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THE GROWTH OF A TURBULENT WAKE IN A DENSITY-STRATIFIED FLUID

by

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NOTATION

a	Stratification parameter $= \frac{1}{\rho_o} \frac{\partial \rho}{\partial y}$
b	Rate of wake growth vertically at $x = 0$
b_1	Initial constant rate of wake growth in the vertical direction at $x = 0$
c	Constant in empirical formulae
D	Observed width of circular jet
g	Gravitational acceleration
n	Constant in empirical formulae
ρ	Fluid density
$\frac{\partial \rho}{\partial y}$	Vertical density gradient
$\rho_o, \bar{\rho}$	Fluid density at elevation of center of wake or that of mixed fluid inside the wake
t	Time after agitation of pendulum
t_{col}	Time of collapse
t'	Time,after time of collapse
$\sqrt{\overline{r^2}}$	Root-mean-square of the radius of the turbulent wake in pure water
$\sqrt{\overline{u'^2}}$	Turbulence intensity
$\sqrt{\overline{u_c'^2}}$	Turbulence intensity at time of collapse
R_i	Richardson number
R_{i_c}	Critical Richardson number

U_m	Mean velocity of advance
U_{max}	Maximum velocity at axis of jet
x,y	Coordinate axes; x-axis is horizontal and y-axis is vertically upwarJ with origin at center of wake
x_o	Horizontal width at time of collapse
y_o	Vertical extent of wake boundary at t = 0
y_{max}	Maximum vertical extent of wake boundary at x = 0 and $t = t_{col}$

ABSTRACT

The force of gravity causes a turbulent wake in a density-stratified fluid to eventually cease its vertical growth and then to collapse towards its horizontal midplane. In the present investigation this phenomenon has been studied experimentally. The turbulent wake was created by means of a spiral paddle, agitated by a pendulum-type arrangement outside a transparent lucite tank. Data were obtained from tracings of the motion pictures taken by a 16 mm movie camera. Both the pendulum arrangement and the paddle diameter were varied to find the possible influence of the experimental conditions.

It was observed that the initial rate of growth in the vertical direction is constant, depending primarily on the density gradient and the agitation mechanism (i.e. pendulum and paddle diameter). This initial rate of growth of the wake, the maximum vertical thickness of the wake, the time at which collapse begins and the turbulence intensity within the wake at that time, were all correlated with the Vaisala frequency, resulting in three important constants which seemed to be independent of the experimental conditions.

During the collapse the same three stages of collapse could be distinguished as had been defined previously by Wu; some differences in the post-collapse behavior were observed, however.

INTRODUCTION

The wake of a body moving in a fluid with a vertical density distribution is considerably different than that of the same body moving in a fluid having no density gradient. In the case of a density gradient, the initial expansion of the mixed fluid is quickly followed by a collapse in the vertical direction, which is accompanied by a further spreading in the horizontal direction. This phenomenon is caused by the force of gravity. The volume of the fluid in the wake behind the body has a more or less constant density due to mixing; driven by the hydrostatic pressures outside the wake, the constant density fluid is forced to seek its own density level in the surrounding fluid.

Only recently, several investigators have carried out laboratory investigations in order to understand the interaction of the turbulent three-dimensional wake with a linear vertical-density gradient. Schooley and Stewart (Reference 1) first studied this phenomenon in 1962 with a model self-propelled body and measured the vertical collapse and the horizontal spreading by introducing dye into the wake behind the body. Wu (Reference 2) in 1965 studied certain aspects of the wake in density-stratified fluid after collapse. He considered only gravitational and inertial effects and obtained in that way a good basic understanding of the collapse and spreading of the wake. Moreover he analyzed the internal waves generated by the wake collapse phenomenon (Reference 6).

In 1966, Stockhausen, Clark, and Kennedy (Reference 3) investigated the momentumless three-dimensional wake in a stratified fluid with a linear density gradient. Their experiments were also conducted with a self-propelled model in a large tank. Variations in salinity were measured in the zone of mixing by means of specially designed conductivity probes. In this way, normalized isochlor distributions for transverse cross-sections at various distances behind the model were deduced, from which information could be obtained about the extent of the mixed region.

In the present study, particular attention is given to the growth of the turbulent wake in the density-stratified fluid up to the moment of collapse. For some of the runs, information is obtained after collapse in order to check the validity of some of Wu's findings (Reference 2).

EQUIPMENT AND EXPERIMENTAL TECHNIQUE

Tank and Stratified Fluid

The experiments were carried out in a transparent tank 48 inches deep, 43 inches long, and 12.5 inches wide.

The vertical density stratification was obtained by introducing salt water through a diffusor at the bottom of the tank. A plastic screen cloth assured a uniform outflow from the diffusor. In order to prevent air bubbles from disturbing the stratification during filling, a 1/4-inch brass tube was mounted at the top of the diffusor, allowing the air bubbles to pass through the wall outside the lucite tank (See Figure 1). The density stratification was controlled by mixing different amounts of sodium chloride in water. The successive layers of salt water were 1 inch thick and alternately colored with red dye. In this way, 24 layers were brought into the tank. The idealized, initial vertical density distribution of the fluid in the tank is a stepped line; but, as has been verified by Wu (Reference 2), leaving the fluid overnight assures a linear density stratification through molecular diffusion.

Paddle Mixer

The turbulent wake was generated by means of a spiral paddle which was known to generate good small scale turbulence (see Reference 4). A spiral paddle, 12-3/8 inches long, with a diameter of 2 inches was first used. It consisted of a brass spiral frame (one turn in 1.5 inches) on which a screen was soldered. The frame was fixed in place by three discs (see Figure 2). The paddle was supported across the tank, 20 inches above the bottom of the tank, by a 1/4-inch O.D. brass tube, in such a way that the axis of rotation of the paddle is perpendicular to the front and back wall of the lucite tank. In the brass tube many small openings were drilled. A rod 1/8-inch in diameter, which had been painted heavily with blue dye dissolved in polyvinyl alcohol, was inserted into the brass tube immediately preceding a test. It took several minutes for the dye to dissolve, during which time the turbulence created by the insertion of the rod was already damped out as could be clearly observed by the manner in which the heavier blue dye came out of the openings of the brass tube. The paddle was then agitated by means of a pendulum-type arrangement at the back of the tank. For the first set of experiments, this pendulum had an effective arm of 21 inches and a weight of 1.5 pounds. After release, the pendulum described an arc of approximately 270°, and on the way back slightly less than that. For the second set of experiments a shorter pendulum was used. This pendulum had an effective arm of 1 foot and a weight of 1.26 pounds. The third series of experiments were carried out with the long pendulum and a spiral paddle having a diameter of 4 inches (see Figure 2).

Data Reduction

The wake created in the above-mentioned way was more or less two-dimensional, at least until the moment of collapse. The boundary of this wake was clearly visible against the background of the color-layer stratification system. The growth and collapse of this wake was photographed by a 16 mm movie camera. With the help of coordinate lines drawn on the wall of the tank, wake profiles could be traced. The time could be read accurately to within 1/100 of a second from a clock which was also photographed. A set of sample profiles for the wake generated with the 2-inch-diameter paddle and the long pendulum is given in Figure 3.

It was a disadvantage that the change in wake thickness after $t = 0$ was small in comparison with the height of the wake at $t = 0$. Moreover, irregularities in the wake shape due to the presence of eddies increased the difficulty of taking accurate measurements, and resulted in a scatter of the data points along the course of the curves (see Figures 5, 6 and 7).

RESULTS

Phase I of this study was performed with the 2-inch-diameter spiral paddle, agitated by the long pendulum. Nine tests were carried out and analyzed: one in pure water and eight with various linear density gradients. The particular method of introducing the salt water in layers allowed a minimum value for $a = \frac{1}{\rho_o} \frac{\partial \rho}{\partial y}$ of 0.002.

In pure water, where the turbulent wake is more or less circular in cross-section, the area of the wake was determined from tracings of the movie-film by means of a planimeter. From the measured area, the root-mean-square of the radius of the wake was computed and plotted versus time, as in Figure 5. Time $t = 0$ is thereby taken as the instant the movement of the pendulum stopped completely, which could be determined from the film. The curve shows a continuously decreasing rate of wake growth in pure water; initially the rate of growth is almost constant.

Tracings were also drawn from the boundary of the wake in the tests with the linear density gradients. With the aid of these tracings, the vertical ordinate, y, of the wake boundary at $x = 0$ was determined. For a definition sketch of the coordinate axes, see Figure 4. The vertical ordinates, y, were plotted versus time in Figure 5, where the different tests are identified by the value for $a = \frac{1}{\rho_o} \frac{\partial \rho}{\partial y}$. Although there occurs a certain amount of scatter due to the way in which y is determined, all curves show the same tendency. Not only does the rate of growth in the vertical direction diminish, but the wake height reaches a maximum and decreases afterwards. It is readily seen that as the density gradient increases (i.e., increasing value for "a"), the rate of growth of the wake, the maximum wake height y_{max}, and the time t_{col}, in which this maximum height was reached, all decrease.

In order to find the possible influence of the pendulum arrangement, Phase II of the study was conducted. The arm of the pendulum therefore was shortened to approximately one-half of the length used in Phase I. Consequently, the time of agitation decreases by about 10 percent. With this new arrangement, five tests were performed: one in pure water and four with various linear density gradients. The results of Phase II are given in Figure 6.

Since the paddle size could also have an effect on the phenomenon, a paddle with a 4-inch diameter was built and six tests were carried out with this paddle, using the long pendulum (Phase III). Figure 7 shows the results of this phase of the study. Curves of the same type are obtained, with a larger initial height of the wake, y_o.

ANALYSIS OF DATA

A closer look at Figures 5, 6 and 7 immediately reveals certain features of the growth of the turbulent wake in the vertical direction. It is apparent from these curves that in the majority of the tests, the rate of growth during the initial stage is almost a constant, indicated by the slope, b_1, of the straight line fitted through the first-measured points. The actual values of b_1 for all tests are given in Table 1. With the aid of b_1, the data can be replotted in a dimensionless way, which gives a better picture of what is actually taking place. When plotting $\frac{y-y_o}{y_o}$ versus $\frac{b_1}{y_o} t$ the straight line sections coincide on a 1:1 slope (see Figures 5a, 6a and 7a). Depending on the density gradient, expressed non-dimensionally in this case as $\frac{y_o}{b_1}\sqrt{ag}$, the individual curves branch off after some time, showing more clearly that the maximum height, y_{max}, increases with a decreasing density gradient. Moreover, the time of collapse, t_{col}, increases with decreasing density gradient. To find out more about the relationship between the maximum height and the density gradient, $\frac{y_{max} - y_o}{y_o}$ is plotted versus $\frac{b_1}{y_o\sqrt{ag}}$ for all available tests. The result is given in Figure 8. The relationship is linear and the slope is equal to 0.8. This means that

$$\frac{b_1}{(y_{max} - y_o)\sqrt{ag}} = 1.25 \qquad [1]$$

for all tests, independent of pendulum length or weight, paddle diameter, or density gradient. This relation is especially important because now it is possible to extrapolate this line to values of "a" which occur in other circumstances.

Another linear relationship can be found if $\frac{y_{max} - y_o}{y_o}$ is plotted versus $t_{col} \frac{b_1}{y_o}$ (see Figure 9). The slope of this line is 0.64, which means that

$$\frac{y_{max} - y_o}{t_{col} \cdot b_1} = 0.64 \qquad [2]$$

Combining the results of [1] and [2], another important constant is obtained:

$$t_{col} \sqrt{ag} = 1.25 \qquad [3]$$

Equation [3] could also have been obtained if $t_{col} \sqrt{ag}$ is computed for all tests and plotted. Figure 10 gives this plot and the average value of $t_{col} \sqrt{ag}$ is again 1.25. It can be seen that there is a certain amount of scatter, but it is difficult to precisely determine t_{col} in the experiment.

DISCUSSION

Growth of the Wake

In order to obtain a better understanding of what is actually taking place during the growth period of a turbulent wake in a density-stratified fluid, the generation of this wake in the present experiments will be discussed first. As soon as the pendulum starts its swinging motion, a large scale eddy is created which is circular in shape (see Figure 11a). This eddy grows in size through the diffusion of turbulence within it. Although the shape of the wake is initially almost circular, the wake flattens as buoyancy starts playing a role. In fact, at t = 0, when the pendulum stops its motion, the wake shape is not quite circular (see Figure 11b), perhaps because of the more effective turbulent diffusion in the horizontal direction. The wake still grows in the vertical direction, but slightly less rapidly than in the horizontal direction. The energy-containing eddies, which cause the growth of the wake by their entrainment of surrounding fluid, are being suppressed more and more near the top and bottom of the wake. Finally, at the time of collapse, the vertical height and the density difference inside and outside the wake become so large that the stabilizing gravity force is strong enough to suppress the turbulence completely in the vertical direction. At this moment there is only growth in the horizontal direction (see Figure 11c). After collapse, the gravity-induced forces become predominant and the mixed fluid tends to flow sideways to the plane of equilibrium. The motion of the wake profile thereafter depends on the degree of mixing and the density gradient (Figure 11d).

