AD635938

TECHNICAL REPORT 503-1

HIGHER ORDER WAVE THEORY FOR SLENDER SHIPS

By

B. Yim

February 1966

DDC
RECEIVED
AUG 2 1966
C

HYDRONAUTICS, incorporated
research in hydrodynamics

Research, consulting, and advanced engineering in the fields of NAVAL and INDUSTRIAL HYDRODYNAMICS. Offices and Laboratory in the Washington D. C. area Pindell School Road Howard County, Laurel Md

HYDRONAUTICS, Incorporated

TECHNICAL REPORT 503-1

HIGHER ORDER WAVE THEORY FOR SLENDER SHIPS

By

B. Yim

February 1966

The research reported herein was carried out under the Bureau of Ships General Hydrodynamics Research Program, administered by David Taylor Model Basin.

Prepared Under

Office of Naval Research
Department of the Navy
Contract No. Nonr-4677(00)

Reproduction in whole or in part is permitted for any purpose of the United States Government.

TABLE OF CONTENTS

	Page
SUMMARY	1
INTRODUCTION	2
SISOV'S HIGHER ORDER WAVE THEORY	5
COORDINATES DEFORMATION AND FORMULATION OF PROBLEM	9
SOLUTION FOR EACH ORDER POTENTIAL	12
SLENDER SHIPS WITH ALMOST FLAT BOTTOM	16
THE LINE INTEGRAL ON THE FREE SURFACE	18
WAVE RESISTANCE	20
ORDER ANALYSIS AND DEPENDENCE ON k_o	26
SINGULARITIES AT THE BOW AND STERN	31
CONCLUDING REMARKS	33
REFERENCES	35

NOTATION

$f(x,z)$	Function representing the ship surface
$F(x,y)$	Function representing the free surface
g	Acceleration of gravity
g_j	Defined in [7], [9], and [11]
G	Green's function
H	Draft of the ship
k_o	$= g/V^2$
L	Length of the ship
ℓ	Intersection of the plane $\zeta = 0$ and $\eta = f(\xi,\zeta)$
$\bar{n}$	Normal vector on the surface into the flow
p	Pressure
q_j	Defined in [8], [10], and [12]
R	Wave resistance
S_F	Free surface
S_s	Ship surface
V	Uniform velocity at infinity
$O\text{-}x,y,z$	Right handed rectangular cartesian coordinates as shown in Figure 1
ω	$\equiv (\xi - x_1) \cos\theta + (\eta - y_1) \sin\theta$
μ	Fictitious friction force
ϵ	Small parameter representing the beam length ratio of a ship
ξ,η,ζ	Transformed coordinate system

Subscript

j	represents coefficients of ϵ^j in the expansion of the attached quantity

Superscript

*	represents the corresponding quantity in the transformed space

SUMMARY

The free surface condition of the surface wave is considered on the free surface itself instead of the mean free surface, by the use of a coordinate transformation, together with the scheme of a systematic expansion in a small parameter. Thus a higher order ship wave theory is developed. The most common practical case of a slender ship with an almost flat bottom is especially treated in detail. The lowest order result is the same as that given by Michell's theory. The next higher order potential and wave resistance are derived explicitly.

INTRODUCTION

Since Kelvin (1887) found the solution for linear water surface waves and Michell (1898) formulated the wave resistance due to a thin surface ship advancing in an inviscid fluid, many experimental and theoretical works on this subject have been performed by various ship hydrodynamists. Th. waves and the wave resistance of many ships have been calcul..ted and compared with experiments. The theory of minimum wave resistance has been developed and waves from the bow, the stern and the shoulder of a ship have been analyzed.

These theories are based on solutions of the Laplace equation with linearized boundary conditions on both the free surface and the ship surface. The theories are mathematically elegant and certainly very significant in the development of the ship wave theory. However, the general agreement with experiment has been relatively very poor (especially for practical ships), as compared with other linear theories such as have been developed in aerodynamics and for cavity flows.

To improve this situation the development of higher order wave theory has been suggested in terms of a systematic expansion in small parameters (Stoker, 1957). Sisov (1961) actually formulated the second order wave resistance for a thin ship where the beam-length ratio is considered to be a fundamental small parameter as in Michell's theory. To improve the comparison between wave theory and experiment, the streamline tracing technique was recommended (Inui, 1957), and also a slender ship

theory has been developed (Vossers, 1962; Tuck, 1963; Maruo, 1962; Joosen, 1964). Wehausen (1964) has also considered an exact formulation of the ship wave problem and showed that for a thin ship with small draft, the most important higher order effect can be represented by a line integral along the intersection of the ship surface and the water surface.

Although no proof has been made for the convergence of the expansion of the potential in power series of a small parameter as in Sisov or Wehausen's work, this expansion scheme is common in applied mathematics. (Lighthill, 1954, Wehausen, 1960, Van Dyke, 1965). The usual expansion problems are made more difficult in the case of ship waves because of the unknown position of the free surface boundary where it is necessary to apply the free surface boundary condition. In addition, the free surface is internally bounded by the ship surface where the usual rigid boundary condition must be applied.

To resolve the difficulty connected with the unknown free surface position, here we first transform the coordinates such that $\zeta = 0$ always represents the free surface. Then, we apply the scheme of an expansion in small parameters to all of the physical quantities; in this development, first the beam-length ratio was assumed small and then, in addition, the draft-length ratio was assumed small (i.e., the ship is thin and then slender). After substituting the relevant series in our governing equation and the boundary conditions, we equate all the same order terms in each equation. Then the governing equation which is the Laplace equation in the physical space becomes, in the transformed

space, Laplace equations in the lower orders, and Poisson's equations in the higher orders. In each order, the problem is a well defined linear boundary value problem whose solution is available by the use of Green's theorem, with one well known Green's function which is equivalent to a potential due to a source under the linearized free surface. The higher order solutions depend upon the lower order solutions. Therefore, we first solve the lowest order problem, and use this solution for the next higher order problem, and so on. Thus, the potential and the wave resistance for both thin ships and slender ships are formulated explicitly here in both the first and the next higher order.

The lowest order solution is the same as Michell's solution whether the ship is thin or slender, as Wehausen showed. The next higher order solution is relatively simple especially for the case of fast slender ships. This is due to three effects: the effect from the line integral (earlier mentioned) which is due to the free surface bounded internally by the ship; the effect from the change of submergence due to the wave; and the bottom effect. The second order solutions for the cases of thin ships, and of slow and medium speed slender ships (ordinary merchant ships) are a little more complicated since the nonlinear effect of free surface needs to be included. Singularities at the bow and stern, especially at the intersection with the free surface are carefully analyzed. For simplicity, no trim nor sinkage is considered, although these effects can be added without too much difficulty.

