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ERROR ANALYSIS OF RC CIRCUITS COMMONLY USED FOR TIMING

by
William E. Ryan
Ira R. Marcus

July 1966

U.S. ARMY MATERIEL COMMAND
HARRY DIAMOND LABORATORIES
WASHINGTON, D.C. 20438

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RC timers

RC error analysis

This report analyzes various RC timing circuits commonly used in ordnance electronic timers. Timing errors are computed as a function of the component tolerances, specified to account for possible manufacturing, aging, and temperature changes. The effect of capacitor leakage is also computed.

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ABSTRACT

This report analyzes various RC timing circuits commonly used in ordnance electronic timers. Timing errors are computed as a function of the component tolerances, specified to account for possible manufacturing, aging, and temperature changes. The effect of capacitor leakage is also computed.

1. INTRODUCTION

RC electronic timers are used in ordnance devices as medium precision timers to perform a variety of functions. This report analyzes the most commonly used RC timing circuits. The object of the analysis is to provide the designer with a means for easily estimating timing errors of a particular circuit as a function of the tolerances specified for the component parameters. These tolerances may be specified to account for manufacturing variations, temperature variations and aging. This paper is meant to be a convenient reference, for particular cases, as follows.

Case 1. An RC circuit charges from a supply voltage V to a threshold voltage aV. Given the expected variations in the value of the supply voltage, the RC time constant, and the detection level aV, compute the timing variation. The cases where aV is dependent, and independent, of the supply voltage are examined. See figure 1.

Case 2. The timing error in an RC circuit due to the leakage of charge from the charging capacitor is computed. The leakage may occur during the entire cycle, or begin sometime during the cycle. This case includes internal capacitor leakage and leakage into the detector prior to firing. See figures 4 and 5.

Case 3. The timing error due to variations in the initial charge on the capacitor is computed. See figure 11.

Case 4. Adjustable RC timers using charged capacitors as references and where set time is adjustable by precharging the timing capacitor are examined. The timing errors due to leakage of the reference capacitor and the timing capacitor and the errors due to variation in reference amplitude and setting amplitude are computed. See figure 13.

Case 5. Variations in the periods of astable and monostable multivibrators are analyzed.

Case 6. A worst-case analysis is made of pulse-synchronized RC pulse divider circuits, and relationships are derived for reliable operation. See figure 16.

2. DERIVATION OF ERROR EQUATIONS

Case 1. RC Timer with Level Detector. A very common RC timing circuit operates on the principle shown in figure 1. A capacitor charges from a supply voltage V through a fixed resistor. The time period is ended when the voltage on the capacitor reaches a value determined by the voltage detector. Figure 2 shows typical circuits which operate on this principle. Figure 1 shows the charging curve. In this class of circuit, the parameters which determine the timing are the supply voltage, the RC product, and the level detector. All are allowed certain tolerances due to manufacturing spreads, aging, or temperature dependence. The detector level may be independent of, or proportional to, the supply voltage. The variations in parameters are represented as follows. The supply voltage is $V(1 \pm \nu)$ where $\nu = \Delta V/V$. The time constant is $\tau(1 \pm \mu)$ where $\mu = \Delta\tau/\tau$ and $\tau = RC$. The threshold voltage of the detector $= aV(1 \pm \alpha)$ where $\alpha = \Delta a/a$. If V_r is the nominal voltage of the voltage detector, then $a = V_r/V$.

We first consider the detector level independent of the supply voltage. The triggering time is $T(1 \pm \delta)$ where $\delta = \Delta T/T$. The basic charging equation is $v = V(1 - e^{-t/\tau})$. Therefore the minimum-triggering-time equation is

$$aV(1 - \alpha) = V(1 + \nu)\left[1 - e^{-\frac{T(1-\delta)}{\tau(1-\mu)}}\right] \qquad (1)$$

where the applied voltage is a maximum, and the RC product and the threshold detector are at a minimum value. Since the differentials are much less than unity, the second-order terms can be dropped and the following approximations made:

$$\frac{1-\delta}{1-\mu} \simeq (1 - \delta)(1 + \mu) \simeq 1 - \delta + \mu \qquad (2)$$

and

$$e^{-\frac{T}{\tau}(1-\delta+\mu)} = 1 - \frac{a(1-\alpha)}{(1+\nu)} = 1 - a(1 - \alpha - \nu) \qquad (3)$$

The maximum triggering time occurs when the supply voltage is a minimum and the RC product and the threshold voltage are at a maximum value. Thus,

$$aV(1 + \alpha) = V(1 - \nu)\left[1 - e^{-\frac{T(1+\delta)}{\tau(1+\mu)}}\right] \tag{4}$$

where

$$e^{-\frac{T}{\tau}(1+\delta-\mu)} = 1 - a(1 + \alpha + \nu) \tag{5}$$

Dividing (3) by (5) and then dividing numerator and denominator by (1-a) gives

$$e^{-\frac{T(2\mu-2\delta)}{\tau}} = \frac{1 + \frac{a(\alpha+\nu)}{1-a}}{1 - \frac{a(\alpha+\nu)}{1-a}} \tag{6}$$

The nominal charging time T is obtained from

$$aV = V\left(1 - e^{-\frac{T}{\tau}}\right) \tag{7}$$

Therefore,

$$e^{-\frac{T}{\tau}} = 1 - a \tag{8}$$

Substituting (8) into (6) yields

$$(1 - a)^{2\mu-2\delta} = \frac{1 + \frac{a(\alpha+\nu)}{1-a}}{1 - \frac{a(\alpha+\nu)}{1-a}} \tag{9}$$

Taking the natural log of both sides,

$$2(\mu-\delta)\,\ln(1-a) = \ln \frac{1 + \frac{a(\alpha+\nu)}{1-a}}{1 - \frac{a(\alpha+\nu)}{1-a}} \tag{10}$$

and since $\ln\left(\frac{1+x}{1-x}\right) \simeq 2x$ for $x << 1$,

then

$$2(\mu-\delta)\,\ln(1-a) = \frac{2a(\alpha+\nu)}{1-a} \quad \text{for } \alpha + \nu << \frac{1-a}{a} \tag{11}$$

Solving for δ

$$\frac{\Delta T}{T} = \delta = \mu + \frac{a(\alpha+\nu)}{(1-a)\ln\frac{1}{1-a}} \tag{12}$$

or,

$$\delta = \mu + (\alpha+\nu)\, f(a) \tag{13}$$

where

$$f(a) = \frac{a}{(1-a)\ \ln\frac{1}{1-a}}$$

Therefore the time deviation is the sum of the RC product deviation and the sum of the level detector and supply voltage deviation times f(a). The f(a) is always greater than unity (fig. 3).

We now consider the case where the detector voltage is proportional to the supply voltage. Circuits a and c on figure 2 are examples of this type of detector.

Relationships similar to (1) and (4) are

$$aV(1-\alpha)(1\pm\nu) = V(1\pm\nu)\left[1 - e^{-\frac{T(1-\delta)}{\tau(1-\mu)}}\right] \tag{14}$$

for the minimum time and

$$aV(1+\alpha)(1\pm\nu) = V(1\pm\nu)\left[1 - e^{-\frac{T(1+\delta)}{\tau(1+\mu)}}\right] \tag{15}$$

for the maximum time. Simplifying, (14) and (15) become

$$e^{-\frac{T}{\tau}(1-\delta+\mu)} = 1 - a(1-\alpha) \tag{16}$$

and

$$e^{-\frac{T}{\tau}(1+\delta-\mu)} = 1 - a(1+\alpha) \tag{17}$$

Solving for δ,

$$\delta = \frac{\Delta T}{T} = \mu + \frac{a\,\alpha}{(1-a)\,\ln\frac{1}{1-a}} = \mu + \alpha\, f(a) \qquad (18)$$

Thus if α equals zero, the time error is equal to the RC tolerance μ.

<u>Case 2. Timing Errors Due to Capacitor Leakage and Detector Leakage.</u> In every RC timing circuit, the capacitor has an associated leakage resistance which causes a timing error (fig. 4,5). In addition, most voltage detectors leak current into the detector prior to firing (fig. 6). The detector leakage can be approximated by a resistance switched into the circuit when $v = V_o$.

As before, the charging equation is

$$V_r = V\left(1-e^{-\frac{T}{RC}}\right) \qquad (19)$$

and the charging time is

$$T_r = \tau\,\ln\frac{V}{V-V_r} = \tau\,\ln\frac{1}{1-a} \quad \text{where } a = \frac{V_r}{V} \qquad (20)$$

By applying Thevenin's Theorem at terminals A-A′ (fig. 5), the permanent leakage resistance of the capacitor may be combined with the series resistance R. This results in R, V, and τ being replaced by rR, rV, and $r\tau$, when R_2 is not in the circuit.

$$V_r = rV\left(1 - e^{-\frac{T_1}{r\tau}}\right) \qquad (21)$$

$$T_1 = r\tau\,\ln\frac{rV}{rV-V_r} = r\tau\,\ln\frac{r}{r-a} \qquad (22)$$

where

$$r = \frac{R_1}{R_1+R} \quad \text{and } 1 > r > a > 0 \qquad (23)$$

The relative increase in charging time due to the capacitor leakage resistance is

$$\delta_1 = \frac{T_1 - T_r}{T_r} = \frac{r \ln \frac{r}{r-a} - \ln \frac{1}{1-a}}{\ln \frac{1}{1-a}} = \frac{\ln \frac{r^r(1-a)}{(r-a)^r}}{\ln \frac{1}{1-a}} \tag{24}$$

We now consider what happens when resistor R_2 is switched into the circuit at voltage V_o, that is, when the voltage detector begins to leak. This occurs when

$$V_o = rV\left(1 - e^{-\frac{T_o}{r\tau}}\right) \tag{25}$$

and time T_o is $T_o = r\tau \ln \frac{r}{r-b}$, where $b = \frac{V_o}{V}$ and $1 > a > b \geq 0$

Thevenin's Theorem is now applied at terminals B-B′ (fig. 5) to combine the series and shunt resistors. The rV and rR are replaced by prV and prR, where $p = R_2/(R_2+rR)$.

The expression for the charging curve between T_o and T_2 is

$$V_r = V_o + (prV - V_o)\left(1 - e^{-\frac{T_2 - T_o}{pr\tau}}\right) \tag{26}$$

The solution for T_2 is

$$T_2 = T_o + pr\tau \ln \frac{prV - V_o}{prV - V_r} \tag{27}$$

Substituting the expression for T_o,

$$T_2 = r\tau \ln \frac{r}{r-b} + pr\tau \ln \frac{pr-b}{pr-a} \tag{28}$$

$$T_2 = r\tau \ln \frac{r(pr-b)^p}{(r-b)(pr-a)^p} \tag{29}$$

The total increase in charging time due to the leaky capacitor and detector is

$$\delta_{12} = \frac{T_2 - T_r}{T_r} = \frac{r\tau \, \ell n \, \frac{r(pr-b)^p}{(r-b)(pr-a)^p} - \tau \, \ell n \, \frac{1}{1-a}}{\tau \, \ell n \, \frac{1}{1-a}} \tag{30}$$

$$\delta_{12} = \frac{\ell n \, \frac{(1-a)r^r \, (pr-b)^{pr}}{(r-b)^r \, (pr-a)^{pr}}}{\ell n \, \frac{1}{1-a}} \tag{31}$$

where $1 > r > pr > a > b \geq 0.$

If R_1 is very large so that $r \simeq 1$, the timing error due only to the detector leakage is

$$\delta_2 \doteq \frac{\ell n \, \frac{(1-a)(p-b)^p}{(1-b)(p-a)^p}}{\ell n \, \frac{1}{1-a}} \tag{32}$$

Figures 7, 8, 9, 10 show plots of typical design values.

