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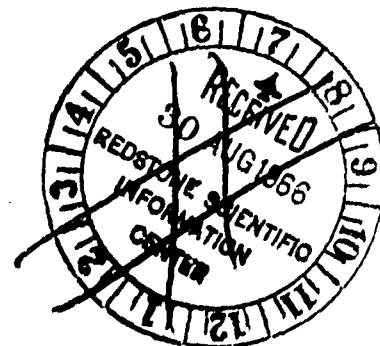
THE HYPERSONIC WAKE PROGRAM
I. A FORTRAN PROGRAM
FOR THE INTEGRAL SOLUTION

by

James E. Wollrab

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THE HYPERSONIC WAKE PROGRAM
I. A FORTRAN PROGRAM
FOR THE INTEGRAL SOLUTION

by

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DA Project No. 1X242104D226
AMC Management Structure Code No. 5241.12.132

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ABSTRACT

A FORTRAN program is explicitly listed for the basic integral solution of the wake of a hypersonic blunt or conical reentry body. The near wake pressure gradient, nonequilibrium chemistry, and laminar-turbulent transition are included.

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SYMBOLS

<u>Text</u>	<u>Program*</u>	<u>Definition</u>
a		Base area, feet ² , Equation (70).
A', A''		Constants, Equations (60) and (61).
	ALT	Altitude, feet
C _D	CD	Drag coefficient, Equation (70).
$\bar{c}_p$		Specific heat at constant pressure of air mixture, °K, Equation (15).
c _p ⁽ⁱ⁾		Specific heat of species i, °K, Equation (15).
D		Drag, pounds, Equation (69).
D _{ijave}		Average binary diffusion coefficient between species i and j, Equation (8).
d	LD	Base diameter of body, feet.
E		Electronic energy, feet ² per second ² , Equation (54).
E _{chem}		Zero-point energy, feet ² per second ² , Equation (54).
e _{int}		Internal energy, feet ² per second ² per unit mass, Equation (4).
E _{vib} ⁽ⁱ⁾		Vibrational energy, feet ² per second ² (equal to Planck's constant times $\gamma^{(i)}$).
g		Electronic degeneracy, Equation (54).
H		Total enthalpy, feet ² per second ² , Equation (57).

*See footnote at end of symbols.

<u>Text</u>	<u>Program*</u>	<u>Definition</u>
h	LHXN (h(x, n))	Enthalpy, feet ² per second ² , Equation (53).
H _{nn0}	HNNO	($\partial^2 H / \partial n^2$) ₀ , Equation (43).
$\tilde{I}$		Unit tensor, Equation (6).
K		Thermal conductivity, Equation (7).
k		Boltzmann's constant, Equation (54).
Le	LEO (Le ₀)	Lewis number, Equation (10).
M	W	Molecular weight of atmosphere model, Equation (74).
M _∞	MIN	Free-stream Mach number, Equation (64).
m		Transformation variable, Equation (23).
M ⁽ⁱ⁾	FM	Mass of ith species, Equation (54).
n		Transformed radial coordinate, Equation (24).
p	PRESS	Pressure, pounds per foot ² , Equation (6).
Pr	PRO (Pr ₀)	Prandtl number, Equation (10).
q		Dynamic pressure, pounds per foot ² , Equation (70).
$\bar{q}$		Heat flux vector, Equation (4).
R		Radial distance to shock, feet, Equation (69).
r		Radial coordinate, feet, Equation (25).
T	TEMP	Temperature (kinetic), °K Equation (75).
T _m	TEMPM	Molecular-scale temperature, Equation (76).

*See footnote at end of symbols.

<u>Text</u>	<u>Program*</u>	<u>Definition</u>
t		Time, seconds.
u		Velocity in downstream (x) direction, feet per second, Equation (9).
u_{∞}	UIN	Free-stream velocity, feet per second.
V		Diffusion velocity, feet per second, Equation (5).
v		Velocity in the radial (r) direction, feet per second, Equation (9).
$\vec{v}$		General fluid velocity, feet per second, vector, Equation (1).
x		Downstream coordinate, feet, Equation (9).
x_t	XT	Laminar turbulent transition coordinate, feet, Equation (64).
Δx		Increment of x, feet.
x_c	START X	Initial x value, feet.
	STOP X	Terminal x value, feet.
X_i		End-point altitudes, feet, Equation (76).
Y	Y	Transition coordinate parameter, Equation (64).
Y_i		End-point temperatures, Equation (77).
Z_0	ZOE ₁	Compressibility factor on the axis.
A, B, C, D, E, F, K, L, N, P		Constants (Paragraph 4).
B, C, D, E _i , H _j		Atmosphere constants, Equations (77) through (81).
a	ALPH	Species concentration, Equation (11).

*See footnote at end of symbols.

<u>Text</u>	<u>Program*</u>	<u>Definition</u>
δ	DEL	Radial distance in x, r coordinate system, feet, Equation (72).
δ_m	DELM2 (δ_m^2)	Transformed distance to outer edge of wake, feet, Equation (25).
θ	THETA	Momentum thickness, feet, Equation (17).
θ_E		Energy thickness, feet, Equation (18).
$\theta^{(i)}$		Concentration thickness, feet, Equation (19).
Λ_c	DELTAC	Constant depending on initial conditions, Equation (44).
$\Lambda_e^{(i)}$		Defined by Equation (59).
μ	MU	Viscosity coefficient, $\frac{\text{lb-sec}}{\text{ft}^2}$, Equation (6).
$\gamma^{(i)}$		Vibrational frequency of i^{th} molecular species, Equation (54).
ρ	DENSG	Density, Equation (1), $\frac{\text{lb-sec}^2}{\text{ft}^4}$
ϕ_N	PHI	Cone half-angle, degrees.
ϕ	S2B	Shock half-angle, degrees, Equation (67).
ϕ	. . .	Coordinate in initial cylindrical coordinate system (not used in equations).

SUBSCRIPTS AND SUPERSCRIPTS

i	I	Index for eight chemical species.
c	C	Initial x coordinate for calculation.

*See footnote at end of symbols.

<u>Text</u>	<u>Program*</u>	<u>Definition</u>
e	E	Values at $n = 1$.
0	0	Values at $n = 0$.
∞	IN	Free-stream conditions.

*Only a few representative program symbols are listed; however, unlisted symbols can be deduced from the text equations. The units used in the program are feet, pounds, seconds, and degrees Kelvin. Corresponding units are assumed throughout the text unless noted otherwise.

1. Introduction

The analysis presented in this report is essentially a programming of the hypersonic axisymmetric wake problem as formulated by Bloom and Steiger¹ and Ness and Fanucci.² The object of this work is to use the fundamental wake analysis as a basis for an extension of the problem to include contributions from ablation and base mixing and to determine their effects on the initial wake conditions and concentrations of radiating species. The FORTRAN program is in Appendix A. To be complete and internally consistent, the program is described in detail beginning with the basic conservation equations. The solutions are obtained for an air model of eight species using nonequilibrium chemistry. The downstream pressure gradient in the wake and the laminar-turbulent transition are included. However, a number of important assumptions have been employed to make the analysis tenable. The units used in the program are feet, pounds, seconds, and degrees Kelvin. Corresponding units are assumed throughout the text, unless noted otherwise.

2. Definition of the Flow Field

The basic regions of the flow field accompanying a hypersonic body as it enters the atmosphere have been extensively studied both experimentally and theoretically. Air is heated irreversibly and compressed by the strong detached bow shock associated with a blunt body. The fluid entering the boundary layer overexpands around the rear edge of the body. In this base-flow region, the fluid is recompressed and turned back parallel to the axial direction producing a secondary shock which is weaker than the main bow shock. Fluid which is not captured by the recirculating base flow passes through a high pressure, minimum diameter neck and into the viscous wake. A conical body is characterized by an attached shock which produces much less heating in the inviscid portion of the wake and by a thicker boundary layer.

The inner viscous wake may be divided into two regions distinguished by the presence of an axial pressure gradient. The near wake extends from the vicinity of the neck to the downstream coordinate at which the pressure has essentially decayed to the ambient value. The far wake is then controlled primarily by diffusion processes. Large pressure gradients and high temperatures characterize the initial portions of the near wake. The outer inviscid wake passes through the bow shock and the recompression shock to be "swallowed" by the radial growth of the inner wake at some downstream coordinate determined by the flight

characteristics and the laminar-turbulent transition point of the inner wake.

3. Basic Conservation Equations, Transformations, and Boundary Conditions

When both the chemical and fluid-mechanical properties of the flow are considered, conservation relations can be written for the momentum, energy, and chemical species. In addition, an equation expressing global continuity (conservation of mass) is valid. Using concise vector notation, the continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{\mathbf{v}}) = 0. \quad (1)$$

Because the calculations are done under the assumption of steady flow, all time derivatives vanish.

$$\nabla \cdot (\rho \bar{\mathbf{v}}) = 0 \quad (2)$$

Similarly, the momentum, energy, and species equations are respectively

$$\nabla \cdot (\rho \bar{\mathbf{v}} \bar{\mathbf{v}} + \tilde{\mathbf{p}}) = 0 \quad (3)$$

$$\nabla \cdot \left(\frac{1}{2} \rho |\bar{\mathbf{v}}|^2 \bar{\mathbf{v}} + \rho e_{\text{int}} \bar{\mathbf{v}} + \nabla \cdot \tilde{\mathbf{p}} + \bar{\mathbf{q}} \right) = 0 \quad (4)$$

$$\nabla \cdot \rho^{(i)} (\bar{\mathbf{v}} + \bar{\mathbf{v}}^{(i)}) = \left(\frac{\partial \rho^{(i)}}{\partial t} \right)_{\text{chem}} \quad (5)$$

$\tilde{\mathbf{p}}$ is the pressure tensor, $\bar{\mathbf{q}}$ is the heat flux vector, and $\bar{\mathbf{v}}^{(i)}$ is the diffusion velocity.

$$\tilde{\mathbf{p}} = p \tilde{\mathbf{I}} - \mu \left[\nabla \bar{\mathbf{v}} + (\nabla \bar{\mathbf{v}})^{\dagger} - \frac{2}{3} (\nabla \cdot \bar{\mathbf{v}}) \tilde{\mathbf{I}} \right] \quad (6)$$

$$\bar{\mathbf{q}} = -K \nabla T + \sum_i \rho^{(i)} \bar{\mathbf{v}}^{(i)} h^{(i)} \quad (7)$$

$$\nabla(i) = -D_{ij\text{ave}} \nabla a^{(i)} / a^{(i)} \quad (8)$$

Superscript i represents the i^{th} chemical species of the reacting air mixture. $D_{ij\text{ave}}$ is the average binary diffusion coefficient between species i and j . $\tilde{\gamma}$ represents the unit tensor, K is the thermal conductivity, and e_{int} represents the internal energy per unit mass of the air mixture. The remaining notation can be obtained from the list of symbols.

