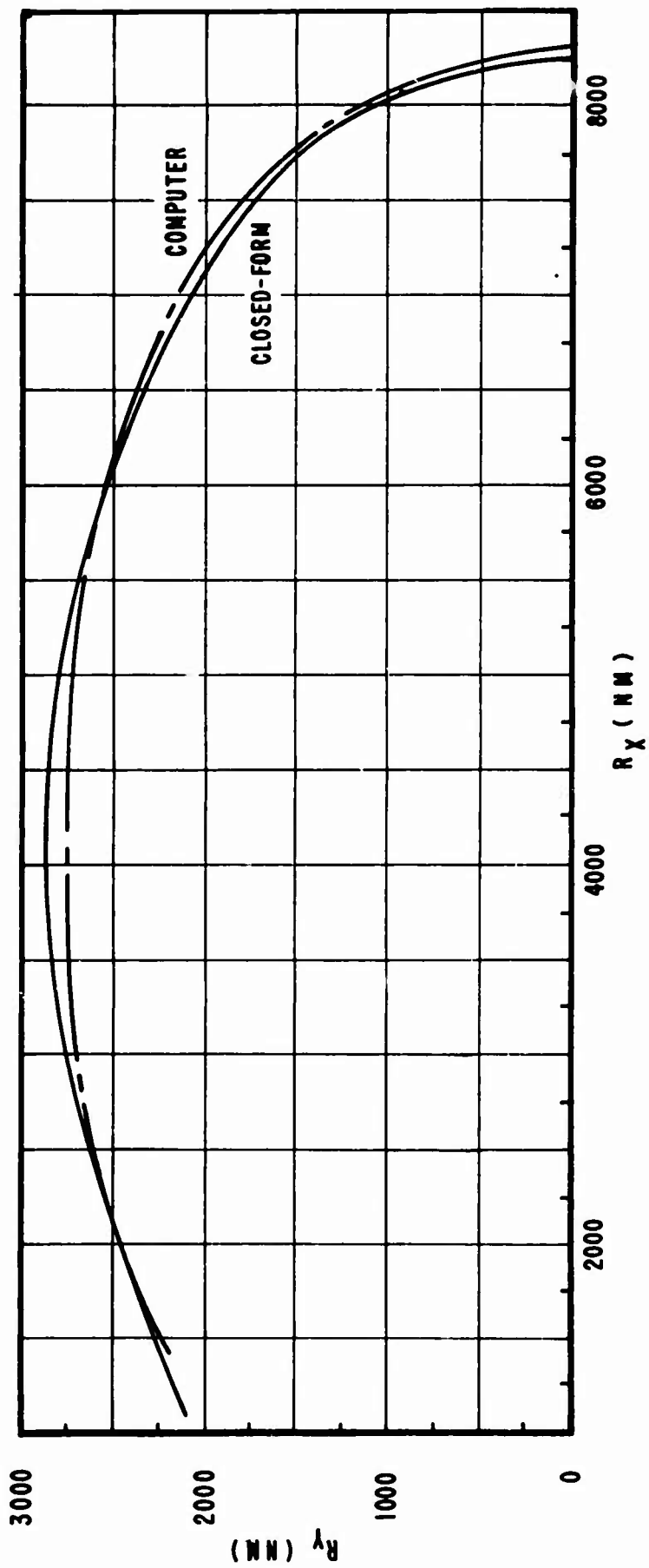


Figure 31. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.89$ ,  $L/D = 3.0$

