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THE WATER ENTRY PROBLEM WITH BIBLIOGRAPHY

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UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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ABSTRACT: This report is a survey of the water entry problem and its various mathematical formulations. Included are discussions on the incompressible case, the compressible case, the linearized incompressible, the linearized compressible, reflected linearized incompressible, Lagrangian, two-dimensional and axially-symmetric cases. Also discussed are the source and dipole formulations, calculation of added masses, similar solutions, problems related to water entry, and a note of caution concerning the conservation of mass in an infinite region. Finally, a fairly extensive bibliography is given primarily covering theoretical work on the water entry and related problems.

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The Water Entry Problem with Bibliography

In this report the mathematical formulation of the water entry problem is presented. A number of different physical assumptions are made and the mathematical model for each set of assumptions is derived. This work will serve as an aid for future work in both the analytical and numerical treatment of the water entry problem. This work was sponsored under task number ORD 035 105/R109 01 01 Prob 003.

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Captain, USN
Commander

W. R. Thickstun

W. R. THICKSTUN
By direction

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1. INTRODUCTION

Physically speaking, the problem of water entry is the problem of determining the forces, moments and pressures acting upon a body during its initial impact with the water surface. Since the forces are quite high during the initial impact phase, these forces can have important effects on the missile shell, its internal components and on the subsequent trajectory of the missile.

The physical problem of water entry can be expressed in mathematical terms in many different ways according to the assumptions made. This report is intended to present several mathematical formulations of the physical water entry problems. After these formulations are made, various aspects of the problem are discussed such as added mass, similar solutions, and various problems related to water entry. Finally, a bibliography is given which is a fairly complete list of theoretical papers on water entry plus some references to experimental work.

2. INCOMPRESSIBLE FORMULATION

The assumptions for the mathematical model we wish to consider first are:

(1) the water is non-viscous

(2) the water is incompressible

(3) surface tension is negligible

(4) the air density is negligible

(5) the air-water interface is at constant pressure

(6) the entering body is rigid

(7) the entering body's trajectory is prescribed and the body is not rotating

(8) the water is undisturbed at great distances from the entering

body.

Assumption (1) gives us a velocity potential ϕ in D, the interior of the fluid, i.e. $\vec{q} = -\nabla\phi$. Assumption (2) and the equation of continuity ($\nabla \cdot \vec{q} = 0$) give us Laplace's equation, $\nabla^2\phi = 0$ in D, for the velocity potential. Assumption (6) gives us the boundary condition on S_E, the surface of the entering body: $\frac{\partial\phi}{\partial n} = -\vec{v}(t) \cdot \vec{n}$. The vector $\vec{n}$ is the unit vector pointing into the fluid and normal to the body surface. The vector $\vec{v}(t)$ is the velocity of the entering body as given by assumption (7). For coordinates fixed relative to the water, $\vec{v}(t)$ would be a non-zero vector. Assumption (5) (and related assumptions (3) and (4)) with Bernoulli's equation for unsteady flow gives:

$$(2.1) \qquad \frac{p}{\rho} + \frac{1}{2}(\nabla\phi)^2 - \frac{\partial\phi}{\partial t} - gz = c(t) \quad \text{on } S_F,$$

where S_F is the air-water interface or free surface, ρ is the density of the water, g is the acceleration due to gravity, z is the distance from the initial plane (with down being positive) and c(t) is a constant with respect to the space variables. Using assumption (8) and normalizing the air pressure to zero, we obtain

$$(2.2) \qquad \frac{1}{2}(\nabla\phi)^2 - \frac{\partial\phi}{\partial t} - gz = 0.$$

We need one more condition (a kinematic condition) to determine the shape of the free surface with time. Suppose the free surface has the equation $F(x, y, z, t) = 0$. Consider at time t any fixed point (x, y, z) on S_F (satisfying $F = 0$). The velocity at this point is well defined, say (u, v, w). At time Δt later, the point (x, y, z) will be at the point $(x + u\Delta t, y + v\Delta t, z + w\Delta t)$. This latter point is on the free surface at time $t + \Delta t$, i.e., it satisfies

$$(2.3) \qquad F(x + u\Delta t, y + v\Delta t, z + w\Delta t, t + \Delta t) = 0$$

If F is a smooth surface, i.e. $F \epsilon C'$, then

$$(2.4) \qquad F(x + u\Delta t, y + v\Delta t, z + w\Delta t, t + \Delta t) =$$

$$F(x, y, z, t) + \frac{\partial F}{\partial t}\Delta t + \frac{\partial F}{\partial x}(u\Delta t) + \frac{\partial F}{\partial y}(v\Delta t) + \frac{\partial F}{\partial z}(w\Delta t) + R(\Delta t)$$

where

$$\frac{R(\Delta t)}{\Delta t} \to 0 \quad \text{or} \quad \Delta t \to 0.$$

Therefore, letting $\Delta t \to 0$, we have $\frac{\partial F}{\partial t} + \vec{q} \cdot \nabla F = 0$[1] which must be satisfied by $F(x, y, z, t)$. This plus the initial condition $F(x, y, z, 0) = f(x, y, z)$, (a known function) completely determine F.

[1] For alternative forms of this equation, see Appendix A.

Summary: Incompressible flow; coordinates fixed in water

$$\phi \equiv 0 \text{ for } t \leq 0$$

$$\nabla^2\phi = 0 \text{ in } D$$

$$\phi \to 0 \text{ as } R \to \infty \text{ for any } t$$

(2.5)
$$\frac{\partial\phi}{\partial n} = -\vec{v} \cdot \vec{n} \text{ on } S_E$$

$$\frac{1}{2}(\nabla\phi)^2 - \frac{\partial\phi}{\partial t} - gz = 0 \text{ on } S_F$$

$$\frac{\partial F}{\partial t} + \nabla\phi \cdot \nabla F = 0 \text{ on } S_F$$

$$F(x, y, z, 0) = f(x, y, z) \text{ on } S_F \text{ (f given)}$$

Also see Fig. 1.

