STATEMENT NO. 1

Distribution of This Document is Unlimited

HYDRONAUTICS, incorporated
research in hydrodynamics

Research, consulting, and advanced engineering in the fields of NAVAL and INDUSTRIAL HYDRODYNAMICS. Offices and Laboratory in the Washington, D. C., area: Pindell School Road, Howard County, Laurel, Md.

HYDRONAUTICS, Incorporated

TECHNICAL REPORT 233-11

17 ¢ 500

THEORY OF EROSION

By

A. Thiruvengadam

March 1967

Prepared Under

Office of Naval Research
Department of the Navy
Contract No. Nonr 3755(00)(FBM)
NR 062-293

TABLE OF CONTENTS

BLANK PAGE

LIST OF FIGURES

NOTATION

I_e	Intensity of erosion
i	Average depth of erosion
t	Time of erosion
S_e	Erosion strength
p_c	Collapse pressure of the bubble or impact pressure of the drop
ρ_l	Density of the liquid
C_l	Speed of sound in the liquid
I_i	Intensity of impact
I_c	Intensity of collapse
r	The radial distance from the surface
A	A proportionality constant with length as dimension
n	The attenuation exponent
η	The efficiency of absorption of the impact energy by the material
I_{max}	The maximum intensity of erosion at a given intensity of collapse I_c
t_1	The time at which the intensity of erosion reaches a maximum; at $t = t_1$, $I_e = I_{max}$.
η_1	The efficiency corresponding to the time t_1
$\bar{I}$	The normalized intensity of erosion, I_e/I_{max}

τ	The normalized time, t/t_1
$\bar{\eta}$	The normalized efficiency, η/η_1

$k = \frac{d\bar{\eta}}{d\tau}$ at $\tau = 1$

α	The Weibull's shape parameter
r_1	The cumulative depth of erosion at time t_1
$\bar{r}$	The normalized cumulative depth of erosion, r/r_1

BLANK PAGE

SUMMARY

An elementary theory of erosion is derived based on the assumptions of "accumulation" and "attenuation" of the energies of impact causing erosion. This theory quantitatively predicts the relative intensity of erosion as a function of relative time and this prediction is in fair agreement with experimental observations. Since the intensity of collision, the distance of shock transmission and the material failure are all statistical events, a generalization of the elementary theory is suggested.

Some of the practical results of this theory are the predictions of the cumulative depth of erosion, the determination of erosion strength and the method of correlation with other parameters such as liquid properties and hydrodynamic factors. Modifications of this theory for brittle and viscoelastic materials are also suggested.

Note: An abstract of this report was published by the Jet Propulsion Laboratory, California Institute of Technology, Pasadena following a lecture on this subject on December 30, 1966 at the Turbine Erosion Conference organized by JPL.

INTRODUCTION

Recent research on cavitation erosion has led to the logical definition of the intensity of erosion as the power absorbed by a unit area of the eroded material (1,2). This definition was successfully used to estimate the intensities of erosion of specimens in laboratory test devices as well as practical field systems and to compare their relationships (2,3). Based on this definition a nomogram has been prepared to help designers select proper materials (3). It has also been pointed out that the same approach may be adopted for other erosion problems such as turbine erosion and rain erosion (4). However, these simple methods of solution to erosion problems are limited by the fact that the rate of erosion is time dependent. The importance of the effects of test duration (or operating time in field installations) was recognized and highlighted in a series of recent publications (2,5,6,7 and 8). These results are further confirmed by Plesset and Devine (9) and Hobbs (10) for cavitation erosion. Such effects are also observed in liquid impact erosion such as turbine erosion (11,12), jet impact erosion (13) and rain erosion (14). It is necessary to understand and predict these non-linear time effects quantitatively in order to achieve meaningful correlations in the laboratory and to extrapolate laboratory data to field systems. The following theories are developed with such objectives.

