BOEING SCIENTIFIC RESEARCH LABORATORIES DOCUMENT • D1-82-0529

BOEING

SCIENTIFIC RESEARCH LABORATORIES

An Experimental Evaluation of Various Rigidity Models

Also available from the author

J. F. Kenney

J. Gauger

M. A. Shea

D. F. Smart

GEO-ASTROPHYSICS LABORATORY MAY 1966

D1-82-0529

AN EXPERIMENTAL EVALUATION OF VARIOUS RIGIDITY MODELS

by

J. F. Kenney, J. Gauger[†]
Geo-Astrophysics Laboratory
Boeing Scientific Research Laboratories
Seattle, Washington 98124

M. A. Shea and D. F. Smart

Air Force Cambridge Research Laboratories
Cambridge, Massachusetts

May, 1966

[†]Douglas Advanced Research Laboratory
Huntington Beach, California

ABSTRACT

Atmospheric cosmic-ray fluxes have recently been measured over large portions of the globe on a series of flights of a United States Air Force jet aircraft. The measured fluxes have been plotted as a function of the vertical cutoff rigidity for several different models.

The rigorous trajectory calculations of Shea et al. give good agreement with the counting rates. The Makino model also orders these data equally well, and therefore must be within a few percent of the value determined by rigorous orbit calculations taken over the portions of the globe covered in this survey.

The other models are demonstrably worse, and a discussion of where these models are inadequate is given. Particular attention is paid to the use of the L parameter as a substitute for rigorous integration procedures. It is shown both experimentally and theoretically that the L parameter is not a good demographic parameter for galactic cosmic radiation at the lower latitudes, but should be sufficiently accurate at higher latitudes for most purposes.

TABLE OF CONTENTS

LIST OF FIGURES

INTRODUCTION

Shortly after the first cosmic ray measurements were made a latitude effect was noted, i.e., the counting rate of any detector was found to diminish as one approached the equator. It soon became obvious, however, that the counting rate could not be described as a simple function of latitude, since longitude effects were also present. Measurements have now been made of the cosmic-ray intensity as a function of position on the surface of the earth for the past few decades, and various cutoff rigidity models have been advanced to order these data. In the last few years, we have made measurements of the cosmic radiation over large portions of the earth's surface using different types of detectors. These data from a moderated BF_3 counter will be described and compared with various aspects of several different rigidity calculations.

EQUIPMENT

The $B^{10}F_3$ tube is brass-walled, 2" in outer diameter, and has an active length of 20". It is moderated by 2" of paraffin and shielded by cadmium so that it responds primarily to nucleons in the 1 - 5 Mev range. The electronic circuitry is straightforward and requires no special description.

The equipment was mounted in a KC-135 jet aircraft operated by the Upper Atmosphere Physics Laboratory of the Air Force Cambridge Research

Laboratories, Bedford, Massachusetts, United States Air Force Office of Aerospace Research.

In this type of aircraft the fuel tanks are physically in the bottom of the fuselage so there is no fuel above or surrounding the counters. Careful measurements made with different quantities of fuel in the various tanks have shown that the fuel load does not influence the measurements.

Various paths taken thus far by the aircraft for which we have analyzed data are shown in Figure 1 to give an indication of the geographic extent of the coverage of these flights. The data were taken in the far Pacific in 1962, and over the South American region in the winter and later in the summer of 1964. Pressure, altitude and position were derived from the navigator's flight log.

DATA

Since considerable geographic and altitude coverage was made on the series of flights, some sort of grouping had to be done in order to present the data. Consequently, all data taken as the aircraft passed through a two-degree latitude and two-degree longitude square were grouped together and treated as if they had been taken at the center of that block. All data were then reduced to a standard altitude of 35,000 feet. The data were then compared to the various cosmic-ray cutoff rigidities assigned to the geographic coordinates at the center of the block.

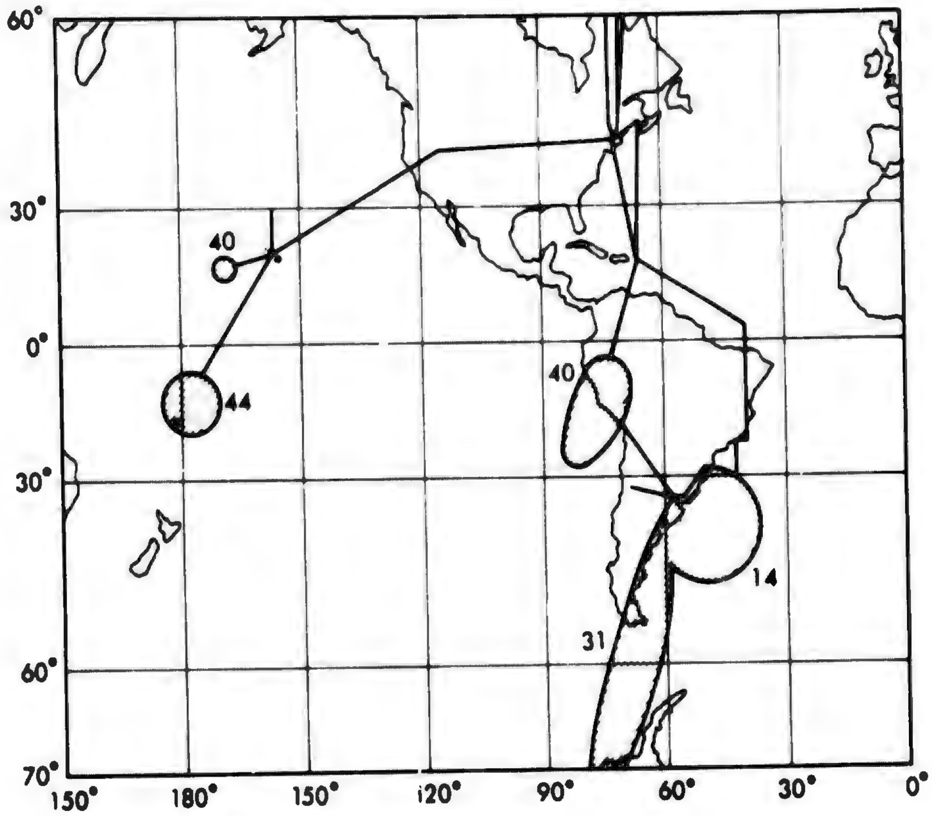

Fig. 1. Region of earth covered in the cosmic-ray survey. Approximate flight times in hours are given in each shaded area.

