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DISTRIBUTION OF THE FIRING ENERGY FOR THE SQUIB MK 1 MOD 0

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FOREWORD

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INTRODUCTION

The energy required to ignite electro-explosive devices (EED's) of a given type will vary from device to device. It is not possible to determine the energy required to fire a given device. The best that can be done is to predict the probability that a randomly selected device will fire at a specified energy.

An accidental firing of an EED by electromagnetic radiation occurs when the induced current exceeds the current required to fire that particular EED. The probability of an accidental firing may be calculated from the distribution of induced current and the distribution of current required to fire a randomly selected device. This has been discussed by Richter and Carson [reference 1]. Thus, any analysis of accidental firings of EED's will require information on the distribution of currents needed to fire EED's.

Approximately 8000 Squib Mark 1 Model 0 devices were tested by the Naval Ordnance Laboratory (NOL) in order to determine the distribution of energies required to ignite these squibs. The results of that test and the analysis of the data have been reported by Hampton and Ayres [reference 2]. Most of the data, particularly in the tails of the distribution, were obtained according to the Bartlett plan. The Bartlett plan used specifies the testing of devices at an energy level until two reversals are observed. A reversal is a non-fire where fires are predominant or vice versa. The NOL analysis indicated that the log-logistic distribution was not completely satisfactory, but it provided a better fit than the log-normal distribution. The NOL analysis was based on maximum likelihood estimates of the probability of firings from the Bartlett plan data. In a subsequent study [reference 3], NOL noted that a bias in these estimated firing probabilities exists where data are collected by the Bartlett plan. In fact, it will be shown that these estimates are biased high by approximately a factor of two at extremely low firing probabilities. It is this lower tail of the firing sensitivity distribution which is of most interest in HERO analyses since most accidental firings will occur when the current required to fire an EED is low. Thus, it appeared desirable to re-analyze the Squib MK 1 Mod 0 firing data using an unbiased estimator for the firing probabilities.

SUMMARY

Neither the log-normal nor the log-logistic provides an adequate fit over the entire range of the Squib Mk 1 Mod 0 data, ~~as stated in reference [2].~~ Using unbiased estimates of firing probabilities, both of these distributions fit the lower 95% of the firing sensitivity data reasonably well. The log-normal gives a slightly better fit to these data. It is illustrated that the upper 5% of the firing energies generally contribute a negligible amount to the probability of an accidental firing. These data indicate that the log-normal distribution provides a satisfactory approximation when calculating the probability of an accidental firing.

Since both the log-normal and log-logistic fit the Squib Mk 1 Mod 0 firing data about equally well, use of the log-normal distribution is recommended in HERO computations where the distribution of the induced currents also must be considered since this leads to a simpler procedure for estimating accidental firing probabilities.

DISCUSSION

Let r represent the number of firings and n the total number of EED's tested at a particular energy level. If EED's are tested at a specified energy level until r firings occur, an unbiased estimate of the probability of firing, p, is given by $(r-1)/(n-1)$, for $r \geq 2$, see the Appendix. For the Squib Mk 1 Mod 0 data, r = 2 with the Bartlett plan used by NOL. The unbiased estimate of the firing probability is $1/(n-1)$; whereas NOL used $2/n$. The NOL estimates in the lower tail are high by a factor of $(\frac{r}{r-1})(\frac{n-1}{n})$, which is approximately a factor of two for r = 2 and large n. Hence, the NOL analysis of the firing sensitivity distribution is based on biased estimates of these probabilities. The biased maximum likelihood estimator does not recognize that testing was terminated with a firing. Therefore, the observed percentage of firings will be high. On the other hand, if a fixed sample size of n EED's are tested at a specified current, and r fire, then the maximum likelihood estimate, r/n, provides an unbiased estimate of the firing probability. The distinction is: in the first case the number of firings, r, is fixed and the number of EED's tested, n, is a random variable; in the second case, the number of EED's tested, n, is fixed and the number of fires, r, is a random variable.

A description of the experiment is given in ref. [2]. The data are reproduced in Table 1 for convenient reference. Unbiased estimates of the firing probabilities from the Bartlett plan (negative binomial) data are provided by $1/(n-1)$, where n is the total number of devices tested at a given energy level. Sufficient detail was not given in [2] in order to determine how the pilot test data were collected. It was assumed that the sample size was fixed for the pilot test data. The NOL estimates were all calculated by, r/n, where r is the number of fires. This estimator is unbiased only for the binomial (pilot test) data if the number of tests, n, are fixed for each energy level in advance of the test. An unbiased estimate of the firing probability at the lowest energy level in Table 1 could not be obtained since it was not known to us whether or not testing terminated with the single firing that occurred.

