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A GENERALIZATION OF TANG, STATZ AND deMARS THEORY OF MULTIMODE OSCILLATIONS IN A SOLID-STATE LASER

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S U M M A R Y

The Tang, Statz and deMars theory of multimode oscillations of a solid-state laser, under stationary conditions, has been extended to treat the cases of cavities with lossy end mirrors or with frequency-dependent losses.

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A GENERALIZATION OF TANG, STATZ AND deMARS THEORY OF MULTIMODE OSCILLATIONS IN A SOLID-STATE LASER

1 - Introduction.

The theory of multimode oscillations of a solid-state laser in a Fabry-Perot cavity has been developed by Tang, Statz and deMars in two basic papers [1,2]. Other work is reported in Refs. 3 to 6.

In Ref.1 only longitudinal modes are considered, which implies dependence of the field quantities on a single coordinate, say z, while transverse modes are taken into account in Ref.2. Here we will refer only to a part of the theory developed in Ref. 1, namely to the stationary case.

In Ref.1, the assumption is made that each mode consists of a plane wave of constant amplitude, bounching back and forth between the two end mirrors. Such an assumption implies that the modes do not suffer substantial losses localized at the end mirrors, but only absorption losses, or, in general, losses uniformly distributed along the cavity. In many practical cases, however, losses at the end mirrors may be substantial. In many Q-switch cavities, for example, only one mirror has high reflection coefficient. Accordingly, any longitudinal mode should be described by two plane waves with amplitudes which are not constant but increase exponentially in the direction of propagation, so that the net gain per transit of each wave compensates for the losses at the end mirrors.

The generalization to the case of lossy end mirrors will be worked out in sec.3.

Another assumption made in Ref.1 is that the losses of the cavity are the same for all longitudinal modes. Such an assumption is quite well acceptable when the resonator is of the Fabry-Perot type completely filled with active material. But it is no longer acceptable in other cases, for example for the resonators designed for mode-selection purposes.

The case of losses dependent on frequency will be treated in sec.4.

In sec.2, we will report, summarizes, the theory of Tang, Statz and deMars, in a little more general form than in Ref.1, suitable for the successive applications. The results obtained in Ref. 1 are rederived, with some minor corrections.

2 - Summary of the previous theory

The starting point of the Tang, Statz and deMars theory is represented by the two following rate equations, relating the spatially varying difference in population $n(z,t)$ between the upper and the lower maser states, with the number of photons $N_m(t)$ in the m-th mode, and the spatially varying energy density $P_m(z,t)$ which induces the stimulated emission in the m-th mode:

$$(2.1) \qquad \frac{\partial}{\partial t} n(z,t) = - \frac{1}{\tau} \left[n(z,t) - \bar{n} \right] - \sum_m^{(j)} D g_m n(z,t) \, P_m(z,t)$$

$$(2.2) \qquad \frac{\partial}{\partial t} N_m(t) = - \gamma_m N_m(t) + D g_m \int_0^L n(z,t) \, P_m(z,t) \, dz \, .$$

Here $\bar{n}$ denotes the steady-state value of the inversion of population in the absence of stimulated emission; τ the spontaneous radiation time; g_m a constant which takes into account the shape of the emission line of the material; Dg_m a constant relating the stimulated emission to $n(z,t)$ and $P_m(z,t)$; γ_m a time constant which takes into account the losses of the cavity in the m-th mode. The summation appearing in (2.1) is intended to be made over all the oscillating modes, j in number, while the integral appearing in (2.2) in to be made over the length of the active material, extending from $z = 0$ to $z = L$. Finally, the units are so chosen that the following relation holds:

$$(2.3) \qquad \int_0^L P_m(z,t)\, dz = L\, N_m(t)$$

Under stationary conditions ($\partial/\partial t = 0$, $n(z,t) = n(z)$, etc.) and at not too high values of the pump power, one derives from (2.1):

$$(2.4) \qquad n(z) = \bar{n}/\left[1 + \tau D \sum_m^{(j)} g_m P_m(z)\right] \cong \bar{n} \left[1 - \tau D \sum_m^{(j)} g_m P_m(z)\right]$$

while (2.2) may be written, on account of (2.4), as

$$(2.5) \qquad G_m^{(j)}\, N_m = 0$$

where

$$(2.6) \qquad G_m^{(j)} = - \gamma_m + \bar{n} D g_m \int_0^L \left[1 - \tau D \sum_s^{(j)} g_s N_s p_s(z)\right] p_m(z)\, dz$$

and $p_m(z)$ is defined by

(2.7) $$P_m(z) = N_m\, p_m(z)$$

By putting

(2.8) $$\int_0^L p_s(z)\, p_m(z)\, dz = L\, F_{sm}$$

and recalling (2.3), Eq.(2.6) becomes

(2.9) $$G_m^{(j)} = -\gamma_m + \bar{n} D g_m L \left[1 - \tau D \sum_s^{(j)} g_s F_{ms} N_s\right]$$

For the oscillating modes, $N_m \neq 0$ and (2.5) yields:

(2.10) $$G_m^{(j)} = 0 \qquad \text{(for the oscillating modes)}$$

For the non-oscillating modes, $N_m = 0$, Eq.(2.5) does not give information about $G_m^{(j)}$, but it may be shown that

(2.11) $$G_m^{(j)} < 0 \qquad \text{(for the non-oscillating modes).}$$

From the preceding equations one can derive the value α_j of the excess pump power, normalized with respect to the pump power necessary to set into oscillation the lowest-threshold mode (the 'fundamental' mode), as a function of the number j of oscillating modes.

In general, the normalized excess pump power α is related to $\bar{n}$ by the expression

(2.12) $$\alpha = \frac{\bar{n} D L g_0}{\gamma_0} - 1$$

where index zero refers to the fundamental mode. We define as α_j the value of α for which j modes oscillate and the next one, the (j+1)-th, is just at threshold.

To derive the value of α_j, one notes that, for $\alpha = \alpha_j$, (2.10) holds not only for the oscillating modes, but also for the (j+1)-th mode which is just at threshold.

By taking into account (2.9), (2.10) reads

$$(2.13) \qquad \bar{n} D g_m L \left[1 - \tau D \sum_s^{(j)} g_s F_{ms} N_s\right] = \gamma_m$$

where both indices m and s refer to oscillating modes, and

$$(2.14) \qquad \bar{n} D g_{j+1} L \left[1 - \tau D \sum_s^{(j)} g_s F_{(j+1)s} N_s\right] = \gamma_{j+1}$$

for the (j+1)-th mode. By putting

$$(2.15) \qquad \frac{\gamma_m}{g_m} = \frac{\gamma_0}{g_0} (1+\Delta_m)$$

and recalling (2.12), Eqs.(2.13) and (2.14) become

$$(2.16) \qquad \tau D \sum_s^{(j)} g_s F_{ms} N_s = 1 - \frac{1}{1+\alpha} (1+\Delta_m)$$

and

$$(2.17) \qquad \tau D \sum_s^{(j)} g_s F_{(j+1)s} N_s = 1 - \frac{1}{1+\alpha} (1+\Delta_{j+1})$$

respectively. Clearly, (2.16) represents a system of j linear equations in the j unknown intensities N_s of the oscillating modes. By solving it, one obtains N_s as a function of α. Upon introduction of the solutions of (2.16), (2.17) becomes an equation for α, whose solution is α_j.