The typical effect of the test duration on the rate of erosion of metals is shown in Figure 1. When the importance of this effect was recognized, this relationship shown in Figure 1 was divided into four zones, namely:

1. Incubation zone,
2. Accumulation zone,
3. Attenuation zone, and
4. Steady state zone.

(Figure 2). The names of these zones were selected to represent the most significant physical mechanism controlling that particular zone. Particularly the "accumulation" zone and the "attenuation" zone are of specific interest for the present discussion. In the "accumulation" zone, it was thought the increasing rate of damage was due to cumulative energy storage and more efficient energy absorption in the material due to fatigue. Whereas in the "attenuation" zone, the bubble collapse energy was thought to be attenuated before reaching the surface due to the erosion of the surface material. It was also noted that the beginning of the attenuation zone was characterized by the formation of deep craters on the eroded surface.

Subsequently two more significant papers appeared; one by Plesset and Devine (9) in which the authors emphasized the hydrodynamic interaction of the surface roughness on the bubble collapse and the other by Heymann (15) in which the material response was the primary consideration. In addition, Hobbs (10) as well as

Plesset and Devine (9) emphasized the "flat" peak which they thought was more steady with respect to time than the so called "steady state zone." The theories developed in this paper not only take into account both the material response and the hydrodynamic attenuation of the shock, but also explain the observations of Hobbs (10) and Plesset and Devine (9) about the "flat peaks." Furthermore the theory provides quantitative results that could be used for engineering calculations.

ELEMENTARY THEORY

The erosion of materials such as cavitation erosion or drop impact erosion is caused by the localized high impact pressures close to the surface of the material (Figure 3). It is generally known that this process of erosion is controlled by two opposing phenomena. The first is the change in the efficiency of absorption of the impact energy by the material and how this efficiency of energy absorption changes with respect to time after repeated indentations. The second is the attenuation of the impact pressures due to increasing distance of travel as the surface gets eroded. The elementary theory is developed by making two basic assumptions for the above two phenomena combined with the definition of the intensity of erosion.

Definition of the Intensity of Erosion

The intensity of erosion is defined as the power absorbed by unit eroded area of the material (2,3 and 4) and is given by

$$I_e = \frac{r\,S_e}{t} \qquad [1]$$

where

r/t is the rate of depth of erosion, and

S_e is the erosion strength.

If the rate of depth of erosion is expressed in differential form then

$$I_e = S_e \frac{dr}{dt} \qquad [2]$$

where r is the mean depth of erosion as measured from the original surface of the material (Figure 3).

The Intensity of Collapse (or Collision)

The intensity of collapse (or collision) depends up on the parameters controlling the dynamics of the bubble or drop. For example the intensity of bubble collapse is defined as the power transmitted by unit surface area of the bubble (16) and is given by

$$I_c = \frac{p_c^{\,2}}{\rho_\ell C_\ell} \qquad [3]$$

where

p_c is the collapse pressure of the bubble, and

$\rho_\ell C_\ell$ is the acoustic impedance of the liquid.

Similarly the intensity of collision of water drops may be defined either with water hammer pressure instead of the collapse pressure or from the energy flux of all the drops on unit area of eroded surface.

Basic Assumptions

The two basic assumptions of the theory are as follows:

1. The attenuation of the intensity of collapse or collision is assumed to be inversely proportional to the nth power of the radial distance. This may be mathematically stated as

$$I_i = \frac{A^n I_c}{(r+r_c)^n} \qquad [4]$$

where

I_i is the intensity of impact,

I_c is intensity of collapse or collision,

r is the radial distance (Figure 3),

A is a proportionality constant with length as dimension, and

n is the attenuation exponent.

2. The second assumption is that the intensity of erosion is related to the intensity of impact in the following manner

$$I_e = \eta I_i \qquad [5]$$

where η is a material property governing the efficiency of energy absorption. It is also further assumed that the efficiency of absorption, η is time dependent

$$\eta = \eta(t)$$

Differential Equation of Erosion

Combining Equations [2], [4] and [5] the following differential equation of erosion may be derived (See Appendix A)

$$\frac{dI_e}{dt} + k \frac{I_e^{\frac{2n+1}{n}}}{\eta^{\frac{1}{n}}} - \frac{I_e}{\eta} \frac{d\eta}{dt} = 0 \qquad [6]$$

where

$$k = \frac{n}{(A^n I_c)^{1/n} S_e}$$

Since the experimental observations show the intensity of erosion reaches a maximum and then decreases, the above differential equation is normalized with respect to this maximum intensity of erosion, I_{max} and the time t_1 at which this maximum occurs.