SISOV'S HIGHER ORDER WAVE THEORY

We consider the right-handed rectangular coordinate $O\text{-}x,y,z$ or $O\text{-}x_1,y_1,z_1$ and the surface ship $y = f(x,z)$ as in Figure 1, in a homogeneous, inviscid, irrotational, and infinitely deep fluid with a free surface. The flow at infintiy is considered to be uniform with the velocity U, and there exists a perturbation velocity potential φ, which satisfies

$$\nabla^2\varphi = 0 \qquad [1]$$

and the proper boundary conditions on the free surface and the ship surface, with $\nabla\varphi = 0$ and $\varphi = 0$ at infinity. Then by Green's formula

$$\varphi = \frac{1}{4\pi}\iint_S \Big[\varphi(x_1,y_1,z_1)\, G_n(x_1,y_1,z_1;x,y,z) - \varphi_n(x_1,y_1,z_1)\, G(x_1,y_1,z_1;x,y,z)\Big]\, dS \qquad [2]$$

where the subscript n indicates a partial derivative in the direction of the inner normal into the fluid at the whole boundary $S(x_1,y_1,z_1)$ including the free surface S_F and the ship surface S_S in Figure 1. G is the Green's function, which is harmonic everywhere in the fluid except at a point $(x,y,z) = (x_1,y_1,z_1)$ where it has a singularity like

$$\left.\begin{aligned} & 1 \Big/ \left\{ (x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 \right\}^{\frac{1}{2}} \\ \text{and} \qquad & \nabla G = 0 \quad , \quad \text{and} \quad G = 0 \quad \text{at infinity in } z < 0 \end{aligned}\right\} \qquad [3]$$

The boundary conditions for φ on the free surface $z = F(x,y)$ are: the kinematic condition

$$(V-\varphi_x)\, F_x - \varphi_y F_y + \varphi_z = 0 \qquad [4a]$$

and the dynamic condition

$$F(x,y) - \frac{V}{g}\varphi_x + \frac{1}{2g}\left(\varphi_x^2 + \varphi_y^2 + \varphi_z^2\right) = 0 \qquad [4b]$$

The boundary condition on the ship surface $y = f(x,z)$ is the kinematic condition

$$\left.\begin{aligned} & (V-\varphi_x) f_x + \varphi_y - \varphi_z f_z = 0 \\ \text{or} \qquad & \varphi_n - \vec{n} \cdot \vec{V} = 0 \end{aligned}\right\} \qquad [5]$$

For convenience we do not consider here any trim or sinkage of the ship. We consider a thin ship symmetric with respect to the $y = 0$ plane, with small slopes to the $y = 0$ plane, and a small value of the half beam-length ratio, ϵ; and assume the series,

$$\varphi(x,y,z,\epsilon) = \epsilon\varphi_1(x,y,z) + \epsilon^2\varphi_2(x,y,z) + \cdots$$

$$F(x,y,\epsilon) = \epsilon F_1(x,y) + \epsilon^2 F_2(x,y) + \cdots$$

$$f(x,z) = \epsilon f_1(x,z) \qquad [6]$$

By substitution of [6] in [4a], [4b], and [5], and using Taylor's expansion, we obtain (see e.g. Sisov, 1961) comparing the terms of ϵ^j,

$$k_o\varphi_{jz} + \varphi_{jxx} = p_j(x,y) \qquad \text{on } S_{PF} \qquad [7]$$

$$(k_o = g/V^2)$$

for the free surface condition

$$\varphi_{jy} = q_j(x,z)$$

$$\varphi_{jy} \equiv 0 \qquad \text{on } \left\{y = 0;\ \text{outside of } S_{pys} \text{ in the fluid}\right\} \qquad [8]$$

for the ship surface condition, where S_{PF} and S_{pys} indicate the projection of S_F and S_s on $z = 0$ and $y = 0$ planes respectively,

$$p_1(x,y) \equiv 0 \qquad [9]$$

$$q_1(x,z) = -Vf_{1x} \qquad [10]$$

$$p_2(x,y) = \frac{1}{V}\left(\varphi_{1x}^2 + \varphi_{1y}^2 + \frac{1}{2}\varphi_{1z}^2\right)_x - \frac{V}{g}\varphi_{1x}\left(k_o\varphi_{1z} + \varphi_{1xx}\right)_z \qquad [11]$$

$$q_2(x,z) = \left(\varphi_{1x} f_1\right)_x + \left(\varphi_{1z} f_1\right)_z \qquad [12]$$

In general, $p_j(x,y)$ and $q_j(x,z)$ for $j \geq 2$ are represented in terms of known functions including the obtained lower order solutions $\{\varphi_i \; ; \; i < j\}$.

Sisov (1961) obtained two Green's functions for the free surface singularities on $z = 0$ and for the ship surface singularities on $y = 0$ respectively, both satisfying the first order free surface condition [7] and [9] except at the singularities. Thus, if we consider that the boundary conditions everywhere on the $z = 0$ plane and the $y = 0$, $z \leq 0$ half plane are given as in [7] and [8], respectively, the combination of the two Green's functions will lead us to the solution of Sisov (1961).

The problem, however, is that the free surface is only outside of the ship boundary and the actual submergence of the ship depends upon the wave height. Thus, the boundary conditions on S_{PF} and S_{pys} are not necessarily those given everywhere on the $z = 0$ plane and on the half $y = 0$ ($z \leq 0$) plane. In addition, in practice, our ships are not thin and the slope to the x,z plane is not everywhere small.

To overcome these difficulties we will try another formulation of the same problem with a different approach.

COORDINATES DEFORMATION AND FORMULATION OF PROBLEM

We consider a transformation of the (x,y,z) coordinates to the (ξ,η,ζ) coordinates by

$$\left.\begin{aligned} x &= \xi \\ y &= \eta \\ z &= \zeta + F(x,y) \end{aligned}\right\} \qquad [13]$$

Thus, $\zeta = 0$ represents the free surface $z = F(x,y)$. Of course, at the beginning, we do not know the wave height $F(x,y)$. Our governing equations and the boundary conditions [1], [4] and [5] can be represented in terms of the ξ,η,ζ coordinates through substitution of

$$\left.\begin{aligned} &\varphi(x,y,z) = \varphi\{\xi,\eta,\zeta + F(x,y)\} = \varphi^*(\xi,\eta,\zeta) \\ &\varphi_x = \varphi^*_\xi - \varphi^*_\xi - \varphi^*_\zeta F_x \\ &\varphi_{xx} = \varphi^*_{\xi\xi} - 2\varphi^*_{\xi\zeta}F_x - \varphi^*_\zeta F_{xx} + \varphi^*_{\zeta\zeta}F_x^2 \\ &\varphi_y = \varphi^*_\eta - \varphi^*_\zeta F_y \\ &\varphi_{yy} = \varphi^*_{\eta\eta} - 2\varphi^*_{\eta\zeta}F_y - \varphi^*_\zeta F_{yy} + \varphi^*_{\zeta\zeta}F_y^2 \\ &\varphi_z = \varphi^*_\zeta \\ &\varphi_{zz} = \varphi^*_{\zeta\zeta} \end{aligned}\right\} \qquad [14]$$

in [1], [4], and [5]. Now if we use the series [6] in the transformed space

$$\left.\begin{aligned} &\Delta\varphi_j^* = d_j(\xi,\eta,\zeta) \\ &k_o\varphi_{j\zeta}^* + \varphi_{j\xi\xi}^* = p_j^*(\xi,\eta) \qquad \text{on} \qquad S_F^* \\ &\varphi_{j\eta}^* = q_j^*(\xi,\zeta) \qquad \text{on} \qquad S_s^* \end{aligned}\right\} \qquad [15]$$

where S_F^* and S_s^* indicate the corresponding surfaces of S_F and S_s in the transformed space.

