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General Reciprocity Parameter

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The reciprocity parameter used in the reciprocity calibration of electroacoustic transducers is derived for the general case of arbitrary boundary conditions and arbitrary definitions of responses. A general reciprocity parameter is defined as the ratio of the volume velocity emanating from a surface defined in the transmitting response to the resulting pressure averaged over a surface defined in the receiving response. It is shown how the general parameter concept leads directly and simply to the various special parameters for the cases of spherical waves, cylindrical waves, plane waves, diffuse sound, couplers, and tubes. Application of the concept to *in situ* and nearfield conditions is also described.

INTRODUCTION

THE reciprocity parameter J used in the reciprocity calibration of electroacoustic transducers is defined by $J=M/S$, where M is the receiving sensitivity and S is the transmitting response of a reciprocal trandsucer. Obviously, J depends on how M and S are defined—that is, on what electrical and acoustical parameters are chosen to describe the input and output of the transducer and on what the boundary conditions on the medium are.

Since MacLean[1] and Cook[2] first applied the reciprocity principle to the calibration of electroacoustic transducers in 1940, there has been a continuing development of reciprocity parameters to fit special conditions. MacLean's original reciprocity calibration was under far-field–free-field conditions and, although it is not usually identified as such, his parameter should be called the *spherical-wave reciprocity parameter*. Cook's work pertained to what is now generally called *coupler reciprocity*. Beranek[3] extended the coupler method to include standing-wave tubes. In 1949, Simmons and Urick[4] developed a *plane-wave reciprocity parameter*. In 1961, Bobber and Sabin[5] contributed a *cylindrical-wave reciprocity* and Diestel[6] a *diffuse-sound reciprocity*. Most recently, Beatty[7] reports on two parameters used in *tube reciprocity*. His tube reciprocity differs from Beranek's in that Beatty uses plane, progressive waves rather than standing waves.

The development of each of these parameters has, for the most part, been independent of the others. There have been some exceptions. Sabin[5], in one of the two methods used to derive the cylindrical-wave parameter, used the spherical-wave parameter. Trott[8] has also shown how the plane-wave and cylindrical-wave parameters can be derived from power relationships if the spherical-wave parameter is known.

Actually, all the reciprocity parameters have a common meaning and physical definition as acoustic transfer admittances. Wathen-Dunn[9] identified J as the transfer admittance between two points in an acoustic medium. Although he allowed arbitrary boundary conditions of the medium, Wathen-Dunn did assume that the reference pressures in the definitions $M=E/p_m$ and $S=p_s/I$ were pressures at a point, thus limiting J to a transfer admittance between points. This author[5] developed the cylindrical-wave reciprocity parameter in terms of the acoustic transfer admittance between two parallel lines. Among the various reciprocity methods,

[1] W. R. MacLean, "Absolute Measurement of Sound without a Primary Standard," J. Acoust. Soc. Am. 12, 140–146 (1940).
[2] R. K. Cook, "Absolute Pressure Calibration of Microphones," J. Acoust. Soc. Am. 12, 415–420 (1941).
[3] L. L. Beranek, *Acoustic Measurements* (John Wiley & Sons, Inc., New York, 1949).
[4] B. D. Simmons and R. J. Urick, "Plane Wave Reciprocity Parameter and Its Application to Calibration of Electroacoustic Transducers at Close Distances," J. Acoust. Soc. Am. 21, 633–635 (1949).
[5] R. J. Bobber and G. A. Sabin, "Cylindrical Wave Reciprocity Parameter," J. Acoust. Soc. Am. 33, 446–451 (1961).
[6] H. G. Diestel, "Reciprocity Calibration of Microphones in a Diffuse Sound Field," J. Acoust. Soc. Am. 33, 514–518 (1961).
[7] L. G. Beatty, "Reciprocity Calibration in a Tube with Active-Impedance Termination," J. Acoust. Soc. Am. 39, 40–47 (1966).
[8] W. J. Trott, "Reciprocity Parameters Derived from Radiated Power," J. Acoust. Soc. Am. 34, 989–990(L) (1962).
[9] W. Wathen-Dunn, "On the Reciprocity Free-Field Calibration of Microphones," J. Acoust. Soc. Am. 21, 542–546 (1949).

