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DEPOSITION OF PARTICLES IN A TURBULENT SLOT FLOW

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INTRODUCTION

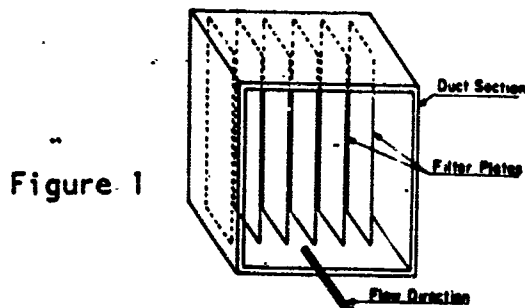
In recent years military equipment designers have become more and more interested in the behavior of dust particles in an air stream. The military environment is often a dusty one, and dust is well known as a destroyer of combustion engines. Several general types of air cleaners have been used to clean the air for use in powered ground equipment. These cleaners have usually been based on inertial, electrostatic, or impingement principles, and their design has required compromises, particularly in the areas of filtration efficiency, pressure drop, size, weight, complexity, and service life. The advent of gas turbine engines in military equipment, increased use of aircraft in the dusty region near the ground, and the general push toward increased efficiency with decreased size and weight, has brought air cleaner designers to the limits of available basic knowledge. One of the most lucrative areas of ignorance tempting the researcher in fluid dynamics is the behavior of dust particles in the boundary layer. Fluid dynamicists have vigorously attacked the question "what happens to a fluid in the vicinity of a wall, where viscous shearing forces are relatively large?" Prandtl's boundary layer hypothesis, and later work by such able researchers as von Karman, have permitted the aerodynamicist to work in an atmosphere of relative certitude. The laminar portions of the boundary layer are susceptible to mathematical solution, although the procedures are somewhat sophisticated. Turbulent boundary layers are less amenable to analysis, although statistical techniques offer some help in their description. When one adds solid dust particles to these already-complex flow phenomena, it is obvious that theoretical analyses will be difficult. Of equal difficulty are the techniques needed to measure the motion of the small dust particles. Optical methods used for studying larger particle movement are totally inadequate to observe particles of the order of one micron in diameter.

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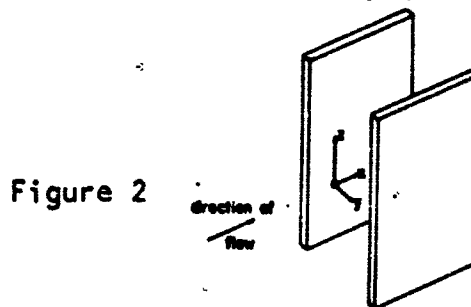
The vortex separators being studied by Dr. Hans J.P. von Ohain and his associates at the USAF Aerospace Research Laboratories, the parallel-plate dust separators studied by Dr. Franz J. Jaklitsch at ATAC, and the improvements on impingement filters by Mr. Richard W. Child of the ATAC Propulsion Systems Laboratory and his associates, all involve motion of dust and air in the boundary layer. These efforts have stimulated the present study, which is pointed specifically at the geometrically-simplest device, the Jaklitsch parallel-plate filter.

A suitable theoretical development has been achieved. It is based on the earlier work of the Prandtl and von Karman schools, applying additional statistical correlation techniques. For convenience, the development assumes spherical particles of fixed density compared to the fluid density. There does not appear to be any insurmountable barrier to handling variations from these assumptions. It is also assumed that particle density is small enough that the particles behave as individuals, with negligible influence from other particles. This assumption is basic to the theoretical development and also to the experimental apparatus. Variation from this assumption appears to be difficult indeed, and is left to other researchers. Fortunately there appear to be many areas of practical interest within existing assumptions. The instrumentation uses a carefully-collimated light beam of high intensity, interrogated at right angles by two carefully-collimated optical seekers of known separation distance. These seekers use electronic light-amplification techniques to provide particle count and particle transit-time information from even the smallest particles. These data are collected on a digital tape recorder. The raw data tapes are subsequently processed in a general-purpose digital computer to determine the desired concentration and velocity coefficients.

It has become apparent that the instrumentation system, the Dobbs Optical Anemometer, is a powerful new tool for research in fluid dynamics. For instance, it can, by using very small particles as tracers, give extremely accurate measurements of both mainstream and boundary-layer flow. It can, in an expanded form, resolve all velocity components of interest, including turbulence as well as average velocities. These can be measured with no appreciable instrumentation influence on the flow.



