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RE-ENTRY HEATING SIMULATION (U)

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U. S. NAVAL ORDNANCE LABORATORY

WHITE OAK, MARYLAND

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RE-ENTRY HEATING SIMULATION

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ABSTRACT: This report describes a method of calculating aerodynamic heating for the purpose of evaluating materials for use as possible re-entry heat shields. The procedure may be described as a simplified re-entry flight simulation. This report gives in detail the assumptions made, equations used, and the form of presentation of the results. It does not give the results of such calculations.

U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

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This report is devoted to the presentation of a method of calculation used in the study of the feasibility of various heat shield materials for atmospheric re-entry.

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Captain, USN
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By direction

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RE-ENTRY HEATING SIMULATION

INTRODUCTION

1. This report presents the calculation procedure in use at the present time at this Laboratory to determine heat transfer and heat conduction on missile re-entry with the purpose of evaluating various materials as heat shields.

The basic philosophy of these calculations is to compute simulated re-entry flights for each material under investigation. It is assumed for this purpose, that the dependence of re-entry velocity and direction is adequately given by assuming an optimal elliptic space trajectory on a non-rotating earth. Further, that the re-entry path is a straight line and that only the drag is significant. The boundary layer flow around the body is that for a perfect gas with laminar, transition, and turbulent regions as described in reference (2). The coefficient of heat transfer, h, at the stagnation point is based on the properties of the air at the surface. At other points on the body, h is taken as approximately independent of wall temperature. The body is assumed to be a hemisphere-cylinder. The heat conduction into the solid surface is taken to be one-dimensional, normal to the wall. The heat conduction is calculated by a direct finite difference method taking into account the variation of thermal properties with temperature. Each such calculation thus furnishes for a given flight the time history of the temperature and heat flux to a chosen point on the body. Thus the peak temperature in particular is obtained for a given flight, material, shield thickness, and body point. Hence many such calculations furnish the variation in peak temperatures in terms of the flight parameters, thermal properties, and thickness.

2. Vacuum Trajectory

The heating of any re-entry body depends basically upon its weight loading, W/C_DA, (or equivalently upon $K=C_DA\, g/W$), its re-entry velocity V_E, and re-entry angle θ_E. In the following the ballistic factor of Allen and Eggers, reference (1) will be denoted by $K_\theta=K/\sin\theta_E$, (i.e., "K" based on θ) to distinguish it from the K used in this report.

For unpowered flight in free space the trajectory is taken to be a minimum energy ellipse for a non-rotating spherical Earth. This reduces the specification of the two parameters V_E and θ_E to the single parameter, R, the range. One has

$$V_E^2 = U^2 \frac{\sin \nu}{1 + \sin\nu}, \qquad (2.1)$$

(2.2) $$\theta_E = \frac{\pi}{4} - \frac{\nu}{2} \quad ,$$

where ν is the angle subtended at the center of the Earth by $\frac{1}{2}R$, i.e., the semi-range angle, and U is the parabolic velocity for the Earth.

3. Re-entry Trajectory

It is considered adequate to use a straight line re-entry trajectory, inclined at the angle θ_E, for heating calculations. Taking the drag as $\frac{1}{2}C_D A \rho V^2$, the equations of motion are easily written as

(3.1) $$dH/dt = -V \sin \theta_E,$$

(3.2) $$d^2H/dt^2 = - K_\rho \rho (V \sin \theta_E)^2,$$

where V is the velocity, and H the height along the path as functions of the time, t, and ρ is the density of the atmosphere. For an isothermal atmosphere

(3.3) $$\rho = \rho_{SL} e^{-H/H_R} \quad ,$$

these equations are readily integrated in closed form. However, to obtain results for a sufficient number of time intervals to use for heat conduction calculations, it is just as useful to use a Runge-Kutta numerical integration. The sea level density, $\rho_{SL} = .00259$ slug ft^3, and $H_R = 30,500$ ft. It should be mentioned here that the atmosphere is assumed isothermal only in the trajectory calculation, not in the heat calculation. The choice of 30,500 feet for H_R was based on the fact that at this height the density of the ARDC atmosphere is 1/e-th of ρ_{SL}. It has more recently become clear that computed values of heat transfer based on this value are much too low. In fact the heat transfer is quite sensitive to the value of H_R. A better choice can be made after the computations are performed by noting at what altitude maximum heating approximately occurs and then choosing ρ_{SL} and H_R so that a best fit to the true density is obtained at this height.

