NRL Report 6451

Determination of Fixed Base Natural Frequencies of Dual Foundation Shipboard Equipments by Shake Tests

L. P. Petak and G. J. O'Hara

Structures Branch
Mechanics Division

August 23, 1966

NAVAL RESEARCH LABORATORY
Washington, D.C.

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Abstract: This report investigates the application of resonance testing to determine the fixed base natural frequencies of an in-place shipboard equipment-foundation system which is supported at two base points. Translation of the base points can cause the structure to rotate as it translates. A method was previously proposed which required the simultaneous use of two shakers capable of phase control and a range of force ratios at each driving frequency; this method would be quite difficult and expensive to apply in the field. A new technique is proposed herein which requires only one shaker to be applied at a time, thereby reducing the difficulties and restrictions considerably. An application of the technique to a theoretical system, and an analog simulation of a shake test utilizing the new method are presented.

INTRODUCTION

The determination of the fixed base natural frequencies* of in-place structures has been a recurrent problem to engineers interested in structural dynamics, particularly from the point of view of shock analysis. For example, a shock design-analysis method developed at the Naval Research Laboratory (1) for shipboard equipment requires the use of design shock spectra in the calculation of stresses and deflections of a contemplated equipment for which no measured shock spectrum exists. These shock design curves are developed either by an inverse application of the design-analysis method itself (using measured responses obtained from realistic shock tests on in-place equipment-foundation systems) or from actual shock spectra obtained from realistic shock tests. Either method requires that the fixed base natural frequencies of the in-place system be known, along with information on mode shapes and modal masses. The frequencies can be determined experimentally or by calculations. Calculations can become cumbersome and have inherent errors for complicated structures; so a convenient experimental method which can be implemented directly upon the in-place system is desirable; it can also serve as a check upon calculated values.

At present, the most convenient method for obtaining these fixed base frequencies is that of shake tests. This involves exciting the structure with a sinusoidal force at various frequencies and recording the response at significant locations. If the structure were truly attached to a fixed base, peaks due to resonance would occur in the response at the fixed base natural frequencies of the structure. Unfortunately, however, few structures res[illegible] bases which are infinitely stiff and heavy, and this condition creates a problem in apply[illegible] field test techniques. This is especially true in shipboard situations where in-place equipment-foundation systems have base supporting structures which are relatively flexible, such as decks and bulkheads.

A recent NRL report (2) examined this problem, and concluded that it is impossible to determine fixed-base frequencies of in-place structures by the commonly accepted test means; *i.e.*, driving on the in-place equipment and seeking nulls at its base for various frequencies. A dynamic chain type of structure was examined, and those frequencies which can be found by driving at various locations were discussed. This report (2) also proposed a technique for finding fixed-base frequencies for this type of structure.

NRL Problem F02-18; Projects SF 013-10-01-1790 and RR 009-03-45-5757. This is an interim report on one phase of the problem; work is continuing. Manuscript submitted June 22, 1966.

*A fixed base natural frequency is a natural frequency which would exist if the base were infinitely stiff and heavy.

The new technique is this: Drive at a point on or below the base of the equipment-foundation system and record the responses of the base and a point on the equipment. Note prominent valleys in the response of the base; these points occur at the fixed base frequencies of the system above the base, and of the system below the driving point. Plot the ratio of equipment response to base response. The fixed base frequencies of interest are the points where prominent peaks in the ratio plot coincide with valleys in the base response plot. This method eliminates the extraneous fixed base frequencies of the system below the driving point.

In a section of the previous report, a structure supported at two base points was discussed. Translation of the base points can cause the structure to rotate as well as translate. A technique using two shakers with variable force and phase control was proposed to find its fixed base frequencies; and a few of the inherent difficulties were pointed out.

The purpose of this report is to examine this problem in detail and remove some of the restrictions and difficulties.

THE PROBLEM

A special, simplified, case in the class of problems dealing with finding the fixed base frequencies of structures which rest on two foundations will be discussed first. The motive for this is to allow the reader to follow the mathematical argument more easily, and to discern without entangling perplexity the extension of the ideas to more complicated cases.

Consider a structure such as shown in Fig. 1. This configuration with unequal parameters allows small rotation coupled with translation. The upper massless bar with lumped masses represents the equipment-and-foundation system of Fig. 2; the lower bar, masses, and springs represent the supporting structure. All joints are considered pinned.

The equipment-and-foundation system has two natural frequencies and two associated mode shapes. These mode shapes cause displacements across springs K_1 and K_2 and resultant spring forces. In Reference 2 it was stated that if the entire system (Fig. 1) is excited at points C and D with forces having magnitudes proportional to the respective modal spring forces and the driving frequency equaling the modal frequency for the upper system, then it will act as a fixed base system and the motion of points C and D will tend to vanish as this condition is approached.

The method of Reference 2 would yield correct results, but difficulties in application are apparent. For the case illustrated, two shakers are required, one at C and one at D. The shakers

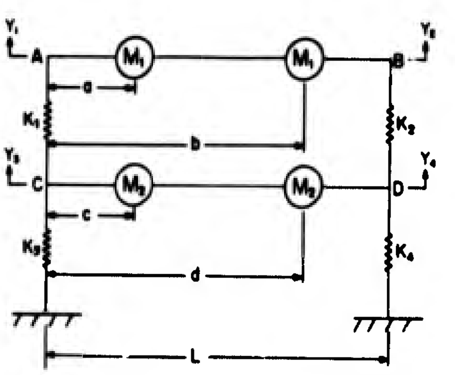


Fig. 1 – Simulated shipboard system with equipment-and-foundation system in place

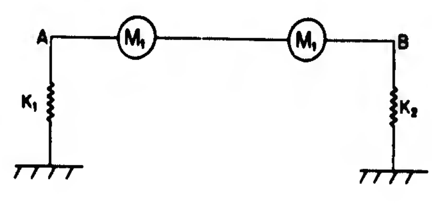


Fig. 2 – The equipment-and-foundation system of interest for which fixed base natural frequencies are desired

must not only sweep through the frequency range of interest, but at each frequency they must also sweep through a range of force ratios. They must (for this undamped system) operate in phase to obtain one frequency and out of phase to obtain the other. The system of interest will act as if on a fixed base only when the driving frequency, force ratio, and phase are simultaneously coincident with the respective quantities for a fixed base natural mode of the system. This of course presents obvious difficulties which would make such field measurements costly and time consuming.

Further attention was applied to this problem, and, as a result, a much more convenient technique has been devised.