Aerodynamic Filter as Proposed by
Dr. F. Jaklitsch (U.S. PAT. No. 2,967,386)



Slot Flow as Set up
for Experiment

ANALYTICAL APPROACH

A 1/4-inch wide by 6-inch high slot, 32 inches long, set in the center of a 6-inch square duct was chosen for the reasons that: (1) It was sufficiently large to work in; (2) It was long enough to insure fully developed turbulent flow profiles in the slot; (3) The height-to-width ratio, 24 , was large enough to permit assumption of two-dimensional flow at the center; (4) The resulting test section was small enough to be built without undue problems (In practice, this has proved only marginally true.); (5) The Reynolds number of the flow, based on the hydraulic diameter of the slot, would be high enough for turbulence.

The range of flow velocities chosen, 20 - 200 fps, was based on the results of the earlier work at ATAC (2). Lower velocity flows did not cause deposition of particles, and higher velocity flows apparently tended to strip the deposited material back off the walls. The resulting range of variation of the flow Reynolds number is 2500 to 25000.

For this velocity range in air under approximately standard conditions it is reasonable to assume incompressible flow. Since a filter based on this principle would normally be a steady flow device, flow was assumed time-steady in the mean. Finally, a particle size range of 0.1 to 10 micron was chosen. Larger particles than this would be more influenced by gravitational forces than by fluid turbulence. Smaller particles present additional experimental problems.

In general, no problem in turbulent fluid mechanics has yet been solved without the use of empirical data at some point. The descriptive equations inevitably contain more unknowns than the number of distinct relationships among them. Such problems have usually been attacked by assuming that, despite the lack of an applicable uniqueness theorem, the Navier-Stokes equations hold, instantaneously, for turbulent flow. This assumption is implicitly made here. In other work involving discrete particles a microscopic viewpoint has often been taken, considering the behavior of the individual particle. In all cases statistical concepts must be applied to account for the random behavior of the flow and the particles in it. Here the concentration by weight of the particulate material is assumed to be low enough (below 1%) so that the behavior of the fluid will not be significantly affected by the particles. At the same time, due to their small size, the numerical density of particles in the flow (on the order of 10^5 /cc) is high enough to permit a continuum approach to the variation in particle density in the flow.

The most applicable previous work on the deposition of particles out of a turbulent flow onto parallel bounding surfaces was done by Friedlander (4) in 1954. He worked with flow through tubes and based his analysis on a paper by Lin, Moulton, and

Putnam(5) in 1953 which was concerned with transfer of material out of solution on to the walls. The analytical approach in both cases was to divide the flow into a viscous sublayer, a turbulent core, and a buffer layer between them. Eddy diffusivity was assumed to occur, decreasing in effect, all the way to the wall. The main stream concentration was assumed to remain constant in the direction of flow. Although this last assumption was applicable in these papers, it is precisely the change in concentration in the direction of flow that is of concern in this paper.

Applying the continuum approach to the particle concentration in the flow, consider an elementary volume of space. For the x direction the net convective change is:

$$\left[CU\Delta y\Delta z + \frac{\partial}{\partial x} (CU) \Delta y\Delta z\Delta x \right] - CU\Delta y\Delta z = \frac{\partial}{\partial x} (CU) \Delta y\Delta z\Delta x \quad (1)$$

The net diffusive change is:

$$\left[D \frac{\partial C}{\partial x} \Delta y\Delta z + \frac{\partial}{\partial x} \left(D \frac{\partial C}{\partial x} \right) \Delta y\Delta z\Delta x \right] - D \frac{\partial C}{\partial x} \Delta y\Delta z = \frac{\partial}{\partial x} \left(D \frac{\partial C}{\partial x} \right) \Delta y\Delta z\Delta x \quad (2)$$

Summing for all three coordinate directions and simplifying, for an incompressible fluid the conservation of suspension equation is:

$$\frac{\partial}{\partial x_i} (CU_i) + \frac{\partial}{\partial x_i} \left(D \frac{\partial C}{\partial x_i} \right) = - \frac{\partial C}{\partial t} \quad (3)$$