As already noted, in Ref.1 the following assumptions were made:

a) Each mode consists of two plane waves of constant amplitudes,

b) The quantity γ_m is independent of index m, $\gamma_m = \gamma$.

According to assumption a), one has

$$(2.18) \qquad p_m(z) = 1 - \cos(2k_m z) \quad , \quad \exp(2ik_m L) = 1$$

$$(2.19) \qquad F_{sm} = F_{ms} = 1 + \frac{1}{2}\delta_{sm}$$

where k_m denotes the wavenumber of the m-th mode, and $\delta_{sm} = 1$ for $s = m$, $\delta_{sm} = 0$ for $s \neq m$. Eq.(2.16) becomes

$$(2.20) \qquad \frac{1}{2}\tau D\, g_m N_m + \tau D \sum_s^{(j)} g_s N_s = 1 - \frac{1}{1+a}(1+\Delta_m)$$

which, solved for N_m, gives (*)

$$(2.21) \quad N_m = \frac{2}{\tau D g_m}\left[1 - \frac{1}{1+a}(1+\Delta_m) - \frac{2}{2j+1}\sum_s^{(j)}\left\{1 - \frac{1}{1+a}(1+\Delta_s)\right\}\right]$$

In view of (2.19), (2.17) reads

$$(2.22) \qquad \tau D \sum_s^{(j)} g_s N_s = 1 - \frac{1}{1+a}(1+\Delta_{j+1})$$

Introduction of (2.21) into (2.22) yields

$$(2.23) \quad \frac{2}{2j+1}\sum_s^{(j)}\left\{1 - \frac{1}{1+a}(1+\Delta_s)\right\} = 1 - \frac{1}{1+a}(1+\Delta_{j+1})$$

(*) System (2.20) is easily solved by noting that the summation over index s is independent of index m.

which, solved for α, gives

$$\alpha = \alpha_j = (2j+1)\,\Delta_{j+1} - 2\sum_s^{(j)} \Delta_s \tag{2.24}$$

This expression of α_j has a general form. In Ref.1 the particular case was considered where g_m is given by

$$g_m = \left[1 + \left(\frac{\nu_m - \nu_0}{\Delta\nu}\right)^2\right]^{-1} = \left[1 + m^2\left(\frac{\delta\nu}{\Delta\nu}\right)^2\right]^{-1} \tag{2.25}$$

where $\Delta\nu$ denotes the half-power line width of the laser material (centered on the frequency ν_0) and the mode frequencies ν_m are equispaced by $\delta\nu$. The quantity $\Delta\nu$ depends on the temperature. A typical value of $(\delta\nu/\Delta\nu)^2$ at room temperature is 10^{-4}.

On account of (2.25), and in view of assumption b), one has

$$\Delta_m = m^2\left(\frac{\delta\nu}{\Delta\nu}\right)^2 \tag{2.26}$$

Note that, for symmetry reasons, the number of oscillating modes is always odd, and that, if the m-th mode is specified by the index m, defined by $\nu_m - \nu_0 = m\,\delta\nu$, the index m varies, for the oscillating modes, from $-(j-1)/2$ to $(j-1)/2$. Eq.(2.24) then gives

$$\alpha_j = \left(\frac{\delta\nu}{\Delta\nu}\right)^2 \frac{j+1}{12}\,(4j^2 + 11\,j + 3) \tag{2.27}$$

3 - The influence of lossy end mirrors.

When the end mirrors are lossy, assumption a) of the preceding section must be given up, and replaced by the assumption that the two plane waves belonging to the m-th mode gain in traveling through the cavity an amount which compensates for the losses at the end mirrors.

With reference to Fig.1, let us denote by ρ' the (amplitude) reflection coefficient of the left-hand mirror and by ρ'' that of the right-hand mirror. Further, denote by W_m^+ (W_m^-) the complex amplitude at the left-hand (right-hand) mirror of the left-to-right (right-to-left) plane wave belonging to the m-th mode. The

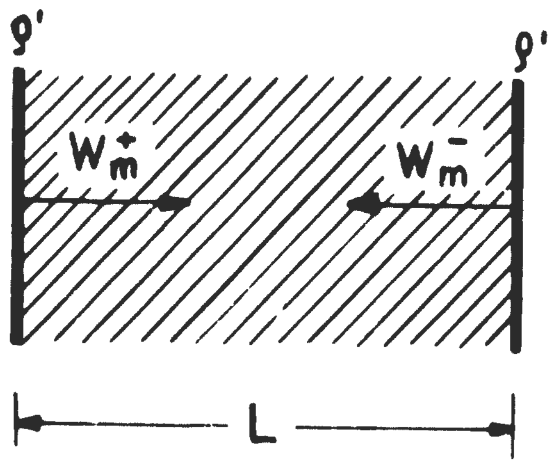

Fig.1 - A conventional Fabry-Perot resonator. The (amplitude) reflection coefficients are ρ' on the left and ρ'' on the right.

complex amplitude $W_m(z)$ of the m-th mode at a point z inside the resonator will be written as

$$(3.1)\quad W_m(z) = W_m^+ \exp(ik_m z+\beta_m z) + W_m^- \exp\left[-ik_m(z-L) - \beta_m(z-L)\right]$$

where β_m denotes the gain (in neper) per unit length. At $z = 0$ and $z = L$, one has

$$(3.2)\quad \begin{aligned} W_m^+ &= \rho' \, W_m^- \exp(ik_m L+\beta_m L) \\ W_m^- &= \rho'' \, W_m^+ \exp(ik_m L+\beta_m L) \end{aligned}$$

respectively. Hence the oscillation and resonance conditions are obtained in the form

$$(3.3) \qquad \rho'\rho'' \exp(2ik_m L + 2\beta_m L) = 1$$

Assuming ρ' and ρ'' to be real and of the same sign (*), Eq.(3.3) yields

$$(3.4) \qquad \begin{aligned} &\rho'\rho'' \exp(2\beta_m L) = 1 \\ &\exp(2ik_m L) = 1 \end{aligned}$$

From the first Eq.(3.4) one derives the obvious result that β_m has the same value for all the oscillating modes:

$$(3.5) \qquad \beta_m = \beta$$

By using the first Eq.(3.2), (3.1) may be written as

$$(3.6) \qquad W_m(z) = W_m^- \exp(ik_m L+\beta L) \left[\rho' \exp(ik_m z+\beta z) + \exp(-ik_m z-\beta z)\right]$$

while, by using the second Eq.(3.2), (3.1) may be written as

$$(3.7) \qquad W_m(z) = W_m^+ \exp(ik_m L+\beta L) \left[\exp\left\{ik_m(z-L)+\beta(z-L)\right\} + \right.$$
$$\left. + \rho' \exp\left\{-ik_m(z-L) - \beta(z-L)\right\}\right]$$

(*) This assumption does not limit the validity of the results. For, if ρ' and ρ'' are complex, the first Eq.(3.4) must be written for their moduli, while their phases, which would appear in the second Eq.(3.4), would only shift, by a constant amount equal to their sum, the mode frequencies without changing their difference.