THEORY AND ANALYSIS

Consider again the structure shown in Fig. 1. The equations of motion for free vibration may be written as

$$\left[\left(\frac{L-a}{L}\right)^2 + \left(\frac{L-b}{L}\right)^2\right] M_1\ddot{y}_1 + \left[\frac{a}{L}\left(\frac{L-a}{L}\right) + \frac{b}{L}\left(\frac{L-b}{L}\right)\right] M_1\ddot{y}_2 + K_1\,(y_1 - y_3) = 0$$

$$\left[\frac{a}{L}\left(\frac{L-a}{L}\right) + \frac{b}{L}\left(\frac{L-b}{L}\right)\right] M_1\ddot{y}_1 + \left[\left(\frac{a}{L}\right)^2 + \left(\frac{b}{L}\right)^2\right] M_1\ddot{y}_2 + K_2\,(y_2 - y_4) = 0$$

$$\left[\left(\frac{L-c}{L}\right)^2 + \left(\frac{L-d}{L}\right)^2\right] M_2\ddot{y}_3 + \left[\frac{c}{L}\left(\frac{L-c}{L}\right) + \frac{d}{L}\left(\frac{L-d}{L}\right)\right] M_2\ddot{y}_4 - K_1(y_1 - y_3) + K_3 y_3 = 0 \tag{1}$$

$$\left[\frac{c}{L}\left(\frac{L-c}{L}\right) + \frac{d}{L}\left(\frac{L-d}{L}\right)\right] M_2\ddot{y}_3 + \left[\left(\frac{c}{L}\right)^2 + \left(\frac{d}{L}\right)^2\right] M_2\ddot{y}_4 - K_2\,(y_2 - y_4) + K_4 y_4 = 0.$$

By making the substitutions,

$$\alpha = \left[\left(\frac{a}{L}\right)^2 + \left(\frac{b}{L}\right)^2\right] \qquad \delta = \left[\left(\frac{c}{L}\right)^2 + \left(\frac{d}{L}\right)^2\right]$$

$$\beta = \left[\frac{a}{L}\left(\frac{L-a}{L}\right) + \frac{b}{L}\left(\frac{L-b}{L}\right)\right] \qquad \epsilon = \left[\frac{c}{L}\left(\frac{L-c}{L}\right) + \frac{d}{L}\left(\frac{L-d}{L}\right)\right], \tag{2}$$

$$\gamma = \left[\left(\frac{L-a}{L}\right)^2 + \left(\frac{L-b}{L}\right)^2\right] \qquad \eta = \left[\left(\frac{L-c}{L}\right)^2 + \left(\frac{L-d}{L}\right)^2\right],$$

and the usual solution assumption

$$y_j = \bar{y}_j \sin\,(\omega t + \phi), \tag{3}$$

a set of algebraic equations results, whose characteristic equation is

$$\psi_4 = \begin{vmatrix} -\gamma M_1\omega^2 + K_1 & -\beta M_1\omega^2 & -K_1 & 0 \\ -\beta M_1\omega^2 & -\alpha M_1\omega^2 + K_2 & 0 & -K_2 \\ -K_1 & 0 & -\eta M_2\omega^2 + K_1 + K_3 & -\epsilon M_2\omega^2 \\ 0 & -K_2 & -\epsilon M_2\omega^2 & -\delta M_2\omega^2 + K_2 + K_4 \end{vmatrix} = 0. \tag{4}$$

Similarly, the characteristic equation of the upper system, shown in Fig. 2, can be shown to be

$$\psi_2 = \begin{vmatrix} -\gamma M_1\omega^2 + K_1 & -\beta M_1\omega^2 \\ -\beta M_1\omega^2 & -\alpha M_1\omega^2 + K_2 \end{vmatrix} = 0. \tag{5}$$

Thus, the complete system frequencies are the roots of Eq. 4; the fixed base natural frequencies of the system of interest are the roots of Eq. 5.

If sinusoidal forces, $P_C \sin \omega t$ and $P_D \sin \omega t$, are applied at points C and D, respectively, of Fig. 1, the equations of motion become

$$\begin{aligned}
&\gamma M_1\ddot{y}_1 + \beta M_1\ddot{y}_2 + K_1\,(y_1 - y_3) = 0\\
&\beta M_1\ddot{y}_1 + \alpha M_1\ddot{y}_2 + K_2\,(y_2 - y_4) = 0\\
&\eta M_2\ddot{y}_3 + \epsilon M_2\ddot{y}_4 - K_1\,(y_1 - y_3) + K_3\,y_3 = P_C \sin \omega t\\
&\epsilon M_2\ddot{y}_3 + \delta M_2\ddot{y}_4 - K_2\,(y_2 - y_4) + K_4 y_4 = P_D \sin \omega t.
\end{aligned} \tag{6}$$

The solution is of the form

$$y_i = \bar{y}_i \sin \omega t, \tag{7}$$

so that the responses at the base points are

$$\begin{aligned}
\bar{y}_C &= \frac{1}{\psi_4}\,[(-\delta M_2\omega^2 + K_2 + K_4)\;\psi_2 - K_2^2\,(-\gamma M_1\omega^2 + K_1)]\;P_C\\
&+ \frac{1}{\psi_4}\,[(\epsilon M_2\omega^2)\;\psi_2 + K_1\,K_2\,(\beta M_1\omega^2)]\;P_D\\
\bar{y}_D &= \frac{1}{\psi_4}\,[(\epsilon M_2\omega^2)\;\psi_2 + K_1\,K_2\,(\beta M_1\omega^2)]\;P_C\\
&+ \frac{1}{\psi_4}\,[(-\eta M_2\omega^2 + K_1 + K_3)\;\psi_2 - K_1^2\,(-\alpha M_1\omega^2 + K_2)]\;P_D.
\end{aligned} \tag{8}$$

Let

$$\begin{aligned}
\chi_{CC} &= [(-\delta M_2\omega^2 + K_2 + K_4)\;\psi_2 - K_2^2\,(-\gamma M_1\omega^2 + K_1)]\\
\chi_{CD} = \chi_{DC} &= [(\epsilon M_2\omega^2)\;\psi_2 + K_1\,K_2\,(\beta M_1\omega^2)]\\
\chi_{DD} &= [(-\eta M_2\omega^2 + K_1 + K_3)\;\psi_2 - K_1^2\,(-\alpha M_1\omega^2 + K_2)];
\end{aligned} \tag{9}$$

then the response equations reduce to

$$\begin{aligned}
\bar{y}_C &= \frac{\chi_{CC}}{\psi_4}\,P_C + \frac{\chi_{CD}}{\psi_4}\,P_D\\
\bar{y}_D &= \frac{\chi_{CD}}{\psi_4}\,P_C + \frac{\chi_{DD}}{\psi_4}\,P_D.
\end{aligned} \tag{10}$$

The results of previous work (2) indicate that a condition of interest is that for which the base responses simultaneously become zero. This gives:

$$\frac{\chi_{CC}}{\psi_4} P_C + \frac{\chi_{CD}}{\psi_4} P_D = 0$$
$$\frac{\chi_{CD}}{\psi_4} P_C + \frac{\chi_{DD}}{\psi_4} P_D = 0. \tag{11}$$