4. Stagnation Point Heat Transfer

At any given height along the trajectory the velocity V (denoted in this section by u_∞) is known from the trajectory calculation. The free stream pressure, P_∞, and temperature, T_∞, are obtained from a polynomial fit to atmosphere data, see Appendix. Next the density, sound speed, and Mach number are obtained from

(4.1) $$\rho_\infty = P_\infty / R\,T_\infty$$

(4.2) $$a_\infty = \sqrt{\gamma R\,T_\infty}$$

(4.3) $$M_\infty = u_\infty / a_\infty \,.$$

R, the gas constant for air is taken here as 1716 $ft^2/sec^2\ {}^oF$, and γ (the specific heat ratio) is taken as 1.4.

Next it is assumed that the flow over the body is that of a perfect gas and that the stagnation temperature is given by the adiabatic formula

(4.4) $$T_o' = T_\infty\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)$$

Further, the losses through the bow shock yield a stagnation pressure given by the Rayleigh formula

(4.5) $$P_o' = P_\infty\left[\frac{(\gamma+1)}{2} M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{2\gamma M_\infty^2-(\gamma-1)}{\gamma+1}\right]^{-\frac{1}{\gamma-1}}$$

for the supersonic portion of the re-entry and the adiabatic formula

(4.6) $$P_o' = P_o = P_\infty\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}$$

for the subsonic portion.

Finally the heat transfer coefficient, h, is given by

(4.7) $$h = .763(Pr)^{.4}\,\frac{k_W}{\nu_W^{1/2}}\,\frac{(1.1)^{1/2}}{D^{1/2}}\,u_\infty^{1/2} \,,$$

where .763 is Sibulkin's constant for an axial symmetric body, Pr the Prandtl number, is taken as .72, D is twice the nose radius of curvature, taken as two feet in our calculations, and the constant 1.1 is the value of the dimensionless velocity gradient at the stagnation point. The particular value taken here is based on the experiments of Korobkin, reference (3).

The wall values, k_w and ν_w for air, are computed from the Sutherland Law, with

(4.8) $$\nu_w = \mu_w/\rho_w \quad . \quad \rho_w = P_o'/R\,T_w$$

(4.9) $$k_w = 2.445 \times 10^{-7} \frac{T_w^{3/2}}{T_w + T_{SU}} \; , \quad T_{SU} = 198.6^{\circ}R$$

(4.10) $$\mu_w = 2.27 \times 10^{-8} \frac{T_w^{3/2}}{T_w + T_{SU}} \; .$$

With h found, the heat transfer and heat conduction phases of the calculation are coupled through the relation for the heat flux, q_{BL}', using

(4.11) $$q_{BL}' = h(T_o' - T_w) - \sigma\epsilon T_w^4 \; ,$$

where T_w is the outer surface or wall temperature, $\sigma = 4.8\text{x}10^{-13}$, the Stefan–Boltzmann constant, and ϵ is the emissivity of the wall material.

5. Heat Conduction

The heat conduction calculation proper as distinguished from the heat transfer calculation assumes a one-dimensional flow of heat normal to the surface according to the usual Fourier equation which may be written as

(5.1) $$C(T)\rho \frac{\partial T}{\partial t} = -\frac{\partial q}{\partial y} \; , \quad q = -K(T)\frac{\partial T}{\partial y} \; ;$$

where ρ, assumed constant, is the density of the material, $C(T)$ is the specific heat at constant pressure, as a function of temperature, T; $K(T)$ is the thermal conductivity; q is the heat flux at each depth, y, measured from the outer wall surface; and, t, is the time. At the initial time, $t = 0$, an initial temperature $T(y)$ is assumed known, (e.g., $T = 460^{\circ}$Rankine) and at subsequent times the equation is to be solved with the boundary conditions at the outside and inside walls stated for given flux, i.e.,

$$(5.2) \qquad \begin{aligned} q(0,t) &= q'_{BL}(t), \quad \text{at } y = 0 \\ q(x,t) &= 0 \quad\quad\;\; , \quad \text{at } y = x \end{aligned}$$

where x is the total thickness of the shield.