$$\left.\begin{aligned} d_1 &= 0 \\ p_1^* &= 0 \\ q_1^* &= -Vf_{1\xi}^* \end{aligned}\right\} \qquad [16]$$

$$\left.\begin{aligned} &f^*(\xi,\zeta) \equiv f^*\{x,z - F(x,y)\} \equiv f(x,z) \\ &F(\xi,\eta) \equiv F(x,y) = \frac{V}{g}\varphi_x - \frac{1}{2g}(\varphi_x^2 + \varphi_y^2 + \varphi_z^2) \\ &F_1(\xi,\eta) = \frac{V}{g}\varphi_{1\xi}^* \qquad \text{on} \qquad \zeta = 0 \end{aligned}\right\} \qquad [17]$$

$$
\left.
\begin{aligned}
d_2(\xi,\eta,\zeta) &= 2\varphi^*_{1\xi\zeta}F_{1x} + 2\varphi^*_{1\eta\zeta}F_{1y} + \varphi^*_{1\zeta}\left(F_{1xx} + F_{1yy}\right) \\
p^*_2(\xi,\eta) &= \frac{1}{V}\left(\varphi^{*2}_{1\xi} + \varphi^{*2}_{1\eta} + \frac{1}{2}\varphi^{*2}_{1\zeta}\right)_\xi + \left(2\varphi^*_{1\xi\zeta}F_{1x} + \varphi^*_{1\zeta}F_{1xx}\right) \\
q^*_2(\xi,\zeta) &= \left(\varphi^*_{1\xi}f^*_{1\xi} + \varphi^*_{1\zeta}f^*_{1\zeta}\right) + Vf^*_{1\zeta}F_{1x} + \varphi^*_{1\zeta}F_{1y}
\end{aligned}
\right\} \quad [18a]
$$

$$
F_2(x,y) = \frac{V}{g}\varphi_{2x} - \frac{1}{2g}\left(\varphi_{1x}^2 + \varphi_{1y}^2 + \varphi_{1z}^2\right) = \frac{V}{g}\varphi^*_{2\xi} + \frac{V}{g}\varphi^*_{1\zeta}F_{1x}
$$

$$
- \frac{1}{2g}\left(\varphi^{*2}_{1\xi} + \varphi^{*2}_{1\eta} + \varphi^{*2}_{1\zeta}\right) \qquad [18b]
$$

d_j, p^*_j, q^*_j for $j \geq 2$ are represented in terms of known functions including the obtained lower order solutions, $\{\varphi^*_i \; ; \; i < j\}$. We drop the superscript * except in the case of confusion.

For a general ship which has a small draft-length ratio as well as a small beam-length ratio, the boundary condition corresponding to [8] and the order analysis needs to be dealt with carefully. The exact boundary condition on the ship surface in x,y,z coordinates is

$$
\varphi_n = -\frac{Vf_x}{\sqrt{1+f_x^2+f_z^2}} \quad \text{on} \quad S_s \qquad [19]
$$

By transformation to the ξ,η,ζ coordinates

$$\varphi_\nu = \frac{\sqrt{1+f_\xi^2+f_\zeta^2}}{1+\left(f_\xi - f_\zeta F_x\right)^2+f_\xi^2}\left\{\varphi_\zeta F_y - F_x\left(\varphi_\zeta f_\xi + \varphi_\xi f_\zeta\right) - Vf_\xi + Vf_\zeta F_x\right\} \quad \text{on } S_s^* \tag{20}$$

SOLUTION FOR EACH ORDER POTENTIAL

The solution for each j can be expressed using Green's formula in the transformed space,

$$\varphi_j = \frac{1}{4\pi}\iint_{S^*}\left[\varphi_j(x_1,y_1,z_1)\, G_\nu(x_1,y_1,z_1;\xi,\eta,\zeta) - \varphi_{j\nu}(x_1,y_1,z_1)\, G(x_1,y_1,z_1;\xi,\eta,\zeta)\right] dS$$

$$- \frac{1}{4\pi}\iiint_{D^*} G\,\Delta\varphi_j\, d\tau \tag{21}$$

where the Green's function G satisfies

$$\left.\begin{aligned} G_{\xi\xi} + k_o G_\zeta &= 0 \quad \text{on} \quad \zeta = 0 \\ G_\eta &= 0 \quad \text{on} \quad \eta = 0 \end{aligned}\right\} \tag{22}$$

in addition to the usual properties previously mentioned in [3]. Such a Green's function $G(x_1, y_1, z_1; \xi, \eta, \zeta)$ is well known to represent the potential due to a point source located at (x_1, y_1, z_1), (see e.g., Lunde, 1952).

$$G(x_1, y_1, z_1; \xi, \eta, \zeta) = \frac{1}{r_1} - \frac{1}{r_2}$$

$$- \frac{k_o}{\pi} \mathrm{Re} \int_{-\pi}^{\pi} \int_{0}^{\infty} \frac{\sec^2\theta \, e^{-k|\zeta+z_1| + ik\omega}}{k - k_o \sec^2\theta - i\mu \sec\theta} \, dk d\theta$$

$$= \frac{1}{r_1} - \frac{1}{r_2} + 4k_o \int_{-\pi/2+\delta}^{\pi/2} e^{-k_o \sec^2\theta \{|\zeta+z_1|\}} \sec^2\theta \sin(k_o \omega \sec^2\theta) d\theta$$

$$- \frac{2}{\pi} \int_{-\pi/2+\delta}^{\pi/2+\delta} d\theta \int_{0}^{\infty} \frac{dm \, e^{-m\omega\{k_o \sec^2\theta \sin(m|\zeta+z_1| - m\cos(m|\zeta+z_1|)\}}}{k_o^2 \sec^4\theta + m^2}$$

[23]

where

$$r_1^2 = (\xi - x_1)^2 + (\eta - y_1)^2 + (\zeta - z_1)^2$$

$$r_2^2 = (\xi - x_1)^2 + (\eta - y_1)^2 + (\zeta + z_1)^2$$

$$\delta = \arctan\left(\frac{\eta - y_1}{\xi - x_1}\right)$$

$$\omega = (\xi - x_1)\cos\theta + (\eta - y_1)\sin\theta$$

μ = fictitious friction force which is put to zero after integration.