The following notation will be used:

x_i = log energy at the ith level

p_i = unbiased estimate of the firing probability at the ith energy level

y_i = standardized normal deviate corresponding to p_i.

If the data fit a log-normal distribution, then

$$y = \frac{x - \mu_x}{\sigma_x} = \frac{-\mu_x}{\sigma_x} + \frac{x}{\sigma_x} , \qquad (1)$$

where μ_x is the average firing log-energy and σ_x is the standard deviation of firing log-energy. That is, y versus x is a linear relationship with intercept, $-\mu_x/\sigma_x$, and slope, $1/\sigma_x$. Estimates of μ_x and σ_x can be obtained from the fit of the simple linear regression of y_i versus x_i.

If the data fit a log-logistic distribution, then

$$\log_{10}\left(\frac{p_i}{1-p_i}\right) = a + bx_i \qquad (2)$$

where a and b are constants which can be estimated by simple linear regression.

TABLE 1. Firing information for Squib Mk 1 Mod 0

Delivered energy ($\log_{10}$ millijoules)	Bartlett plan		Pilot tests		Unbiased estimate
	Fires	Fails	Fires	Fails	
.2528	2486	2			.9996
.2340	617	2			.9984
.2151	460	2			.9978
.1964	819	2			.9988
.1778	61	2			.9839
.1596	118	2			.9916
.1408	37	2			.9737
.1229	34	2			.9714
.1045	86	2			.9885
.0864	14	2			.9333
.0846			53	9	.8548
.0678	9	2			.9000
.0508	5	2			.8333
.0422			125	53	.7022
.0009			67	124	.3508
- .0158			2	3	.[illegible]000
- .0191	2	23			.0417
- .0357	2	31			.0312
- .0405			1	46	.0213
- .0531	2	281			.0035
- .0696	2	344			.0029
- .0862	2	234			.0043
- .1024	2	441			.0023
- .1186	1	1270			--

Ordinary logit and probit analysis techniques are not applicable to the negative binomial (Bartlett plan) data obtained at NOL and presented in ref. [2]. An unweighted least squares procedure was used to fit the log-normal and log-logistic distributions to the data. The way these tests were conducted, the data in the tails of the distribution possess smaller variances. In a weighted least squares analysis, the ends of the distribution would dominate the analysis and tend to cause a straight line fit regardless of the type of distribution. Hence, an unweighted least squares analysis was used in order to detect non-linearity.

RESULTS

The unbiased estimates of the firing probabilities are plotted in Figure 1. The normal deviate, y_i, is plotted against log-energy. A straight line here would indicate that the data fit a log-normal distribution. $Log_{10}[p_i/(1-p_i)]$ is plotted in Figure 2 against log-energy. A straight line here would indicate that the data fit a log-logistic distribution. As can be seen from Figures 1 and 2 and verified by statistical tests, neither the log-normal nor log-logistic distribution provides a good fit over the entire range of firing energy.

For the HERO problem, most accidental firings will occur with EED's possessing low firing energies. Hopefully, for any given set of conditions, the probability of an accidental firing will be low. That is, the probability that the firing energy is less than the induced energy [1/] will generally be low. A satisfactory estimate of firing probability can be obtained by fitting the lower part of the firing sensitivity distribution. In fact, either a log-normal or log-logistic can be used to fit all but the upper 5% of these firing data. Using the data with estimated firing probabilities below 95% resulted in an extremely good fit to the log-normal distribution. The non-linearity (quadratic effect) in the lower 95% of the distribution was significant at only the 20% level. That is, if the relationship is linear (log-normal distribution), the amount of non-linearity observed here could be attributed to chance alone approximately 80% of the time from similar experiments. The log-logistic did not fit quite as well, its non-linearity was significant at the 40% level. That is, 60% of the

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[1/] The term "induced energy" is used here for purposes of illustration, since the firing probabilities were estimated as a function of energy. The energy induced results from the current induced by electromagnetic radiation. Mathematically, it makes no difference whether energy or current is used here as long as a comparable characteristic (current or energy) is used for both the firing distribution and the distribution induced by electromagnetic radiation.