Assuming instantaneous concentration and velocity can be expressed as:

$$C = \bar{C} + c \quad (4)$$

$$U = \bar{U} + u \quad (5)$$

and using Reynolds rules for averaging, the turbulent flow form of equation (3) is:

$$\frac{\partial \bar{C}}{\partial t} + \bar{U}_i \frac{\partial \bar{C}}{\partial x_i} = \frac{\partial}{\partial x_i} \left(D \frac{\partial \bar{C}}{\partial x_i} - \overline{cu_i} \right) \quad (6)$$

Consider Figure 2. For a slot flow $\bar{U}_y = \bar{U}_z = 0$. Since the flow is two-dimensional and there is no change along the z-coordinate,

$$\frac{\partial \bar{C}}{\partial z} = 0, \quad \frac{\partial \overline{cu}}{\partial z} = 0$$

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Since the flow is time-steady in the mean $\frac{\partial \bar{c}}{\partial t} = 0$

The conservation of suspension equation for a turbulent slot flow is then

$$\bar{u} \frac{\partial \bar{c}}{\partial x} - D \left(\frac{\partial^2 \bar{c}}{\partial x^2} + \frac{\partial^2 \bar{c}}{\partial y^2} \right) + \frac{\partial}{\partial x} (c\bar{u}) + \frac{\partial}{\partial y} (c\bar{v}) = 0 \quad (7)$$

I II III IV V

provided D , the molecular diffusion coefficient is assumed constant.

Equation (7) can be further reduced by considering the relative magnitude of the terms. Experimental data on slot flow⁽⁶⁾ indicates term IV of equation (7) to be no larger than about one per-cent of term I. Experimental data on particle deposition rates⁽⁴⁾ indicates that term I is small, but that term II must approach zero. The high value of the Schmidt number in this case (nominally 2.5×10^4 to 5×10^6) implies a very flat concentration profile falling off very sharply close to the wall. Terms III and V then should have high values, essentially equal to each other, at the edge of the viscous sublayer. From the foregoing it may be concluded that terms I, III, and V of equation (7) should be retained. In dimensionless form, this gives the following approximate equation for the particle concentration in a turbulent slot flow:

$$V \frac{\partial \psi}{\partial \xi} - \frac{1}{R^* Sc} \frac{\partial^2 \psi}{\partial \eta^2} + \frac{\partial g}{\partial \eta} = 0 \quad (8)$$

To solve this, expressions for the velocity, V , and the correlation function g are needed. Pai found a solution in polynomial form for V . This is (7):

$$V = A + B\eta^2 + C\eta^n \quad (9)$$

where A , B , C , n are constants whose exact form and derivation are not of concern here.

The correlation coefficient, g , must be measured experimentally. This is one of the aims of this work. The resulting curve will be described by a polynomial in η . With this, any particular solution for ψ , the dimensionless mean concentration (no matter how peculiar) will enable the following transformation to reduce equation (8) to a homogenous equation:

$$\psi = \psi_p + \Psi \quad (10)$$

The resulting equation is:

$$\frac{\partial^2 \Psi}{\partial \eta^2} - R^* Sc (A + B\eta^2 + C\eta^n) \frac{\partial \Psi}{\partial \xi} = 0 \quad (11)$$

Separation of variables reduces this to two ordinary differential equations, one with variable coefficients, which may be solved by ordinary methods. Substituting for Ψ ,

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$$\Psi = \theta(\xi) \phi(\eta) \quad (12)$$

These are, where K is a constant,

$$\theta' + K\theta = 0 \quad (13)$$

$$\phi'' + K R^* Sc(A + B\eta^2 + C\eta^n) \phi = 0 \quad (14)$$

The critical points in this procedure are the determination of an expression for g and the construction of a matching particular solution for Ψ . Without experimental data, further meaningful discussion is difficult at this point.

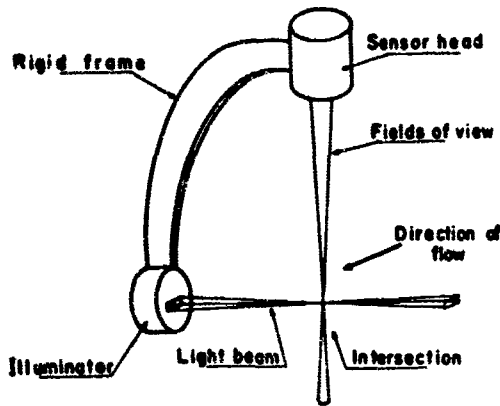
EXPERIMENTAL APPROACH

Many physical quantities must be measured in an experiment of this sort. This discussion, however, is limited to those involving the particle concentration and flow velocity. The quantities which must be measured in order to solve and evaluate equation (8) are: (1) the mean particle concentration at a series of points across the channel; (2) the average product of the particle concentration variation and the y -direction velocity variation at a similar series of points.