In cases where several layers of different materials are considered, equation (5.1) becomes subscripted with C_i, ρ_i, K_i, where $i = 1,2,\ldots,M$ for each of M materials; thus separate tables $K_i(T)$, $C_i(T)$ of material properties must be used for each material in the respective layers, i. At the interface between two such layers one must demand that the flux across the interface from the left equal the flux out to the right, i.e., $q_L = q_R$.

Equation (5.1) is solved by an implicit first order finite difference method. In finite difference form the heat balance at the j-th point of some finite number of discrete points may be written in terms of the neighboring points $j-1$ and $j+1$ as

$$(5.3) \qquad C_j \rho_j \frac{\overset{n}{T}_j - \overset{o}{T}_j}{t} = - \frac{q_{j+1/2} - q_{j-1/2}}{\Delta y} \; ,$$

$$q_{j-1/2} = - \frac{K_j + K_{j-1}}{2} \; \frac{\overset{n}{T}_j - \overset{n}{T}_{j-1}}{\Delta y} \; ,$$

where the super, o, stands for "old", i.e., at time t, and the super, n, stands for "new", i.e., at time $t + \Delta t$.

At any instant, (5.3) is seen to constitute a system of equations for the new unknown temperatures in terms of the known old temperatures. This system can be assumed linear if one regards the K's as slowly varying functions of T, and hence be evaluated for $\overset{o}{T}$. Two of the equations in the system (5.3) must have a different form, namely at the inner and outer boundaries, $j=0$ and $j=N$ for $N+1$ points. These are taken as

$$(5.3a) \qquad C_N \rho_N \frac{\overset{n}{T}_N - \overset{o}{T}_N}{\Delta t} = - \frac{q'_{BL} - q_{N-1/2}}{1/2 \Delta y} \; , \qquad T_N = T_W$$

$$(5.3b) \qquad C_o \rho_o \frac{\overset{n}{T}_o - \overset{o}{T}_o}{t} = - \frac{q_{1/2} - q_o}{1/2 \Delta y} \; , \qquad q_o = 0 \; .$$

The flux from the boundary layer, q_{BL}, less the radiative flux, is given by

$$(5.4)\qquad q'_{BL} = q_{BL} - \sigma e \overset{n}{T}_W^{4} ,$$

$$q_{BL} = h(T'_o - \overset{n}{T}_W) ,$$

where h is the boundary layer heat transfer coefficient, and T'_o is the stagnation temperature.

It is to be noticed that q'_{BL} in (5.4) is not known until the new wall temperature $\overset{n}{T}_W$ has been found, but the system (5.3), (5.3a), (5.3b) cannot be solved until q'_{BL} is prescribed; thus an iterative procedure is resorted to, using $\overset{o}{T}_W$ in (5.4) as a first approximation to $\overset{n}{T}_W$. Then the temperatures $\overset{n}{T}_j$ including $\overset{n}{T}_W$ so found can be used to enter (5.4) again. This is repeated three or four times until the difference $|\overset{n}{T}_W^{(k)} - \overset{n}{T}_W^{(k-1)}|$ between the k-th and (k-1)st iterations is less than some chosen value (1/10 degree Rankine in most cases). If the process fails to converge, or is converging so slowly that more than 10 iterations will be required, the program starts over with the old values $\overset{o}{T}_j$ and uses a new Δt one half the old one. If repeated attempts lead to $\Delta t < 1/1000$ second, the case is stopped.

6. Maximum Heating Point Heat Transfer

It is to be recognized that although the stagnation point on a body is normally expected to be a place where aerodynamic heating is most severe, i.e., is locally a point of maximum heating, other points or zones may actually be heated more because of poor shape (e.g., extreme curvature or corners) or because the boundary layer flow may become and remain turbulent for an appreciable time. A detailed study of the heating history at every point on the body may be obtained by performing a boundary layer calculation on the body for every step in the time history of the simulated flight. Such calculations have been made at this Laboratory using the boundary layer formulation given in the report of J. Persh, reference (2). The details will be omitted here. It must be noted that this method utilizes a heat transfer coefficient, h, analogous to the stagnation point value given by (4.7) in that it depends on the wall values K_W, ν_W which in turn depend on the unknown wall temperature T_W. The variation of h with T_W over the range $T_{su} < T_W < 16\ T_{su}$ can be seen from equation (4.7) to be at most 15 per cent. This fact makes it possible to deal with an h based on some constant T_W in investigations where errors of 15 per cent are satisfactory. This assumption of an h, independent of wall temperature, was adopted so that detailed boundary layer calculations could be made for many trajectories independently of heat conduction calculations. Thus it became possible to show at what point in most cases the peak heating would be expected to occur. With this information one can then use the values of $h(t)$ at this point alone, together with the adiabatic temperature, T_{ad}, to obtain the heat flux q'_{BL}. T_{ad} is obtained from an equation analogous to equation (4.4), namely the relation,

