Technical Report 167

THE ELASTIC CONSTANTS, STRENGTH AND DENSITY OF GREENLAND SNOW AS DETERMINED FROM MEASUREMENTS OF SONIC WAVE VELOCITY

by

James L. Smith

Code 1

NOVEMBER 1965

U.S. ARMY MATERIEL COMMAND
COLD REGIONS RESEARCH & ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

Errata - Technical Report 167

p. iii, Figure 19 caption should read:

Rate of change in velocity vs density, Greenland snow

p. 5, Table I, column 2, line 5 5673 should read:

5763

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PREFACE

The work described in this report was performed on Project 50b Dynamic Properties of Greenland Snow at Camp Century, Greenland during the 1963-64 summer seasons.

The study was performed by Mr. J. L. Smith under the general direction of Mr. A. Wuori, Chief, Applied Research Branch, of the Experimental Engineering Division, Mr. K. A. Linell, Chief. Appreciation is expressed to Mr. North Smith, Applied Research Branch, not only for assistance in the tests but also for discussion which aided in interpretation of the data. Dr. Tung-Ming Lee, Expert to CRREL, reviewed the report for technical content and his comments and advice are appreciated.

USA CRREL is an Army Materiel Command laboratory.

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SUMMARY

The measurement of sonic wave velocities on undisturbed samples of Greenland snow was accomplished using piezo-electric transducers in conjunction with a soniscope which provided the exciting source and the time measuring device.

These velocity measurements were correlated with density determinations, crushing strength tests, and the elastic constants of the Greenland snow samples for a density range of 0.4 to 0.9 g/cm^3.

Empirical formulae were derived that present the density, crushing strength, and the elastic constants as functions of sonic wave velocity.

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INTRODUCTION

The desirability of performing non-destructive tests on snow is emphasized by the fact, as stated by Bader (1962), that it has been found to be almost impossible to obtain or prepare identical snow samples for laboratory investigation.

Seismic measurements have been used successfully in determining the elastic constants of snow and correlated with depth and density (Bentley,1957). Lee (1961) utilized the fact that the propagation of sonic wave depends on the elastic constants to illustrate an increase in wave velocity with an increase in density for high density snow.

A pulse propagation technique was used to demonstrate a relationship and establish a correlation between wave velocity and crushing strength for processed snow from measurements of wave velocity, density, and temperature (Smith,1963).

This report deals with the determination of the density, crushing strength, and the elastic constants for naturally compacted Greenland snow from measurements of sonic wave velocity and temperature.

EXPERIMENTAL FACILITIES

During the 1963 field season at Camp Century on the Greenland Ice Cap an inclined drift was constructed on a 3:1 slope to a vertical depth of 300 feet, providing a means of obtaining undisturbed naturally compacted snow samples over a wide density range.

A snow laboratory was made at the level of the floor of an existing tunnel by excavating a room 8 x 8 x 20 feet into the wall of the tunnel. The temperature in the snow laboratory ranged from -15 to -13C during the testing period. All snow samples that were obtained from the inclined drift were stored in the snow laboratory for at least 12 hours before measurements were taken, in order to reach the laboratory temperature.

The sonic measurements reported here were taken from snow blocks with dimensions of at least 1.5 ft cube or 3.0 inch diam core samples. Cylinders 2.0 in. in diameter with a length to diameter ratio of 2.5 to 1 were then made from these samples for density and crushing strength determinations.

METHOD OF TESTING

Sonic velocity measurements

Specimens were obtained from the wall of the inclined drift at several elevations. These samples were cut with a chain saw and shaped into rectangular blocks with a crosscut saw.

Care was exercised to mark the sample so that the sonic measurements could be taken in its original vertical and horizontal direction.

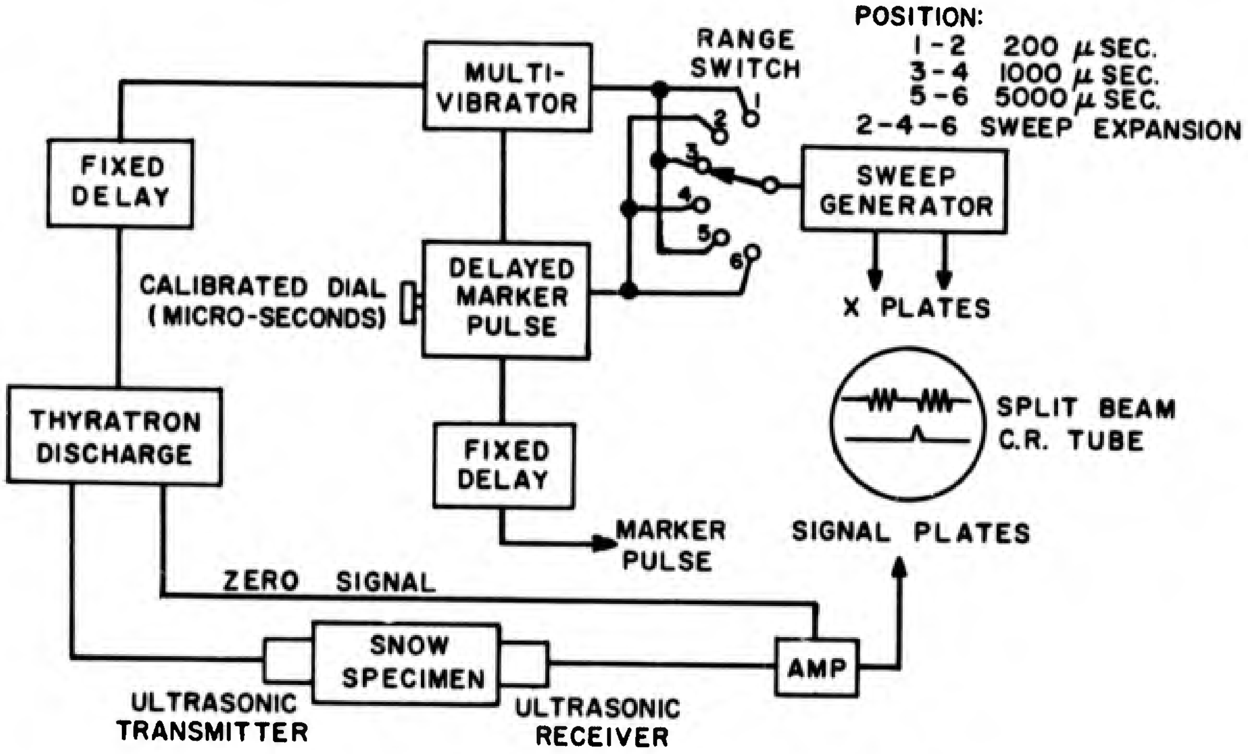

Figure 1. Block diagram of the soniscope.

