NAVAL AIR DEVELOPMENT CENTER

Johnsville, Warminster, Pennsylvania

REPORT NO. NADC-AE-6709 18 APR 1967

DIRECTIONAL CHARACTERISTICS OF SLOTTED PIEZOELECTRIC CERAMIC TUBES

FINAL REPORT
FOUNDATIONAL RESEARCH TASK NO. ZR011-01-01
Work Unit No. EY-5-01

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DEPARTMENT OF THE NAVY
U. S. NAVAL AIR DEVELOPMENT CENTER
JOHNSVILLE
WARMINSTER, PA. 18974

Aero-Electronic Technology Department

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Single element and line transducers of various dimensions were fabricated from radially polarized, slotted piezoelectric ceramic tubes. The tubes exhibited multilobe directivity characteristics, in a plane normal to the axis of the tubes, at certain discrete frequencies. The shape of the directivity patterns are explained in terms of circumferential extensional vibrations with geometrically determined nodal positions. In addition, unidirectional response patterns were found at frequencies that were below the fundamental breathing mode of the tubes.

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SUMMARY

INTRODUCTION

This report presents the results of an experimental investigation of the directional response characteristics displayed by thin-walled piezoelectric ceramic tubes with a finite discontinuity in the shell. It has been observed that line transducers employing cylindrical elements showed deviations from the expected axial symmetry in their acoustic response patterns, at certain discrete frequencies, if the elements had been damaged by cracking. These deviations appeared in the receiving patterns as varying degrees of directionality. If the cause of the directional effects could be determined, this information could be of use in the design of relatively small directional transducers. Consequentially, an experimental research project was initiated to investigate these directional effects by slotting ceramic tube transducers along their length and measuring their acoustical characteristics.

SUMMARY OF RESULTS

Slotted cylindrical tube transducers of various dimensions were fabricated and their acoustic properties measured over a frequency range of 3 to 80 khz. Several trends in the behavior of the tubes were recognizable. First, the tubes exhibited multilobe directivity characteristics, in a plane normal to their longitudinal axis, at certain discrete frequencies. These frequencies have been shown to be related to the circumferential extensional mode of vibration.

Secondly, the experiments established that the directivity patterns were unidirectional with maximum response occurring 180 degrees away from the slot at frequenices corresponding to the radial extensional mode of vibration of the tubes. The third trend observed indicated that the directional patterns existed at certain frequencies that were well below those frequencies corresponding to the radial resonance of the tubes.

CONCLUSIONS

1. Slotted tube transducers which provide directional characteristics, at particular frequencies, can be designed from mechanical resonance information. These transducers can produce directional beam patterns without the use of reflectors or large transducer arrays. Since these tubes are open to water on all surfaces (free flooding), they are not affected by large hydrostatic pressures. Designs of this type might be useful in the construction of deep operating, lightweight directional transducers.

2. Unidirectional patterns occurred at the frequency of radial extensional resonance of the tubes. Front-to-back ratios ranged in values between 10 and 30 decibels. Although the value of the front-to-back

ratio can not be predicted accurately, these unidirectional characteristics may be useful in applications where directionality is required at only one frequency and over a relatively narrow bandwidth.

3. In the case of circumferential extensional resonances in complete tubes, the positions of the radial nodes of vibration are determined by the point of application of the driving force. Experimental measurements made on slotted tubes show that the radial nodes for these tubes are determined by the position of the slot and are independent of the position of the driving force.

RECOMMENDATIONS

It is recommended that further investigation of the directional characteristics of slotted tubes, at the frequencies below their radial extensional resonance, be carried out to determine the mode of vibration which causes the unidirectional effects at these frequencies. If this knowledge is obtained, it may be possible to design small lightweight transducers that would provide consistent and predictable directional characteristics at relatively low frequencies.

It is also recommended that the existence of circumferential extensional vibrations with geometrically determined nodal positions in slotted tubes be verified by investigating the extensional vibrations of incomplete metal rings and tubes.

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LIST OF FIGURES

LIST OF SYMBOLS

A = a constant determined by the energy input

R = the mean radius of the tube

U_r = radial component of displacement of any point on the tube

U_θ = tangential component of such displacement

c = compressional velocity of sound in the material of the tube

f = frequency

n = integer equal to one half the number of nodes

θ = the angle, measured from midway between two radial nodes

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APPARATUS AND PROCEDURE

PIEZOELECTRIC CERAMICS USED

Twenty single element slotted tubes and two line transducers were used in the investigation. Eighteen of the tubes were made from a barium titanate compound containing 4 percent by weight of lead titanate. Two of the tubes were made from a lead-titanate, lead-zirconate compound which is available commercially under the trade name of "Electramite 1400."

The slots extended over the length of the tubes and through the full thickness of the shells as shown in figure 1. Each tube had silver electrodes on its inside and outside lateral surfaces and polarization was everywhere radial. All tubes were waterproofed completely by encapsulating them in polyurethane or vinyl, and they were used in a free-flooded condition.

The two line transducers were made from barium-titanate tubes. One of these used eight tubes, each being 1-3/8 inches in diameter by 1-3/8 inches long with a wall thickness and slot width of 1/8 inch. A line transducer consisting of five tubes, each being 2 inches in diameter by 1-3/4 inches long with a wall thickness and slot width of 1/8 inch was also tested. In both cases the tubes were joined end to end, with the slots aligned, and were isolated mechanically from one another as shown in figure 2. The ends of the lines were open, allowing the center to flood with water.

Sixteen of the single element tubes were used to investigate the acoustic behavior as a function of the diameter, length, wall thickness, and slot width. The tubes were divided into sets of four, each set possessing identical dimensions except for the one under investigation. Diameters varied from 2 to 6 inches, lengths from 1/2 inch to 4 inches, wall thicknesses from 1/8 inch to 1/2 inch, and slot widths from 1/8 inch to 1 inch.

EXPERIMENTAL MEASUREMENTS

The acoustic characteristics of the slotted tube transducers were measured in the sonar calibration tanks at the U.S. Naval Air Development Center, Johnsville, Warminster, Pennsylvania. These measurements included receiving response as a hydrophone, transmitting response as a projector, impedance under water load, and directivity patterns. Those measurements of particular interest in this investigation were the directivity patterns that express the ratio in db of the response in any direction to the response in the direction of maximum sensitivity. For a receiver or hydrophone, the response is the open circuit voltage out of the transducer and for a transmitter or projector the response is the acoustic pressure measured at a distance of 1 yard from the projector.

Most electro-acoustic transducers are reciprocal in that they obey the reciprocity principle. These transducers have the same directivity patterns for transmission as reception. Measurements have shown that slotted tube transducers are reciprocal. In addition to the measurements listed above, the "In Vacuo" resonant frequencies of the slotted tubes were measured on a vector impedance locus plotter.

Orientation of the tubes for the acoustic measurements was such that the Z-axis of a left-handed coordinate system would coincide with an axis through the center of the tube. Patterns were then measured in the XY-plane. The tube transducers were suspended at a mean depth of 6 feet below the surface of the water. Acoustic measurements quoted in this report were obtained using a pulsed-cw technique[1].

RESULTS AND DISCUSSION

MULTILOBE DIRECTIVITY PATTERNS

1. Background

The slotted tubes exhibited multilobe directivity patterns at certain discrete frequencies. Experimental measurements of resonant frequencies and mode shapes indicate that the patterns are due to circumferential extensional vibrations.

The lack of axial symmetry due to the discontinuity in the shell makes an analytical analysis of the extensional vibrations of slotted tubes extremely difficult. Analysis for a closed or complete tube, under certain limiting conditions, is treated in several classical works[2] [3] [4] [5].

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1. *Wallace, John D. and McMarrow, E. W., Jan 1961; Sonar Transducer Pulse Calibration System; JASA, Vol 33, No. 1, p 75-84.*
2. *Love, A. E. H., 1943; A Treatise on the Mathematical Theory of Elasticity; Cambridge University Press, London; p 287-292, 451-454.*
3. *Rayleigh, J. W. S., 1945; The Theory of Sound; Dover Publications, New York; p 304-305, 383-388.*
4. *Lamb, H., 1960; The Dynamical Theory of Sound; Dover Publications, New York; p 135-140.*
5. *Kuhl, W., 1942; Measurements for the Theory of Natural Oscillations of Circular Rings of Arbitrary Thickness; AKUSTISCHE ZEITSCHRIFT, Vol 7, p 125-152.*

When a complete tube vibrates in one of its circumferential extensional modes, one section of the tube elongates circumferentially while the adjacent section contracts. The number of sections and their position relative to the position of the driving force depends on the particular mode of vibration. This type of motion is oscillatory and can be described by considering that the middle surface of "central line" of the tube alternately undergoes extension and compression in equal amounts, Thus, the mean circumference of the tube remains unchanged so that the circumferential vibrations are not coupled with longitudinal vibrations and are therefore independent of the length of the tube.