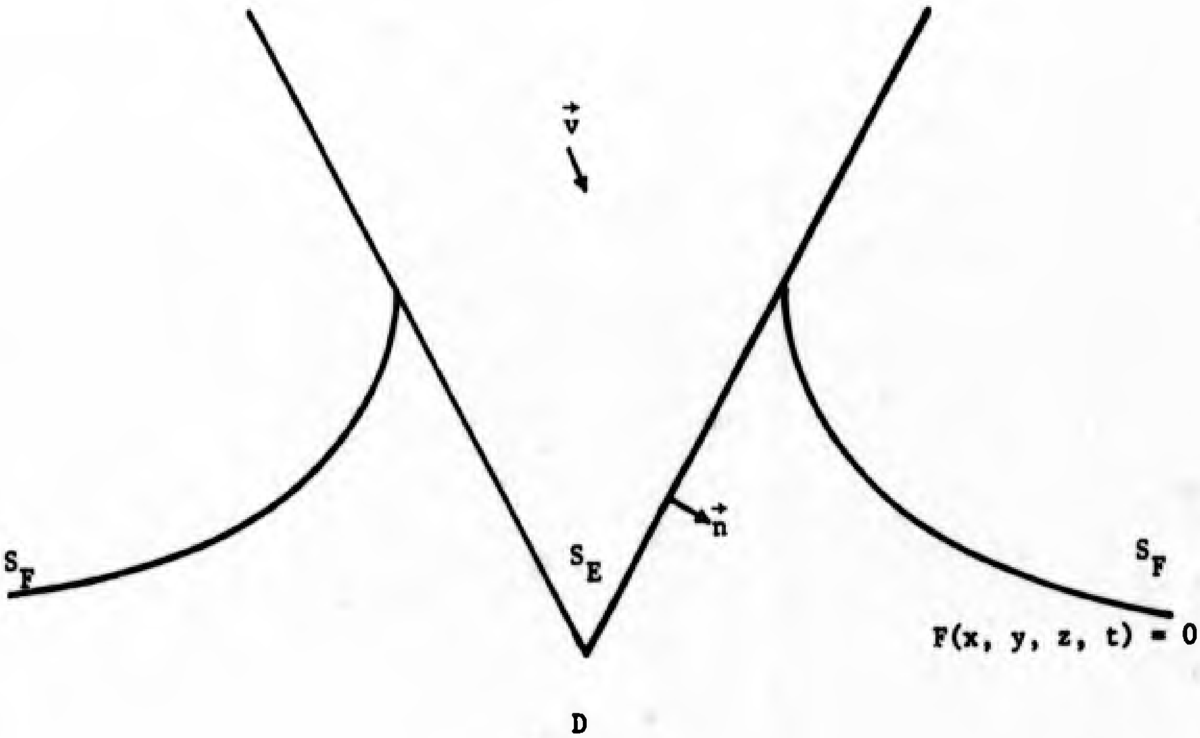

Fig. 1

$$\nabla^2\phi = 0$$

$$\vec{q} = -\nabla\phi$$

Bernoulli's equation: $\frac{1}{2}(\nabla\phi)^2 - \frac{\partial\phi}{\partial t} - gz = 0$

As an alternative to coordinates fixed in the water, one may use coordinates fixed relative to the entering body. In the latter case, the solid then has a zero velocity while the fluid has a velocity $-\vec{v}(t)$ at infinity.

For the velocity $\vec{v}(t)$ with components (v_x, v_y, v_z), we may consider the following change of variables:

$$\bar{x} = x - v_x t$$

$$\bar{y} = y - v_y t$$

$$\bar{z} = z - v_z t$$

$$\bar{t} = t$$

$$\bar{\phi} = \phi(x, y, z) - (v_x x + v_y y + v_z z).$$

Note that the change in the dependent variable ϕ was such that the gradient of $\bar{\phi}$ gives the velocity of a particle as measured in the moving coordinate system. An alternative change is $\bar{\phi}(\bar{x}, \bar{y}, \bar{z}, \bar{t}) = \phi(x, y, z, t)$ wherein the gradient does not directly give the velocity.

Now for Bernoulli's equation:

$$(2.7) \qquad \frac{\partial \phi}{\partial t} = \frac{\partial \bar{\phi}}{\partial t} = \frac{\partial \bar{\phi}}{\partial t}\frac{\partial \bar{t}}{\partial t} + \frac{\partial \bar{\phi}}{\partial \bar{x}}\frac{\partial \bar{x}}{\partial t} + \frac{\partial \bar{\phi}}{\partial \bar{y}}\frac{\partial \bar{y}}{\partial t} + \frac{\partial \bar{\phi}}{\partial \bar{z}}\frac{\partial \bar{z}}{\partial t}$$

$$= \frac{\partial \bar{\phi}}{\partial \bar{t}} - \frac{\partial \bar{\phi}}{\partial \bar{x}} v_x - \frac{\partial \bar{\phi}}{\partial \bar{y}} v_y - \frac{\partial \bar{\phi}}{\partial \bar{z}} v_z$$

and

$$(2.8) \qquad \frac{\partial \phi}{\partial x} = \frac{\partial \bar{\phi}}{\partial x} + v_x = v_x + \frac{\partial \bar{\phi}}{\partial \bar{x}}\frac{\partial \bar{x}}{\partial x} + \frac{\partial \bar{\phi}}{\partial \bar{y}}\frac{\partial \bar{y}}{\partial x} + \frac{\partial \bar{\phi}}{\partial \bar{z}}\frac{\partial \bar{z}}{\partial x} = v_x + \frac{\partial \bar{\phi}}{\partial \bar{x}}.$$

In (2.8) $\frac{\partial \phi}{\partial x} = \frac{\partial \bar{\phi}}{\partial \bar{x}}$ so that the gradient is unchanged in form in the two systems. Therefore, rewrite (2.7) as:

$$(2.9) \qquad \frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial \bar{t}} - \vec{v} \cdot \nabla \bar{\phi}$$

and (2.8) as:

$$\nabla\phi = \vec{v} + \nabla\bar{\phi} \; . \tag{2.10}$$

Thus Bernoulli's equation becomes:

$$\frac{1}{2}(\nabla\bar{\phi} + \vec{v})^2 - \frac{\partial\bar{\phi}}{\partial\bar{t}} + \vec{v}\cdot\nabla\bar{\phi} - g(\bar{z} + v_z\bar{t}) = 0 \tag{2.11}$$

All equations are easily summarized. Bars are dropped.

Summary: Incompressible flow, coordinates fixed in body.

$$\nabla^2\phi = 0 \text{ in } D$$

$$\phi \to -\vec{v}(t) \cdot \vec{R} \text{ as } R \to \infty$$

$$\frac{\partial\phi}{\partial n} = 0 \text{ on } S_E$$

$$\frac{1}{2}(\nabla\phi + \vec{v})^2 - \frac{\partial\phi}{\partial t} + \vec{v} \cdot \nabla\phi - g(z + v_z t) = 0 \quad (2.12)$$

$$\frac{\partial F}{\partial t} + (\nabla\phi + \vec{v}) \cdot \nabla F = 0 \text{ on } S_F$$

$$F(x, y, z, 0) = f(x, y, z) \text{ on } S_F$$

Also see Fig. 2.