Accordingly, one can write, by using (3.6),(3.7) and (3.4),

$$P_m(z) = W_m(z)\, W_m^*(z) = \tag{3.8}$$

$$= W_m^- W_m^{+*} \exp(ik_m L+\beta L)\left[\rho' \exp(2\beta z)+\rho'' \exp\left\{-2\beta(z-L)\right\}+2 \cos(2k_m z)\right]$$

Recalling (2.3) and (2.7), one can write

$$P_m(z) = f\left[\rho' \exp(2\beta z) + \rho'' \exp\left\{-2\beta(z-L)\right\} + 2 \cos(2k_m z)\right] \tag{3.9}$$

where

$$\frac{1}{f} = \frac{1}{2\beta L}\,(\rho'+ \rho'')(e^{2\beta L} - 1) \tag{3.10}$$

or also, in view of the first of (3.4),

$$\frac{1}{f} = \frac{1}{-\ln(\rho'\rho'')}\left[\frac{1}{\rho''} - \rho'' + \frac{1}{\rho'} - \rho'\right] \tag{3.11}$$

By using (3.8) and neglecting terms of the order of $1/k_m^2 L^2$ with respect to unity, (2.8) yields

$$F_{sm} = f^2\left\{\frac{1}{4\beta L}\,(\rho'^2+ \rho''^2)(e^{4\beta L}-1) + 2 + 2\delta_{sm}\right\} = F_{ms} \tag{3.12}$$

which may be written as

$$F_{sm} = F + 2f^2\, \delta_{sm} \tag{3.13}$$

with

$$F = f^2\left[\frac{1}{-\ln(r' r'')}\left(\frac{1}{r''} - r'' + \frac{1}{r'} - r'\right) + 2\right] \tag{3.14}$$

where now r' and r'' denote the power reflection coefficients of the left-hand mirror and of the right-hand mirror respectively ($r' = \rho'^2$, $r'' = \rho''^2$).

Substitution of (3.13) into (2.16) gives

$$(3.15) \qquad 2\tau Df^2 g_m N_m + \tau D F \sum_s^{(j)} g_s N_s = 1 - \frac{1}{1+\alpha}(1+\Delta_m)$$

Hence (see the footnote on p.6),

$$(3.16) \qquad N_m = \frac{1}{2\tau Df^2 g_m}\left[1 - \frac{1}{1+\alpha}(1+\Delta_m) - \frac{F}{2f^2+jF}\sum_s^{(j)}\left\{1 - \frac{1}{1+\alpha}(1+\Delta_s)\right\}\right]$$

Then, (2.17) reads

$$(3.17) \qquad \frac{F}{2f^2+jF}\sum_s^{(j)}\left[1 - \frac{1}{1+\alpha}(1+\Delta_s)\right] = 1 - \frac{1}{1+\alpha}(1+\Delta_{j+1})$$

which, solved for α, gives

$$(3.18) \qquad \alpha = \alpha_j = \left[1 + j\frac{F}{2f^2}\right]\Delta_{j+1} - \frac{F}{2f^2}\sum_s^{(j)}\Delta_s$$

The 'reflection-loss function' $F/2f^2$ turns out to depend on r' and r''. However, by inspection of a plot of the curves $F/2f^2 =$ = constant in the plane r', r'' (Fig.2), it appears that the reflection-loss function virtually depends only on r' or r'', whichever has the smaller value. Accordingly, one can approximate the function $F/2f^2$ by the function $F_1(r)$, shown in Fig.3, where r denotes the smaller one of the reflection coefficients of the two mirrors.

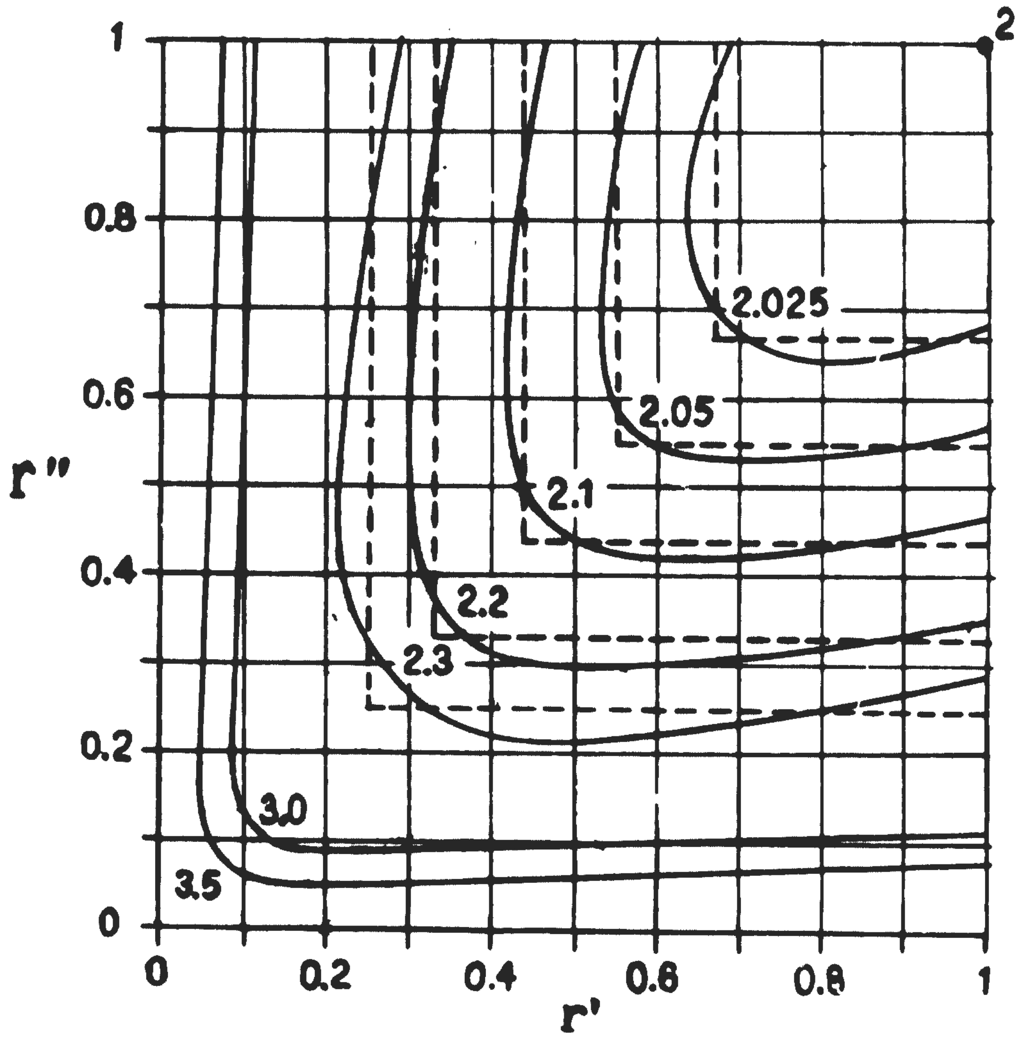

Fig.2 - Plot of the 'reflection-loss function' in the plane r',r''. Here r' (r'') denotes the power reflection coefficient of the left-hand (right-hand) mirror. Solid lines represent curves $F/2f^2$ = constant, dashed lines represent curves F_1=constant.

Let us now consider the case when g_m is given by (2.25) and therefore, on account of (2.15), Δ_m is given by (2.26). Eq.(3.18) becomes

$$(3.19) \qquad \alpha_j = \left(\frac{\delta\nu}{\Delta\nu}\right)^2 \frac{j+1}{2}\left[\frac{j+1}{2} + \frac{j}{3}(j+2)\,\frac{F}{2f^2}\right]$$

(recall that the (j+1)-th mode is specified by the value $m=(j+1)/2$, as already noted). When β tends to zero, $F/2f^2$ tends to 2 and (3.19) tends to (2.27).