The resonant frequency of a closed thin-walled tube of small axial height vibrating in a circumferential extensional mode is given by Love[2] as:

$$f = \frac{c}{2\pi R} \sqrt{n^2 + 1} \qquad (1)$$

f = the resonant frequency

R = the mean radius of the tube

c = compressional velocity of sound in the material of the tube

n = integer equal to one half the number of nodes

This expression indicates that the resonant frequencies depend only on the mean radius and the velocity of sound in the tube.

In the case of circumferential extensional modes of vibration, displacements are predominantly tangential with a radial component occurring because of transverse contraction. Expressions for the radial and tangential components of vibration are given by:

$$U_r = [A \cos n\ \theta] e^{jwt} \qquad (2)$$

$$U_\theta = [-A\ n \sin \theta] e^{jwt}$$

Where

U_r = the radial component of displacement of any point on the tube

U_θ = tangential component of such displacement

A = a constant determined by the energy input

n = zero or an integer, corresponding to the modes

θ = the angle, measured from midway between two radial nodes or from the point of application of a radial driving force

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[2] *See pg 2.*

When $n = 0$, the tangential component U_θ is zero and the radial component U_r is independent of θ. Thus, the vibrations reduce to the radial extensional case, where the motion at all points on the tube is in phase, equal, and radial. This mode is sometimes called the "hoop" or "breathing" mode and will be discussed in more detail in a later portion of the report. For higher order modes, the radial component U_r, varies sinusoidally with θ and all portions of the ring do not vibrate in phase. When $n = 1$ for example, nodes of radial motion occur at $\theta = 90°$ and 270°, and one-half the tube vibrates in opposite phase to the other half. For $n = 2$, the nodes of radial motion occur at $\theta = 45°$, 135°, 225°, 315°, and the tube vibrates in four sections. An exaggerated representation of the way the tube vibrates at a given instant of time is shown in figures 3 and 4.

2. Acoustical Properties Predictable

When a free flooded, thin walled, piezoelectric tube of small axial height is used as an underwater acoustic transducer, the acoustic coupling to the water is provided by the radial component of displacement. For tubes vibrating in the circumferential modes, the change in phase of the vibration at each radial node should produce a minimum in the response at each of these points in the directivity patterns. The directivity patterns predicted for the $n = 1$ and $n = 2$ modes discussed previously would have the forms shown in figure 5.

Thus, if a tube transducer is made to vibrate in a circumferential extensional mode, it is possible to predict analytically the shape of the directivity pattern and the approximate frequency at which it will occur. In addition, the equations representing the radial component of displacement show that the distribution of this displacement does not depend on wall thickness. Rayleigh[3] has shown from energy considerations that the amplitude of the displacement should decrease with increasing wall thickness.

3. Results of the Acoustic Measurement of Slotted Tubes

The problem of the slotted tube is difficult to solve analytically. However, the predominance of circumferential extensional vibrations in the frequency range of interest may be inferred from an analysis of the mechanical and acoustical measurements.

Figures 6, 7, 8, and 9 are directivity patterns for a slotted tube transducer that has a diameter of 6 inches, a height of 4 inches, and a wall thickness and slot width of 1/8 inch. With the exception of the variations introduced by the presence of the slot, the shape of the

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[3] *See pg 2.*

patterns and the frequencies at which they occurred were explainable on the basis of circumferential extensional vibrations with geometrically determined nodal positions. In contrast to closed tubes, the angle θ is not measured from the point of application of the radial driving force, but is measured from the slot. If it were not for this fact, the directivity patterns of the radially polarized slotted tubes would have been omnidirectional at all frequencies, as is the case for radially poled closed tubes.

Figures 10, 11, and 12 show the decrease in amplitude of radial displacement with an increase in wall thickness as predicted by Rayleighs'[3] analysis of circumferential extensional vibrations. These patterns are for three separate single element slotted tube transducers vibrating in the dipole mode ($n = 1$). Each tube had a diameter of 6 inches, a height of 2 inches, and a slot width of 1/4 inch. Their wall thicknesses were 1/4, 3/8, and 1/2 inch, respectively. These patterns show that an increase in thickness of the tubes is accompanied by a corresponding decrease in the magnitude of the minimum values in the dipole patterns.

The predominance of the circumferential extensional vibrations in slotted tubes was further verified by "In Vacuo" resonance measurements. To accomplish this, the resonant frequencies in air of five radially poled closed tubes were determined from motional impedance measurements. Slots were then cut in the tubes, and their mechanical resonances were remeasured. Additional resonant frequencies were found to occur and their values were predictable from the equation for the circumferential resonances in closed tubes. A sample set of measurements is given in table I.

UNIDIRECTIONAL CHARACTERISTICS

The second trend in the behavior of the slotted tubes was that the directivity patterns were unidirectional, with maximum response occurring 180 degrees away from the slot, at frequencies corresponding to the radial extensional or "hoop" resonance of the tube. Front-to-back ratios ranged in value from 8 decibels to better than 30 decibels. For a receiver, the front-to-back ratio is defined as the ratio in decibels of the open circuit voltage produced in response to sound pressure waves arriving in the direction of maximum sensitivity to that produced for waves incident in the direction of minimum sensitivity. The degree of directionality was related to the size of the slot and thickness of the shell as well as to the stiffness of the encapsulant. No analytical expression relating directionality to these parameters has been found.

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[3] *See pg 2.*

However, the experimental results indicate that directionality can be improved by minimizing the wall thickness and using low durometer plastic encapsulants for waterproofing. Measurements of the variation in directionality with slot width showed that a slot width to diameter ratio of between 1/16 and 1/32 was optimum for tubes with diameters from 1 to 6 inches.

Figures 13 through 16 give examples of the type of directivity patterns obtained at the hoop resonance of slotted tubes. The pattern in figure 13 is for a single element tube, 6 inches in diameter, 4 inches long, with a slot width and wall thickness of 1/8 inch. This tube was encapsulated in a low durometer vinyl, and the slot was not filled with the encapsulant. Figure 14 shows a pattern for a tube of like dimensions that was encapsulated in a stiff (80 durometer) polyurethane. In this case the slot was filled with the plastic so that the encapsulation was continuous around the shell.

Figures 15 and 16 are the patterns for the 8- and 5-element line transducers discussed earlier in the report. The 8-element line was encapsulated in low durometer vinyl and the 5-element line in high durometer polyurethane.

Unidirectional patterns were also detected at one frequency that was far below the extensional resonances and substantially above all but the high overtones of the flexural resonances of the tubes. Figures 17 and 18 show examples of these patterns for the 8- and 5-element line transducers, respectively. This behavior could not be explained on the basis of any of the types of vibration considered in this investigation. The frequency where these patterns occurred was inversely proportional to the radius of the tubes. An empirically derived relationship given below gives an approximate indication of the frequency at which the directional effect occurs:

$$f = \frac{K}{R} \tag{3}$$

where

f = frequency in khz

R = mean radius

K = proportionality constant equal to 8.5 khz-in. for $BaTiO_3$ tubes

Directionality at these frequencies had a much lower front-to-back ratio and was, in general, less consistent than the directional effect at the hoop resonance. The relatively low frequency, however, makes these effects of greater interest.