The sonic wave velocity was measured using piezo-electric transducers in conjunction with a soniscope, which provided the exciting source and the time measuring device. The electric pulses generated in the control unit of the soniscope are transformed into bursts of sound, lasting a few hundred microseconds, at the contact surface of the snow sample. The travelling time through or across the sample to a receiver transducer was measured by means of a calibrated time scale on the soniscope screen. The wave velocities were determined from the time needed for the distance travelled. A block diagram of the soniscope is shown on Figure 1.

Density determinations

The test specimens were measured using a Starrett vernier caliper for the diameter and a Starrett vernier height gage for the length. The specimens were weighed to the nearest 0.05 gram. The density reported for each block is the average of the several specimens prepared from each block for crushing strength tests.

Crushing strength tests

The crushing strengths were determined using a constant velocity motorized press. The apparatus has a swivel head to assure loading normal to the surface of the sample. The pressure during compression was measured by a 2000-lb capacity load cell and recorded by a Speedomax recorder, which is a component part of the apparatus.

The specimens were prepared with a length to diameter ratio of approximately 2.5 to 1 and the loading rate was sufficiently high to make the measurements independent of the loading rate. Bader (1962) states that "crushing strengths must be measured at high rates of loading to eliminate the effects of plastic yielding (testing time of the order of 10 seconds)."

The ends were melted slightly by touching them on a hot plate with the length of the sample lined against a vertical guide to assure parallel ends as well as provide smooth and hardened end surfaces.

TEST RESULTS

Longitudinal, shear, and Rayleigh wave velocity were measured on each block of snow obtained from the wall of the inclined drift. Specimens for density and crushing strength tests were prepared from each block. The results of these determinations are tabulated in Appendix A.

Additional specimens for density and crushing strength determinations were obtained from the floor of the inclined drift at its termination in the summer of 1963, using the 3-in. diam snow auger to a density of 0.71 g/cm³. These cores were reduced to the 2.0-in. diam sample size for testing. The results of these tests are also included in Appendix A.

In the summer of 1964, the inclined drift was extended and snow blocks were obtained to a density of 0.735 g/cm³. Again additional samples were recovered from the floor of the inclined drift using the 3-in. diam snow auger to a density of 0.790 g/cm³. Also, core specimens from the thermal drill project were utilized to extend the study to a density of 0.914 g/cm³. These samples were used to complete the correlation of sonic wave velocity with density and the elastic constants. These results are tabulated in Appendix B.

Crushing strength

The graph of crushing strength vs density (Fig. 2) shows that the crushing strength is nearly a linear function of density in the range of densities tested.

An empirical relationship obtained by the method of least squares from the data shown on Figure 2 for the density range of 0.4 to 0.7 g/cm³ is

$$\sigma_c = 1542 (\rho - 0.40)$$

where:

σ_c is the crushing strength in psi, and

ρ is the density in g/cm³.

All data have been corrected to -10C employing a relationship used by Butkovich (1956):

$$\log \frac{\sigma_2}{\sigma_1} = 0.16 \log \frac{T_2}{T_1}$$

where:

σ_1 is the strength at T_1.

The graphs of crushing strength vs sonic wave velocity (Fig. 3) can be fitted by straight lines. The empirical relationships established are

$$\sigma_c = \frac{C_1 - 3042}{18.3} \quad \text{for} \quad C_1 < 8000$$

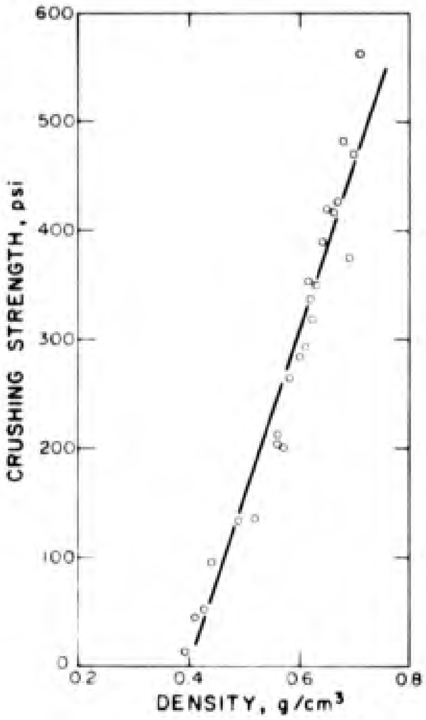

Figure 2. Crushing strength vs density, Greenland snow. Each point is the mean value for at least 5 tests from each block and 2 tests from each core.

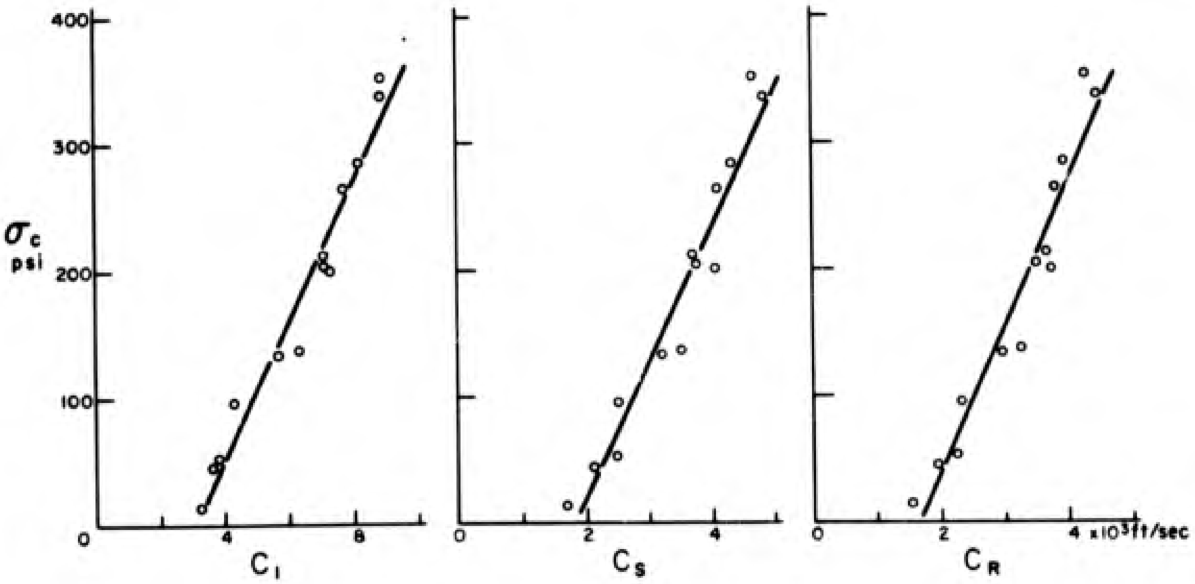

Figure 3. Crushing strength vs sonic wave velocity for Greenland snow. Note expanded scale for C_S and C_R.

$$\sigma_c = \frac{C_S - 1802}{9.2} \qquad \text{for } C_S < 4500$$

$$\sigma_c = \frac{C_R - 1648}{8.7} \qquad \text{for } C_R < 4000$$

where

σ_c is expressed in psi,
C_l is the longitudinal wave velocity in ft/sec,
C_S is the shear wave velocity in ft/sec, and
C_R is the Rayleigh wave velocity in ft/sec.