$G_{x_1}(x_1, y_1, 0; \xi, \eta, \zeta)$ represents the potential due to a Kelvin's source or a pressure point on the free surface. Using the conditions [22] for Green's function and the boundary conditions for φ_j, we obtain

$$\int_{S_F^*} \varphi_j G_\nu - \varphi_{j\nu} G \; dS = - \int_{S_F^*} \varphi_j G_{z_1} - G\varphi_{jz_1} \; dx_1 dy_1$$

$$= \int_{S_F^*} \frac{1}{k_o} \frac{\partial}{\partial x_1} \left(\varphi_j G_{x_1} - \varphi_{jx_1} G \right) + \frac{1}{k_o} p_j^* G \; dx_1 dy_1$$

$$= - \frac{1}{k_o} \int_{\ell} \varphi_j G_{x_1} - \varphi_{jx_1} G \; dy_1 + \frac{1}{k_o} \int_{S_F^*} p_j^* G \; dx_1 dy_1 \qquad [23^*]$$

where ℓ represents the intersection of S_F^* and S_s^*. Therefore we can write,

$$\varphi = \Sigma \epsilon^j \varphi_j = \frac{1}{4\pi} \Sigma \epsilon^j \int_{S_s^*} \varphi_j G_\nu - \varphi_{j\nu} G \; dS$$

$$+ \frac{2}{k_o} \epsilon \int_o^L \varphi_j G_{x_1} - \varphi_{jx_1} G \; \frac{df_1}{dx_1} \, dx_1 + \int_{S_F^*} p_j^* \frac{G dx_1 dy_1}{k_o}$$

$$- \int_{D^*} G d_j d\tau \qquad [24]$$

Now we may consider a Taylor's expansion of the Green's function at $\eta = 0$,

$$\left.\begin{aligned} G(x_1, \epsilon f_1, z_1; \xi, \eta, \zeta) &= G(x_1, 0, z_1; \xi, \eta, \zeta) + \frac{\epsilon^2 f_1^2}{2} G_{y_1 y_1} + \cdots \\ G_\nu(x_1, \epsilon f_1, z_1; \xi, \eta, \zeta) &= \frac{-G_{x_1} f_{x_1} + G_{y_1} - G_{z_1} f_{z_1}}{\sqrt{1 + f_{x_1}^2 + f_{z_1}^2}} \\ &= \epsilon f_1 G_{y_1 y_1}(x_1, 0, z_1; \xi, \eta, \zeta) + O(\epsilon^2), \text{etc.} \end{aligned}\right\} \quad [25]$$

Since we do not know S_s^* a priori we consider the sequence of ship surfaces which converges to S_s^*, or

$$S_{os}^* , S_s^* \cdots S_s^* . \quad [26]$$

where S_{os}^* represents the ship surface without any wave. Thus for each φ_j we can have a sequence of solutions which may eventually have the domain of integration S_s^*. The problem for the first order potential φ_1 on S_{os}^* is exactly the same as the first order problem in the case of Sisov's theory.

SLENDER SHIPS WITH ALMOST FLAT BOTTOM

When we consider that the draft of the ship is of the same order as the beam in [24], the draftwise integration of the integrand $O(\epsilon^j)$ becomes $O(\epsilon^{j+1})$. Thus, in [24], the lowest order of φ is $O(\epsilon^2)$ on and outside of the ship. If the ship has almost a flat bottom, φ_v is $O(\epsilon)$ only near the ship sides (from [20]).

With all these considerations including Equations [24] - [26], if we collect the coefficients of the same order ϵ^j from [24] we can have φ_j for each j. Thus the lowest order solution is

$$\varphi = \epsilon^2\varphi_2 = -\frac{V\epsilon^2}{2\pi}\int_0^L\int_{-H_1}^{0} f_{1x_1}\, G(x_1, 0, \epsilon z_1; x, y, z)\, dz_1\, dx_1 \qquad [27]$$

where $H = \epsilon H_1$ and L indicates the draft and the length of the ship respectively.

The next higher order solution would be

$$\varphi = \epsilon^2\varphi_2 + \epsilon^3\varphi_3 = \frac{V\epsilon}{2\pi}\iint_{S^*_{p\eta s}} f^*_{x_1}(x_1, z_1)\, G[x_1, 0, z_1; x, y, z - F_1(x,y)]\, dz_1\, dx_1$$

$$- \frac{\epsilon^3}{2\pi}\left[\int_0^L f_1(x_1, -H)\varphi_2(x_1, 0, -H)\, G_{z_1}(x_1, 0, -H; x, y, z)\, dx_1\right.$$

$$\left. = \frac{1}{K_0}\int_0^L \left\{\varphi_2(x_1, f_1, 0)\, G_{x_1}(x_1, f_1, 0, x, y, z) - \varphi_{2x_1}(x_1, f_1, 0)\, G\right\}\frac{df_1}{dx_1}\, dx_1\right] \qquad [28]$$

The first term of the right hand side in [28] is different from [27] only in the domain of integration. Thus, the influence of the change of the submerged ship surface due to the wave appears in the change of $f_{1x}(x,z)$ on S^*_{os} into $f_{1\xi}(\xi,\zeta)$ on S^*_{1s} which is bounded by a straight plane $\zeta = 0$ at the exact free surface and the wavy ship bottom in the ξ,η,ζ space. If the ship is wall sided, the only influence is from the vicinity of the bottom but not from the free surface in S^*_{1s}.

The second term in the right hand side of [28] is the flat bottom effect. Here the Taylor expansion of φ which is symmetric with respect to $\eta = 0$ is used. It represents a distribution of vertical doublets with strength proportional to $f_1\varphi_2$.

The third integral in the right hand side of [28] is the line integral along the line of intersection between the ship surface and the free surface. This indicates the line distribution of singularities on the free surface, and will be investigated further in the following section. This is particularly important because even for a thin ship with deep draft, φ will be of the same order as the case of the small draft when the ship speed is low.

The wave height is $0(\epsilon^3)$, and from [17]

$$F_3(\ ,y) = \frac{\nu}{g}\varphi_{3x}(x,y,0) \qquad [29a]$$

sufficiently apart from the slender ship. On the ship surface, it may be written

$$F_2(x,f_1) = \frac{1}{g}\left\{V\varphi_{2x}(x,f_1,0) - \frac{V^2}{2} f_{1x}^2\right\} \qquad [29b]$$

except near the bow and the stern.

THE LINE INTEGRAL ON THE FREE SURFACE

The third integral in the right hand side of Equation [28]

$$I = \frac{1}{k_o}\int_o^L \left\{\varphi_2(x_1,f_1,0)G_{x_1}(x_1,f_1,0,\xi,\eta,\zeta) - \varphi_{2x_1} G\right\}\frac{df_1}{dx_1}\,dx_1 \qquad [30a]$$

can be interpreted as a potential due to the distribution of doublets and sources on a line $\eta = f_1(\xi,0)$ whose strengths are proportional to $\varphi_2 f_1 x_1 \; 1/k_o$ and $\varphi_{2x_1} f_{1x_1}$ respectively. It is well known that a doublet distribution on the free surface is the same as a pressure distribution on the free surface (Wehausen, 1959; Ursell, 1960). Therefore a distribution of sources on the free surface can also be interpreted in terms of a corresponding distribution of pressure on the free surface. For example, we consider a smooth distribution of pressure $p(\xi,\eta)$ on a certain domain D on the free surface $\zeta = 0$ of an otherwise uniform flow, and $p = 0$ on the boundary of D. Then

$$\varphi = \frac{V}{4\pi g}\iint_D pG_{x_1}\,dx_1\,dy_1 = \frac{1}{4\pi}\frac{V}{\rho g}\iint_D p_{x_1} G\,dx_1\,dy_1$$

Namely, the potential anywhere in the fluid can be expressed either by a doublet distribution on $\zeta = 0$ with strength proportional to $p(\xi,\eta)$ or a source distribution on $\zeta = 0$ with strength proportional to $p_\zeta(\xi,\eta)$. Singularities on the free surface need special attention because it is known that when a pressure point or a point doublet is located on a free surface, the flow behavior is singular not only at the point itself but also on its path line and the corresponding wave resistance blows up (Lamb, 1932; Ursell, 1960). The same kind of singular behavior takes place for a point source on the free surface and the corresponding wave resistance becomes also infinity (Yim, 1966).