In order to have a solution other than the trivial one, the determinant of the coefficients of the P's must vanish. Therefore,

$$\frac{\chi_{CC}\,\chi_{DD} - \chi_{CD}^2}{(\psi_4)^2} = 0. \tag{12}$$

Evaluating the numerator gives

$$\begin{aligned}\chi_{CC}\,\chi_{DD} - \chi_{CD}^2 = {} & (-\delta M_2\omega^2 + K_2 + K_4)(-\eta M_2\omega^2 + K_1 + K_3)(\psi_2)^2 \\ & -K_1^2\,(-\alpha M_1\omega^2 + K_2)(-\delta M_2\omega^2 + K_2 + K_4)\psi_2 \\ & -K_2^2\,(-\gamma M_1\omega^2 + K_1)(-\eta M_2\omega^2 + K_1 + K_3)\psi_2 \\ & +K_1^2\,K_2^2(-\gamma M_1\omega^2 + K_1)(-\alpha M_1\omega^2 + K_2) \\ & -(\epsilon M_2\omega^2)^2\,(\psi_2)^2 - 2K_1K_2\,(\beta M_1\omega^2)(\epsilon M_2\omega^2)\psi_2 \\ & -K_1^2\,K_2^2\,(\beta M_1\omega^2)^2;\end{aligned} \tag{13}$$

and since in Eq. 5

$$\psi_2 = (-\gamma M_1\omega^2 + K_1)(-\alpha M_1\omega^2 + K_2) - (\beta M_1\omega^2)^2,$$

Eq. 13 reduces to

$$\begin{aligned}\chi_{CC}\,\chi_{DD} - \chi_{CD}^2 = {} & \{[(-\delta M_2\omega^2 + K_2 + K_4)(-\eta M_2\omega^2 + K_1 + K_3) - (\epsilon M_2\omega^2)^2]\psi_2 \\ & -K_1^2\,(-\alpha M_1\omega^2 + K_2)(-\delta M_2\omega^2 + K_2 + K_4) \\ & -K_2^2\,(-\gamma M_1\omega^2 + K_1)(-\eta M_2\omega^2 + K_1 + K_3) \\ & -2K_1\,K_2\,(\beta M_1\omega^2)(\epsilon M_2\omega^2) + K_1^2\,K_2^2)\}\,\psi_2.\end{aligned} \tag{14}$$

The enclosed term is simply ψ_4, by means of Eq. 4. Thus,

$$\chi_{CC}\,\chi_{DD} - \chi_{CD}^2 = (\psi_4)(\psi_2), \tag{15}$$

so Eq. 12 now becomes

$$\frac{(\psi_4)(\psi_2)}{(\psi_4)^2} = 0, \tag{16}$$

or

$$\frac{\psi_2}{\psi_4} = 0. \tag{17}$$

There are three frequencies at which simultaneous nulls can occur: λ_1 and λ_2, the fixed base frequencies of the upper system; and infinite frequency, since the order of ψ_4 is greater than that of ψ_2. Associated with each of these frequencies is a force ratio which will satisfy Eqs. 11. Simultaneous base nulls will occur *only* when these frequency and force ratio relationships are satisfied by the driving forces. Since this occurrence is unique, it could be utilized to obtain the desired frequencies, as was previously proposed. However, a field test which attempted to obtain simultaneous base nulls would obviously be cumbersome to perform. The response expressions, Eqs. 10, have been re-examined to determine if a more convenient relationship exists between the desired frequencies and the system response.

If in Eqs. 10 the ratio of $\bar{y}_C$ to $\bar{y}_D$ were taken, then

$$\frac{\bar{y}_C}{\bar{y}_D} = \frac{\chi_{CC} P_C + \chi_{CD} P_D}{\chi_{CD} P_C + \chi_{DD} P_D}. \tag{18}$$

This ratio will be ***independent*** of the P's when

$$\frac{\chi_{CC}}{\chi_{CD}} = \frac{\chi_{CD}}{\chi_{DD}}. \tag{19}$$

However, this is exactly

$$\chi_{CC}\,\chi_{DD} - \chi_{CD}^2 = (\psi_4)(\psi_2) = 0. \tag{20}$$

Therefore, the ratio of $\bar{y}_C$ to $\bar{y}_D$ is independent of the P's at the complete system natural frequencies and at the fixed base natural frequencies of the upper system. Since the ratio is independent of the P's at these frequencies, it makes no difference if one of the P's is zero. This provides a convenient technique for finding the desired fixed base natural frequencies.

TECHNIQUE

On the system of Fig. 1 apply the driving force at C and sweep through the frequency range. Monitor the response at C, at D, and the ratio of the response at C to the response at D. The system natural frequencies will show as resonant peaks in the plots of the responses at C and D. The ratio

$$\frac{\bar{y}_C}{\bar{y}_D} = \frac{\chi_{CC}}{\chi_{DC}} \tag{21}$$

has also been obtained.

Repeat the above procedure with the force applied at D. Again; the system frequencies appear as resonant peaks in the direct response plots. The ratio

$$\frac{\bar{y}_C}{\bar{y}_D} = \frac{\chi_{CD}}{\chi_{DD}} \tag{22}$$

has also been obtained.

Now, superpose the ratio plots (Eqs. 21 and 22). The curves will cross at frequencies where the two ratios are equal. These points must be at either the complete system frequencies or the fixed base frequencies of the upper system, by reason of Eqs. 19 and 20. The complete system frequencies are found by examination of the responses at *C* and *D*; crossings at these frequencies can be eliminated from consideration, then the remaining crossings must occur at the desired fixed base frequencies.

THEORETICAL APPLICATION

To illustrate the technique proposed in the preceding section, the following values were assigned to the parameters of the system in Fig. 1:

$$M_1 = 1 \quad K_1 = 1 \quad K_2 = \frac{11}{25} K_1$$

$$M_2 = 2M_1 \; K_3 = 2K_1 \; K_4 = \frac{33}{25} K_1.$$

The system natural frequencies for these parameters are

$$\phi_1 = 0.596 \text{ rad/sec}$$

$$\phi_2 = 1.154 \text{ rad/sec}$$

$$\phi_3 = 1.896 \text{ rad/sec}$$

$$\phi_4 = 3.717 \text{ rad/sec},$$

and the fixed base natural frequencies of the upper system are

$$\lambda_1 = 0.775 \text{ rad/sec}$$

$$\lambda_2 = 2.569 \text{ rad/sec}.$$

Proceeding with the proposed method, a theoretical shake test was performed upon the structure.