The leakage resistance of the detector can be approximated by assuming a straight line between the time leakage starts until V_r is reached, then

$$\bar{R} = \frac{V_r - V_o}{2} \Big/ \frac{I_L - 0}{2} = \frac{V_r - V_o}{I_L} \tag{33}$$

where I_L is the leakage at V_r.

<u>Case 3. Timing Changes Due to Variations in Initial or Residual Charge on the Timing Capacitor.</u> In some timing circuits, due to the special requirements of the circuit, the timing capacitor starts off with an initial charge on the capacitor. In many timing oscillators, the first cycle has the capacitor at no precharge, but in all subsequent cycles the capacitor has an initial charge. Most unijunction oscillators fall into the latter class. A large class of RC interval timers which have adjustable time intervals precharge the capacitor to adjust the interval. The charging equation for a capacitor starting from a nonzero voltage is

$$v = V_i + (V - V_i)\left(1 - e^{-\frac{t}{RC}}\right) \tag{34}$$

where V_i is the initial charge voltage. This charging curve is shown in figure 11.

If V_i is positive, then the cycle is shortened and

$$V_r = V_i + (V-V_i)\left(1 - e^{-\frac{(T-\Delta T)}{\tau}}\right) \tag{35}$$

so that

$$T - \Delta T = \tau \, \ell n \frac{V-V_i}{V-V_r} \tag{36}$$

If $\frac{V_i}{V} = k$, then,

$$T - \Delta T = \tau \, \ell n \frac{1-k}{1-a} \tag{37}$$

Since $\delta = \frac{\Delta T}{T}$, then,

$$\frac{T-\Delta T}{T} = \frac{T(1-\delta)}{T} = 1 - \delta = 1 - \frac{\ell n(1-k)}{\ell n(1-a)}$$

and

$$\delta = \frac{\ell n(1-k)}{\ell n(1-a)} \tag{38}$$

These timing errors are shown in figure 12.

Case 4. Voltage Settable RC Timers which Use a Charged Capacitor as a Voltage Reference. An RC circuit capable of wide application, for quick-set, adjustable timing for short intervals is shown in figure 13. A capacitor C_r is charged to a reference voltage V_r. An RC network is charged to a voltage depending upon the time interval desired. When timing is to begin, both terminals are disconnected from the setter. The timing capacitor discharges toward the reference capacitor voltage. When the voltages coincide, the comparator puts out a signal which is normally derived from discharging C_T, C_r, or both. The variation in V_r, V, and τ from the desired values and the leakage of capacitor C_r cause system timing error.

The ideal case gives

$$V_r = V e^{-T/\tau} \quad \text{and} \quad T = \tau \, \ell n \frac{V}{V_r} = \tau \, \ell n \frac{1}{a} \tag{39}$$

However, the errors in the setting mechanism and the leakage of capacitor C_r yield the relation

$$V_r(1\pm\epsilon)\, e^{-\frac{T}{\tau_r}(1\pm\delta)} = V(1\pm\nu)\, e^{-\frac{T(1\pm\delta)}{\tau(1\pm\mu)}} \qquad (40)$$

where

$$\epsilon = \frac{\Delta V_r}{V_r}\,, \quad \nu = \frac{\Delta V}{V}\,, \quad \mu = \frac{\Delta\tau}{\tau}$$

Leakage resistance of C_T is included in $\Delta\tau$. If ϵ and ν are independent variables, then the maximum-charging-time equation is

$$V_r(1-\epsilon)\, e^{-\frac{T}{\tau_r}(1+\delta)} = V(1+\nu)\, e^{-\frac{T(1+\delta)}{\tau(1+\mu)}} \qquad (41)$$

and the minimum-charging-time equation is

$$V_r(1+\epsilon)\, e^{-\frac{T}{\tau_r}(1-\delta)} = V(1-\nu)\, e^{-\frac{T(1-\delta)}{\tau(1-\mu)}} \qquad (42)$$

Substituting $\frac{V}{V_r} = e^{T/\tau}$ in the maximum-charging-time equation gives

$$\frac{1-\epsilon}{1+\nu}\, e^{-\frac{T}{\tau_r}(1+\delta)} = e^{\frac{T}{\tau}\left[1-\frac{1+\delta}{1+\mu}\right]} \simeq e^{-\frac{T}{\tau}(\delta-\mu)} \qquad (43)$$

Multiplying by $e^{\delta T/\tau_r}$, taking the natural log of both sides, solving for δ yields

$$\delta_+ \simeq \frac{\tau}{\tau_r-\tau} + \frac{\mu\,\tau_r}{\tau_r-\tau} - \frac{\tau_r}{\tau_r-\tau}\,\frac{\tau}{T}\,\ln(1-\epsilon-\nu) \qquad (44)$$

$$\delta_+ \simeq \frac{1}{1-\frac{\tau}{\tau_r}}\left[\frac{\tau}{\tau_r} + \mu + \frac{\tau}{T}(\nu+\epsilon)\right] \qquad (45)$$

Since $\frac{\tau}{T} = \frac{1}{\ln\frac{1}{a}}$, (45) becomes

$$\delta_+ \cong \frac{1}{1 - \frac{\tau}{\tau_r}} \left[\frac{\tau}{\tau_r} + \mu + \frac{\nu+\epsilon}{\ln \frac{1}{a}} \right] \tag{46}$$

where μ, ν, and ϵ are positive quantities in the equation, the ν and μ are maximum positive deviations, and ϵ is a maximum negative deviation. By similar reasoning using the minimum-charging-time equation (42),

$$\delta_- \simeq \frac{1}{1 - \frac{\tau}{\tau_r}} \left[\frac{\tau}{\tau_r} - \mu - \frac{\nu+\epsilon}{\ln \frac{1}{a}} \right] \tag{47}$$

where μ, ν, and ϵ are positive quantities in the equation, ν and μ are now maximum negative deviations, and ϵ is a maximum positive deviation.

It is noted that if ν and ϵ deviations are not independent, but instead equal in sign and magnitude, then

$$\delta_\pm \sim \frac{1}{1 - \frac{\tau}{\tau_r}} \left[\frac{\tau}{\tau_r} \pm \mu \right] \tag{48}$$

Case 5. Time Interval Changes in Astable and Unistable Multivibrators. Astable and unistable multivibrators have wide application for time bases and time delays. Both circuits utilize similar means for determining the time period. Figure 14 shows a waveform common to these circuits. The time T is proportional to the RC of the circuit and also dependent upon V, V_r, and V_o. The V, V_r, and V_o may vary due to temperature and component variation. We now compute the error in T due to these variations.

The basic charging equation is

$$v = V_o + (V-V_o) \left(1-e^{-\frac{t}{RC}}\right) \tag{49}$$

and

$$\frac{t}{\tau} = \ln \left[1 + \frac{v-V_o}{V-v}\right] \tag{50}$$

At $t = T$, $v = V_r$

$$\frac{T}{\tau} = \ln\left[1 + \frac{V_r - V_o}{V - V_r}\right] = \ln\left[1 + \frac{A}{B}\right] \tag{51}$$

where

$$A = V_r - V_o \quad \text{and} \quad B = V - V_r .$$

Let the variations in A and B be expressed as

$$A \pm \Delta A = A(1 \pm \alpha) \qquad \text{and} \qquad B \pm \Delta B = B(1 \pm \beta)$$

then

$$T \pm \Delta T = \tau \ln\left[1 + \frac{A(1\pm\alpha)}{B(1\pm\beta)}\right] \tag{52}$$

Therefore, if $\alpha = \beta$, T does not vary; however, if α and β are independent, then the maximum triggering time is obtained when α has plus sign and β has minus sign. Then

$$T + \Delta T = \tau \ln\left[1 + \frac{A(1+\alpha)}{B(1-\beta)}\right] \tag{53}$$

$$\frac{T}{\tau}(1+\delta_+) = \ln\left[1 + \frac{A}{B}(1 + \alpha + \beta)\right] \tag{54}$$

where

$$\delta = \frac{\Delta T}{T} \quad \text{and} \quad \frac{1+\alpha}{1-\beta} = 1 + \alpha + \beta \quad \text{for } \alpha \ll 1 \text{ and } \beta \ll 1.$$

Dividing equation (54) by (51) and solving for δ

$$\delta_+ = \frac{\ln\left[1 + \frac{A}{A+B}(\alpha+\beta)\right]}{\ln \frac{A+B}{B}} \simeq \frac{A(\alpha+\beta)}{(A+B)\ln \frac{A+B}{B}} \tag{55}$$

For minimum triggering time

$$\frac{T}{\tau}(1-\delta_-) = \ln\left[1 + \frac{A(1-\alpha)}{B(1+\beta)}\right] \tag{56}$$

Solving for δ_-,

$$\delta_- \simeq \frac{\ln\left[\frac{B+A}{B+A(1-\alpha-\beta)}\right]}{\ln\frac{A+B}{B}} = \frac{\ln\left[\frac{1}{1-\frac{A}{A+B}(\alpha+\beta)}\right]}{\ln\frac{A+B}{B}} \tag{57}$$

$$\delta_- \simeq \frac{\ln\left[1+\frac{A}{A+B}(\alpha+\beta)\right]}{\ln\frac{A+B}{B}} \tag{58}$$

$$\delta_- \simeq \delta_+ \simeq \frac{A(\alpha+\beta)}{(A+B)\ln\frac{A+B}{B}} \tag{59}$$

Figure 15 is a plot of this function for several typical values.

Case 6. Pulse-Synchronized RC Pulse-Divider Circuits. Many pulse-divider circuits are based upon the principle of synchronizing an RC interval timing circuit as shown in figure 16. Timing pulses are superimposed upon the RC charging curve and fire the detector after a given amount of pulses. The capacitor discharges through the detector giving an output pulse and resetting the capacitor to ground. The process begins again. Thus an RC timer with a controlled tolerance can be synchronized to input pulses and exactly divide these pulses by a given number. The following analysis will determine what the RC tolerance can be and still divide precisely.

We assume the incoming pulses are accurately spaced in time but that the amplitude may vary. The amplitude of this synchronizing pulse is $P(1\pm\theta)$. The supply voltage V also may vary. Though the detector threshold voltage is proportional to the supply voltage, the proportionality constant may vary. This threshold voltage is $aV(1\pm\epsilon)$. Finally, the RC product may vary so that the capacitor charging equation is

$$v_c = V\left(1 - e^{-t/\tau(1\pm\mu)}\right) \tag{60}$$

where

$$\theta = \frac{\Delta P}{P}, \quad \epsilon = \frac{\Delta aV}{aV}, \quad \mu = \frac{\Delta\tau}{\tau}$$

The trigger voltage $= P(1\pm\theta) + V\left(1 - e^{-\frac{nT}{\tau(1\pm\mu)}}\right)$ (61)

where n is the number of P pulses and T is now the precise time distance between them.