By an integration of [30a] by parts

$$I = \frac{1}{k_o} \varphi_2 f_{1X} G(x_1, f_1, 0, \xi, \eta, \zeta) \Big|_{x_1=0}^{L}$$

$$- \frac{1}{k_o} \int_0^L G \left\{ \frac{\partial}{\partial x_1} (\varphi_2 f_{1 x_1}) + \varphi_{2 x_1} f_{1 x_1} \right\} dx_1 \qquad [30b]$$

neglecting higher order terms. The first term of the right hand side in [30b] indicates the potential due to a point source located at $(x_1, f_1, 0)$ on the free surface. This would induce the singular behavior mentioned before unless $f_{1 x_1} = 0$ at $x_1 = 0$ and $x_1 = L$. Therefore, in this report we will consider the ship surface $y = f(x,z)$ which is sufficiently smooth in $-\infty < x < \infty$,

near the free surface. In an engineering approximation of the line integral for the lower orders, the free surface waterline of a ship of finite entrance angle may be approximated by a cusped smooth waterline sufficiently close to the original one, since there can never exist such singular phenomena in the actual case of real fluid. At present, no meaningful mathematical limit of the wave resistance or the wave height due to either a point source or a point doublet on the free surface is available. This is discussed more fully later in the section entitled Singularities at the Bow and Stern.

WAVE RESISTANCE

The wave resistance R can be calculated by the integration of pressure p from the Bernoulli equation,

$$p = \rho \left\{ V\varphi_x - zg - \frac{1}{2} (\varphi_x^2 + \varphi_y^2 + \varphi_z^2) \right\} \qquad [31]$$

over the wetted ship surface,

$$R = \iint_{S_S} p \frac{f_x}{\sqrt{1 + f_x^2 + f_z^2}} dS$$

$$= \int_0^L \int_{-H}^{F(x,z)} 2\rho \left\{ V\varphi_x - zg - \frac{1}{2} (\varphi_x^2 + \varphi_y^2 + \varphi_z^2) \right\} f_x dx\, dz \qquad [32]$$

If we put

$$R = \epsilon^2 R_2 + \epsilon^3 R_3 + \epsilon^4 R_4 + \epsilon^5 R_5 + \cdots \qquad [33]$$

the lowest order wave resistance for a thin ship will be

$$R_2 = \int_0^L \int_{-H}^0 2\rho V \varphi_{1x} f_{1x} \, dx \, dz \qquad [34]$$

since

$$\int_0^L f_{1x} \, dx = 0$$

This leads to the Michell's wave resistance formula. The lowest order wave resistance due to a slender ship is

$$R_4 = \int_0^L \int_{-H_1}^0 2\rho \left\{ V \varphi_{2x} - \frac{1}{2} V^2 f_1 x^2 \right\} f_{1x} \, dx \, dz \qquad [35]$$

in which only the part due to regular waves will remain. Thus this is also the same as the Michell's wave resistance.

The next higher order wave resistance of a slender ship with the almost flat bottom is

$$R_6 = 2\int_0^L\int_{-H_1}^0 \rho V \varphi_{3x} f_{1x}\,dx\,dz + 2\int_0^L\int_0^{F_2(x,z)} \rho V \varphi_{2x} f_{1x}\,dx\,dz$$

$$- 2\int_0^L\int_0^{F_2(x,y)} \rho\left[zg + \frac{1}{2}V^2 f_{1x}^2\right] f_{1x}\,dx\,dz \qquad [36]$$

where φ_2, φ_3, and F_2 are given in [27], [28], and [29b] respectively. If we consider that f_{1x} is almost independent of z near the free surface, the last integral will become

$$-\int_0^L \rho\left(F_2 g + V^2 f_{1x}^2\right) F_2 f_{1x}\,dx$$

From [28], [35] and [36], we can write

$$R = \epsilon^4 R_4 + \epsilon^6 R_6$$

$$= 2\rho \iint_{S^*_{1pns}} V f_1^* \xi \, d\xi \, d\zeta \; \frac{V\epsilon^2}{2\pi} \iint_{S^*_{1pns}} f_1^* x_1 \; G_\xi \left| x_1, 0, z_1 ; \xi, 0, \zeta \right| dx_1 \, dz_1$$

$$-2\rho V \iint_{S_{opys}} f_{1x} \, dx \, dz \; \frac{\epsilon^4}{2\pi} \left[\int_0^L f_1(x_1, -H) \varphi_2(x_1, 0, -H) G_{z_1 x}(x_1, 0, -H; x, 0, z) \, dx_1 \right.$$

$$\left. + \frac{1}{k_0} \int_0^L G_x \left\{ \varphi_2(x_1, 0, 0) f_{1 x_1 x_1} + 2\varphi_{2\,x_1}(x_1, 0, 0) f_{1 x_1} \right\} dx_1 \right]$$

$$- \int_0^L \rho \left(F_2 g + V^2 f_{1x}^2 \right) F_2 f_{1x} \, dx \qquad [37]$$

Since $f_{.x} = 0$ on the bottom, the integration over the bottom is omitted. However, in actual calculation it may be better consider the formal consideration of the bottom integral to eliminate the calculation of local disturbance, because if we consider the wave resistance due to a singularity distribution through Lagalley's theorem the contribution from the local disturbance is zero (see e.g., Lunde, 1957). By the same reason

we consider the effect of φ_{2x} on the line source distribution on the free surface

$$-2\rho\int_0^L \frac{\varphi_{2x}(x,0,0)}{k_0}\left\{\varphi_2(x,0,0)\, f_{1xx} + \varphi_{2x}(x,0,0) f_{1x}\right\}dx$$

$$= -2\rho\int_0^L \frac{1}{V}\left(F_2 g + \frac{1}{2}V^2 f_{1x}^2\right)\left\{\frac{\varphi_2}{k_0} f_{1xx} + V\left(F_2 + \frac{1}{2k_0} f_{1x}^2\right) f_{1x}\right\}dx \qquad [38]$$