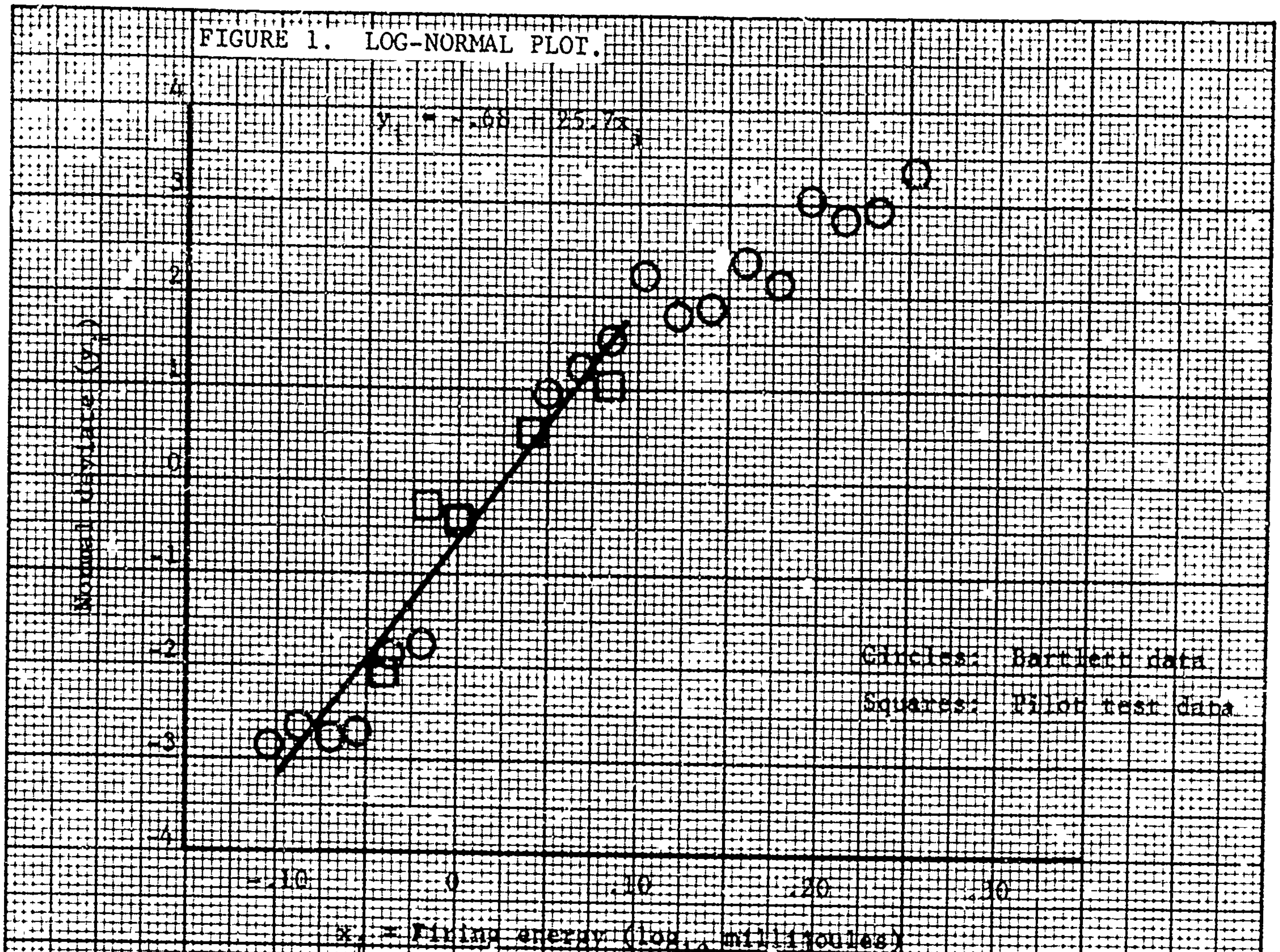

FIGURE 1. LOG-NORMAL PLOT.