RIGIDITY CALCULATIONS

There are several ways of calculating rigidity, ranging from the rigorous integration of orbits to the simple dipole approximation. The general Stoermer expression for the cutoff rigidity $P_D(r,\lambda,E,\phi)$ in a dipole of magnetic moment M is given by

$$P_D(r,\phi,E,\lambda) = \frac{M \cos^4\lambda}{r^2[1 + \{1 - \sin E \cos\phi \cos^3\lambda\}^{\frac{1}{2}}]^2} \qquad (1)$$

where r is the distance from the center of the dipole to the point where P is measured, λ is the magnetic latitude, E is the zenith angle measured from the vertical, and ϕ is the azimuthal angle measured from the east. In the special case of the vertical cutoff rigidity at the surface of the earth, Equation (1) reduces to

$$P_D = P_e \cos^4\lambda \qquad (2)$$

where $P_e = M/4r_\oplus^2$, and $r_\oplus$ is the radius of the earth.

L PARAMETER

Recently, Lin et al. [1963] have demonstrated that, at least for some limited coverage in longitude and for data taken by satellites, the L parameter has some merit as a substitute for exact rigidity calculations. It is a rather natural development to go from the geomagnetic dipole latitude to an invariant latitude using the usual transform that $1/L = \cos^2\lambda_L$.

This would then give rigidity measurements which are simply inversely proportional to L^2,

$$P_L = P_e L^{-2} . \tag{3}$$

QUENBY AND WEBBER CALCULATIONS

Quenby and Webber [1959] have earlier considered the non-dipole character of the earth's internal field and deduced approximate corrections which must be applied to the dipole vertical cutoff rigidities in order to obtain a better agreement with observed values. These corrections have been published for every 10° of latitude and 15° of longitude. Interpolation procedures have been used to evaluate the vertical cutoff rigidity values at other points on the earth's surface.

QUENBY AND WENK CALCULATIONS

Quenby and Wenk [1962] improved this method by considering the shielding effect of the solid earth on the particle trajectories in the penumbral region. Modification of the previous calculations were made assuming a non-dipole field and making approximations in the penumbral region. The data published by these authors have also been interpolated to obtain values of the vertical cutoff rigidity for each two-degree square on the earth's surface.

MAKINO CALCULATIONS

Makino [1963] modified the Quenby-Webber approximations by introducing different penumbral corrections than those of Quenby-Wenk, and by introducing an empirical eastward shift of the impact point. Sauer and Ray [1963] have given a theoretical basis to this eastward correction by the concept of a guiding center for the particle as is used in trapped radiation studies. The eastward longitude correction was empirically determined to be $16^\circ\cos^3\lambda$, where λ is the geomagnetic latitude.

SHEA, SMART AND M^C^CRACKEN CALCULATIONS

It has long been recognized that the rigorous way to compute cutoff rigidities is to integrate out the orbits of particles of different rigidities. The drawback to this method is that a vast amount of computer time is necessary, even for the fastest modern-day digital computers. Nevertheless, Shea et al. [1965] have computed the vertical cutoff rigidity for many points on the earth's surface by determining the trajectories of cosmic ray protons outward from the earth's surface to 25 earth radii. The computer program used in these calculations [McCracken et al. 1962] utilizes a sixth-degree simulation of the geomagnetic field. Both the Finch and Leaton [1957] and the Jensen and Cain [1962] field coefficients have been used. The rigidity spectrum was investigated down to at least 1 BV below the last allowed trajectory at intervals as small as 0.1 BV in order to consider the effects of the penumbra. An effective cutoff rigidity was determined by summing over all allowed rigidities.

Since a vast amount of computer time would be involved in making these computations all over the earth's surface, only selected points have been computed thus far, primarily along the cosmic-ray equator, at IQSY stations, and along the routes of cosmic-ray survey ships and aircraft.

ANALYSIS OF DATA

In order to compare the data against the various rigidity calculations, one must reduce all data taken at different altitudes to a standard altitude and also remove the time variation incurred by taking the data over a two-year span.

A plot of the counting rate versus rigidity should ideally fall on a single-valued curve if the counting rate can be expressed as a simple function of rigidity and if we have computed the rigidity properly. The fact that the counting rate does not fall on a single-valued curve may be due to several factors:

(1) the statistical nature of the data and the grouping techniques that we used,

(2) improper altitude corrections,

(3) uncorrected time variations, and

(4) an inexact rigidity model.

Since we are attempting to evaluate the relative merits of the different rigidity models, the uncertainties introduced by the first three sources of error must be small in comparison with the differences due to the different rigidity models. An attempt must be made to evaluate the amount of error due to each of these causes.

In general, the statistical error associated with each of these points is quite small due to the very high counting rate of the apparatus and the relatively large amount of time spent in each 2° square. Statistical errors for each point are, in general, much less than one percent.