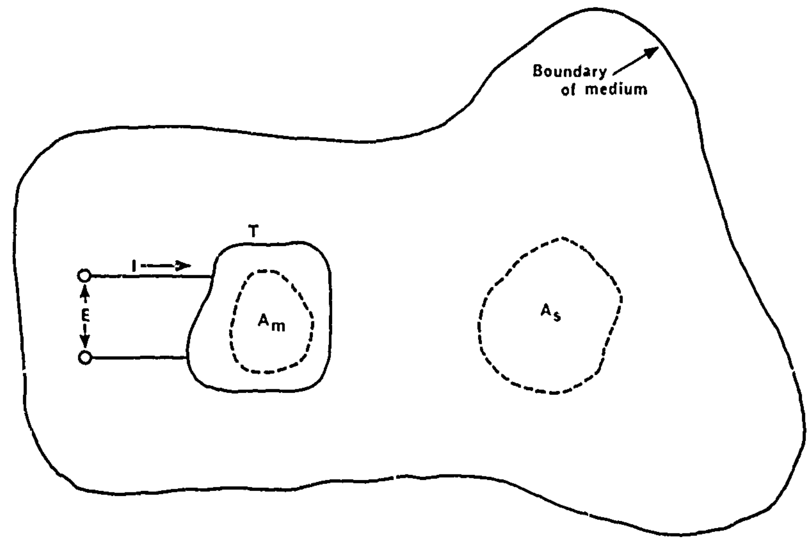

FIG. 1. Transducer T of arbitrary shape in a medium with an arbitrary boundary; A_s is the area over which the transmitted pressure is measured, A_m is the area over which the received pressure is measured, E is voltage, and I is current.

the sound pressures p_m and p_s in different cases are pressures at a point, along a line, over a plane, and the average pressure in a room or tank.

A unifying concept for all reciprocity parameters is derived in terms of a general reciprocity parameter, where the boundary conditions of the medium, the reference pressures used in defining M and S, and the transducer configuration are all arbitrary. It is also shown how a general parameter can be used to derive special parameters—in some cases with considerable savings in analytical labor—and how the general parameter concept is used to measure J for boundary conditions that preclude computation.

I. DERIVATION OF THE GENERAL RECIPROCITY PARAMETER

Consider a reciprocal, electroacoustic transducer T of arbitrary shape in a medium with arbitrary boundaries as shown in Fig. 1. The current through and the voltage across the transducer are I and E, respectively. The receiving sensitivity of T in the most general case is the ratio of an output voltage or current to an input pressure or velocity. Although any of the four combinations of these output/input parameters can be used, we are interested only in the voltage/pressure ratio that is usually used to express the sensitivity of a microphone or hydrophone. For both theoretical and practical reasons, the output voltage E is always an open-circuit voltage E_0. The input pressure may be defined in any of the several ways mentioned in the Introduction—that is, at a point, along a line, etc. In some cases, the reference pressure is the pressure that exists when T is absent; in other cases, it is the real applied pressure p_a at the transducer diaphragm.

The ratio E_0/p_a is called a *pressure sensitivity*. The pressure sensitivity usually is measured and used only when the dimensions of the transducer are small as compared with a wavelength and the transducer has an acoustic impedance equal to or greater than the characteristic impedance of the medium. Under these conditions, the difference between the applied pressure and the pressure in the absence of the transducer is negligible.

Let $\bar{p}_m$ be defined as the average input or received pressure at A_m when the transducer is either absent or present. For specific cases, the definition of M will state or imply which condition applies. Then $M=E_0/\bar{p}_m$. The place A_m at which the pressure acts can be a point, a line, or a surface, and may be at an arbitrary location. In practice, of course, A_m is usually a point and is called the *acoustic center* of T, but this is only a practice and not a requirement. There is not necessarily any relation between A_m and the actual size or shape of T. When the medium is unbounded, $\bar{p}_m$ may be a free-field pressure, but not necessarily a farfield pressure, as is illustrated later in a nearfield case.

The transmitting response S is defined as the ratio $\bar{p}_s/I_1$, where $\bar{p}_s$ is the output or transmitted pressure and I_1 is the input current when T is the only source in the medium. Again, we could use output velocity and input voltage, but choose pressure and current for simplicity and conformity with common practice. The pressure $\bar{p}_s$ is the pressure at some arbitrary place A_s in the medium averaged over A_s. As with A_m, A_s can be a point, a line, or a surface.

In the analysis to follow, both A_m and A_s will be understood to be a continuous closed surface. When either is referred to as a point, it is intended to mean an infinitesimally small spherical surface. Likewise, a line is a cylindrical surface of infinitesimal diameter and a plane is actually two parallel planes separated by an infinitesimal distance. The ends of the lines and the edges of the planes are part of the closed surface but are of negligible area.

Having defined M and S for general boundary conditions, we can now find a general reciprocity parameter J, where

$$J=M/S=(E_0/\bar{p}_m)/(\bar{p}_s/I_1). \qquad (1)$$

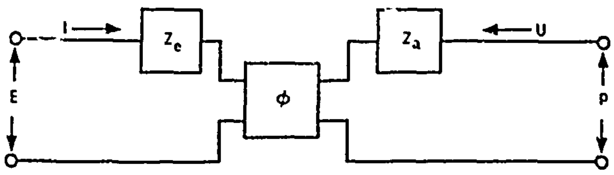

FIG. 2. Network representation of the electroacoustic system; I is current, E is voltage, p is the pressure at A_s, U is the volume velocity from A_s, Z_e is the electrical impedance when $U=0$, Z_a is the acoustic impedance when $I=0$, ϕ is the electroacoustic transfer impedance.