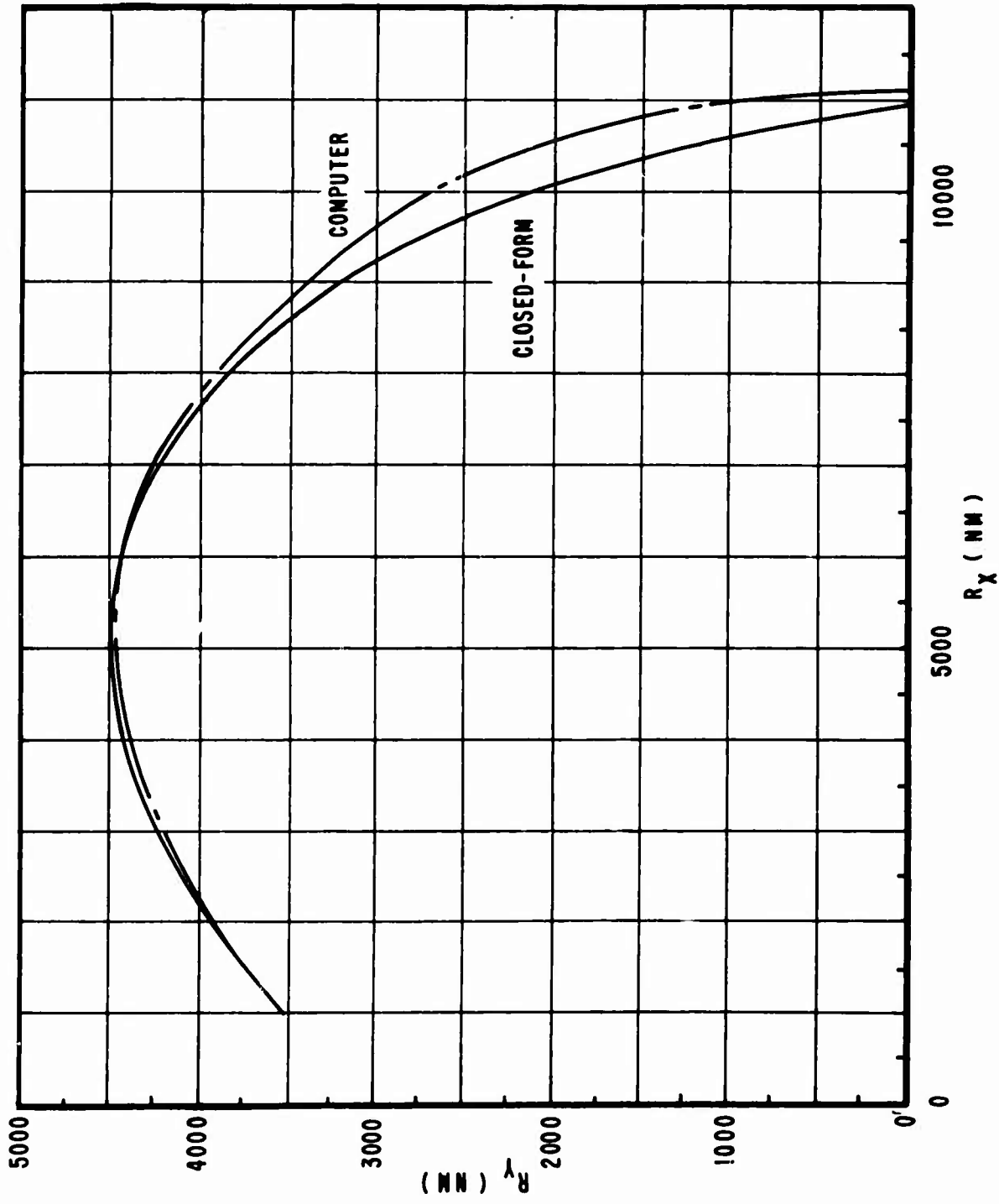


Figure 32. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.89$ ,  $L/D = 4.0$