TABLE I

COMPARISON OF CALCULATED AND MEASURED RESONANT FREQUENCIES (KHZ) FOR $BaTiO_3$ TUBE (IN AIR), BEFORE AND AFTER SLOTTING

	Circumferential Extensional Resonances			
	Before Slotting		After Slotting	
Mode (n)	Calculated	Experimental	Calculated	Experimental
0	30	29.6	30	32.3
1			42	44.4
2			66	65
3			93.5	97.6

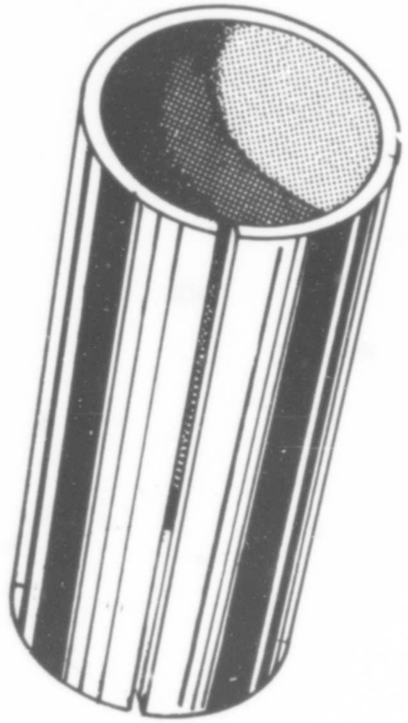


FIGURE 1 - Slotted Cylinder

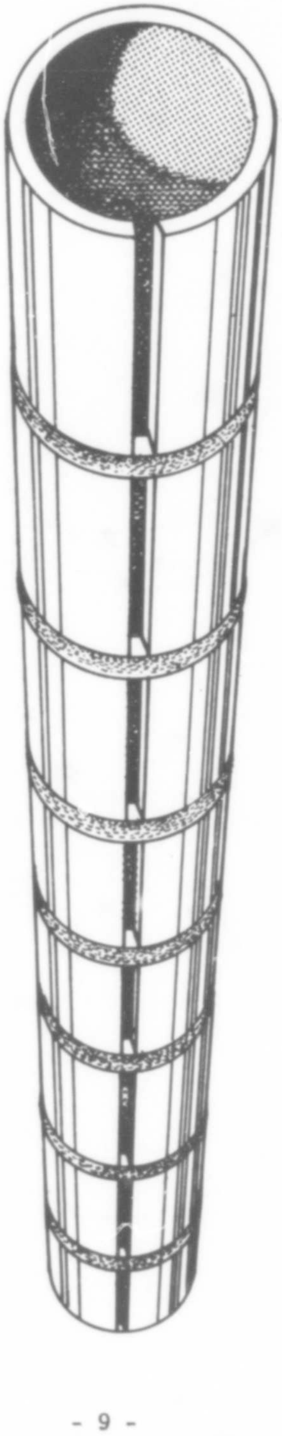


FIGURE 2 - Line Hydrophone

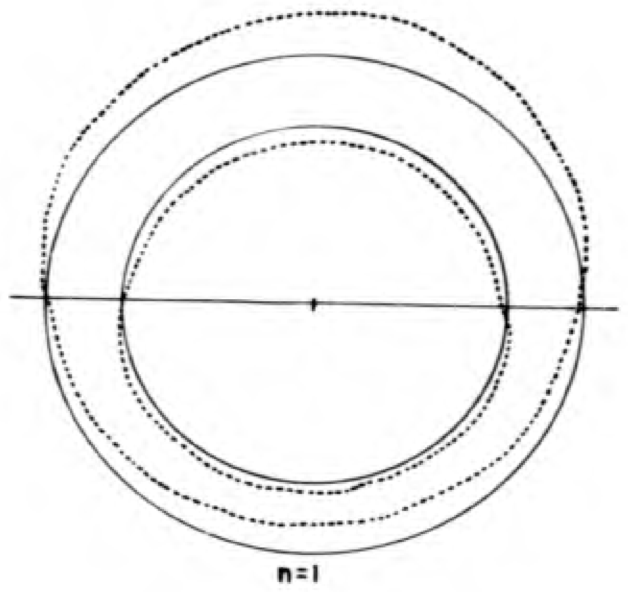


FIGURE 3 - Instantaneous Distribution of Amplitude of Vibration (n = 1)

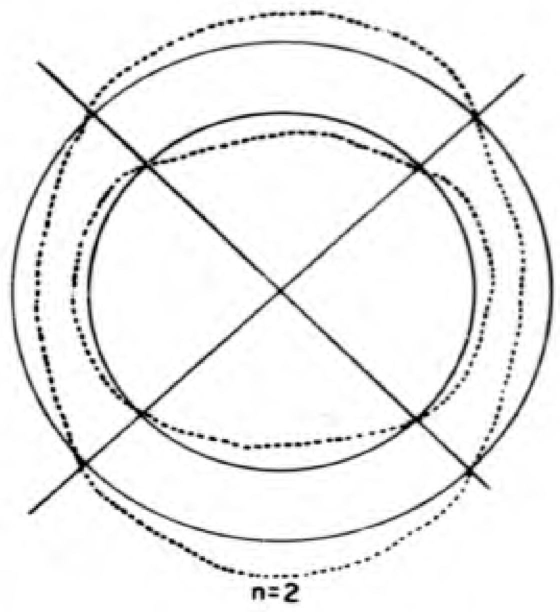
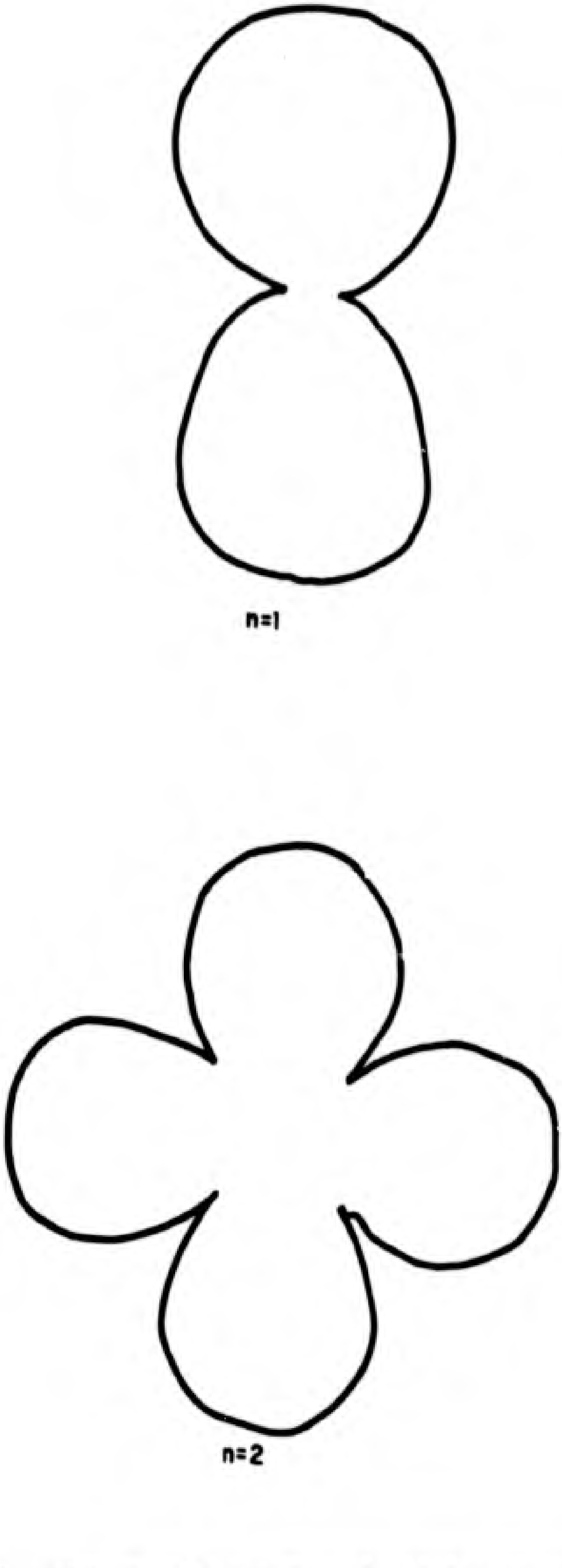