The voltage during the previous pulse $= P(1\pm\theta)+V\left(1-e^{-\frac{(nT-T)}{\tau(1\pm\mu)}}\right)$ (62)

Requirements for proper operation are:

(a) Since $P > 0$, the lowest threshold voltage must be greater than the greatest capacitor voltage at time nT,

$$aV(1-\epsilon) > V\left(1 - e^{-\frac{nT}{\tau(1-\mu)}}\right) \tag{63}$$

If this condition did not hold, the capacitor voltage and not the P pulse would trigger the detector before nT.

(b) The smallest P pulse plus the smallest RC voltage at nT must be larger than the highest detector voltage. This insures firing on the nth pulse.

$$P(1-\theta) + V\left[1 - e^{-\frac{nT}{\tau(1+\mu)}}\right] > aV(1+\epsilon) \tag{64}$$

(c) The sum of the P pulse and the largest RC voltage at time (nT-T) must be less than the lowest detector voltage. This insures not firing on the (n-1) pulse.

$$aV(1-\epsilon) > P(1+\theta) + V\left(1 - e^{-\frac{nT-T}{\tau(1-\mu)}}\right) \tag{65}$$

Dividing by V and defining $p = \frac{P}{V}$, equations (63), (64), and (65) become

$$a(1-\epsilon) > 1 - e^{-\frac{nT}{\tau(1-\mu)}} \tag{66}$$

$$a(1-\epsilon) > p(1+\theta) + 1 - e^{-\frac{(nT-T)}{\tau(1-\mu)}} \tag{67}$$

$$p(1-\theta) + 1 - e^{-\frac{nT}{\tau(1+\mu)}} > a(1+\epsilon) \tag{68}$$

When equations (67) and (68) are added, the following inequality is obtained

$$e^{-\frac{nT-T}{\tau(1-\mu)}} - e^{-\frac{nT}{\tau(1+\mu)}} > 2p\,\theta + 2a\epsilon \tag{69}$$

This means that the lowest charging voltage at the nth pulse minus the highest charging voltage at the (n-1)th pulse must be greater than the sum of the voltage error band of the sync pulse and the voltage error band of the detector.

Adding (66) and (67) and rearranging, an additional inequality is obtained

$$p(1-\theta) > 2a\epsilon + e^{-\frac{nT}{\tau(1+\mu)}} - e^{-\frac{nT}{\tau(1-\mu)}} \tag{70}$$

Thus the minimum sync pulse voltage must be greater than the sum of the voltage threshold band of the detector plus the charging voltage error band at the nth pulse.

Consider the minimum and maximum RC charging curves at times (n-1)T and nT (fig. 17). The following relationships hold.

$$v_c - v_b = e^{-\frac{nT-T}{\tau(1-\mu)}} - e^{-\frac{nT}{\tau(1+\mu)}} \tag{71}$$

$$(v_d - v_a) - (v_b - v_a) - (v_d - v_c) = e^{-\frac{nT-T}{\tau(1-\mu)}} - e^{-\frac{nT}{\tau(1+\mu)}} \tag{72}$$

If

$$v_b - v_a \simeq v_e - v_d \tag{73}$$

$$(v_d - v_a) - (v_e - V_c) \simeq e^{-\frac{nT-T}{\tau(1-\mu)}} - e^{-\frac{nT}{\tau(1+\mu)}} \tag{74}$$

and substituting in (69)

$$(v_d - v_a) > 2p\theta + 2a\epsilon + (v_e - v_c) \tag{75}$$

Thus the sum of the error bands, or tolerances, must be less than the difference in the charging voltage at the nth and (n-1)th pulses.

We will now determine the values of v_d and τ which will make $(v_d - v_a)$ a maximum for a given n, so that the allowed sum of the error bands is maximized. Normalized equations are used. Let

$$f = v_d - v_a = e^{-\frac{(n-1)T}{\tau}} - e^{-\frac{nT}{\tau}} \qquad (76)$$

Differentiating and equating to zero,

$$\frac{df}{d\tau} = \frac{(n-1)T}{\tau^2} e^{-\frac{(n-1)T}{\tau}} - \frac{nT}{\tau^2} e^{-\frac{nT}{\tau}} = 0 \qquad (77)$$

When multiplied by $\frac{\tau^2}{T} e^{nT/\tau}$, (77) reduces to

$$(n-1)\, e^{\frac{T}{\tau}} - n = 0 \qquad (78)$$

$$\frac{T}{\tau_{opt}} = \ln \frac{n}{n-1} \qquad (79)$$

Thus the optimum time constant is equal to

$$\tau_{opt} = \frac{T}{\ln \frac{n}{n-1}} \qquad (80)$$

Substituting τ_{opt} in the nominal charging equation and solving for the normalized v_d yields

$$v_{d(opt)} = 1 - \left(\frac{n-1}{n}\right)^n \qquad (81)$$

The maximum sum of the error bands, the maximum $(v_d - v_a)$, is now calculated by substituting (80) in (76).

$$f_{max} = v_d - v_a = e^{-\frac{(n-1)T}{\tau_{opt}}} - e^{-\frac{nT}{\tau_{opt}}} \qquad (82)$$

Since

$$1 - v_{d(opt)} = \left(\frac{n-1}{n}\right)^n = e^{-\frac{nT}{\tau_{opt}}} \qquad (83)$$

then

$$f_{max} = \left[\left(\frac{n-1}{n}\right)^n\right]^{\frac{n-1}{n}} - \left(\frac{n-1}{n}\right)^n \qquad (84)$$

This reduces to

$$f_{max} = (v_d - v_a)_{max} = \frac{(n-1)^{n-1}}{n^n} \qquad (85)$$

Optimum values of v_d and $\frac{T}{\tau}$ are given in table I for various values of n, along with the maximum permissible sum of the error bands.

Table I. Optimum Voltage and Time Constant for Maximum Error Allowances

n	$(v_d-v_a)_{max}$ (sum of error bands)	$(v_d)_{opt}$	$(\frac{T}{\tau})_{opt}$
2	.2500	.750	.693
3	.1481	.704	.405
4	.1055	.684	.288
5	.0819	.672	.223
6	.0670	.665	.182
7	.0567	.660	.154
8	.0491	.656	.133
9	.0433	.654	.118
10	.0349	.651	.105

3. CONCLUSIONS

The RC time circuits analyzed in this report reappear in ordnance timing devices. This report places the analyses of these circuits under a single cover and is meant to be a reference so that recalculation of the error equations each time the circuits are used is no longer required.

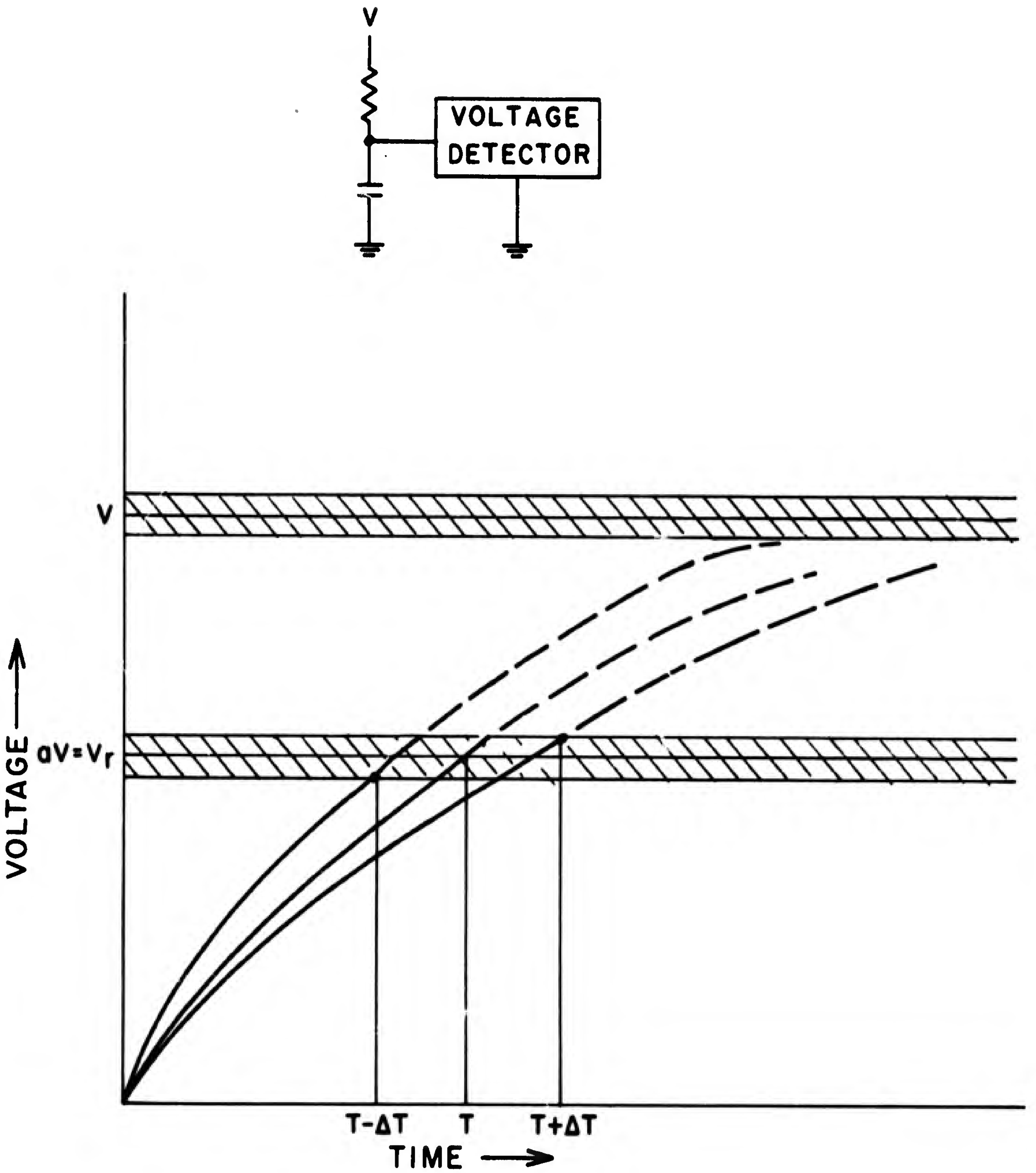


Figure 1. Case 1. Variation of Supply voltage, RC constant, and voltage detection point.