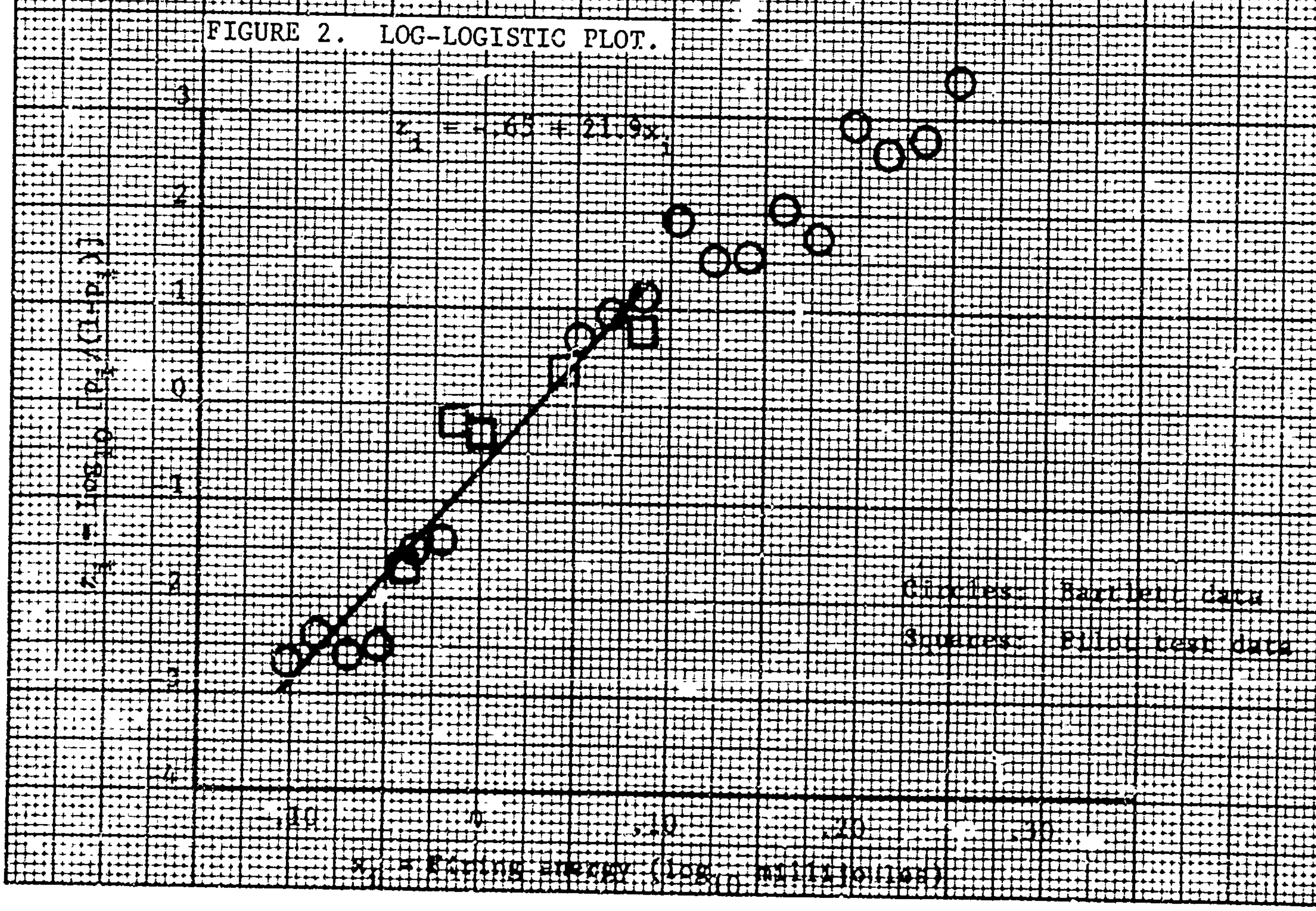

FIGURE 2. LOG-LOGISTIC PLOT.

time a worse fit would be expected due to chance variation if the distribution is truly log-logistic.

Using the log-normal or log-logistic distribution obtained from firing probabilities below 95% extrapolated over the entire range of firing energy will generally result in a negligible over-estimate of the probability of accidental firings. The observed firing energies in the upper tail are slightly higher than those predicted by extending the fit from the observed firing probabilities below 95%. A few examples will illustrate that the upper tail of the firing sensitivity distribution is generally unimportant in HERO problems. Using the Squib Mk 1 Mod 0 data below the 95% firing level resulted in an estimated average firing log-energy of $\bar{x}_2 = .026$ with an estimated standard deviation of $s_2 = .039$. Suppose for a given set of conditions that the estimates of the mean and standard deviation of the induced log-energy are $\bar{x}_1$ and s_1. It appears from analyses of HERO research test data, Naval Weapons Laboratory, that the log-normal distribution is a satisfactory approximation to the distribution of induced current. Since induced energy is proportional to the induced current, induced energy would be described by a log-normal distribution. Then, the difference between induced log-energy and firing log-energy, $x_1 - x_2$, follows a normal distribution with an estimated mean of $(\bar{x}_1 - \bar{x}_2)$ and estimated standard deviation of $\sqrt{s_1^2 + s_2^2}$. An accidental firing occurs when the induced log-energy, x_1, exceeds the firing log-energy, x_2. The approximate estimated probability of an accidental firing is