FIGURE 4 - Instantaneous Distribution of Amplitude of Vibration (n = 2)



n=1

n=2

FIGURE 5 - Theoretically Predicted Receiving Patterns

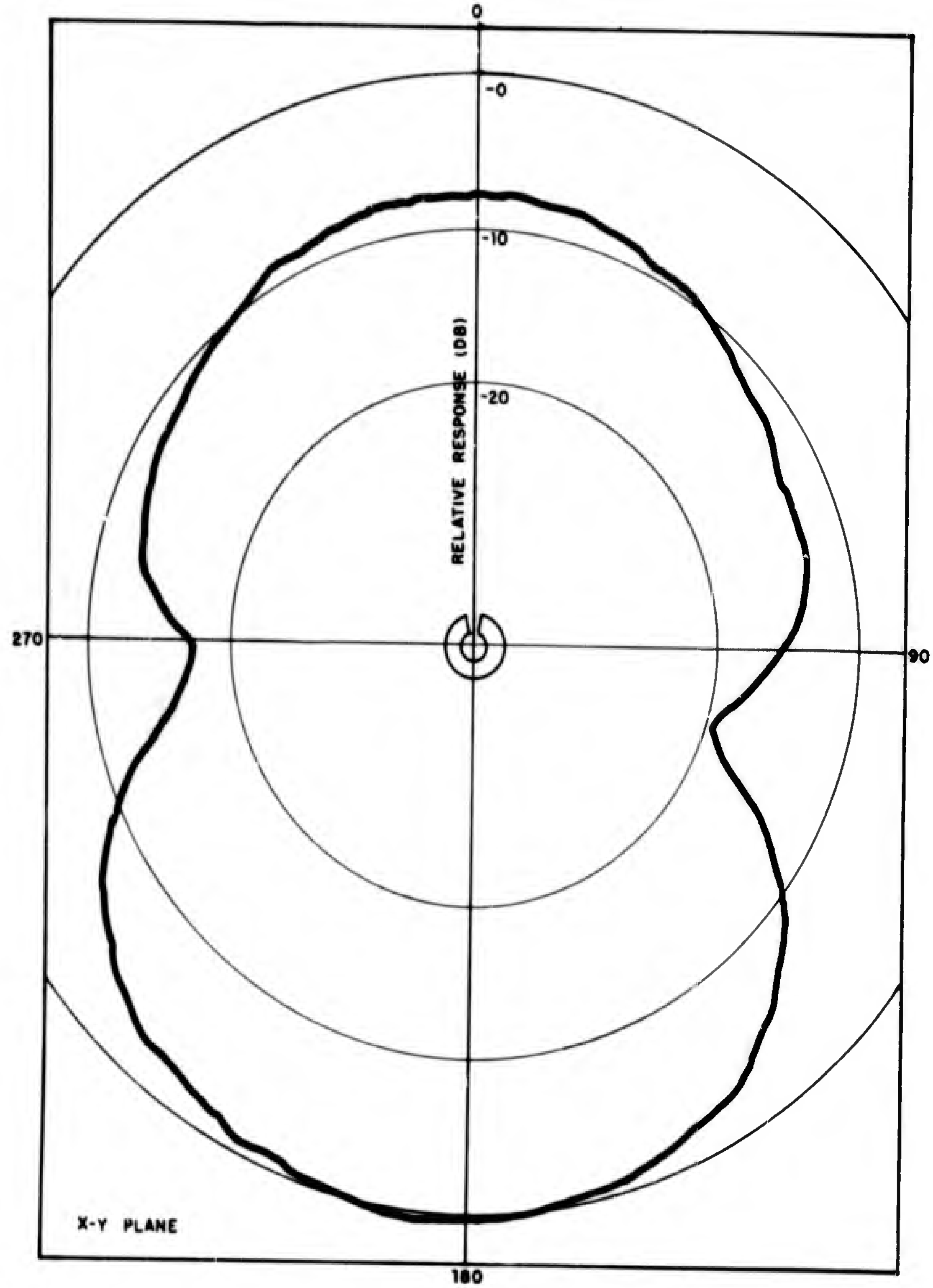


FIGURE 6 - Receiving Pattern (n = 1)

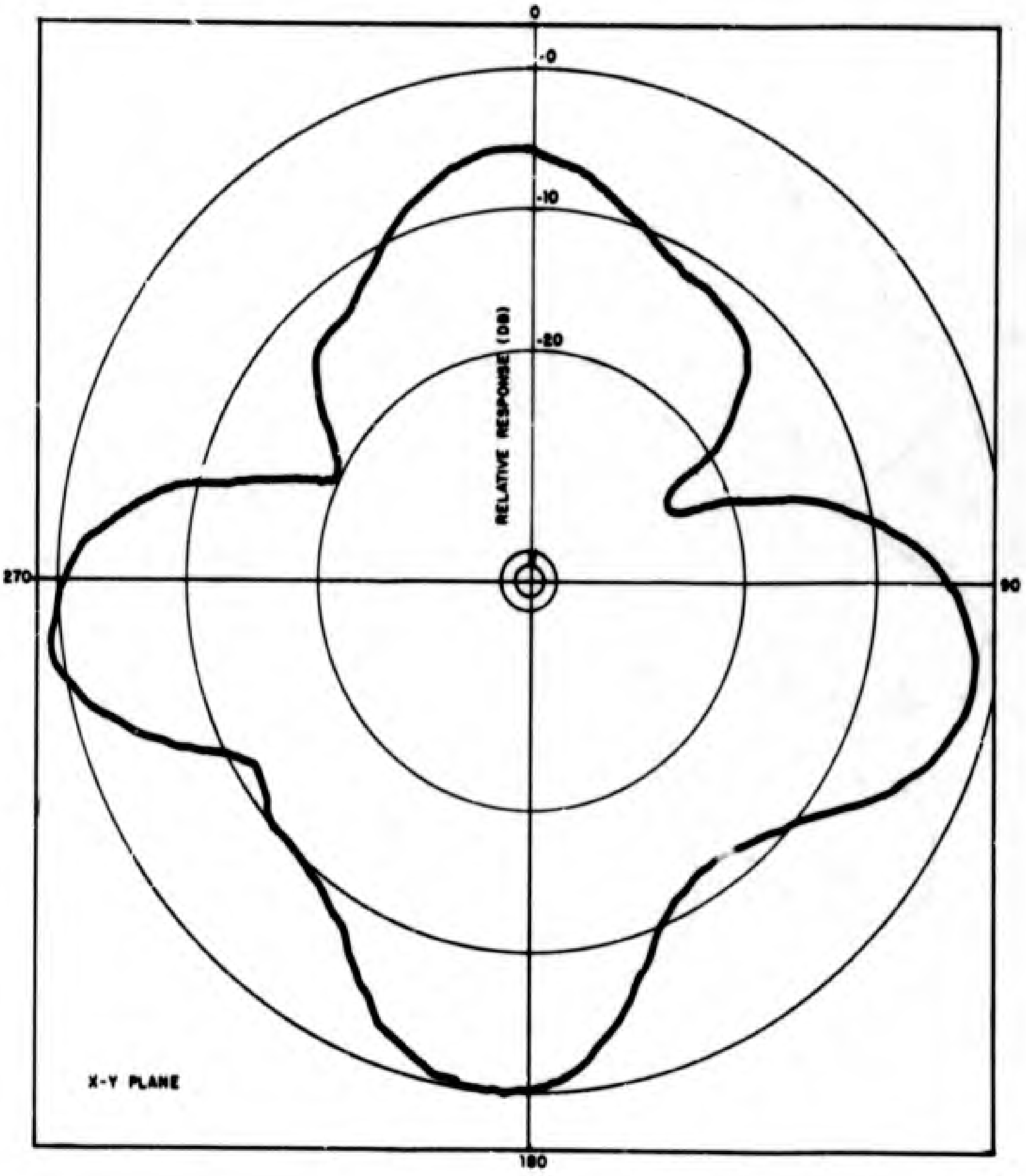


FIGURE 7 - Receiving Pattern (n = 2)