In the following section, the instant of collapse and the horizontal spreading will be considered in detail.

Instant of Collapse

A rough explanation of the growth process of the wake in the vertical direction can also be attempted in terms of turbulent energy. A water particle near the boundary of the turbulent wake moves as part of a turbulent eddy or puff. Considering only its vertical component, its kinetic energy per unit volume can be described as

$$K.E. = \rho_o \overline{u'^2} \qquad [4]$$

where ρ_o is the density of the midplane, assuming that the fluid inside the wake is fully mixed, and $\sqrt{\overline{u'^2}}$ is the turbulence intensity in the vertical direction.

Because of the presence of a density gradient outside the wake, the same particle becomes surrounded by lighter fluid when crossing the boundary. Assuming again a completely mixed wake, the density difference at the wake edge can be computed as

$$\Delta\rho = \frac{\partial\rho}{\partial y} y \qquad [5]$$

With respect to the midplane, this particle thus has a potential energy expressed as

$$\text{P.E.} = \frac{\partial\rho}{\partial y}\, y\, g\, y \qquad [6]$$

At the instant of collapse, when the motion in the vertical direction has stopped, the kinetic energy of the fluid near the wake boundary is assumed to be proportionate to the potential energy gained due to displacement from the midplane. This results in the following expression:

$$\frac{\text{K.E.}}{\text{P.E.}} = \frac{\rho_o\, \overline{u_c'^{2}}}{y_{max}^{2}\, \frac{\partial\rho}{\partial y}\, g} = \text{constant} \qquad [7]$$

This can be further reduced to

$$\frac{\sqrt{\overline{u_c'^{2}}}}{y_{max}\sqrt{ag}} = \text{constant} \qquad [8]$$

For the present experiments, y_{max} has been measured for a certain $\sqrt{ag}$. To evaluate the turbulence intensity $\sqrt{\overline{u_c'^{2}}}$, however, a little more has to be said about the turbulence intensity in an axially symmetric wake <u>in pure water</u>.

A fully turbulent wake is separated from the surrounding nonturbulent fluid by an irregular turbulent front in which large scale eddies convect the fully turbulent fluid outward and simultaneously bring nonturbulent fluid inward. Thus the large scale eddies control the rate of spreading of the turbulent wake, while the small scale eddies bring the entrained fluid to a turbulent state. Because of dissipation and energy transfer to the entrained nonturbulent fluid, the turbulence level inside the wake will lower continuously. From the principle of Reynolds number similarity, the velocity of advance of the turbulent front is proportional to the root-mean-square turbulent velocity :

$$\frac{1}{\sqrt{\overline{u'^2}}} \frac{d\sqrt{\overline{r^2}}}{dt} = \text{constant} \qquad [9]$$

Townsend (Reference 5) has verified this equation and found the constant to be 0.51. The constant will be approximately the same for an axially symmetric wake, so

$$\sqrt{\overline{u'^2}} \approx 2\,\frac{d\sqrt{\overline{r^2}}}{dt} \qquad [10]$$

Let us now return to the present study where there is also the effect of stratification. In order to find a suitable value of $\frac{d\sqrt{\overline{r^2}}}{dt}$ at the instant of collapse, let us try to understand the behavior of the mixed region in the turbulent wake during the initial stage of wake growth. In the early wake, turbulent mixing is the dominant feature of the motion, and the wake expands in all directions. If a particle is considered at some x and y (see Figure 4), then the mean turbulent motion is almost radial. This radial turbulent vector has a y-component vertically upward for the particle shown in Figure 4 and a horizontal component in the -x direction. Until the instant of collapse, the x-component is affected very little by the density gradient, while the upward motion is suppressed by the gravity-induced force due to the density gradient. That is, a particle on the x-axis is not significantly influenced by gravitational effects during the period of vertical wake height increase. During the collapse of the wake, however, the gravity forces start to influence the behavior of the mixed region by returning the fluid containing the displaced salinity toward a level of density equilibrium, where it spreads laterally. This means that during the collapse stage a change in the horizontal spreading is to be expected. Before collapse, however, the horizontal spreading should be the same as in the pure water case. This concept will be checked now with the aid of experimental results. In Figure 12,

$$\frac{\sqrt{\overline{r^2}} - \sqrt{\overline{r_o^2}}}{\sqrt{\overline{r_o^2}}}$$

versus time for the pure water case and $\frac{x-x_o}{x_o}$ versus time for several runs in density-stratified fluid are plotted. The plots show that indeed in the beginning there doesn't appear to be much difference between the wake growth in fresh water and that in density-stratified fluid. At the moment of collapse, the line is dotted and the slope of the line is very close to the slope in the pure water case. The curvature changes after collapse, as can be clearly seen in Figure 12. This is due to the sudden lateral surge when the heavier fluid flows down and sideways to its own density level. Note that the change in curvature is larger and earlier with increasing $a = \frac{1}{\rho_o}\frac{\partial\rho}{\partial y}$ as was to be expected. Since it has now been shown that the turbulence intensity that would have existed in the vertical direction at y_{max} and at t_{col} in the case of density-stratified flow can be found by calculating the slope of the pure water $\sqrt{\overline{r^2}}$ versus time-curve at t_{col}, the ratio, in Equation [8], can be computed for each run. These values of $\frac{\sqrt{\overline{u'^2_c}}}{y_{max}\sqrt{ag}}$ are tabulated in Table 1 and appear to be more or less constant, with an average value of 0.36. In general, then, it can be stated that approximately

$$\frac{\sqrt{\overline{u'^2_c}}}{y_{max}\sqrt{ag}} = 0.36 \qquad [11]$$

Let us compare this value with values which can be analyzed from data obtained by other investigators. Both Kennedy et al (Reference 3) and Schooley (Reference 1) have done experiments on the wake of a self-propelled body moving in a fluid with a vertical density gradient. From their data the following results can be computed for comparison.

Kennedy et al	Schooley and Stewart
Slope at t_{col} = 0.61 ips	Slope at t_{col} = 2.71 cm/sec
$\sqrt{\overline{u_c'^2}}$ = 1.22 ips	$\sqrt{\overline{u_c'^2}}$ = 5.42 cm/sec
a = 0.003 ft^{-1}	a = 0.1585 ft^{-1}
$\dfrac{y_{max}}{y_o}$ = 2.16	$\dfrac{y_{max}}{y_o}$ = 4.32
y_o = 3 in	y_o = 1.1 cm
y_{max} = 6.5 in	y_{max} = 4.75 cm
$\sqrt{ag}$ = 0.31 sec^{-1}	$\sqrt{ag}$ = 2.26 sec^{-1}
$\dfrac{\sqrt{\overline{u_c'^2}}}{y_{max}\sqrt{ag}} = 0.604$	$\dfrac{\sqrt{\overline{u_c'^2}}}{y_{max}\sqrt{ag}} = 0.504$
$t_{col}\sqrt{ag} = 3.04$	$t_{col}\sqrt{ag} = 2.26$
$\dfrac{b_1}{(y_{max}-y_o)\sqrt{ag}} = 1.2$	$\dfrac{b_1}{(y_{max}-y_o)\sqrt{ag}} = 1.26$

Stockhausen, Clark and Kennedy deduced the size of the mixed region from isochlor maps obtained from measurements using conductivity probes. With the occurrence of internal waves, however, this method is a rather inaccurate one. A plot of the heights and widths of the envelopes of the zone of disruption versus distance behind the model was estimated, from which the above-mentioned values could be computed. In spite of the inaccurate way of determining the height and width of the wake, the results are not too different from those in the present study. The dimensionless time of collapse will be somewhat larger for the case with a self-propelled body because at t = 0 in the present study the wake already had a certain finite size.

Schooley and Stewart used a much larger density gradient than in our studies. In relation to the large a-value, the value for $\frac{y_{max}}{y_o}$ seems rather large, but is due to the high turbulence velocities. The dimensionless time of collapse is larger again than in the present study, as has been explained above. The value of 1.26 for $\frac{b_1}{(y_{max}-y_o)\sqrt{ag}}$ agrees well with the value 1.25 in the present study. In both studies the value for $\frac{\sqrt{\overline{u_c'^2}}}{y_{max}\sqrt{ag}}$ is slightly higher (by about 50 percent) than in the present case. The reason may be sought in the actual density distribution inside the wake. Figure 13 shows the difference between the theoretically assumed density distribution and a possible actual density distribution. Of course, the actual density distribution in a wake behind a self-propelled body may be somewhat different from that resulting from the use of a spiral paddle. The degree of mixing in the wake is a problem which is very important when one starts to compare one experimental study with another. The effect of the degree of mixing should be studied further in the future.

Townsend, in Reference 7, states a criterion for the existence of turbulent motion in the presence of a sharp density interface. Townsend checked his considerations with a series of experiments in which a liquid jet was injected horizontally along the interface between a dense solution of salt in water. From a cinė record, estimates could be made of the velocity and width of the jet. It was found that entrainment of fluid by the jet almost ceased when the Richardson number (which increases with distance from the nozzle) exceeded 0.3. Beyond this point, the velocity and cross-section of the jet remained nearly constant, indicating very little entrainment and presumably very little turbulent motion. The Richardson number criterion, then, determines the conditions under which the stability of the water reaches a sufficient value so that turbulence cannot act against it and vertical motion ceases entirely (see also Reference 8).

Townsend computed the critical Richardson number in his case as

$$R_1 = 0.064\ g\ [(\rho_1 - \rho_2)/\bar{\rho}]\ (D/U_m^2) \qquad [12]$$

where

D was the observed width, and

U_m the mean velocity of advance.

When applying this to the present study, this Richardson number becomes

$$R_{i_c} = 0.064\ g\ \frac{\frac{\partial\rho}{\partial y}\, y_{max}\, \frac{1}{\rho_o} \cdot 2 \cdot y_{max}}{\overline{u'^2}} \cdot \frac{\overline{u'^2}}{U_m^{\ 2}} \qquad [13]$$

Substituting Equation [11] gives

$$R_{i_c} = 0.128 \cdot \left(\frac{1}{0.36}\right)^2 \cdot \frac{\overline{u'^2}}{U_m^{\ 2}} \qquad [14]$$

From data given by Townsend (Reference 5) the ratio of $\overline{u'^2}$ and $U_m^{\ 2}$ can be estimated as

$$\frac{\sqrt{\overline{u'^2}}}{U_{max}} = 0.22 \text{ and } \frac{U_m}{U_{max}} = 0.45 \qquad [15]$$

The above ratios combined and squared results in

$$\frac{\overline{u'^2}}{U_m^{\ 2}} = 0.24 \qquad [16]$$

The critical Richardson number for the present study is then computed as

$$R_{i_c} = 0.128 \times 7.7 \times 0.24 = 0.24 \qquad [17]$$

Townsend (Reference 7) found experimentally the value 0.3 as the critical Richardson number. This agreement is remarkably close.

Horizontal Spreading

With respect to the horizontal extent of the wake (see Figure 12), Kennedy stated that the horizontal extent appears to increase almost linearly with time. From Figure 12 it is clear in our case, too, that initially the horizontal extent of the wake grows linearly with time. However, as time goes on, the curve becomes convex. During the time of collapse, the curvature changes and becomes concave for a short time interval, then changes again into a convex curve. Finally, the relationship becomes more or less linear; again afterwards, of course, the curve becomes once more convex since due to inadequate mixing, the wake never extended horizontally all the way to the sidewalls of the tank. All these changes in curvature are demonstrated by one example given in Figure 14. In Figure 12 the initial changes can be observed more accurately for several other runs. The above-mentioned changes in curvature can only be observed if many data points are taken. If there are only several data points or one neglects small changes in curvatures, then the curve can be approximated by:

1. A straightline section through the first-measured points,
2. A convex part of the curve, changing into
3. Another straightline section, and
4. Once more a convex part.

This configuration can be seen from Figure 14. Moreover the curve of horizontal width of the wake versus time obtained by Schooley and Stewart has these same general features.