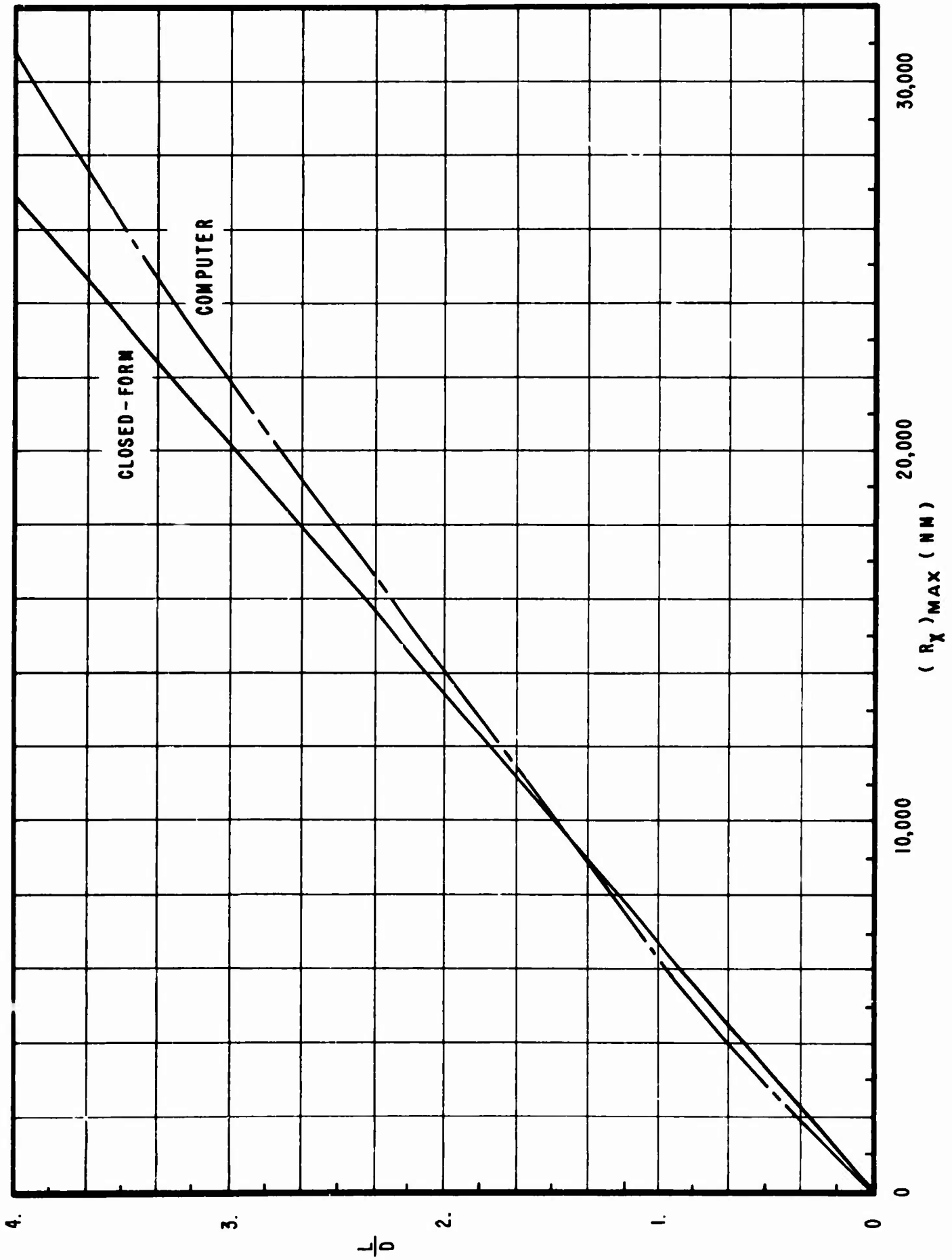


Figure 33. Comparison Between Closed-Form and Computer Maximum Longitudinal Range Values,  $V_1/V_c = 0.99$

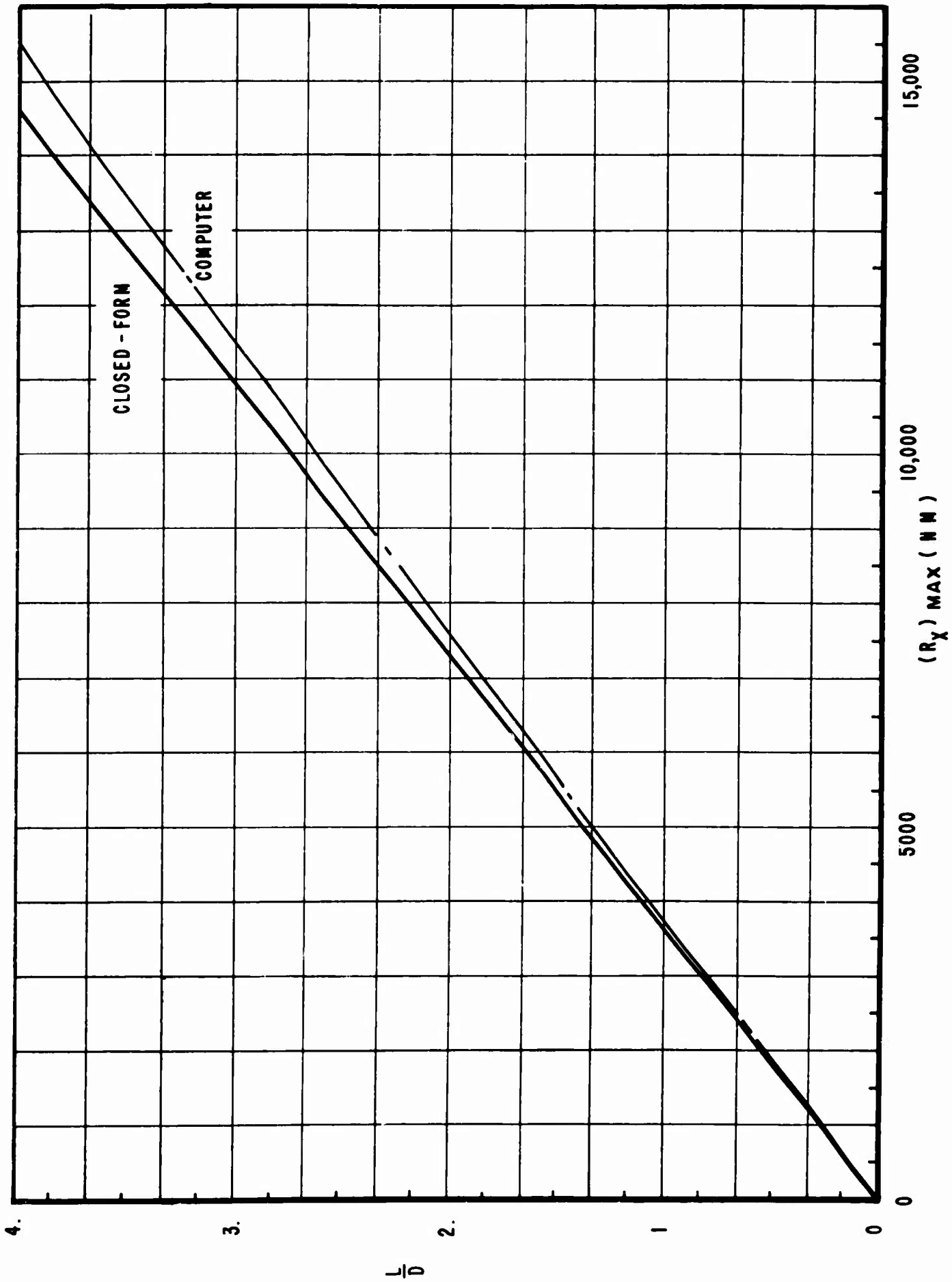


Figure 34. Comparison Between Closed-Form and Computer Maximum Longitudinal Range Values,  $V_1/V_c = 0.94$