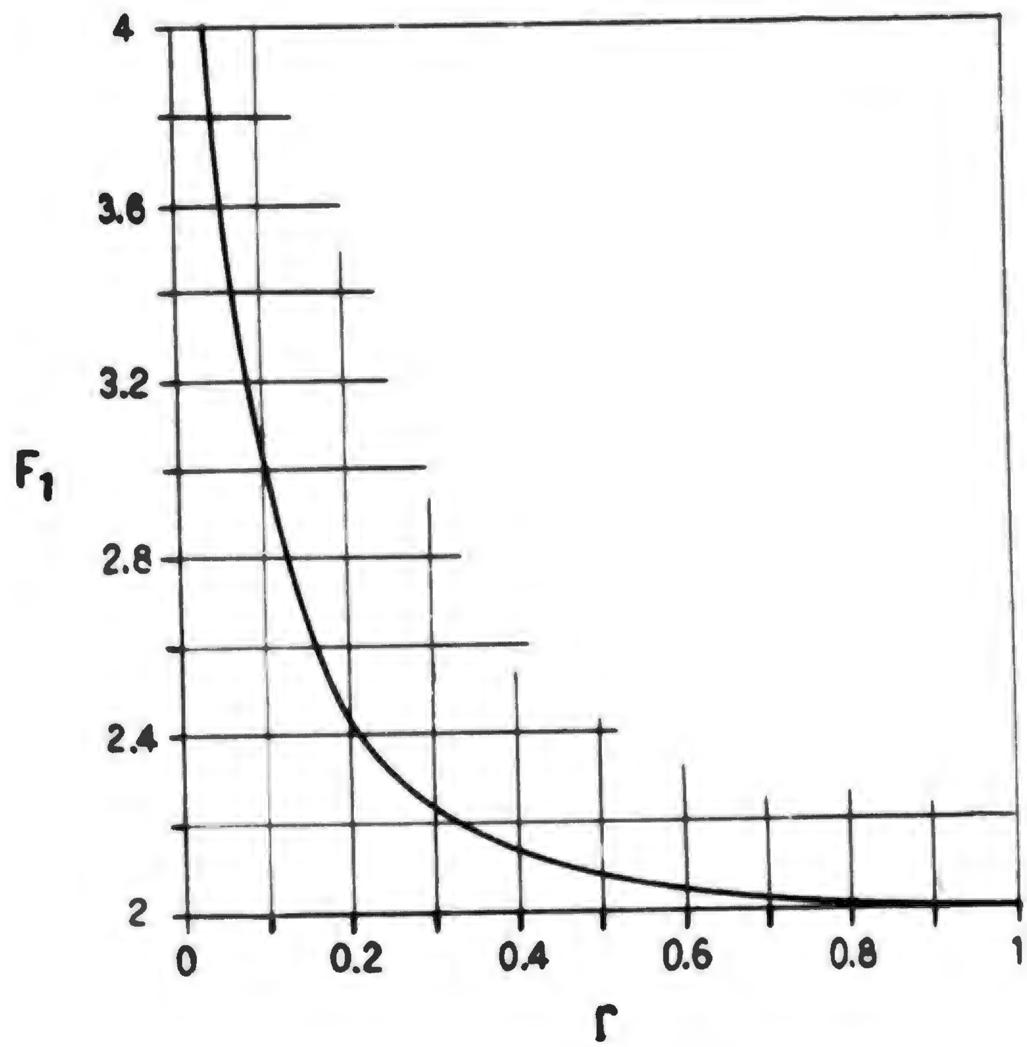

Fig.3 - Plot of the approximated reflection-loss function F_1, versus the smaller one of the two reflection coefficients.

Fig.4 shows a_j versus j for a few values of the parameters. In particular, the case $F/2\ell^2 = 3.3$ is considered, which corresponds to r = 0.07, a value obtaining for ruby, with (at least) one end face left uncoated.

It appears from Fig.4, or from a discussion of (3.19), that, when j modes are oscillating at the same time, the increase of normalized excess power necessary to set into oscillation the (j+1)-th mode has a higher value in the presence of losses than in the absence of losses.

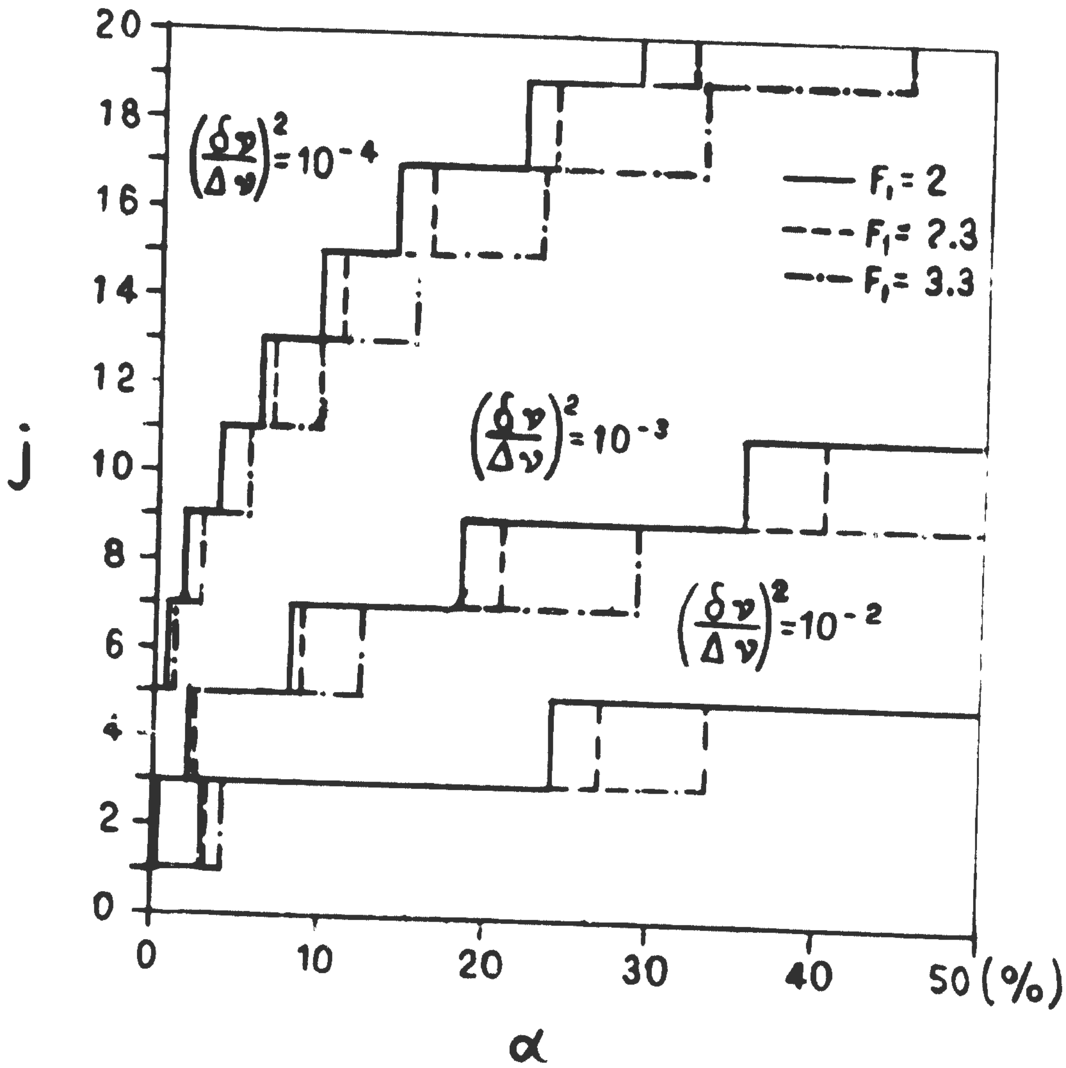

Fig.4 - Number of oscillating modes for various values of the normalized excess pump power α, in a conventional cavity. The temperature dependence occurs through the quantity $\Delta\nu$, and loss dependence through F_1 ($F_1=2$: high reflectivity at both end mirrors; $F_1 = 3.3$: one or both end faces left uncoated).