The response at *C*, response at *D*, and the ratio of the response of *C* to that at *D* for a driving force applied at *C* are shown in Figs. 3, 4, and 5, respectively. These graphs are plotted as absolute amplitude *versus* driving frequency since this is the most probable way that real data would be presented. Figures 6, 7, and 8 present the respective quantities for a driving force applied at *D*. The two ratio plots are superposed to give the graph of Fig. 9. Inspection of this graph shows six points where the ratios are equal. The frequencies corresponding to these six crossing points are

$$\omega = 0.60 \text{ rad/sec}$$

$$\omega = 0.77 \text{ rad/sec}$$

$$\omega = 1.15 \text{ rad/sec}$$

$$\omega = 1.90 \text{ rad/sec}$$

$$\omega = 2.57 \text{ rad/sec}$$

$$\omega = 3.72 \text{ rad/sec}.$$

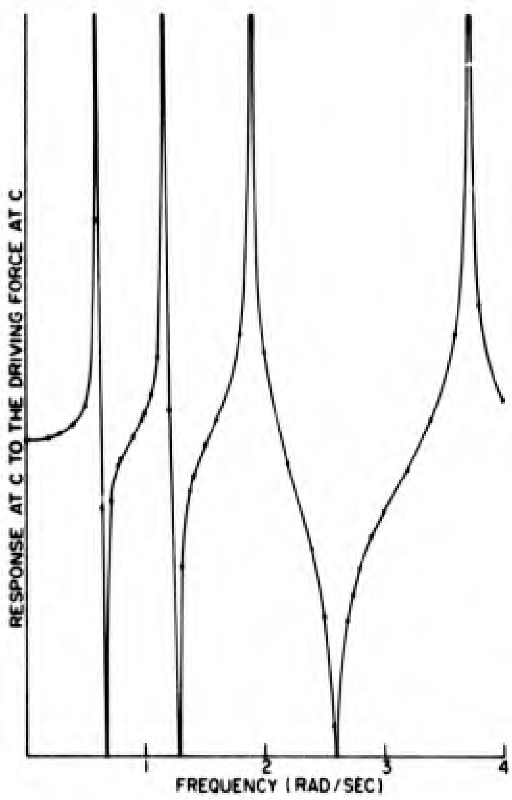


Fig. 3 – The response at *C* to the driving force at *C* in the system of Fig. 1

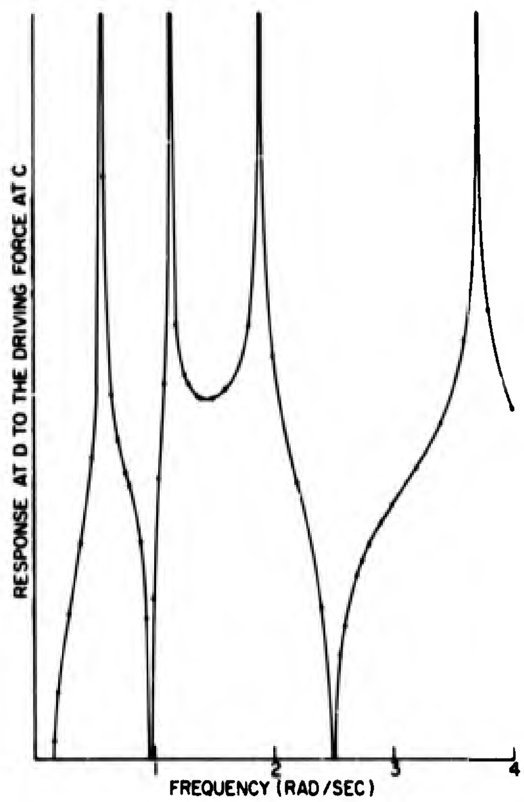


Fig. 4 – The response at *D* to the driving force at *C*

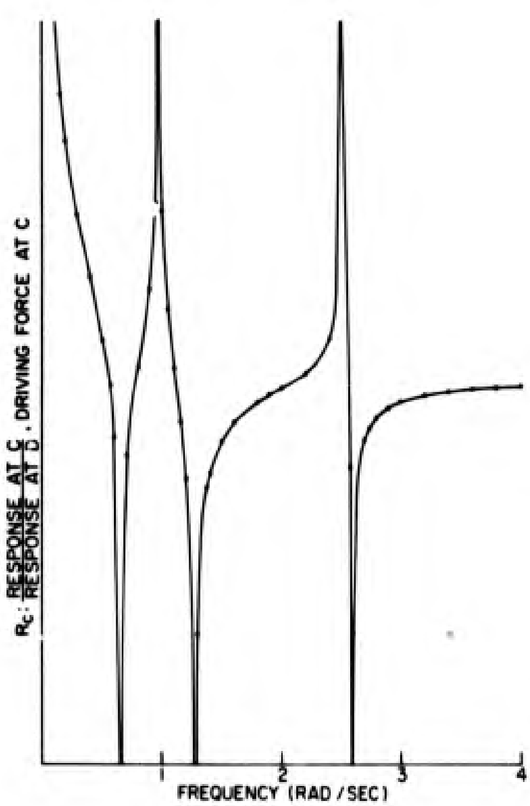


Fig. 5 – The ratio of the response at *C* to that at *D* when the driving force is at *C*