Foldy and Primakoff[10] have proved that the reciprocity theorem applies for the general case of a linear, passive, reversible electroacoustic transducer immersed in a bounded or unbounded medium where the medium itself is reciprocal. Therefore, a system as shown in Fig. 1 can be depicted by a network as shown in Fig. 2 and described by the two equations

$$\bar{p}=Z_aU+\phi I, \tag{2}$$

$$E=\phi' U+Z_eI, \tag{3}$$

where $\bar{p}$ is the average presusre over A_s and U is the net volume velocity emanating from A_s, or $\int_{A_s}u dA_s$, where u is the linear velocity normal to the surface. The coefficients ϕ and ϕ' are the electroacoustic transfer impedances of the system and $\phi=\phi'$ because the system is reciprocal. A restriction should be noted at this point. The transducer T cannot have both electrostatic and electromagnetic couplings. Foldy and Primakoff included this restriction in their proof, and others[11,12] have demonstrated that a combination system is not reciprocal.

The coefficient Z_e in Eq. 3 is the electrical input impedance of the transducer when $U=0$ or when the system is driven only electrically.

From the definition, U has a positive value when a source is located inside A_s and sound emanates outward, and it has a negative value when a net velocity flows inward to an energy sink. The volume velocity U_s is zero when sound merely passes through A_s. The coefficient Z_a is the acoustic impedance at A_s when $I=0$, or when the system is driven only acoustically by a source at A_s and the electrical terminals of transducer T are open. If A_s is visualized as an imaginary transducer, Z_a is the acoustic radiation impedance of A_s.

If we drive the system electrically and $U=0$, Eq. 2 becomes

$$\bar{p}_s=\phi I_i. \tag{4}$$

If we drive it acoustically with an imaginary source at A_s and open-circuit the electrical terminals, $I=0$ and Eq. 3 becomes

$$E_0=\phi U, \tag{5}$$

and

$$\phi=\bar{p}_s/I_i=E_0/U. \tag{6}$$

Multiplying the right-hand member of Eq. 6 by $(\bar{p}_m/\bar{p}_m)$ produces

$$\bar{p}_s/I_i=(E_0/\bar{p}_m)(\bar{p}_m/U). \tag{7}$$

From Eqs. 1 and 7,

$$M/S=U/\bar{p}_m=J. \tag{8}$$

Equation 8 becomes the definition of the general reciprocity parameter. It is the ratio of the volume velocity emanating from a surface A_s (defined by S) to the resulting pressure $\bar{p}_m$ at A_m (defined by M). Thus, it is an acoustic transfer admittance, but in a reverse sense. That is, although associated with the transducer T, the parameter J describes a transfer function of a wave starting at A_s and traveling to T. If $\bar{p}_m$ is defined as a pressure in the absence of T, then J can have an alternate meaning. If U' is the velocity emanating from an imaginary source at A_m, and $\bar{p}'$ is the resulting pressure at A_s, then, according to the acoustical reciprocity theorem,

$$U'/\bar{p}'=U/\bar{p}_m=J. \tag{9}$$

Thus, J can also be given by the transfer admittance $U'/\bar{p}'$. As we shall see, this alternate definition of J can be useful.

II. DERIVATION OF SPECIAL RECIPROCITY PARAMETERS

Given Eq. 8, the derivation of all the special parameters becomes relatively straightforward. Although Wathen-Dunn[9] has used a similar approach for the spherical-wave case, it is reexamined here to justify the use of point transducers. The author[5] has already used the general parameter concept in deriving the cylindrical-wave parameter, and it is not repeated here.

A. Spherical Waves

The case of the conventional spherical-wave parameter J_{sph} is shown schematically in Fig. 3. The trans-

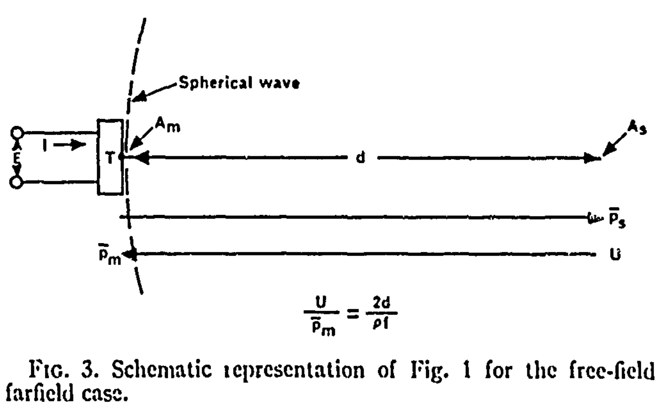

FIG. 3. Schematic representation of Fig. 1 for the free-field farfield case.