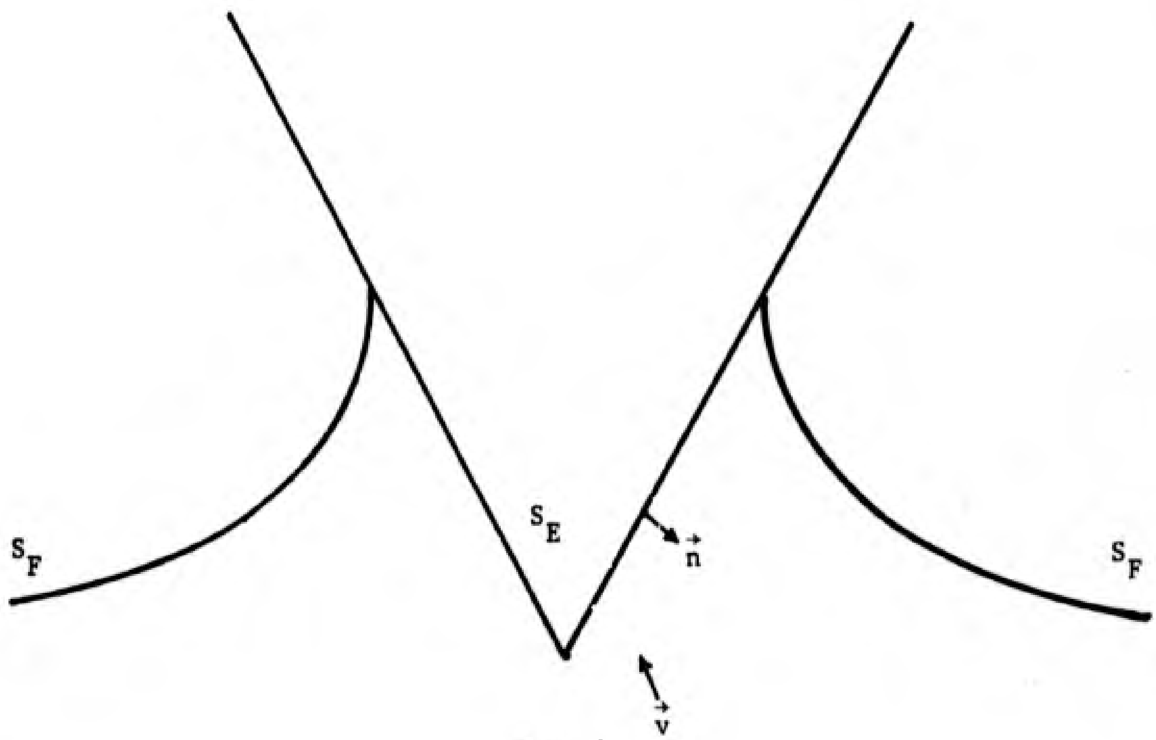

Fig. 2

$$\nabla^2 \phi = 0$$

$$\vec{q}_m = -\nabla \phi$$

3. COMPRESSIBLE FORMULATION

All of the assumptions used for the incompressible formulation are retained except, of course, that assumption (2) is replaced by:

(2b) the water is compressible.

Two additional assumptions are convenient:

(9) gravity effects are negligible

(10) $p = f(\rho)$ (e.g., an isentropic fluid)

Assumption (9) is for simplicity and is nearly true. If assumption (10) (the polytropic assumption) is not used, it is necessary to add an equation of state and an energy equation to the system.

The following set of equations may be obtained:

(3.1) $\vec{q} = -\nabla\phi$ (viscosity neglected)

(3.2) $\nabla \cdot \vec{q} = -\frac{1}{\rho}\frac{D\rho}{Dt}$ (continuity equation)

where $\frac{D}{Dt}$ is the material derivative $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{q} \cdot \nabla$

(3.3) $\nabla^2\phi = -\frac{1}{\rho}\frac{D\rho}{Dt}$ (3.1 and 3.2)

(3.4) $a^2 \equiv \frac{Dp}{D\rho}$ (definition of sound velocity)

(3.5) $a^2 = f'(\rho)$ (polytropic assumption (10))

(3.6) $\frac{1}{\rho}\frac{D\rho}{Dt} = \frac{1}{\rho}\frac{D\rho}{Dp}\frac{Dp}{Dt} = \frac{1}{\rho}\frac{1}{a^2}\frac{Dp}{Dt} = \frac{1}{a^2}\frac{DP}{Dt}$ (3.4 where $P \equiv \int \frac{dp}{\rho}$)

(3.7) $\nabla^2\phi = \frac{1}{a^2}\frac{DP}{Dt}$ (3.3 and 3.6).

From integration of the momentum equation we have Bernoulli's equation (gravity neglected by assumption (9))

(3.8) $$\int \frac{dp}{\rho} + \frac{1}{2} q^2 - \frac{\partial \phi}{\partial t} = C(t) .$$

For conditions at rest at infinity, we have:

(3.9) $$P + \frac{1}{2} q^2 - \frac{\partial \phi}{\partial t} = C .$$

Then

(3.10) $$- \frac{DP}{Dt} = + \frac{\partial}{\partial t} \left(\frac{1}{2} q^2 + \frac{\partial \phi}{\partial t} \right) + \vec{q} \cdot \nabla \left(\frac{1}{2} q^2 + \frac{\partial \phi}{\partial t} \right)$$

(3.11) $$- \frac{DP}{Dt} = \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial}{\partial t} \left(\frac{1}{2} \vec{q} \cdot \vec{q} \right) + \vec{q} \cdot \nabla \left(\frac{\partial \phi}{\partial t} \right) + \vec{q} \cdot \nabla \left(\frac{q^2}{2} \right).$$

The second and third terms on the right are both equal to $\vec{q} \cdot \frac{\partial q}{\partial t}$; thus:

(3.12) $$\nabla^2 \phi - \frac{1}{a^2} \frac{\partial \phi}{\partial t} = \frac{2}{a^2} \vec{q} \cdot \frac{\partial q}{\partial t} + \frac{1}{a^2} \vec{q} \cdot \nabla \left(\frac{q^2}{2} \right) .$$

Equation 3.12 may be written throughout in terms of ϕ. Using three dimensional cartesian coordinates and expanding 3.12, we may summarize.

Summary: Compressible flow, coordinates fixed in fluid, gravity neglected, polytropic fluid.

$$(3.13)\quad \frac{\partial^2\phi}{\partial x^2}\left(1-\frac{\phi_x^{\,2}}{a^2}\right)+\frac{\partial^2\phi}{\partial y^2}\left(1-\frac{\phi_y^{\,2}}{a^2}\right)+\frac{\partial^2\phi}{\partial z^2}\left(1-\frac{\phi_z^{\,2}}{a^2}\right)-\frac{1}{a^2}\frac{\partial^2\phi}{\partial t^2}$$

$$= 2\frac{\phi_x\phi_y}{a^2}\frac{\partial^2\phi}{\partial x\partial y}+2\frac{\phi_y\phi_z}{a^2}\frac{\partial^2\phi}{\partial y\partial z}+\frac{\phi_z\phi_x}{a^2}\frac{\partial^2\phi}{\partial z\partial x}$$

$$+ 2\frac{\phi_x}{a^2}\frac{\partial^2\phi}{\partial x\partial t}+2\frac{\phi_y}{a^2}\frac{\partial^2\phi}{\partial y\partial t}+2\frac{\phi_z}{a^2}\frac{\partial^2\phi}{\partial z\partial t}$$

in D. (cf. von Mises [78], p. 72).

Boundary conditions are as in the incompressible case:

$$\phi \equiv 0 \text{ for } t \leq 0$$

$$\frac{\partial\phi}{\partial n} = -\vec{v}\cdot\vec{n} \text{ on } S_E$$

$$\phi \to 0 \text{ or } R \to \infty$$

$$(3.12)\quad \frac{1}{2}(\nabla\phi)^2+\frac{\partial\phi}{\partial t} = 0 \text{ on } S_F$$

$$\frac{\partial F}{\partial t} + \nabla\phi\cdot\nabla F = 0 \text{ on } S_F$$

$$F(x, y, z, 0) = f(x, y, z) \text{ on } S_F$$

also:

$$p = f(\rho),\quad a^2 = f'(\rho),$$

$$\int_{ps}^{p}\frac{dp}{\rho}+\frac{1}{2}(\nabla\phi)^2-\frac{\partial\phi}{\partial t} = 0$$ at all points of the fluid (ps = surface pressure).

4. LINEARIZED INCOMPRESSIBLE FORMULATION

In the analysis of incompressible water entry, the boundary condition on S_F may give difficulty becuase of the non-linear term $(\nabla\phi)^2$. Consequently, a linearized formulation is of some interest.

The problem may be linearized in several ways with respect to time, with respect to velocity, with respect to slenderness of the body, or with respect to several of these simultaneously. For the incompressible case, time seems the most logical.

Therefore, we linearize with respect to time, i.e. find a solution which is valid for small values of time. We assume a solution

(4.1) $$\phi(x, y, z, t) = u_1(x, y, z, t)\tau + u_2(x, y, z, t)\tau^2 + \theta(\tau^3).$$

where τ is a small but fixed interval of time and t is restricted to $0 \le t \le \tau$. Then $\nabla^2\phi = 0$ implies $\nabla^2 u_1 = 0$, $\nabla^2 u_2 = 0$, ...