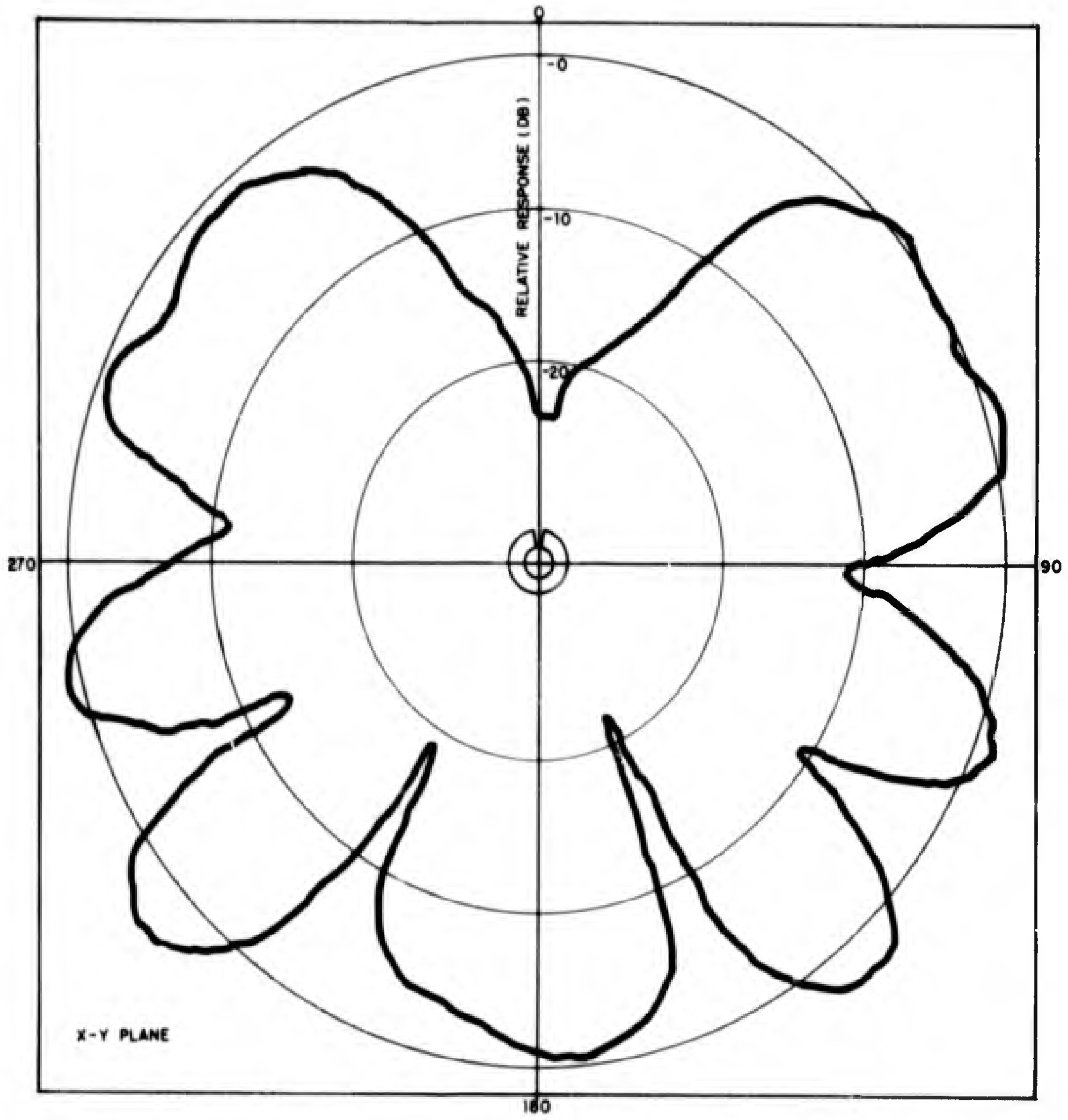


FIGURE 8 - Receiving Pattern (n = 4)

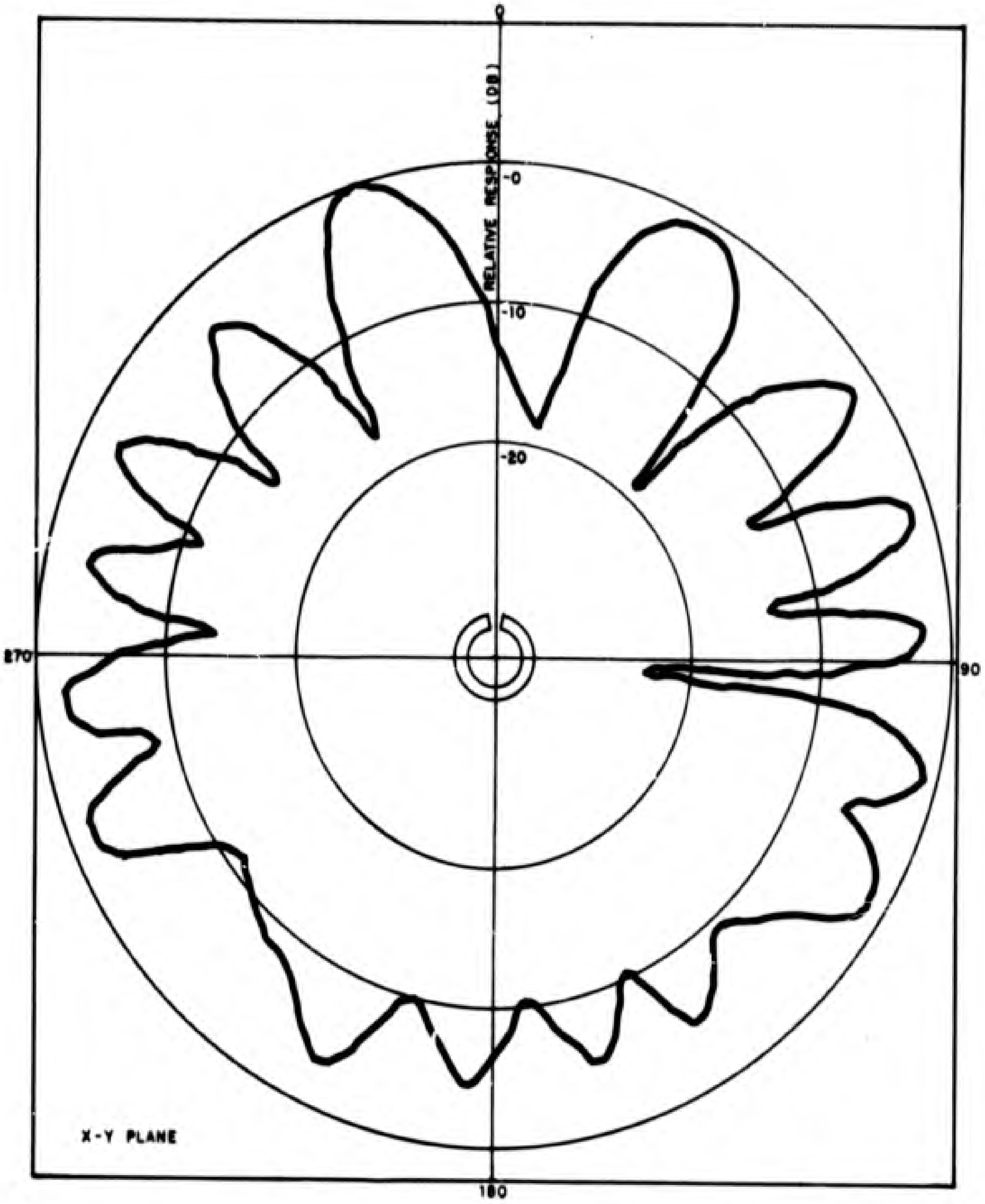


FIGURE 9 - Receiving Pattern (n = 7)

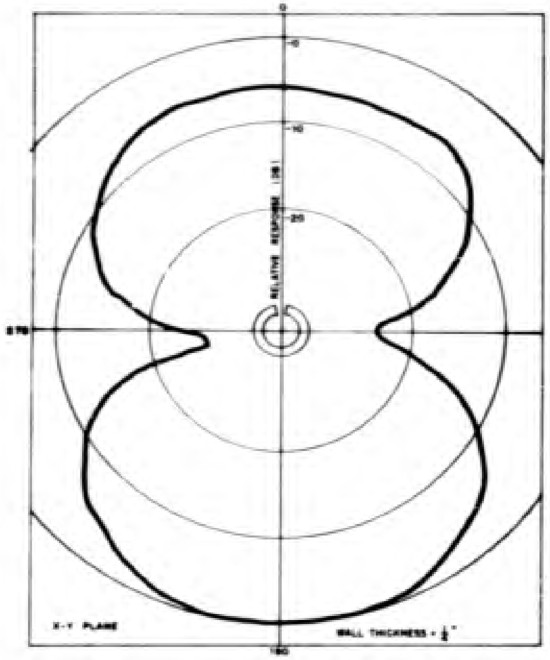


FIGURE 10 - XJD-360-1B Receiving Pattern Showing Effects of Variation in Wall Thickness

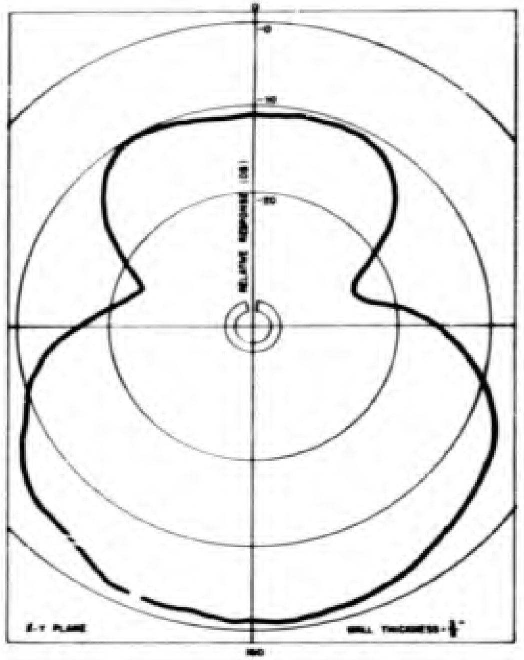


FIGURE 11 - XJD-360-1C Receiving Pattern Showing Effects of Variation in Wall Thickness