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[10] H. Primakoff and L. L. Foldy, "A General Theory of Passive, Linear, Electroacoustic Transducers and the Electroacoustic Reciprocity Theorem II," J. Acoust. Soc. Am. 19, 50–58 (1947).

[11] E. M. McMillan, "Violation of the Reciprocity Theorem in Linear Passive Electromechanical Systems," J. Acoust. Soc. Am. 18, 344–347 (1946).

[12] R. J. Bobber and C. L. Darner, "A Linear Passive Nonreciprocal Transducer," J. Acoust. Soc. Am. 26, 98(L) (1954).

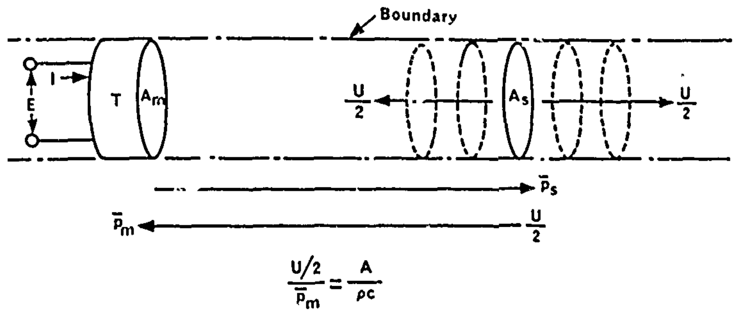


FIG. 4. Schematic representation of Fig. 1 for the plane-wave case.

mitted pressure $\bar{p}_s$ is defined as the pressure at a point in the far field. Therefore, U in Eq. 8 is the velocity emanating from a point. The pressure $\bar{p}_m$ is the free-field, plane-wave pressure in the absence of the transducer. True plane waves are, of course, impossible to achieve in a free field. What is actually done is to use long distances between the source and receiver so that the radius of curvature of the spherical wave is large and the receiver intercepts a spherical-wave segment that is indistinguishable from a plane wave. Thus, $\bar{p}_m$ becomes the pressure at a point in a spherical wave of large radius and is no longer an average pressure. The parameter J_{sph} is then the ratio of the volume velocity emanating from a point in a free field to the pressure produced at a second distant point. This ratio is well-known as[13,14]

$$U/\bar{p}_m = 4\pi d/\rho ck = 2d/\rho f = J_{sph}, \qquad (10)$$

where d is the distance between points, ρ is the density of the medium, c is the speed of sound, k is the wave-number, and f is the frequency. In some derivations of J_{sph} in the literature, a point transducer is substituted for the real transducer and the conclusion is drawn that $J_{sph} = 2d/\rho f$ is valid only for small transducers. Here, we see that substituting a point transducer for T means only that we use the alternate concept of J as given by Eq. 9 and the substitution in no way limits the shape or size of T. A point is used because $\bar{p}_s$ is defined as the pressure at a point.

B. Plane Waves

The plane-wave or one-dimensional case is shown schematically in Fig. 4. Here, $\bar{p}_s$ is averaged over a plane A equal in area to the diaphragm of the transducer and parallel to the diaphragm. Boundary conditions are chosen so that only plane waves are propagated. The pressure and volume velocity amplitude will then be the same at all points and the ratio, or acoustic-wave impedance, will be $\rho c/A$. However, the volume velocity in the wave moving to the left in Fig. 4 contains only half of the total velocity U emanating from the imaginary source at A_s. Now, $\bar{p}_m$ is the pressure in the plane of the transducer and in the absence of the transducer. Thus,

$$\bar{p}_m/\tfrac{1}{2}U = \rho c/A, \qquad (11)$$

or

$$J_{pl} = U/\bar{p}_m = 2A/\rho c. \qquad (12)$$

For the ideal case, p_m and p_s would be uniform over A. In practice, however, J_{pl} may be used in nearfield measurements of large piston transducers, and average values $\bar{p}_m$ and $\bar{p}_s$ are actually measured.