(4.2) $$\phi(\infty) = 0 \text{ implies } u_1(\infty) = 0, \quad u_2(\infty) = 0$$

(4.3) $$\frac{\partial\phi}{\partial n} = \tau\frac{\partial u_1}{\partial n} + \tau^2\frac{\partial u_2}{\partial n} + O(\tau^3) = -\vec{v}\cdot\vec{n}\,.$$

Expand $\vec{v}$ in powers of τ, $\vec{v} = \vec{v}_1\tau + \vec{v}_2\tau^2 + O(\tau^3)$ and $\frac{\partial u_1}{\partial n} = \vec{v}_1 \cdot \vec{n}$, $\frac{\partial u_2}{\partial n} = \vec{v}_2 \cdot \vec{n}$, ... For $\vec{v}(t) = \vec{v}(0) + \vec{v}'(0)t + O(t^2)$, we are free to close $\vec{v}_1 \equiv \vec{v}(0)$, $\vec{v}_2 = \vec{v}'(0)$, ... In this case, we have $\frac{\partial u_1}{\partial n} = \vec{v}_0 \cdot \vec{n}$, $\frac{\partial u_2}{\partial n} = \vec{v}_0{}' \cdot \vec{n}$, ... On S_F: $\frac{1}{2}(\nabla\phi)^2 - \frac{\partial\phi}{\partial t} = 0$ (gravity neglected) becomes:

(4.4) $$\frac{1}{2}\left(\tau\nabla u_1 + \tau^2\nabla u_2 + O(\tau^3)\right)^2 - \tau\frac{\partial u_1}{\partial t} - \tau^2\frac{\partial u_2}{\partial t} + O(\tau^3) = 0$$

or

(4.5) $$\frac{\partial u_1}{\partial t} = 0, \quad \frac{\partial u_2}{\partial t} = \frac{1}{2} (\nabla u_1)^2, \cdots$$

Integrating the condition on u_1, we have u_1 = constant for all time. However $\phi \equiv 0$ for $t \leq 0$; therefore, the constant is zero and

$$u_1 \equiv 0 \text{ on } S_F \text{ for all } t.$$

The equation $\frac{\partial F}{\partial t} + \nabla\phi \cdot \nabla F = 0$ after expanding F as

(4.6) $$F(x, y, z, t) = \tau F_1(x, y, z, t) + \tau^2 F_2(x, y, z, t) + O(\tau^3),$$

becomes

(4.7) $$\tau\frac{\partial F_1}{\partial t} + \tau^2\frac{\partial F_2}{\partial t} + O(\tau^3) = -(\tau\nabla u_1 + \tau^2\nabla u_2 + O(\tau^3))\cdot(\tau F_1 + \tau^2 F_2 + O(\tau^3))$$

Therefore $\frac{\partial F_1}{\partial t} = 0$, $\frac{\partial F_2}{\partial t} = u_1 \cdot F_1$ on S_F. That is, F_1 is invariant in time. Taking F_1 to be the initial plane, we have that F_1 is always the initial plane.

Summary: Linearized, incompressible, gravity neglected

First approximation u_1

$$u_1 \equiv 0 \quad \text{for} \quad t \leq 0$$

$$\nabla^2 u_1 = 0 \quad \text{in} \quad D$$

$$u_1(\infty) = 0 \tag{4.8}$$

$$\frac{\partial u_1}{\partial n} = -\vec{v}_0 \cdot \vec{n} \quad \text{on} \quad S_E$$

$$u_1 \equiv 0 \quad \text{on} \quad S_{F_1}$$

$$F_1 = x \equiv 0 \quad \text{for} \quad S_{F_1}$$

Bernoulli's equation becomes $\frac{p}{\rho} + \frac{\partial u_1}{\partial t} = 0$

Second approximation u_2

$$u_2 \equiv 0 \quad \text{for} \quad t \leq 0$$

$$\nabla^2 u_2 = 0 \quad \text{in} \quad D$$

$$u_2(\infty) = 0$$

$$\frac{\partial u_2}{\partial n} = \vec{v}_0{}' \cdot \vec{n} \quad \text{on} \quad S_E \tag{4.9}$$

$$\frac{\partial u_2}{\partial t} = \frac{1}{2} (\nabla u_1)^2 \quad \text{on} \quad S_{F_2}$$

$$\frac{\partial F_2}{\partial t} = \nabla u_1 \cdot F_1 \quad \text{on} \quad S_{F_2}$$

Bernoulli's equation becomes $\frac{p}{\rho} + \frac{1}{2} (\nabla u_1)^2 + \frac{\partial u_2}{\partial t} = 0.$

Linearizing assumptions used above may be summarized as:

$$\phi = u_1\tau + u_2\tau^2 + O(\tau^3)$$

(4.10) $$\vec{v} = \vec{v}_0\tau + \vec{v}_0\tau^2 + O(\tau^3)$$

$$F = F_1\tau + F_2\tau^2 + O(\tau^3).$$

Alternatively, one may linearize with respect to velocity. Let $V \equiv \max |\vec{v}|$ and

(4.11) $$\phi = u_1V + u_2V^2 + O(V^3)$$

(4.12) $$\vec{v} = \vec{v}_1V \quad (\text{where } |\vec{v}_1| \leq 1)$$

(4.13) $$F = F_1V + F_2V^2 + O(V^3)$$

If we linearize with respect to body slenderness S where S = cone or wedge angle or ratio of width to length for a finite body, we have

(4.14) $$\phi = u_1S + u_2S^2 + O(S^3)$$

(4.15) $$\vec{v} = \vec{v}_1S$$

(4.16) $$F = F_1S + F_2S^2 + O(S^3)$$

The final equations for u_1, using V and S are the same as for τ except for the condition on $\vec{v}$. The latter condition becomes:

(4.17) $$\frac{\partial u_1}{\partial n} = -\vec{v} \cdot \vec{n}$$

for the cases using V and S. If desired, the linearization τ also may use the same condition.

In the second approximation

$$\frac{\partial u_2}{\partial n} = 0 \tag{4.18}$$

for the V and S linearizations.

5. LINEARIZED COMPRESSIBLE FORMULATION

In considering compressible water entry, these are non-linear terms appearing in both the boundary condition on S_F and in the equation for ϕ in the interior of the fluid. One may linearize with respect to τ, V, or S as for the incompressible case. The resulting equations may be summarized.

Summary: Linearized, compressible, gravity neglected

First approximation:

$$\nabla^2 u_1 - \frac{1}{a_0^2}\frac{\partial u_1}{\partial t} = 0$$

$$u_1 = 0 \quad \text{for} \quad t \leq 0$$

$$\frac{\partial u_1}{\partial n} = -\vec{v}\cdot\vec{n} \quad \text{on} \quad S_E \quad (-\vec{v}_0 \cdot n \quad \text{for} \quad \tau \text{ if desired})$$

(5.1) $$u_1(\infty) = 0$$

$$u_1 = 0 \quad t \geq 0 \quad \text{on} \quad S_{F_1}$$

$$F_1 = x \equiv 0 \quad \text{for} \quad S_{F_1}$$

Bernoulli's equation becomes $\int_{ps}^{p} \frac{dp}{\rho} - \frac{\partial u_1}{\partial t} = 0$ (or $P = \frac{p_s}{\rho_0} - \frac{\partial u_1}{\partial t}$)