$$T_{ad} = T_e\left(1 + \frac{\gamma-1}{2} M_e^2\right) , \tag{6.1}$$

where T_e, M_e are temperature and Mach number external to the boundary layer at the body point in question. The heat flux at this body point is then calculated as part of the heat conduction calculation described in paragraph 5 using the equation analogous to (5.4), namely,

$$q'_{BL} = h(T_{ad} - T_W) - \sigma\epsilon T_W^4 \quad . \tag{6.2}$$

7. Calculation Procedure

Calculations of this type have been performed on the IBM 704 electronic calculator using the FORTRAN coding system. For this purpose two main programs were written: First a boundary layer program including sub-programs for the vacuum trajectory, and the re-entry trajectory. This uses as input the parameters, W/C_DA, and R, and furnishes as output a deck of cards containing h and T_{ad} at one given body point (at or near maximum heating point) for each time step down the trajectory. Second, a heat conduction program including sub-programs for either computing stagnation point heat transfer, or for reading cards of h and T_{ad} as input. The latter program in addition, has as input the thermal properties $K(T)$, $C_p(T)$, ρ, ϵ, and the shield thickness x, or several sets of properties and thicknesses if the shield is a layer composite.

To evaluate a single layer material, separate runs on the machine are made with a very thin layer (effectively of zero thickness) for different loading factors, W/C_DA, and the peak wall temperature found for each W/C_DA. From this information one can readily interpolate by hand to find that value of W/C_DA for which the wall would just break down or melt if the temperature T_m, of thermal instability is known. This yields the highest value of W/C_DA, for a given range, for which the material can be permitted to reach radiative equilibrium with the atmosphere. This value actually depends only upon the emissivity and the melting point, T_m.

If this calculation be repeated for several ranges, R, one obtains on a plot of R versus W/C_DA a curve that may be referred to as a zero thickness (or zero shield weight) curve.

The same type of calculation is then repeated for a large thickness (effectively an infinite thickness) and one obtains a curve that may be referred to as an infinite thickness (or weight) curve. This curve shows those pairs of values of R and W/C_DA for which the maximum heat sink abilities (if any) of the material are utilized and represent the ultimate performance the material is capable of exhibiting.

Intermediate curves for given shield weight per unit area or for given thickness may be calculated in the same way.

APPENDIX

Formulas used to approximate a standard atmosphere:

(A.1) $$T_\infty = a_o - a_1 H + a_2 H^2 - a_3 H^3 + a_4 H^4 - a_5 H^5 ,$$

where T_∞ is the temperature in degrees Kelvin at height H in kms. The values for $H < 15.7$ km are

$a_o = 292.943$	$a_3 = 0.237434$
$a_1 = 6.70606$	$a_4 = 0.0174843$
$a_2 = 1.08387$	$a_5 = 0.00408653$

For $H \geqslant 15.7$ km the values used are

$a_o = 295.457$	$a_3 = 0.014460238$
$a_1 = 14.158767$	$a_4 = 1.22412 \cdot 10^{-4}$
$a_2 = 0.74888548$	$a_5 = 2.8784 \cdot 10^{-7}$

The pressure, P_∞ in lbs. per ft^2 is given by

(A.2) $$P_\infty = 10332.2 \cdot 10^x$$

$$x = -0.014 - 0.06104H + (0.064 + .00024H) x \sin(\frac{18-H}{S})$$

$$S = 9.549296 \text{ for } H < 18 \text{ km.}$$

$$S = 6.162149 \text{ for } H \geqslant 18 \text{ km.}$$

This fit was an interim fit obtained from the Army Ballistic Missile Agency in 1957 and used for these calculations until August 1959.

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