The physical coordinate system consists of a downstream coordinate x , a radial coordinate r , and an angular coordinate ϕ . However, the cylindrical symmetry of the problem removes any ϕ dependence. It is also possible to apply boundary layer approximations if the gradients normal to the flow direction are stronger than those along the flow direction.

a. Momentum

$$\frac{\partial}{\partial x} (r\rho u^2) + \frac{\partial}{\partial r} (r\rho uv) - \frac{\partial}{\partial r} \left(r\mu \frac{\partial u}{\partial r} \right) + \frac{\partial}{\partial x} (r\rho) = 0 \quad (9)$$

b. Energy

$$\begin{aligned} \frac{\partial}{\partial x} (r\rho uH) + \frac{\partial}{\partial r} (r\rho vH) &= \frac{\partial}{\partial r} \left(\frac{\mu}{Pr} r \frac{\partial H}{\partial r} \right) \\ + \frac{\partial}{\partial r} \left[\frac{\mu}{Pr} (Pr-1) r u \frac{\partial u}{\partial r} \right] &+ \frac{\partial}{\partial r} \left[\frac{\mu}{Pr} (Le-1) r \sum_i h^{(i)} \frac{\partial a^{(i)}}{\partial r} \right] \end{aligned} \quad (10)$$

c. Chemical Species

$$\begin{aligned} \frac{\partial}{\partial x} (r\rho u a^{(i)}) + \frac{\partial}{\partial r} (r\rho v a^{(i)}) \\ - \frac{\partial}{\partial r} \left(r\mu \frac{Le}{Pr} \frac{\partial a^{(i)}}{\partial r} \right) &= \left(r \frac{\partial \rho^{(i)}}{\partial t} \right)_{\text{chem}} \end{aligned} \quad (11)$$

d. Continuity

$$\frac{\partial}{\partial x} (r\rho u) + \frac{\partial}{\partial r} (r\rho v) = 0 \quad (12)$$

Using the expression for the temperature gradient

$$\nabla T = \frac{1}{\bar{c}_p} \nabla \left(H - \frac{u^2}{2} \right) - \frac{1}{\bar{c}_p} \sum_i h^{(i)} \nabla a^{(i)} \quad (13)$$

and Equation (8) for the diffusion velocity, the heat flux vector may be written as

$$\bar{q} = -\frac{\mu}{Pr} \nabla H + \frac{\mu}{Pr} \nabla \left(\frac{u^2}{2} \right) + \frac{\mu}{Pr} (1 - Le) \sum_i h^{(i)} \nabla a^{(i)} \quad (14)$$

$$\tau_p = \sum_i a^{(i)} c_p^{(i)}. \quad (15)$$

The conservation equations can now be integrated across the wake from $r=0$ to $r=\delta$. $\delta(x)$ represents the wake thickness as a function of the downstream coordinate x . Using Leibnitz's rule for the derivative of an integral, the continuity equation becomes

$$(r\rho v)_e = (r\rho u)_e \frac{d\delta}{dx} - \frac{d}{dx} \int_0^\delta r\rho u dr. \quad (16)$$

Further defining the integrals

$$\theta = \int_0^\delta \frac{\rho u}{\rho_e u_e} \left(1 - \frac{u}{u_e} \right) r dr \quad (17)$$

$$\theta_E = \int_0^\delta \frac{\rho u}{\rho_e u_e} \left(1 - \frac{H}{H_e} \right) r dr \quad (18)$$

$$\theta^{(i)} = \int_0^\delta \frac{\rho u}{\rho_e u_e} \left(a_e^{(i)} - a^{(i)} \right) r dr \quad (19)$$

the integral form of the momentum, energy, and chemical species equations in the x, r system is

$$\frac{d\theta}{dx} + \theta \frac{d}{dx} (\ln \rho_e u_e^2) - \left(\int_0^\delta \frac{\rho u}{\rho_e u_e} r dr \right) \frac{d}{dx} \ln u_e \quad (20)$$

$$- \frac{\delta^2}{2 \rho_e u_e^2} \frac{dp}{dx} = 0$$

$$\rho_e u_e \theta_E = \text{constant} \quad (21)$$

$$\frac{d\theta^{(i)}}{dx} + \theta^{(i)} \frac{d}{dx} (\ln \rho_e u_e) - \left(\int_0^\delta \frac{\rho u}{\rho_e u_e} r dr \right) \frac{da_e^{(i)}}{dx} = \quad (22)$$

$$- \frac{1}{\rho_e u_e} \int_0^\delta \left(\frac{\partial \rho}{\partial t} \right)_{\text{chem}} r dr.$$

The transformed radial coordinate n is now introduced by the Dorodnitsen transformation through the relationships

$$mdm = \frac{\rho}{\rho_e} r dr \quad (23)$$

$$m = n \delta_m \quad (24)$$

which lead to

$$\delta_m^2 n dn = \frac{\rho}{\rho_e} r dr. \quad (25)$$

The effect of this transformation is to remove the radial density dependence from the calculations by introducing the x, n coordinate system in place of the cylindrical x, r system.

Several sets of boundary conditions apply to the concentrations, velocity, and enthalpy at the axis ($n = 0$) and on the outer edge ($n = 1$).

$n = 0$:

$$a^{(i)} = a_0^{(i)}(x) ; \left(\frac{\partial a^{(i)}}{\partial n} \right)_0 = 0 \quad (26)$$

$$u = u_0(x) ; \left(\frac{\partial u}{\partial n} \right)_0 = 0 \quad (27)$$

$$H = H_0(x) ; \left(\frac{\partial H}{\partial n} \right)_0 = 0 \quad (28)$$

$n = 1$:

$$a^{(i)} = a_e^{(i)}(x) ; \left(\frac{\partial a^{(i)}}{\partial n} \right)_e = \left(\frac{\partial^2 a^{(i)}}{\partial n^2} \right)_e = 0 \quad (29)$$

$$u = u_e(x) ; \left(\frac{\partial u}{\partial n} \right)_e = \left(\frac{\partial^2 u}{\partial n^2} \right)_e = 0 \quad (30)$$

$$H = H_e ; \left(\frac{\partial H}{\partial n} \right)_e = \left(\frac{\partial^2 H}{\partial n^2} \right)_e = 0 \quad (31)$$

These boundary conditions follow from a consideration of the physical model of the symmetrical wake. From these conditions it is possible to formulate profile expressions of the proper form

$$\frac{a^{(i)}(x, n) - a_0^{(i)}(x)}{a_e^{(i)}(x) - a_0^{(i)}(x)} = \frac{u(x, n) - u_0(x)}{u_e(x) - u_0(x)} = 6n^2 - 8n^3 + 3n^4. \quad (32)$$

For the enthalpy, $\left(\frac{\partial^2 H}{\partial n^2} \right)_0$ is included by using a fifth-degree polynomial.

$$H = H_0 + (H_e - H_0)(10n^3 - 15n^4 + 6n^5) + \frac{1}{2} H_{nn0}(n^2 - 3n^3 + 3n^4 - n^5) \quad (33)$$

The momentum Equation (9), the energy Equation (10), and the chemical species Equation (11) are next specialized to $n = 0$ and $n = 1$ after transformation to the x, n system.

$n = 0$:

$$\frac{du_0}{dx} = \frac{24\mu_0}{\rho_e u_0 \delta_m^2} (u_e - u_0) - \frac{1}{\rho_0 u_0} \frac{dp}{dx} \quad (34)$$

$$\begin{aligned} \frac{dH_0}{dx} = \frac{2\mu_0}{\rho_e u_0 \delta_m^2} & \left[\frac{H_{nn0}}{Pr_0} + 12 \left(\frac{Pr_0 - 1}{Pr_0} \right) u_0 (u_e - u_0) \right. \\ & \left. + 12 \left(\frac{Le_0 - 1}{Pr_0} \right) \left(\sum_i a_e^{(i)} h_0^{(i)} - h_0 \right) \right] \end{aligned} \quad (35)$$

$$\frac{da_0^{(i)}}{dx} = \frac{24\mu_0 Le_0}{\rho_e u_0 \delta_m^2 Pr_0} \left(a_e^{(i)} - a_0^{(i)} \right) + \frac{1}{u_0} \left[\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)^{(i)} \text{chem} \right]_0 \quad (36)$$

$n = 1$:

$$\frac{du_e}{dx} = - \frac{1}{\rho_e u_e} \frac{dp}{dx} \quad (37)$$

$$\frac{da_e^{(i)}}{dx} = \frac{1}{u_e} \left[\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)^{(i)} \text{chem} \right]_e \quad (38)$$

Because the stagnation enthalpy is assumed to be constant along the outer edge of the wake, the energy equation is automatically satisfied.

The transformed integral momentum equation is

$$\frac{d\theta}{dx} = - \theta \frac{d\ln \rho_e}{dx} + \left[2\theta + \frac{\delta^2}{2} - \frac{\delta_m^2}{10} \left(4 + \frac{u_0}{u_e} \right) \right] \frac{1}{\rho_e u_e^2} \frac{dp}{dx} \quad (39)$$

where

$$\delta_m^2 = \frac{210 \theta}{\left(1 - \frac{u_0}{u_e} \right) \left(10 + 11 \frac{u_0}{u_e} \right)} \quad (40)$$

The integral form of the energy Equation (21) remains unchanged and the constant is given by the value of $\rho_e u_e \theta_E$ at the initial coordinate of the calculation.

$$\rho_e u_e \theta_E = (\rho_e u_e \theta_E)_c \quad (41)$$

where

$$\theta_E = \frac{\delta_m^2}{22} \left[\frac{1}{210} \left(1 - \frac{H_0}{H_e} \right) \left(349 + 311 \frac{u_0}{u_e} \right) - \frac{H_{nn_0}}{840 H_e} \left(43 + 23 \frac{u_0}{u_e} \right) \right] \quad (42)$$

The shape parameter H_{nn_0} is then

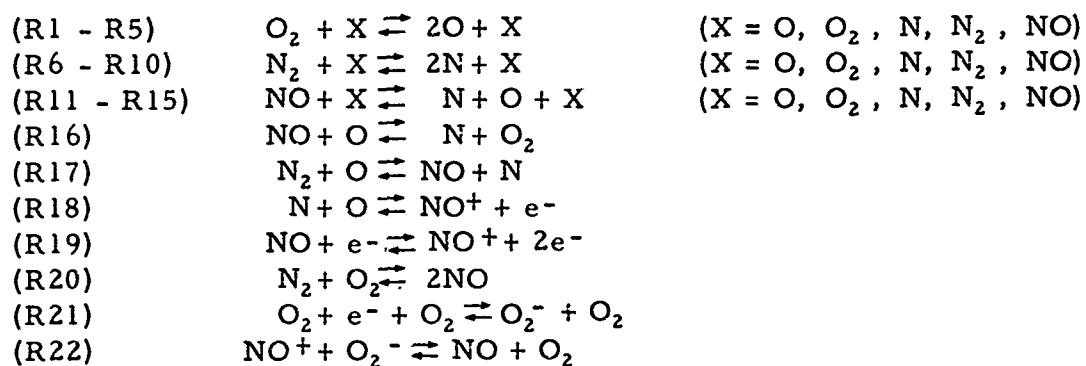
$$H_{nn_0} = 4(H_e - H_0) \left[\frac{349 u_e + 311 u_0}{43 u_e + 23 u_0} \right] - \frac{(\rho_e \delta_m^2)_c \Lambda_c}{\rho_e \delta_m^2 (43 u_e + 23 u_0)} \quad (43)$$

Λ_c depends on the initial conditions as indicated by the subscript c.

$$\Lambda_c = 4(H_e - H_{0c}) (349 u_e + 311 u_0)_c - (H_{nn_0})_c (43 u_e + 23 u_0)_c \quad (44)$$

4. Reacting Air Mixture

A reacting air mixture containing eight chemical species is used to represent the fluid chemistry. The air species are N_2 , O_2 , N, O, NO^+ , O_2^- , NO, and e^- . The 22 allowed chemical reactions involving these eight species are