Second approximation:

$$\nabla^2 u_2 - \frac{1}{a^2}\frac{\partial u_2}{\partial t} = \frac{1}{a^2}\frac{\partial}{\partial t}(\nabla u_1)^2$$

$$u_2 = 0 \quad \text{for} \quad t \leq 0$$

$$u_2(\infty) = 0$$

(5.2) $$\frac{\partial u_2}{\partial n} = 0 \quad \text{on} \quad S \quad (= -\vec{v}_0{}' \cdot \vec{n} \quad \text{for} \quad \tau \quad \text{if desired})$$

$$\frac{\partial u_2}{\partial t} = \frac{1}{2}(\nabla u_1)^2 \quad \text{on} \quad S_{F_2}$$

$$\frac{\partial F_2}{\partial t} = \nabla u_1 \cdot F_1 \quad \text{on} \quad S_{F_2}$$

6. REFLECTED LINEARIZED FORMULATION (INCOMPRESSIBLE)

Since the free surface is initially a plane and remains plane in the linearized problem, and since the value of the potential is zero on this plane, the potential $u_1(x, y, z, t)$ may be continued into the upper half plane by reflection. That is suppose that x is the vertical coordinate and the positive direction is downward. Let

$$(6.1)\qquad \begin{aligned} \bar{u}(x, y, z, t) &\equiv - u_1(-x, y, z, t) \quad \text{for} \quad x \le 0 \\ \bar{u}(x, y, z, t) &\equiv u_1(x, y, z, t) \quad \text{for} \quad x \ge 0 \end{aligned}$$

then $\bar{u}$ will satisfy Laplace's equation in the exterior of S_E and its reflection S.

The above formulation has a physical interpretation. Consider the velocities obtained from the $\bar{u}(x, y, z, t)$ potential:

$$(6.2)\qquad \begin{aligned} v_x &= \bar{u}_x(x, y, z, t) = u_{1x}(x, y, z, t) \\ v_y &= \bar{u}_y(x, y, z, t) = u_{1y}(x, y, z, t) \qquad x \ge 0 \\ v_z &= \bar{u}_z(x, y, z, t) = u_{1z}(x, y, z, t) \end{aligned}$$

$$(6.3)\qquad \begin{aligned} v_x &= \bar{u}_x(x, y, z, t) = +u_{1x}(-x, y, z, t) \\ v_y &= \bar{u}_y(x, y, z, t) = -u_{1y}(-x, y, z, t) \qquad x \le 0 \\ v_z &= \bar{u}_z(x, y, z, t) = -u_{1z}(-x, y, z, t) \end{aligned}$$

Thus at a point (x, y, z) below the surface and at the corresponding point (-x, y, z) above the surface, the v_x components are exactly equal and the v_y, v_z components are equal in magnitude but opposite in sign.

The boundary conditions for $\frac{\partial \bar{u}}{\partial n}$ on the reflection S_E' can be obtained from the above velocity equations. Since the problem is linear, the velocity

$\vec{v}_0$ can be broken into horizontal $\vec{v}_H$ and vertical $\vec{v}_V$ components which can be treated independently. The normal $\vec{n}$ may be similarly broken into $\vec{n}_H$ and $\vec{n}_V$. Then $\vec{v} = \vec{v}_H + \vec{v}_V$ on S_E becomes $\vec{v} = \vec{v}_H - \vec{v}_V$ on S_E', and $\vec{n} = \vec{n}_H + \vec{n}_V$ on S_E becomes $\vec{n} = -\vec{n}_H + \vec{n}_V$ on S_E'. We, therefore, obtain

$$(6.4) \qquad \frac{\partial\phi}{\partial n}\Big|_{S_E'} = -\nabla\phi \cdot \vec{n}\Big|_{S_E'} = -(\vec{v}_H - \vec{v}_V) \cdot (-\vec{n}_H + \vec{n}_V)$$

$$= +\vec{v}_H + \vec{v}_V = -\vec{v}_H \cdot \vec{n}\Big|_{S_E'} + \vec{v}_V \cdot \vec{n}\Big|_{S_E'} .$$

The case of vertical entry $(\vec{v}_H = 0)$ for t fixed is then the same boundary value problem obtained from a completely immersed body S_ES_E' moving through a stationary fluid with a velocity $\vec{v}_V$. The case $\vec{v}_V = 0$ is the same problem as that obtained from a body with S_E moving with a velocity $\vec{v}_H$ and S_E' moving with a velocity $-\vec{v}_H$. If the coordinates are transformed so that the fluid is moving, then the vertical entry case corresponds to a fluid moving upward with velocity $\vec{v}_V$. The case $\vec{v}_V = 0$ corresponds to a fluid moving with velocity $\vec{v}_H$ in one half of the space and with velocity $-\vec{v}_H$ in the other half. for either moving or stationary fluid the general entry consists of the superposition of the two flows.

An advantage of the reflected problem is that a mixed Neumann-Dirichlet problem becomes a pure Neumann problem. An advantage for the vertical entry case is that the problem is reformulated in terms of an equivalent problem whose solution may be known.

Summary: Linearized, incompressible, reflected

$$\bar{u} \equiv 0 \quad \text{for} \quad t \leq 0$$

$$\nabla^2 \bar{u} = 0 \quad \text{in } D \text{ and } D'$$

(6.5) $$\bar{u}(\infty) = 0$$

$$\frac{\partial \bar{u}}{\partial n} = -\vec{v}_0 \cdot \vec{n} \quad \text{on} \quad S_E$$

$$\frac{\partial \bar{u}}{\partial n} = -\vec{v}_H \cdot \vec{n} + \vec{v}_V \cdot \vec{n} \quad \text{on} \quad S_E'.$$

7. TWO-DIMENSIONS AND AXIAL SYMMETRY

In the two-dimensional and the axial-symmetric cases, a stream function can be introduced. For two dimensions, a stream function ψ may be defined by

$$\frac{1}{\rho}\frac{\partial\psi}{\partial y} = v_x = -\frac{\partial\phi}{\partial x}$$

(7.1)
$$-\frac{1}{\rho}\frac{\partial\psi}{\partial x} = v_y = -\frac{\partial\phi}{\partial y}$$

This change of variable is possible with either the exact and the linearized boundary conditions and either compressible or incompressible flow. In the case of the reflected, linearized problem for incompressible flow, the transformation converts the Neumann problem to a Dirichlet problem.

The techniques of complex variables may be applied to the two-dimensional case by introducing the complex potential $w = \phi + i\,\psi$.

For the axially-symmetric case, the stream function may be defined by:

(7.2)
$$\frac{1}{r}\frac{\partial\psi}{\partial r} = v_z = -\frac{\partial\phi}{\partial z}$$

$$-\frac{1}{r}\frac{\partial\psi}{\partial z} = v_r = -\frac{\partial\phi}{\partial r}$$

This change of variable is possible with either the exact or linearized boundary condition for incompressible flow. Such a change of variable does not appear feasible for the compressible case.

8. LAGRANGIAN FORMULATION

In the Lagrangian formulation of fluid mechanics, one sets up a coordinate system which follows the motion of the particles. One may visualize the problem geometrically by considering a coordinate system, say (x, y, z)

cartesian coordinates, embedded in a fluid. As the fluid moves the embedded system distorts into another coordinate system. At any later time, a given fluid particle can be represented either in the distorted coordinate system in a coordinate system which has remained fixed with time. The problem is a Lagrangian formulation is to find the equation of transformation between the distorted system and the fixed system. Let (x, y, z) be the initial coordinates of a particle and let (ξ, η, ζ) be its coordinates at time t. Then we wish to find ξ, η, and ζ as functions of x, y, z and t:

$$\begin{aligned} \xi &= \xi(x, y, z, t) \\ \eta &= \eta(x, y, z, t) \qquad (8.1) \\ \zeta &= \zeta(x, y, z, t) \end{aligned}$$

In addition to the dependent variables ξ, η, ζ, we also take the density ρ and pressure p as dependent variables:

$$\begin{aligned} \rho &= \rho(x, y, z, t) \qquad (8.2) \\ p &= p(x, y, z, t) \ . \end{aligned}$$

Five equations necessary to define ξ, η, ζ, ρ and p can be obtained from the conservation of momentum (3), conservation of mass (1), and a polytropic relation $p = f(\rho)$ between pressure and density (1).