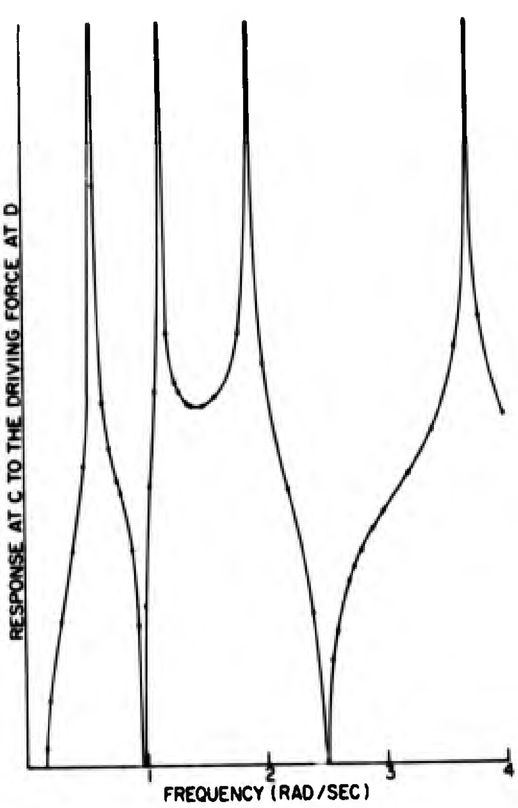


Fig. 6 – The response at *C* to the driving force at *D* in the system of Fig. 1

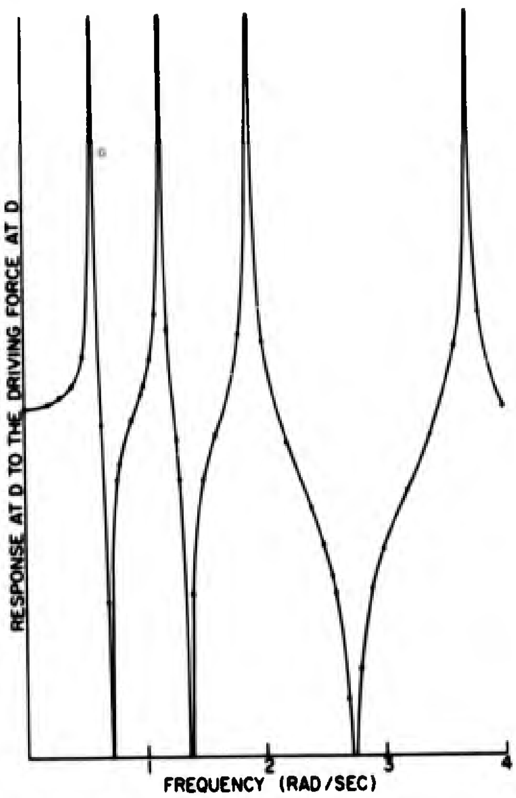


Fig. 7 – The response at *D* to the driving force at *D*

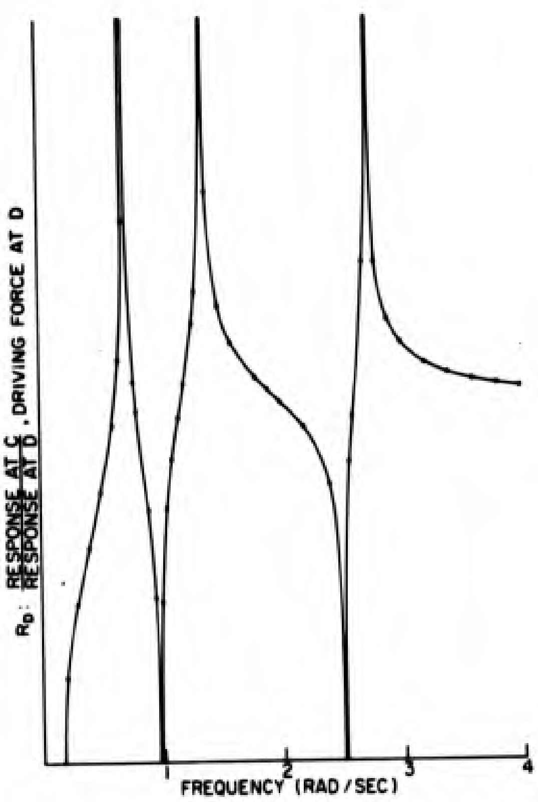


Fig. 8 – The ratio of the response at *C* to that at *D* when the driving force is at *D*

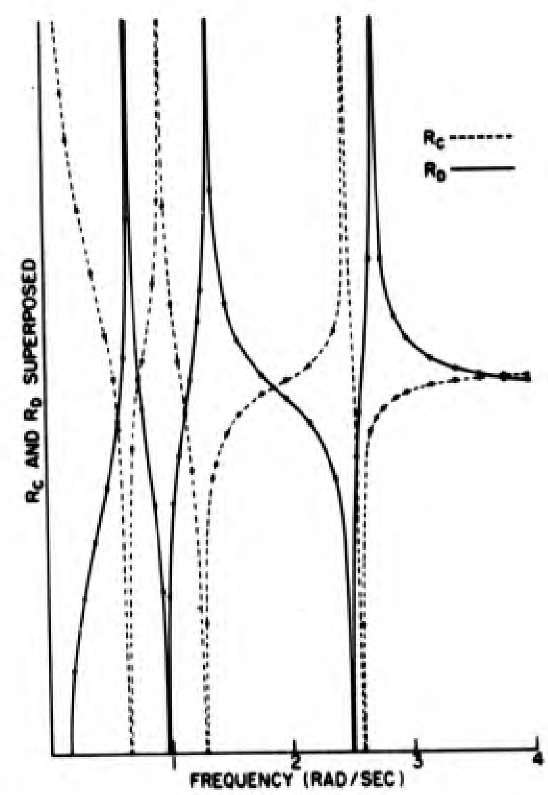


Fig. 9 – The ratio plots, Figs. 5 and 8, superposed to determine at which frequencies the ratios are of equal value

These six frequencies include the four complete system frequencies and the two upper system fixed base frequencies. However, the complete system frequencies can be determined by examination of the direct response plots. Resonant peaks in those plots occur at

$$\omega_1 = 0.60 \text{ rad/sec}$$

$$\omega_2 = 1.15 \text{ rad/sec}$$

$$\omega_3 = 1.90 \text{ rad/sec}$$

$$\omega_4 = 3.72 \text{ rad/sec.}$$

Now, eliminating crossings at these frequencies from consideration; two crossing points remain at $\omega = 0.77$ and $\omega = 2.57$. These are the fixed base natural frequencies of the upper system, the system of interest.

A MORE GENERAL CASE

A more general situation encountered in the practice of resonance testing on in-place structures is found in Fig. 10. This system combines the characteristics of the two-base model with those of the chain model. Once again, the upper system (Fig. 11) is the system of interest for which the fixed base natural frequencies must be determined. When the analyses of the preceding sections is extended to this type of problem, it is found that the behavior of the system is also similar to that of the dynamic chain, and the procedure necessary to find the desired frequencies combines the techniques for each of the two models.

In the dynamic chain, driving forces must be applied at or below the base. If driving forces are applied at points *C* and *D* of Fig. 10, the ratio of the response of *C* to that of *D* will become independent of the ratio of the driving forces at the complete system frequencies, at the fixed base frequencies of the system above points *C* and *D*, and at the fixed base frequencies of the

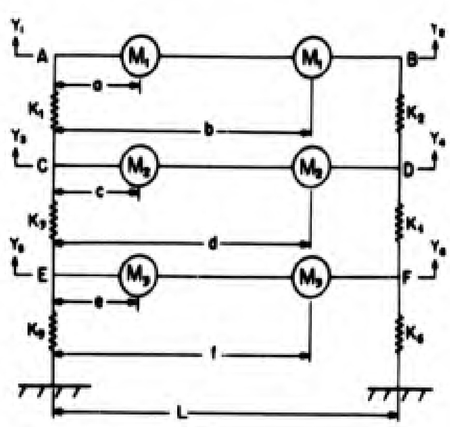
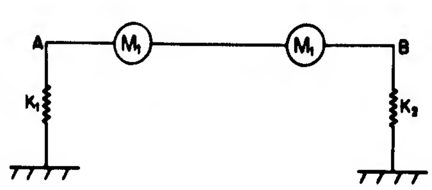


Fig. 10 – A more general system with the system of interest in place

Fig. 11 – The system of interest for which fixed base natural frequencies are desired