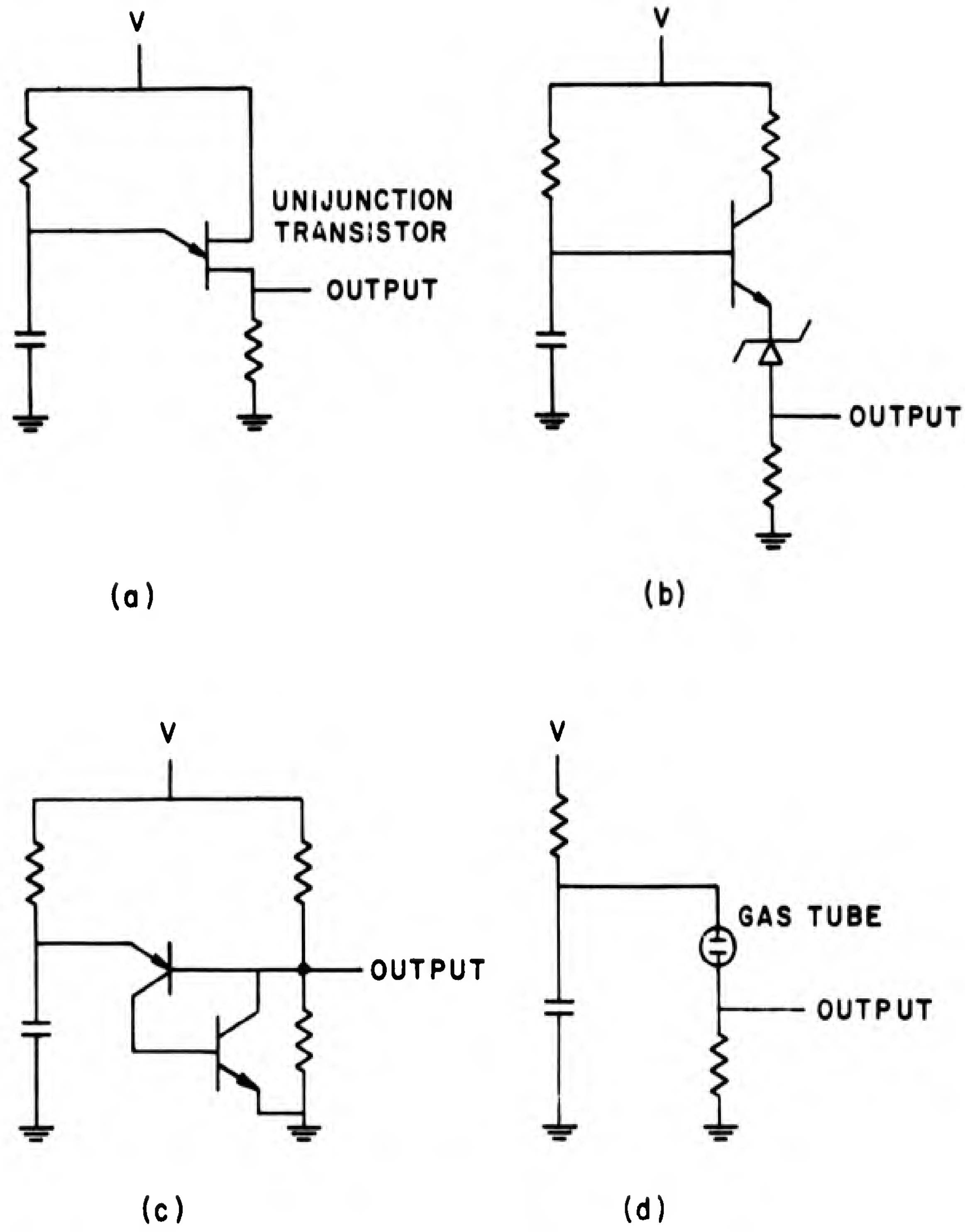


Figure 2. Examples of Case 1 type circuits.

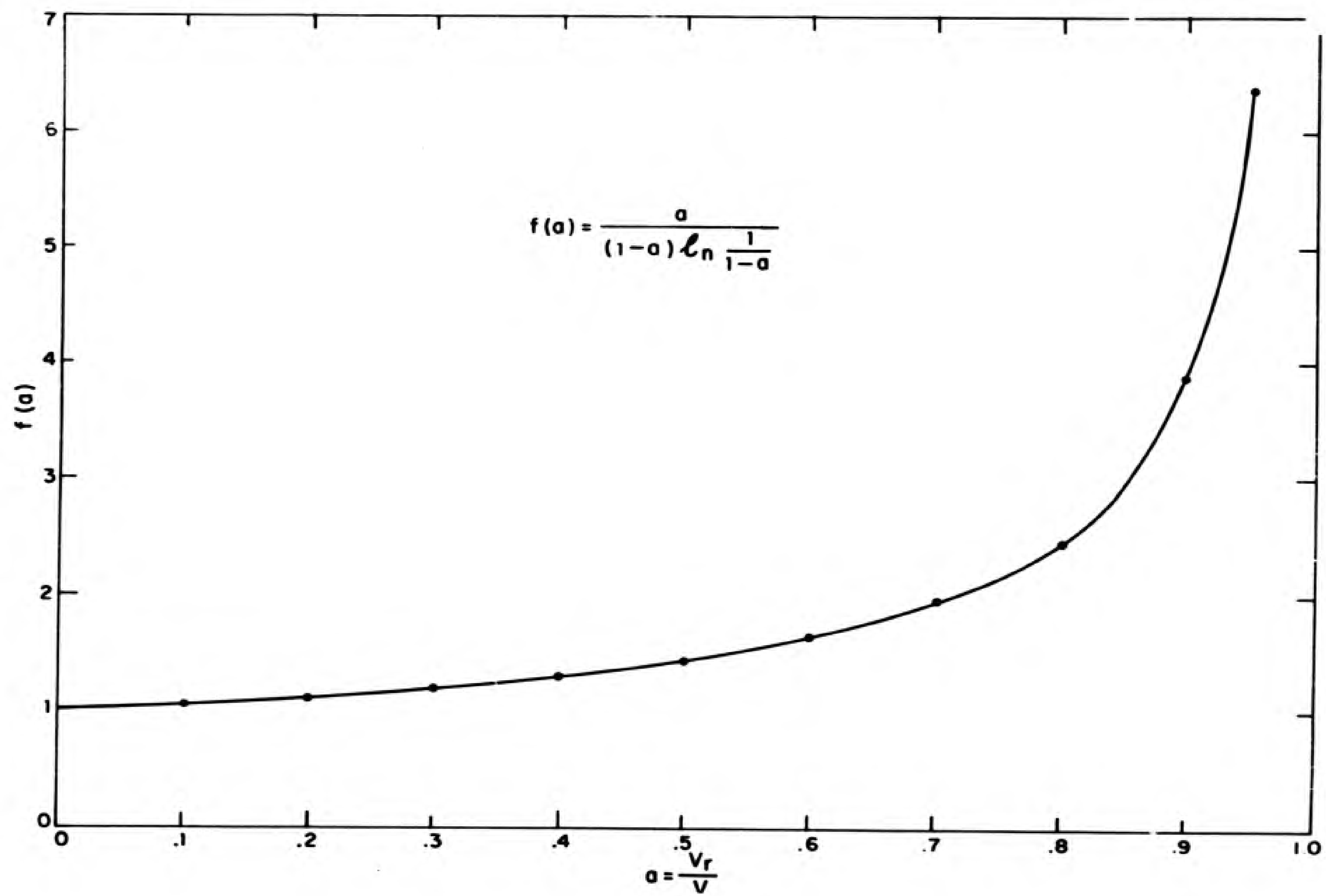


Figure 3. Case 1. Plot of a versus f(a).

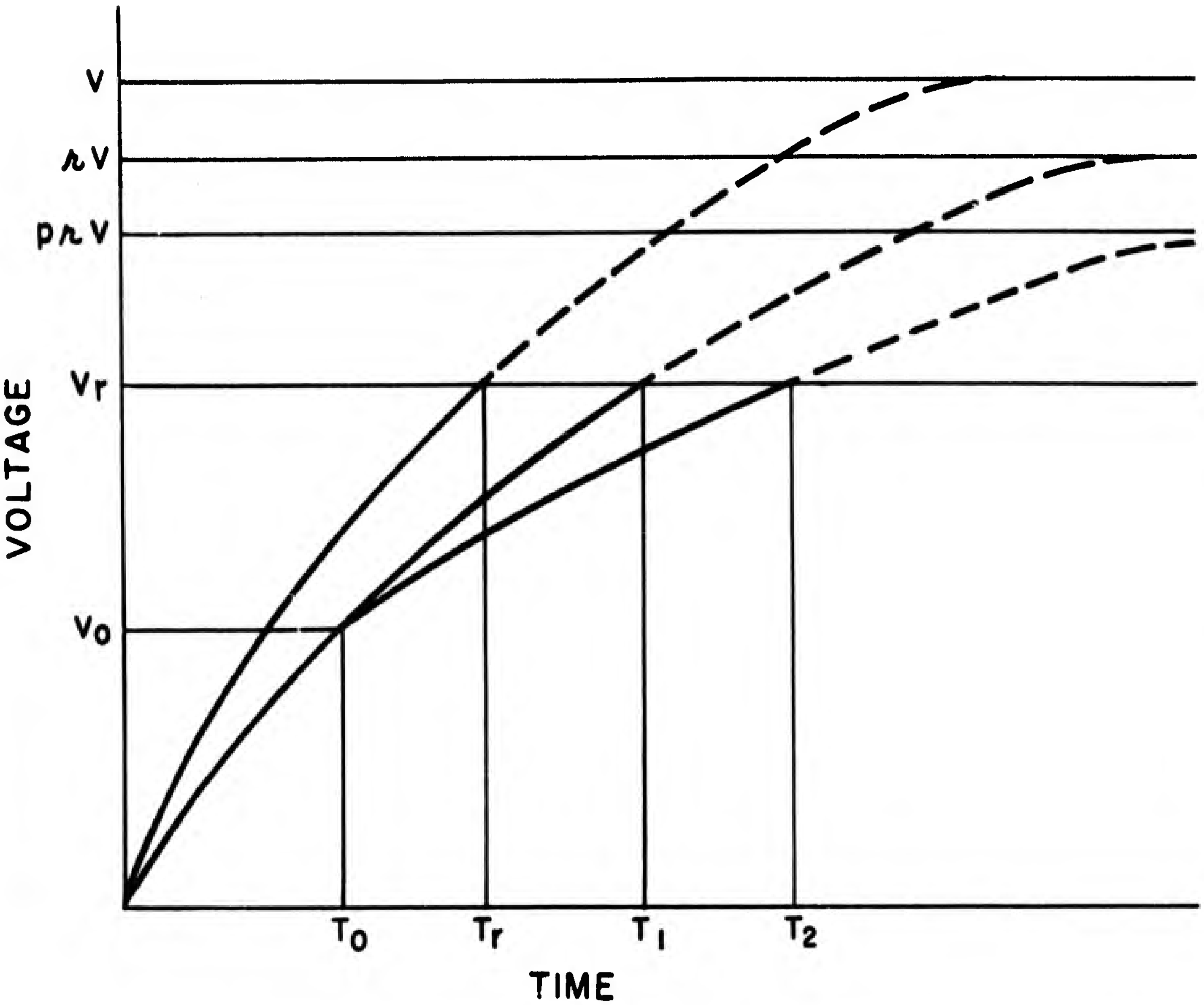


Figure 4. Case 2. Leakage errors due to capacitor leakage and detector leakage.