C. Coupler

Coupler reciprocity systems have very small chambers. The chambers are so small, in fact, and the boundaries are of such high impedance, that it is valid to assume that the instantaneous pressure in the chamber is everywhere the same, so that we have a zero-dimensional case. A precise definition of where $\bar{p}_s$ and $\bar{p}_m$ are measured becomes unnecessary. The pressure resulting from any volume velocity is found from the acoustical compliance of the system—that is, from the parallel combination of the medium and boundaries, including other transducers in the chamber. Thus, $U/\bar{p}_m = \omega C$. In most cases, the controlling compliance is the compliance C of the medium. The reciprocity parameter is then

$$M/S = J_{cpl} = \omega C. \qquad (13)$$

The compliance C must be measured or computed for the chamber with or without the transducers present, depending on how M and S are defined. The reciprocal transducer is, or is not, present, depending on whether $\bar{p}_m$ is the applied diaphragm pressure or the pressure in the absence of T. The two other transducers of the usual reciprocity trio—that is, the sound source and the hydrophone—are, or are not, present, depending on whether $\bar{p}_s$ is defined as the chamber pressure in, or not in, the presence of the two transducers.

D. Diffuse Sound

In Diestel's diffuse-sound reciprocity method, the transducers are placed in those parts of a reverberant chamber where a diffuse sound field exists—that is, where the time average of the mean-square sound pressure is everywhere the same and the flow of energy in

[13] H. F. Olson, *Acoustical Engineering* (D. Van Nostrand Co., Inc., Princeton, N. J., 1957), Sec. 2.2.
[14] L. E. Kinsler and A. R. Frey, *Fundamentals of Acoustics* (John Wiley & Sons, Inc., New York, 1962), Sec. 7.7.

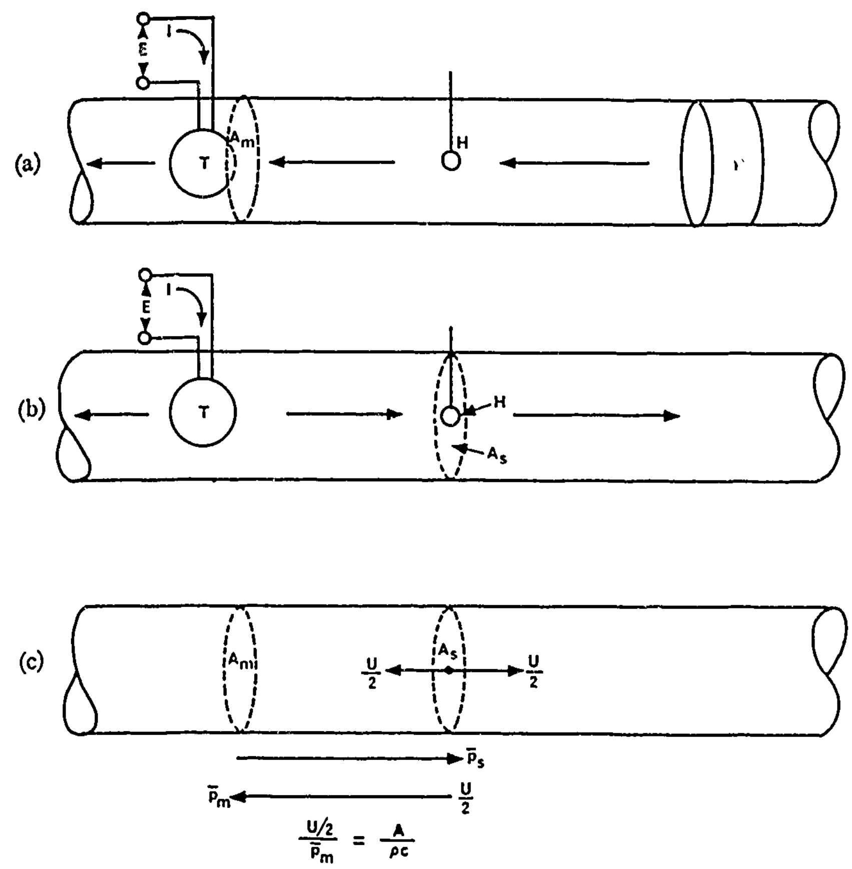


FIG. 5. Beatty's tube-reciprocity arrangement (a) when the reciprocal transducer T is receiving and (b) when T is transmitting; (c) is a schematic representation of Fig. 1 for the Beatty tube reciprocity case.