The errors caused by grouping the data in 2° squares can also be estimated. We shall see that the data are reasonably fit to the rigidity parameter by an exponential function of the form

$$I = I_0 e^{-P/P_0} \tag{4}$$

where I is the counting rate of the detector, P is the rigidity, and I_0 and P_0 are characteristic intensities and rigidities respectively. The characteristic rigidity, P_0, will be shown to be approximately 8 BV, and the relation of the rigidity to latitude can be estimated for the purposes of error analysis by the dipole model as shown in Equation (2). The change of the experimental counting rate ΔI, due to a corresponding latitude change $\Delta\lambda$, can then be easily shown to be

$$\frac{\Delta I}{I} = 4P_e P_0^{-1} \cos^3\lambda \sin\lambda \, \Delta\lambda \, . \tag{5}$$

In both polar and equatorial regions very little counting rate change is incurred from one extreme of a 2° square to the other due to the $\cos^3\lambda$ and $\sin\lambda$ terms respectively in Equation (5). The maximum error involved in one of these 2° squares will occur at some intermediate latitude determined by the point at which $\frac{d}{d\lambda}\left\{\frac{\Delta I}{I\Delta\lambda}\right\}$ is zero, or 30°. At that point the error involved in grouping the data will be approximately four percent.

The data taken at different altitudes were normalized to a standard altitude of 35,000 feet by empirically determined mean free paths, using data from the flight log as the basis for the altitude information. Uncertainty in the altitude of the aircraft and errors in the altitude corrections can introduce errors of the order of one to two percent.

The method used to remove the time variations was decided to be the following. The data taken on each of the three sets of flights were grouped together with no time corrections and fitted to the various rigidity models by the method of least squares. It was arbitrarily decided to normalize the other epochs to the 1962 data. Consequently, each experimental point gathered in one of the other epochs was multiplied by the ratio of the best fit curves for 1962, and the other epoch, evaluated at the rigidity of the location for each rigidity model. The normalized data were then all lumped together and curve-fitted again to each individual rigidity model.

The technique has the advantage of guaranteeing that the data taken during the different epochs were normalized in such a fashion that each group of data would show least scatter about the best fit straight line through a semilog plot of intensity versus rigidity for all the rigidity models. It suffers from the disadvantage of not removing shorter period time variations such as diurnal variations. Periods of abnormal variation, such as Forbush decreases, were eliminated from the data and points taken near or above the knee were also eliminated in this treatment.

To check on the total error introduced by uncorrected time variations, uncertainty in aircraft altitude, and all other sources of experimental error, a series of flights was held in the summer of 1964 along the same flight path near the equator off the west coast of Peru. The data in this region, which should have remained the same on a day-to-day basis, varied by amounts of about 2.5 percent.

Hence, we are led to the conclusion that the accuracy of the data is not limited by simple statistical considerations, but rather by other factors such as uncorrected time variations, uncorrected altitude variations, and data grouping. A realistic figure for the average error of each experimental point would then be about two or three percent.

RESULTS

The data which were normalized as described in the previous section have been plotted on a semilogarithmic scale against the rigidity calculated at the center of each 2° square according to the different models in Figures 2 through 7.

The normalized data were also curve-fitted by the method of least squares using statistical errors for the weights to an exponential function in rigidity

$$I = I_0 \exp(-P/P_0), \tag{6}$$

and the root mean square of the deviation of the experimental points from the best fit curve was determined for each model. The characteristic flux I_0, characteristic rigidity P_0, and rms deviation for each model is presented in Table I.

TABLE I

Rigidity Model	I_0 Counts sec^{-1}	P_0 BV	RMS Deviation Counts sec^{-1}
Dipole	49.05	8.36	1.49
Quenby-Webber	47.08	9.39	1.26
Quenby-Wenk	55.89	8.85	1.18
Makino	56.35	8.83	0.83
$15L^{-2}$	47.92	8.95	1.25
Shea-Smart	57.32	8.63	0.87