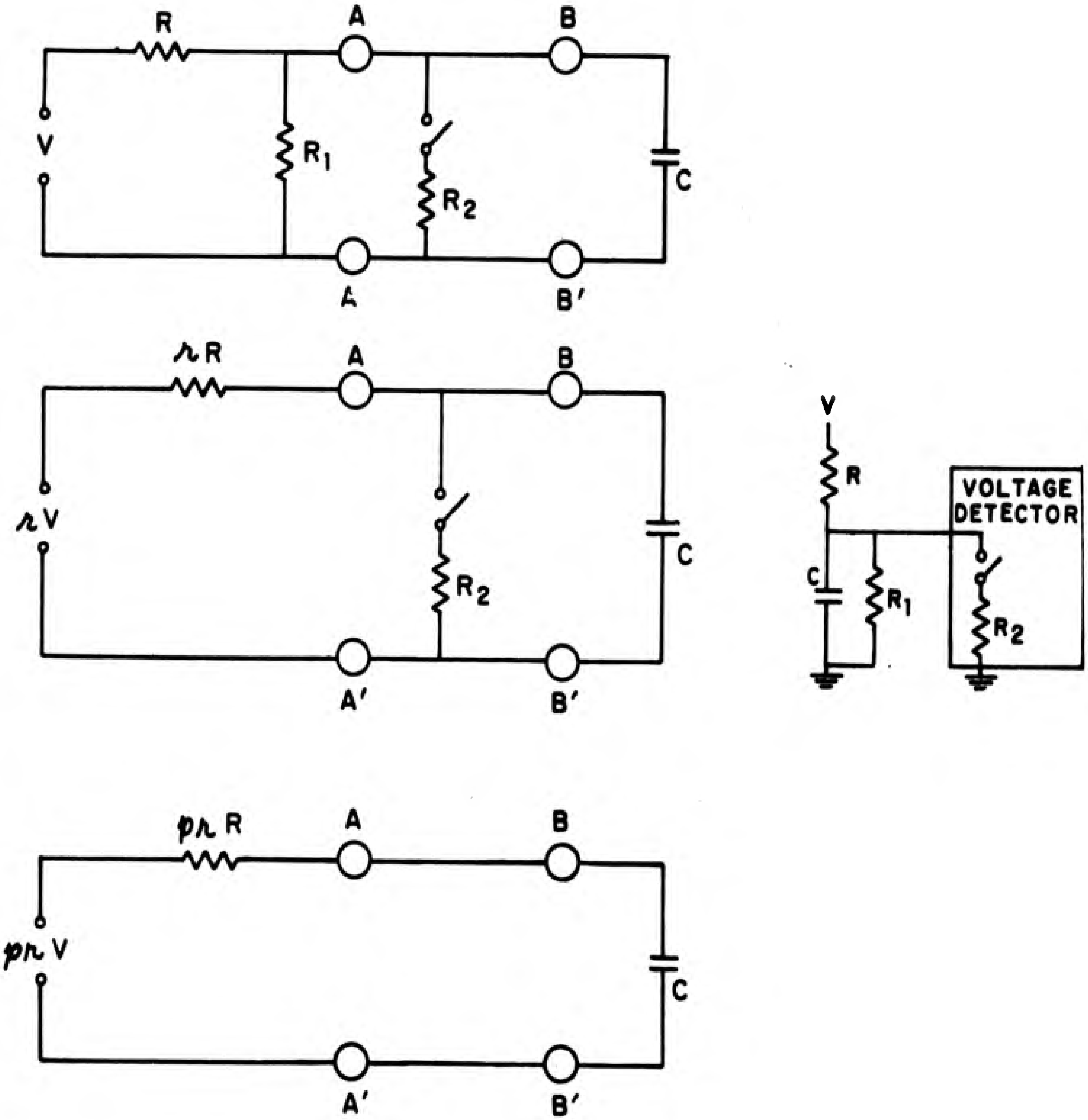


Figure 5. Case 2. Equivalent circuits used in leakage formula derivation.

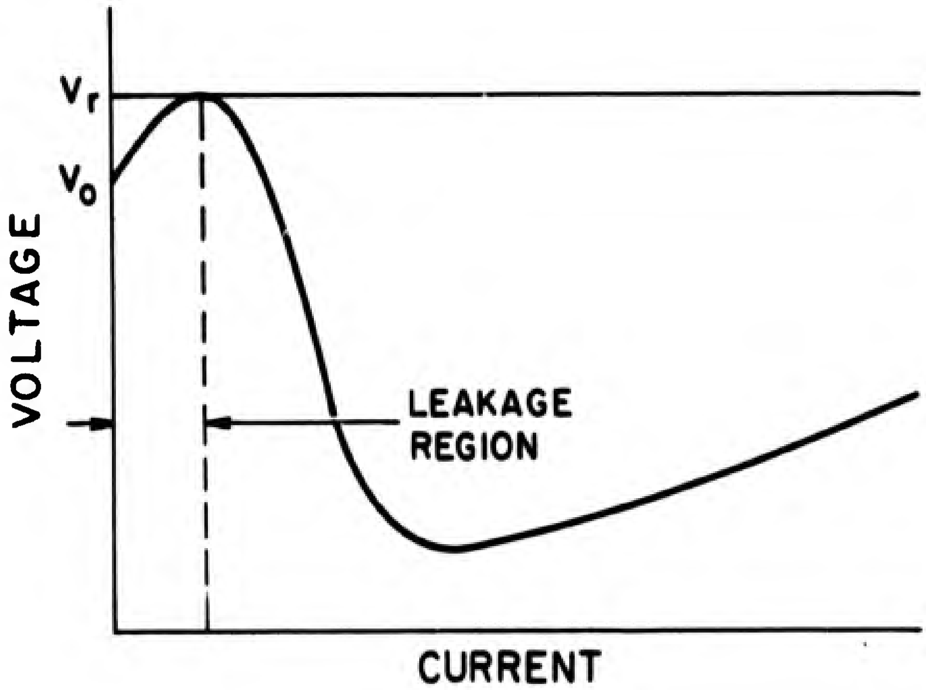


Figure 6. Case 2. Typical detector characteristics.

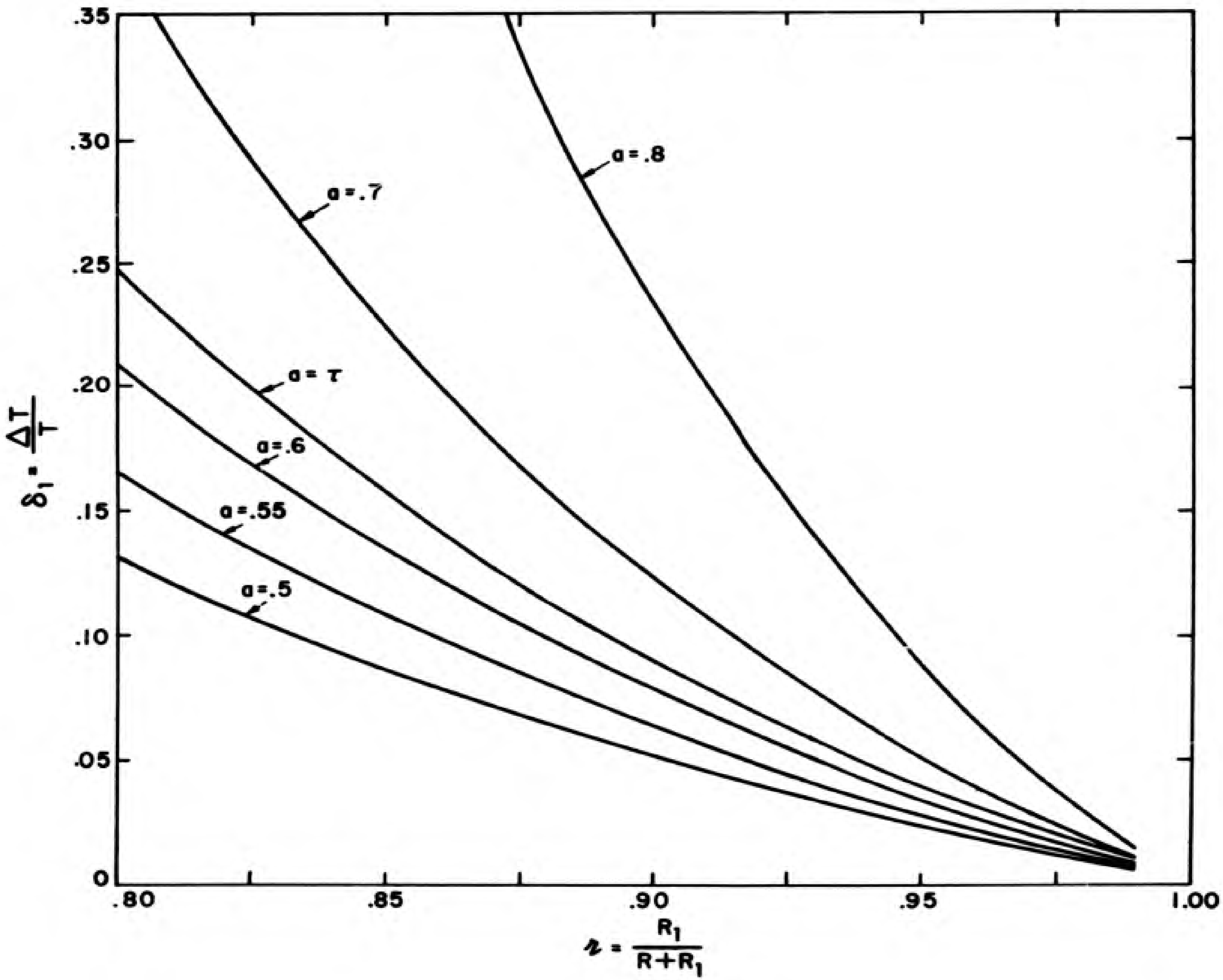


Figure 7. Case 2. Effects of capacitor leakage on timing circuits.