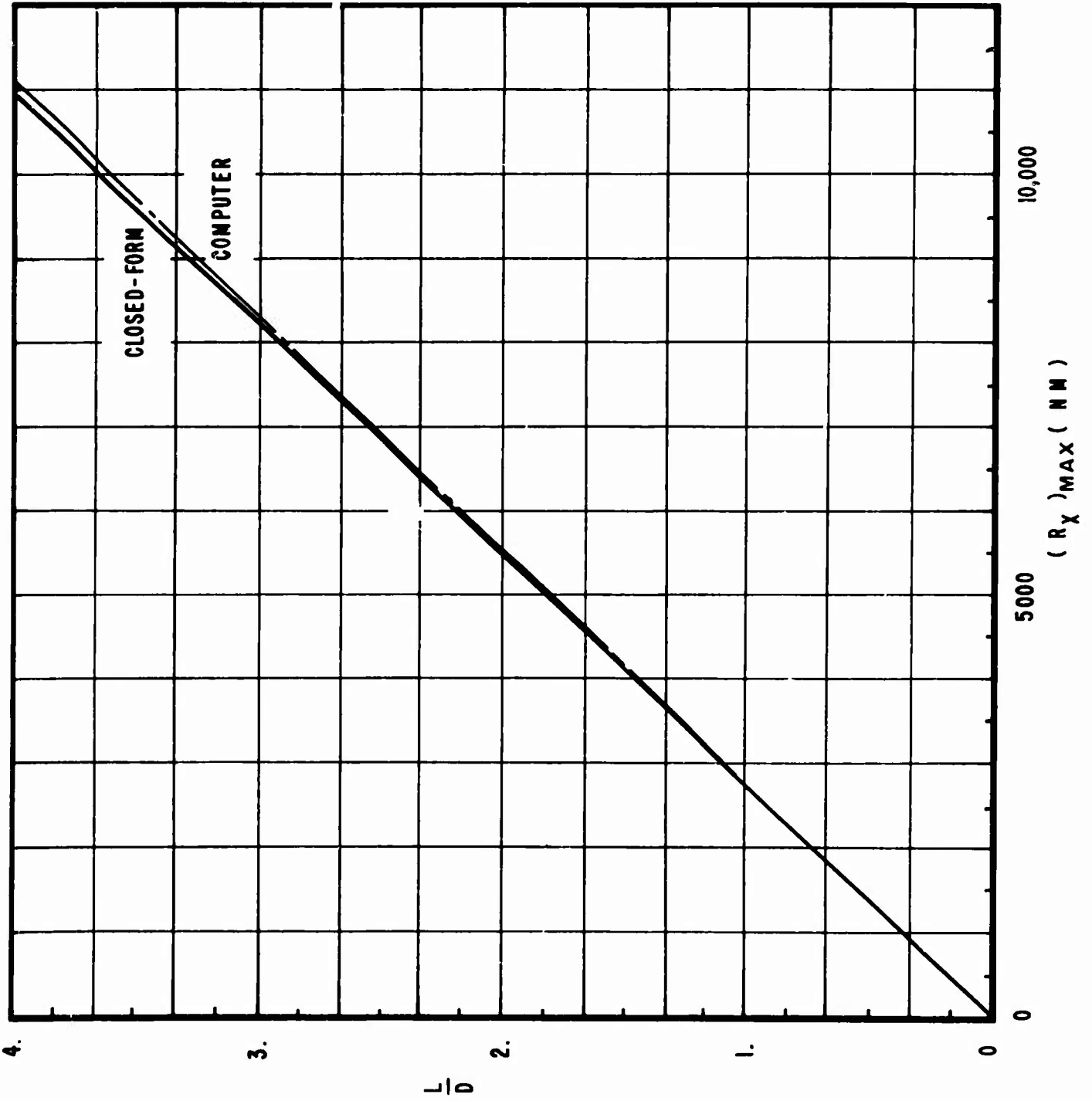


Figure 35. Comparison Between Closed-Form and Computer Maximum Longitudinal Range Values,  $V_1/V_C = 0.89$