Search of the literature disclosed no existing method of measuring these. The discrete nature of the particles indicated the quantities could not be measured directly but would have to be computed from counts taken over finite space and time intervals. The problem then resolved itself into one of how to restrict these intervals to small enough values so that measurement of the microstructure of the flow would not be averaged out. It was also necessary, due to the random nature of the data to be observed, that the measurement produce the least possible disturbance in the flow.

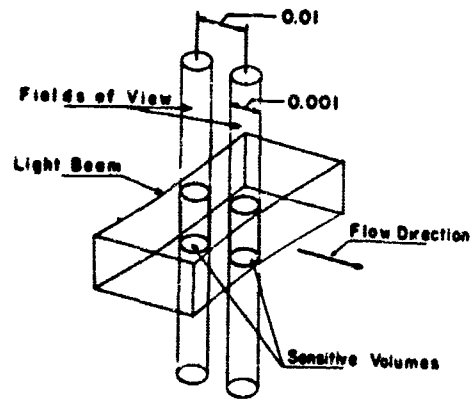
After considering and discarding a number of counting probe designs it was decided to attempt the design of an instrument based on light-scattering. This would entirely circumvent the problem of disturbing the flow when measuring it. Light-scattering particle counters draw the mixture of fluid and particles through a known, illuminated volume (normally several cc), at a known rate, and count all particles in the stream. The optical anemometer, as the proposed instrument became known, would not have this control over the fluid stream. As shown in Figure 3, the sensitive volume would be defined by the intersection of a light beam and a field of view in the fluid stream itself. This sensitive volume had to be kept as small as possible to satisfy the basic requirement for a point measurement. The instrument also had to measure the "instantaneous" flow velocity at the point where the particle count was being taken. An approximately cylindrical volume 0.001-inch in diameter and

0.010-inch in length was chosen as best meeting the previously stated requirements, allowing statistically significant particle counts with the chosen particle densities, and allowing sharp optical definition. As shown in Figure 4, two such volumes 0.010-inch apart, aligned with the principal flow direction, are used to time those particles which intercept both volumes. The average transit time of these during the period of measurement is taken as the flow velocity. All particles passing through one of the volumes are counted to obtain the particle count. A short time period, τ , on the order of 0.01 seconds is used for each such count with its accompanying velocity measurements. A long time period, T , on the order of 30 seconds, is used for the complete measurement at given point in the flow. Each T period is made up of several thousand τ periods, each representing an individual concentration measurement. All data are automatically recorded as they are taken on digital tape in suitable form for direct computer processing.



Sensor System

Figure 3



Intersection of light beam and fields of view

Figure 4

The concentration calculated from a single τ period is

$$C = \frac{N}{A \tau \dot{U}} \quad (15)$$

where

$$\dot{U} = \frac{\sum U_i}{N_p} \quad (16)$$

The mean concentration is then

$$\bar{C} = \frac{\sum C}{N_c} \quad (17)$$

The mean velocity, for this purpose,

$$\bar{U} = \frac{\sum \dot{U}}{N_c} \quad (18)$$

and the concentration and velocity variations for a given τ period are

$$c = C - \bar{C} \quad (19)$$

$$u = \dot{U} - \bar{U} \quad (20)$$

The mean product of these is

$$\overline{cu} = \frac{\sum cu}{N\tau} \quad (21)$$

The product which is needed, however, is \overline{cv} . Since v cannot be measured with the present instrumentation, it is necessary to turn to the literature for a way out of this difficulty.