Table I gives the density determinations, the sonic velocity measurements, and the results of the crushing strength tests for each block of undisturbed snow.

Table I. Density, sonic wave velocities, and crushing strength of Greenland snow at -10C.

ρ (g/cm³)	C_l (ft/sec)	C_S (ft/sec)	C_R (ft/sec)	σ_c (psi)
0.39	3358	1728	1581	16.6
0.41	3636	2127	1943	45.8
0.425	3865	2480	2251	51.9
0.44	4305	2486	2297	97.7
0.49	5673	3235	2950	134.7
0.52	6355	3567	3280	137.9
0.56	7128	3740	3470	203.9
0.56	7063	3706	3689	212.1
0.57	7307	4039	3732	200.0
0.58	7692	4111	3804	264.5
0.60	8261	4335	3970	284.4
0.615	8882	4660	4330	353.2

Density

Results of measurements of sonic wave velocity and density for the 1963 field season are plotted on Figure 4. Computing the best fit between sonic wave velocity and density by the method of least squares yields

$$\rho = \frac{C_l + 6285}{24282} \qquad \text{for } C_l < 8000$$

$$\rho = \frac{C_S + 2911}{12231} \qquad \text{for } C_S < 5000$$

$$\rho = \frac{C_R + 2773}{11477} \qquad \text{for } C_R < 4600$$

where: ρ is expressed in g/cm³ and the sonic wave velocity in ft/sec.

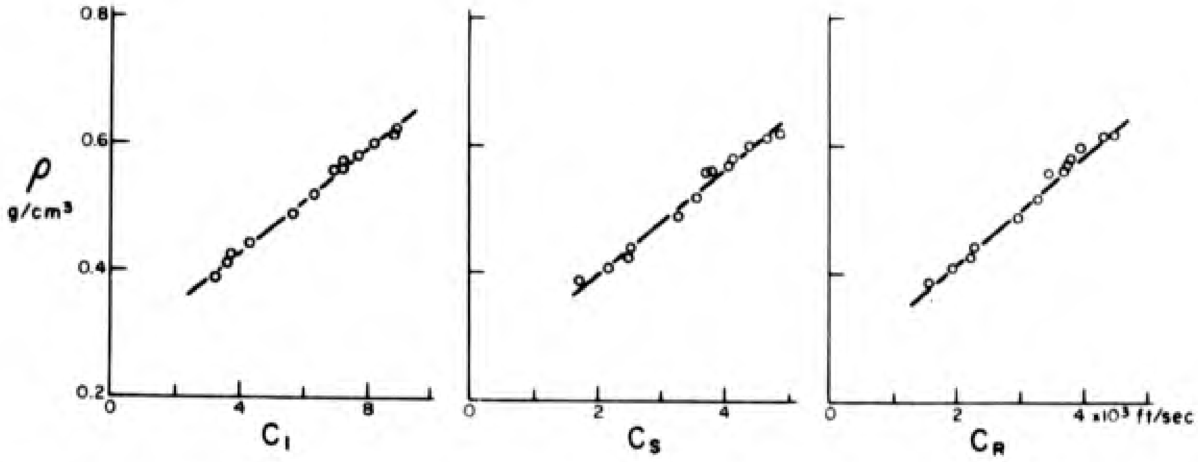

Figure 4. Density vs sonic wave velocity for Greenland snow, 1963 field season. Note expanded scale for C_S and C_R.

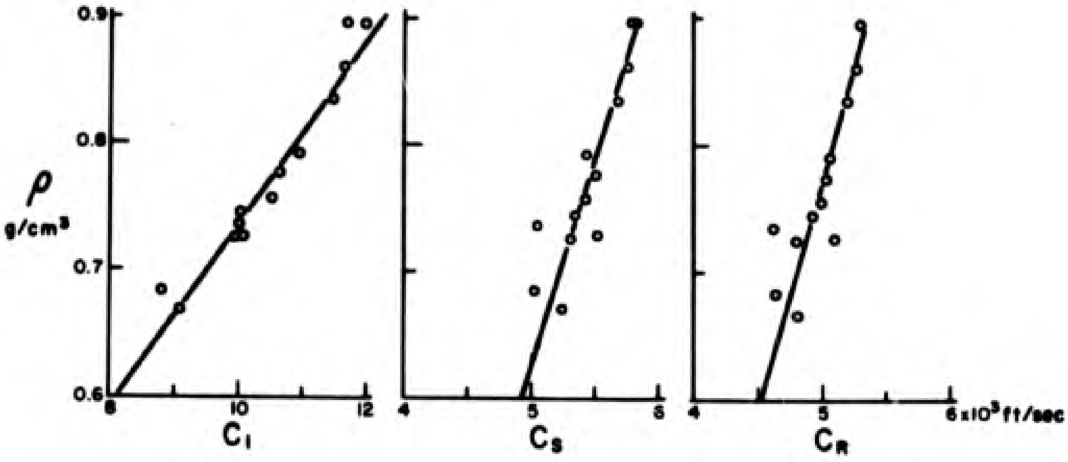

Figure 5. Density vs sonic wave velocity for Greenland snow, 1964 field season. Note expanded scale for C_S and C_R.

Results of measurements of sonic wave velocity and density for the 1964 field season are plotted on Figure 5 and the empirical equations derived from the data are:

$$\rho = \frac{C_l + 120}{13781} \quad \text{for } C_l > 8000$$

$$\rho = \frac{C_S - 3106}{3025} \quad \text{for } C_S > 5000$$

$$\rho = \frac{C_R - 2872}{2762} \quad \text{for } C_R > 4600.$$

Figures 6, 7 and 8 show the results of density vs sonic velocity measurements for the 1963 and 1964 field seasons combined.