Collapsing Stages

With data about the horizontal extent of the wake available, a comparison can be made with the experiments of Wu (Reference 3). With no turbulence present in his experiments he defined three processes of collapse: initial, principal, and final stages. He derived empirical formulae to describe the wake collapse of the first two stages. The initial collapse stage he expressed as follows:

$$\frac{x-x_o}{x_o} = c\left(t'\sqrt{\frac{1}{\rho_o}\frac{\partial\rho}{\partial y}g}\right)^n \qquad [18]$$

where x_o is the horizontal extent at the moment of collapse, and t' the time after collapse. The constants "c" and "n" can be found when $\frac{x-x_o}{x_o}$ is plotted versus $t'\sqrt{\frac{1}{\rho_o}\frac{\partial\rho}{\partial y}g}$. For the present study, this is carried out for three experiments of Phase I, one of Phase II and one of Phase III (see Figure 15). The average slope of the straightline segments is 0.94 and the average dimensionless time reading for their intersections on the line $\left(\frac{x-x_o}{x_o}\right) = 0.1$ is 0.6. The values for "c" and "n" can now be determined to be:

Present study	Wu's experiments
$c = 0.16$	$c = 0.29$
$n = 0.94$	$n = 1.08$

In order to find the relationship for the principal collapse $\frac{x}{x_o}$ is plotted versus $t'\sqrt{\frac{1}{\rho_o}\frac{\partial\rho}{\partial y}g}$. The straightline segments shown in Figure 16 have an average slope of 0.34, while the average of their intersection with the line $\frac{x}{x_o} = 1$ is at $t'\sqrt{\frac{1}{\rho_o}\frac{\partial\rho}{\partial y}g} = 0.95$. The expression for the principal stage of collapse can then be given as

$$\frac{x}{x_o} = c\left(t'\sqrt{\frac{1}{\rho_o}\frac{\partial\rho}{\partial y}g}\right)^n \qquad [19]$$

where the values for the constants are $c = 1.02$ and $n = 0.34$, while Wu found $c = 1.03$ and $n = 0.55$.

The reason that the "c" and "n" values in the present study are different from the values Wu found is undoubtedly that in his experiments, the gravitational effect was made to be predominant throughout the motion. In the present study, however, both turbulence and gravity play an important role. Another reason is that in Wu's experiments the fluid inside the wake was more completely mixed and had a density very close to ρ_o. As has been mentioned before, the density distribution inside the wake looks approximately like the distribution given in Figure 13.

The above-mentioned reasons also help explain the difference in area measurements of the wake in both studies. Wu found, in the absence of turbulence, no change in the wake volume during collapse, and thus concluded that the mixed fluid inside the wake maintains its density during the wake collapse. As a comparison, the areas for several tests in the present study have been plotted in a convenient way in Figure 17. Before collapse the area increases linearly with time. Then the curve becomes convex and undergoes several undulations, presumably due to internal waves. The area of the wake still seems to increase considerably after the wake has collapsed. Besides these two reasons:

1. The presence of turbulence, and

2. The density of fluid inside the wake is not constant, there is a third important reason (experimental) for the apparent area increase:

3. As time increases, the wake created by the agitation of the paddle becomes less two-dimensional so that the actual rate of growth in volume may be less than the apparent rate as observed through the front wall.

CONCLUSIONS

The growth of a wake in a density-stratified medium can be studied in detail by means of a spiral paddle agitated by a pendulum-type arrangement. In the vertical direction the rate of growth of the wake is initially constant, but finally ceases, whereupon the wake collapses vertically. This initial rate of growth, the maximum vertical extent of the wake, and the time of collapse have been related to the Vaisala frequency. Independent of the size of the spiral paddle and the type of pendulum arrangement, two empirical constants have been determined:

$$\frac{b_1}{(y_{max}-y_o)\sqrt{\frac{1}{\rho_o}\frac{\partial\rho}{\partial y}g}} = 1.25$$

and

$$t_{col} \sqrt{\frac{1}{\rho_o} \frac{\partial \rho}{\partial y} g} = 1.25$$

Besides accurate measurements of the vertical extent of the wake a precise description is also given of the horizontal spreading of the wake. These measurements were used to estimate the turbulence intensity at the time of collapse in the presence of a density gradient. It was found that the ratio of the kinetic energy and the gain of potential energy at the moment of collapse is a constant which may be expressed as follows:

$$\frac{\sqrt{\overline{u'^2_c}}}{y_{max} \sqrt{\frac{1}{\rho_o} \frac{\partial \rho}{\partial y} g}} = 0.36$$

There is a need for a parameter to take into account the degree of mixing inside the wake. This is especially important when comparing different experimentally obtained data. After collapse, the horizontal spreading was analyzed for several runs. The occurrence of three stages of collapse as observed by Wu, was checked in the present study where gravitational forces as well as turbulence play an important role. All three states were clearly noticeable. The constants in the empirical formulae describing the initial and principal stages of collapse were found to be somewhat different from those found by Wu, due to incomplete mixing of the fluid inside the wake and the presence of turbulence.

The volume of the wake after collapse seemed to increase further, possibly as a consequence of the presence of turbulence.

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TABLE 1

Experimental Results

	a	y_o	y_{max}	t_{col}	b_1	$t_{col}\sqrt{ag}$	$\sqrt{\overline{u_c'^2}}$	$\frac{\sqrt{\overline{u_c'^2}}}{y_{max}\sqrt{ag}}$
	ft^{-1}	inch	inch	sec	ips	—	ips	
Phase I	0.000	1.9	—	—	0.35	—	—	—
	0.002	1.825	2.86	5.25	0.35	1.33	0.197	0.272
	0.00273	1.56	2.37	5.0	0.3	1.48	0.211	0.30
	0.00273	1.71	2.49	4.1	0.32	1.21	0.274	0.272
	0.00542	1.50	2.11	3.25	0.276	1.35	0.355	0.403
	0.00537	1.54	2.26	3.25	0.302	1.35	0.355	0.378
	0.01074	1.61	2.01	2.4	0.244	1.4	0.456	0.388
	0.0210	1.60	1.81	1.5	0.17	1.23	0.584	0.393
	0.0203	1.63	1.9	1.3	0.24	1.05	0.616	0.40
Phase II	0.0000	1.77	—	—	0.46	—	—	—
	0.00545	1.60	2.19	2.75	0.36	1.12	0.33	0.36
	0.00547	1.47	2.21	2.75	0.35	1.12	0.33	0.36
	0.0109	1.63	2.12	1.5	0.343	0.89	0.53	0.42
	0.0217	1.66	2.0	1.5	0.27	1.25	0.53	0.32
Phase III	0.00000	3.075	—	—	0.685	—	—	—
	0.00545	3.22	3.68	2.5	0.26	1.05	0.68	0.442
	0.00547	2.80	3.21	1.75	0.255	0.73	0.80	0.59
	0.0211	2.96	3.22	1.3	0.31	1.07	0.94	0.355
	0.0423	2.48	2.72	1.1	0.42	1.28	1.0	0.315
	0.0432	2.64	2.92	1.0	0.33	1.18	1.04	0.304

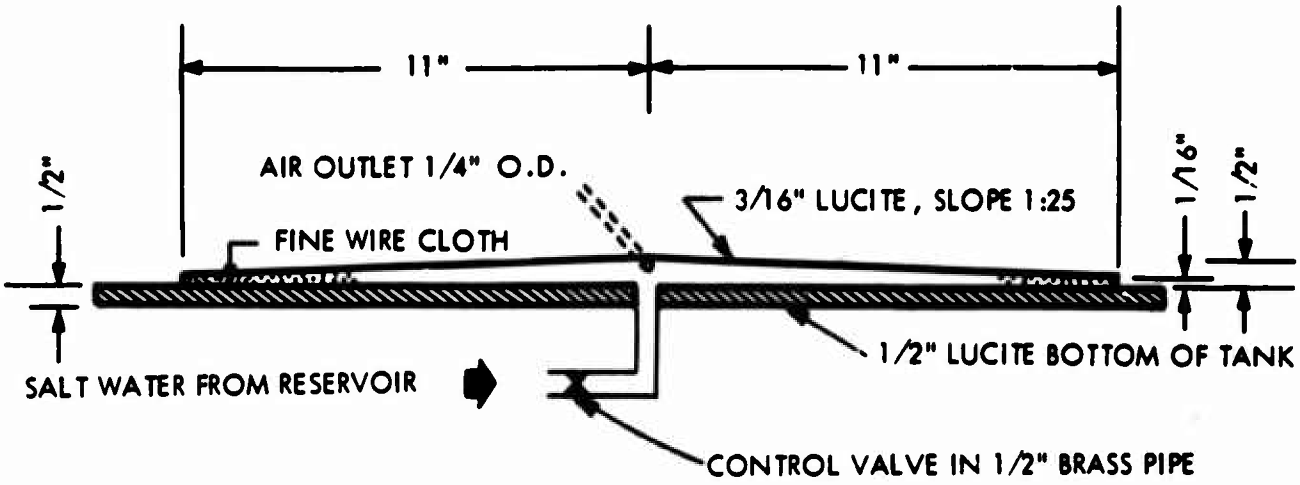

FIGURE 1 - DETAIL OF SALT WATER DIFFUSER

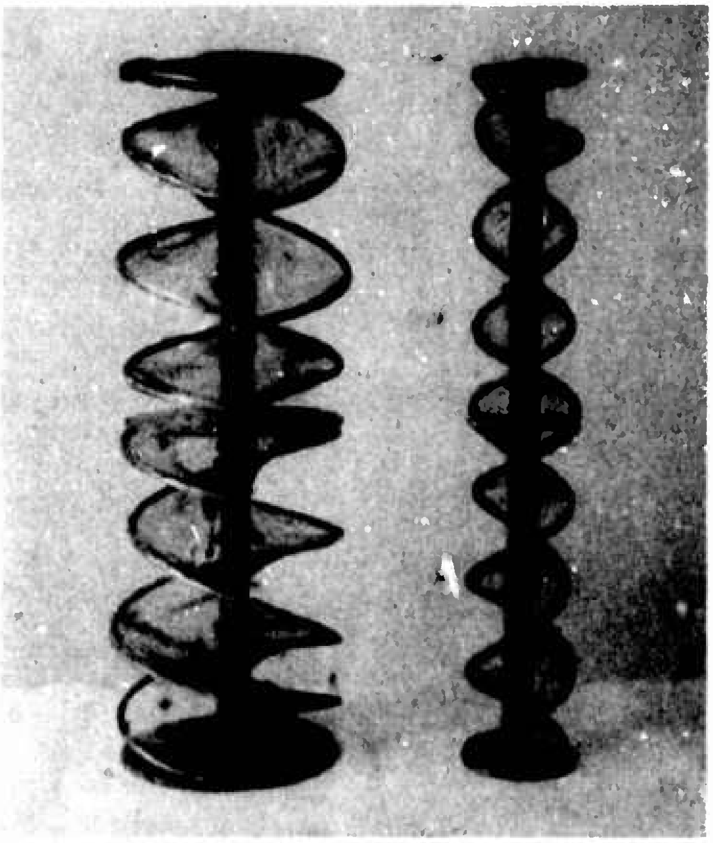

FIGURE 2 - SPIRAL PADDLES, 4 IN AND 2 IN DIAMETERS

t = 0.25 SEC

t = 1.50 SEC

t = 3.5 SEC

t = 7 SEC

FIGURE 3 - SAMPLE PICTURES OF SPREADING AND COLLAPSE OF WAKE FOR $a = 0.0054$ (FRONT VIEW)