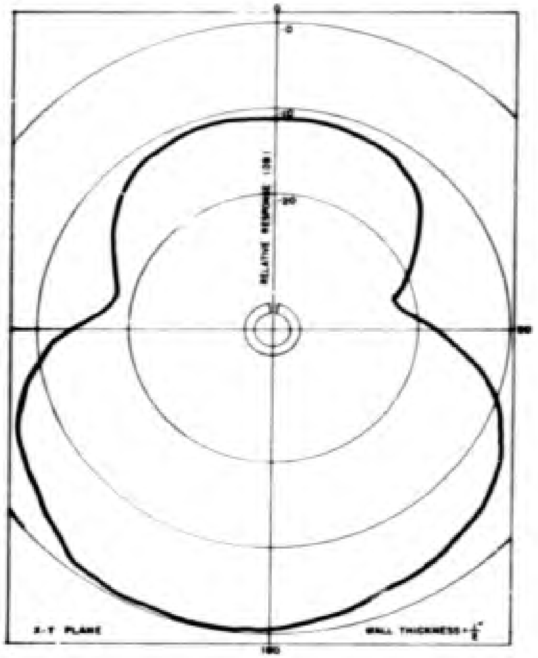


FIGURE 12 - XJD-360-1D Receiving Pattern Showing Effects of Variation in Wall Thickness

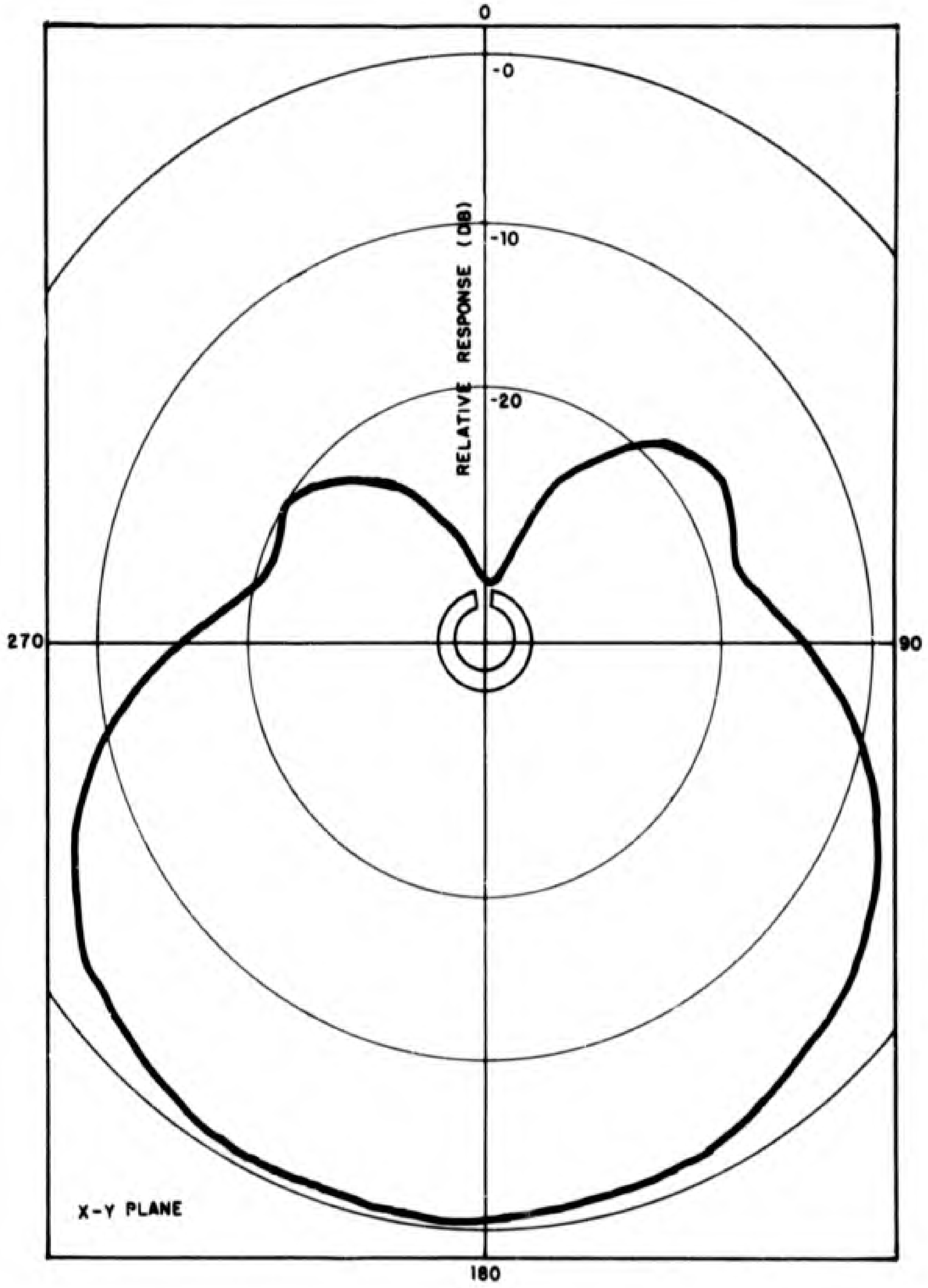


FIGURE 13 - XJD-344-4 Receiving Pattern (n = 0)

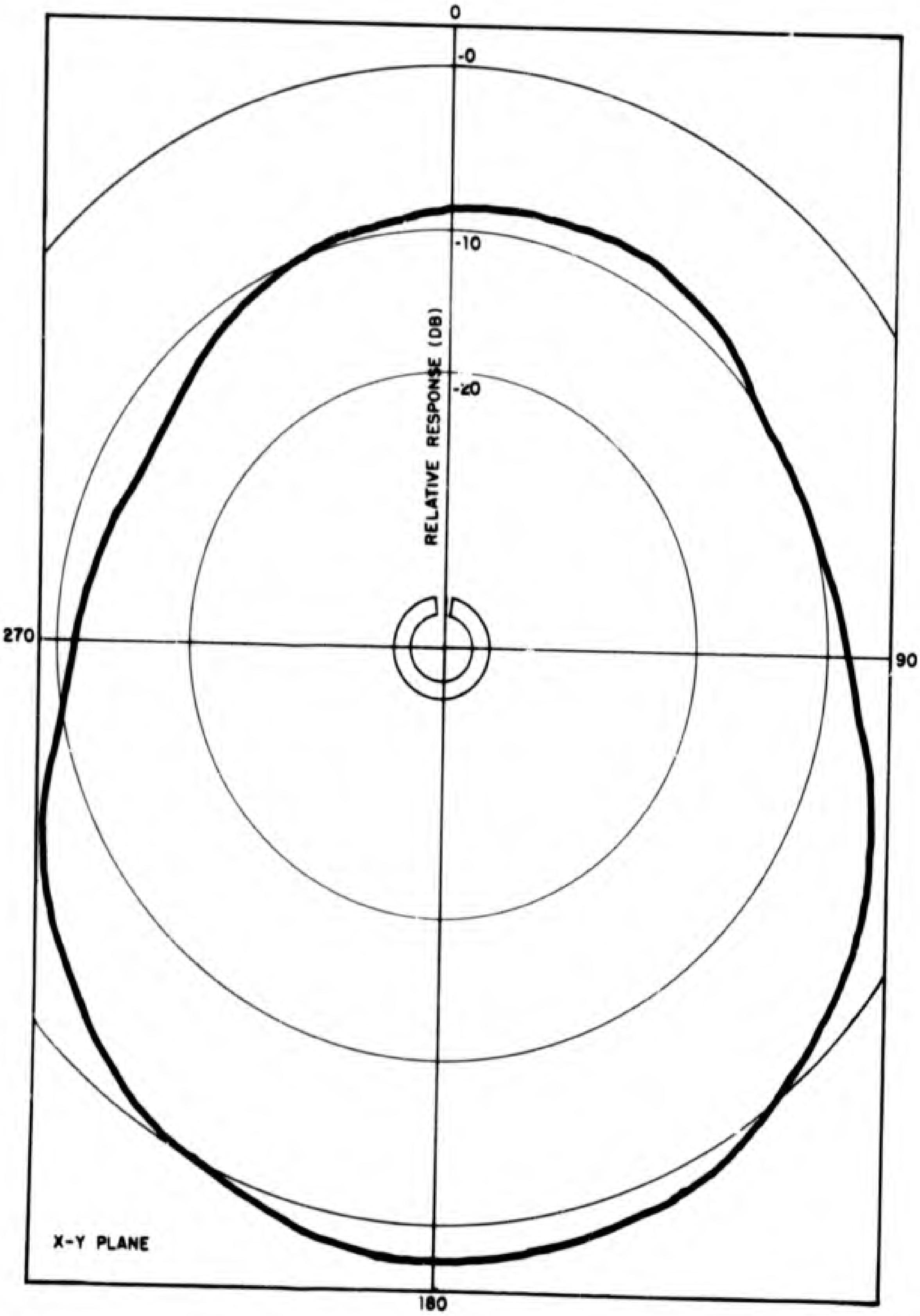


FIGURE 14 - XJD-344-3 Receiving Pattern (n = 0)