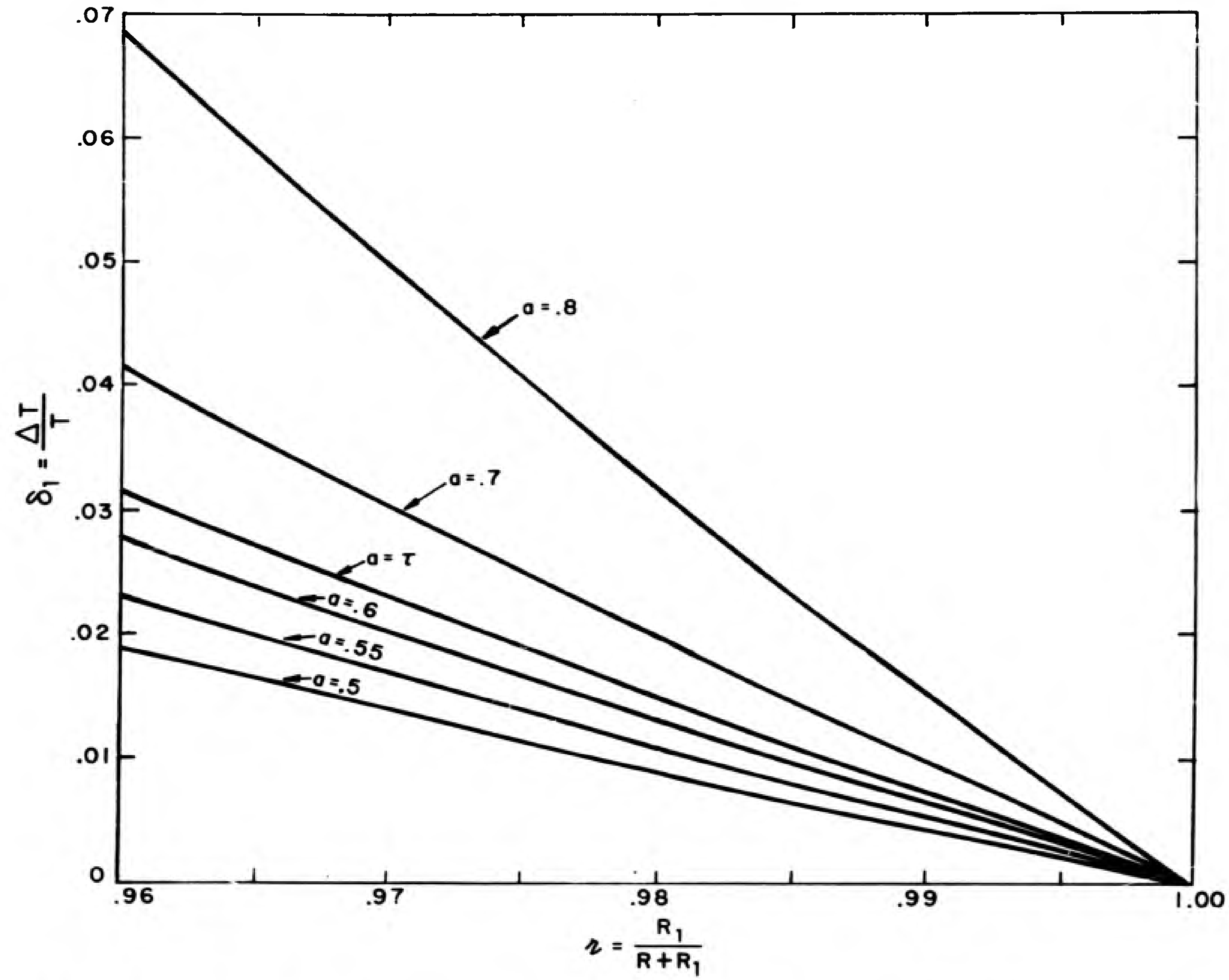


Figure 8. Case 2. Effects of capacitor leakage on timing accuracy.

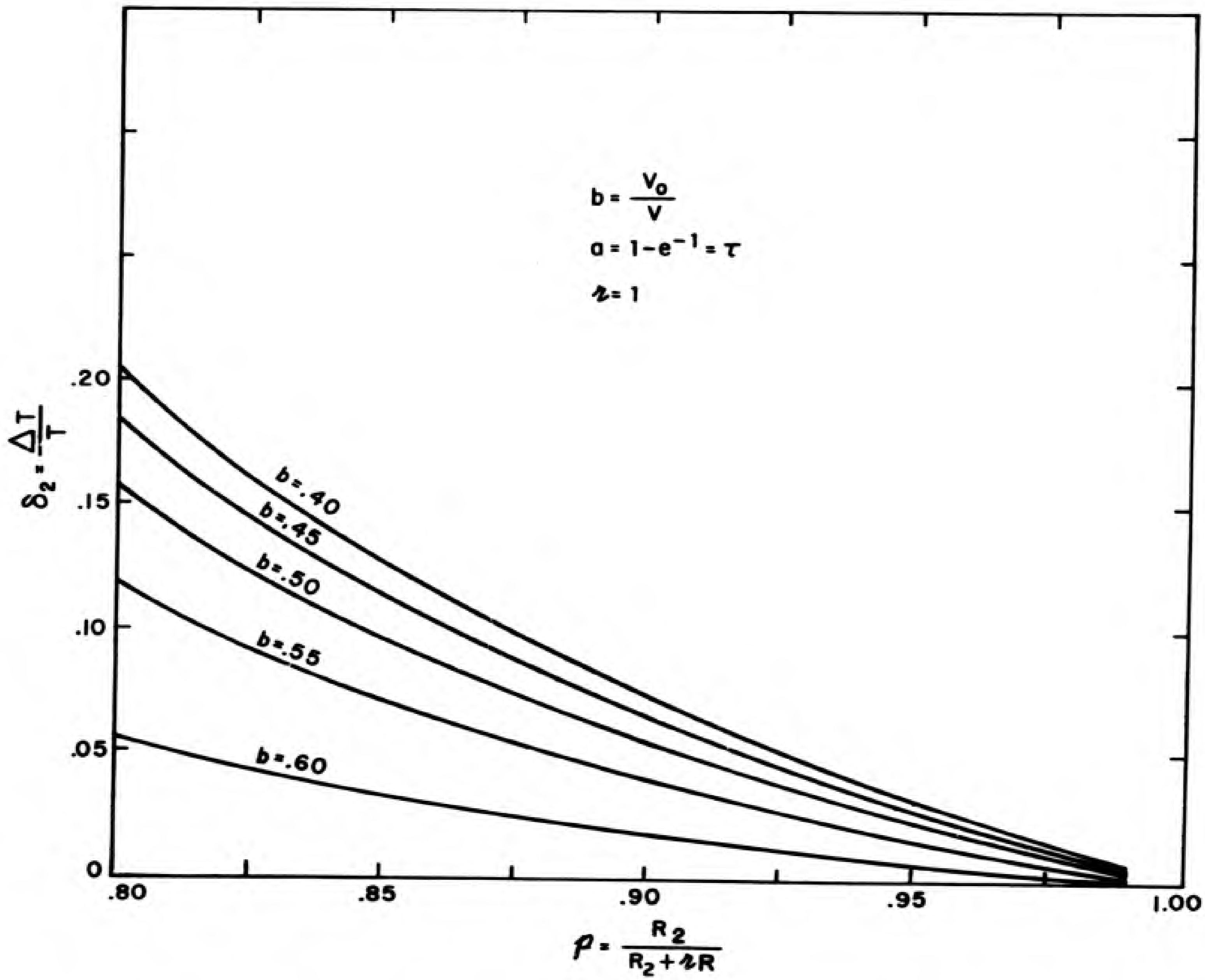


Figure 9. Case 3. Effects of detector leakage on timing accuracy.

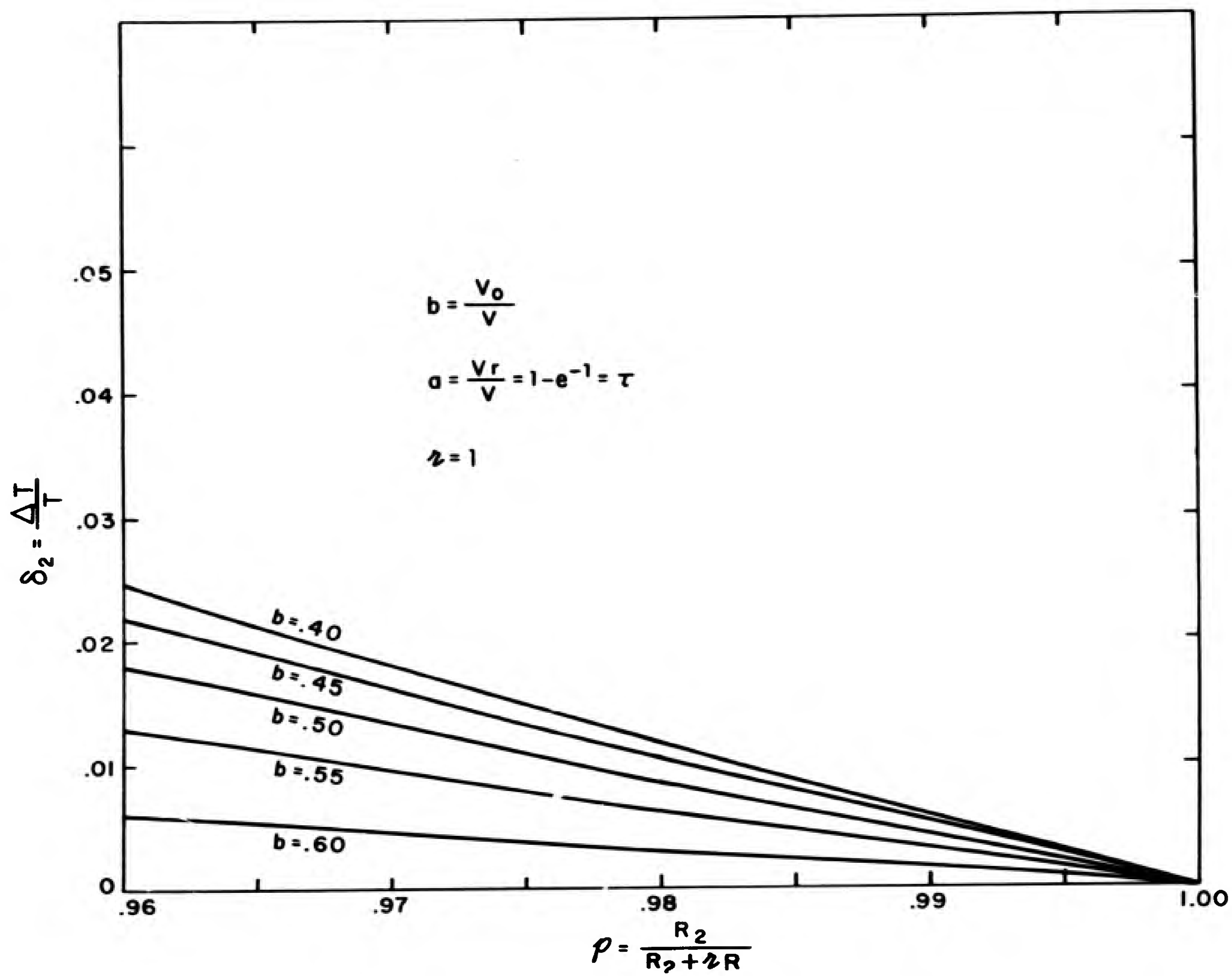


Figure 10. Case 3. Effects of detector leakage on timing accuracy.