Constants involved in the chemical kinetics are given by Ness and Fanucci.²

The chemical kinetics enter the analysis through the chemical production terms $\left(\frac{\partial \rho(i)}{\partial t} \right)_{\text{chem}}$ in the law of mass action. The partition

functions are derived by assuming equilibrium for all degrees of freedom. The resulting expressions are

$$\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_{\text{chem}}^{(\text{O})} = A + \frac{m^{(\text{O})}}{m^{(\text{NO})}} (C - D - E) - \frac{m^{(\text{O})}}{m^{(\text{NO}^+)}} F \quad (45)$$

$$\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_{\text{chem}}^{(\text{N})} = B + \frac{m^{(\text{N})}}{m^{(\text{NO})}} (C + D + E) - \frac{m^{(\text{N})}}{m^{(\text{NO}^+)}} F \quad (46)$$

$$\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_{\text{chem}}^{(\text{O}_2)} = -A + \frac{m^{(\text{O}_2)}}{m^{(\text{NO})}} (D - \frac{1}{2} L) - \frac{m^{(\text{O}_2)}}{m^{(\text{O}_2^-)}} (N - P') \quad (47)$$

$$\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_{\text{chem}}^{(\text{N}_2)} = -B - \frac{m^{(\text{N}_2)}}{m^{(\text{NO})}} (E + \frac{1}{2} L) \quad (48)$$

$$\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_{\text{chem}}^{(\text{NO})} = E + L - C - D - K + \frac{m^{(\text{NO})}}{m^{(\text{O}_2^-)}} P' \quad (49)$$

$$\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_{\text{chem}}^{(\text{NO}^+)} = F + \frac{m^{(\text{NO}^+)}}{m^{(\text{NO})}} K - \frac{m^{(\text{NO}^+)}}{m^{(\text{O}_2^-)}} P' \quad (50)$$

$$\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_{\text{chem}}^{(\text{e}^-)} = \frac{m^{(\text{e}^-)}}{m^{(\text{NO}^+)}} F + \frac{m^{(\text{e}^-)}}{m^{(\text{NO})}} K - \frac{m^{(\text{e}^-)}}{m^{(\text{O}_2^-)}} N \quad (51)$$

$$\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_{\text{chem}}^{(\text{O}_2^-)} = N - P' \quad (52)$$

The constants A, B, C, D, E, F, K, L, N, and P' are given by Ness and Fanucci.²

The total static enthalpy h of the air mixture is

$$h = \sum_i a^{(i)} h^{(i)} = H - \frac{u^2}{2} \quad (53)$$

The individual species states enthalpy is a function of the static temperature and molecular and atomic constants only.

$$h^{(i)} = \frac{7}{2} \frac{kT}{m^{(i)}} + \frac{E_{vib}^{(i)}}{m^{(i)} \left(e^{\frac{E_{vib}^{(i)}}{kT}} - 1 \right)} + \frac{1}{m^{(i)}} \left[\frac{\sum_{\ell} g_{\ell}^{(i)} E_{\ell}^{(i)} e^{\frac{-E_{\ell}^{(i)}}{kT}}}{\sum_{\ell} g_{\ell}^{(i)} e^{\frac{-E_{\ell}^{(i)}}{kT}}} \right] + \frac{E_{chem}^{(i)}}{m^{(i)}} \quad (54)$$

From the equation of state,

$$p = \rho kT \sum_i \frac{a^{(i)}}{m^{(i)}} \quad (55)$$

differentiation leads to

$$\frac{d \ln \rho e}{dx} = \frac{dp}{dx} - \frac{d \ln T_e}{dx} - \frac{\sum_i \frac{1}{m^{(i)}} \frac{da_e^{(i)}}{dx}}{\sum_i \frac{a_e^{(i)}}{m^{(i)}}} \quad (56)$$

The temperature derivative can be obtained from

$$H_e = \sum_i a_e^{(i)} h_e^{(i)} + \frac{u_e^2}{2} \quad (57)$$

Substitution of the $h_e^{(i)}$ values into Equation (57) and differentiation gives

$$\frac{d \ln T_e}{dx} = - \frac{A' + \frac{A''}{kT_e}}{\sum_i \frac{a_e^{(i)}}{m^{(i)}} \left[\left(\Lambda_e^{(i)} \right)^2 + \Lambda_e^{(i)} \frac{E_{vib}^{(i)}}{kT_e} + \frac{7}{2} \right] + \sum_i \frac{5}{2} \frac{a_e^{(i)}}{m^{(i)}}} \quad (58)$$

$$\Lambda_e^{(i)} = \frac{\frac{E_{vib}^{(i)}}{kT_e}}{e^{\frac{E_{vib}^{(i)}}{kT_e}} - 1} \quad (59)$$

$$A' = \sum_i \frac{1}{m^{(i)}} \left(\frac{7}{2} + \Lambda_e^{(i)} \right) \frac{da_e^{(i)}}{dx} + \sum_i \frac{5}{2} \frac{1}{m^{(i)}} \frac{da_e^{(i)}}{dx} \quad (60)$$

$$A'' = u_e \frac{du_e}{dx} + \sum_i \frac{E_{chem}^{(i)}}{m^{(i)}} \frac{da_e^{(i)}}{dx} \quad (61)$$

5. Laminar-Turbulent Transition and the Pressure Gradient

The laminar-turbulent transition is treated by changing the viscosity model. The laminar portion of the wake is governed by the Sutherland equation.

$$\mu_0 = 3.05 \times 10^{-8} \frac{T_0^{3/2}}{T_0 + 110} \quad (62)$$

The turbulent viscosity is

$$\mu_0 = 0.02 \delta_m \rho_e (u_e - u_0) \quad (63)$$

The transition coordinate is computed using the equation formulated by Zeiberg.³

$$x_t = \frac{\mu_\infty Y}{\rho_\infty u_\infty} \left[-\frac{5}{M_\infty^2} + \frac{6 \left(1 + \frac{5}{M_\infty^2} \right) M_\infty^2 \sin^2 \phi}{M_\infty^2 \sin^2 \phi + 5} \left(\frac{6}{7 M_\infty^2 \sin^2 \phi - 1} \right)^{5/7} \right] \quad (64)$$

Y is defined by

$$M_\infty \leq 8 : \log_{10} Y = 4.744 + 0.124 M_\infty + 0.00976 M_\infty^2 \quad (65)$$

$$M_\infty > 8 : \log_{10} Y = 5.48 + 0.11 M_\infty \quad (66)$$

A cubic can be written for the shock angle ϕ

$$\begin{aligned} & (\sin^2 \phi)^3 - \left[\frac{M_\infty^2}{M_\infty^2} + 2 + 1.4 \sin^2 \phi_c \right] (\sin^2 \phi_N)^2 \\ & + \left[\frac{2M_\infty^2}{M_\infty^4} + 1 + \left(1.44 + \frac{0.4}{M_\infty^2} \right) \sin^2 \phi_N \right] \sin^2 \phi - \frac{\cos^2 \phi_N}{M_\infty^4} = 0 \end{aligned} \quad (67)$$

ϕ_N is the cone half-angle.

For a blunt body (detached bow shock) $\phi_N \approx \frac{\pi}{2}$ and $\phi = \frac{\pi}{2}$. For a cone, ϕ is obtained as the intermediate root of Equation (67) which has three positive roots for reasonable cone angles.

The pressure gradient in the near wake is expressed by the equation

$$\frac{p}{p_\infty} = \frac{0.0665 M_\infty^2}{x/d} + 1 \quad (68)$$

Initial calculations were begun at a coordinate downstream of the neck ($x > 0$) to prevent the first term from becoming infinite.

From the momentum theorem the drag can be used to give an initial value for the momentum thickness.

$$D = 2\pi \int_0^R (p_\infty - p) r dr + 2\pi \int_0^R \rho u (u_\infty - u) r dr \quad (69)$$

R is the radial distance to the shock. It is assumed that the total drag occurs in the area up to $R = \delta$ and that $p = p_\infty$ for $R > \delta$.

$$D = c_D q a = c_D \left(\frac{\rho_\infty u_\infty^2}{2} \right) \frac{\pi d^2}{4} \quad (70)$$

The initial momentum thickness is then

$$\theta_c = \frac{\rho_\infty}{(\rho_e)_c} \frac{c_D d^2}{16} + \frac{(p_c - p_\infty) \delta_c^2}{2 (\rho_e)_c u_e^2} \quad (71)$$

Since

$$\delta = \sqrt{2} \delta_m \left[\int_0^1 \frac{\rho_e(x)}{\rho(x, n)} n dn \right]^{1/2}, \quad (72)$$

$$\theta_c = \frac{\frac{\rho_\infty}{(\rho_e)_c} \frac{c_D d^2}{16}}{\left[1 - \frac{210 (p - p_\infty)}{\rho_e u_e^2 \left(1 - \frac{u_0}{u_e} \right) \left(10 + 11 \frac{u_0}{u_e} \right)} \int_0^1 \frac{\rho_e}{\rho} n dn \right]_c} \quad (73)$$

6. Sequence of Calculations

A printout of the FORTRAN source program is given in Appendix A. In the right-hand margin references are made to the equations used in the calculations. Important program symbols are defined in the list of symbols and correlated with the symbols used in the text.

The input data include α values for O_2 , N_2 , O , N , NO , O_2^+ , NO^+ , and e^- and α_e values for O_2 and N_2 . The remaining initial α_e values have been set equal to zero internally. Initial u_0 , H_0 , T_0 , ϕ_n , Z_0 , H_{nn0} , and x values as well as Pr_0 , Le_0 , c_D , d , u_∞ , and final x values are used as input. The altitude must be provided to initiate the atmospheric subroutine for the calculation of free-stream temperature, pressure, density, viscosity, Reynolds number, and speed of sound.⁴ This routine was adopted from an IBM 1620 program for the calculation of atmospheric properties up to 2320 kilofeet. A polynomial function of temperature as a function of geopotential altitude is used up to approximately 295 kilofeet. Below this altitude it is assumed that air has a constant molecular weight of 28.9644. Geometric altitude is employed above 295 kilofeet. For these higher altitudes the molecular weight is given by the expression

$$M = 32.154 + AKM \left[-0.034513 + AKM (2.0326 \times 10^{-6} + 2.1032 \times 10^{-8} AKM) \right] \quad (74)$$

where AKM is the altitude in kilometers. Molecular-scale temperature ($^{\circ}R$), the rate of change of temperature with geopotential altitude, and the initial pressure values ($lb-ft^{-2}$) are listed for 23 altitude values in the atmosphere table.