If we add [38] in [37] the effect of the local disturbance due to the surface line integral is eliminated within $O(\epsilon^5)$, and subtracting [38] from the last integral of [37] we obtain,

$$R = \frac{4\rho}{\pi}\left\{\int_0^{\pi/2}\epsilon^2(C_1^2 + S_1^2)\sec^3\theta\, d\theta\right.$$

$$- 2\int_0^{\pi/2}\epsilon^4 (C_0C_b + S_0S_b)\sec^5\theta\, d\theta$$

$$\left. - 2\int_0^{\pi/2}\epsilon^4 (C_0C_F + S_0S_F)\sec^3\theta\, d\theta\right\}$$

$$+ \epsilon^4\rho\int_0^L\left\{\frac{\varphi_2(x_1,0,0)}{k_0} f_{1x_1x_1}\left(F_2\frac{g}{V} + \frac{V}{2} f_x^2\right) + \left(F_2 g + V^2 f_{1x}^2\right) F_2 f_{1x}\right.$$

$$\left. + \frac{V^2}{2k_0} f_{1x}^6\right\} dx \qquad [39]$$

where

$$\left.\begin{matrix} C_o \\ S_o \end{matrix}\right\} = \iint_{S_{opys}} dx\, dz V k_o f_{1x} e^{k_o z \sec^2\theta} \left\{\begin{matrix}\cos \\ \sin\end{matrix}\right\} (k_o x \sec\theta)$$

$$\left.\begin{matrix} C_1 \\ S_1 \end{matrix}\right\} = \iint_{S^*_{1p\eta s}} d\xi\, d\zeta V k_o f_{1\xi} e^{k_o \zeta \sec^2\theta} \left\{\begin{matrix}\cos \\ \sin\end{matrix}\right\} (k_o \xi \sec\theta)$$

$$\left.\begin{matrix} C_b \\ S_b \end{matrix}\right\} = \int_o^L k_o^2 f_1(x,-H)\varphi_2(x,0,-H)\, e^{-k_o H \sec^2\theta} \left\{\begin{matrix}\cos \\ \sin\end{matrix}\right\} (k_o x \sec\theta)dx$$

$$\left.\begin{matrix} C_F \\ S_F \end{matrix}\right\} = \int_o^L \left\{\varphi_2(x,0,0) f_{1xx} + 2\varphi_{2x} f_{1x}\right\} \left\{\begin{matrix}\cos \\ \sin\end{matrix}\right\} (k_o x \sec\theta)dx$$

Thus the higher order wave resistance of slender ships $O(\varepsilon^5)$ is from the change of the ship submergence due to the wave, the bottom effect, and the line integral due to the free surface bounded internally by the ship surface. These higher order effects are functions of the lowest order wave height and the lowest order potential, which are zero at infinity.

ORDER ANALYSIS AND DEPENDENCE ON k_o

The selection of terms in each higher order for thin ships and slender ships in the above analysis has been strictly formal under the assumptions that series expansions [6] converge for any small ϵ, and all the variables behave properly. However this analysis would be incomplete if we did not consider the actual situation which is highly dependent on the Green's function [23] and the parameter k_o. In practice, for ordinary merchant ships (low and medium speed ships) and fast ships, the magnitude of the following parameters, viz, Froude number $F_r = 1/\sqrt{k_o L}$, $k_o L_o \equiv k_1$, beam-length ratio, and draft-length ratio are as shown in Table 1.

TABLE 1

Magnitude of Ship Parameters

	Ordinary Merchant Ships	Fast Ships
F_r	0.33 — 0.18	0.33 — 1.5
$1/k_o L \equiv 1/k_1$	0.11 — 0.033	0.11 — 2.2
$k_o L \equiv k_1$	9 — 30	9 — 0.45
Half-Beam Length Ratio	0.08 — 0.05	0.05 — 0.03
Draft-Length Ratio	0.07 — 0.04	0.07 — 0.04

Thus, when we consider physical quantities, non-dimensionalized with respect to L and V, it is easy to see that k_1 as a factor to a term in our analysis affects the order of magnitude of the term in the case of ordinary merchant ships since it is $O(1/\epsilon)$. On the other hand, for fast ships k_1 may be considered as of $O(1)$.

As for the Green's function, we may separately estimate the magnitudes of the local disturbance and the regular part from the following simple ship. Namely, when we consider a wedge ship whose waterline slope is

$$f_x(x,z) = \left\{ \begin{array}{ll} = a_o = O(\epsilon) \text{ in } & 0 < x < c \\ = 0 & c < x < 1 - c \\ = -a_o & 1 - c < x < 1 \end{array} \right\} \quad \begin{array}{l} \text{and} \\ 0 < -z < d \ , \end{array}$$

the regular part of φ_x on $y = 0$, $0 < x < 1 - c$ is

$$\varphi_{x,reg} = \frac{4a_o}{\pi} \int_0^{\pi/2} e^{k_1 z \sec^2\theta} \left(1 - e^{-k_1 d \sec^2\theta}\right) \sin(k_1 x \sec\theta)\, d\theta$$

Since

$$\int_0^{\pi/2} \sin(k_1 x \sec\theta)\, d\theta = -\frac{\pi}{2} \int_0^{k_1 x} Y_0(t)\, dt$$

which is O(1) for all $k_1 x$,

$$|\varphi_{x,reg}| \leq 2a_o \min\left(e^{k_1 z}, 1 - e^{-k_1 d}\right) \left| \int_0^{k_1 x} Y_o(t)\, dt \right|$$

Since, for fast ships, $k_1 d = O(\epsilon)$, we obtain $1 - e^{-k_1 d} = O(\epsilon)$ in this case. Therefore we can write

$$\varphi_{x,reg} \begin{cases} = O(\epsilon) & \text{for low and medium speeds} \\ = O(\epsilon^2) & \text{for high speeds} \end{cases}$$

The local disturbance on the free surface is known to be of the same order near the body as the regular wave although it decays very rapidly in the far field. It is evident from the expression for φ that the derivative with respect to any coordinate non-dimensionalized by L increases the factor k_1 while the integration reduces this factor.

From the first two terms of Green's function [23], $\varphi_o = a_o \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$, it is not difficult to show that

$$\varphi_{ox} = \int_{1-c}^{1} \int_{0}^{d} G_{cx}\, dx_1\, dz_1 - \int_{1-c}^{1} \int_{0}^{d} G_{c}\, dx_1\, dz_1$$

and φ_{cxx} are $O(\epsilon^2 \log \epsilon)$ near the ship. It is also easy to see

that the continuous variation of the waterline slope does not affect the order of magnitude of the physical variables since the influence on the physical quantities from the sudden change of the slope at the bow and stern dominates the other terms for ordinary merchant ships and is of the same order as the other terms in the case of fast ships.

From the boundary condition, we know that φ_z is of the same order as φ_{xx}/k_1 on the free surface and is equal to zero on the ship's flat bottom. We also know, from the boundary condition on the ship surface, that $\varphi_y = O(\epsilon)$ near the ship sides.

Thus we may tabulate the magnitudes of some physical quantities near the ship surface, except within a distance ϵ^2 from the bow and stern as shown in Table 2.

TABLE 2

Order of Magnitudes of Some Physical Quantities

	Ordinary Merchant Ship	Fast Ships
$1/k_1$	$O(\epsilon)$	$O(1)$
φ	$O(\epsilon^2 \log \epsilon)$	$O(\epsilon^2 \log \epsilon)$
φ_x, φ_z	$O(\epsilon)$	$O(\epsilon^2)$
Wave Height $= \varphi_x/k_1$	$O(\epsilon^2)$	$O(\epsilon^2)$
φ_y	$O(\epsilon)$	$O(\epsilon)$
$\varphi_{xx}, \varphi_{yy}, \varphi_{zz}$	$O(1)$	$O(\epsilon^2)$
$\varphi_{yx}, \varphi_{zx}$	$O(1)$	$O(\epsilon^2)$

Therefore the higher order potential [28] and wave resistance [39] in the case of slender ships are only valid for fast ships. For slender ships at low Froude numbers, namely for ordinary merchant ships, the higher order potentials are rather the same as for the case of thin ships shown in [24].