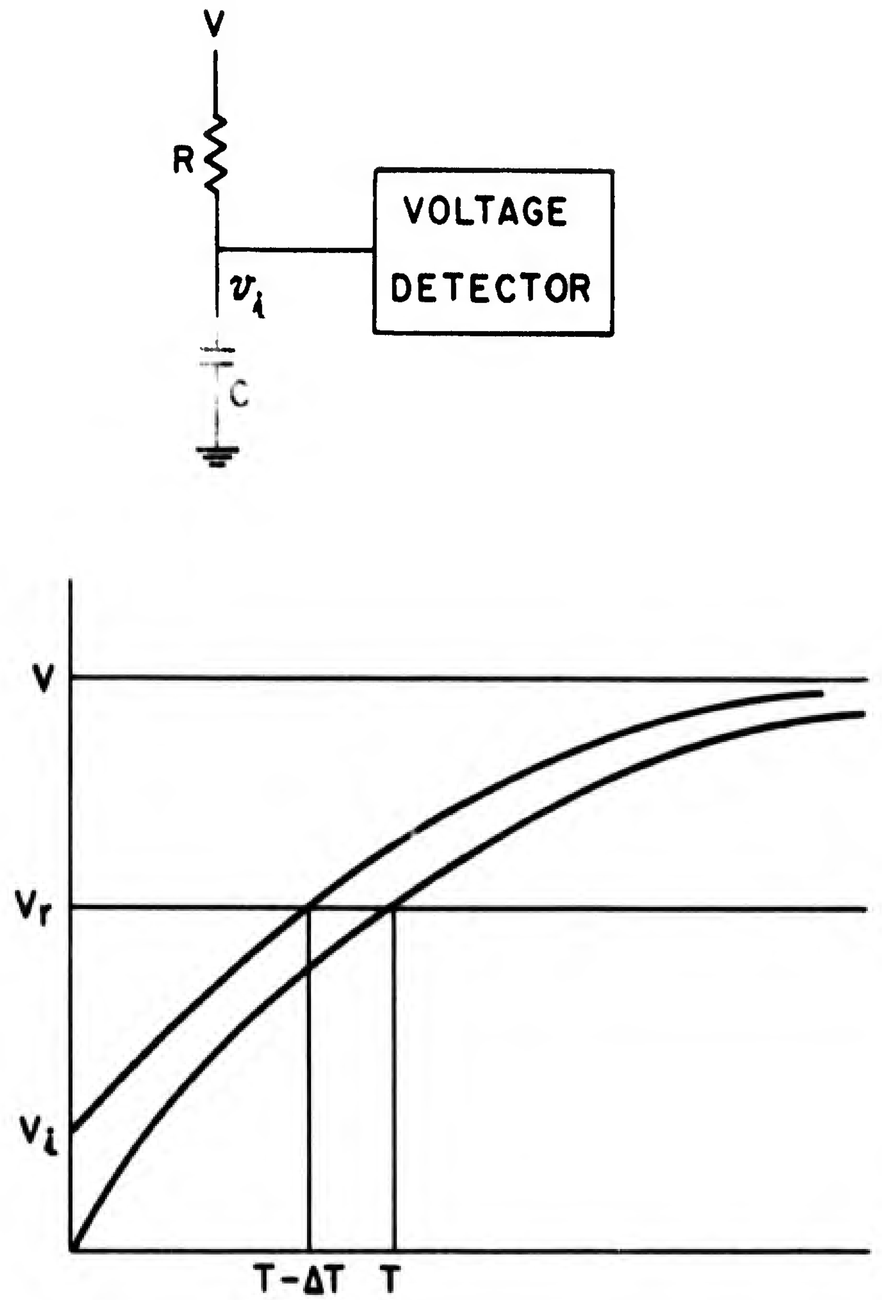


Figure 11. Case 3. Timing changes due to initial charge on the timing capacitor.

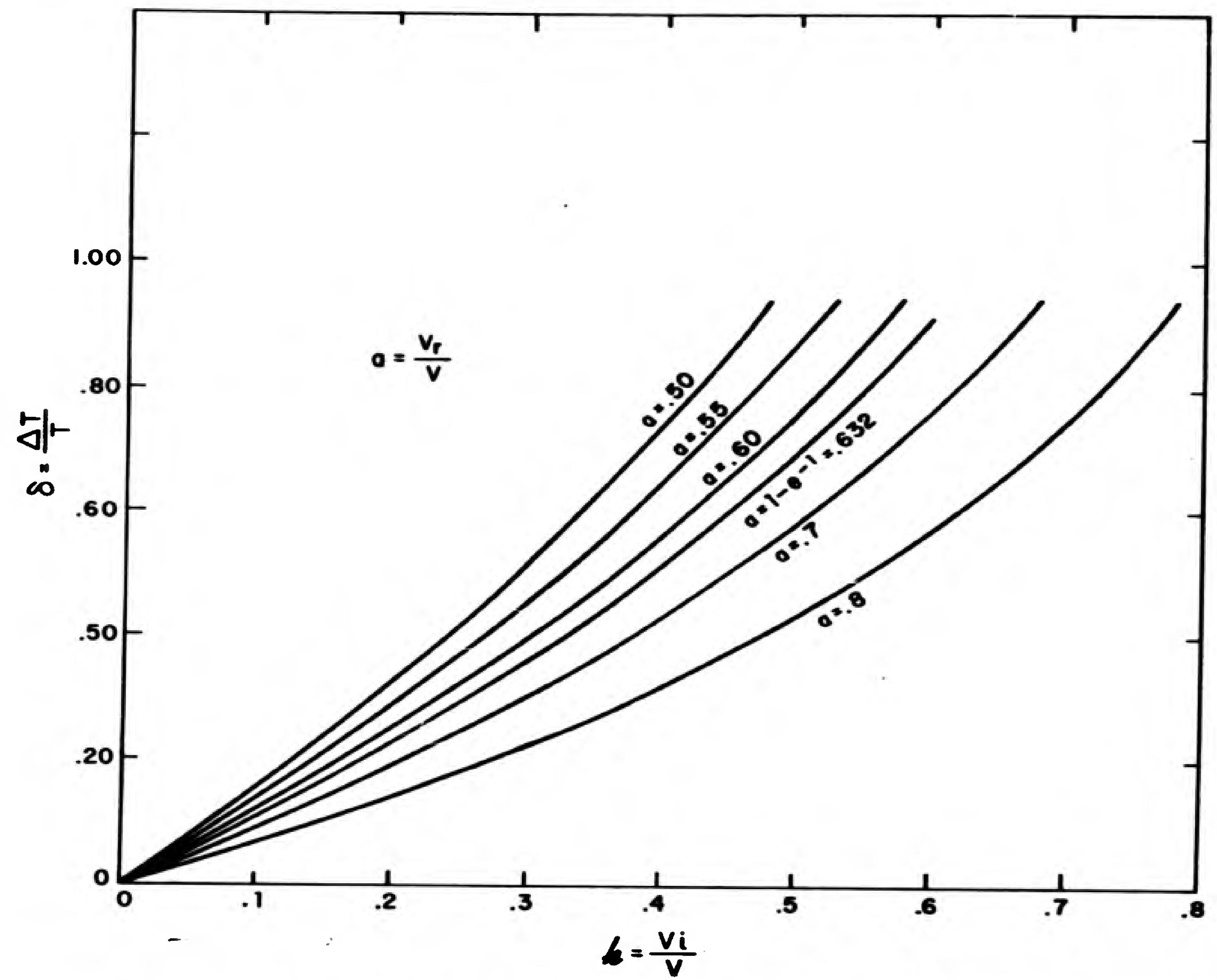


Figure 12. Case 3. Change in time due to variation in precharge or capacitor.

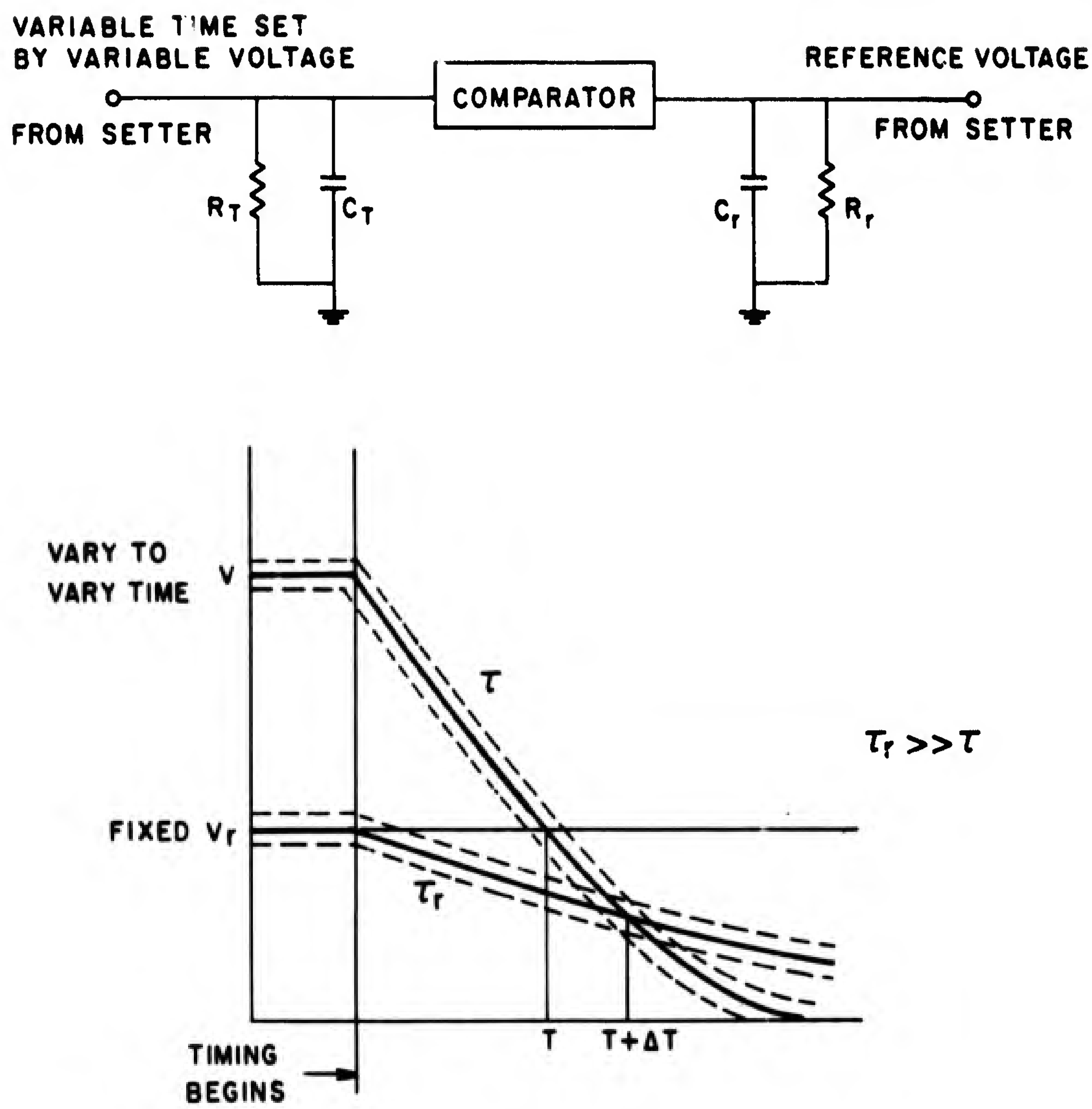


Figure 13. Case 4. Voltage settable RC timer with a charged reference capacitor.

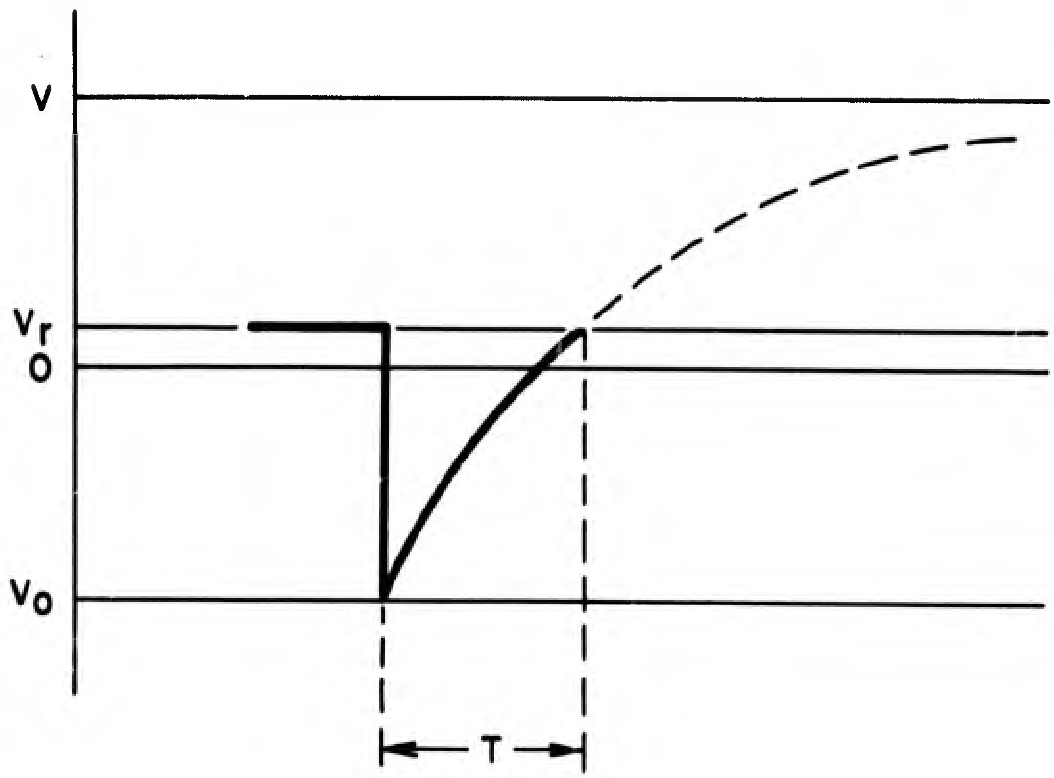


Figure 14. Case 5. Time interval changes in astable and unistable multivibrators.