Elastic constants

Utilizing the ratio of the Rayleigh or shear and longitudinal wave velocity (C_R/C_l or C_S/C_l) that depends only on Poisson's ratio (Leslie, 1949) (Fig. 9) the values of Poisson's ratio were obtained for each undisturbed snow sample tested. A plot of Poisson's ratio (ν) vs density (ρ) is shown on Figure 10. Computing the best straight line fit by the method of least squares yields

$$\nu = 0.174 + 0.186\,\rho \quad \text{for } 0.4 < \rho < 0.9.$$

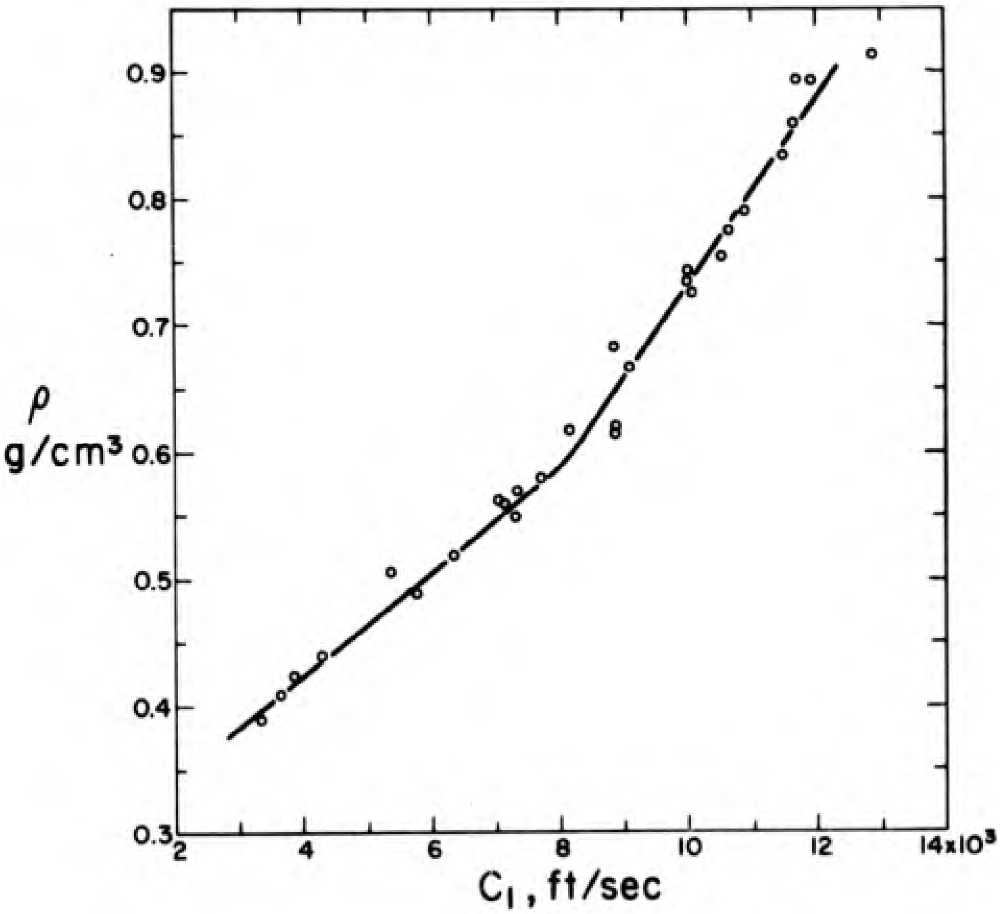

Figure 6. Density vs longitudinal wave velocity for Greenland snow.

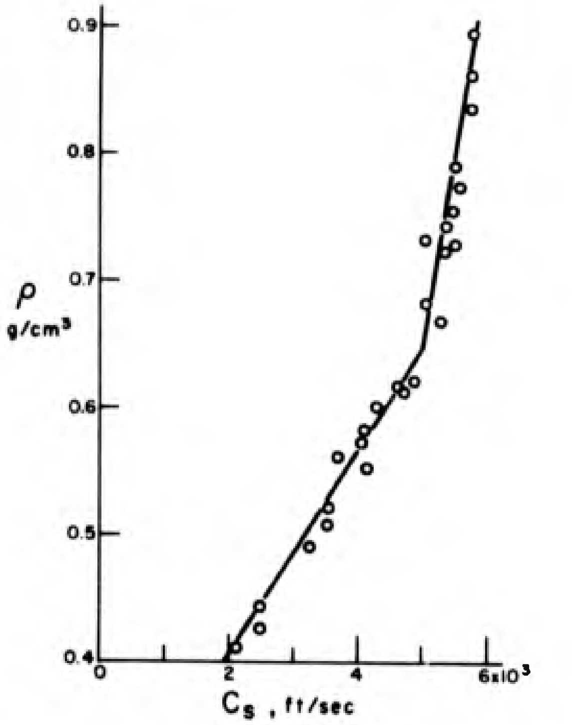

Figure 7. Density vs shear wave velocity for Greenland snow.

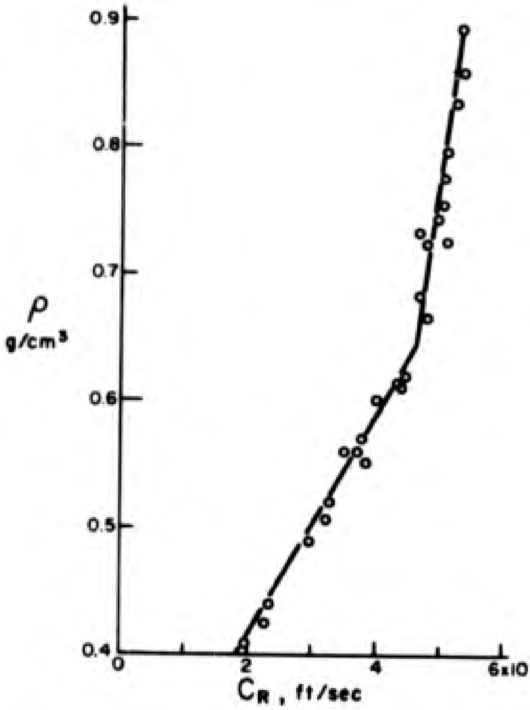

Figure 8. Density vs Rayleigh wave velocity for Greenland snow.