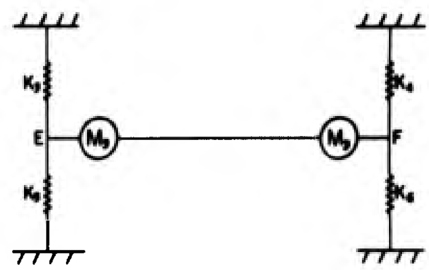


Fig. 12 – The system located below the driving points C and D

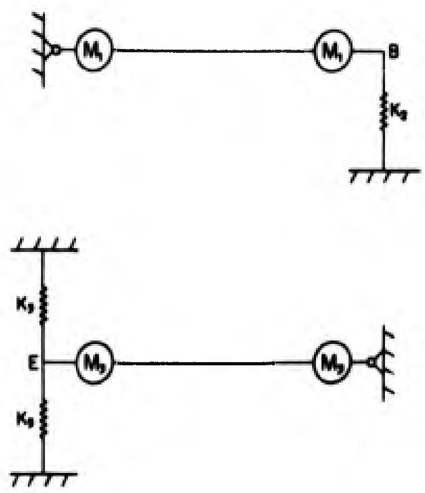


Fig. 13 – The system located above the response points A and D, and the system located below the driving points C and F

system below points C and D. (The system below points C and D is shown in Fig. 12.) The problem remaining is to separate the desired frequencies from the extraneous frequencies of the complete system and the system below points C and D. Consider the driving forces still at C and D. The ratio of the response of A to that of B will become independent of the ratio of the driving forces (at C and D) only at the complete system frequencies and at the fixed base frequencies of the system below points C and D. These frequencies can be eliminated from consideration, and those remaining as crossings on the superposed C-D ratio plot will be the fixed base natural frequencies of interest. Thus, by using the dynamic chain characteristics to select driving and response points, and the techniques used previously in this report to determine the frequencies at which the particular response ratios become independent of their excitation ratios, it is possible to find the desired frequencies.

A general rule for this type of structural system is that if two points, a and b, on the structure are excited with sinusoidal force P_a and P_b, and the responses of two points, 1 and 2, located above the driving points are recorded, then the ratio of the responses, R_1/R_2, will become independent of the ratio of the driving forces, P_a/P_b, at the fixed base frequencies of the system above the response points (as if the response points were fixed against translation), at the fixed base frequencies of the system below the driving points (as if the driving points were fixed against translation), and at the complete system frequencies. (For example, suppose that driving forces were placed at points C and F on the structural system of Fig. 10 and the responses of points A and D were monitored. Then the ratio of the responses, R_A/R_D, would become independent of the ratio of the driving forces, P_C/P_F, at the frequencies of the complete system and at the frequencies of the systems shown in Fig. 13). If the response points are located below the driving points, then the ratio of the responses will become independent of the ratio of the driving forces at the fixed base frequencies of the system below the response points, at the fixed base frequencies of the system above the driving points, and at the complete system frequencies.

DISCUSSION

A comparison of the experimental technique presented herein, as applied to the special case, with that proposed in Ref. 1 shows this to be a much more convenient method. Two frequency sweeps are required; however, only one shaker is applied for each sweep. There is no need to sweep through a band of force ratios (including phase), along with frequency, as was required by the previous method. The response measurements needed can be acquired in the routine manner, and the ratios easily calculated manually or electronically.

In the examples presented here the complete systems have been mounted on a fixed base. However, the technique can also be applied to a free-free complete system with the same results.

SUMMARY AND CONCLUSIONS

Design shock spectra are based on measured responses from realistic shock tests on in-place equipment-foundation systems, and the methods used to derive these design curves require that the fixed base natural frequencies of the in-place system be known. The most convenient method to date for obtaining these frequencies has been through shake tests. However, the past method for finding the frequencies of a certain type of structure, one which rests on two base points which are not fixed and rotates as it translates was rather cumbersome to apply. This report presents a new technique which is much more convenient to implement, and describes how it can be applied to analyze a more general class of structures.

The new technique is this: choose two driving points, (m,n), at or below the base points, (j,k), of the equipment-foundation system for which fixed base frequencies are desired. Choose two points, (r,s), on the system of interest; the responses of these two points along with those of the base points will be necessary to the analysis. Apply a sinusoidal force to one of the driving points and record the responses of the specified points. Plot the ratio of the motion of one base point to that of the other, R_j/R_k; and also the ratio of the motion of one equipment point to that of the other, R_r/R_s. Apply the exciting force at the other driving point and repeat the procedure; plot the same response ratios as before. Superpose the two base point response ratio plots and note the frequencies where crossings occur. These crossings will be at the complete system frequencies, at the fixed base natural frequencies of the system above the base points (the desired frequencies), and at the fixed base frequencies of the system below the driving points. Next, superpose the two equipment point response ratios and once again note the frequencies where crossings occur. These crossings will be at the complete system frequencies, the fixed base frequencies of the system above the equipment response points, and at the fixed base frequencies of the system below the driving points. When the frequencies of the complete system, and of the system below the driving points are known, they may be eliminated from consideration in the set of frequencies noted previously. The frequencies remaining in that set will be the desired fixed base natural frequencies.

Difficulties in placing the shaker and obtaining necessary force levels, as discussed in Ref. 1, remain. However, the difficulties which ensued from the use of two shakers simultaneously have been eliminated. This new technique will conveniently provide the desired fixed base natural frequencies.

REFERENCES

1. Belsheim, R.O., and O'Hara, G.J., "Shock Design of Shipboard Equipment, Part I – Dynamic Design-Analysis Method," NRL Report 5545, Sept. 1960

2. Petak, L.P., and Kaplan, R.E., "Resonance Testing in the Determination of Fixed Base Natural Frequencies of Shipboard Equipment," NRL Report 6176, Dec. 1964

Appendix

ANALOG SIMULATION OF A RESONANCE TEST

An analog computer study was performed to show that the proposed method, utilizing ratio crossing points to obtain certain frequencies, would be feasible in a pseudo-realistic situation. The special case was used to illustrate the technique; parameters used for the theoretical application in the text were scaled to provide convenient, realistic natural frequencies, and the computer was programmed for the equations of motion of the system. A small amount of damping was added to assure stability of the computer solution. Response data was obtained with a commercial impedance plotter, modified to provide two direct responses and their ratio. With this device it was possible to sweep through a frequency range automatically at a predetermined rate of speed and obtain strip chart recordings of the three quantities; or to step through the frequency range at convenient increments and to obtain point by point readings of the desired quantities with a digital voltmeter. The curves presented here were read and plotted at one cycle increments.