all directions is equally probable. Note that with this definition the diffuse field will not exist near a transducer unless the transducer is small, has high acoustic impedance, and is passive. The pressures $\bar{p}_s$ and $\bar{p}_m$ are then diffuse sound pressure at any point in the diffuse sound field. The diffuse-field reciprocity parameter J_{df} is then the diffuse sound pressure $\bar{p}_m$ produced by a point source of volume velocity U. The ratio $U/\bar{p}_m$ is found from two known relationships. The total power W radiated by a point source is[13,14]

$$W=(\rho ck^2/4\pi)U^2, \tag{14}$$

where the wavenumber $k=\omega/c=2\pi/\lambda$, and U is a root-mean-square value. The diffuse sound pressure p_{df} in a reverberant chamber is given as[15]

$$\bar{p}_m{}^2=\bar{p}_{df}{}^2=4W\rho c/K, \tag{15}$$

where K is the chamber absorption constant. Thus, from Eqs. 14 and 15,

$$J_{df}=U/\bar{p}_m=(K/4\pi)^{\frac{1}{2}}/\rho f. \tag{16}$$

The constant K can be expressed as a function of the chamber volume V, the reverberation time T, and the speed of sound c. In MKS units,[15,16]

$$K=55.3(V/cT). \tag{17}$$

Substituting Eq. 17 in Eq. 16, gives Diestel's expression:

$$J_{df}=(2.1/\rho f)(V/cT)^{\frac{1}{2}}. \tag{18}$$

E. Tube

In Beatty's tube reciprocity, the transducer diaphragms are not of the same shape and area as the cross section of the tube. Both radial and longitudinal motions are present. The boundary conditions are, therefore, different from and more complex than those for the plane-wave case. It is a three-dimensional case, although the diameter of the tube is restricted to about $\lambda/3$.

A simplified picture of tube-reciprocity conditions is given here. The reader is referred to Beatty's paper for more detail. The conditions are shown in Figs. 5(a) and (b). The usual trio of transducers is represented by the source projector P, the reciprocal transducer T, and the hydrophone H. The arrows indicate the directions of sound-energy flow in plane progressive waves. The radial modes are present only near the transducers. Active electroacoustic absorbers terminate the tube and effectively place the terminal boundary an infinite distance away—to the left in Fig. 5(a) and in both directions in Fig. 5(b).

The receiving sensitivity of T is defined in terms of the plane-wave pressure in the plane A_m in the absence of, or without the effect of, the presence of T.

The total real pressure at any position in the tube is the sum of the longitudinal plane-wave mode and the

[15] L. L. Beranek, *Acoustics* (McGraw-Hill Book Co., Inc., New York, 1954), Secs. 10.11, and 10.13.
[16] R. W. Young, "Sabine Reverberation Equation and Sound Power Calculations," J. Acoust. Soc. Am. 31, 912-921 (1959).

radial modes that are present only near the transducers. Since the plane-wave mode suffers no attenuation, the position of A_m is arbitrary.

The transmitting response of T is defined in terms of the pressure at the plane A_s of the plane progressive waves traveling toward the right in Fig. 5(b). As with A_m, the position of plane A_s is arbitrary. The hydrophone H is assumed so small that it does not disturb the plane waves.

From the general reciprocity-parameter concept, the reciprocity parameter becomes the transfer admittance $U/\bar{p}_m$ depicted in Fig. 5(c); U is the volume velocity emanating from A_s, and $\bar{p}_m$ is the plane-wave pressure at A_m. Figure 5(c) represents conditions that do not differ from those for the plane-wave case. The tube-reciprocity parameter, therefore, is $2A/\rho c$. We see, then, that the chocie of the plane-wave reference pressures at A_m and A_s eliminates the complications introduced by the radial modes.

Beatty also describes a second (less useful) tube-reciprocity parameter that is applicable when the boundary conditions for T transmitting differ from those for T receiving. The direct approach using the general reciprocity parameter cannot be made under such conditions.

III. MEASURED AND *IN SITU* PARAMETERS

It follows from the general approach to reciprocity parameters that a reciprocity calibration can be performed under any boundary conditions. The initial assumptions of a linear, passive, reversible system must prevail, but these conditions are the rule rather than the exception. The usual problem is not whether a system is reciprocal, but how to determine the reciprocity parameter. In some cases of complicated or unknown boundary conditions, it is impossible to calculate the parameter. An alternative, then, is to measure J. In some special cases, the volume velocity being emitted by a transducer can be determined. The pressure produced at any place in the medium can be measured with a calibrated hydrophone. Thus, the ratio $J=U/\bar{p}_m$ can be measured.

A second method is to conduct a reciprocity calibration of a hydrophone whose calibration is already known and then solve for the unknown parameter J. That is, the rôles of M as the unknown and J as a known quantity can be reversed in the familiar equation

$$M=[(E_1E_2/E_3I)\,J]^{\frac{1}{2}} \qquad (19)$$

for determining the sensitivity M of a hydrophone from reciprocity-calibration measurements. The need for a calibrated hydrophone in both methods might seem to eliminate the need for, or usefulness of, a reciprocity calibration. That is, if a calibrated hydrophone is already available, why use a reciprocity calibration? One answer is that sometimes the boundary conditions are more stable over long periods of time (months, or even years) than is a hydrophone calibration. This is true of some ocean-bottom locations, for example. In such cases, an *in situ* reciprocity parameter can be measured with the aid of a calibrated hydrophone, and thereafter a reciprocity calibration can be made independently of the stability of the hydrophone calibration. A system exploiting the *in situ* reciprocity technique for calibration on the ocean bottom has been built, but not used.[17]