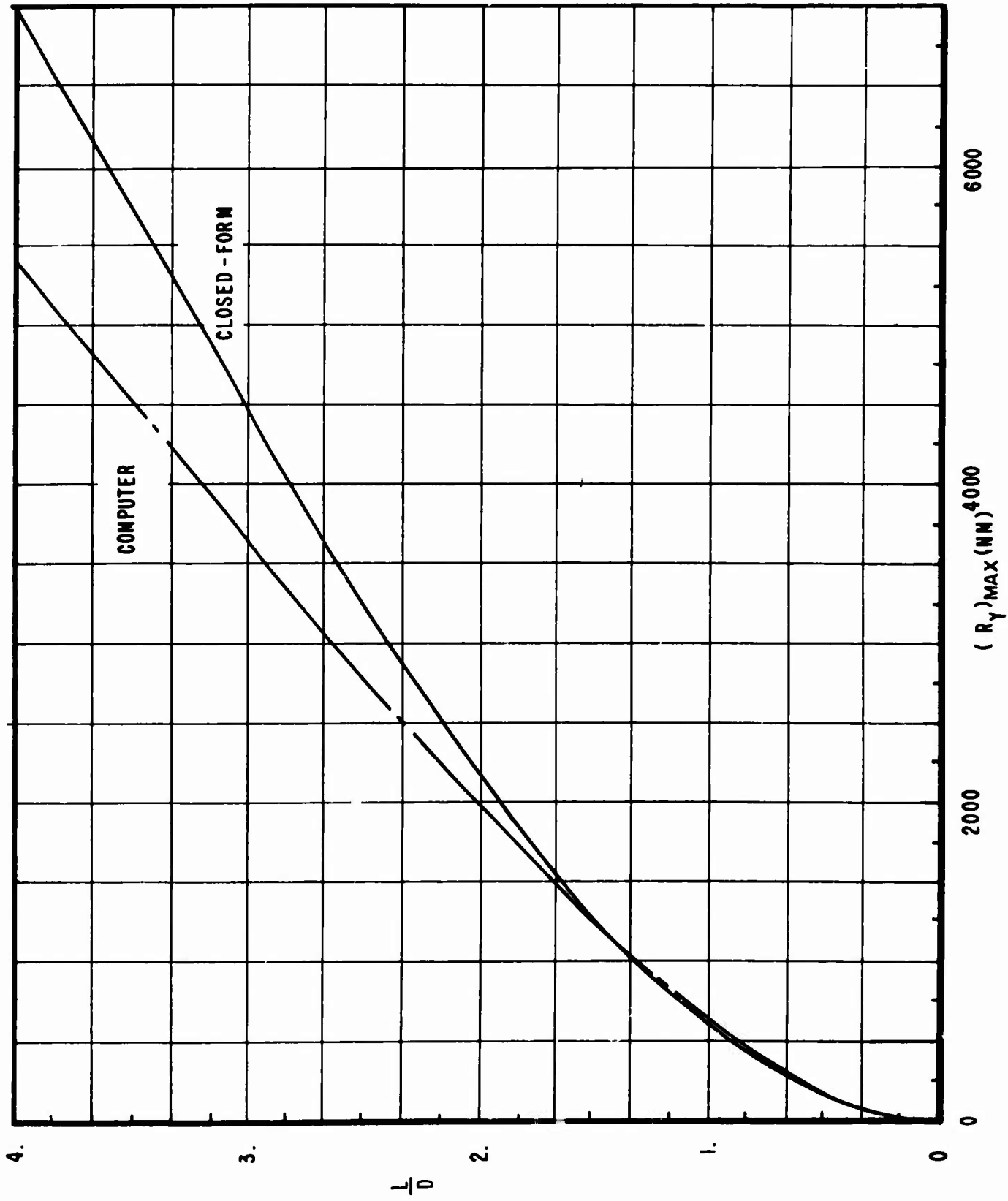


Figure 36. Comparison Between Closed-Form and Computer Maximum Lateral Range Values,  $V_1/V_c = 0.99$