The kinetic temperature is obtained from the molecular-scale temperature through the equation

$$T = \frac{T_m M}{28.9644} \quad (75)$$

where the molecular-scale temperature can be obtained from the relation

$$T_m(X) = D + BX + \sum_i E_i |X - X_i| \quad (76)$$

The X_i 's are altitudes at the end points of straight line segments over which the variation of altitude and molecular-scale temperature is assumed linear. The constants D , B , and E_i are functions of the X_i and the end-point temperatures which are denoted as Y_i .

$$H_j = -\frac{1}{2} \left(\frac{Y_j - Y_{j-1}}{X_j - X_{j-1}} \right) \quad (j = 2 \dots 23) \quad (77)$$

$$D = C + H_2 X_1 + H_{23} X_{23} \quad (78)$$

$$C = \frac{Y_1 + Y_{23}}{2} \quad (79)$$

$$B = -H_2 - H_{23} \quad (80)$$

$$E_i = H_j - H_{j+1} \quad (81)$$

Once the free-stream conditions are known, the laminar-turbulent transition coordinate is calculated as a function of the free-stream conditions and M_∞ using Equation (64). The axial viscosity is obtained from the Sutherland Equation (62) for the laminar region and from Equation (63) in the turbulent zone. Constants A through P' which appear in the chemical production terms are evaluated on the axis using ρ_0 , T_0 , and α_0 , . . . and at the edge of the wake using ρ_e , T_e , and α_e , Density species derivatives in Equations (45) through (52) are then calculated at the axis and on the outer edge. From the profiles for species concentrations, velocity, and enthalpy, $\alpha(x,n)$, $u(x,n)$, and $H(x,n)$ can be evaluated. $T(x,n)$ is then obtained by iteration of Equation (53) using the individual species enthalpies given in Equation (54). Once the pressure is evaluated, from Equation (68), $p(x,n)$ is given by Equation (55).

At this point, θ_c is calculated from Equation (73). This is only done for the initial coordinate. Once δ_m is obtained from Equation (40), the derivatives of $\alpha_0^{(i)}$, $\alpha_e^{(i)}$, p , u_0 , and u_e in Equations (36), (38), (68), (34), and (37), respectively, allow these parameters to be calculated at the next downstream coordinate. Equations (58) and (56) give $d \ln T_e / dx$ and $d \ln p_e / dx$, and δ is obtained by integration of Equation (72). These lead to $d\theta/dx$ in Equation (39). Λ_c and H_{nn0} from Equations (44) and (43) allow dH_0/dx to be determined using Equation (35).

Finally, T_0 can be evaluated using the new values of H_0 , u_0 , and $\alpha_0^{(i)}$, and the calculations are then repeated for an incremented value of x . In this calculation, x was incremented uniformly as $\Delta x = 0.1$. Better results may be obtained by using smaller increments for small values of x ; however, the error was not large enough to justify a procedure of this type.

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Appendix A

FORTRAN PROGRAM

The hypersonic WAKE program described in the text is given along with the required subroutines. Comments appearing in the right-hand margin refer to equations discussed in the text. Some important symbols are defined in the list of symbols; however, most of the quantities can be identified by reference to the sequence of calculation (Paragraph 6) and the original equations.

```

C      ---HYPERSONIC WAKE CALCULATION ---
C
C      DIMENSION ALPHE(8),ALPHO(8),FM(8),FN(4),ALPHE1(8),ALPHO1(8),H(4),
1T(4),A(2),B(2),C(2),D(2),E(2),F(2),K(2),L(2),N(2),PP(2),
2ALPH(8),TOE(2),RHOE(2),      UOE(2),LHOE(2),HOF(2),ZOF(2),
3AA(23,4),HH(23),RHOE1(2),UOF1(2),HOE1(2),
4ZOE1(2),ALPHXN(8,4),UXN(4),HXN(4),LHXN(4),TXN(4),HSUBS(8),RHOEXN(4
5),PPTSBO(8),PPTSBE(8),FK(4),DALPHO(8),DALPHE(8)
C      DIMENSION PPTO(2),PPTN(2),PPTO2(2),PPTN2(2),PPTNO(2),PPTM(2),
1PPTO2M(2),PPTNOP(2)
C      DIMENSION COE(51),ROOTR(50),ROOTI(50),RUSED(3),BETA(3)
C      DATA FM/ 16.,14.,32.,30.,30.,32.,54171181E-4,28./,EP19,EP20,EP21,
1EP22/4*1./,R,GA,S/1545.31,32.1741,20855490./
C      --- ** 1962 ATMOSPHERE PROPERTIES ---
C      DATA AA/0.,36088.8,65616.,104985.6,154197.6,170601.6,200128.8,
1259183.2,291148.,295272.,328080.,360888.,393696.,492120.,524928.,
2557736.,623352.,754584.,984240.,1312320.,1640400.,1968480.,
32296560.,518.67,389.97,389.97,411.57,487.17,487.17,454.77,325.17,
4325.17,325.17,379.17,469.17,649.17,1729.17,199.17,2179.17,
52431.17, 2791.17,3295.17,3889.17,4357.17,4663.17,4861.17,
6-3.566203E-3,0.,5.486467E-4,1.536211E-3,0.,1.097293F-3,-2.194587F-
73,0.,0.,1.6459E-3,2.743233E-3,5.486467E-3,10.97293E-3,8.2297E-3,
85.486467E-3,3.840527E-3,2.743233E-3,2.194587F-3,1.810534E-3,
91.42648E-3,93226993E-3,365149E-3,0.,2116.2,471.5867,114.4748,
A18.14354,2.302997,1.232752.,3814048.,02161063.,00436286.,0034331,
B6.28109E-4,1.53598E-4,5.26658E-5,1.0571E-5,7.7157E-6,5.8324E-6,
C3.5197E-6,1.4537E-6,3.9344E-7,8.4176E-8,2.2884E-6,7.2059E-9,0./
C      REAL K,L,N,LHOE,MUO,LD,MUIN,MIN,LHXN,LEO
C      COMMON FN,RHOEXN,RHOE
C      1 READ (5,2) ALPHO
C      READ(5,2) ALPHE(3), ALPHE(8), UOE(1), HOE,      TOE(1), PHI, ALT
C      READ(5,2) ZOE(1),      HNNO,      PRO,      LEO, CD, LD,      UIN, STARTX
C      READ(5,2) STOPX,      EXOUT
C      EXOUT=0.0 FOR DX=1.0 OUTPUT-----NOT=0.0 FOR DX=0.1 EXTRA OUTPUT
C      2 FORMAT (8E10.8)
C      FN=0.
C      FN(2)=.33333333
C      FN(3)=.66666666
C      FN(4)=1.
C      ALPHE=0.
C      ALPHE(2)=0.
C      ALPHE(4)=0.
C      ALPHE(5)=0.
C      ALPHE(6)=0.

```

```

ALPHE(7)=0.
DX=.1
FNN=1.
X=STARTX
MM=1
NN2=0
NN3=0
HH(1)=0.
DO 1002 KK=2,23
  HH(KK)=-.5*(AA(KK,2)-AA(KK-1,2))/(AA(KK,1)-AA(KK-1,1))
  RB=HH(2)-HH(23)
  CC=(AA(1,2)+AA(23,2))/2.
  DD=CC+HH(2)*AA+HH(23)*AA(23,1)
  IF(ALT-295272.)1003,1003,1004
1003 ALT=(20855490.*ALT)/(20855490.+ALT)
  GO TO 1005
1004 ALTF=ALT
1005 FF=0.
DO 1006 KK=2,22
  EE=HH(KK)-HH(KK+1)
  FF=EE*((ALTF-AA(KK,1))*(ALTF-AA(KK,1))**.5+FF.
  TEMPM=DD+BB*ALTF+FF
  WO=28.9644
  IF(ALT-305114.4)1007,1007,1008
1007 W=28.9644
  GO TO 1009
1008 AKM=ALT/3280.8333
  W=32.154+AKM*(-.034513+AKM*(2.0326E-6+2.1032E-8*AKM))
1009 TEMP=TEMPM*W/WO
  IF(ALT-295272.)1010,1010,1016
1010 DO 1011 KK=2,23
  IF(ALTF-AA(KK,1))1012,1011,1011
1011 CONTINUE
1012 KK=KK-1
  DALF=ALTF-AA(KK,1)
  IF(AA(KK,3))1013,1014,1015
1013 PRESS=AA(KK,4)/(AA(KK,2)/TEMP)**(.0187434/(-AA(KK,3)))
  GO TO 1019
1014 V1=(.0187434/AA(KK,2))*DALF
  PRESS=AA(KK,4)*(1./EXP(V1))
  GO TO 1019
1015 PRESS=AA(KK,4)*(AA(KK,2)/TEMP)**(.0187434/AA(KK,3))
  GO TO 1019
1016 DO 1017 KK=10,23

```

EQUATION 77
 EQUATION 80
 EQUATION 79
 EQUATION 78

EQUATION 81

EQUATION 74
 EQUATION 75

```

1017 CONTINUE
1018 KK=KK-1
      DALT=ALT-AA(KK,1)
      XXX=S/(S-AA(KK,2)/AA(KK,3)+AA(KK,1))
      Y1=XXX*(ALOG((AA(KK,2)/AA(KK,3)+DALT)/(S+ALT))-ALOG((AA(KK,2)/
      1AA(KK,3))/(S+AA(KK,1))))
      Y2=-S*DALT/((S+AA(KK,1))*(S+ALT))
      Y3=XXX*(Y1+Y2)
      PRESS=AA(KK,4)/EXP(WO/AA(KK,3)*Y3/R)
1019 DENS=PRESS/(53.352*TFMPM)
      G=GA*(S/(S+ALT))**2
      DENS=DENS/32.174
      IF(ALT-295272.)1021,1021,1022
1021 CS=49.1*TEMPM**5
      SS=198.72
      RRR=7.3025E-7
      V'S= ' BB*TEMP**1.5)/(TFMP+SS)
      VISG=VIS/G
      MIN=UIN/CS
      RE=DENS*CS/VIS
      TEMPK=TEMP/1.8
      GO TO 1023
1022 VISG=0.
      MIN=0.
      RE=0.
      TEMPK=0.
      CS=0.
1023 PIN=PRESS
      TIN=TEMPK
      RHIN=DENSG
      MUIN=VISG
      UOE(2)=UIN
      TOE(2)=TIN
      ZOE(2)=1.
      P=(.0665*MIN**2*LD/X+1.)*PIN
      DO 10231 I=1,2
C      --EQUATION OF STATE--
10231 RHOE(I)=3.23E-4/ZOE(I)*P/TOE(I)
      WRITE(6,101)
101  FORMAT(1H1)
      WRITE(6,102)
102  FORMAT(19H INITIAL CONDITIONS)
      WRITE(6,106)

```

EQUATION 68

```

106 FORMAT (3H0 X15X2HU014X2HUE14X2HT014X2HTE14X2HH014X2HHE)
    WRITE (6,104)X,UOE,TOE,H0E
107 FORMAT(11H0 ALPHAO(O)7X9HALPHAO(N)7X10HALPHAO(O2)6X10HALPHAO(N0)6X
    11HALPHAO(N0+)5X11HALPHAO(O2-)5X10HALPHAO(E-)6X10HALPHAO(N2))
    WRITE(6,104)ALPHO
108 FORMAT(8E16.8)
    WRITE(6,105)
109 FORMAT(11H0 ALPHAE(O)7X9HALPHAE(N)7X10HALPHAE(O2)6X10HALPHAE(N0)6X
    11HALPHAE(N0+)5X11HALPHAE(O2-)5X10HALPHAE(E-)6X10HALPHAE(N2))
    WRITE(6,104)ALPHE
110 FORMAT(6H0 M(O)12X4HM(N)12X5HM(O2)11X5HM(N0)11X6HM(N0+)10X6HM(O2-)
    11X5HM(E-)11X5HM(N2))
    WRITE(6,104)FM
111 FORMAT(6H0 EP1912X4HEP2012X4HEP2112X4HEP2212X2HCD14X2HLD14X4HUINF1
    12X4HTINF)
    WRITE(6,104)EP19,EP20,EP21,EP22,CD,LD,UIN,TIN
112 FORMAT(6H0 PINF12X6HRHOINF10X5HMUINF11X4HMINF12X2HN114X2HN214X2HN3
    114X2HN4)
    WRITE(6,104)PIN,RHIN,MUIN,MIN,FN
113 FORMAT(5H0 PHI13X3HALT13X2HZ014X2HZE14X4HRH0012X4HRH0E12X4HHNNO12X
    13HPRO)
    WRITE (6,104) PHI,ALT,ZOE,RH0E,HNNO,PRO
114 FORMAT (3H0 W15X2HRE14X2HCS)
    WRITE(6,104) W,RE,CS
115 FORMAT(5H0 LEO13X5HSTOPX11X2HDX)
    WRITE(6,104)LEO,STOPX,DX
116 FORMAT(6,101)
    F90=90./57.295779+.0001
    PHI1=PHI/57.295779
    IF (PHI,NF,90.)GO TO 121
    S2B=1.
    GO TO 122
121 COE(4)=-COS(PHI1)**2/MIN**4
    COE(3)=(2.*MIN**2+1.)/MIN**4+(1.44+.4/MIN**2)*SIN(PHI1)**2
    COE(2)=-((MIN**2+2.)/MIN**2+1.4*SIN(PHI1)**2)
    COE(1)=1.
    CALL RTSOLV(COE,3,ROOTR,ROOTI)
    DO 112 I=1,3
    IF (ABS(ROOTI(I)).GT..0000001)GO TO 120