Let us now substitute (3.18) into (3.16). We easily obtain

(3.20)
$$N_m = \frac{1}{2\tau D f^2 g_m} \frac{1}{1+\alpha_j} (\Delta_{j+1} - \Delta_m)$$

and therefore

(3.21)
$$\frac{N_m}{N_o} = \frac{g_o}{g_m} \frac{\Delta_{j+1}-\Delta_m}{\Delta_{j+1}}$$

Thus, the intensity of the m-th mode at the value of the pump power for which the (j+1)-th mode is just at threshold, is nearly proportional to the difference $\Delta_{j+1}-\Delta_m$. The situation is sketched in Fig.5, where the dashed line represents Δ_m plotted versus m. When the mode labelled (j+1) breaks into oscillation, to find the relative intensities of different modes one has to draw the line P_{j+1}, P'_{j+1}; then, the intensity of the m-th mode is proportional to the length $A_m B_m$.

If (2.25) and (2.26) hold, (3.21) may be written as

$$\frac{N_m}{N_o} = \left[1 + m^2\left(\frac{\delta\nu}{\Delta\nu}\right)^2\right]\left[1 - \left(\frac{2m}{j+1}\right)^2\right] \tag{3.22}$$

Note that losses do not appear in (3.22). Hence, the relative intensities of the j oscillating modes do not depend on the loss coefficients. What depends on the loss coefficients is the value of the pump power necessary to set into oscillation the fundamental mode, as well as the normalized excess pump power necessary to set into oscillation the j modes.

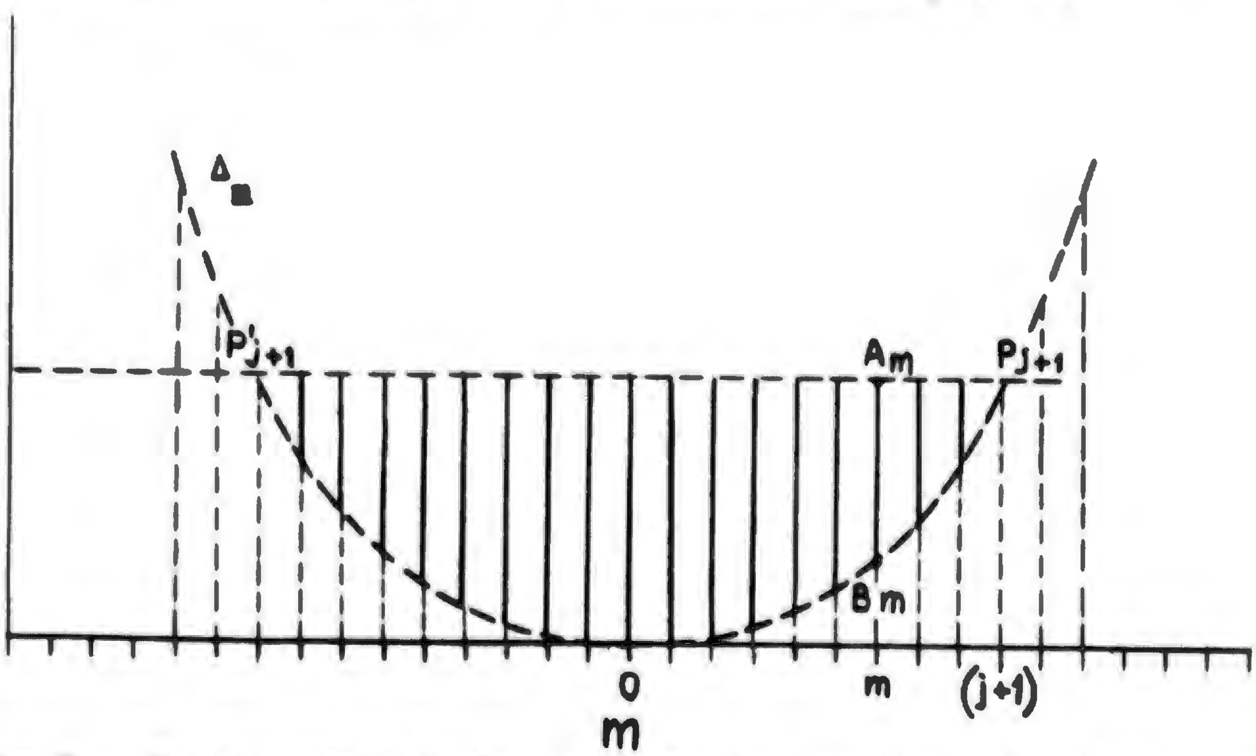

Fig.5 - Sketch illustrating how to derive the relative intensities of the j oscillating modes, for $\alpha=\alpha_j$, from the plot of Δ_m versus m.

4 - Frequency-dependent reflection losses

Assume now the reflection coefficient of one mirror, say ρ', to be independent of the frequency (without loss of generality, ρ' may be assumed to be real) and the other one, that is ρ'', to be of the form (*)

$$\rho'' = \rho''_m \exp(i\varphi_m) \tag{4.1}$$

where index m refers to the m-th mode, and ρ''_m is assumed to be real with the same sign as ρ'. On account of (4.1),(3.4) must be replaced by

$$\begin{aligned} &\rho' \rho''_m \exp(2\beta_m L) = 1 \\ &\exp(2ik_m L + i\,\varphi_m) = 1 \end{aligned} \tag{4.2}$$

According to the first Eq.(4.2), it turns out that (3.5) ceases to hold. Again, (3.8) must be replaced by

$$P_m(z) = W^-_m W^{+*}_m \exp(ik_m L+\beta_m L)\Big[\rho' \exp(2\beta_m z) + \tag{4.3}$$
$$+\rho''_m \exp\{-2\beta_m(z-L)\} + 2\cos(2k_m z)\Big]$$

(*) When a reflection coefficient depends on the frequency, in general its phase is not the same at all frequencies, and therefore it must be written in the form (4.1). This is the case, for example, for the Kleinman and Kisliuk arrangement (7). However, the influence of the phase φ_m regards only the resonance frequencies and their spacing, but not the reflection-loss function.