$$P(x_1 \geq x_2) = P(x_1 - x_2 \geq 0) \quad .$$

The approximation results from extending the log-normal distribution from the lower 95% of the firing sensitivity distribution over the entire range of firing energy. Letting $N[\mu,\sigma^2]$ denote a normal distribution with mean, μ, and variance, σ^2,

$$P(x_1 - x_2 \geq 0) = \int_0^\infty N[(\mu_1 - \mu_2),(\sigma_1^2 + \sigma_2^2)]dx \approx \int_u^\infty N[0,1]dy \ , \quad (3)$$

where

$$u = \frac{0 - (\bar{x}_1 - \bar{x}_2)}{\sqrt{s_1^2 + s_2^2}} \ .$$

For large sample sizes, this probability can be approximated using tables of the standardized normal distribution.

The above formula shows how firing probabilities can be estimated if both the distribution of log-induced energy and the distribution of log-firing energy are normal. If the log-logistic distribution is used for firing energy, then a complicated numerical integration procedure would have to be performed. Since both the log-normal and log-logistic fit the Squib Mk 1 Mod 0 firing data about equally well, use of the log-normal distribution is recommended since this leads to a simpler procedure for estimating accidental firing probabilities.

It is of interest to determine the contribution of the upper 5% of the firing sensitivity distribution to the probability of an accidental firing. Consider the worst condition where all EED's in the upper 5% of the distribution would fire at the same energy obtained at the 95% level of the distribution. For the Squib, Mk 1 Mod 0, it is estimated that 95% of the EED's would fire at a log-energy below

$$\bar{x}_2 + 1.645\ s_2 = .026 + 1.645(.039) = .09 \ .$$

To obtain an upper bound on the importance of the upper 5% of the firing sensitivity distribution, suppose that the remaining 5% all fire at a log-energy of .09. The data show that the upper 5% of the log-firing energies are higher than .09. Hence, fewer accidental firings actually would occur. Under this assumption, the importance of the upper tail is magnified. Even under this assumption, Table 2 shows that the contribution of the upper tail to the probability of an accidental firing is negligible. Several values of $\bar{x}_1$ were chosen to provide a wide range of probabilities which might occur for an accidental firing. The value of $s_1 = .100$ for the standard deviation of induced log-energy was chosen as a somewhat typical value from HERO research test data collected at the Naval Weapons Laboratory. This value corresponds to a coefficient of variation, (standard deviation ÷ mean) x 100%, of approximately 25% for induced energy. The probability that the induced log-energy, x_1, exceeded .09, $P(x_1 \geq .09)$, was then obtained from tables of the standardized normal. The probability of a firing log-energy of .09 is .05 under the assumptions. Hence, the probability that an EED with a firing log-energy above .09 results in an accidental firing is $.05 \times P(x_1 \geq .09)$. This provides an upper bound to the contribution to the probability of a HERO accident from the EED's with the highest 5% of firing energies. An examination of Table 2 indicates that, for HERO-type probability calculations, it is not of primary importance for the distribution used to describe EED firing sensitivities to fit real well in the upper tail.

It is of interest to compare the probabilities of an accident based on the log-normal and the log-logistic firing sensitivity distribution. It appears that a satisfactory fit of the data for HERO predictions is obtained from the

TABLE 2. Contribution of the upper 5% of firing currents to the probability of an accidental firing.

Average induced energy (log milijoules) $\bar{x}_1$	Approximate prob. of accident $P(x_1 - x_2 \geq 0)$ 1/	Upper bound for prob. of accident from EED's with $x_2 \geq .09$ 2/
- .100	1.20×10^{-1}	1.44×10^{-3}
- .200	1.74×10^{-2}	9.33×10^{-5}
- .300	1.16×10^{-3}	2.40×10^{-6}
- .400	3.43×10^{-5}	2.40×10^{-8}

1/ Calculated from (3) with the distribution of firing energy extrapolated from the lower 95% over the whole range of firing energy.