```

SOLUTION OF EQUATION 67

```

112 CONTINUE
GO TO 115
120 S2B=1.
GO TO 122
115 DO 114 I=1,2
DO 114 J=2,3
IF(ROOTR(I).LE.ROOTR(J))GO TO 114
TMP=ROOTR(I)
ROOTR(I)=ROOTR(J)
ROOTR(J)=TMP
114 CONTINUE
S2B=ROOTR(2)
122 IF(MIN.LE.8.)GO TO 116
Y=EXP(2.30258509*(5.48+.11*MIN))
GO TO 123
116 Y=EXP(2.30258509*(4.744+MIN*(.124+.00976*MIN)))
123 XT=MUIN/RHIN*Y/UIN*(-5./MIN**2+6.*(1.+5./MIN**2)*MIN**2*S2B/(MIN**
12*S2B+5.))*(6./(7.*MIN**2*S2B-1.))*((5./7.))
100 IF(X.GF.XT)GO TO 117
MUO=3.05E-8*(TOE/(TOE+110.))
GO TO 118
117 MUO=.02*SQRT(DELM2)*RHOE(2)*(UOE(2)-UOE)
118 DO 9 I=1,2
GO TO (3,5),I
3 DO 4 J=1,8
4 ALPH(J)=ALPHO(J)
GO TO 7
5 DO 6 J=1,8
6 ALPH(J)=ALPHF(J)
7 TMP1= 5.+3.*EXP(-228./TOE(I))+EXP(-326./TOE(I))
TMP2=1.-EXP(-2274./TOE(I))
TMP3=3.+2.*EXP(-11390./TOE(I))
TMP4=TOE(I)*(-1.5)
TMP5=SQRT(TOE(I))
TMP6=1.-EXP(-3395./TOE(I))
TMP7=1.-EXP(-2740./TOE(I))
TMP8=1.+EXP(-178./TOE(I))
TMP9=1.-EXP(-1728./TOE(I))
XP=-59365./TOE(I)
IF(XP.GE.-88.)AND(XP.LE.88.)GO TO 8
IF(XP)10,10,11
10 XP=-88.
GO TO 8
11 XP=88.

```

EQUATION 66

EQUATION 65
EQUATION 64

EQUATION 62

EQUATION 63

A OF PARAGRAPH 4

8 A(I)=1.003E18*RHOE(I)*TMP4*(5.*ALPH(I)/8.*ALPH(3)/16.*ALPH(2)/21.+
1ALPH(8)/42.*ALPH(4)/45.)*(0.2345*ALPH(3)*TMP5*EXP(XP)*TMP2/TMP3*
-TMP1-RHOE(I)*ALPH(1)**2)

XP=-113200./TOE(I)
IF(XP.GE.-88..AND.XP.LE.88.)GO TO 12
IF(XP)13,13,14
13 XP=-88.
GO TO 12

B OF PARAGRAPH 4

14 XP=88.
12 B(I)=1.147E19*RHOE(I)*TMP4*(5.*ALPH(2)/28.*ALPH(8)/56.*ALPH(1)/96.+
1+ALPH(3)/192.*ALPH(4)/180.)*(3.747*ALPH(8)*TMP5*EXP(XP)*TMP6-RHOE(I)
21)*ALPH(2)**2)

XP=-75310./TOE(I)
IF(XP.GE.-88..AND.XP.LE.88.)GO TO 15
IF(XP)16,16,17
16 XP=-88.
GO TO 15

C OF PARAGRAPH 4

17 XP=88.
15 C(I)=2.15E19*RHOE(I)*TMP4*(ALPH(4)/30.*ALPH(1)/16.*ALPH(2)/14.+
1ALPH(3)/32.*ALPH(8)/28.)*(0.234*ALPH(4)*TMP5*EXP(XP)*TMP7/TMP8*
2TMP1-RHOE(I)*ALPH(2)*ALPH(1))

D(I)=3.45E10*RHOE(I)*TMP5*EXP(-3120./TOE(I))*(.994*ALPH(4)*ALPH(1)
1*EXP(-15945./TOE(I))/TMP2*TMP7/TMP8*TMP3/TMP1-ALPH(2)*ALPH(3))
XP=-37890./TOE(I)
IF(XP.GE.-88..AND.XP.LE.88.)GO TO 18
IF(XP)19,19,20
19 XP=-88.
GO TO 18

E OF PARAGRAPH 4

20 XP=88.
18 E(I)=4.78E11*RHOE(I)*(16.04*ALPH(1)*ALPH(8)*EXP(XP)/TMP7*TMP6/TMP1
1*TMP8-ALPH(2)*ALPH(4))

XP=-32130./TOE(I)
IF(XP.GE.-88..AND.XP.LE.88.)GO TO 21
IF(XP)22,22,23
22 XP=-88.
GO TO 21

F OF PARAGRAPH 4

23 XP=88.
21 F(I)=1.696E24*RHOE(I)*TMP4*(8.04E-12*ALPH(1)*ALPH(2)*TOE(I)/TMP6*
1EXP(XP)/TMP1-ALPH(5)*ALPH(7))

XP=-107440./TOE(I)
IF(XP.GE.-88..AND.XP.LE.88.)GO TO 24
IF(XP)25,25,26
25 XP=-88.
GO TO 24

24

```

26 XP=88.
24 K(I)=1.765E30*RHOE(I)*TMP4*ALPH(7)*(1.88E-12*ALPH(4)*TOE(I)**1.5
  1/TMP6*EXP(XP)/TMP8*TMP7-RHOE(I)*ALPH(5)*ALPH(7))
  XP=-43000./TOE(I)
  IF(XP.GE.-88..AND.XP.LE.88.)GO TO 27
  IF(XP)28,28,29
28 XP=-88.
  GO TO 27
29 XP=88.
27 L(I)=1.65E22*RHOE(I)*TOE(I)**(-2.5)*EXP(XP)*(16.15*ALPH(8)*ALPH(3)
  1*EXP(-21945./TOE(I))*TMP6/TMP3*TMP2/TMP7*TMP8-ALPH(4)**2)
  N(I)=4.83E9*RHOE(I)*ALPH(3)*TOE(I)*(2.805E11*RHOE(I)*ALPH(3)*
  1ALPH(7)*TMP4/TMP9*TMP2/TMP3-ALPH(6)*EXP(-5110./TOE(I))
  XP=-102000./TOE(I)
  IF(XP.GE.-88..AND.XP.LE.88.)GO TO 30
  IF(XP)31,31,32
31 XP=-88.
  GO TO 30
32 XP=88.
30 PP(I)=3.44E13*RHOE(I)*(1.98*ALPH(5)*ALPH(6)*EXP(340./TOE(I))*TMP6*
  1TMP9/TMP7*TMP8/TMP2*TMP3-ALPH(4)*ALPH(3)*EXP(XP))
  PPTO(I)=A(I)+FM(1)/FM(4)*(C(I)-D(I)-E(I))-FM(1)/FM(5)*F(I)
  PPTN(I)=B(I)+FM(2)/FM(4)*(C(I)+D(I)+E(I))-FM(2)/FM(5)*F(I)
  PPTO2(I)=-A(I)+FM(3)/FM(4)*(D(I)-EP20*L(I)/2.)-FM(3)/FM(6)*(EP21*
  1N(I)-EP22*PP(I))
  PPTN2(I)=-B(I)-FM(8)/FM(4)*(E(I)+EP20*L(I)/2.)
  PPTNO(I)=-C(I)+D(I)+EP19*K(I)+E(I)+EP20*L(I)+FM(4)/FM(5)*EP22*
  1PP(I)
  PPTM(I)=FM(7)/FM(5)*F(I)+FM(7)/FM(4)*EP19*K(I)-FM(7)/FM(6)*EP21*
  1N(I)
  PPTO2M(I)=EP21*N(I)-EP22*PP(I)
  9 PPTNOP(I)=F(I)+FM(5)/FM(4)*EP19*K(I)-FM(5)/FM(6)*EP22*PP(I)
  DO 321 I=1,4
  DO 311 J=1,8
311 ALPHXN(J,I)=ALPHO(J)+(ALPHE(J)-ALPHO(J))*(6.*FN(I)**2-8.*FN(I)**3+
  13.*FN(I)**4)
  UXN(I)=UOE+(UOE(2)-UOE)*(6.*FN(I)**2-8.*FN(I)**3+3.*FN(I)**4)
  HXN(I)=HOE+(HOE(2)-HOE)*(10.*FN(I)**3-15.*FN(I)**4+6.*FN(I)**5)+
  1HNNO*.5*(FN(I)**2-3.*FN(I)**3+3.*FN(I)**4-FN(I)**5)
321 LHXN(I)=HXN(I)-.5*UXN(I)**2
  DO 33 I=1,4
  TBN=TOE
  IM=1
35 IF(ABS(TRN).LE.39.) TRN=39.
  TMP1=EXP(2274./TBN)-1.