Hence one derives

$$(4.4)\quad p_m(z) = f_m\left\{\rho'\exp(2\beta_m z) + \rho''_m \exp\left[-2\beta_m(z-L)\right] + 2\cos(2k_m z)\right\}$$

where

$$(4.5)\quad \frac{1}{f_m} = \frac{1}{2\beta_m L}(\rho'+\rho'_m)\left[\exp(2\beta_m L) - 1\right] - \frac{1}{k_m L}\sin\varphi_m$$

$$\simeq \frac{1}{2\beta_m L}(\rho'+\rho''_m)\left[\exp(2\beta_m L) - 1\right]$$

On account of the first of (4.2), one can write

$$(4.6)\quad \frac{1}{f_m} \simeq \frac{1}{-\ln(\rho'\rho'_m)}\left[\frac{1}{\rho''_m} - \rho''_m + \frac{1}{\rho'} - \rho'\right]$$

The quantity F_{sm}, defined by (2.8), is now given by

$$(4.7)\quad F_{sm} = F'_{sm} + 2 f_s f_m \delta_{sm}$$

where

$$(4.8)\quad F'_{sm} = F'_{ms} = \frac{f_s f_m}{-\ln(\rho'^2\rho''_s\rho''_m)}\left[\frac{1}{\rho''_s\rho''_m} - \rho''_s\rho''_m + \frac{1}{\rho'^2} - \rho'^2\right] +$$

$$+ \frac{f_s f_m}{\ln(\rho''_s/\rho''_m)}\,\frac{1}{\rho''_m\rho''_s}\,(\rho''^2_s - \rho''^2_m)$$

(terms of the order of $1/k_m L$ are here neglected). Substitution of (4.8) into (2.16) yields:

(4.9) $\quad 2\tau D f_m^2\, g_m N_m + \tau D \sum_s^{(j)} g_s F'_{ms} N_s = 1 - \frac{1}{1+\alpha}(1+\Delta_m)$

It must be noted now that a substantial difference distinguishes the case here treated from the case of the preceding section. In the case examined in sec.3, Δ_m was a monotonically increasing function of $|\nu_m - \nu_o|$, so that, for j oscillating modes, the frequencies were included in the interval $\pm \ell\delta\nu$ around ν_o with $\ell = (j-1)/2$. Here, due to the dependence of γ on the frequency, Δ_m may cease to be a monotonic function of $|\nu_m - \nu_o|$, so that, in order to solve (3.18), one has first to determine which are the j oscillating modes and the (j+1)-th mode.

It seems reasonable to assume, at least if coupling between oscillating and non-oscillating modes ([8]) is neglected, that the sequence in which the modes break into oscillation is determined by the threshold associated with each mode, if it could oscillate alone. Now, the m-th mode is at its threshold for single-mode oscillation when the steady-state value $\bar{n}$ of the inversion of population satisfies the relation

(4.10) $$\frac{\bar{n} D L g_m}{\gamma_m} = 1$$

If $\bar{n}_m$ denotes the value of $\bar{n}$ satisfying (4.10),(4.10) says that γ_m/g_m is proportional to $\bar{n}_m$, or also, recalling (2.15), that $1+\Delta_m$ is proportional to $\bar{n}_m$. The sequence in which the modes enter into oscillation is therefore determined by Δ_m and may be derived from the plot of Δ_m versus frequency. The larger is Δ_m, the larger is the value of $\bar{n}_m$. Accordingly, the j oscillating modes are those associated with the j lowest values of Δ_m.

There follows that, for the oscillating modes, and at not too high values of the pump power, f_m and F'_{sm} do not differ substantially from one another. Consider for example, the function F'_{sm}. The reflection coefficients ρ''_s and ρ''_m on which it depends, may differ substantially from one another, depending on the structure of the resonator: in the Kleinman and Kisliuk arrangement, ρ' may vanish at some frequencies and approach unity at other frequencies. However, low values of ρ'' correspond to high values of γ, according to the relation (*)

$$\gamma = \frac{c}{\mu L} (1 - |\rho'\rho''|) \tag{4.11}$$

where μ denotes the real part of the refractive index of the active material. Assume, for the sake of simplicity, the central frequency ν_0 of the resonance line to be associated with the highest value ρ''_0 of ρ'' (not necessarily close to 1). Modes with ρ'_m much lower than ρ'_0 are associated with large values of Δ_m, while modes with ρ'_m equal to, or not too much lower than ρ'_0, are associated with small values of Δ_m. If the normalized excess pump power is not too high, the oscillating modes correspond to value of Δ, and therefore of γ, close to one another, which implies $\rho'_s \simeq \rho''_m$, and therefore, $f_m \simeq$ constant $= f$, $F'_{sm} \simeq$ constant $= F'$. Accordingly, (4.9) can be rewritten as

$$2\tau D f^2 g_m N_m + \tau D F' \sum_s^{(j)} g_s N_s = 1 - \frac{1}{1+\alpha} (1+\Delta_m) \tag{4.12}$$

(*) To verify this expression, one can make use, e.g., of Eq.(3.13) of Ref.9.

Eq.(4.12) is of the same form as (3.15). Its solution may be therefore written in the form (3.16). Eqs.(3.17), (3.19), (3.20) and (3.21) still hold. Further, the plot of Δ_m versus ν_m (or m) yields at once the relative intensity of the oscillating modes for the pump power value at which a prescribed mode, the (j+1)-th, is just at threshold.

As an example, let us refer to the resonator proposed by Kleinman and Kisliuk, consisting of a passive Fabry-Perot resonator, in series with the laser cavity (Fig.6). If the reflection coefficient ρ (assumed real) of the second mirror is equal to that of the third mirror, and L_1 denotes the length of the second cavity, one has, at frequency ν_m,

$$(4.13) \qquad \rho' \simeq \rho \frac{1 + \exp(2ik_m^{o}L_1)}{1 + \rho^2 \exp(2ik_m^{o}L_1)}$$

where k_m^o/μ denotes the free space wavenumber of frequency ν_m. Hence,

$$(4.14) \qquad \rho'_m \simeq 2\rho \frac{\left|\cos(k_m^o L_1)\right|}{\sqrt{(1+\rho^2)^2 - \left[2\rho \sin(k_m^o L_1)\right]^2}}$$

$$(4.15) \qquad \varphi_m \simeq k_m^o L_1$$

Accordingly, recalling (4.12) and (2.15), one has

$$(4.16) \qquad \Delta_m = m^2\left(\frac{\delta\nu}{\Delta\nu}\right)^2 + \left[1+m^2\left(\frac{\delta\nu}{\Delta\nu}\right)^2\right]\frac{\rho'\rho''_o}{1-\rho'\rho''_o}\left[1 - \frac{\left|\cos(k_m^o L_1)\right|}{\sqrt{1-\rho''^{\,2}_o \sin^2(k_m^o L_1)}}\right]$$

if g_m is still given by (2.25). Clearly, in evaluating the mode separation $\delta\nu$, (4.15) must be taken into account, together with the second Eq.(4.2).

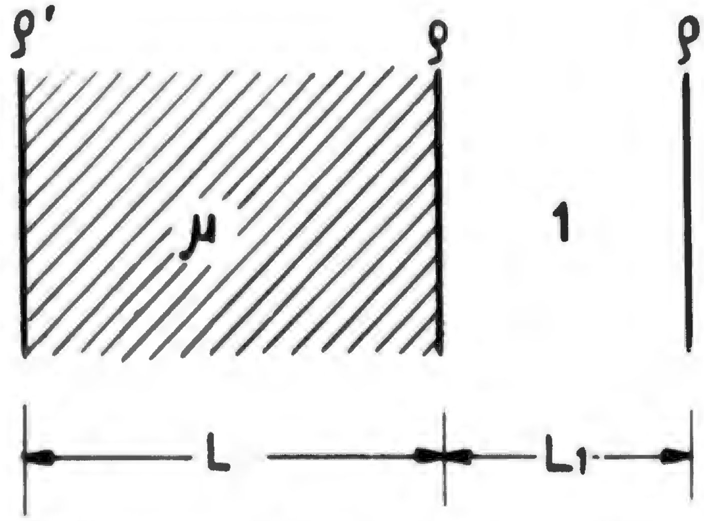

Fig.6 - A Kleinman and Kisliuk resonator.