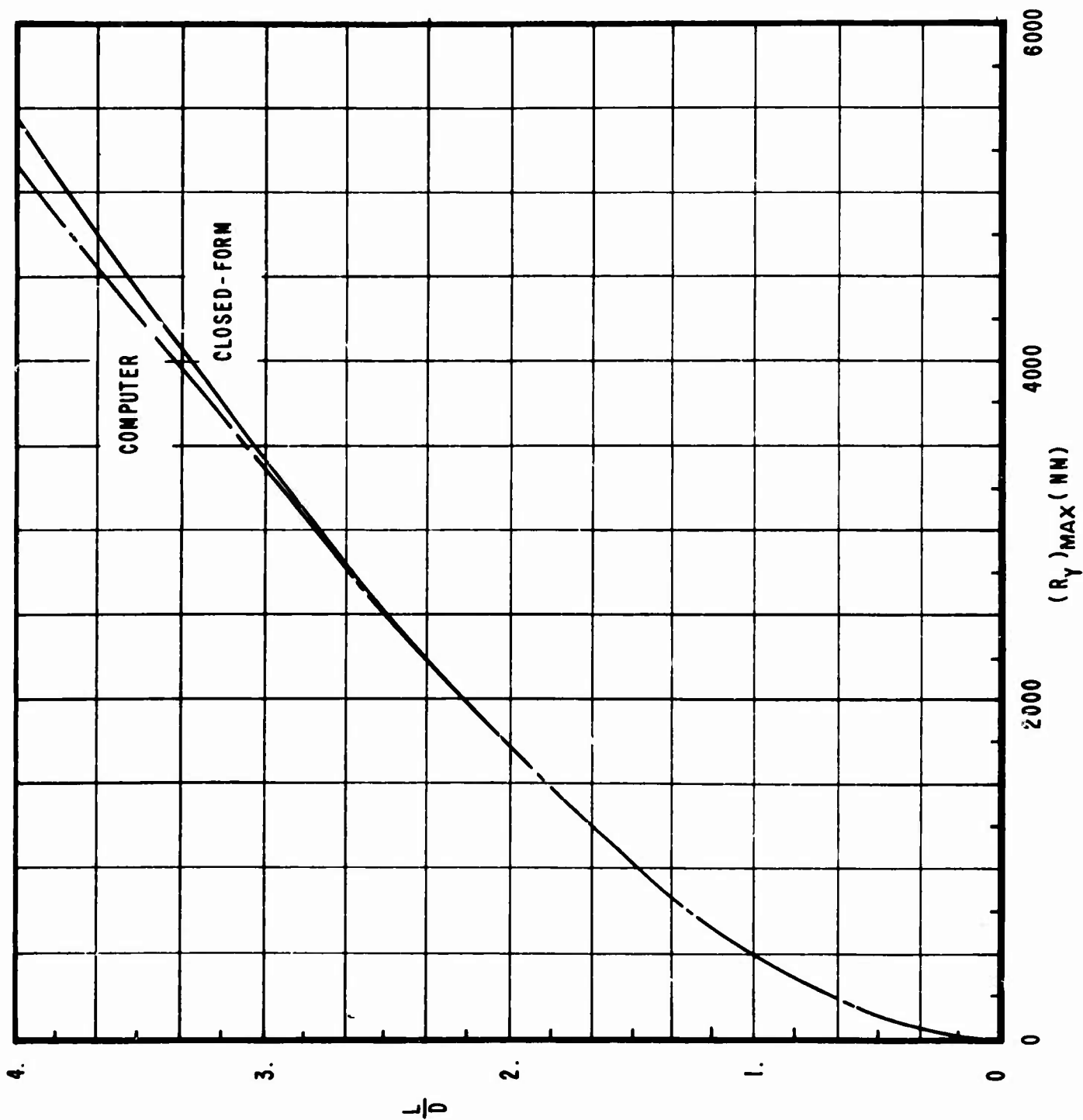


Figure 37. Comparison Between Closed-Form and Computer Maximum Lateral Range Values,  $V_1/V_c = 0.94$

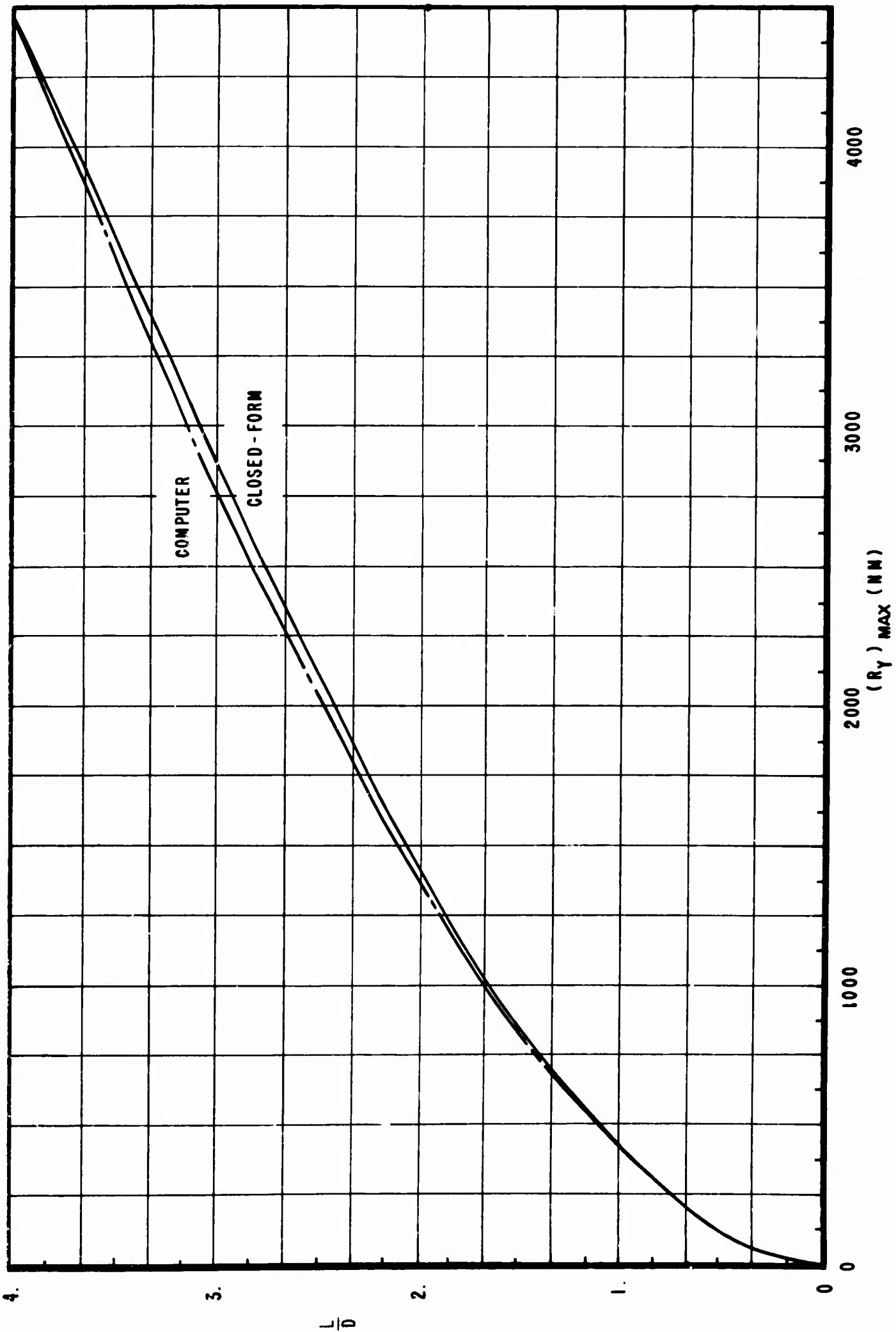


Figure 38. Comparison Between Closed-Form and Computer Maximum Lateral Range Values,  $V_1/V_c = 0.89$