Considering the velocities $\left(\frac{\partial\xi}{\partial t}, \frac{\partial\eta}{\partial t}, \frac{\partial\zeta}{\partial t}\right)$, the accelerations $\left(\frac{\partial^2\xi}{\partial t^2}, \frac{\partial^2\eta}{\partial t^2}, \frac{\partial^2\zeta}{\partial t^2}\right)$, and forces (pressure and potential forces F_ξ, F_η, F_ζ per unit mass) which act on a differential element of volume, we obtain by conservation of momentum:

$$\frac{\partial^2\xi}{\partial t^2} = F_\xi - \frac{1}{\rho}\frac{\partial p}{\partial \xi}$$

$$(8.3) \qquad \frac{\partial^2\eta}{\partial t^2} = F_\eta - \frac{1}{\rho}\frac{\partial p}{\partial \eta}$$

$$\frac{\partial^2\zeta}{\partial t^2} = F_\zeta - \frac{1}{\rho}\frac{\partial p}{\partial \zeta} .$$

(cf. Lamb [63], p. 12)

To eliminate the derivatives of p with respect to dependent variables, we can proceed in two ways. For fixed t, (8.1) can be considered as a change of variables from (x, y, z) to (ξ, η, ζ) and conversely. Then:

$$(8.4) \qquad \frac{\partial p}{\partial x} = \frac{\partial p}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial x} + \frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial x} .$$

Multiplying the equations (8.3) by $\frac{\partial \xi}{\partial x}$, $\frac{\partial \eta}{\partial x}$ and $\frac{\partial \zeta}{\partial x}$, respectively, and adding yields $\frac{\partial p}{\partial x}$. Similarly to obtain equations in $\frac{\partial p}{\partial y}$ and $\frac{\partial p}{\partial z}$.

As a second approach, we write out all equations (8.4)

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial x} + \frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial x}$$

$$(8.4) \qquad \frac{\partial p}{\partial y} = \frac{\partial p}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial y} + \frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial y}$$

$$\frac{\partial p}{\partial z} = \frac{\partial p}{\partial \xi}\frac{\partial \xi}{\partial z} + \frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial z} + \frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial z} .$$

Considering this as a linear system of equations in the unknowns $\left(\frac{\partial p}{\partial \xi}, \frac{\partial p}{\partial \eta}, \frac{\partial p}{\partial \zeta}\right)$, we solve:

$$\frac{\partial p}{\partial \xi} = \frac{\partial(p, \eta, \zeta)}{\partial(x, y, z)} \Big/ \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)}$$

$$(8.5) \qquad \frac{\partial p}{\partial \eta} = \frac{\partial(\xi, p, \zeta)}{\partial(x, y, z)} \Big/ \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)}$$

$$\frac{\partial p}{\partial \zeta} = \frac{\partial(\xi, \eta, p)}{\partial(x, y, z)} \Big/ \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)}$$

where the term $\frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)}$ is the Jacobian of the transformation from (x, y, z) to (ξ, η, ζ).

Thus the momentum equations (8.3) can be written:

$$\left(\frac{\partial^2 \xi}{\partial t^2} - F_\xi\right) \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)} = -\frac{1}{\rho} \frac{\partial(p, \eta, \zeta)}{\partial(x, y, z)}$$

$$(8.6) \qquad \left(\frac{\partial^2 \eta}{\partial t^2} - F_\eta\right) \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)} = -\frac{1}{\rho} \frac{\partial(\xi, p, \zeta)}{\partial(x, y, z)}$$

$$\left(\frac{\partial^2 \zeta}{\partial t^2} - F_\zeta\right) \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)} = -\frac{1}{\rho} \frac{\partial(\xi, \eta, p)}{\partial(x, y, z)} .$$

The conservation of mass can be obtained from the fact that the differential volume element dx dy dz under the transformation (8.3) becomes $\frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)}$ dx dy dz . Thus the conservation of mass becomes:

$$(8.7) \qquad \rho_0 = \rho \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)}$$

where ρ_0 is the density at time $t = 0$. Using (8.7), (8.6) may be written in a simpler form as in the following summary.

Summary: Lagrangians formulation, polytropic fluid

(8.7) $$\rho_0 = \rho \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)}$$

(8.8a) $$\frac{\partial^2 \xi}{\partial t^2} - F_\xi = -\frac{1}{\rho_0} \frac{\partial(p, \eta, \zeta)}{\partial(x, y, z)}$$

(8.8b) $$\frac{\partial^2 \eta}{\partial t^2} - F_\eta = -\frac{1}{\rho_0} \frac{\partial(\xi, p, \zeta)}{\partial(x, y, z)}$$

(8.8c) $$\frac{\partial^2 \zeta}{\partial t^2} - F_\zeta = -\frac{1}{\rho_0} \frac{\partial(\xi, \eta, p)}{\partial(x, y, z)} .$$

Equations (8.7) and (8.8) plus the polytropic equation

(8.9) $$p = f(\rho)$$

yield five equations for the five dependent variables ξ, η, ζ, p, ρ.

In cases where the polytropic equation (8.9) is not available, it is necessary to add one more dependent variable representing the energy, one equation for this variable (conservation of energy) and one equation to replace (8.9) (equation of state). In the use of numerical methods, it is also advisable to add a von Neumann-Richtmyer artificial viscosity (to spread out shock discontinuities).

The boundary conditions involving $p =$ constant on the free surface and normal component of velocity on the body are easily handled in the Lagrangian formulation. The movement of the free surface itself is obtained simply by following the motion of particles originally on the free surface.

The CYCLONE code at the Naval Ordnance Laboratory can handle a axially-symmetric body entering vertically into a compressible fluid. This code makes use of an equation for energy, an equation of state, and an artificial viscosity. The disadvantage of using a numerical treatment of the Lagrangian equations is the lengthly computing time required. (cf. [91]).

9. SOURCE AND DIPOLE FORMULATION

For a body entering an incompressible fluid, the water entry problem is one in potential theory. By Green's third identity (see Courant-Hilbert [31], p. 257 or Kellogg [59], p. 219), any harmonic function in a region can be represented as the sum of a source and a dipole distribution of the boundary of the region providing the harmonic function has a continuous derivative up to and including the boundaries. By considering the exterior and interior boundary value problem for the same boundary, we may eliminate the dipole term. In fact, for a infinte domain, we may specify that the source or dipole terms occur either alone or in any combination.

If we chose a source distribution, we are thus able to write the solution to our problem as:

$$(9.1) \qquad \phi(x, y, z) = \iint\limits_{S_E+S_F} \frac{Q(\xi, \eta, \zeta)}{\sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}} d\sigma \equiv \iint\limits_{B} \frac{Qd\sigma}{r}$$

where (x, y, z) is a point in the fluid, (ξ, η, ζ) is a point on the boundary $(S_E + S_F) \equiv B$, $d\sigma$ is the element of surface area, and $Q(\xi, \eta, \zeta)$ is the unknown source distribution per unit area.