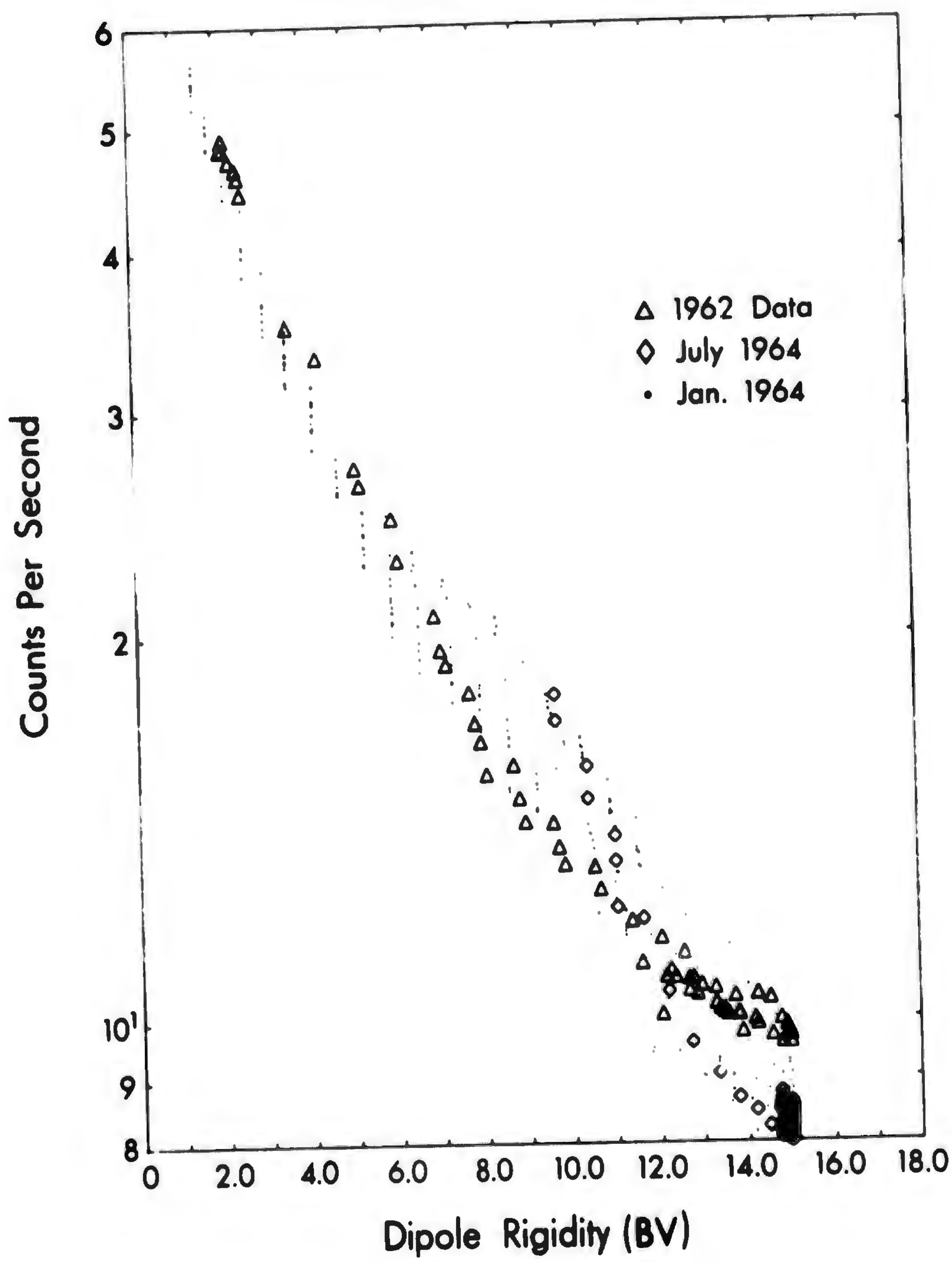

Fig. 2. Corrected counting rate as a function of dipole rigidities.

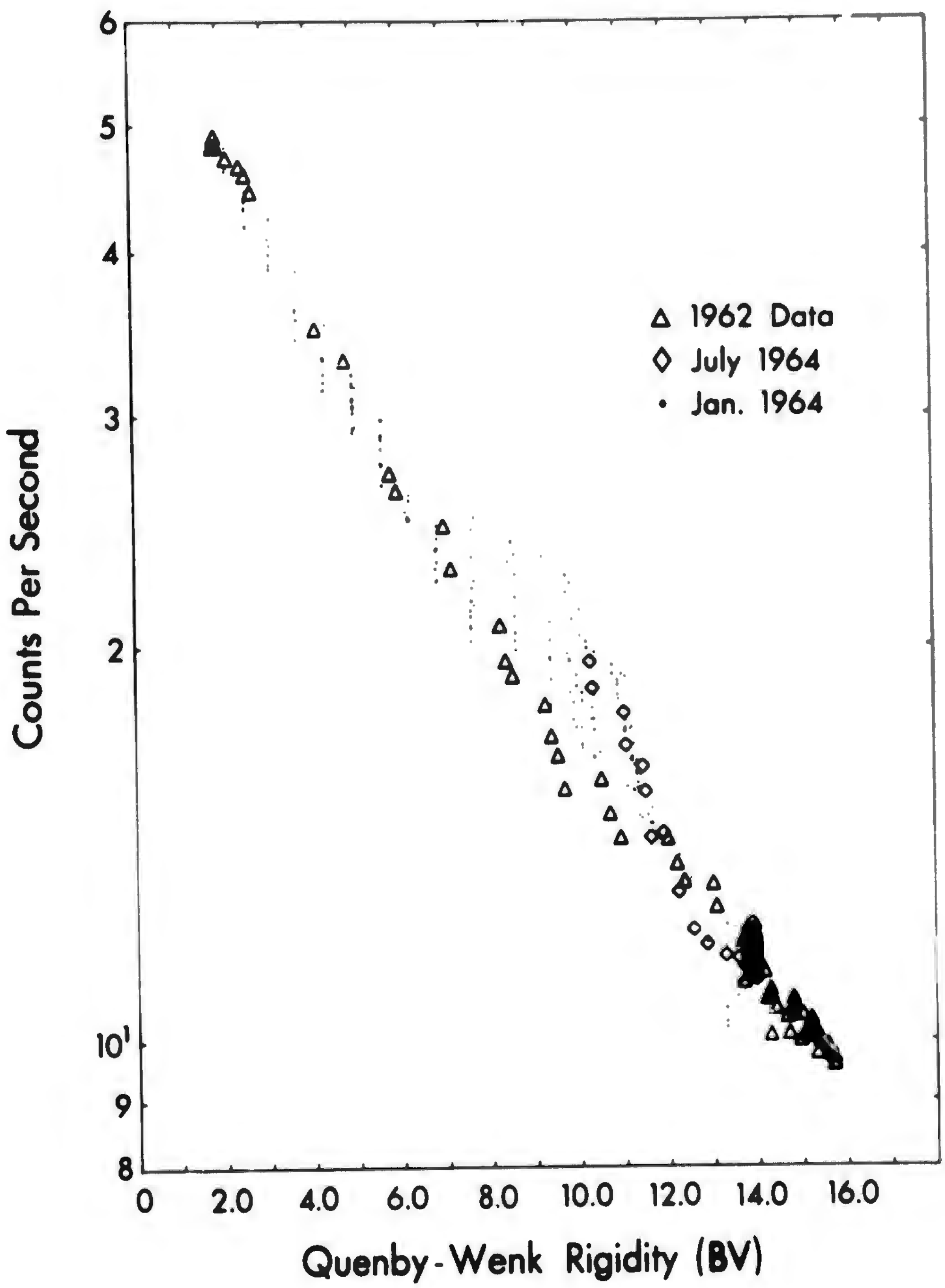

Fig. 3. Corrected counting rate as a function of Quenby-Wenk rigidities.