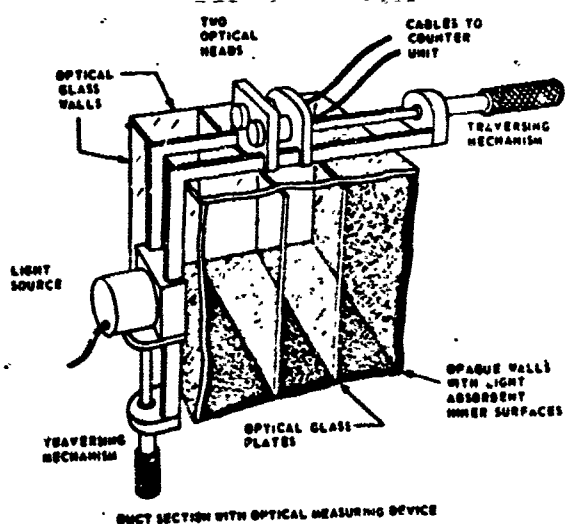
Both $\sqrt{u^2}$ and $\sqrt{v^2}$ have been measured for channel flow⁽⁶⁾. Therefore, for the present, since c is a scalar quantity, it is proposed that the following relationship be used:

$$\overline{cv} = \overline{cu} \frac{\sqrt{v^2}}{\sqrt{u^2}} \quad (22)$$

An improved version of the instrument, which will be able to measure v directly, is already being planned.

The instrument as constructed consists of: (1) traversing and mounting system made up of a 72-inch lathe bed with a traveling carriage permitting the sensor system 40 inches of movement along the bed and 6 inches in each of the other two coordinate directions; (2) a 40-inch long, 6-inch square, glass-walled test section which mounts over the lathe bed; (3) sensor system arranged physically as shown in Figure 3 and schematically as shown in Figure 5 (Two such systems arranged to move independently of each other are planned for an improved version of the instrument.); (4) electronics to count and time the particles and put the information in proper format for recording; (5) digital tape recorder to store the information.

Figure 5



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Once the instrument has been adjusted to scan a chosen point in the flow stream, the sequence of events in making a measurement is as follows: (1) The operator places an End-of-File character on the tape to indicate the start of a measurement.

(2) The operator places a six-character label on the tape to identify the measurement. This step may be repeated as many times as desired to put additional instructions on the tape concerning the particular measurement. This information must be numerical.

(3) The operator starts the measurement period (T).

(4₁) The instrument starts the first counting period (τ_1).

(a₁) The first particle velocity is recorded. The following information is given:

1 A single-character sensor system identifier.

2 A four-character transit time measurement (.50 nanosecond resolution).

3 A nine-character measurement of time from the start of the counting period (2 microsecond resolution).

(a₂) The second particle velocity is recorded.

•
•
•

(a_n) The nth particle velocity is recorded.

(b) The counting period ends.

(c) A six-character counting period identifier is recorded.

(d) A six-character total particle count for the counting period is recorded.

(e) An End-of-Information character is recorded to mark the end of the recording for that counting period.

(4₂) The instrument starts the second counting period (τ_2).

•
•
•

(4_N) The instrument starts the Nth counting period (τ_N).

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(5) The measurement period ends. The sensor system can then be moved to scan the next point where a measurement is to be made.

When a series of measurements have been completed, the taped information is fed directly into a digital computer programmed to compute the desired quantities.

While the instrument was designed to meet the requirements of the problem discussed in this paper, it is evident it has wider applicability. The particles in the flow, themselves of concern here, also might in other problems serve solely as tracers for measuring the turbulent characteristics of the flow. Since no physical probe is used, investigation of flow characteristics is possible in cases which previously were difficult or impossible.

The improved version of the instrument, which is now being planned, will incorporate two independent sensor systems, a more accurate and flexible traversing system to carry them, particle size discrimination, the ability to measure transverse velocity components, and such improvements in electronics as are available at the time it is constructed. This will make possible direct measurement of the transverse velocity variation - concentration variation correlation, measurement of the microscale of turbulence, and measurement of the correlation coefficients for the flow itself.

It is hoped that significant improvements in our understanding of turbulent flow can be made through the use of this development in instrumentation.