The representation for ϕ will automatically satisfy Laplace's equation in the interior of the fluid. To solve for Q it is necessary to substitute (9.1) into the boundary conditions for S_E and S_F and to solve the resulting integral equations. For the solution of these integral equations a numerical approach is desirable. The advantage of solving the problem in this form (numerically or otherwise) is that the unknown source density Q is a function of only two variables for a full three-dimensional problem and a function of only one variable for a two-dimensional or an axially symmetric problem. The saving in computer time and storage is considerable.

10. OUTPUTS AND ADDED MASS

Once the velocity potential ϕ has been obtained, the pressures, forces and moments acting on the solid body can be calculated. Bernoulli's equation:

$$(10.1) \qquad \frac{p}{\rho} + \frac{1}{2}(\nabla\phi)^2 - \frac{\partial\phi}{\partial t} - gz = 0$$

can be used to calculate p, the increase over atmospheric pressure, at any point in the fluid, and, in particular along the solid body. Integration over the body surface then gives the total forces and moments acting on the body.

Other items of interest obtainable from the solution are the shape of the free surface, the contact angle (angle between the solid body and the free surface at the point of contact), and the added mass.

The added mass is of special interest because it provides an alternative method for calculating the total force on the body. We define energy added mass as follows. First, calculate the total kinetic energy of the fluid due to the introduction of the solid body, i.e.

$$(10.2) \qquad \iiint_D \frac{1}{2}\rho(\nabla\phi)^2\, d\tau \ .$$

Then the energy added mass m_E is defined to be that mass of water which if moving with the velocity of the solid body has the kinetic energy given by (10.2). Expressed mathematically, m_E must satisfy:

$$(10.3) \qquad \frac{1}{2} m_E v(t)^2 = \iiint_D \frac{1}{2}\rho(\nabla\phi)\ d\tau \ .$$

(Virtual mass may then be defined as the sum of m_E and m, the mass of the solid body.)

Conservation of energy gives:

$$(10.4) \qquad \frac{1}{2} m v_0^{\ 2} = \frac{1}{2} m v(t)^2 + \frac{1}{2} m_E v(t)^2$$

The total drag force F_D (opposite in direction to $v(t)$) is:

$$(10.5) \qquad F_D = -\frac{d}{dt}(mv(t)).$$

Differentiation of (10.4) gives:

$$(10.6) \qquad 0 = \frac{mvdv}{dt} + m_E \frac{vdv}{dt} + \frac{1}{2}v^2 \frac{dm_E}{dt} \ .$$

Solving for $\frac{mdv}{dt}$ and substituting into (10.5) gives

$$(10.7) \qquad F_D = \frac{\frac{1}{2} v \frac{dm_E}{dt}}{1 + \frac{m_E}{m}}$$

Alternatively, since $v = \frac{dz}{dt}$ and rewriting (10.4) as

$$(10.8) \qquad {v_0}^2 / (1 + \frac{m_E}{m}) = v^2 \ ,$$

we have:

$$(10.9) \qquad F_D = \frac{\frac{1}{2} {v_o}^2 \frac{dm_E}{dz}}{(1 + m_E/m)^2} \ .$$

To put (10.3) and the calculation of m_E into more convenient form, we apply Green's first identity:

$$(10.10) \qquad \iiint_{V_R} (\nabla\phi)^2 \, d\tau = -\iiint_{V_R} \phi(\nabla^2\phi) \, d\tau - \iint_{S_F+S_E+S_R} \phi \frac{\partial\phi}{\partial n} \, d\sigma$$

where $\frac{\partial\phi}{\partial n} = \nabla\phi \cdot \vec{n}$, $\vec{n}$ is the unit normal to the surface directed into the fluid, S_F is the free surface, S_E is the surface of the entering body, S_R is the underwater surface of a large sphere of radius R enclosing S_E, and V_R is the fluid volume enclosed by S_F, S_E and S_R.

The first integral on the right-hand side of (10.10) vanishes since ϕ is harmonic. If ϕ and $\frac{\partial\phi}{\partial n}$ approach zero at infinity at least as fast as a point source, then

$$(10.11) \qquad \iint_{S_R} \phi \frac{\partial\phi}{\partial n} \, d\sigma \rightarrow 0 \text{ as } R \rightarrow \infty \ .$$

Thus combining (10.3), (10.10) and (10.11), we have:

$$(10.12) \quad m_E = \frac{-\rho}{V^2} \iint_{S_F+S_E} \phi \frac{\partial \phi}{\partial n} \, d\sigma \, .$$

One may extend the added mass concept in several different ways. An added mass due to momentum may be defined in a way similar to (10.3) (see Schiffman and Spencer [108]). Also the added mass may be considered as a tensor in either the energy or momentum formulation (see Birkhoff [10] and [11]).

11. GENERAL DISCUSSION

One should note that a solution to the water entry problem has applications in several connections. Of course, the most obvious application is to a torpedo or a missile striking the water surface. Another application is to the forces and moments acting upon a pontoon during seaplane landing. A third application is to ship slamming, that is, the forces exerted on the hull of a ship during heavy seas when the bow of the ship is alternately lifted out and slammed into the sea.

Several alternatives to the source-sink formulation are available. One could assume a dipole (or vortex) distribution over the free surface and the body surface. That is, assume ϕ is of the form:

$$(11.1) \quad \phi = \iint_B Q \frac{\partial}{\partial n} \left(\frac{1}{r}\right) d\sigma$$

Substitute into the boundary conditions and solve for the unknown dipole strengths Q. Again, one could assume a dipole distribution over the body surface and a source distribution over the free surface or a dipole distribution over free surface and a source distribution over the body surface. There are certain reasons for preferring the latter. Again, one could assume the superposition of a source and a dipole distribution over all surfaces. Then by imposing one added condition on either the dipole or the source distribution (or some combination), one may solve for the remaining distribution. This is the most general situation for this class of solutions. The previous cases correspond to an added condition that certain distributions have zero strength.

For some body configurations and some trajectories, it is possible to transform the water entry problem to one wherein time does not explicitly appear. These are the quasi-steady state or "similar" solutions which apply to the wedge or the cone (in two or three dimensions, respectively). If a wedge is entering the water at a constant velocity, it seems possible that the free surface at a time $2t_0$ should be the same as the free surface at time t_0 except for an magnification by a factor of two. The absence of a characteristic length also suggests such a similar solution. One may verify by a change of variables that there, indeed, is a boundary value problem whose solution is the similar solution indicated above. Let us change variables from (x, y, ϕ, F) to $(\xi, \eta, \bar{\phi}, \bar{F})$ in the following way:

$$
\begin{aligned}
x &= Vt\xi \\
y &= Vt\eta \\
\phi &= V^2t\bar{\phi}(\xi, \eta) \\
F &= Vt\bar{F}(\xi, \eta)
\end{aligned}
\tag{11.2}
$$

The first two transformations (11.2) are suggested by the magnification varying linearly with time. The second two transformations are suggested by dimensional consideration of ϕ and F. Substitution of the new variables into the differential equation and all boundary conditions verifies that the water entry problem transforms into one whose solution is independent of time. The solution of the transformed problem is the similar solution.

In a corresponding way a similar solution may be described for a cone in three dimensions. This class of similar solutions can be obtained in either two or three dimensions for either vertical or oblique entry and even for compressible as well as incompressible flow. Another class of similar solutions is obtainable for the wedge or cone when the body is subjected to a constant acceleration. In the latter case, gravity may be taken into account since this is another constant acceleration term. The transformation is of the form $x = at^2\xi, \ldots$. This second class of solutions may find some application in the water exit problem (with constant thrust) whereas the first class is more appropiate for water entry.