```

K OF PARAGRAPH 4

L OF PARAGRAPH 4

N OF PARAGRAPH 4

P' OF PARAGRAPH 4

EQUATION 45
EQUATION 46
EQUATION 47

EQUATION 48
EQUATION 49

EQUATION 51
EQUATION 52
EQUATION 50

EQUATION 32

EQUATION 32
EQUATION 33

EQUATION 53

```

TMP2=EXP(3395./TBN)-1.
TMP3=EXP(2740./TBN)-1.
TMP4=EXP(1728./TBN)-1.
T2=(LHXN(1)-2800./TMP1*2274.*ALPHXN(3,1)-3200./TMP2*3395.*ALPHXN(8
1,1)-2965./TMP3*2740.*ALPHXN(4,1)-2985./TMP2*3395.*ALPHXN(5,1)-
22800./TMP4*1728.*ALPHXN(6,1)-1.663E8*ALPHXN(1,1)-3.62E8*ALPHXN(2,1
3)-3.28E7*ALPHXN(4,1)-3.536E8*ALPHXN(5,1)+1.429E7*ALPHXN(6,1))/
4(3.5*2800.*ALPHXN(3,1)+3.5*3200.*ALPHXN(8,1)+1.4E4*ALPHXN(1,1)+
51.6E4*ALPHXN(2,1)+3.5*2985.*ALPHXN(4,1)+3.5*2985.*ALPHXN(5,1)+
64.074E8*ALPHXN(7,1)+3.5*2800.*ALPHXN(6,1))
TEST=ABS(T2-TBN)/ABS(T2)
IF(TEST-.00001)33,33,36
36 GO TO(341,342),IM
341 TBNM1=TOE
TBN=T2
T1=T2
IM=2
GO TO 35
342 SA=(T2-T1)/(TBN-TBNM1)
SQ=SA/(SA-1.)
TBNP1=SQ*TBN+(1.-SQ)*T2
T1=T2
TBNM1=TBN
TBN=TBNP1
GO TO 35
33 TXN(1)=T2
P=(.0665*MIN**2*LD/X+1.)*PIN
DO 37 I=1,4
37 RHOEXN(1)=P/(8.9517E4*TXN(1)*(ALPHXN(3,1)/32.+ALPHXN(8,1)/28.+
ALPHXN(1,1)/16.+ALPHXN(2,1)/14.+ALPHXN(4,1)/30.+ALPHXN(5,1)/30.+
21820.*ALPHXN(7,1)+ALPHXN(6,1)/32.))
GO TO (38,39),MM
38 CALL IGPAT(0.,1.,.01,ANS,1)
THETA=(RHIN/RHOE(2))*CD/16.*LD**2)/(1.-210.*(P-PIN)/(RHOE(2)*UOE(2)
1**2*(1.-UOE/UOE(2))*(10.+11.*UOE/UOE(2)))*ANS)
39 DELM2=210.*THETA/(1.-UOE/UOE(2))*(10.+11.*UOE/UOE(2))
GO TO(391,392),MM
391 DELM21=DELM2
392 PPTSBO(1)=PPTO(1)
PPTSBE(1)=PPTO(2)
PPTSBO(2)=PPTN(1)
PPTSBE(2)=PPTN(2)
PPTSBO(3)=PPTO2(1)
PPTSBE(3)=PPTO2(2)

```

EQUATION 68

EQUATION 55

EQUATION 73

EQUATION 40

```

PPTSBO(4)=PPTNO(1)
PPTSBF(4)=PPTNO(2)
PPTSBO(5)=PPTNOP(1)
PPTSBF(5)=PPTNOP(2)
PPTSBO(6)=PPTO2M(1)
PPTSBE(6)=PPTO2M(2)
PPTSBO(7)=PPTM(1)
PPTSBE(7)=PPTM(2)
PPTSBO(8)=PPTN2(1)
PPTSBE(8)=PPTN2(2)
DO 40 I=1,8
DALPHO(I)=24./RHOE(2)*MUO/UOE*LEO/(DELM2*PRO)*(ALPHE(I)-ALPHO(I))
1+PPTSBO(I)/UOE
40 DALPHF(I)=PPTSRE(I)/UOE(2)
DO 41 I=1,6
YY=ALPHO(I)
DO 42 I=1,4
FK(I)=DX*(24./RHOE(2)*MUO/UOE*LEO/DELM2*(ALPHE(I)-YY)/PRO+PPTSBO(
I)/UOE)
GO TO (43,43,44,42),I
43 YY=ALPHO(I)+FK(I)/2.
GO TO 42
44 YY=ALPHO(I)+FK(I)
42 CONTINUE
41 ALPHO(I)=ALPHO(I)+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
ALPHO(7)=FM(7)*ALPHO(5)/FM(5)-FM(7)*ALPHO(6)/FM(6)
SUM=0.
DO 45 I=1,7
SUM=SUM+ALPHO(I)
ALPHO(8)=1.-SUM
DO 46 I=1,8
FK(1)=DX*PPTSRE(I)/UOE(2)
46 ALPHE(I)=ALPHE(I)+FK(I)
ALPHE(7)=FM(7)*ALPHE(5)/FM(5)-FM(7)*ALPHE(6)/FM(6)
SUM=0.
DO 47 I=1,7
SUM=SUM+ALPHE(I)
ALPHE(8)=1.-SUM
DPDX=-.0665* PIN*MIN**2*LD/X**2
YY=UOE
XX=X
DO 51 I=1,4
FK(I)=DX*(24./RHOE(2)*MUO/YY*(UOE(2)-YY)/DELM2+.0665/XX**2* PIN/
1RHOE*MIN**2/YY*LD)

```

EQUATION 36

EQUATION 38

dp/dx

du_0/dx

du_e/dx

```
GO TO (52,53,54,51),I
52 XX=X+DX/2.
   YY=UOF+FK(I)/2.
GO TO 51
53 XX=X+DX
   YY=UOF+FK(I)
51 CONTINUE
   DUODX=FK(I)/DX
GO TO(51,512),MM
511 UOE1=UOE
512 UOE=UOE+(FK(I)+2.*FK(2)+2.*FK(3)+FK(4))/6.
   XX=X
   YY=UOE(2)
DO 54 I=1,4
   FK(I)=.0665/RHOF(2)*PIN/YY*MIN**2/XX**2*LD*DX
GO TO (55,55,56,54),I
55 XX=X+DX/2.
   YY=UOF(2)+FK(I)/2.
GO TO 54
56 XX=X+DX
   YY=UOE(2)+FK(I)
54 CONTINUE
   DUEDX=FK(I)/DX
GO TO(54,542),MM
541 UOE1(2)=UOE(2)
   HNNO1=HNNO
   HOF(2)=HOF(2)
542 UOE(2)=UOE(2)+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
   YY=TOE(2)
DO 57 I=1,4
   TMP1=2274./YY
   TMP2=3395./YY
   TMP3=2740./YY
   TMP4=1728./YY
   TMP5=EXP(TMP1)-1.
   TMP6=EXP(TMP2)-1.
   TMP7=EXP(TMP3)-1.
   TMP8=EXP(TMP4)-1.
   FK(I)=-DX*YY*(1.118E-5*UOE(2)/YY*DUEDX+(7./2.*TMP1/TMP5)/32.*
1DALPHE(3)+(7./2.*TMP2/TMP6)/28.*DALPHE(8)+(7./60.*TMP3/(TMP7*30.)+
2366./YY)*DALPHE(4)+(7./60.*TMP2/(30.*TMP6)+3950./YY)*DALPHE(5)+
3(5./32.*+1853./YY)*DALPHE(1)+(7./64.*TMP4/(32.*TMP8)-159.3/YY)*
4DALPHE(6)+(5./28.*+4040./YY)*DALPHE(2)+4550.*DALPHE(7))/((TMP1/
5TMP5)**2+TMP1**2/TMP5+7./2.)*ALPHE(3)/32.+(TMP3/TMP7)**2+TMP3**2
6/TMP7+7./2.)*ALPHE(4)/30.+(TMP2/TMP6)**2+TMP2**2/TMP6+7./2.)*
7(ALPHE(8)/28.*+ALPHE(5)/30.)+(TMP4/TMP8)**2+TMP4**2/TMP8+7./2.)*
```

```

8 ALPHE(6)/32+.5./32.*ALPHE(1)+5./28.*ALPHE(2)+4550.*ALPHE(7))
GO TO (58,58,59,57),I
58 YY=TOF(2)+FK(I)/2.
GO TO 57
59 YY=TOF(2)+FK(I)
57 CONTINUE
DLTEDX=FK(1)/(DX*TOE(2))
TOE(2)=TOE(2)+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
SUM=0.
SUM1=0.
DO 60 I=1,8
SUM=SUM+DALPHE(I)/FM(I)
60 SUM1=SUM1+ALPHE(I)/FM(I)
YY=RHOE(2)
DO 61 I=1,4
FK(I)=DX*YY*(DPDX/P-DLTEDX-SUM/SUM1)
GO TO (62,62,63,61),I
62 YY=RHOE(2)+FK(I)/2.
GO TO 61
63 YY=RHOE(2)+FK(I)
61 CONTINUE
DLREDX=FK(1)/(DX*RHOE(2))
GO TO(61,612),MM
611 RHOE1(2)=RHOE(2)
HOE1=HOE
612 RHOE(2)=RHOE(2)+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
CALL IGRAT(0,1,01,ANS,1)
DEL=SQRT(2.*DELM2*ANS)
YY=THETA
DO 64 I=1,4
FK(I)=DX*(-YY*DLREDX+(2.*YY+DEL**2/2.-DELM2*(4.+UOE/UOE(2))/10.)/
1RHOE(2)*DPDX/UOE(2)**2)
GO TO (65,65,66,64),I
65 YY=THETA+FK(I)/2.
GO TO 64
66 YY=THETA+FK(I)
64 CONTINUE
DTHDX=FK(1)/DX
THETA=THETA+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
GO TO (67,68),MM
67 MM=2
DELTAC=4.*(HOE(2)-HOE1)*(349.*UOE1(2)+311.*UOE1)-HNNO1*(43.*UOE1(
12)+23.*UOE1)
68 HNNO=4.*(HOE(2)-HOE)*(349.*UOE(2)+311.*UOE)/(43.*UOE(2)+23.*UOE)-

```

$\frac{d}{dn} T_e / dx$, EQUATION 58

$\frac{d}{dn} \rho_e / dx$

EQUATION 72

$\frac{d\theta}{dx}$

EQUATION 44

EQUATION 43

```

1RHOE1(2)/RHOE(2)*DELM21/DELM2*DELTAC/(43.*UOE(2)+23.*UOE)
SUM=0.
CALL HSUR(TOE,HSUBS)
DO 69 I=1,8
69 SUM=SUM+HSURS(I)*ALPHE(I)
YY=HOE
DO 70 I=1,4
FK(I)=2./RHOE(2)*MUO/(UOE*DELM2)*(HNNO/PRO+12.*(PRO-1.)/PRO*UOE*
1(UOE(2)-UOE)+12.*(LEO-1.)/PRO*(SUM-YY+UOE**2/2.))*DX
GO TO (71,71,72,70),I
71 YY=HOF+FK(I)/2.
GO TO 70
72 YY=HOF+FK(I)
70 CONTINUE
HOE=HOE+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
TBN=TOE
IM=1
75 TMP11=EXP(2274./TBN)-1.
TMP22=EXP(3395./TBN)-1.
TMP33=EXP(2740./TBN)-1.
TMP44=EXP(1728./TBN)-1.
T2=(HOE-.5*UOE**2-2800./TMP11*2274.*ALPHO(3)-3200./TMP22*3395.*
1ALPHO(8))-2985./TMP33*2740.*ALPHO(4)-2985./TMP22*3395.*ALPHO(5)-
22800./TMP44*1728.*ALPHO(6)-1.663E8*ALPHO -3.62E8*ALPHO(2)-3.28E7*
3ALPHO(4)-3.536E8*ALPHO(5)+1.429E7*ALPHO(6))/(3.5*2800.*ALPHO(3)+
43.5*3200.*ALPHO(8)+1.4E4*ALPHO+1.6E4*ALPHO(2)+3.5*2985.*ALPHO(4)+
53.5*2985.*ALPHO(5)+4.074E8*ALPHO(7)+3.5*2800.*ALPHO(7))
TEST=ABS(T2-TBN)/ABS(T2)
IF (TEST-.00001)74,74,741
741 GO TO(742,743),IM
742 TBNM1=TOE
T1=T2
IM=2
GO TO 75
743 SA=(T2-T1)/(TBN-TBNM1)
SQ=SA/(SA-1.)
TBNP1=SQ*TBN+(1.-SQ)*T2
T1=T2
TBNM1=TBN
TBN=TBNP1
GO TO 75
74 TOE=T2