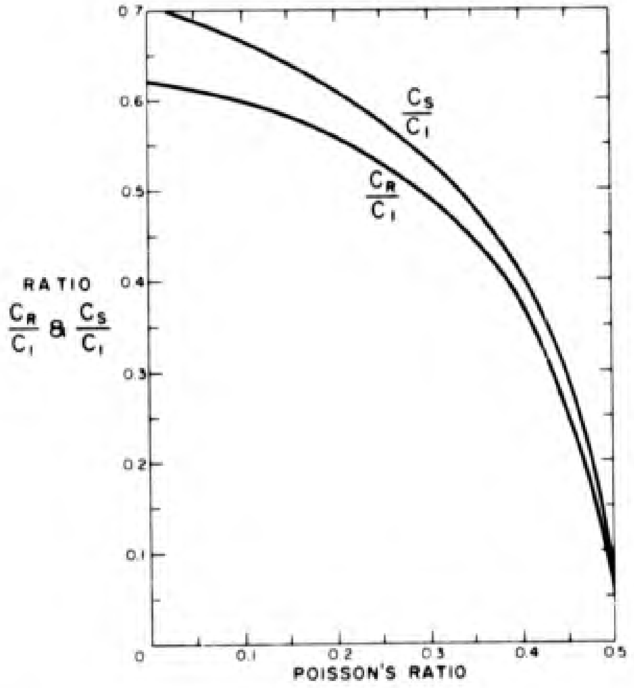

Figure 9. Theoretical relationship between velocity and Poisson's ratio.

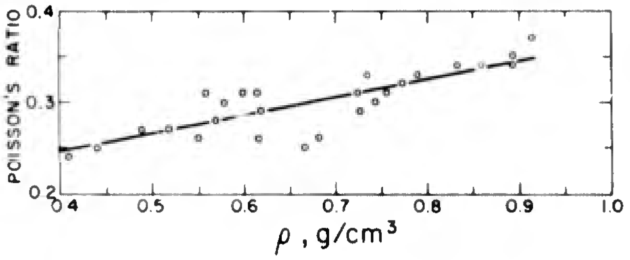

Figure 10. Poisson's ratio vs density for Greenland snow.

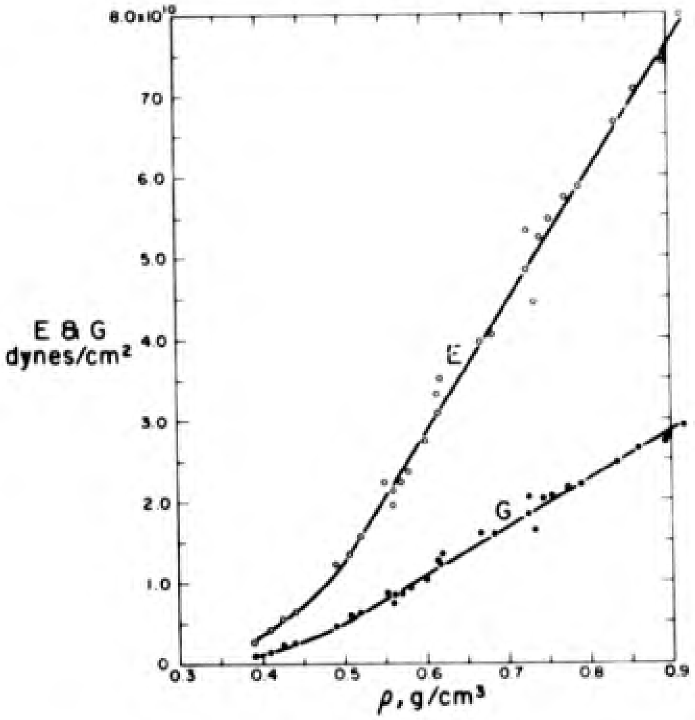

Figure 11. Young's and shear moduli vs density for Greenland snow.

The relationship of longitudinal wave velocity and the elastic constants of a material (Ewing, 1957) is

$$E = C_l^2 \rho \left[\frac{(1+\nu)(1-2\nu)}{(1-\nu)} \right] \quad \text{and}$$

$$G = \frac{E}{2(1+\nu)}$$

where:

C_l is the longitudinal wave velocity in cm/sec ,
ρ is the density in g/cm³,
ν is the Poisson's ratio,
E is the Young's modulus in dynes/cm², and
G is the shear modulus in dynes/cm².

From these relationships the values of E and G were determined for each test sample and are tabulated in Appendix B.

The elastic constants are plotted as functions of density on Figure 11. Most of these data lie in a straight line except those below 0.5 g/cm³ density. The empirical relationships derived from the data are:

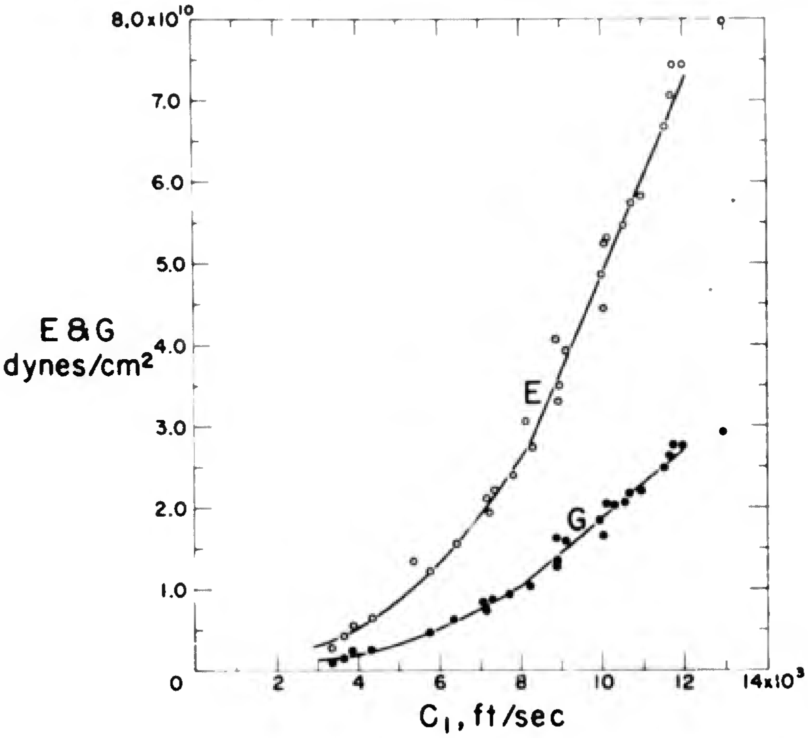

Figure 12. Young's and shear moduli vs longitudinal wave velocity for Greenland snow.