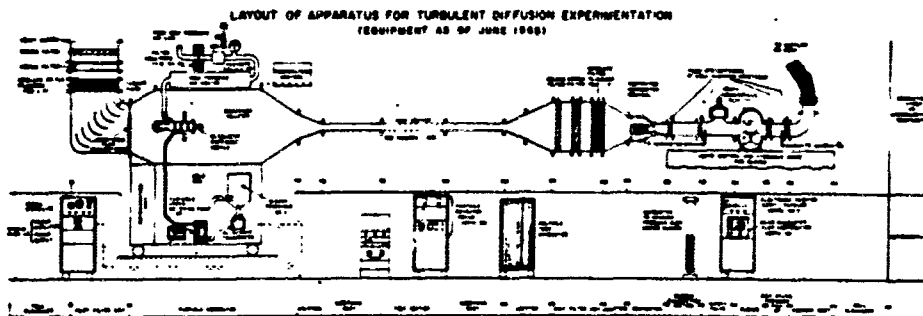


Figure 6

NOMENCLATURE

A	Cross sectional area of sensitive volume perpendicular to main flow (equation 15)
A	Constant (equation 9, 11, 14)
B	Constant (equation 9, 11, 14)
C	Constant (equation 9, 11, 14)
C	Concentration (equations 1, 2, 3, 4, 15, 19)
$\bar{c} = \bar{c}(\xi, \eta)$	Mean concentration
$\bar{c}_0 = \bar{c}(0,0)$	
\mathcal{D}	Molecular diffusion coefficient for particles
K	Constant
N	Number of particles counted during period τ
N_p	Number of particles timed during period τ
N_τ	Number of τ periods in T
$R^* = \frac{U^* \nu_0}{\nu}$	Wall-friction velocity Reynolds Number
$Sc = \frac{\nu}{\mathcal{D}}$	Schmidt number
T	Time for complete measurement at a point
U	Velocity
U_i	Notation representing velocities in the three coordinate directions
\bar{U}	Average velocity of particles timed during τ
U_p	Velocity of an individual particle
$U^* = \frac{\sigma_0}{\rho}$	Wall-friction velocity
\bar{U}	Mean velocity in the x-direction
\bar{U}_i	Notation representing the mean velocities in the three coordinate directions
\bar{U}_y	Mean velocity in the y-direction

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\bar{U}_z	Mean velocity in the z-direction
$V = \frac{\bar{U}}{U^*}$,	Dimensionless mean velocity in the x-direction
c	Variation from the mean concentration
$\overline{cu_i}$	Notation representing the mean products of the concentration variation with the velocity variations in the three coordinate directions
\overline{cu}	Mean product of the concentration variation with the x-direction velocity variation
\overline{cv}	Mean product of the concentration variation with the y-direction velocity variation
$g = \frac{\overline{cv}}{\bar{c}_o U^*}$,	Dimensionless correlation coefficient between the concentration variation and the y-direction velocity variation
n	Integer constant
t	Time
u	Variation from mean velocity in the x-direction
$\overline{u^2}$	Mean value of u^2
v	Variation from mean velocity in the y-direction
$\overline{v^2}$	Mean value of v^2
X	Coordinate of main flow direction
X_i	Notation representing the three coordinate directions
Δx	Incremental length in the x-direction
y	Coordinate perpendicular to the channel walls
y_o	Half width of the channel
Δy	Incremental length in the y-direction
z	Coordinate perpendicular to the main flow direction and parallel to the channel walls
Δz	Incremental length in the z-direction
$\eta = \frac{y}{y_o}$,	Dimensionless y-direction coordinate

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$\theta = \theta \xi$, Function defined by equation (13)

μ Absolute fluid viscosity

$\nu = \frac{\mu}{\rho}$, Kinematic fluid viscosity

$\xi = \frac{x}{y_0}$, Dimensionless x-direction coordinate

ρ Fluid density

σ_0 Shear stress at the wall

τ Time for single counting period

$\phi = \phi(\eta)$ Function defined by equation (14)

$\Psi = \Psi(\xi, \eta)$ Function defined by equation (10)

$\psi = \frac{\bar{c}}{\bar{c}_0}$, Dimensionless mean concentration

ψ_p Particular solution to equation (8)

REFERENCES

1. Jaklitsch, Franz J.; U. S. Patent No. 2,967,586, 10 January 1961.
2. ATAC Laboratories Division Report No. 4396 (Final), dated 18 January 1961.
3. ATAC Laboratories Division Report No. 5001 (First), dated 11 September 1961.
4. Friedlander, Sheldon K.: "Deposition of Aerosol Particles from Turbulent Gases", Dissertation for the Department of Chemical Engineering, University of Illinois, 1954.
5. Lin, C. S., R. W. Moulton, and G. L. Putnam: "Mass Transfer Between Solid Walls and Fluid Streams", *Industrial and Engineering Chemistry*, V. 45, n. 3, March 1953, p. 636 - 646.
6. Laufer, John: "Investigation of Turbulent Flow in a Two-Dimensional Channel", NASA Report TR 1053, 1951.
7. Pai, Shih-I: Viscous Flow Theory, Vol. II, D. Van Nostrand Company, Inc., New York (1957), p. 33 - 37.