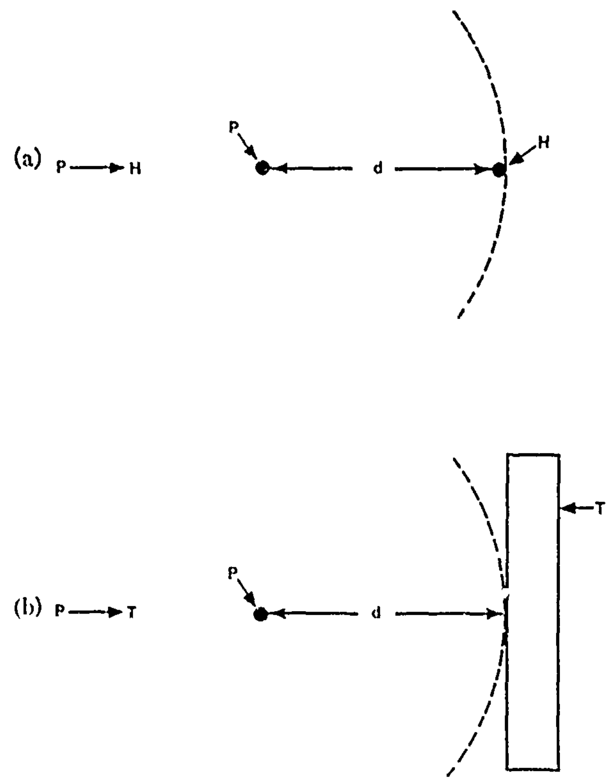
IV. NEARFIELD CASE

The dependence of the reciprocity parameter on the definitions of A_m and A_s in the receiving and transmitting responses of the reciprocal transducer, and the usefulness of the general parameter concept can be further illustrated by considering a nearfield–free-field case. Suppose that the projector P and hydrophone H are points but the reciprocal transducer T is large. Suppose, further, that the distance d used is of such a magnitude that in the P → H measurement P and H are in the far fields of each other, but in the P → T and T → H measurements each is in the near field of T. The situation is depicted in Fig. 6. Now, define the transmitting response S_T of the reciprocal transducer so that the reference transmitted pressure $\bar{p}_s$ is the pressure at the position point of H in Fig. 6(c). Also, define the receiving sensitivity M_T of the reciprocal transducer in terms of a free-field pressure $\bar{p}_m$ in a spherical wave of radius d being emitted from P. Thus, both S_T and M_T have very special definitions and are dependent on the distance d. From the definitions, both A_m and A_s are points in a free field; therefore J is the same as $J_{sph}=2d/\rho f$. Thus, H can be calibrated in the near field of T using the farfield parameter J_{sph}! This is a surprising conclusion, but a valid one. It can be corroborated by considering how P, T, and H are affected in a conventional trio of farfield calibration measurements when d becomes small. When d is very large, the sensitivity of H is given by

$$M_H=[(E_{TH}E_{PH}/E_{PT}I_T)J_{sph}]^{\frac{1}{2}}, \qquad (20)$$

where E is the voltage output of the receiving transducer, I is the current into T, and the subscripts indicate the transducers used in each of the measurements. Now, if the farfield distance d is reduced so that P and H are in the far field of each other but the near field of T, then E_{PH} and J_{sph} would be affected according to the inverse-square law, but the product $E_{PH}J_{sph}$ would be constant. The voltages E_{TH} and E_{PT} would be affected according to some nearfield function of d, the frequency, and the dimensions of T. Since both P and H are points, and both transducer pairs (T–H and P–T) constitute reciprocal systems, the function, whatever it is, will affect E_{TH} and E_{PT} equally and the ratio E_{TH}/E_{PT} will be unaffected. Thus, Eq. 20 produces the same M_H in this nearfield arrangement that it does in the far field.

[17] M. Weinstein, Underwater Systems, Inc., Silver Spring, Md., private communication (1965).