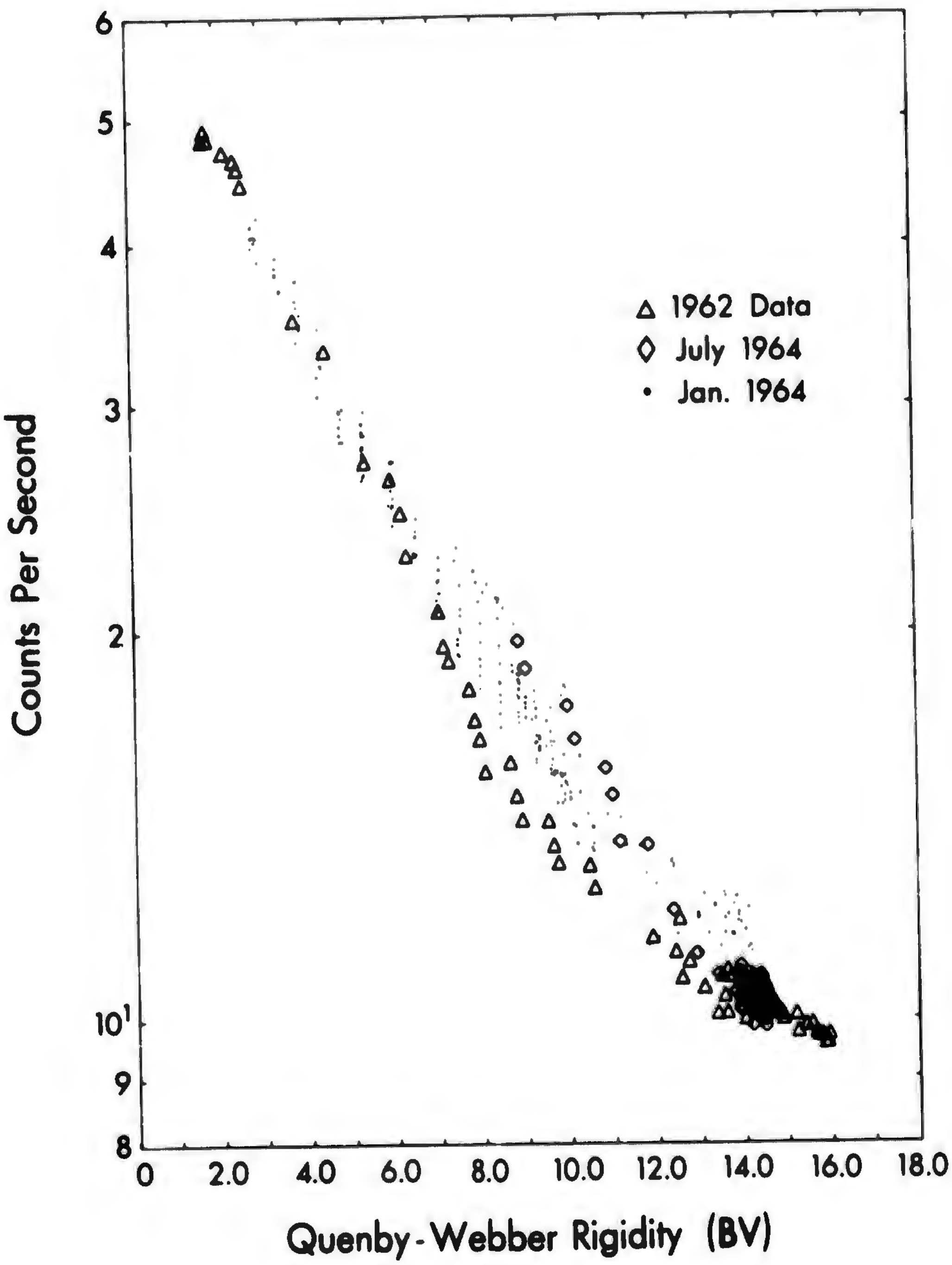

Fig. 4. Corrected counting rate as a function of Quenby-Webber rigidities.