The lowest order non-dimensional wave resistance for ordinary merchant ships is the same as that given in Equation [34], but the magnitude should be listed as $O(\epsilon^4)$, as in the case of fast ships, because the double integral in Equation [34] brings out the factor $1/k_1^2$ which is $O(\epsilon^2)$. The higher order wave resistance from ordinary merchant ships can be obtained from [32]. This will include not only the effects needed for the fast ships in [39] but also the other effects due to the nonlinear terms of the pressure equation [31] viz $-1/2(\varphi_{1X}^2 + \varphi_{1Z}^2)$ which is neglected in [39], as well as additional terms in φ_{2X} shown in [24]. Essentially the expression for the higher order wave resistance of ordinary merchant ships will be the same as for the case of thin ships. However if k_1 is sufficiently large such that $e^{-k_1 H} = O(\epsilon)$, (e.g., $k_1 = 30$, $H = 0.07$, then $e^{-k_1 H} = 0.122$) then the bottom effect and the effect due to the change of the wetted surface of the ship in [24] will become one order higher.

When we use the Taylor series expansion of Green's function [25], we have to notice the parameter $k_1 \epsilon f_1$ where the factor k_1 comes in because of a differentiation of Green's function with respect to y. The parameter $k_1 \epsilon$ varies in the range of 0.45 - 2.4 for ordinary merchant ships. Thus, in this case, the convergence of [25] is unreliable in a certain part of f_1,

and the accuracy of the first order theory (Michell's theory) appears to be grossly affected and the convergence of the higher order theory may be slow. Nevertheless, the situation is not that serious due to the fact that $k_1 \epsilon f_1$ in [25] is a function of x and z which is zero at x = 0 and x = 1, and to the principle that the potential is dominantly influenced by the entrance angle for low Froude numbers.

SINGULARITIES AT THE BOW AND STERN

When the waterline slopes at the bow and the stern change suddenly, φ_{1z} has a logarithmic singularity at the origin while φ_{1x} and φ_{1y} are finite there. This means that the slope of the free surface F_x and F_y at the origin is infinity. In addition φ_{1x} and the higher order derivatives are singular along the bow and the stern. Thus the present analysis cannot hold in the immediate neighborhood of the bow and stern. However the principal effects of this singularity under the free surface is the same as in the flow about a thin wing in an infinite medium without a free surface.

This fact also indicates that the singular behavior of the line integral [30a] due to the point singularity at the origin discussed earlier cannot be analyzed by means of this linear theory. In fact if we consider the line integral in [23*] or [30a], in the physical space then we have

$$I = \frac{1}{k_o} \oint_{\ell} (\varphi_2 G_{x_1} - \varphi_{2x_1} G)\, dy_1$$

$$= \frac{2}{k_o} \int_o^{\ell_1} (\varphi_2 G_{x_1} - \varphi_{2\, x_1} G)\, \frac{df_1}{ds}\, ds$$

where s represents the length of the line ℓ from the bow to each point on ℓ, and ℓ_1 represents the length of ℓ from the bow to the stern of the ship. Integrating by parts we obtain

$$I = \frac{2}{k_o} \left(\varphi_2 G_{x_1}{}^{o}\right) \frac{df_1}{ds} \Bigg|_{s=0}^{s=\ell_1} - \frac{2}{k_o} \int_o^{\ell_1} \left\{ G_{x_1}{}^{o}\, \frac{d}{ds} \left(\varphi_2\, \frac{df_1}{ds}\right) + \varphi_{2\, x_1} G\, \frac{df_1}{ds} \right\} ds$$

where

$$G_{x_1}{}^{o} \equiv \int^{s} G_{x_1}\, ds$$

since $F_x = F_y = \infty$ and therefore $\frac{df_1}{ds} = 0$ at $s = 0$ and $s = \ell_1$.

Hence

$$I = - \frac{2}{k_o} \int_o^{\ell} \left\{ G_{x_1}{}^{o}\, \frac{d}{ds} \left(\varphi_2\, \frac{df_1}{ds}\right) + \varphi_{2\, x_1} G\, \frac{df_1}{ds} \right\} ds \ .$$

In this expression any effect from a point source or a point doublet on the free surface is no longer included.

CONCLUDING REMARKS

In the first order ship theory, whether we start to satisfy the boundary condition on the free surface itself or on the mean free surface does not make any difference in the result. Likewise, whether the free surface is considered to be internally bounded by the ship waterline at the free surface or not does not affect the result of linear theory. However when we develop a higher order theory, it seems necessary to consider these two items very carefully. The application of the free surface condition on the mean free surface cannot lead to the proper treatment of the influence of the change of ship wetted surface caused by surface waves. This difficulty has been resolved by distorting the coordinates. Furthermore the requirement that the free surface be internally bounded by the ship waterline at the free surface makes a large difference in the result obtained. The aforementioned influences appear in the second and higher order terms, and are more important than the usually considered nonlinear effect of the free surface boundary condition which comes in only to the third and higher order terms in the case of fast slender ships, and to the second and the higher order terms in the case of thin ships and common speed slender ships.

The higher order analysis for common speed slender ships (ordinary merchant ships) is rather the same as that for thin ships, and the equations for the second order wave height and

for the second order wave resistance are a little more complicated than those for fast slender ships.

Essentially, the higher order wave theory considered here, is based on the assumption of the regular or asymptotic convergence of the series expansion of physical quantities. In this respect any higher order terms can be obtained from the present analysis. However, in that case, unless we consider the viscosity and the surface tension, the theory higher than the second order would be meaningless.

The numerical computation of the higher order wave form or wave resistance due to a slender ship is not too complicated in this age of high speed computers. It would be extremely interesting to see the numerical results even for the case of individual high order influences such as the line integral effect, the bottom effect, and others.

REFERENCES

Inui, T., "Study of Wave-Making Resistance of Ships," The Society of Naval Architects of Japan, 60th Anniversary Series, Vol. 2, 1957.

Joosen, W. P. A., "The Velocity Potential and Wave Resistance Arising from the Motion of a Slender Ship," Proceedings of International Seminar on Theoretical Wave-Resistance, Vol. II, University of Michigan, 1964.

Lamb, H., "Hydrodynamics," 1932, ed. Dover Publication, New York, 1945.

Lunde, J. K., "On the Theory of Wave Resistance and Wave Profile," Shipsmodelltankens meddelelse, Nr. 10, 1952.

Lighthill, M. J., "Higher Approximation in General Theory of High Speed Aerodynamics," Edited by W. R. Sears, Princeton University Press, 1954.

Maruo, H., "Calculation of the Wave Resistance of Ships, the Draught of Which is Small as the Beam," Soc. of Nav. Arch. of Japan, 1962.