```

```

CALL HSUB( TOE(2), HSUBS )
SUM=0.
DO 76 I=1,8
76 SUM=SUM+ALPHE(1)*HSUBS(1)
HOE(2)=.5*UOE(2)**2+SUM
RHOE=1.23E-4/ZOE*P/TOF
X=STARTX+FNN*DX
IF( EXOUT.EQ.0.0 ) GO TO 90
--- INTERMEDIATE OUTPUT IF EXOUT IS NOT EQUAL TO ZERO ---
WRITE (6,119)
119 FORMAT(21H) INTERMEDIATE RESULTS)
WRITE(6,104) X, P, UOE, TOF, HOE
WRITE(6,104) ALPHO
WRITE(6,104) ALPHE
WRITE(6,104) DALPHO
WRITE(6,104) DALPHE
WRITE(6,104) A, B, C, D
WRITE(6,104) E, F, K, L
WRITE(6,104) N, MUO, PP, PPTO
WRITE(6,104) PPTN, PPT02, PPTN2, PPTNO
WRITE(6,104) PPTM, PPT02M, PPTNOP
WRITE(6,104) DPDX, DUODX, DUEDX, DLTEDX, THETA, DELM2
WRITE(6,104) DLREDX, DEL, DTHDX, DELTAC, S2B, XT, RHOE
WRITE(6,104) ROOTR(1), ROOTI(1), ROOTR(2), ROOTI(2), ROOTR(3), ROOTI(3)
WRITE(6,104) COE(1), COE(2), COE(3), COE(4), UXN
WRITE(6,104) HXN, LHXN
WRITE(6,104) ALPHXN
90 CONTINUE
FNN=FNN+1.
NN2=NN2+1
IF( NN2.NE.10 ) GO TO 77
NN2=0
WRITE (6,1060)
1060 FORMAT(3H0 X15X1HP15X2HU014X2HUE14X2HT014X2HTE14X2HHO14X2HHE)
WRITE(6,104) X,P,UOE,TOF,HOE
WRITE(6,103)
WRITE(6,104) ALPHO
WRITE(6,105)
WRITE(6,104) ALPHE
NN3=NN3+1
IF( NN3.NE.5 ) GO TO 77
NN3=0
WRITE(6,101)
77 IF( (X+.000001).GE.50. ) GO TO 1
GO TO 100
END

```

END PROGRAM

SUBROUTINES

SUBROUTINE HSUB

```

$IRFTC A5 LIST
SUBROUTINE HSUB(T,HSURS)
--
DIMENSION HSURS(8)
XP=2274./T
X=7./2.
HSURS(3)=2800.*T*(X+XP/(EXP(XP)-1.))
XP=3395./T
HSURS(8)=3200.*T*(X+XP/(EXP(XP)-1.))
HSURS(1)=1.4E4*T+1.663E8
HSURS(2)=1.6E4*T+3.62E8
XP=2740./T
HSURS(4)=2983.*T*(X+XP/(EXP(XP)-1.))+3.28E7
XP=3395./T
HSURS(5)=2985.*T*(X+XP/(EXP(XP)-1.))+3.536F8
HSURS(7)=4.074E8*T
XP=1728./T
HSURS(6)=2800.*T*(X+XP/(EXP(XP)-1.))-1.429F7
RETURN
END

```

SUBROUTINE INTEQS

```

$IBFTC A4 LIST
SUBROUTINE INTEQS(XX,YY,NOFO)
--*
COMMON FN(4),RHOEXN(4),RHOE(2)
NOEQ=NOEQ
CALL INTERP(XX,FN,RHOEXN,4,4,ANS,NERR)
YY=RHOE(2)*XX/ANS
RETURN
END

```

```

$IBFTC A1 LIST
SUBROUTINE IGRAT(LLIM,ULIM,DELTA,ANS,NOEQ)
--*
1 LLIM LOWER LIMIT OF INTEGRATION
2 ULIM UPPER LIMIT OF INTEGRATION
3 DELTA STEP SIZE FOR SOLUTION
4 ANS VALUE OF INTEGRAL
5 NOEQ ALL EQUATIONS TO BE INTEGRATED MUST BE
      COMPILED IN A SUBROUTINE CALLED INTEQS.
      NOEQ IS TO BE USED BY THIS SUBROUTINE
      TO SELECT THE PROPER EQUATION FOR
      INTEGRATION IE. NOEQ=1 WILL BE TO INTEGRAT
      EQUATION 1, NOEQ=2 TO INTEGRAT EQUATION
      2, ETC. THE FORM OF INTEQS MUST BE...
      INTEQS(X,Y,NOEQ) WHERE
          1 X INDEPENDENT VARIABLE
          2 Y DEPENDENT VAR. IE. Y=F(X)

```

3 NOEQ SAME AS ABOVE. PROBABLY
USED IN A COMPUTED GO TO.
10 SEPT 64
RALPH SELLERS

C
C
C

```

DIMENSION F(6),U(6),R(3)
DATA (U(I), I=1,6) / .11930959, -.11930959, .33060469, -.33060469,
A .46623476, -.46623476 /
DATA (R(I), I=1,3) / .23395697, .18038079, .85662246E-1
REAL LLIM
5 A = LLIM
DEL = DELTA
LAST = 1
ANS = 0.0
IF(ABS (ULIM-LLIM) - .00001)80,80,10
10 R = A + 4.0*DEL
IF(R-ULIM)40,30,20
20 B = ULIM
30 LAST = 2
40 DO 50 I=1,6
X = (R-A)*U(I) + .5*(A+B)
CALL INTEG(X,F(I),NOEQ)
50 CONTINUE
60 ANS = ANS + (B-A)*(R(1)*(F(1)+F(2)) + R(2)*(F(3)+F(4)) + R(3)*
A (F(5)+F(6)))
GO TO (70,80),LAST
70 A = B
GO TO 10
80 RETURN
END

```

```

$1BFTC A2 LIST
SUBROUTINE INTERP(ARG,XTAB,YTAB,NX,NP,ANS,NERR) SUBROUTINE INTERP
--
1. ARG THE INDEPENDENT VARIABLE FOR THE DESIRED
UNKNOWN.
2. XTAB TABLE OF INDEPENDENT VALUES. MUST BE IN
INCREASING ORDER.
3. YTAB TABLE OF DEPENDENT VALUES.
4. NX NUMBER OF POINTS IN XTAB.
5. NP NUMBER OF POINTS FOR INTERPOLATION.
6. ANS THE DEPENDENT VALUE CORRESPONDING TO THE
VALUE OF ARG.
7. NERR WILL BE SET UNEQUAL ZERO IF ARG IS NOT ON
XTAB. ROUTINE WILL EXTRAPOLATE.
10 SEPT 64
RALPH SELLERS

```

C
C
C
C
C
C
C
C
C
C
C
C
C

```

DIMENSION XTAB(NX),YTAB(NX)
NERR = 0
IH = NP/2
I = 1
IF (XTAB(I) - ARG)30,20,10
10 IH = 0
12 NERR = 01
GO TO 70
20 ANS = YTAB(I)
GO TO 999
30 I = NX
IF (XTAB(I) - ARG)12,20,50
50 L = IH + 1
DO 60 I=L,NX
IF (XTAB(I) - ARG)60,20,70
60 CONTINUE
70 K = I - IH
N = K + NP - 1
ANS = 0.0
IF (N - NX)90,90,80
80 N = NX
K = NX-NP+1
90 DO 120 J=K,N
P = 1.0
DO 110 I=K,N
IF (I-J)100,110,100
100 P = P * (ARG - XTAB(I)) / (XTAB(J) - XTAB(I))
110 CONTINUE
120 ANS = ANS + YTAB(J) * P
999 RETURN
END

$IRFTC A3 LIST
SUBROUTINE RTSOLV (COE,N1,ROOTR,ROOTI)
C
--
DIMENSION COE(51),ROOTR(50),ROOTI(50)
N2=N1 + 1
CTEBB=COE(1)
DO 1 I=1,N2
1 COE(I)=COE(I)/CTEBB
AN30D=ABS(COE(1))
DO 2 J=2,N2
IF (ABS(COE(J))>.GT.*AN30D) AN30D=ABS(COE(J))
FJD=FLOAT(J)
AKD=1.0/(FJD-1.0)

```

SUBROUTINE RTSOLV

```

TMD=AN30D**AKD
2 CONTINUE
DO 3 K=1,N2
  J=K-1
  3 COE(K)=COE(K)/(TMD**J)
  N4=0
  I=N1+1
  19 IF(COE(I))9,7,9
  7 N4=N4+1
  ROOTR(N4)=0.
  ROOTI(N4)=0.
  I=I-1
  IF(N4-N1)19,37,19
  9 CONTINUE
  10 AXR=0.8
  AXI=0.
  L=1
  N3=1
  ALP1R=AXR
  ALP1I=AXI
  M=1
  GO TO 99
  11 BET1R=TEMR
  BET1I=TEMI
  AXR=0.85
  ALP2R=AXR
  ALP2I=AXI
  M=2
  GO TO 99
  12 BET2R=TEMR
  BET2I=TEMI
  AXR=0.9
  ALP3R=AXR
  ALP3I=AXI
  M=3
  GO TO 99
  13 BET3R=TEMR
  BET3I=TEMI
  14 TE1=ALP1R-ALP3R
  TE2=ALP1I-ALP3I
  TE5=ALP3R-ALP2R
  TE6=ALP3I-ALP2I
  TEM=TE5*TE5+TE6*TE6
  TE3=(TE1*TE5+TE2*TE6)/TEM

```

```

TE4=(TE2*TE5-TE1*TE6)/TEM
TE7=TE3+1.
TE9=TE3*TE3-TE4*TE4
TE10=2.*TE3*TE4
DE15=TE7*8ET3R-TE4*8ET3I
DE16=TE7*8ET3I+TE4*8ET3R
TE11=TE3*8ET2R-TE4*8ET2I+8ET1R-DE15
TE12=TE3*8ET2I+TE4*8ET2R+8ET1I-DE16
TE7=TE9-1.
TE1=TE9*8ET2R-TE10*8ET2I
TE2=TE9*8ET2I+TE10*8ET2R
TE13=TE1-8ET1R-TE7*8ET3R+TE10*8ET3I
TE14=TE2-8ET1I-TE7*8ET3I-TE10*8ET3R
TE15=DE15*TE3-DE16*TE4
TE16=DE15*TE4+DE16*TE3
TE1=TE13*TE13-TE14*TE14-4.*(TE11*TE15-TE12*TE16)
TE2=2.*TE13*TE14-4.*(TE12*TE15+TE11*TE16)
TEM=SQRT(TE1*TE1+TE2*TE2)
IF(TE1)113,113,112
113 TE4=SQRT(.5*(TEM-TE1))
TE3=.5*TE2/TE4
GO TO 111
112 TE3=SQRT(.5*(TEM+TE1))
IF(TE2)110,200,200
110 TE3=-TE3
200 TE4=.5*TE2/TE3
111 TE7=TE13+TE3
TE8=TE14+TE4
TE9=TE13-TE3
TE10=TE14-TE4
TE1=2.*TE15
TE2=2.*TE16
IF(TE7*TE7+TE8*TE8-TE9*TE9-TE10*TE10)204,204,205
204 TE7=TE9
TE8=TE10
205 TEM=TE7*TE7+TE8*TE8
TE3=(TE1*TE7+TE2*TE8)/TEM
TE4=(TE2*TE7-TE1*TE8)/TEM
AXR=ALP3R+TE3*TE5-TE4*TE6
AXI=ALP3I+TE3*TE6+TE4*TE5
ALP4R=AXR
ALP4I=AXI