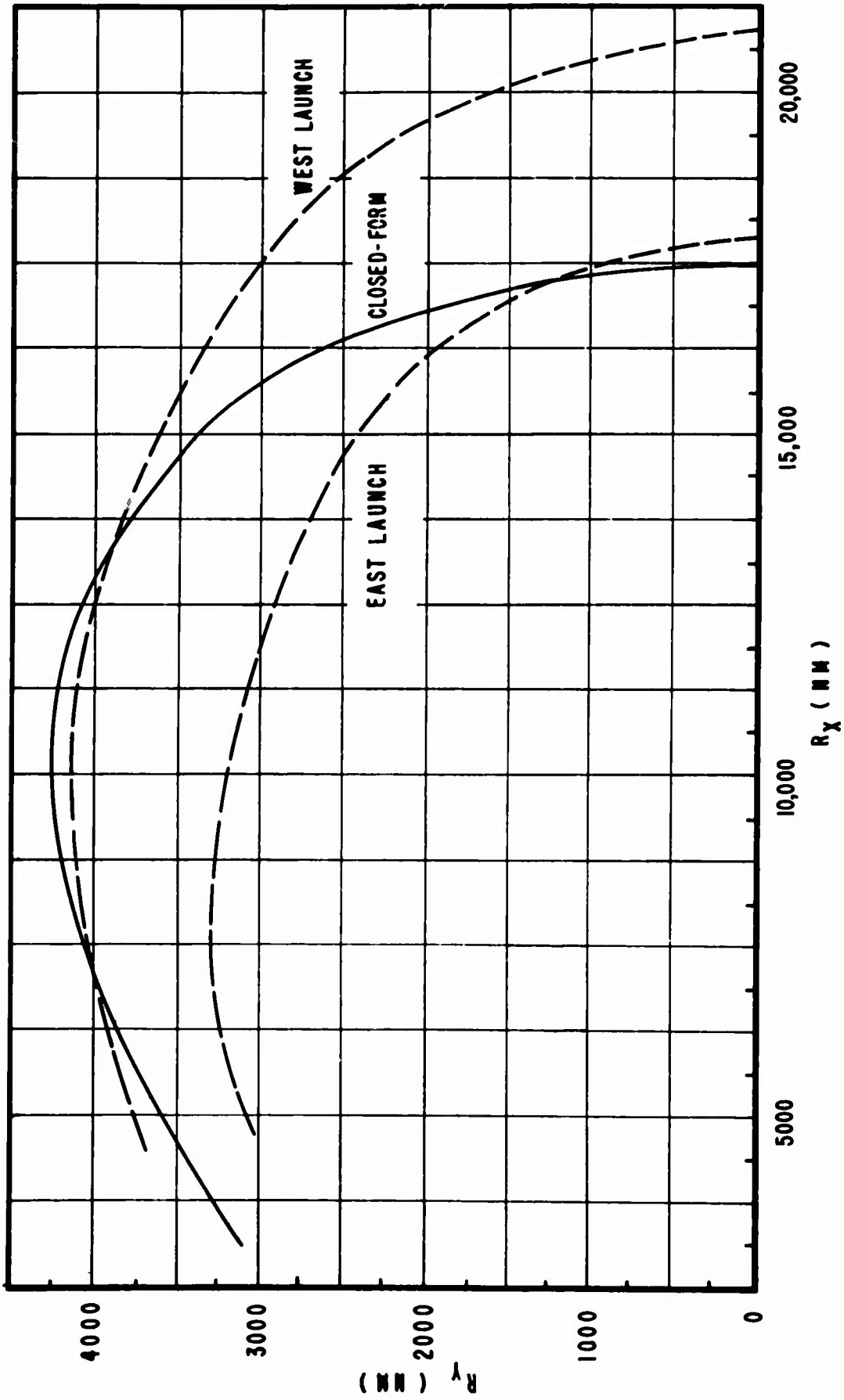


Figure 39. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.983$ ,  $L/D = 3.0$