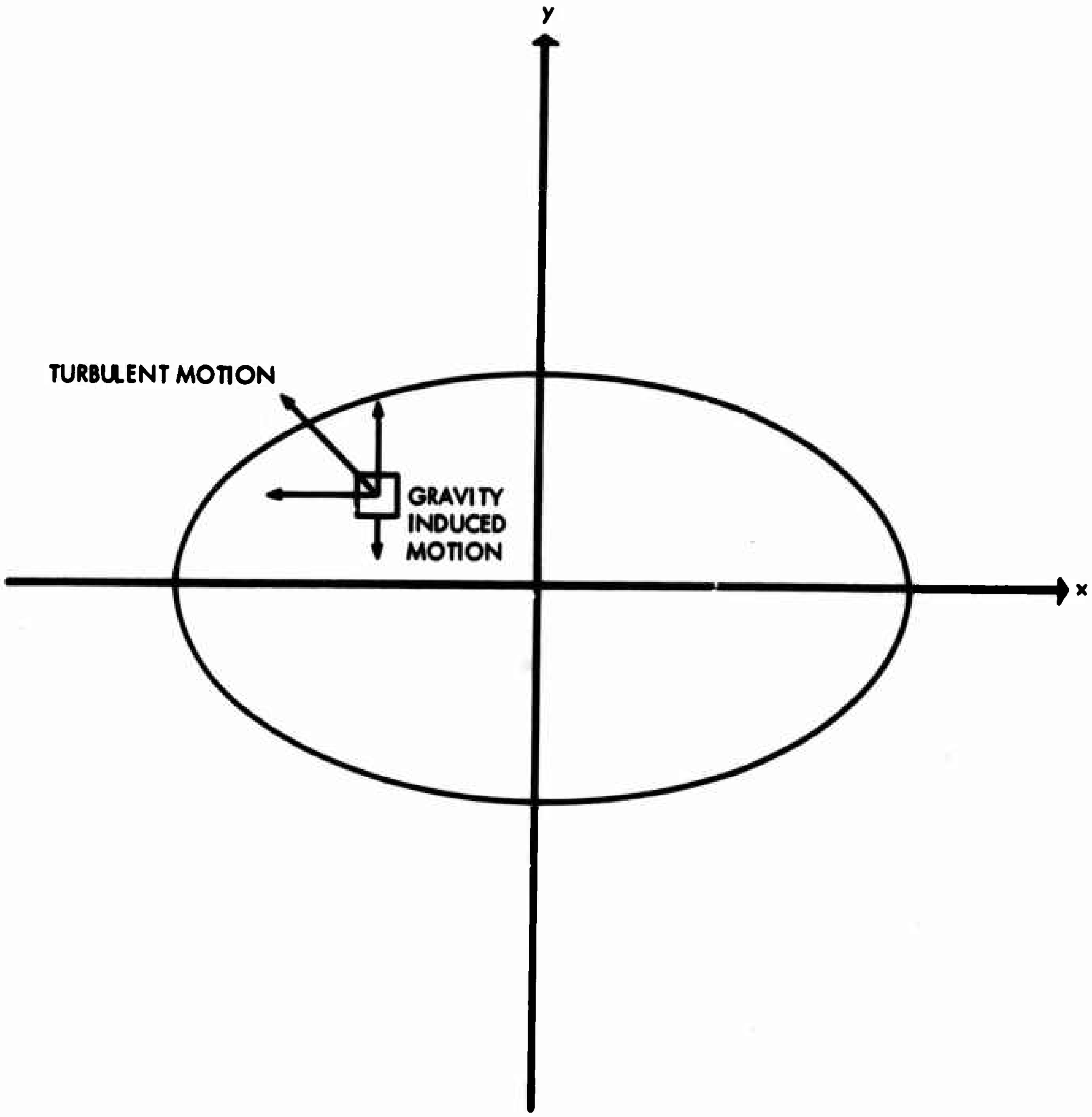

FIGURE 4 - DEFINITION SKETCH OF WAKE

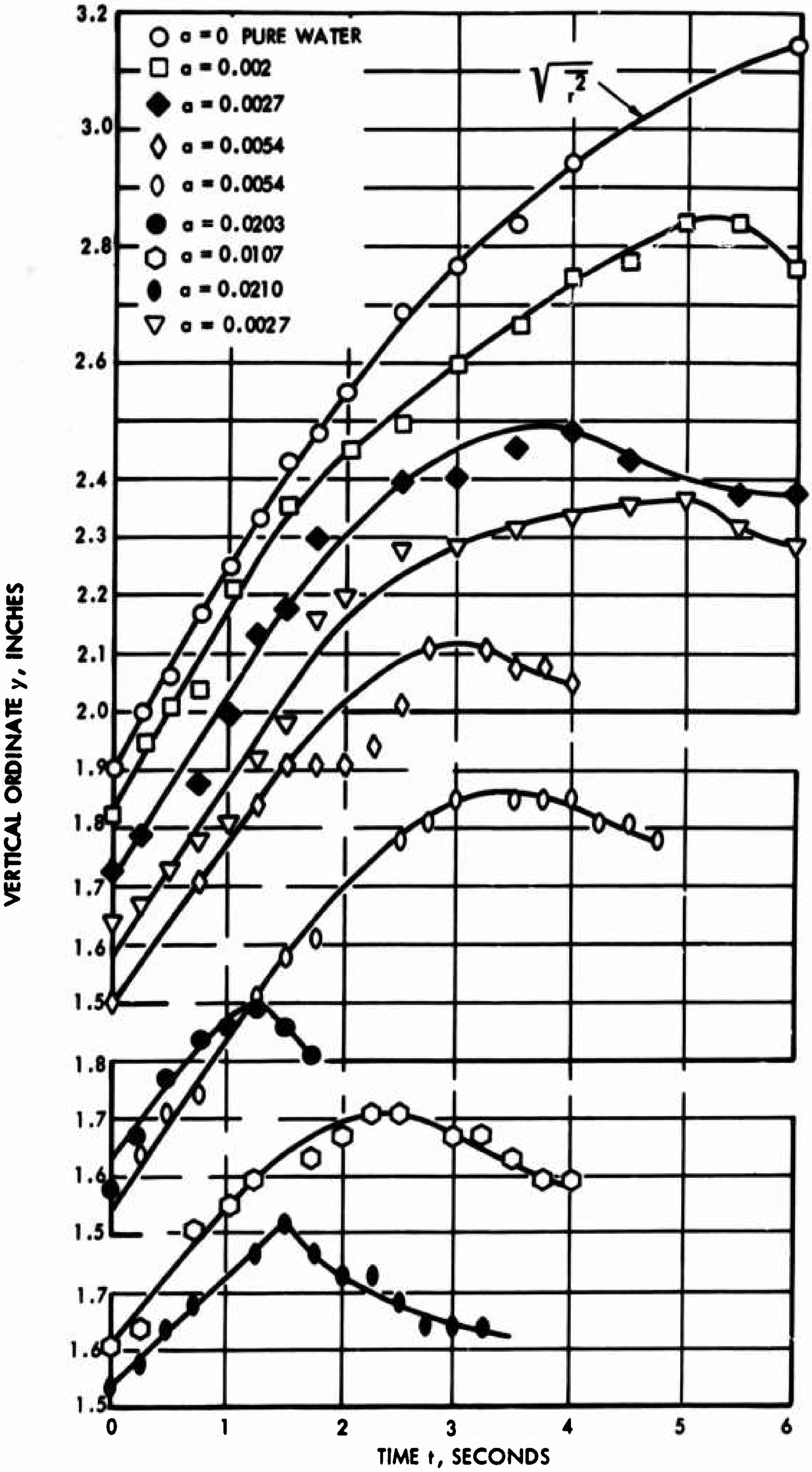

FIGURE 5 - GROWTH AND SUBSEQUENT COLLAPSE IN THE VERTICAL DIRECTION - PHASE I

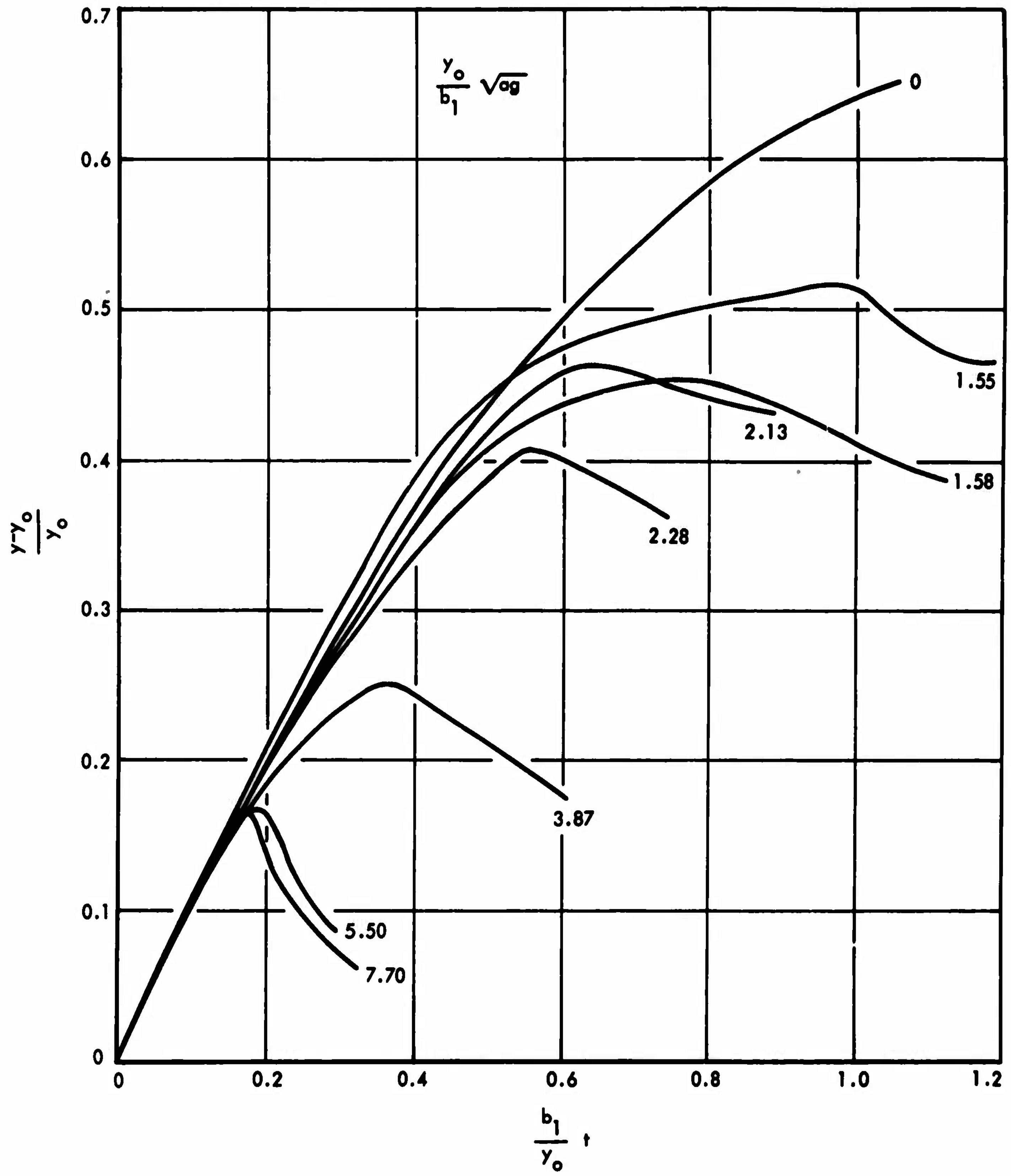

FIGURE 5a - DIMENSIONLESS TIME HISTORY OF VERTICAL ORDINATE OF WAKE - PHASE I

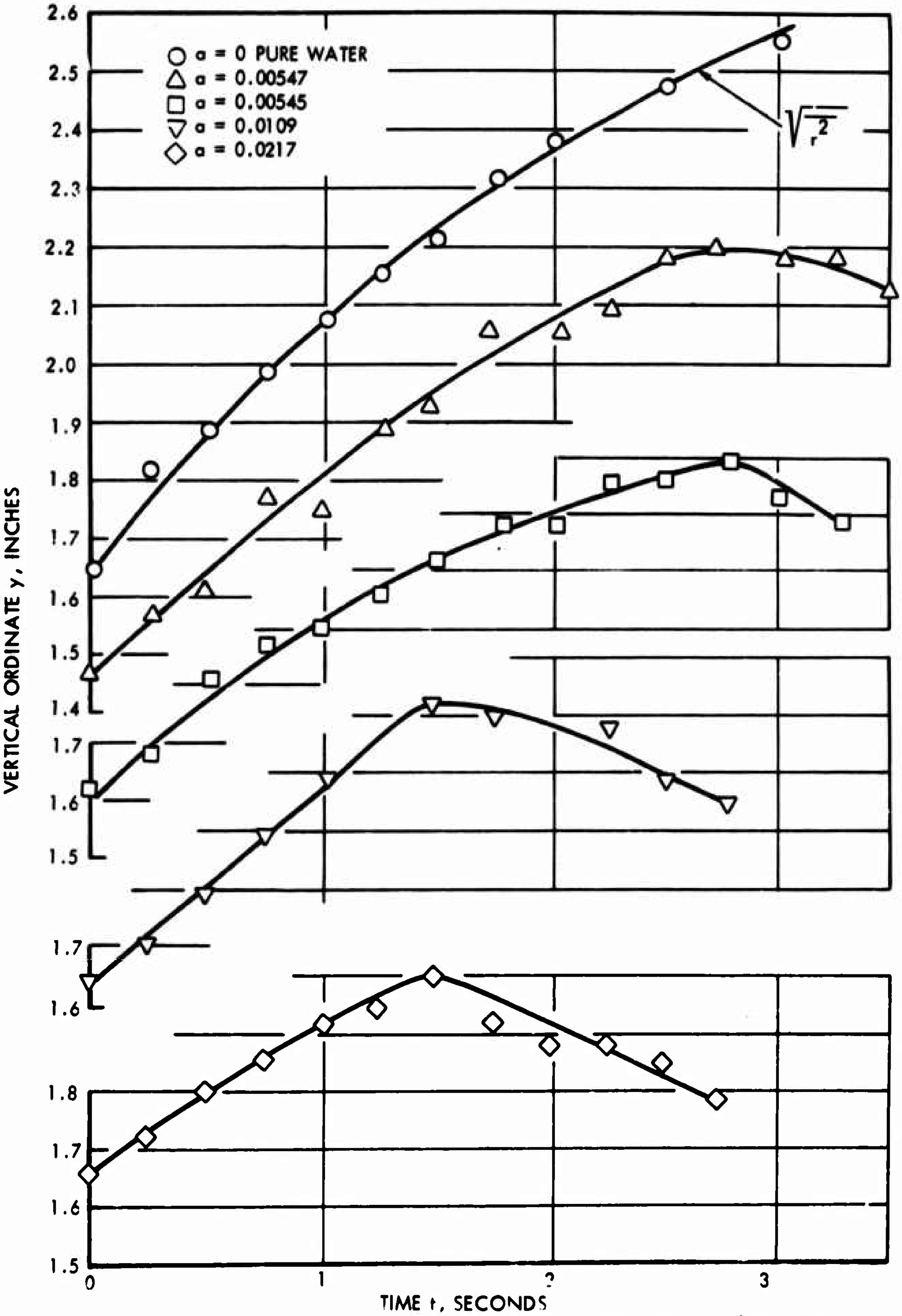

FIGURE 6 - GROWTH AND SUBSEQUENT COLLAPSE IN THE VERTICAL DIRECTION - PHASE II

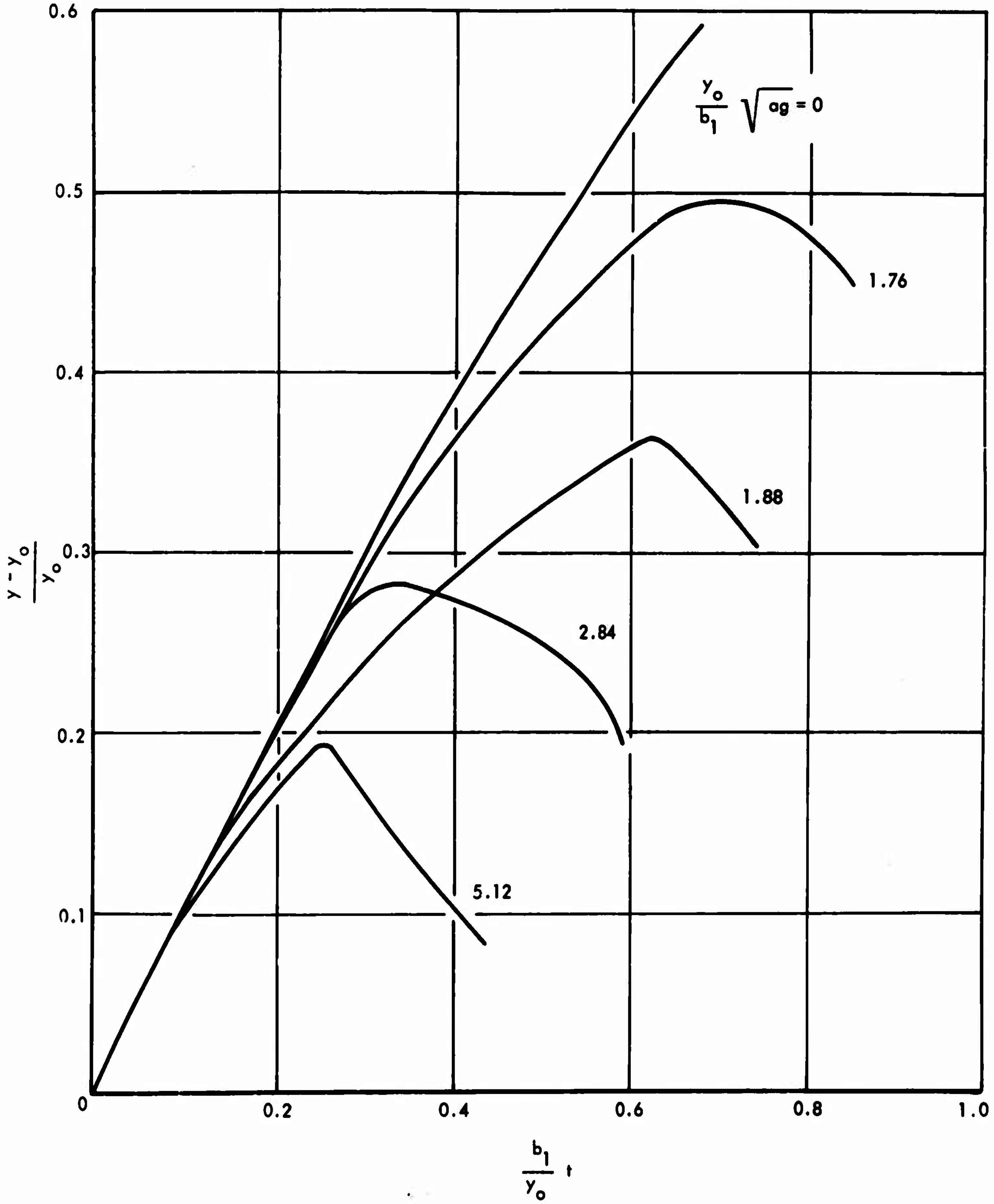

FIGURE 6a - DIMENSIONLESS TIME HISTORY OF VERTICAL ORDINATE OF WAKE - PHASE II

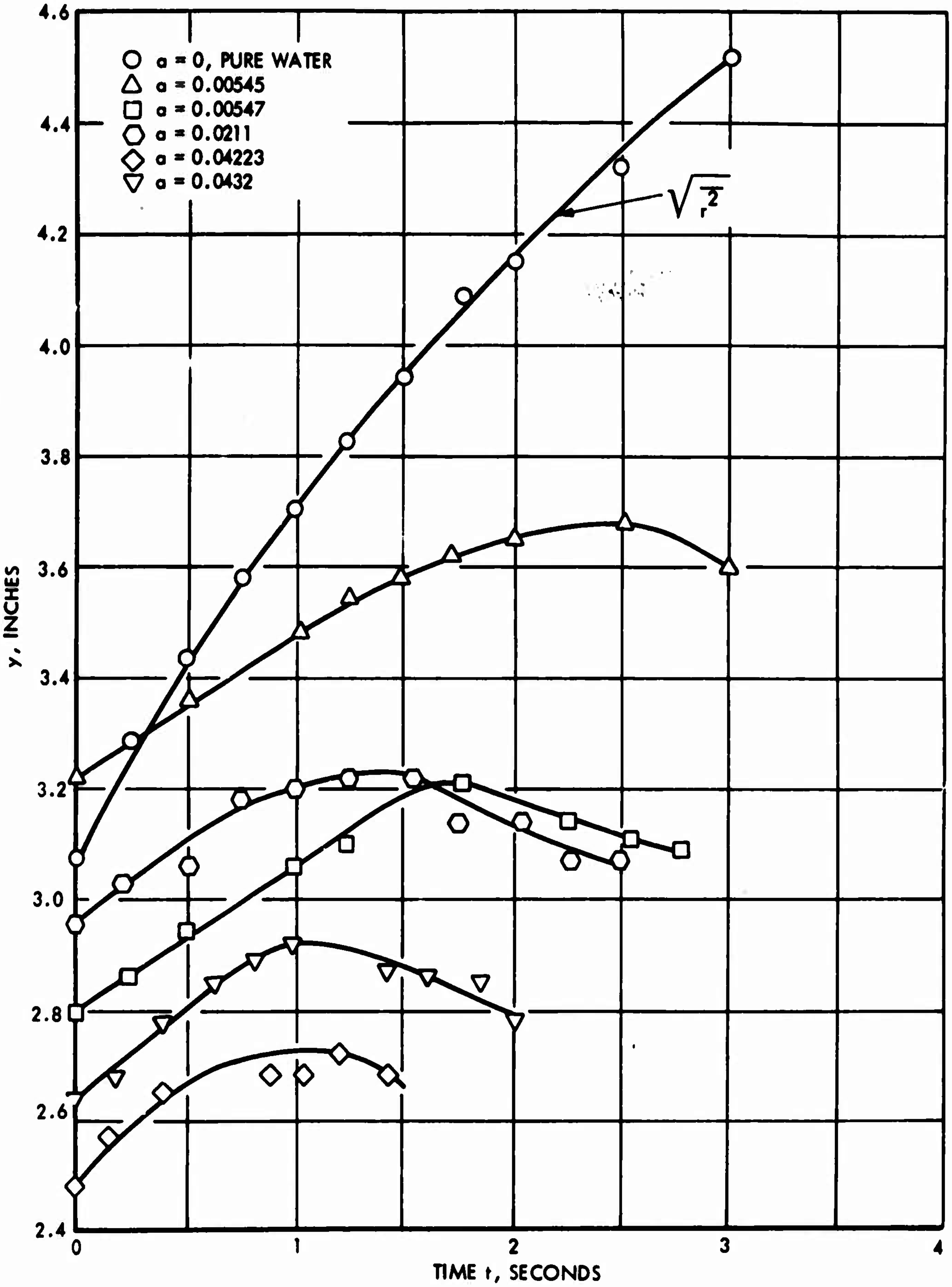

FIGURE 7 - GROWTH AND SUBSEQUENT COLLAPSE IN THE VERTICAL DIRECTION - PHASE III

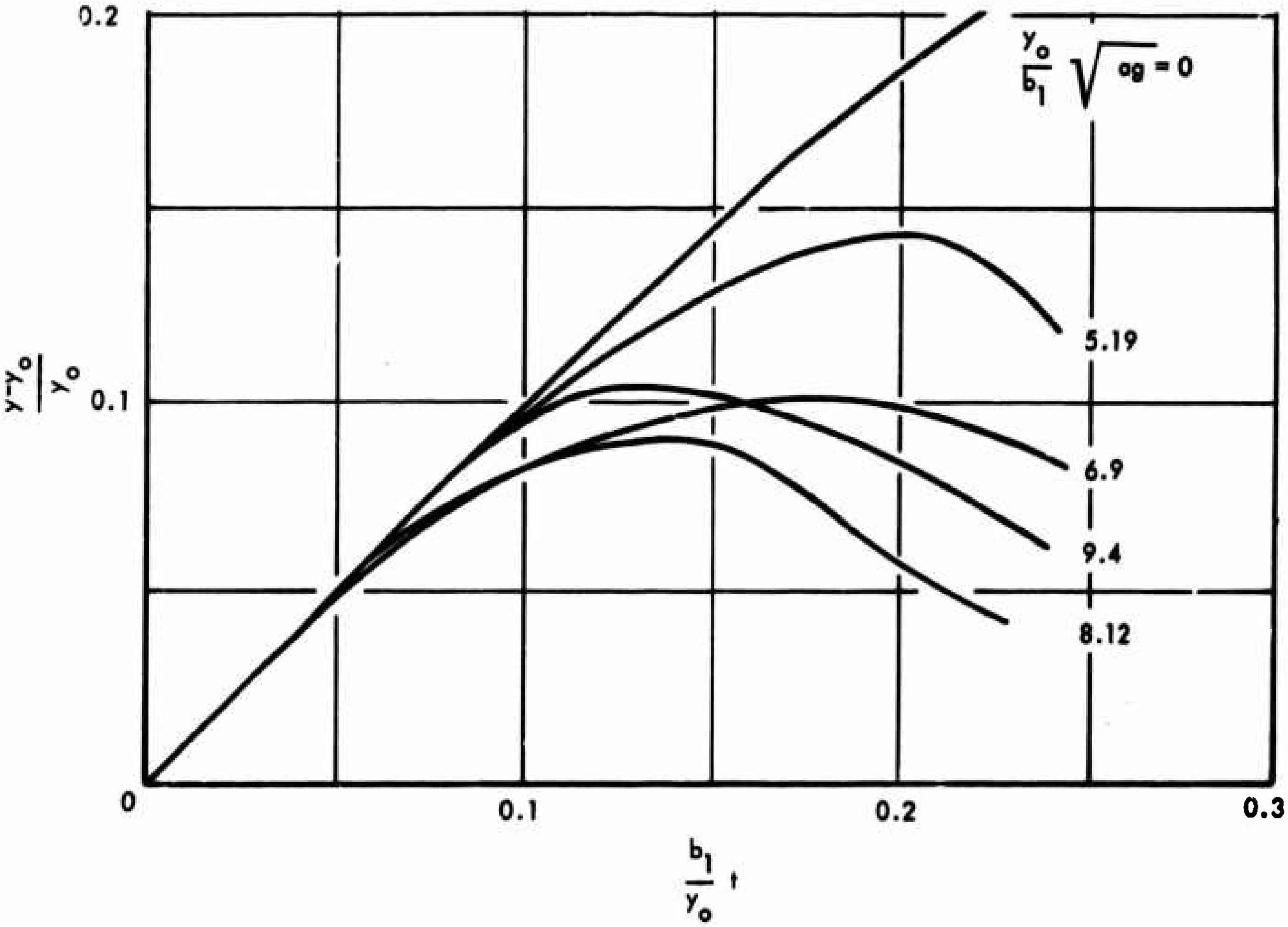