```

```

M=4
GO TO 99
15 N6=1
38 IF(ABS(HELL)+ABS(BELL)-1.E-20)18,18,16
16 TE7=ABS(ALP3R-AXR)+ABS(ALP3I-AXI)
   IF(TE7/(ABS(AXR)+ABS(AXI))-1.E-7)18,18,17
17 N3=N3+1
   ALP1R=ALP2R
   ALP1I=ALP2I
   ALP2R=ALP3R
   ALP2I=ALP3I
   ALP3R=ALP4R
   ALP3I=ALP4I
   BET1R=BET2R
   BET1I=BET2I
   RET2R=RET3R
   RET2I=RET3I
   BET3R=TEMR
   BET3I=TEMI
   IF(N3-100)14,18,18
18 N4=N4+1
   ROOTR(N4)=ALP4R
   ROOTI(N4)=ALP4I
   N3=0
41 IF(N4-N1)30,37,37
37 DO 5 L=1,N2
   ROOTR(L)=ROOTR(L)*TMD
   ROOTI(L)=ROOTI(L)*TMD
   DO 6 M=1,N2
     J=M-1
     COE(M)=(COE(M)*(TMD**J))*CTEBB
   RETURN
30 IF(ABS(ROOTI(N4))-1.E-5)10,10,31
31 GO TO(32,10),L
32 AXR=ALP1R
   AXI=-ALP1I
   ALP1I=-ALP1I
   M=5
GO TO 99
33 BET1R=TEMR
   BET1I=TEMI
   AXR=ALP2R
   AXI=-ALP2I
   ALP2I=-ALP2I
   M=6

```

```

GO TO 99
34 BET2R=TEMR
   BET2I=TEMI
   AXR=ALP3R
   AXI=-ALP3I
   ALP3I=-ALP3I
   L=2
   M=3
99 TEMR=COE(1)
   TEMI=0.0
   DO 100 I=1,N1
      TEL=TEMR*AXR-TEMI*AXI
      TEMI=TEMI*AXR+TEMR*AXI
100 TEMR= TEL+COE(I+1)
      HELL=TEMR
      RELL=TEMI
42 IF(N4)102,103,102
102 DO 101 I=1,N4
      TEM1=AXR-ROOTR(I)
      TEM2=AXI-ROOTI(I)
      TE1=TEM1*TEM1+TEM2*TEM2
      TE2=(TEMR*TEM1+TEMI*TEM2)/TE1
      TEMI=(TEM1*TEM1-TEMR*TEM2)/TE1
101 TEMR=TE2
103 GO TO(11,12,13,15,33,34),M
      END

```

Appendix B

SAMPLE CALCULATIONS

Sample cases involving a blunt body ($\phi_N = 90$ degrees) and a cone ($\phi_N = 20$ degrees) were run at 150 kilofeet and $M_\infty = 17.2$. The calculations began at $x = 3$ and carried through $x = 50$. As an example, the figure compares the axial temperature as a function of x for the blunt body and the cone. The transition from laminar to turbulent flow occurs sooner for the blunt body thereby initiating a rapid decay in axial temperature.

Input data for the cone are listed below. Velocities are in feet per second. Temperature is in degrees Kelvin. Enthalpies are in feet² per second². Altitude, diameter, and x are in feet. EXOUT determines the amount of output data. For minimum output, EXOUT = 0.0 or is omitted. If EXOUT $\neq$ 0.0, intermediate results at each $\Delta x = 0.1$ are printed as listed preceding statement 90 in Appendix A.

Card	Column							
	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
1	$a_0(O)$ 0.232	$a_0(N)$ 0.0132	$a_0(O_2)$ 6.96 E-4	$a_0(NO)$ 6.08 E-3	$a_0(NO^+)$ 1.86 E-5	$a_0(O_2^+)$ 0.0	$a_0(e^-)$ 3.41 E-10	$a_0(N_2)$ 0.748
2	$a_e(O_2)$ 0.232	$a_e(N_2)$ 0.768	$(u_0)_c$ 13,079.0	H_0 1.81646 E8	r_e 1.81646 E8	T_0 4030.0	ϕ_n 20.0	ALT 150,000.0
3	Z_0 1.217	Hnn_0 0.0	Pr_0 1.0	Le_0 1.0	CD 0.104	d 1.0	u_∞ 18,900.0	X_c 3.0
4	Stop X 50.0	EXOUT 0.0						

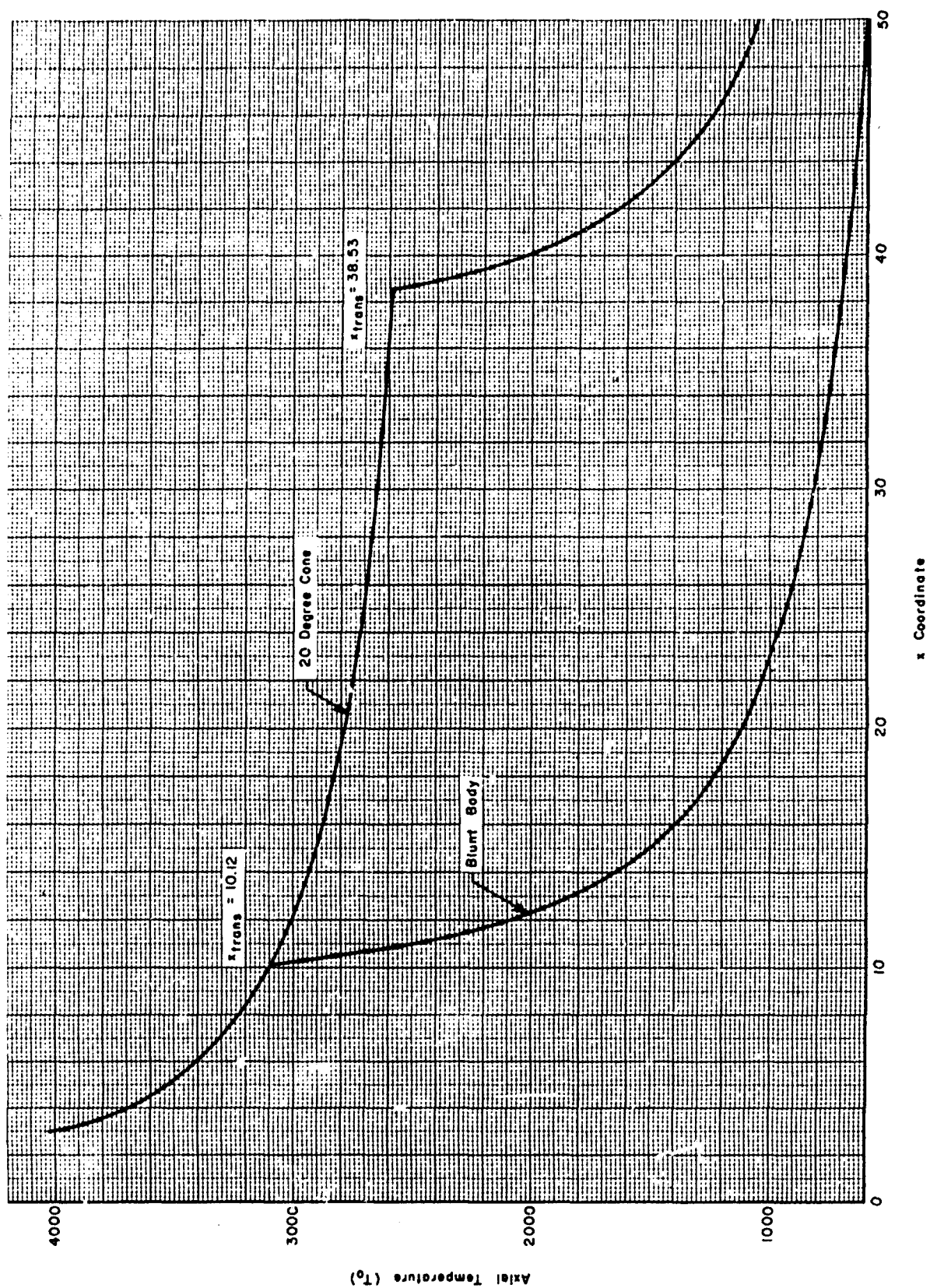


Figure. Axial Temperature Profile ($^{\circ}\text{K}$)

INITIAL CONDITIONS

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X	P	UO	UE	TO	TE	HO	HE
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.23426126E 00	0.11170580E-01	0.20670090E-03	0.27530917E-02	0.21923736E-04	0.22341484E-13	0.39587858E-10	0.75158644E 00
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.97365092E-35	0.42125975E-31	0.23200000E 00	0.18255955E-34	0.	0.	0.	0.76800000E 00
X	P	UO	UE	TO	TE	HO	HE
0.69999999E 01	0.11321448E 02	0.13761912E 05	0.18927205E 05	0.33050843E 04	0.21766693E 03	0.18164585E 09	0.18148677E 09
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.23460232E 00	0.10858068E-01	0.16190571E-03	0.21971828E-02	0.21931233E-04	0.16202855E-13	0.39601379E-10	0.75215858E 00
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.11865357E-34	0.54363545E-31	0.23200000E 00	0.22247619E-34	0.	0.	0.	0.76800000E 00
X	P	UO	UE	TO	TE	HO	HE
0.79999999E 01	0.10248358E 02	0.13848211E 05	0.18930664E 05	0.32246682E 04	0.21155480E 03	0.18164581E 09	0.18148574E 09
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.23485154E 00	0.10626555E-01	0.13172096E-03	0.17862590E-02	0.21797786E-04	0.12105054E-13	0.39360393E-10	0.75258213E 00
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.13414066E-34	0.66056275E-31	0.23200000E 00	0.23151374E-34	0.	0.	0.	0.76800000E 00
X	P	UO	UE	TO	TE	HO	HE
0.49000000E 02	0.40402319E 01	0.17724082E 05	0.18958918E 05	0.10952071E 04	0.16183112E 03	0.18148260E 09	0.18148023E 09
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.69008109E-01	0.31645631E-04	0.16160735E 00	0.46615661E-02	0.36367753E-05	0.19062316E-09	0.65666243E-11	0.76468769E 00
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.21115285E-34	0.42227179E-30	0.23200000E 00	0.39591153E-34	0.	0.	0.	0.76800000E 00
X	P	UO	UE	TO	TE	HO	HE
0.50000000E 02	0.40162599E 01	0.17785059E 05	0.18959076E 05	0.10549995E 04	0.16155300E 03	0.18148222E 09	0.18148019E 09
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.65683020E-01	0.21694389E-04	0.16499310E 00	0.44480612E-02	0.34021324E-05	0.22784827E-09	0.61428651E-11	0.76485072E 00
ALPHA(O)	ALPHA(N)	ALPHA(O2)	ALPHA(N)	ALPHA(N)	ALPHA(O2)	ALPHA(O2)	ALPHA(N2)
0.21155599E-34	0.42997908E-30	0.23200000E 00	0.39666741E-34	0.	0.	0.	0.76800000E 00