2/ Assumes that all EED's with log-firing energy above .09 would fire at .09.

lower 95% of the distribution. The comparison is based on extrapolating the log-normal and log-logistic distributions from the lower 95% of the distributions. Table 3 shows a comparison of the approximate probabilities of HERO accidents based on these two firing sensitivity distributions. The induced energy distribution was taken to be log-normal as before. The probabilities for the log-normal firing sensitivity distribution were taken from Table 2. The probabilities for the log-logistic firing sensitivity distribution were obtained by numerical integration.

It is of interest to note that for these data, the log-normal distribution gives slightly higher probabilities. One objection that has been raised against using the log-normal is that in past analyses by NOL, ref. [4], it has predicted higher firing currents in the lower tail than the log-logistic. These data indicate just the opposite. This illustrates the need for having large sample sizes to estimate the lower tail of the firing sensitivity distribution. Apparently, the variation in the estimates of the parameters of a distribution may have more influence on probability calculations than the form of the distribution used. The close agreement between the probabilities based on the log-normal and log-logistic firing sensitivity distributions is encouraging.

TABLE 3. Approximate probabilities of an accident.

Average induced energy (log millijoules)	Firing sensitivity distribution	
	Log-normal	Log-logistic
- .100	1.20×10^{-1}	1.11×10^{-1}
- .200	1.74×10^{-2}	1.53×10^{-2}
- .300	1.16×10^{-3}	0.96×10^{-3}
- .400	3.43×10^{-5}	2.70×10^{-5}

APPENDIX

The so called negative binomial distribution is obtained when testing is continued until a specified number, r, of events are observed. The probability that a particular order of fires and no fires results in r fires after n tests is $p^r(1-p)^{n-r}$, where p is the probability that an EED will fire at a particular energy. Note that the rth firing occurs on the nth trial. The previous (r-1) firings may occur in any of the previous (n-1) trials. The number of ways that (r-1) firings can occur in (n-1) trials is given by (n-1)!/(r-1)!(n-r)! Then, the probability that n tests are required for r firings is given by the negative binomial distribution

$$P(n|r) = \frac{(n-1)!}{(r-1)!(n-r)!} p^r (1-p)^{n-r} . \qquad \text{(A1)}$$

To show that (r-1)/(n-1) is an unbiased estimator of the firing probability at a given energy, it is necessary to show that the expected value of (r-1)/(n-1) is equal to p.

$$E\left(\frac{r-1}{n-1}\right) = \sum_{n=r}^{\infty} \frac{r-1}{n-1} P(n|r) = \sum_{n=r}^{\infty} \frac{r-1}{n-1} \frac{(n-1)!}{(r-1)!(n-r)!} p^r (1-p)^{n-r}$$

$$= p \sum_{n=r}^{\infty} \frac{(n-2)!}{(r-2)!(n-r)!} p^{r-1} (1-p)^{n-r} \quad \text{for } r \geq 2 .$$

Let s = (r-1) and m = (n-1); then

$$E\left(\frac{r-1}{n-1}\right) = p \sum_{m=s}^{\infty} \frac{(m-1)!}{(s-1)!(m-s)!} p^s (1-p)^{m-s} .$$

Each quantity in the summation is of the same form as (A1). These quantities are the probabilities of observing s firings in m tests. The sum of the probabilities of all possibilities, $m = s, s + 1, s + 2, \ldots,$ is one. Hence, $E[(r-1)/(n-1)]$ is p.

Note that this estimator exists only for $r \geq 2$.

REFERENCES

[1] Richter, J. H. and Carson, R. W. "The Present Status of the HERO Probability Analysis." Tech. Memo W-8/65, U. S. Naval Weapons Laboratory, June, 1965.

[2] Hampton, L. D. and Ayres, J. N. "Characterization of Squib Mk 1 Mod 0: Determination of the statistical model." Naval Weapons Report 7347, U. S. Naval Ordnance Laboratory, March, 1961.

[3] Quarterly Progress Report, TASK NO.-443, 1 April 1963 - 30 June 1963. Explosion Dynamics Division, Explosions Research Department, U. S. Naval Ordnance Laboratory.

[4] Ayres, J. N., Hampton, L. D., and Kabik, I. "The Prediction of Very-Low EED Firing Probabilities." NOLTR63-133, U. S. Naval Ordnance Laboratory, 4 September 1963.