FIGURE 7a - DIMENSIONLESS TIME HISTORY OF VERTICAL ORDINATE OF WAKE - PHASE III

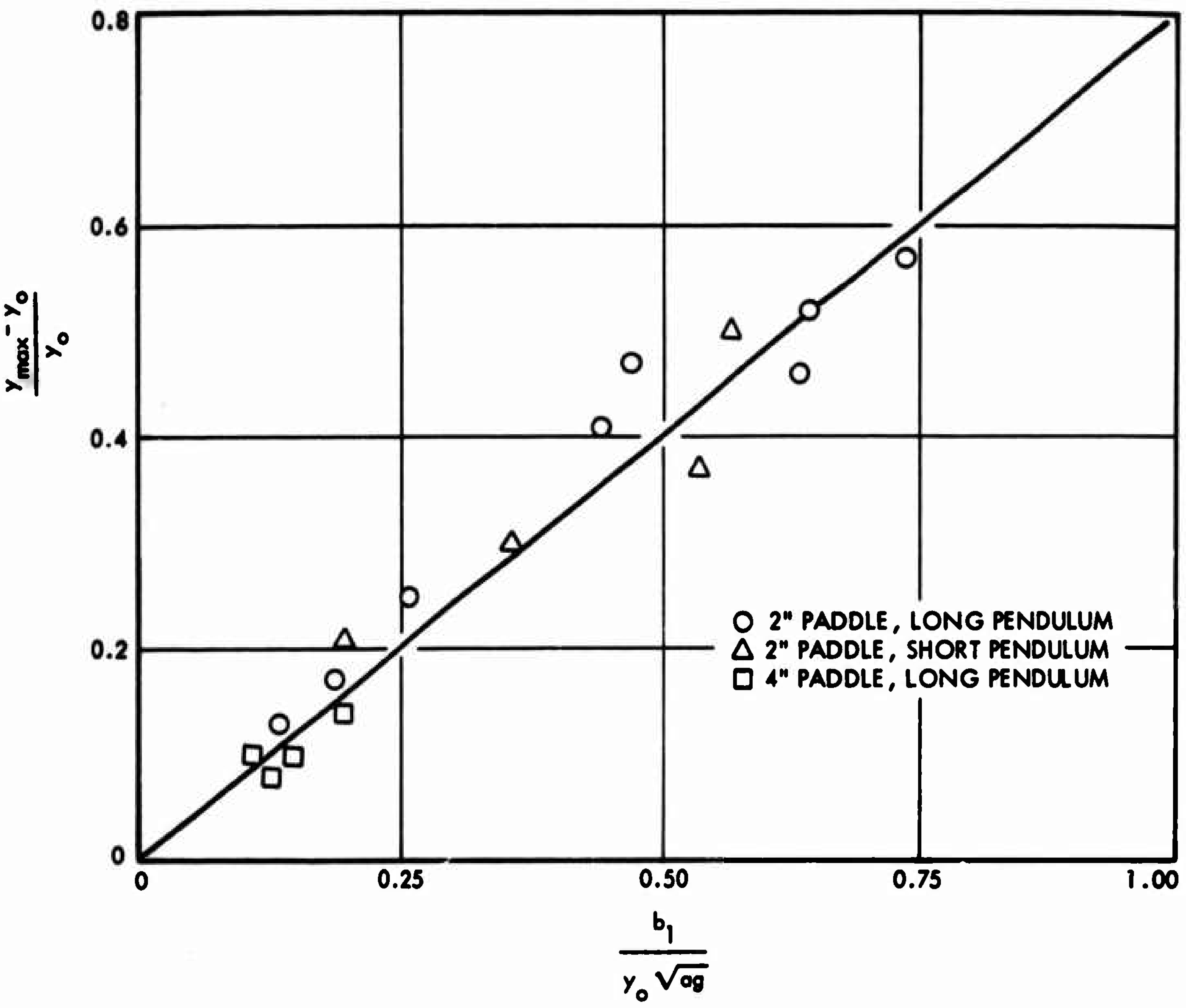

FIGURE 8 - DIMENSIONLESS REPRESENTATION OF DATA

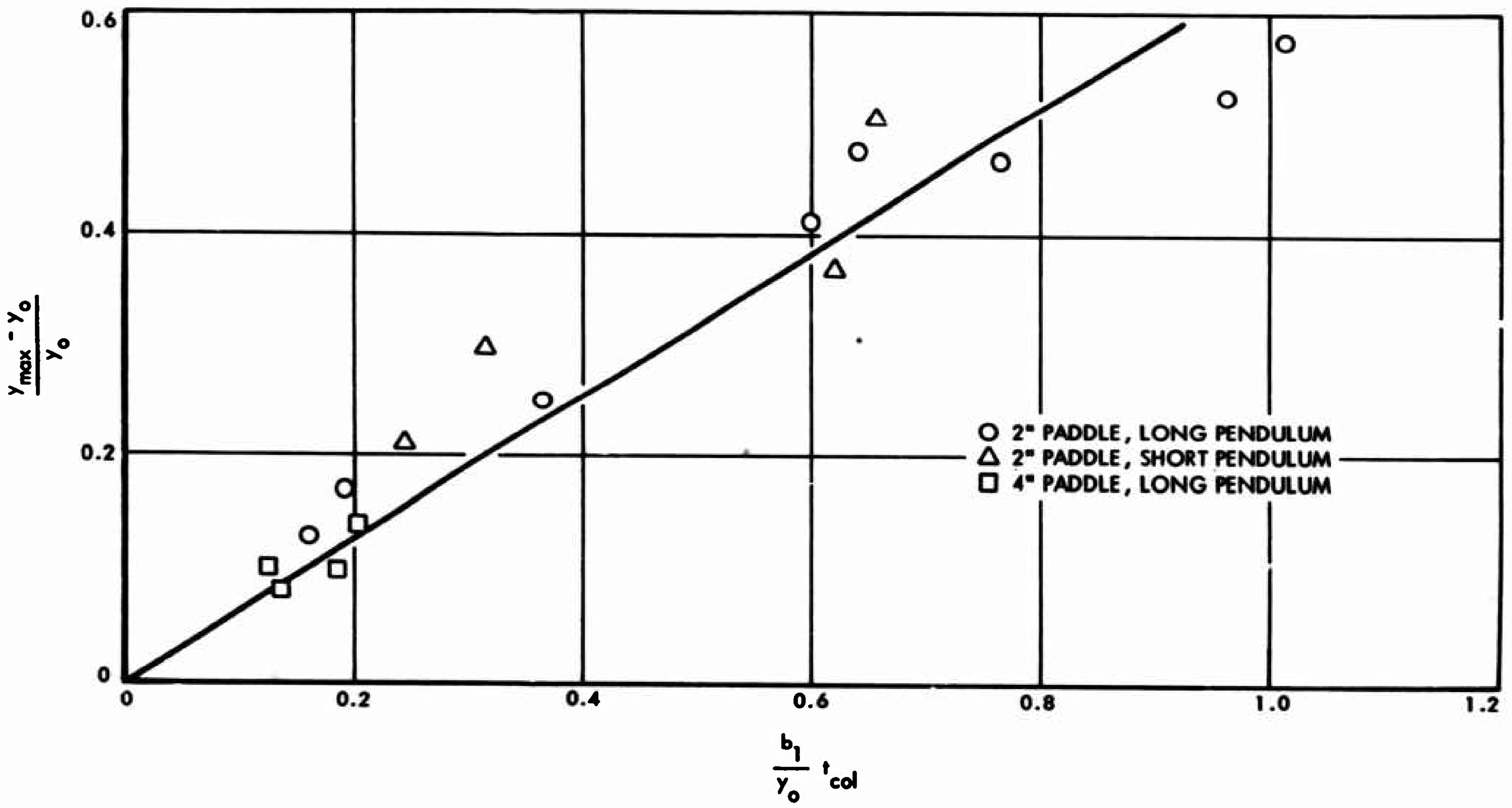

FIGURE 9 - RELATION BETWEEN MAXIMUM VERTICAL EXTENT OF WAKE AND TIME OF COLLAPSE

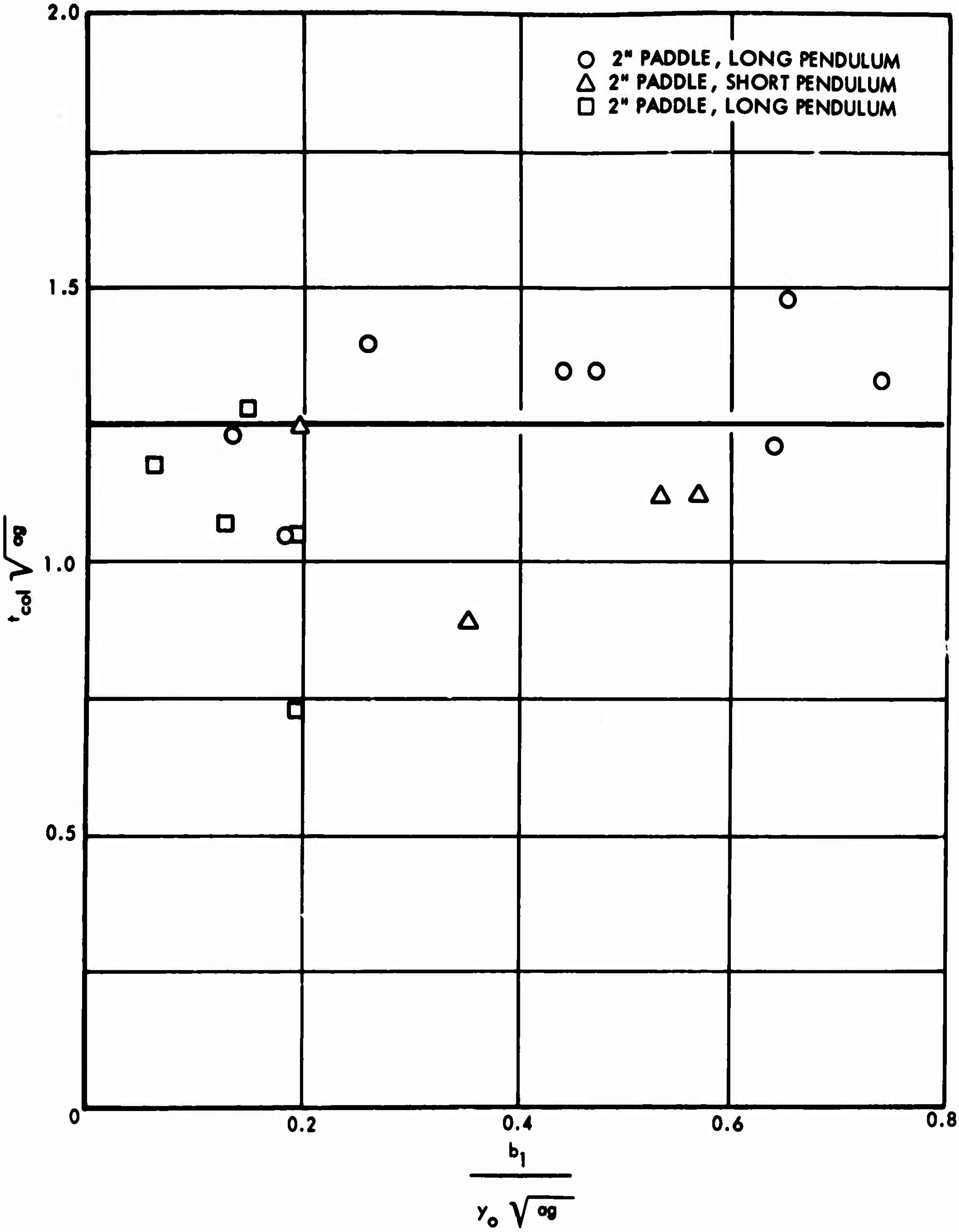

FIGURE 10 - SCATTER IN DIMENSIONLESS TIME OF COLLAPSE

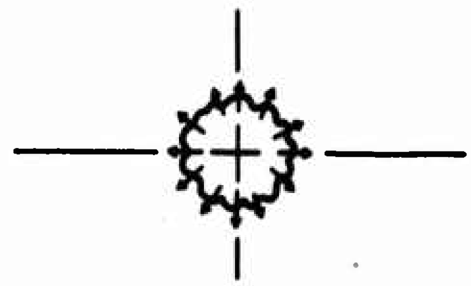

FIGURE 11a - WAKE SHAPE DURING GENERATION

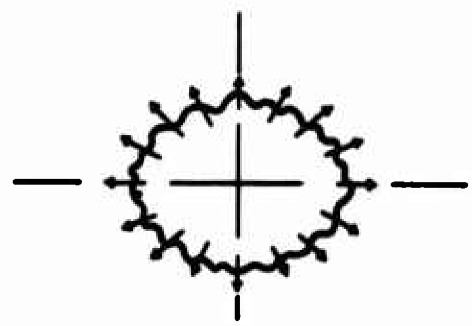

FIGURE 11b - WAKE SHAPE DURING GROWTH

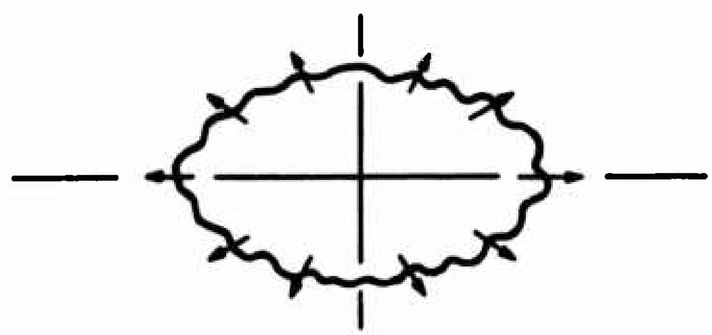

FIGURE 11c - WAKE SHAPE AT TIME OF COLLAPSE

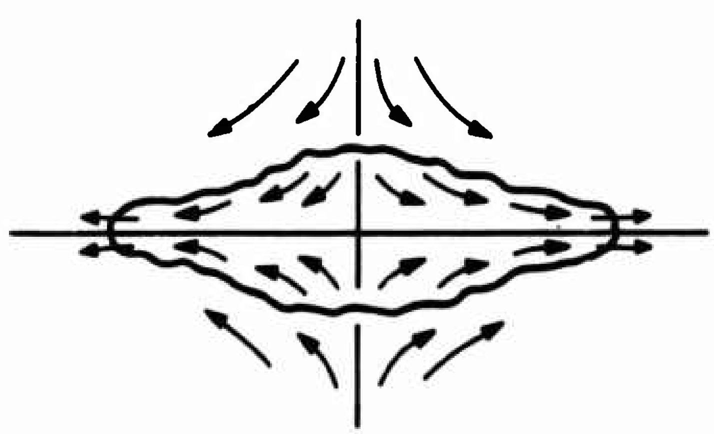

FIGURE 11d - WAKE SHAPE AFTER COLLAPSE

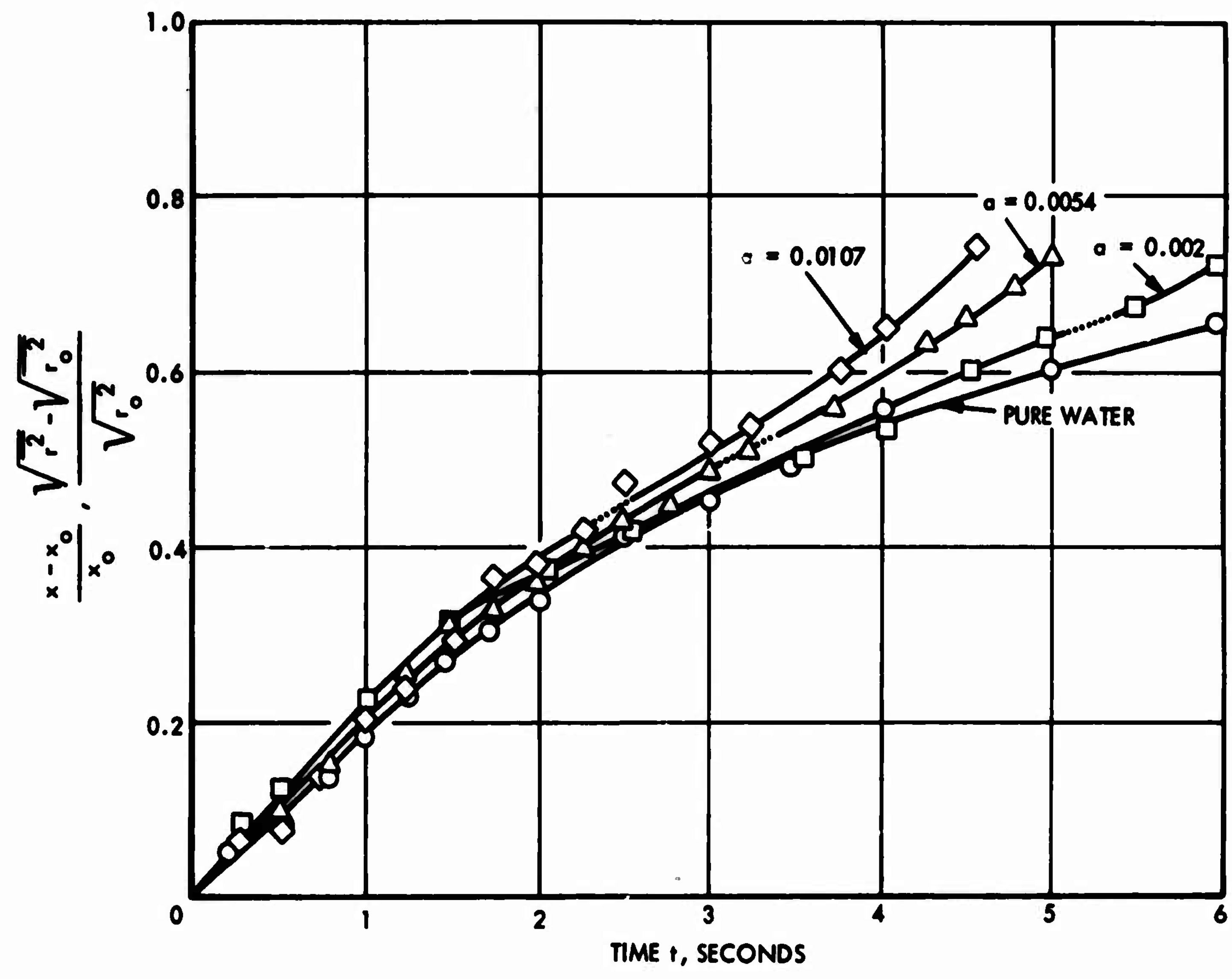