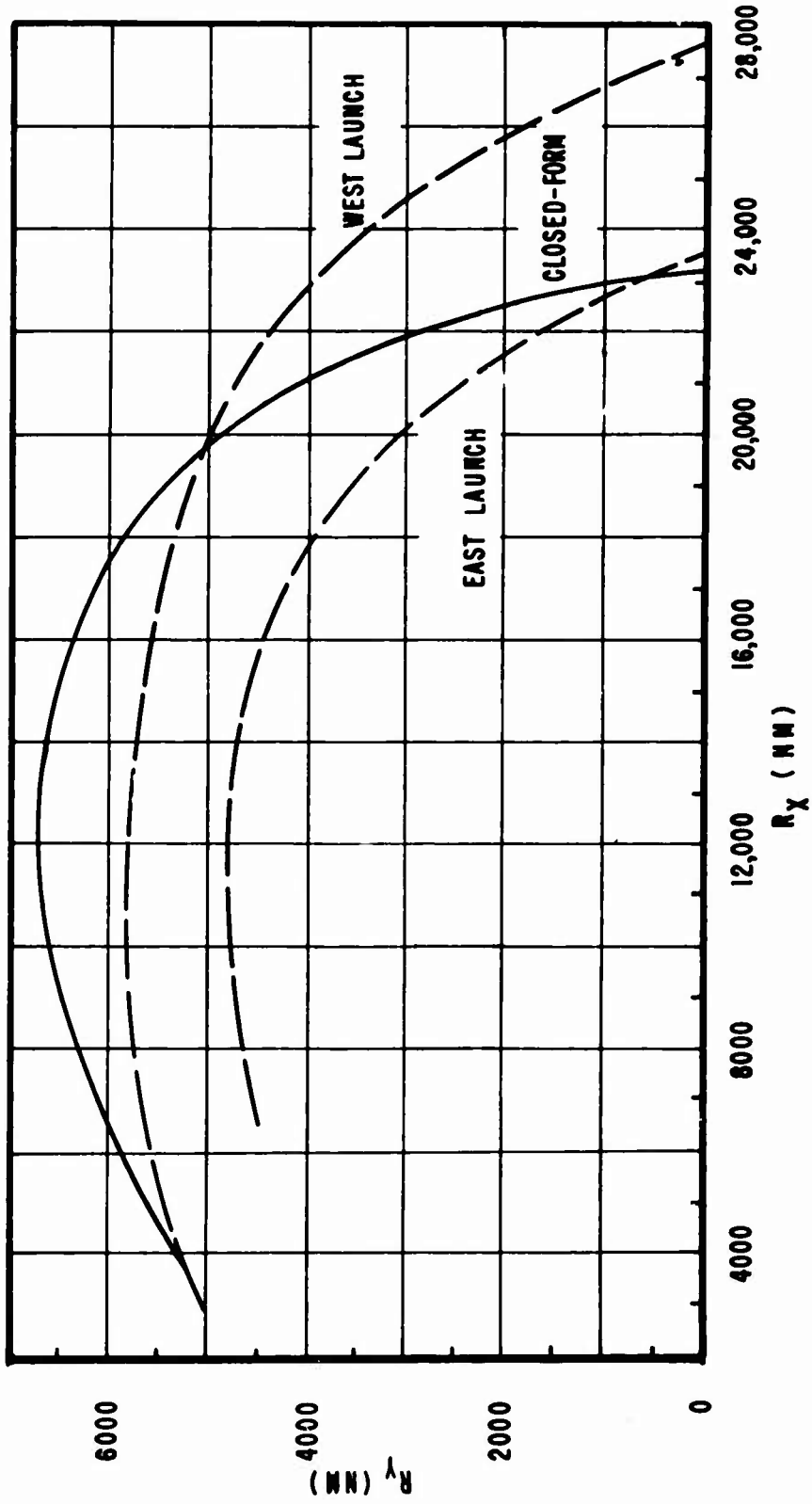


Figure 40. Footprint Comparison Between Closed-Form and Computer Solutions,  $V_1/V_c = 0.983$ ,  $L/D = 4.0$

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author) Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio	2a REPORT SECURITY CLASSIFICATION UNCLASSIFIED
	2b GROUP

3 REPORT TITLE  
A Closed-Form Solution to Lifting Reentry

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)  
August 1964 through March 1965

5 AUTHOR(S) (Last name, first name, initial)  
Bell, Roland N.

6 REPORT DATE December 1965	7a. TOTAL NO. OF PAGES 73	7b. NO. OF REFS 12
--------------------------------	------------------------------	-----------------------

8a. CONTRACT OR GRANT NO.  b. PROJECT NO. 1366  c. Task No. 136602  d.	9a. ORIGINATOR'S REPORT NUMBER(S)  AFFDL-TR-65-65
	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson AFB, Ohio
-------------------------	---

13 ABSTRACT

This report derives closed-form expressions for predicting the longitudinal and lateral range attainable by lifting reentry vehicles. The resultant equations sensitively and accurately define the influence of L/D ratio, bank angle and entry velocity variations over a spectrum of values. To illustrate the usefulness of the method, the derived expressions were used to conduct a parametric reentry study covering a range of L/D ratios from 0.5 to 4.0, bank angles from 0° to 75° and entry velocities from 0.89V<sub>c</sub> to 0.99V<sub>c</sub>. The results of this study are compared with those obtained from a high speed computer study using the same range of reentry conditions. As an aid to future investigators, a series of curves is presented giving longitudinal and lateral range values for various selected L/D, bank angle and entry velocity values. For those wishing to investigate reentry under conditions not covered by these curves, a detailed "recipe" for utilizing the method is included in an appendix. A comparison of the results of this method with those of more rigorous methods for the same reentry conditions shows that the closed-form solution has sufficient accuracy and sensitivity to be of considerable value to those persons requiring a rapid, preliminary estimate of vehicle performance.

UNCLASSIFIED

Security Classification

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Lifting Reentry Maneuverability Closed-Form Approximation Longitudinal and Lateral Range						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parentheses immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through \_\_\_\_\_."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through \_\_\_\_\_."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through \_\_\_\_\_."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical content. The assignment of links, rules, and weights is optional.

UNCLASSIFIED

Security Classification