The driving force was applied at point *C*; the response of *C*, the response at D, and the ratio of the response of *C* to that of *D* are shown in Figs. A1, A2, and A3, respectively. Peaks occur in the direct response plots at 19 cps, 36 cps, 59 cps, and 116 cps; these are the natural frequencies of the complete system.

The driving force was removed from point *C* and applied at point *D*. Once again, the response at *C* and at *D* was plotted. These are shown in Figs. A4 and A5. The ratio of the motion at *C* to that at *D* is shown in Fig. A6. Resonant peaks in Figs. A4 and A5 verify the system natural frequencies chosen before.

The ratio curves of Figs. A3 and A6 are superposed in Fig. A7. Crossing points occur at frequencies of 19 cps, 24.5 cps, 36.5 cps, 57 cps, 79 cps, and about 110 cps. The crossings at four of these frequencies are associated with the complete system natural frequencies and can be eliminated from consideration. Eliminating the crossings at 19 cps, 36.5 cps, 57 cps, and 110 cps leaves the two crossing at 24.5 cps and 79 cps. The new theory states that these are the fixed base natural frequencies of the upper system, the system of interest. Thus the desired fixed base natural frequencies are 24.5 cps and 79 cps.

The calculated undamped natural frequencies of the complete system are

$$f_1 = 19.0 \text{ cps}$$

$$f_2 = 36.7 \text{ cps}$$

$$f_3 = 60.4 \text{ cps}$$

$$f_4 = 118.3 \text{ cps;}$$

and the calculated undamped fixed base natural frequencies of the upper system are

$$f_a = 24.7 \text{ cps}$$

$$f_b = 81.8 \text{ cps.}$$

The new technique has provided the desired fixed base natural frequencies to a good degree of accuracy. The damping, which is unaccounted for in the calculations, undoubtedly caused the lowering of some of the frequencies to create the discrepancies.

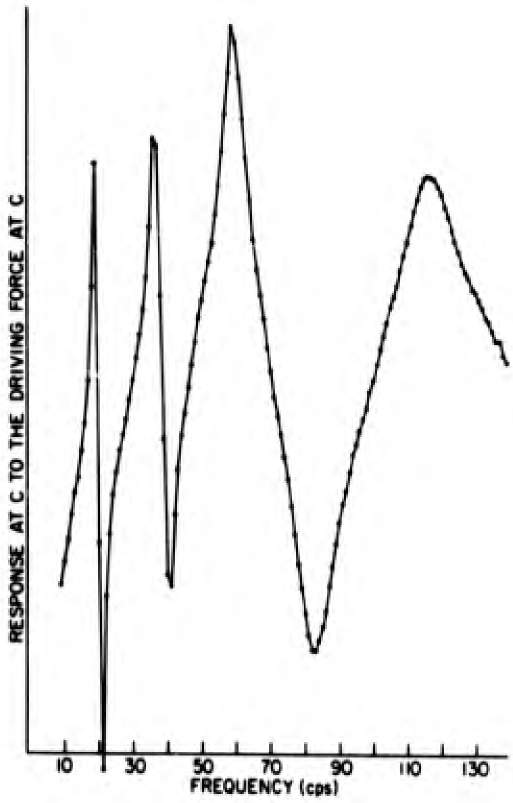


Fig. A1 – The response at *C* to driving force at *C* in the analog simulation of the system shown in Fig. 1

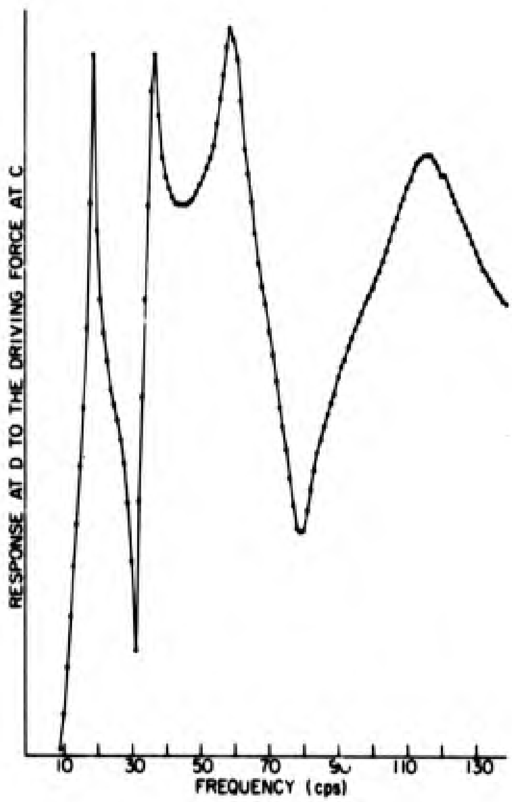


Fig. A2 – The response at *D* to the driving force at *C*

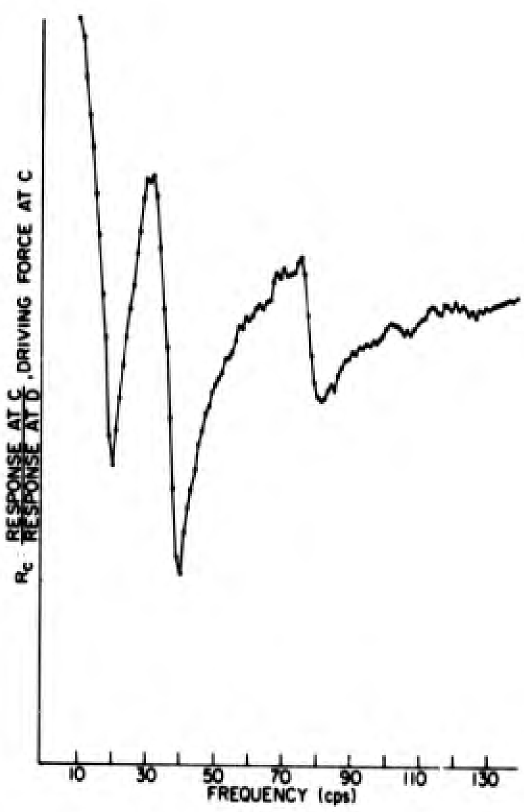


Fig. A3 – The ratio of the response at *C* to that at *D* when the driving force is at *C*