Fig.7 shows Δ_m plotted versus m for the following values of the parameters: $(\delta\nu/\Delta\nu)^2 = 10^{-3}$, $L_1 = (2\mu/11)L$, $\rho_0'' = \rho' = 0.98$. From this plot one derives that the sequence in which the first few modes break into oscillation is: i = 0, $\pm$ 1, $\pm$ 12, $\pm$ 11, $\pm$ 2. Then, with the help of (3.18) one derives the graph of Fig.8 (solid line), where j is plotted versus α_j for the same values of the parameters.

In order to evaluate the mode properties of the Kleinman and Kisliuk arrangement, let us compare the device sketched in Fig.6 with a conventional cavity with $\rho' = \rho'' = 0.98$ and with the same length L of active material. If $L_1 = (2\mu/11)L$, and $(\delta\nu/\Delta\nu)^2 = 10^{-3}$ for the Kleinman and Kisliuk device, one easily derives, for the conventional cavity, $(\delta\nu/\Delta\nu)^2 = 1.2\ 10^{-3}$.

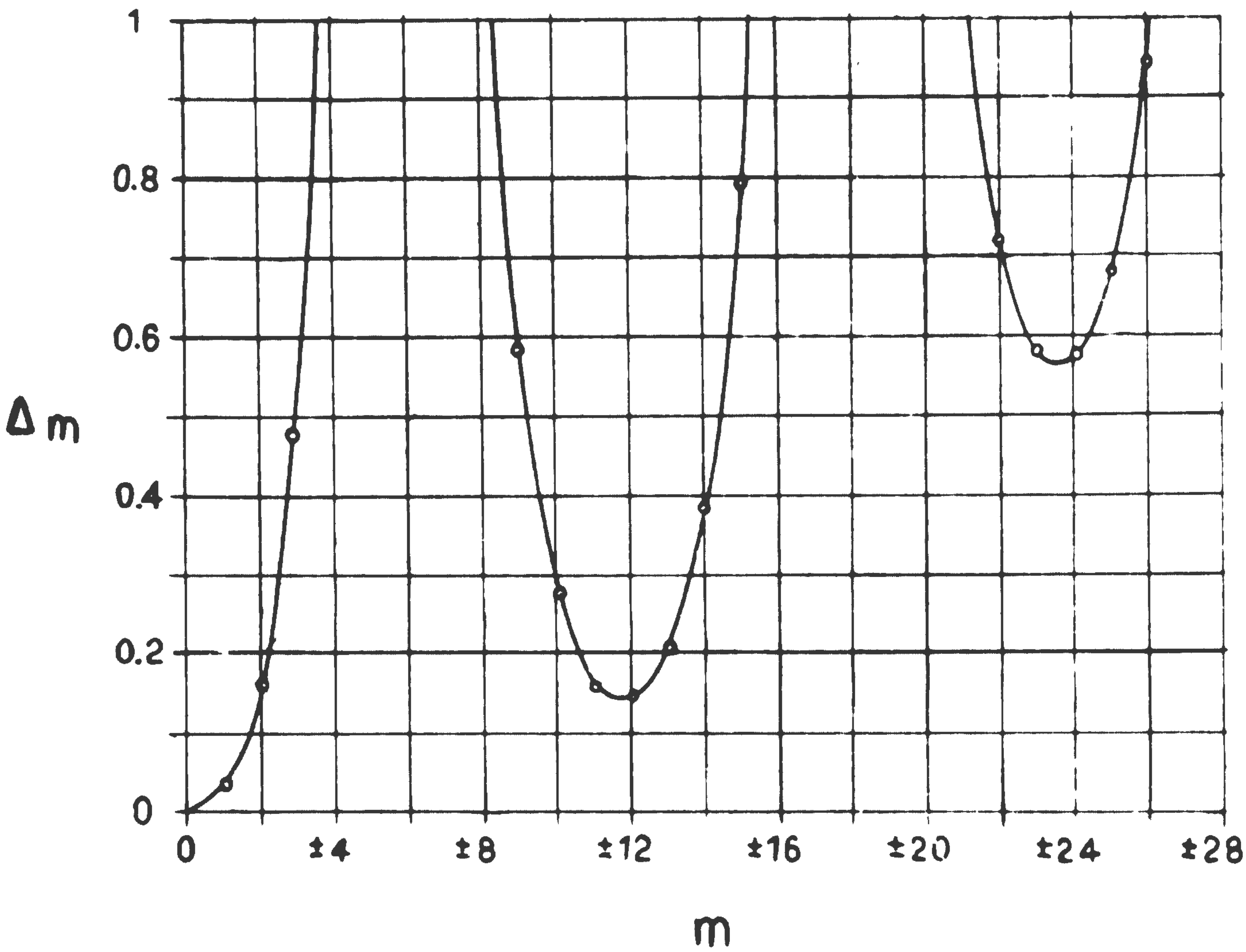

Fig.7 - A plot of Δ_m versus m, for a Kleinman and Kisliuk resonator for the following values of the parameters: $(\delta\nu/\Delta\nu)^2 = 10^{-3}$, $L_1 = 2\mu L/11$, $\rho' = \rho''_o = 0.98$ $(\rho = 0.82)$.

Then, with the help of (2.27) one derives the dashed line of Fig.8, which has to be compared with the solid line of the same figure. Fig.9 shows N_m/N_o in the two cases, with $\alpha_j \simeq$ 110 %.

5 - Conclusion.

The Tang, Statz and deMars theory of multimode oscillations of a solid-state laser, under stationary conditions,