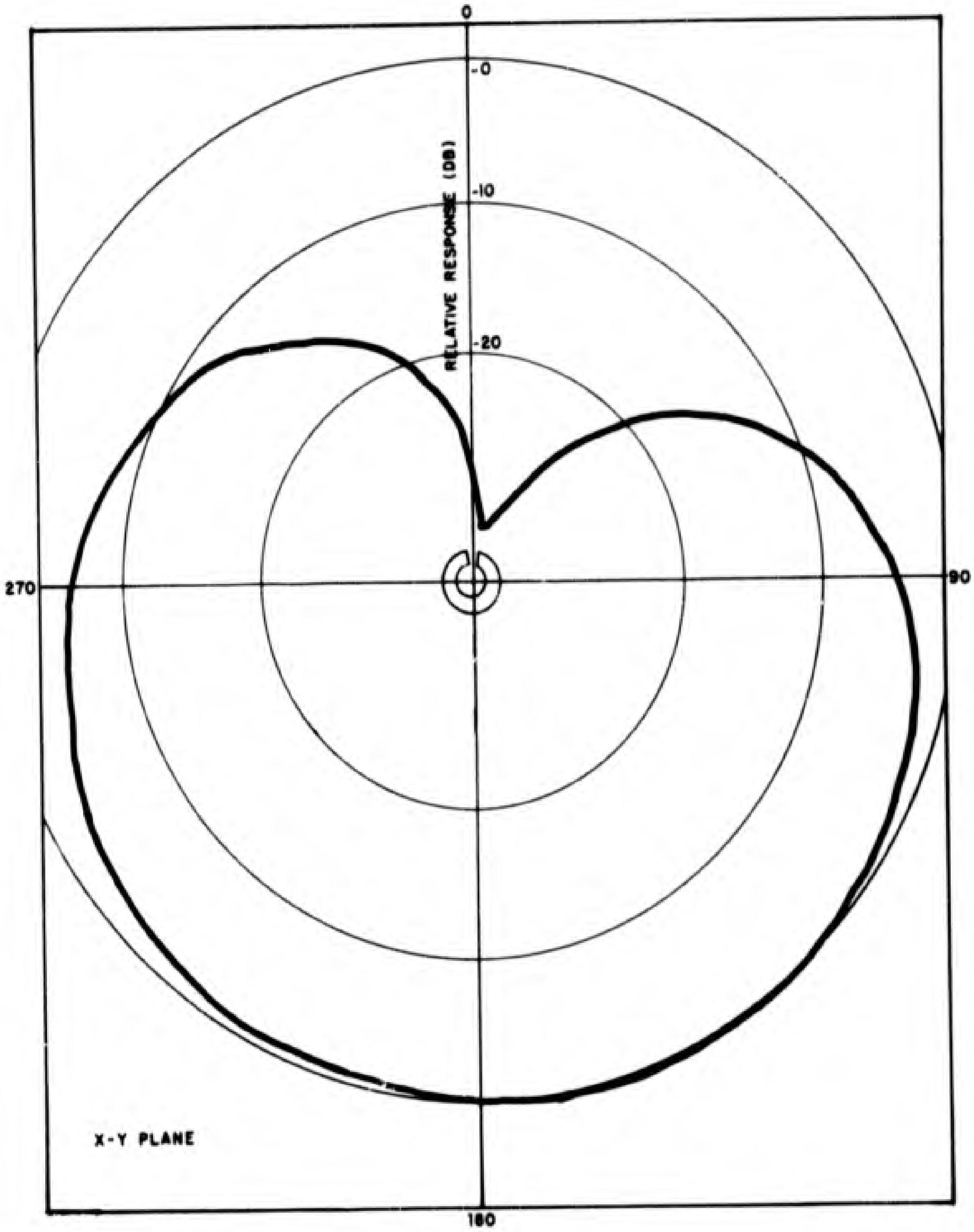


FIGURE 15 - XJD-335-1 Receiving Pattern (n = 0)

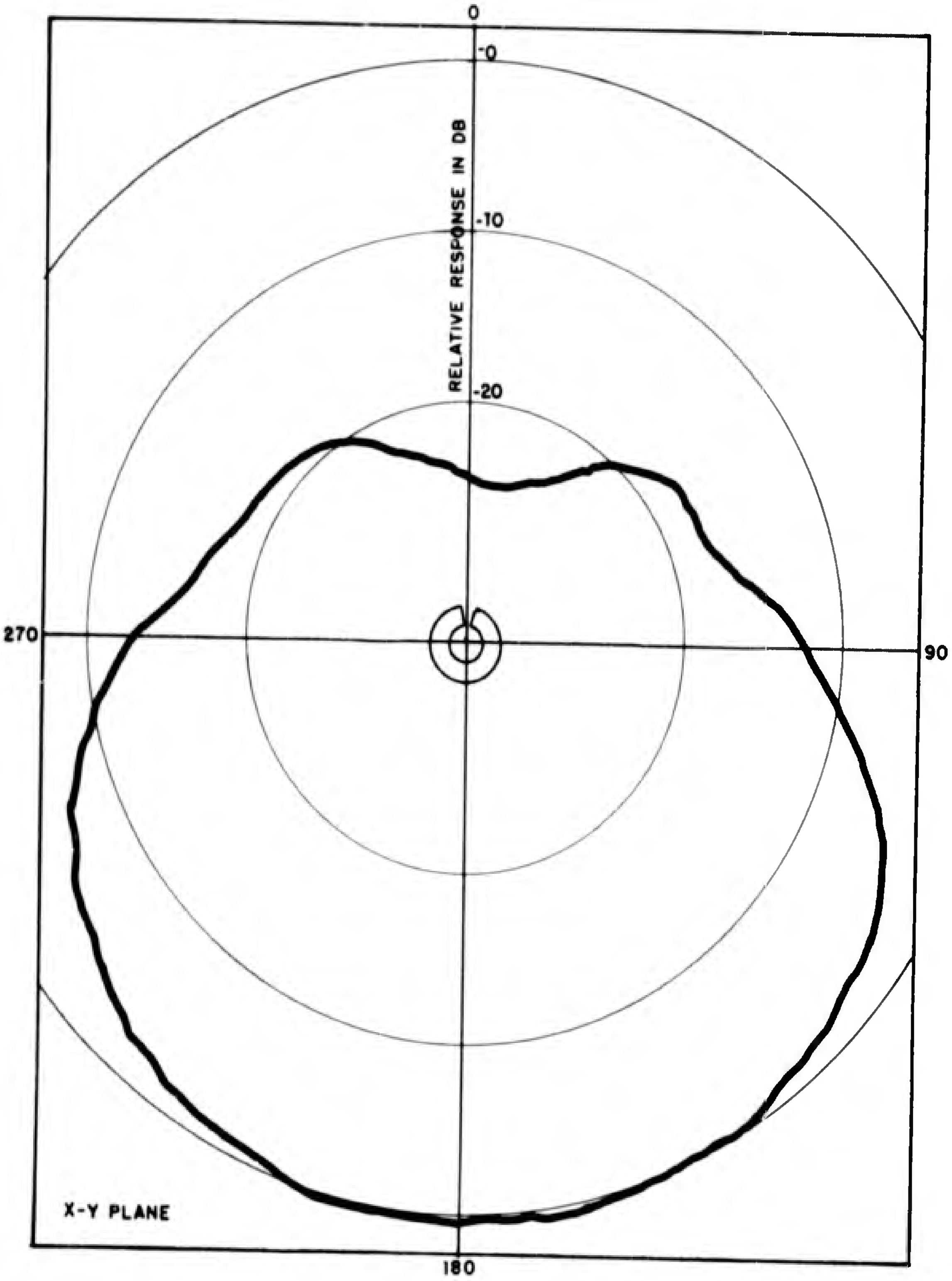


FIGURE 16 - XJD-357-1E Receiving Pattern (n = 0)