An analytic but approximate solution for the total force acting on an entering body can be obtained from the added mass. Direct methods of the calculus of variations may be used to get approximations or bounds for the added mass. Then time differentiation of the added mass will give a total force estimate. Von Karman's approximation (the expanding plate approximation) may be obtained in this manner.

A complete analytic solution is possible for the wedge in linearized, incompressible flow. In this case, the techniques of conformal mapping are used. A Schwarz-Christoffel mapping takes the wedge into a straight line. The solution in the transformed plane plus the mapping yield the complete solution in the original plane. This solution is complicated by inconvenient functions and their inverses occurring in the mapping.

The various formulations of the water entry problem given in this report can be broadened to include still other effects. The effect of air density has been ignored so far except for the requirement of constant surface pressure. If a non-zero air density is assumed, then we have a two fluid problem with the Bernoulli's equation at the free surface replaced by the continuity of the pressure and by the continuity of the normal velocity component at the surface. Another effect which may be included is the coupling between the trajectory of the body and the forces acting upon it. Use of a six-degree of freedom trajectory problem would allow for rotations and accelerations in all dimensions. The water entry formulation would have to be modified to include all of these possibilities. Another effect which might be included is the elasticity of the body. Vibrations and small deformations may not be too difficult. Crushing might be handled by some discontinuous change in body geometry.

Problems related to the physical problem of water entry are (1) the fluid body impacting on a fluid surface (hydroballistic impact as for very high velocity iron pellet striking an iron plate), (2) a solid wedge of included angle α striking a fluid wedge of included angle β (the usual case is $\beta = 180°$; a similar case is $\alpha = 180°$; other cases might correspond, physically, to a wedge striking a water wave, (3) the water exit problem (one should note that the linearized water entry and linearized water exit problems are equivalent in the sense that for corresponding boundary and initial conditions one is the time reversal of the other); (4) jets, cavities, and underwater explosions (these are similar problems in that they involve free surface and constant pressure boundary conditions); (5) the richochet problem (under what conditions will the entering body richochet on the water surface).

With the possible exception of certain singular points, the source and dipole formulation should be able to describe the forces acting upon a body not only during the initial impact phase but also during the phases of cavity formation, fully developed cavity, and full immersion. These calculations of the forces coupled with a trajectory calculation would give a fairly complete picture of water entry. Some of the singular points giving mathematical difficulty might be the point of contact between the rigid body and the free surface and the point of cavity closure. There may also be mathematical instability in the full cavity phase related to the physical instability as the body strikes the cavity walls.

A note of caution should be observed concerning the conservation of mass. A number of writers on water entry have stated that the conservation of mass for an incompressible fluid is equivalent to the statement that the mass of fluid appearing above the initial plane is equal to the mass of fluid displaced by the body below the initial plane. As first pointed out by Charles Weber, in private communication, this equivalence is certainly true in the finite case, i.e. where the amount of fluid is contained within a large tank or other container. However, in the case where we have an infinite ocean, there is no a priori assurance that some of the fluid does not flow off to infinity. The simplest example of fluid flowing to infinity without creation or destruction of mass is that of a bubble expanding in a ocean which is infinite in every direction. A way of side-stepping this question would be to consider the problem of water entry into a large body of water enclosed in a tank. In this case, however, the boundary value problem must be modified. Boundary conditions must then be specified on the walls of the tank (no fluid penetration) as well as the body surface and the free surface.

APPENDIX A

The kimematic surface condition

(A.1) $$\frac{\partial F}{\partial t} + \vec{q} \cdot \nabla F = 0$$

may be put into several alternate forms. First, suppose that we have Cartesian coordinates (x, y, t) fixed relative to the water and the equation of the surface is given in the form

(A.2) $$z = g(x, y, t)$$

where z is in the vertical direction. Then

(A.3) $$F \equiv g(x, y, t) - z, \quad \frac{\partial F}{\partial t} = \frac{\partial g}{\partial t} ,$$

(A.4) $$\nabla F = \vec{i} \frac{\partial g}{\partial x} + \vec{j} \frac{\partial g}{\partial y} - \vec{k} , \quad \vec{q} = -\nabla\phi .$$

Thus,

(A.5) $$\frac{\partial g}{\partial t} = \nabla\phi \cdot (\vec{i} \frac{\partial g}{\partial x} + \vec{j} \frac{\partial g}{\partial y} - \vec{k}).$$

If we introduce $\vec{n}$, the unit normal pointing out of the water, we have

(A.6) $$\vec{n} \equiv \frac{-\vec{i}g_x - \vec{j}g_y + \vec{k}}{\sqrt{(g_x)^2 + (g_y)^2 + 1}} ,$$

and thus,

(A.7) $$\frac{g_t}{\sqrt{(g_x)^2 + (g_y)^2 + 1}} = - \nabla\phi \cdot \vec{n} .$$

Noting that:

(A.8) $\vec{k} \cdot \vec{n} = 1/\sqrt{(g_x)^2 + (g_y)^2 + 1}$,

we finally obtain:

(A.9) $(\vec{k}g_t) \cdot \vec{n} = -\nabla\phi \cdot \vec{n}$.

The last equation has a geometric interpretation. The vertical velocity $(\vec{k}g_t)$ of the surface has its normal component equal to the normal component of the velocity of the fluid particle.

As a second illustration, suppose we have spherical coordinate system (r, ψ, θ) fixed relative to the body (moving velocity $\vec{v}$). Let us further assume that the flow is symmetric about the z-axis so that the motion is independent of the ψ coordinate, and that the equation of the surface given as

(A.10) $r = g(\theta, t)$.

Thus,

(A.11) $F \equiv g(\theta, t) - r, \quad \frac{\partial F}{\partial t} = \frac{\partial g}{\partial t}$

(A.12) $\nabla F = -\vec{i}_r + \frac{\vec{i}_\theta}{r} \frac{\partial g}{\partial \theta}$,

and

(A.13) $\vec{q} = -\nabla\phi - \vec{v}$.

We then obtain:

(A.14) $\frac{\partial g}{\partial t} = (\nabla\phi + \vec{v}) \cdot (-\vec{i}_r + \frac{\vec{i}_\theta}{r} \frac{\partial g}{\partial \theta})$

or

$$\text{(A.15)} \quad \frac{\partial g}{\partial t} = (-\phi_r + \frac{1}{r^2} \frac{\partial \phi}{\partial \theta} \frac{\partial g}{\partial \theta}) + (-v_r) - (\frac{-v_\theta}{r} \frac{\partial g}{\partial \theta})$$

The last equation gives the explicit dependence of the time rate of change $(\frac{\partial g}{\partial t})$ of the r-coordinate in terms of the particle velocity $(-\phi_r, -\frac{1}{r}\frac{\partial \phi}{\partial \theta})$, the slope of the free surface $(\frac{\partial g}{\partial \theta})$, and the negative of the body velocity $(-v_r, -v_\theta)$.

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The following bibliography is a rather complete listing of those papers and reports (primarily theoretical) on water entry which have come to the writer's attention. Also included are papers which discuss topics closely related to water entry. The list of papers on the experimental side of water entry is quite brief. Only a few of the well-known reports have been included here.

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13. ABSTRACT

This report is a survey of the water entry problem and its various mathematical formulations. Included are discussions on the incompressible case, the compressible case, the linearized incompressible, the linearized compressible, reflected linearized incompressible, Lagrangian, two-dimensional and axially-symmetric cases. Also discussed are the source and dipole formulations, calculations of added masses, similar solutions, problems related to water entry, and a note of caution concerning the conservation of mass in an infinite region. Finally, a fairly extensive bibliography is given primarily covering theoretical work on the water entry and related problems.

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