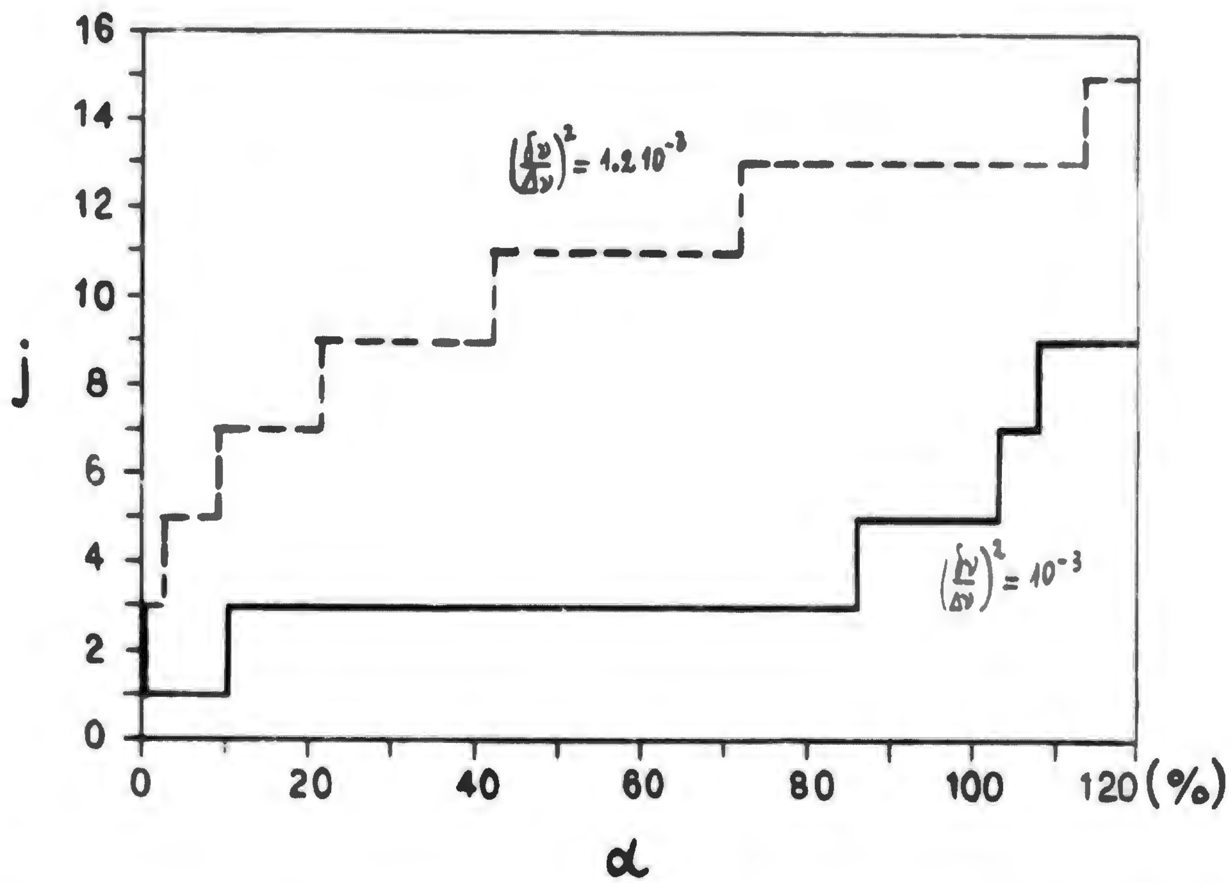

Fig.8 - Number of oscillating modes for variuos values of normalized excess pump power α, for a Kleinman and Kisliuk resonator (solid line) and for a conventional resonator (dotted line) with equal length of active material.

has been extended to treat the cases of cavities with lossy end mirrors (sec.3) or with frequency-dependent losses (sec.4).

In the case of lossy end mirrors, it has been found that the results mostly depend on the smaller one of the reflection coefficients of end mirrors. Substantial loss at (at least) one end mirror changes the value of the pump power necessary to set into oscillation the lowest-threshold mode, but also the value α_j of the normalized excess pump power necessary to get into oscillation j modes. The relative intensity of the oscillating modes turns out not to depend on the losses.

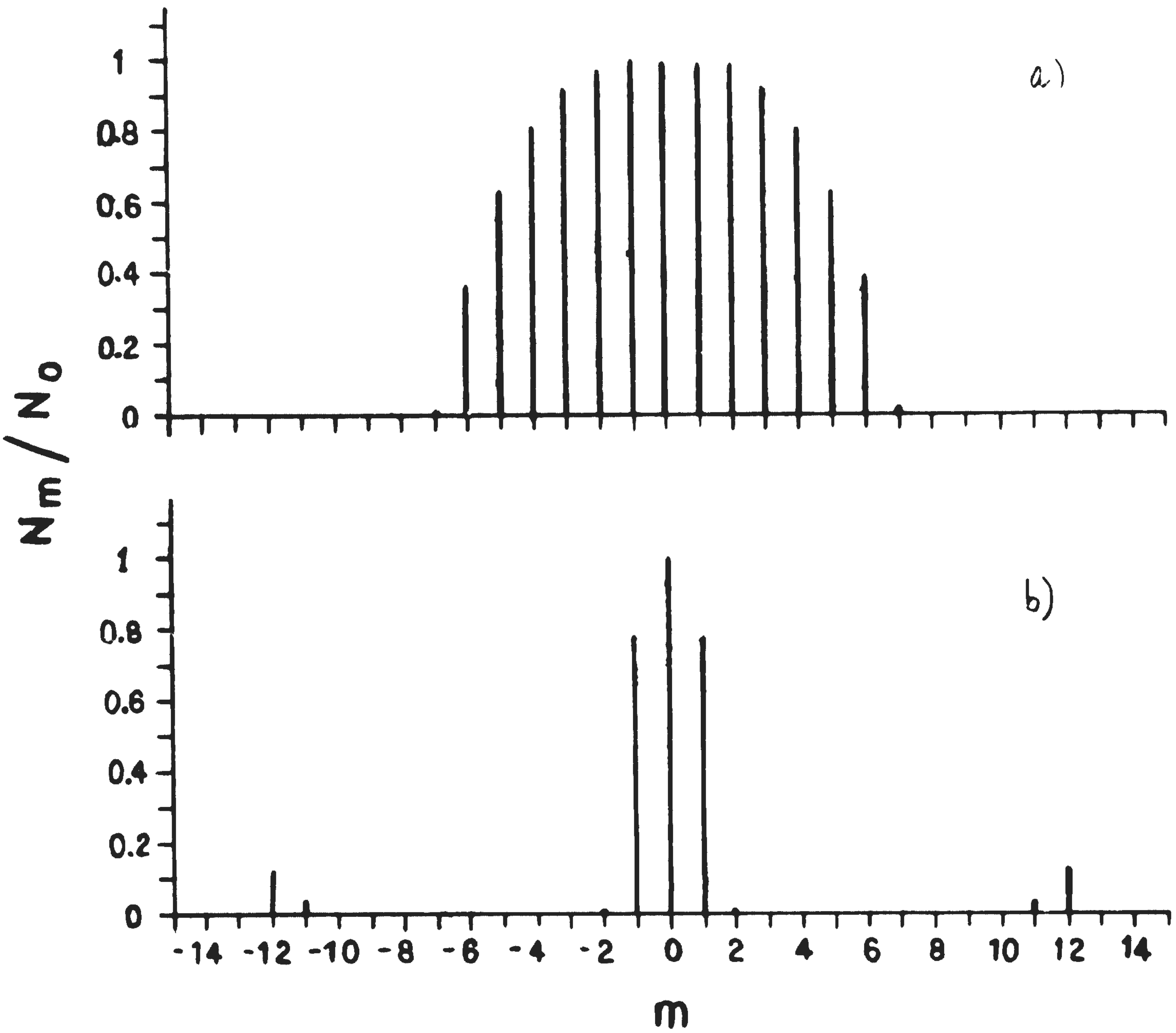

Fig.9 - Relative intensities of oscillating modes, for $a_j \simeq 110$ %, a) in a conventional cavity, b) in a Kleinman and Kisliuk cavity.

As a general result, it has been found that the function Δ_m defined by (2.15) plays an important role. A plot of Δ_m versus frequency allows one to determine the sequence in which the modes break into oscillation, and the relative intensities of the oscillating modes. The normalized excess pump power for which j modes oscillate while the next one is just at threshold, is expressed in terms of Δ_m by Eq.(3.18).

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13 ABSTRACT

The Tang, Statz and deMars theory of multimode oscillations of a solid-state laser, under stationary conditions, has been extended to treat the cases of cavities with lossy end mirrors or with frequency-dependent losses.

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