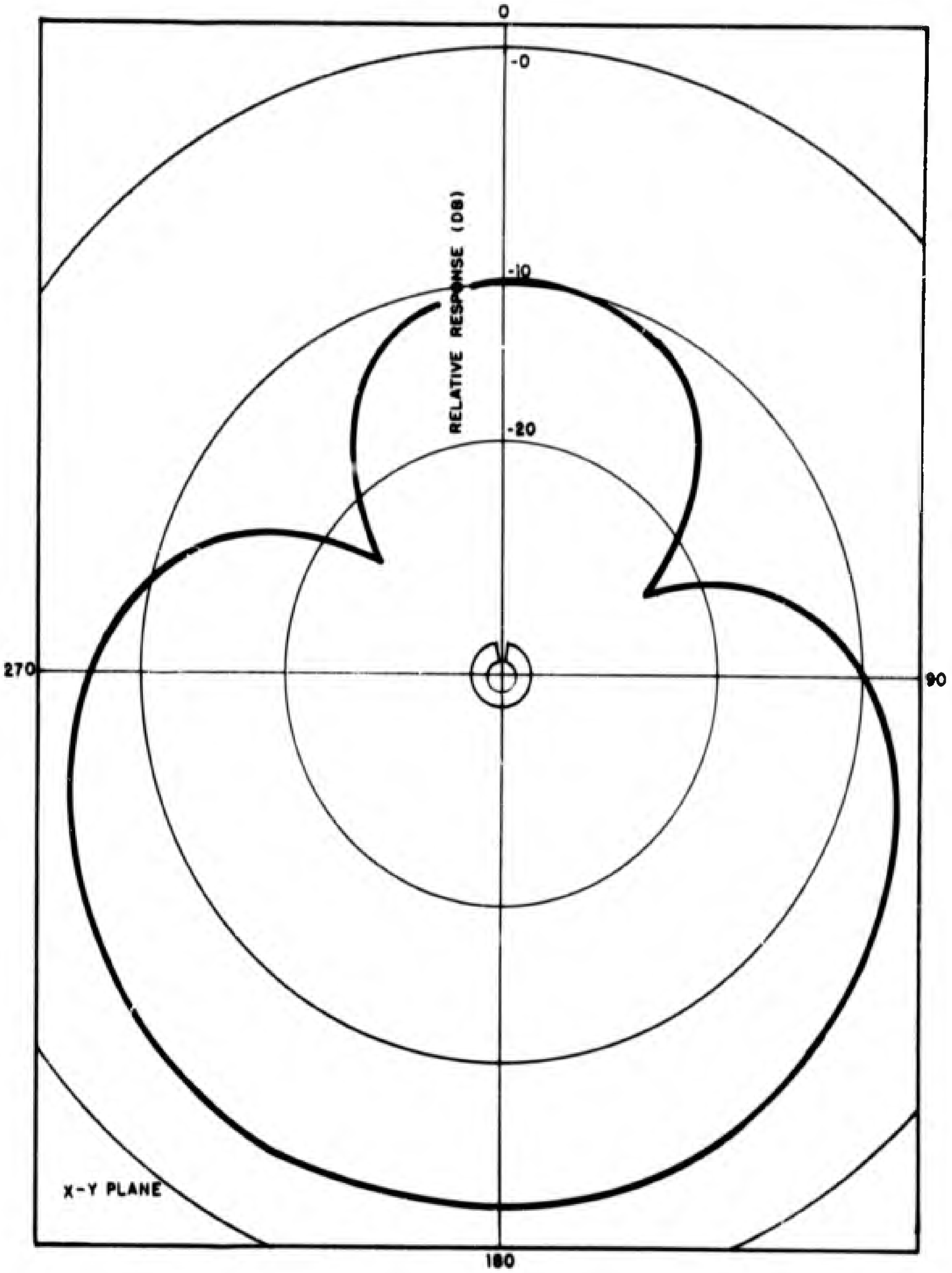


FIGURE 17 - XJD-335-1 Receiving Pattern (f = 11.93 KHZ)

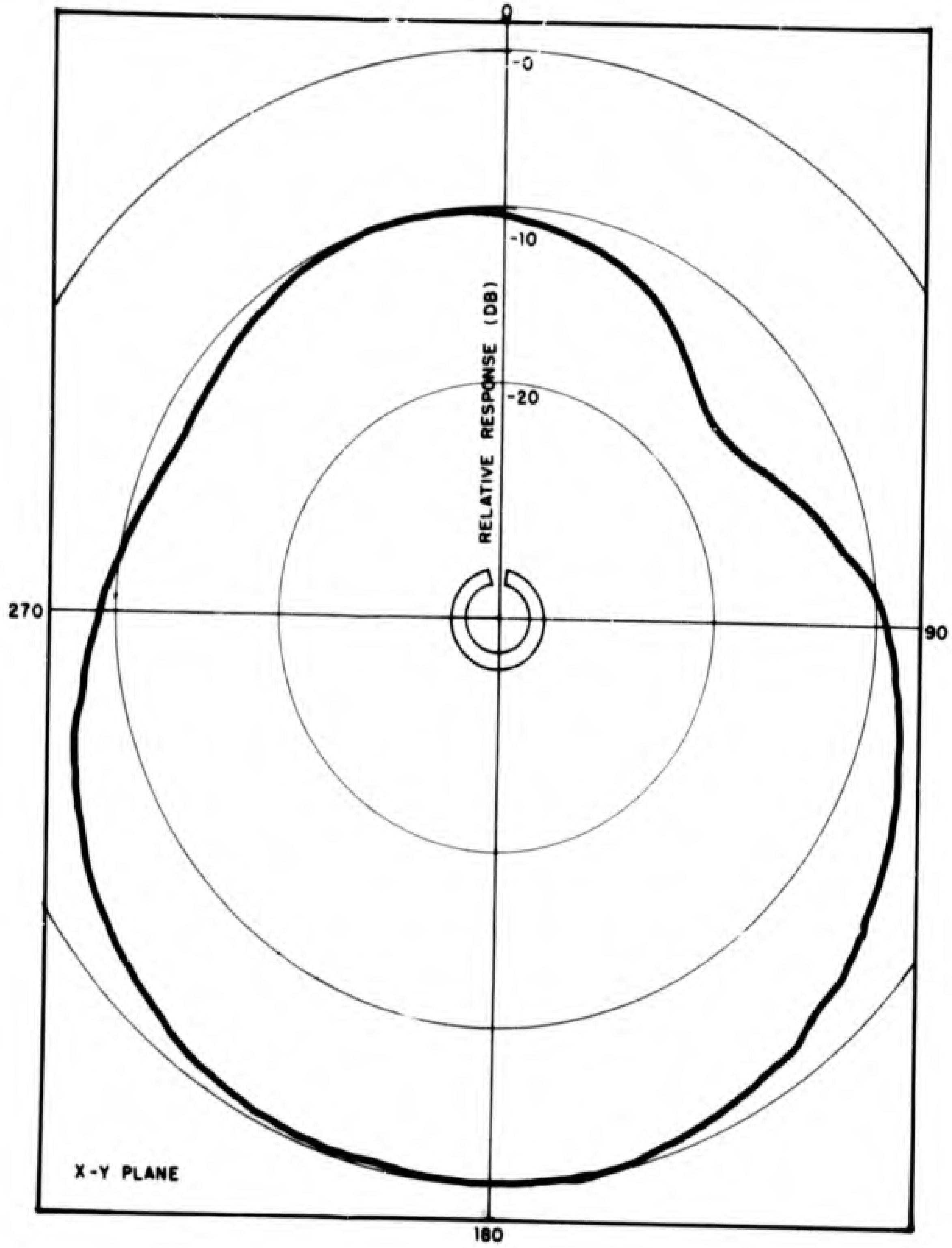


FIGURE 18 - XJD-357-1E Receiving Pattern (f = 8.0 KHZ)

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13. ABSTRACT

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11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

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