$$E = 3.55 \times 10^{11} \rho^{4.89} \qquad 0.4 < \rho < 0.5$$

$$G = 1.29 \times 10^{11} \rho^{4.78}$$

and

$$E = 1.605 \times 10^{11} (\rho - 0.42) \qquad 0.5 < \rho < 0.9 .$$

$$G = 5.84 \times 10^{10} (\rho - 0.41)$$

Figure 12 shows the elastic constants as functions of longitudinal wave velocity. Here the straight line portion begins at approximately 8000 ft/sec and the equations become

$$E = 22.305\, C_l^{2.32} \qquad C_l < 8000$$

$$G = 14.28\, C_l^{2.27}$$

and

$$E = 1.17 \times 10^{7} (C_l - 5735)$$

$$G = 4.01 \times 10^{6} (C_l - 5187). \qquad 8000 < C_l$$

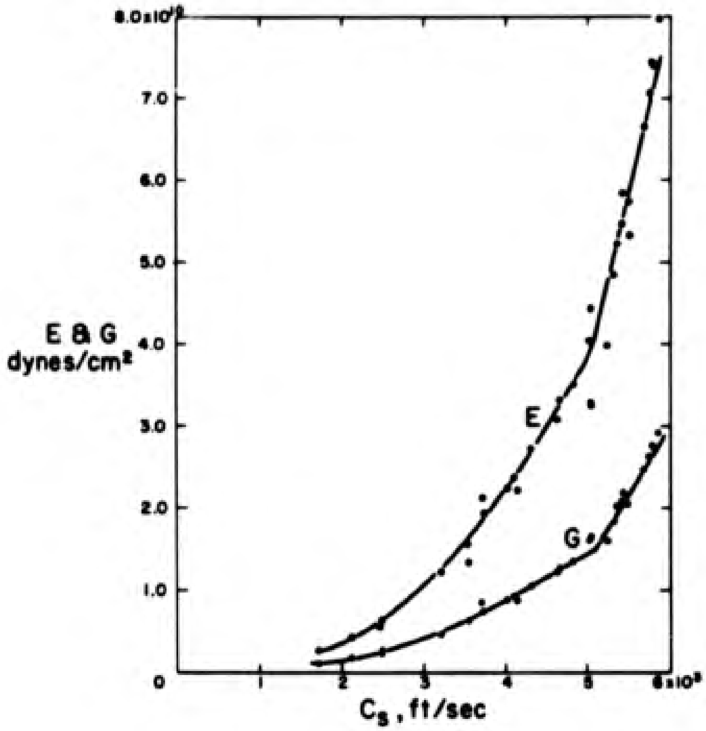

Figure 13. Young's and shear moduli vs shear wave velocity for Greenland snow.

A plot of the shear wave velocity vs the elastic constants is shown on Figure 13. The empirical relationships established are:

$$E = 8.72\ C_S^{2.61}$$

$$G = 4.63\ C_S^{2.57} \qquad C_S < 5000$$

and

$$E = 4.42 \times 10^7\ (C_S - 4176)$$

$$G = 1.55 \times 10^7\ (C_S - 4064), \qquad 5000 < C_S$$

The relationships between the Rayleigh wave velocity and the elastic constants as determined from the data shown on Figure 14 are

$$E = 12.85\ C_R^{2.59}$$

$$G = 8.11\ C_R^{2.53} \qquad C_R < 4600$$

and

$$E = 4.80 \times 10^7\ (C_R - 3840)$$

$$G = 1.69 \times 10^7 (C_R - 3734). \qquad 4600 < C_R$$

where a, b, d, and e are constants,

ρ is the density in g/cm^3

y is the modulus desired (E or G) in dynes/cm^2, and

x is the independent variable (density in g/cm^3 or wave velocity in ft/sec).

The relationship between Poisson's ratio and density was found to be

$$\nu = 0.186\rho + 0.174 \text{ for } 0.4 < \rho < 0.9$$

where

ν is the Poisson's ratio.

The empirical equation derived for the relationship of crushing strength to density has the form

$$\sigma_c = 1542 (\rho - 0.40)$$

where:

σ_c is the crushing strength in psi.

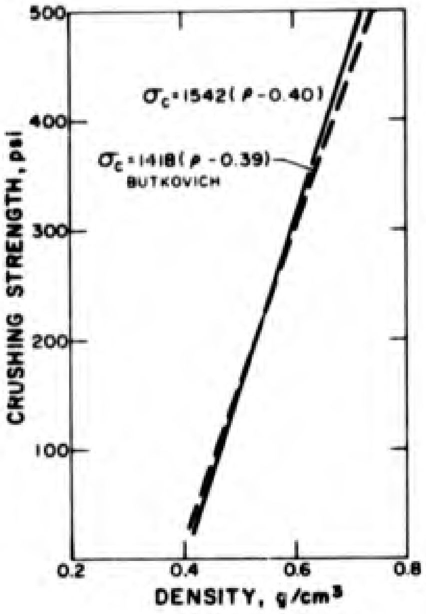

Figure 15. Crushing strength vs density for Greenland snow at -10C.

Butkovich (1956) expressed unconfined compressive strength vs density as a linear relationship for high density snow. The empirical equation obtained was

$$\sigma_c = 1418 (\rho - 0.39)$$

which compares quite favorably with the results from this study. A comparison of these equations is shown on Figure 15.

The empirical relationship between density and longitudinal wave velocity was established as

$$\rho = \frac{C_L + 6285}{24282} \quad \text{for } 0.4 < \rho < 0.6$$

and

$$\rho = \frac{C_L + 120}{13781} \quad \text{for } 0.6 < \rho < 0.9$$

For comparison this curve is shown with a plot of longitudinal wave velocity vs density from Bentley (Fig. 16).

Nakaya (1959) measured Young's modulus for Greenland snow by the vibration method and concluded that the Young's modulus-density relation for the whole range of snow and ice is divided into three parts. The empirical formulae presented for densities between 0.4 and 0.9 g/cm^3 are

$$\log E = 6.35\rho + 6.80 \quad \text{for } 0.25 < \rho < 0.5$$

and

$$E = (16.4\rho - 7.20) \times 10^{10} \quad \text{for } 0.5 < \rho < 0.9.$$

The Young's modulus-density curve (Fig. 17) shows the comparison between the results of Nakaya's study and the relationship established in this report which gives

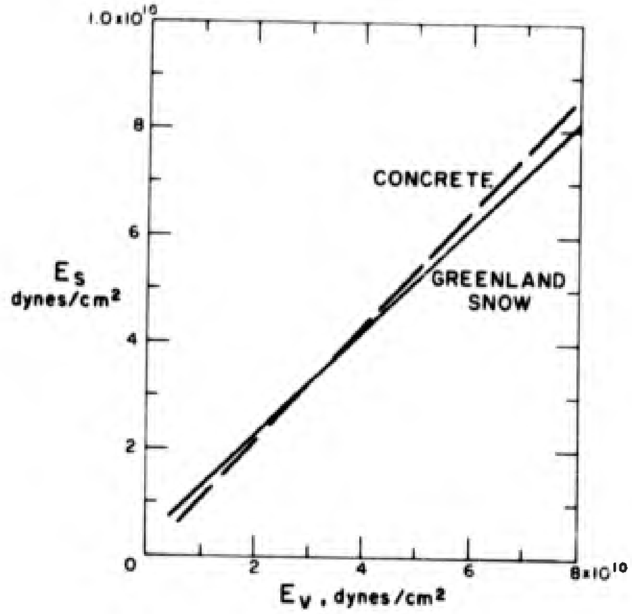

Figure 18. Correlation between vibration and sonic modulus for Greenland snow and concrete.