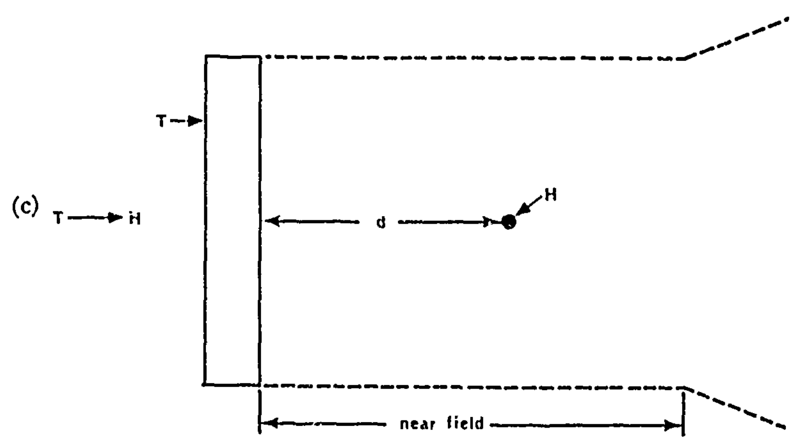


FIG. 6. Three reciprocity-calibration measurements when, because of the large size of T, the distance d is too small to meet farfield criteria.

pressure over the area of T produced by a point source at a distance d." Now, A_s is still a point, but A_m is the area of T. In some simple cases, as when T is a uniform circular piston and A_s is a point on the axis, it is still easy to find J from the well-known formula for the axial pressure p as a function of source velocity u:

$$1/J_x = p/u = (2\rho c/\pi a^2)\{\sin\tfrac{1}{2}k[(a^2+d^2)^{\frac{1}{2}}-d]\}, \qquad (21)$$

where a is the radius of the piston and k is the wavenumber $2\pi/\lambda=\omega/c$.

Equation 21 also is valid in the reciprocal sense—that is, when the axial point is the source and the piston is the receiver. When the reciprocity-calibration formula for M_H is derived for these conditions and definitions, we get

$$M_H = [\alpha(E_{TH}E_{PH}/E_{PT}I_T)J_x]^{\frac{1}{2}}. \qquad (22)$$

The new factor α is defined by $\bar{p}=\alpha p$, where $\bar{p}$ is the average pressure over the area of T (or A_m) in Fig. 6(b), and p is the pressure in the spherical wave of radius d. The difference between $\bar{p}$ and p corresponds to the two different definitions of M_T in this nearfiled case. It also corresponds to the difference in the reference sound pressure acting on T and on H in Figs. 6(a) and (b), and this is why it appears in Eq. 22. From the definition of α, it is evident that α is equal to the ratio of Eq. 21 to the reciprocal of Eq. 10. Thus,

$$\alpha = (1/J_x)/(1/J_{sph}), \qquad (23)$$

or

$$\alpha J_x = J_{sph}. \qquad (24)$$

The relationship in Eq. 24 could have been obtained merely by arguing that M_H must be independent of the calibration technique so that Eqs. 20 and 22 must be identical. The mathematical argument, however, shows that by choosing A_m as the area over T, we needlessely complicate the reciprocity calibration. That

In any practical use of this nearfield reciprocity technique, it must be remembered that the pressure amplitude in the near field of a large transducer varies rapidly with position. Thus, little latitude is allowed in the positions or sizes of P and H.

V. CASE WHEN $A_s \neq A_m$

In all the special cases described, A_s and A_m are equal. That is, both are points, both are planes of equal area, and so forth. Insofar as the definition of $J=M/S$ is concerned, there is no restriction that A_s and A_m be the same; however, if they are not the same, the usefulness of J in a reciprocity calibration diminishes.

To illustrate, consider again the nearfield case in Fig. 6. Instead of the previous definition of the receiving sensitivity M_T of the large reciprocal transducer T, we choose "M_T is the ratio of output voltage to the average is, the right-hand member of Eq. 21 appears in both J_x and α but drops out of the product αJ_x. Therefore, rather than calculating J_x and α separately, and then having the product αJ_x reduce to J_{sph} it is simpler to define M_T and S_T such that J_{sph} applies to begin with. This is what was done in the first nearfield case. In any practical case of a reciprocity-calibration technique, all the reference pressures should be defined for equal areas and A_s should be identical in shape and size to A_m.

VI. SUMMARY

The reciprocity parameter in any reciprocity calibration is the acoustic transfer admittance $U/\bar{p}_m$, where U is the volume velocity emanating from the surface A_s, $\bar{p}_m$ is the average pressure produced by U over the surface A_m, A_s is the surface over which the pressure is averaged in the definition of the transmitting response, and A_m is the surface over which the pressure is averaged in the definition of the receiving sensitivity.

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13. ABSTRACT

The reciprocity parameter used in the reciprocity calibration of electroacoustic transducers is derived for the general case of arbitrary boundary conditions and arbitrary definitions of responses. A general reciprocity parameter is defined as the ratio of the volume velocity emanating from a surface defined in the transmitting response to the resulting pressure averaged over a surface defined in the receiving response. It is shown how the general parameter concept leads directly and simply to the various special parameters for the cases of spherical waves, cylindrical waves, plane waves, diffuse sound, couplers, and tubes. Application of the concept to in situ and nearfield conditions is also described.

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