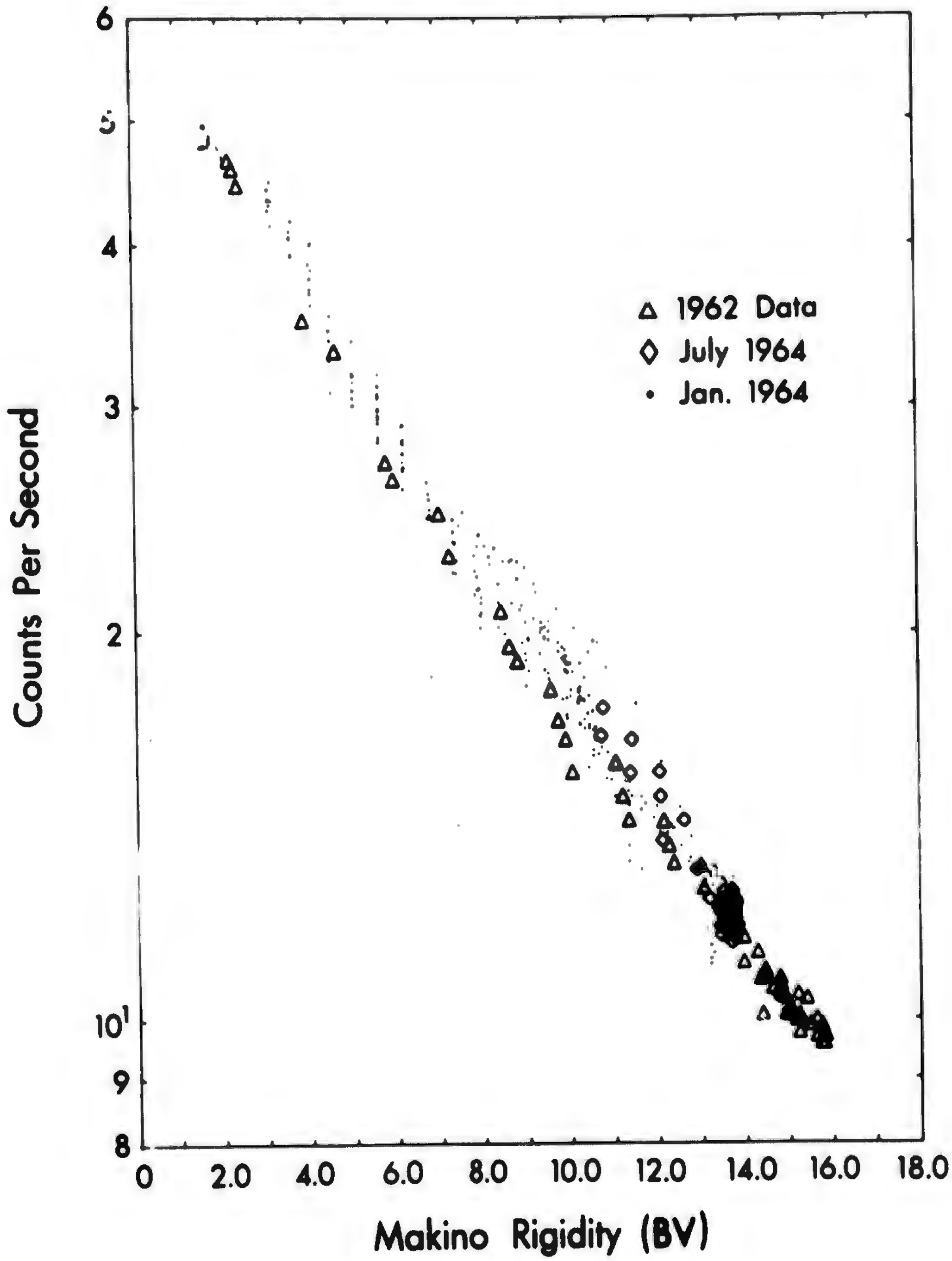

Fig. 5. Corrected counting rate as a function of Makino rigidities.

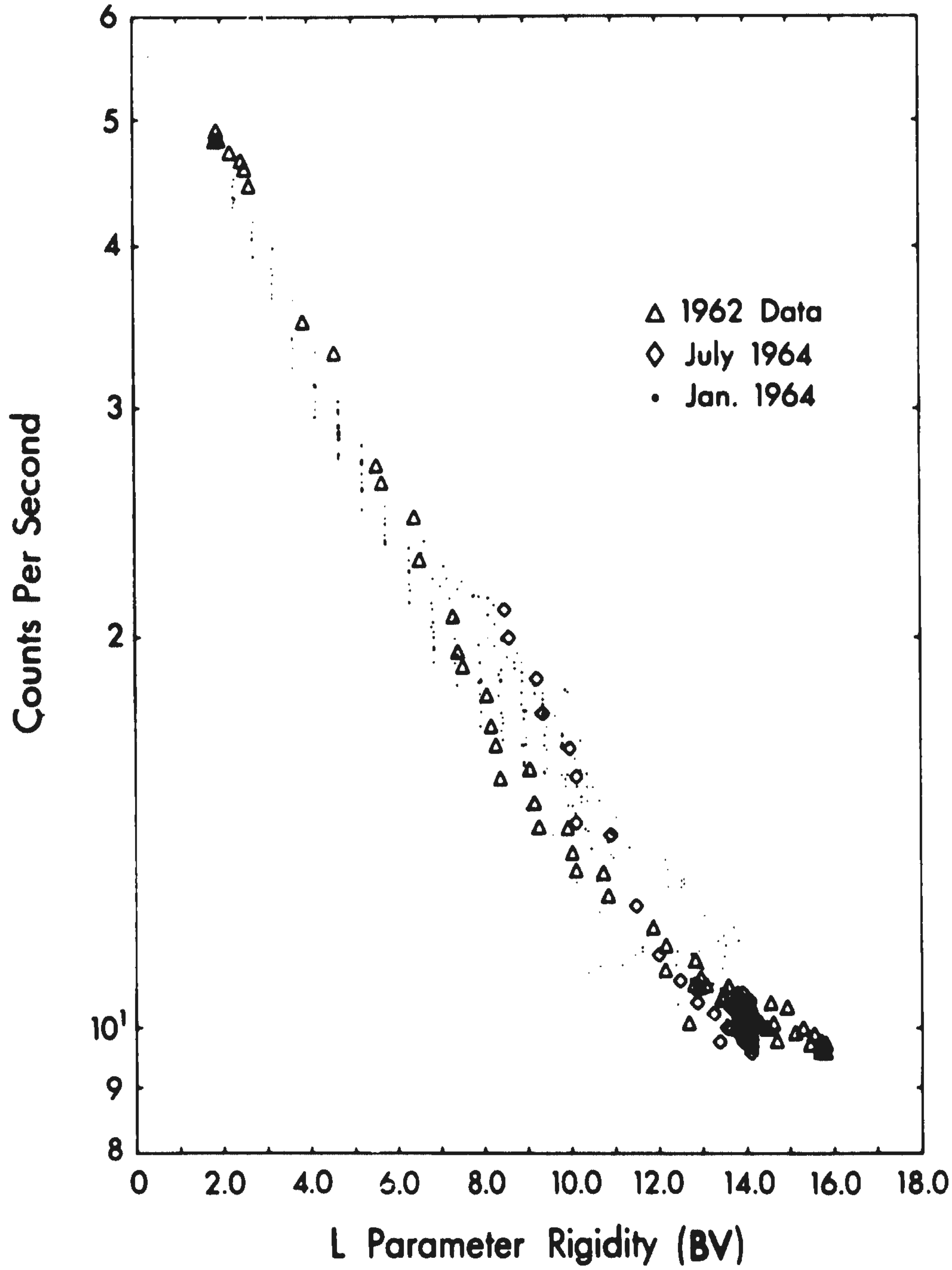

Fig. 6. Correcteu counting rate as a function of L parameter rigidities.