FIGURE 12 - EFFECT OF COLLAPSE ON RATE OF SPREADING IN THE HORIZONTAL DIRECTION

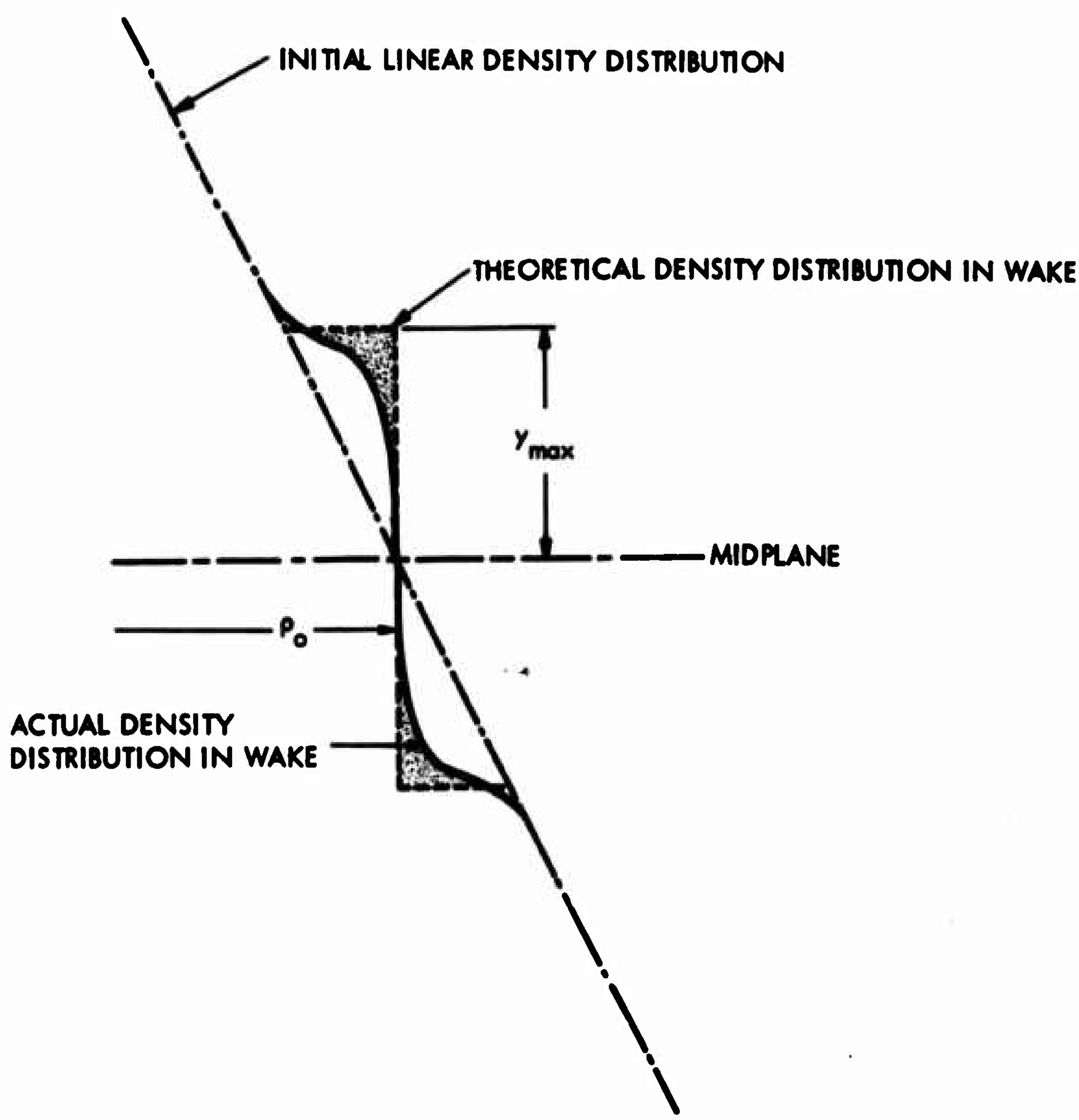

FIGURE 13 - COMPARISON OF THEORETICAL AND ACTUAL DENSITY DISTRIBUTION INSIDE THE WAKE

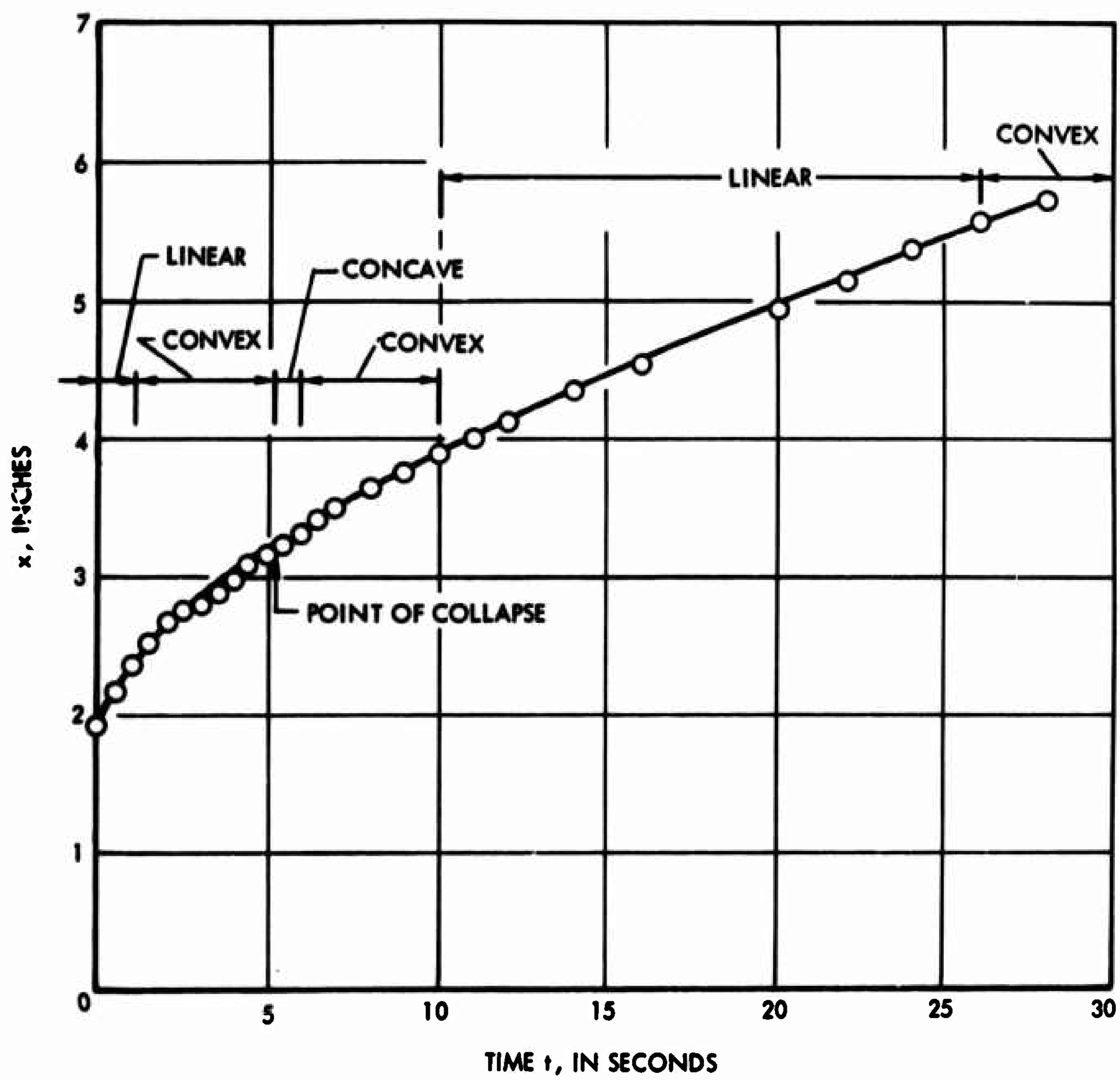

FIGURE 14 - HORIZONTAL SPREADING FOR $a = 0.002$, PHASE I

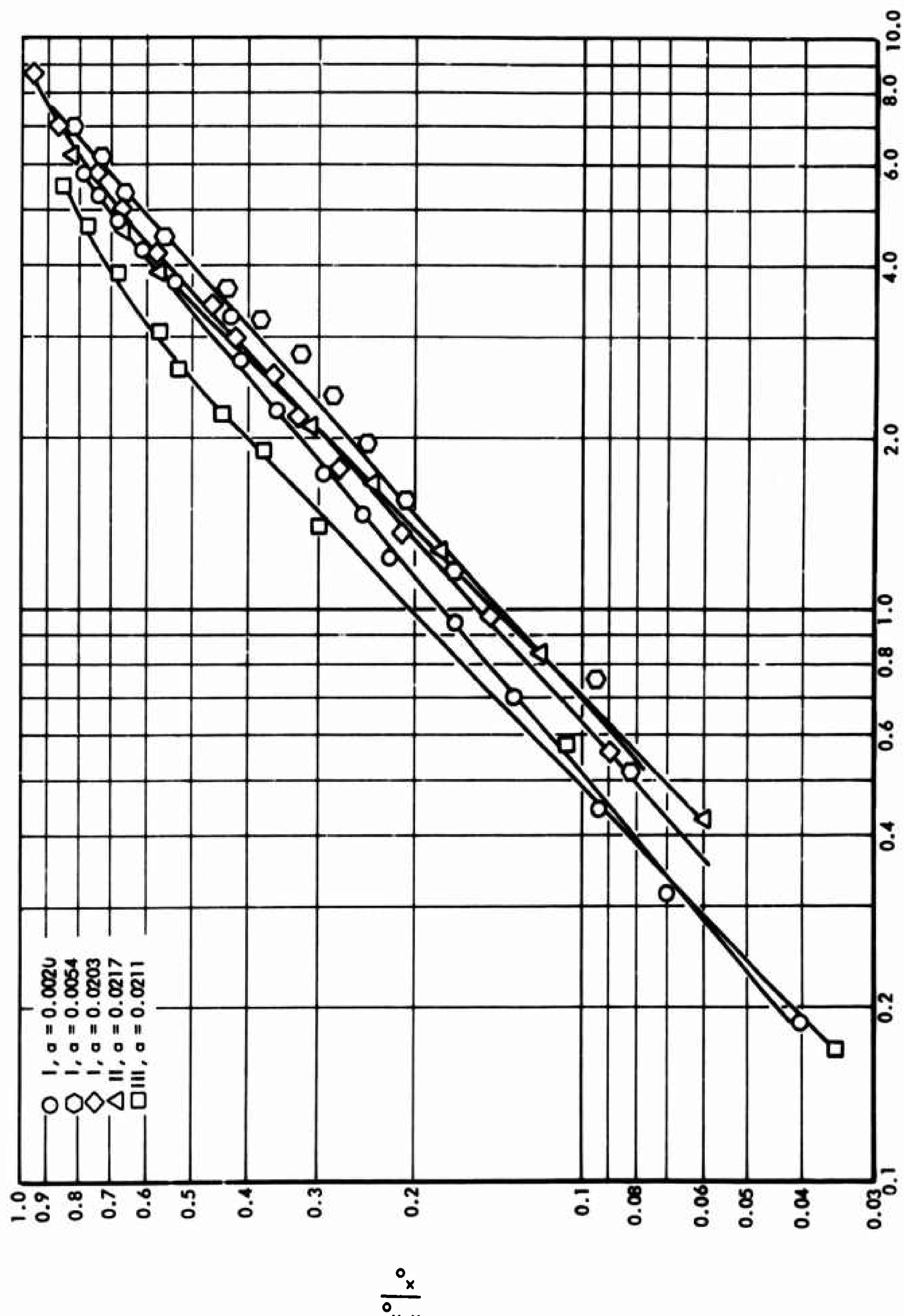

FIGURE 15 - INITIAL STAGE OF WAKE COLLAPSE

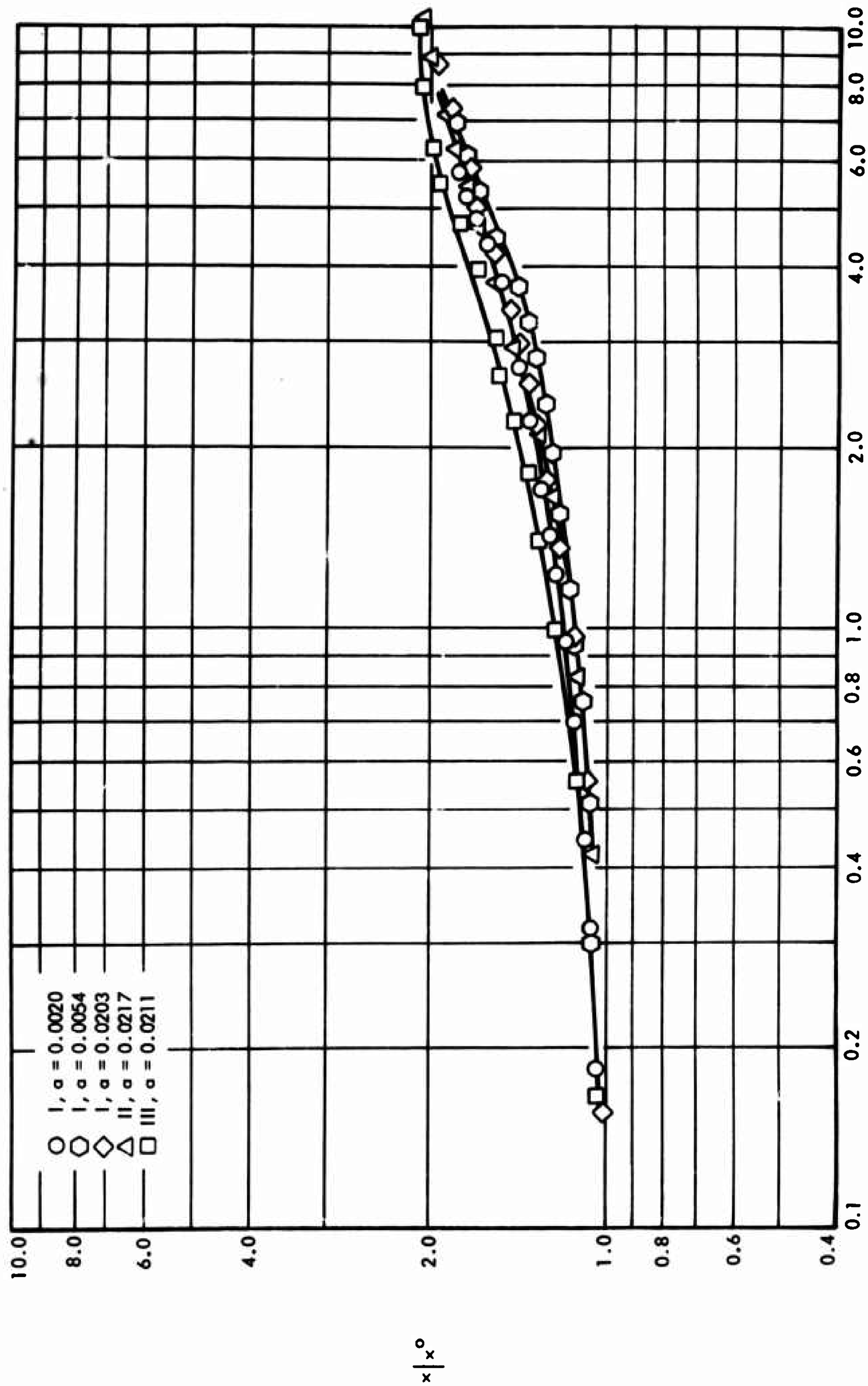

FIGURE 16 - PRINCIPAL STAGE OF WAKE COLLAPSE

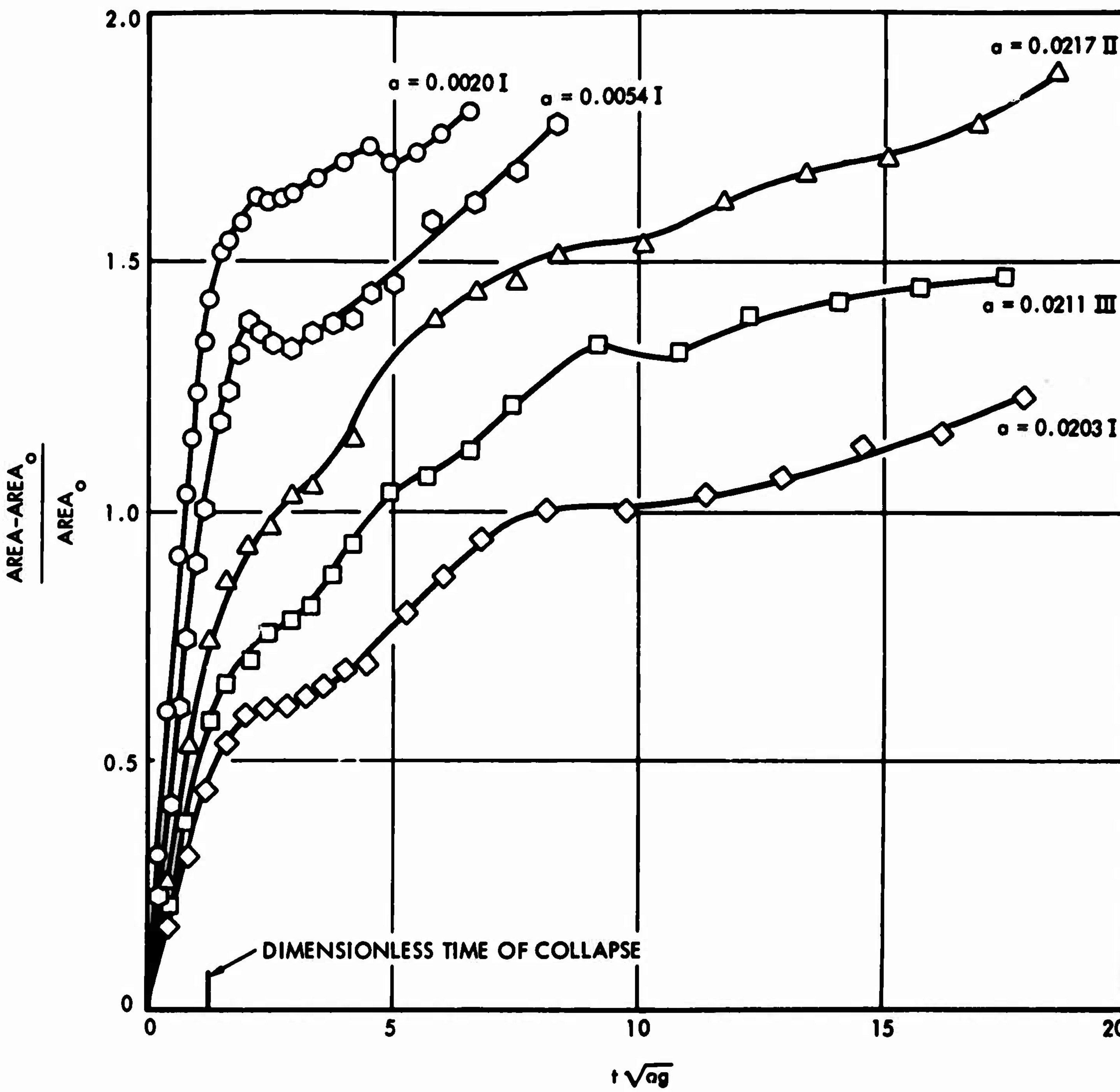

FIGURE 17 - INCREASING AREAS OF WAKE WITH TIME

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13. ABSTRACT The force of gravity causes a turbulent wake in a density-stratified fluid to eventually cease its vertical growth and then to collapse towards its horizontal midplane. In the present investigation this phenomenon has been studied experimentally. The turbulent wake was created by means of a spiral paddle, agitated by a pendulum-type arrangement outside a transparent lucite tank. Data were obtained from tracings of the motion pictures taken by a 16 mm movie camera. Both the pendulum arrangement and the paddle diameter were varied to find the possible influence of the experimental conditions.

It was observed that the initial rate of growth in the vertical direction is constant, depending primarily on the density gradient and the agitation mechanism (i.e. pendulum and paddle diameter). This initial rate of growth of the wake, the maximum vertical thickness of the wake, the time at which collapse begins and the turbulence intensity within the wake at that time, were all correlated with the Vaisala frequency, resulting in three important constants which seemed to be independent of the experimental conditions.

During the collapse the same three stages of collapse could be distinguished as had been defined previously by Wu; some differences in the post-collapse behavior were observed, however.

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	ROLE	WT	ROLE	WT	ROLE	WT
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continuous density gradient						
turbulence						
mixing						
Turbulent Wake						
wake growth						
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