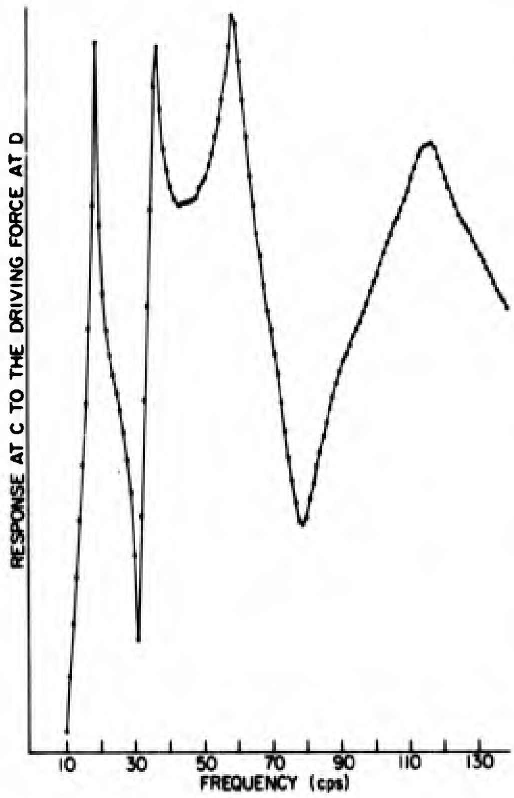


Fig. A4 – The response at *C* to the driving force at *D*

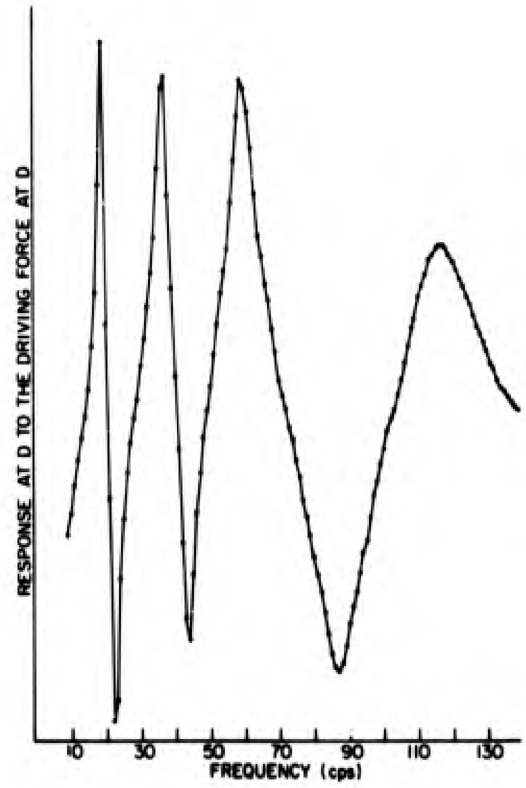


Fig. A5 – The response at *D* to the driving force at *D*

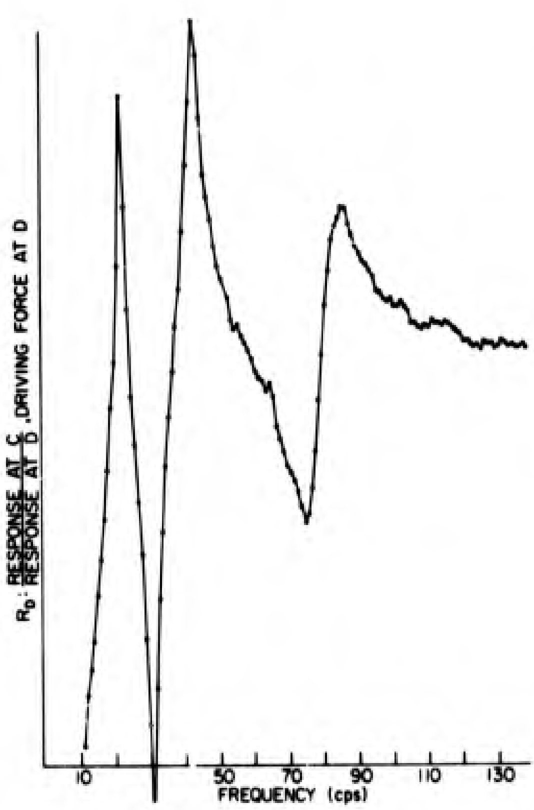


Fig. A6 – The ratio of the response at *C* to that at *D* when the driving force is at *D*

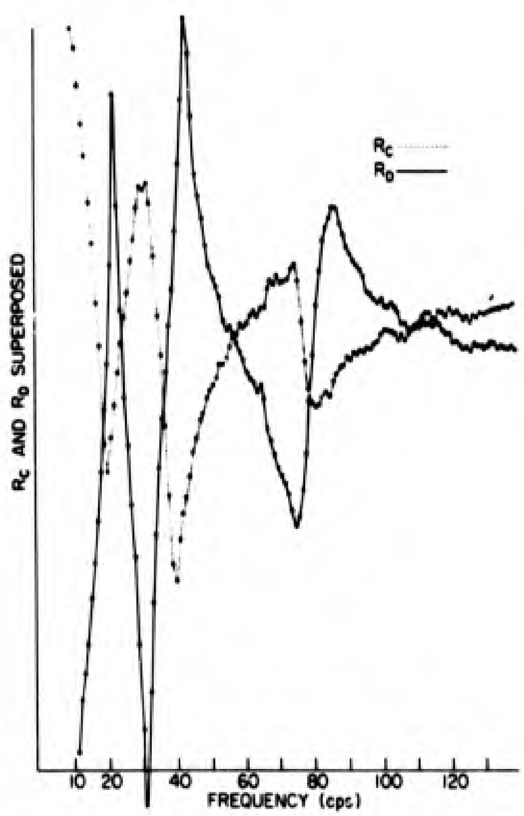


Fig. A7 – The ratio plots, Figs. A3 and A6, superposed to determine at which frequencies the two ratios are of equal value

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(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Research Laboratory Washington, D.C. 20390	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b GROUP

3. REPORT TITLE

DETERMINATION OF FIXED BASE NATURAL FREQUENCIES OF DUAL FOUNDATION SHIPBOARD EQUIPMENTS BY SHAKE TESTS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

An interim report on one phase of the problem.

5. AUTHOR(S) (Last name, first name, initial)

Petak, L.P., and O'Hara, G.J.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August 23, 1966	20	2

8a. CONTRACT OR GRANT NO. NRL Problem F02-18 b. PROJECT NO. SF 013-10-01-1790 c. RR 009-03-45-5757 d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 6451 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Department of the Navy (Office of Naval Research and Naval Ship Systems Command), Washington, D.C. 20360

13. ABSTRACT

This report investigates the application of resonance testing to determine the fixed base natural frequencies of an in-place shipboard equipment-foundation system which is supported at two base points. Translation of the base points can cause the structure to rotate as it translates. A method was previously proposed which required the simultaneous use of two shakers capable of phase control and a range of force ratios at each driving frequency; this method would be quite difficult and expensive to apply in the field. A new technique is proposed herein which requires only one shaker to be applied at a time, thereby reducing the difficulties and restrictions considerably. An application of the technique to a theoretical system, and an analog simulation of a shake test utilizing the new method are presented.

DD FORM 1 JAN 64 1473

14. KEY WORDS	LINK A		LINK B		LINK C	
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In-place equipment						
Shipboard equipment						
Shock spectra						
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