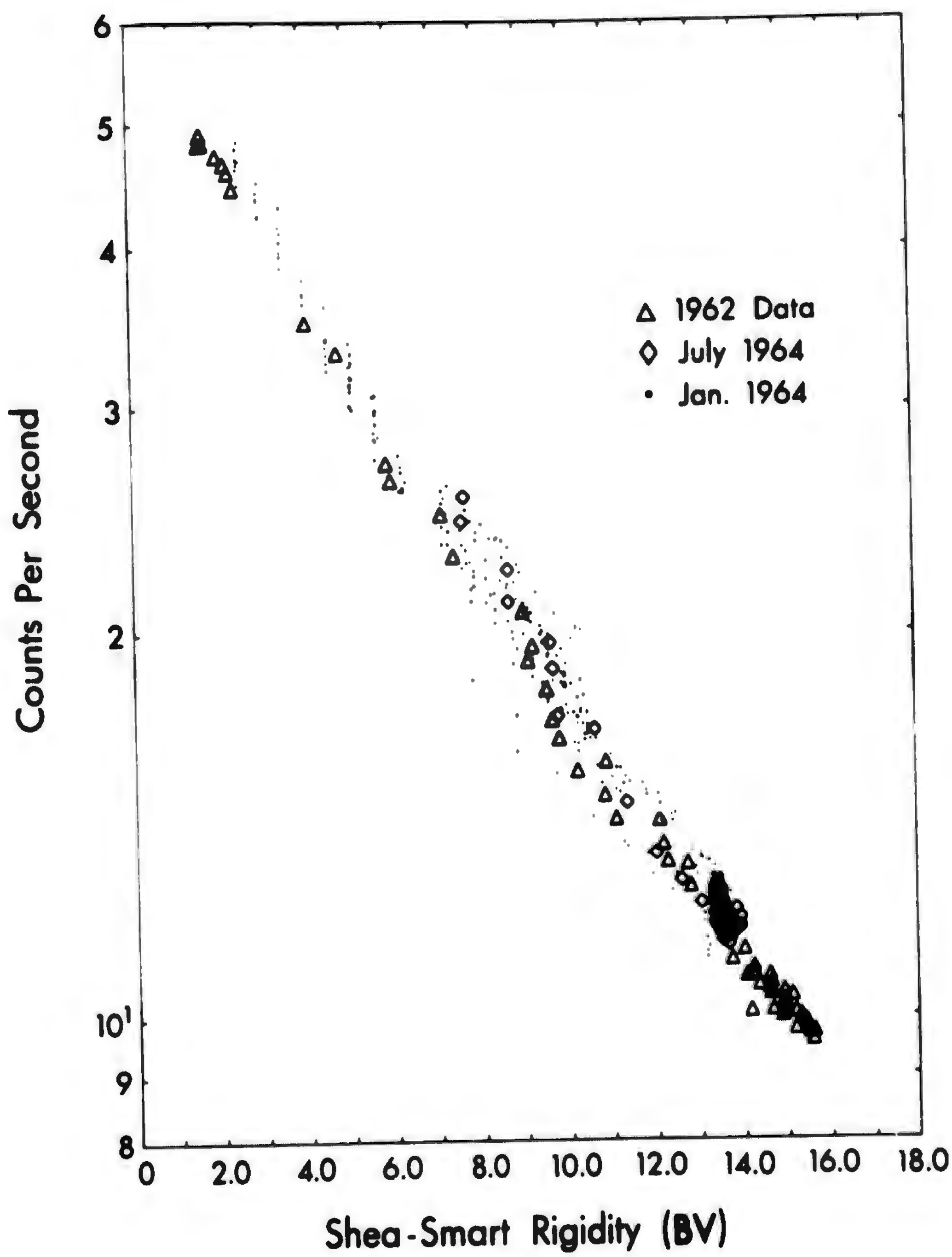

Fig. 7. Corrected counting rate as a function of rigidities computed by exact trajectory calculations.

These rms deviations and the plots shown in Figures 2 through 7 may be used as a basis for evaluating the validity of the different rigidity models.

DIPOLE MODEL

The dipole model has proven to be the worst approximation. This agrees with the results of all other latitude surveys and is to be expected because it employs the least refinement of all the models.

QUENBY-WEBBER AND QUENBY-WENK MODELS

Both of these models represent an improvement over the dipole, with better ordering accomplished by the Quenby-Wenk [1962] model. There is still a considerable scatter of points in the mid-latitude region due, in large part, to inadequate corrections in the region of the Brazilian anomaly. This wide scatter does not show up in the Makino [1963] or the Shea et al. [1963] computations.

MAKINO AND SHEA-SMART-McCRACKEN MODELS

Both the Makino [1963] model and the Shea et al. [1965] calculations order the data equally well, and significantly better than the other models. Since the Shea et al. [1965] rigidities are based on exact trajectory calculations, it is not surprising that they do a good job of ordering the data. The rms error in both cases is about 0.8 counts sec^{-1}, or about

2.5 percent of the median value. In the discussion of experimental errors, we had noted that a reasonable value for the rms error of an experimental point was about two or three percent. Hence, it is concluded that the Shea-Smart-McCracken calculations should be used wherever available, but that the Makino [1963] model may be used for data taken with moderate precision over the portions of the globe covered in this survey.

L PARAMETER CALCULATIONS

The rms scatter of experimental points from the best fit curve was found to be significantly larger for the L parameter than even the Quenby-Wenk [1962] model. It is seen from Figure 6 that this model is worst in the equatorial regions.

The L parameter is primarily useful because it effectively restores dipole symmetry to the geomagnetic field, and because it allows good demographic descriptions to be applied to trapped radiation. Its applicability as a demographic parameter for primary cosmic radiation is questionable since no rigorous theoretical basis has been shown to justify this use of the parameter.

One can try several different approaches in an attempt to justify this use of the L parameter. The simplest way is to start from Stoermer's theory, and generalize to the real field by changing the dipole latitude to an invariant latitude. This invariant latitude is frequently expressed as a function of L, $L = \cos^{-2}\lambda_L$, from which, we derive an L-dependent rigidity,

$$P_L = P_e \cos^4 \lambda_L = P_e L^{-2}. \tag{7}$$

One of the criticisms that must be raised to this method is that it is difficult to talk about the invariant latitude of a point on the surface of the earth where the L parameter is smaller than 1. More fundamentally, perhaps, is the difficulty that the L parameter does not completely restore dipole symmetry to the real field, and hence, the use of dipole-like equations is not strictly valid. This objection is most pronounced in equatorial regions.