Michell, J. H., "The Wave Resistance of a Ship," Philosophical Magazine, 45, 106-122, 1898.

Sisov, V. G., "Contribution to the Theory of Wave Resistance of Ships in Calm Water," Izv. Akad., Nank, SSSR, Dept. of Tech. Sci. Mechanics and Machine Constructions, No. 1, pp. 75-85, 1961. (Bureau of Ships Translation No. 887).

Stoker, J. J., "Water Waves," Interscience Publishers, Inc., New York, 1957.

Tuck, E. O., "The Steady Motion of a Slender Ship," PhD Thesis, Cambridge, 1963.

Ursell, F., "On Kelvin's Ship Wave Pattern," Journal of Fluid Mechanics, Vol. 8, Part 3, 1960.

Van Dyke, M., "Perturbation Methods in Fluid Mechanics," Academic Press, New York, 1964.

Vossers, G., "Some Applications of the Slender Body Theory in Ship Hydrodynamics," PhD Thesis, Delft, 1962.

Wehausen, J. V., "An Approach to Thin-Ship Theory," Proceedings of International Seminar on Theoretical Wave-Resistance, Vol. II, University of Michigan, 1964.

Wehausen, J. V., "Surface Waves," Handbuch der Physik 9, Springer-Verlag, 1960.

Yim, B., "Singularities on the Free Surface and Higher Order Wave Height far Behind a Parabolic Ship," HYDRONAUTICS, Incorporated Technical Report 503-2 in preparation.

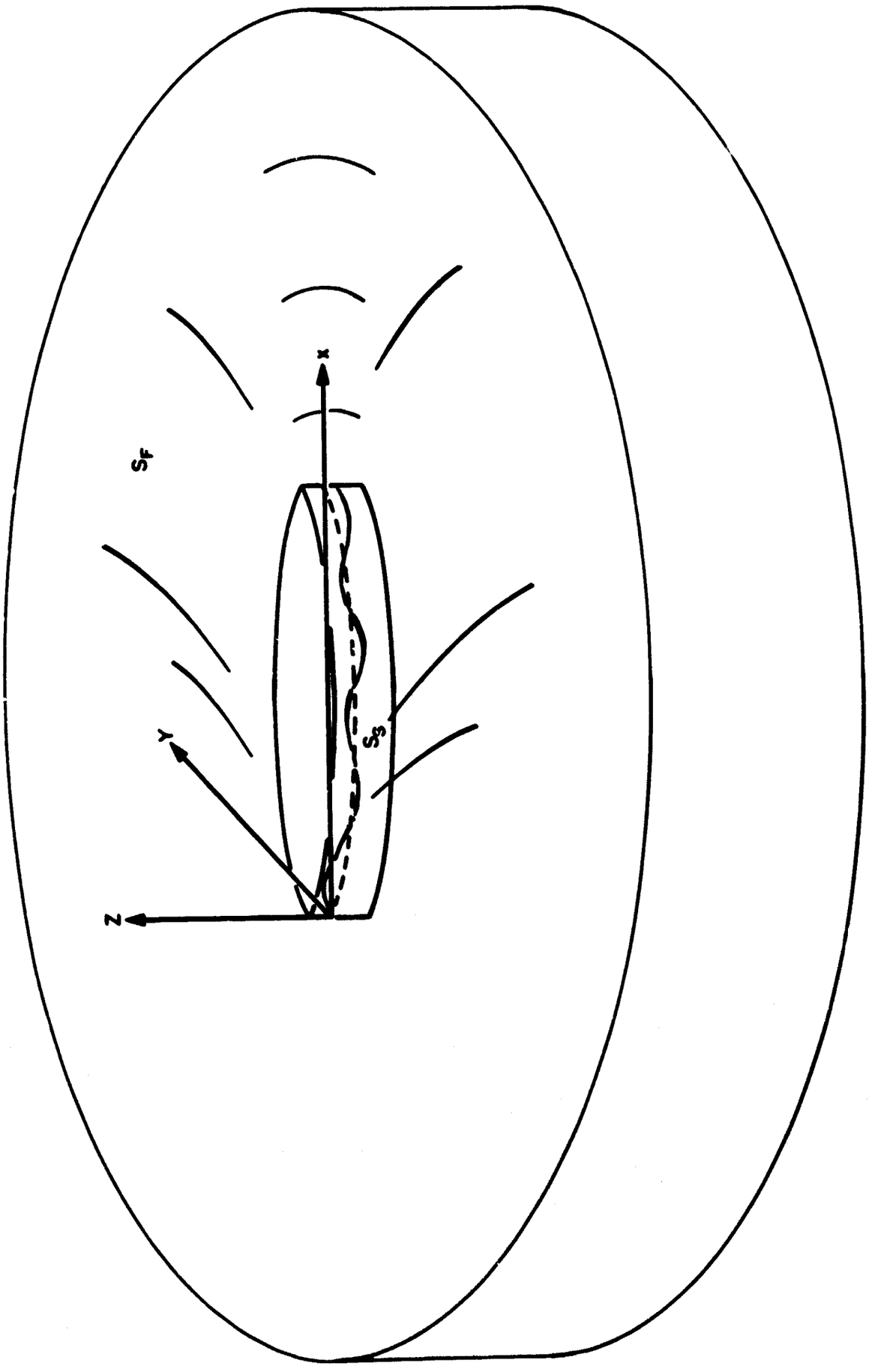

FIGURE 1-SCHEMATIC DIAGRAM FOR A SURFACE SHIP AND THE COORDINATE SYSTEM

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author)	2a REPORT SECURITY CLASSIFICATION
HYDRONAUTICS, Incorporated, Pindell School Road, Howard County, Laurel, Maryland	UNCLASSIFIED
	2b GROUP

3 REPORT TITLE

HIGHER ORDER WAVE THEORY OF SLENDER SHIPS

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report

5 AUTHOR(S) (Last name, first name, initial)

Yim, Bohyun

6 REPORT DATE	7a TOTAL NO OF PAGES	7b NO OF REFS
February 1966	41	16

8a CONTRACT OR GRANT NO	9a ORIGINATOR'S REPORT NUMBER(S)
Nonr-4677(00)	Technical Report 503-1
b PROJECT NO	
c	9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d	

10 AVAILABILITY/LIMITATION NOTICES

Copies of this report may be obtained from the Clearing House for Federal Scientific and Technical Information, Sills Building, Springfield, Virginia 22151.

11 SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY
	Bureau of Ships, Administered by David Taylor Model Basin

13 ABSTRACT

The free surface condition of the surface wave is considered on the free surface itself instead of the mean free surface, by the use of a coordinate transformation, together with the scheme of a systematic expansion in a small parameter. Thus a higher order ship wave theory is developed. The most common practical case of a slender ship with an almost flat bottom is especially treated in detail. The lowest order result is the same as that given by Michell's theory. The next higher order potential and wave resistance are derived explicitly.

DD FORM 1 JAN 64 1473

023558

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
free surface condition Froude number Green's function surface waves potential wave resistance thin ship slender ship						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY.** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity, or other organization *(corporate author)* issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through _____."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through _____."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through _____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.