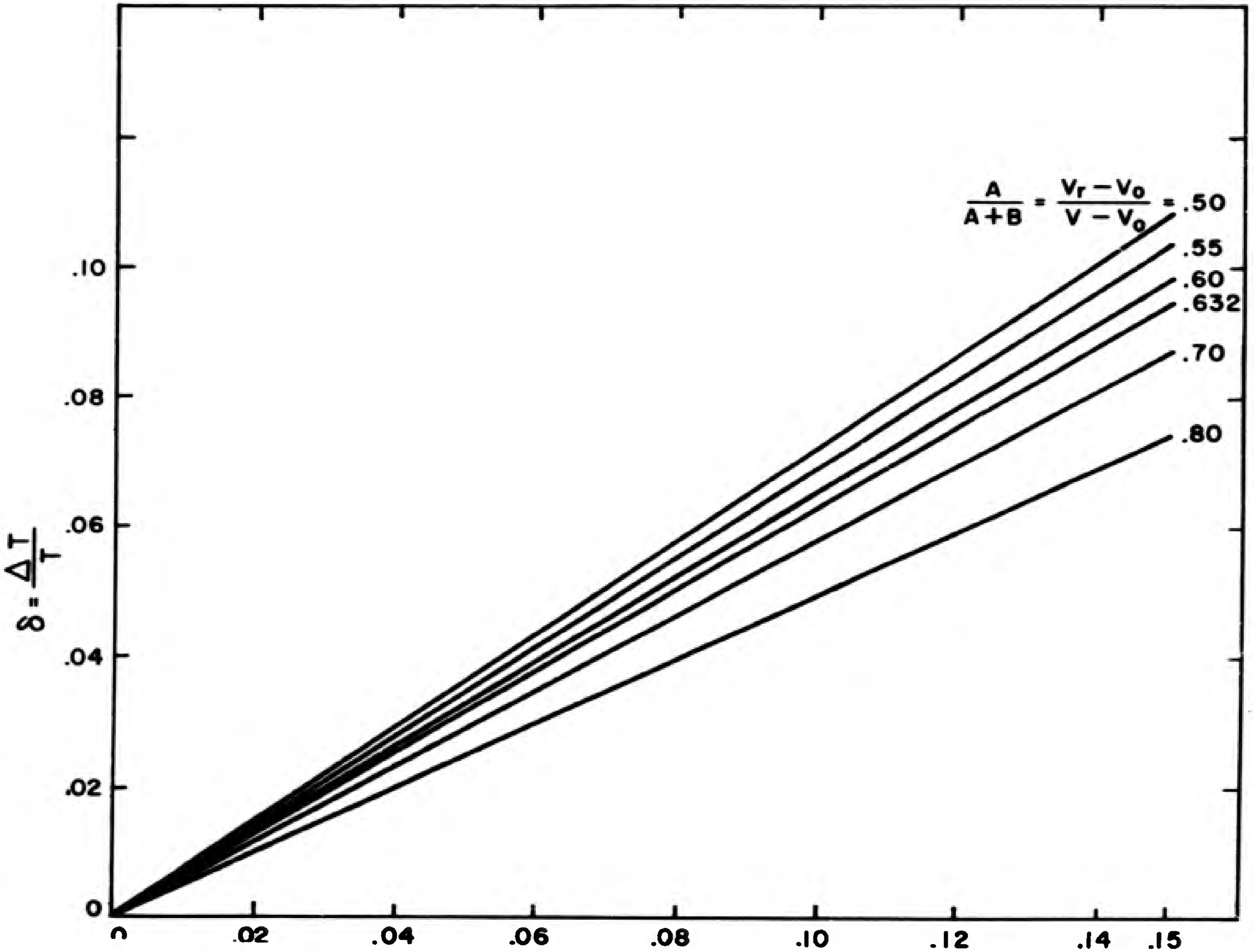
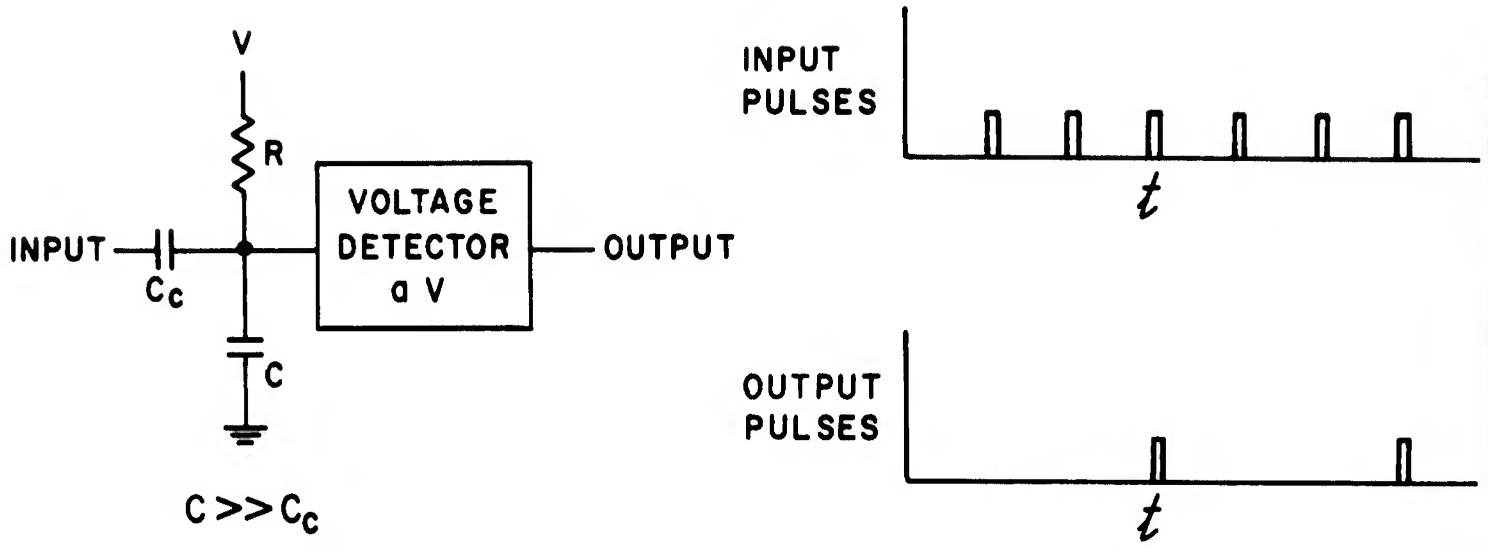


Figure 15. Case 5. Multivibrator accuracy curves.



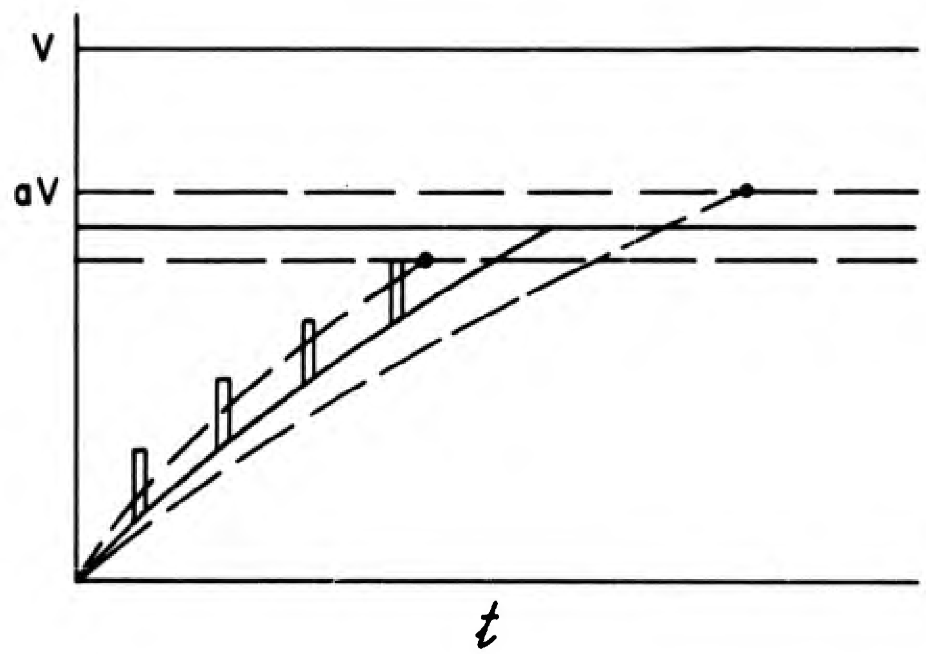


Figure 16. Case 6. Pulse synchronized RC pulse dividers.

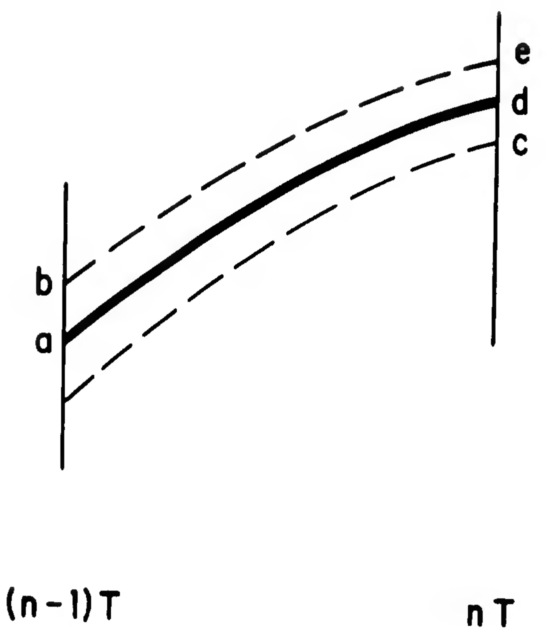


Figure 17. Case 6. Section of capacitor voltage charging curve with allowance for variation in RC time constant.

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13. ABSTRACT

This report analyzes various RC timing circuits commonly used in ordnance electronic timers. Timing errors are computed as a function of the component tolerances, specified to account for possible manufacturing, aging, and temperature changes. The effect of capacitor leakage is also computed.

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