A second way of deriving Equation (7) without using the intermediary of an invariant latitude is to start from the general Stoermer expression in Equation (1). One notes that in the special case of the north-south meridian plane, the cutoff rigidity is constant, and thus the vertical cutoff rigidity is equal to the cutoff rigidity of a particle arriving parallel to the field line. If the guiding center approach could be used, one could then say that this particle would follow the flux tube up to the equatorial region. One notes that the cutoff in the north-south meridian plane at the equator out in the magnetosphere from Equation (1) is simply

$$P_L = \frac{M}{4r_e^2} = P_e L^{-2} \tag{8}$$

where r_e is the place where the flux tube crosses the equator, $P_e = M/4r_\oplus^2$, and $r_e = Lr_\oplus$. Therefore, one arrives at the same result as Equation (7) without having to go through the awkward step of using an invariant latitude. The principal objections to this treatment are that the guiding center approximation is not justified and that errors are introduced by assuming that the impact point and the guiding center coincide. The guiding center approximation is invalid because the adiabatic requirement $\frac{\rho|\vec{\nabla}B|}{|B|} << 1$, that $|\vec{B}|$ be essentially constant over one radius of gyration of the particle is not satisfied. These two objections are most critical for low-latitude regions, and give one an insight into the reasons why the dipole equations cannot be used, even though the L parameter is designed to restore dipole symmetry to the field parameters.

Since rigidities proportional to L^{-2} are based on dipole-like equations, they are least accurate in regions where dipole symmetry cannot be perfectly restored to the real field. At low latitude the guiding center is not near the impact point and hence the L value of the impact point is not a good measure of the field sampled by the incoming particle. In addition, the adiabatic invariant is inviolate over most of the trajectory for particles arriving at high latitudes, but is violated for all the trajectory at low latitudes. As a consequence, there should exist a non-systematic error in the calculation of low-latitude rigidities using the L parameter.

Furthermore, since no penumbral corrections are contained in this model, a systematic error will exist in the mid-latitude regions with the calculated rigidities being too low. It is to be noted that these considerations are borne out by the experimental observation that a large scatter of points exists in the equatorial region, and that the entire curve sags in the mid-latitude region.

ADDITIONAL RESULTS

We have made several equatorial crossings which allow us to locate the cosmic-ray equator with rather high precision. All of these determinations were made from a number of crossings at different times of day along the same longitude. The error assigned to the experimental points was arrived at on the basis of statistical error, as well as consistency from flight to flight, so that the data were checked for internal and external consistency. These crossings were compared with exact rigidity computations of Shea et al. [1965], as well as the location of the minimum value of L, and are shown in Table II.

TABLE II

Longitude	Experimental Cosmic Ray Equator	Shea et al. [1965] Calculations	L Minimum
190°	(1.3 ± 0.5)°S	(1.0 ± 0.5)°S	(1.0 ± 0.5)°N
280°	(10.0 ± 1.0)°S	(10.0 ± 0.5)°S	(11.5 ± 0.5)°S
291°	(8.0 ± 2.0)°S	(8.0 ± 0.5)°S	(13.5 ± 0.5)°S
320°	(1.0 ± 1.0)°N	(1.0 ± 0.5)°N	(7.5 ± 0.5)°S

The agreement between the exact trajectory calculations and the experimental measurements is quite striking. The lack of agreement with the

L parameter is equally striking, and is another indication of the failure of this parameter as a demographic parameter for galactic cosmic radiation at low latitude.

SUMMARY

Measurements of the atmospheric cosmic-ray flux over large portions of the globe have been used to determine the accuracy of various cosmic-ray rigidity models.

The rigorous trajectory calculations of Shea et al. give good agreement with the counting rates. The Makino model also orders these data equally well, and therefore must be within a few percent of the value determined by rigorous orbit calculations taken over the portions of the globe covered in this survey.

The other models are demonstrably worse, and a discussion of where these models are inadequate is given. Particular attention is paid to the use of the ' parameter as a substitute for rigorous integration procedures. It is shown both experimentally and theoretically that the L parameter is not a good demographic parameter for galactic cosmic radiation at the lower latitudes, but should be sufficiently accurate at higher latitudes for most purposes.

REFERENCES

Finch, H. P., and B. R. Leaton, The earth's main magnetic field - epoch 1955, Monthly Notices Roy. Astron. Soc. Geophys. Suppl., 7, 314-317, 1957.

Jensen, D. C., and J. C. Cain, An interim geomagnetic field, J. Geophys. Res., 67, 3568-3569, 1962.

Lin, W. C., D. Venkatesan, and J. A. Van Allen, Latitude survey of cosmic-ray intensity by Explorer 7, October, 1959 to February, 1961, J. Geophys. Res., 68, 4885-4896, 1963.

Makino, T., On the cosmic-ray cutoff rigidities and the earth's magnetic field, Rept. Ionosphere Space Res. Japan, 17, 173-186, 1963.

McCracken, K. G., U. R. Rao, and M. A. Shea, The trajectories of cosmic rays in a high-degree simulation of the geomagnetic field, Mass. Inst. Technol. Res. Lab. Electron. Tech. Rept., 77, August, 1962.

Quenby, J. J., and W. R. Webber, Cosmic-ray cutoff rigidities and the earth's magnetic field, Phil. Mag., 4, 90-105, 1959.

Quenby, J. J., and G. J. Wenk, Cosmic ray threshold rigidities and the earth's magnetic field, Phil. Mag., 7, 1457-1485, 1962.

Sauer, H. H., and E. C. Ray, On cosmic-ray cutoffs, Ann. Phys., 25, 135-142, 1963.

Shea, M. A., D. F. Smart, and K. G. McCracken, A study of vertically incident cosmic-ray trajectories using sixth-degree simulations of the geomagnetic field, AFCRL-65-705 Environmental Res. Paper #141, September, 1965.