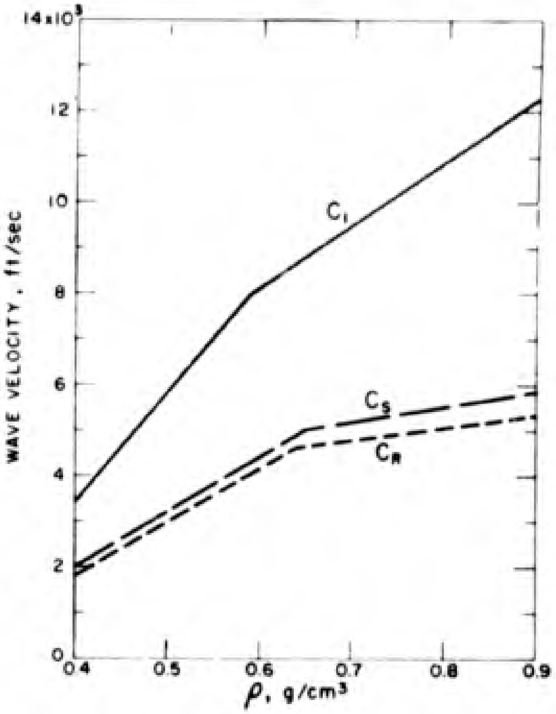

Figure 19. Dependence of the rate of change in velocity on density of Greenland snow.

$$G = 1.29 \times 10^{11} \rho^{4.78}$$

while in the region above 0.5 g/cm³ density the relation is expressed by a straight line of the form

$$E = 1.605 \times 10^{11} (\rho - 0.42) \qquad \text{and}$$

$$G = 5.84 \times 10^{10} \; (\rho - 0.41).$$

The moduli-sonic velocity relations show a similar behavior and are also divided into two regions. For a compressional wave velocity below 8000 ft/sec, a shear wave velocity below 5000 ft/sec and a Rayleigh wave velocity below 4600 ft/sec, the relation is expressed as log-log of the form

$$y = ax^{b}$$

and for the region above these values the relation is expressed as a straight line by

$$y = d\,(x + e)$$

where

y is the modulus desired (E or G) in dynes/cm²,

x is the sonic wave velocity (C_1, C_S, C_R) in ft/sec,

and a, b, d and e are constants.

The transition from one region to the other cannot be abrupt but must be gradual and at or near the stated limits.

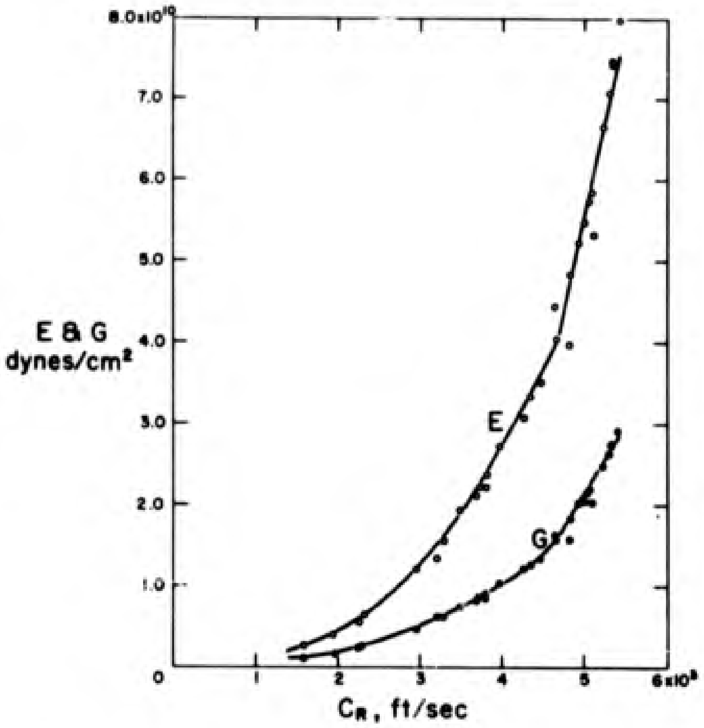

Figure 14. Young's and shear moduli vs Rayleigh wave velocity for Greenland snow.

DISCUSSION

A series of crushing strength tests were conducted at Camp Century, Greenland, in conjunction with a program of determining the sonic wave velocity for naturally compacted snow.

A correlation between wave velocity and crushing strength was established as

$$\sigma_c = \frac{C - C_0}{a}$$

where:

σ_c is the crushing strength in psi,

C is the measured wave velocity in ft/sec, and

C_0 and a are constants depending on the type of wave velocity measured.

In addition, the elastic constants E (Young's modulus) and G (shear modulus) were determined along with the values of Poisson's ratio for Greenland snow with densities from 0.4 g/cm³ to 0.9 g/cm³.

The empirical equations derived to express E or G in terms of either density or wave velocity take the form

$$y = ax^b$$

and

$$y = d(x + e) \text{ for } 0.5 < \rho < 0.9$$

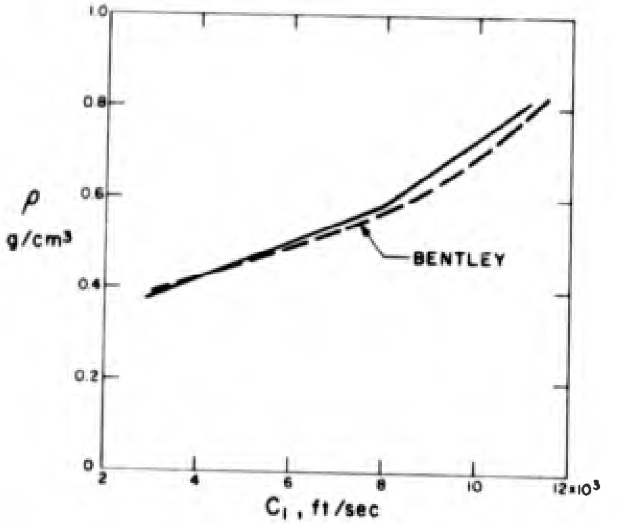

Figure 16. Longitudinal wave velocity vs density for Greenland snow.

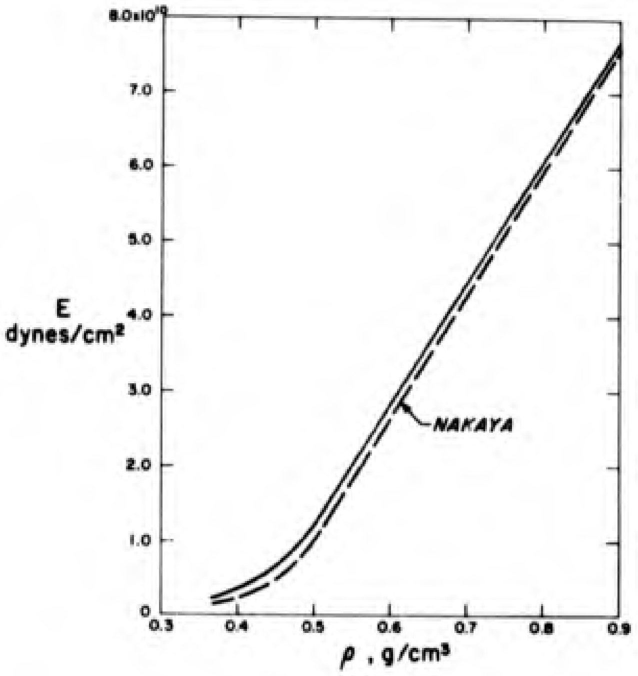

Figure 17. Young's modulus vs density for Greenland snow.

$$E = 3.55 \times 10^{11} \rho^{4.89} \quad \text{for } 0.4 < \rho < 0.5$$

and

$$E = 1.605 \times 10^{11} (\rho - 0.42) \quad \text{for } 0.5 < \rho < 0.9.$$

The correlation between these two methods is in agreement with the results of extensive laboratory tests on concrete specimens (Leslie, 1949) correlating the soniscope method with the standard vibration test procedure (ASTM designation C 215-60) as shown on Figure 18.

CONCLUSIONS

The use of the soniscope as a tool for non-destructive testing on undisturbed snow samples has been demonstrated. The reliability of the results obtained and the empirical relationships established are substantiated by comparison with the results of other investigators.

Density has been presented as a function of sonic wave velocity which was the measurable quantity. Actually the change in wave velocity is dependent on the change in density as illustrated in Figure 19. The rate of change in wave velocity with respect to density decreases at a density between 0.6 and 0.65 g/cm³. Although density measurements can be made accurately by conventional methods, the non-destructive feature of the sonic velocity method may be adopted for in-situ density determinations.

The moduli-density relation obtained for the range of density observed in this study is divided into two regions. In the region below 0.5 g/cm³ the relation is expressed in a log-log form as

$$E = 3.55 \times 10^{11} \rho^{4.89} \quad \text{and}$$

The empirical relationships established from the results of this study allow the determination of the density, crushing strength, and dynamic elastic constants for Greenland snow from measurements of sonic wave velocities.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Snow cover--Greenland						
Geophysical exploration (sonic)						
Snow--Density						
Snow--Strength						
Snow--Elasticity						

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APPENDIX A: TABLE OF TEST DATA

Type of sample	Density g/cm²	Crushing strength psi	Number of tests	Type of sample	Density g/cm²	Crushing strength psi	Number of tests
Block	0.39	16.6	5	Core	0.61	294.5	2
Block	0.41	45.8	12	Core	0.62	338.7	2
Block	0.425	51.9	8	Core	0.62	319.2	2
Block	0.44	97.7	12	Core	0.63	350.3	5
Block	0.49	134.7	10	Core	0.64	389.5	5
Block	0.52	137.9	10	Core	0.65	419.9	3
Block	0.56	203.9	9	Core	0.66	417.8	5
Block	0.56	212.1	10	Core	0.67	428.6	4
Block	0.57	200.0	6	Core	0.68	482.5	5
Block	0.58	264.5	12	Core	0.69	375.0	4
Block	0.60	284.4	11	Core	0.70	470.8	3
Block	0.615	353.2	8	Core	0.71	563.3	2

APPENDIX B: TABLE OF E AND G VALUES

Density	Sonic wave velocity (ft/sec)			Elastic constants (dynes/cm^2)	
g/cm^3	C_1	C_S	C_R	E	G
0.39	3358	1728	1581	2.68 x 10^9	1.01 x 10^9
0.41	3636	2127	1943	4.19	1.69
0.425	3865	2480	2251	5.61	2.43
0.44	4305	2486	2297	6.40	2.57
0.49	5763	3235	2950	1.21 x 10^{10}	4.75
0.508	5370	3560	3200	1.34	6.05
0.52	6355	3567	3280	1.57	6.19
0.551	7300	4150	3800	2.22	8.80
0.56	7128	3740	3470	1.94	7.44
0.56	7063	3706	3689	2.12	8.42
0.57	7307	4039	3732	2.24	8.78
0.58	7692	4111	3804	2.39	9.23
0.60	8261	4335	3970	2.73	1.04 x 10^{10}
0.615	8882	4660	4330	3.33	1.28
0.617	8100	4610	4260	3.08	1.22
0.62	8891	4835	4462	3.51	1.36
0.668	9100	5250	4810	3.99	1.60
0.683	8820	5020	4650	4.05	1.61
0.725	9975	5330	4820	4.85	1.85
0.728	10100	5500	5100	5.32	2.06
0.735	10010	5050	4630	4.45	1.66
0.744	10030	5360	4940	5.24	2.02
0.756	10570	5430	5000	5.48	2.08
0.774	10640	5500	5050	5.75	2.18
0.790	10940	5440	5070	5.86	2.20
0.834	11500	5690	5220	6.67	2.49
0.860	11650	5750	5290	7.08	2.64
0.894	11980	5800	5310	7.43	2.75
0.895	11700	5790	5310	7.45	2.78
0.914	12900	5850	5390	7.99	2.92

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3. REPORT TITLE

THE ELASTIC CONSTANTS, STRENGTH, AND DENSITY OF GREENLAND SNOW AS DETERMINED FROM MEASUREMENTS ON SONIC WAVE VELOCITY

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Technical Report

5. AUTHOR(S) *(Last name, first name, initial)*

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13. ABSTRACT

The measurement of sonic wave velocities on undisturbed samples of Greenland snow was accomplished using piezo-electric transducers in conjunction with a soniscope which provided the exciting source and the time measuring device. These velocity measurements were correlated with density determinations, crushing strength tests, and the elastic constants of the samples for a density range of 0.4 to 0.9 g/cm^3. Empirical formulas were derived that present the density, crushing strength, and the elastic constants as functions of sonic wave velocity.

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