At

$$t = t_1$$

$$I_e = I_{max}$$

$$\frac{dI_e}{dt} = 0$$

$$\eta = \eta_1$$

The normalized parameters are given below

$$\tau = \frac{t}{t_1} \; ; \; \bar{I} = \frac{I_e}{I_{max}} \; , \; \bar{\eta} = \frac{\eta}{\eta_1} \; ;$$

and

$$\bar{k} = \left. \frac{d\bar{\eta}}{d\tau} \right|_{\tau=1}$$

The normalized differential equation of erosion is given by (See Appendix A)

$$\frac{d\bar{I}}{d\tau} + \bar{k} \frac{\bar{I}^{\frac{2n+1}{n}}}{\bar{\eta}^{1/n}} - \frac{\bar{I}}{\bar{\eta}} \frac{d\bar{\eta}}{d\tau} = 0 \qquad [7]$$

The general solution of this normalized differential equation is (See Appendix A)

$$\bar{I} = \frac{\bar{\eta}}{\left[1 + \bar{k} \frac{n+1}{n} \int_1^{\tau} \bar{\eta} d\tau \right]^{\frac{n}{n+1}}} \qquad [8]$$

In order to calculate $\bar{I}$ as a function of τ, the value of n and the function $\eta(t)$ must be known. In under water explosions, it is known that the shock pressure attenuates inversely as the distance travelled (17). Since the intensity varies as the square of the shock pressure, the value of n may be assumed to be 2. However calculations for n = 3 and n = 4 have also been made for the purposes of comparison as discussed later.

The nature of the function $\eta(t)$ may be inferred from the definition of the erosion strength, S_e (4) as the energy absorbed by unit volume of the material up to complete fracture and removal from the surface as a result of the repeated action of the erosive forces. If the unit volume of the material fractured is considered as a fatigue specimen, then the efficiency of absorption is associated with the probability of failure of the unit volume after a given time. There are several statistical distributions of the life of the materials under fatigue. The one that is more general and that has wide acceptance is the Weibull distribution (18) of the type

$$\eta = 1 - \exp(-\tau^{\alpha}) \qquad [9]$$

where α is the Weibull shape parameter and it depends upon the material as well as the stress level. As can be seen, the above distribution becomes a simple exponential distribution when $\alpha = 1$ and the Rayleigh distribution when $\alpha = 2$ (19). It corresponds to a nearly normal distribution when $\alpha = 3.57$ (19). Figure 4 shows the variation of the relative intensity of erosion, $\bar{I}$ as a function of relative time, τ for different values of α in Equation [9] and for n = 2. Figures 5 and 6 show the same relationships for n = 3 and for n = 4. While α has a large effect on the results n does not as can be seen in Figure 7 which shows a comparison for one value of $\alpha = 3.57$.

COMPARISON OF ELEMENTARY THEORY WITH EXPERIMENTAL DATA

The early results that led to the recognition of these time effects (5) shown in Figure 1 were obtained in cavitation erosion tests with a magnetostriction apparatus using 304 stainless steel and water at various amplitudes. These data are reduced in the non-dimensional form and plotted in Figure 8 along with the theoretical curve corresponding to the value of $\alpha = 3.57$ (which is a nearly normal distribution). The agreement of experimental results with theory is very good for $\tau > 1$. However for values of $\tau < 1$ the experimental results do not agree so well with the theory.

While the above comparison shows that much more remains to be done by way of correlating experimental results with the theory, the general trend is extremely encouraging. In addition, the controversy about the "flat peak" is resolved since the curve for the lowest amplitude which exhibited the so called "flat peak" falls in line with the other data as well as with the theory.

GENERAL THEORY OF EROSION

In the elementary theory, only the efficiency function $\eta(t)$ was assumed to be a statistical distribution. However, the intensity of collapse, I_c, as well as the distance, r, are both statistical quantities. For example the intensity of collapse depends upon the bubble size distribution (or drop size distribution) and the value of r would depend upon the roughness distribution on the surface, Figure 9. Furthermore, the number of bubbles collapsing in a given area may be affected by the hydrodynamic interaction of the roughness of the surface which in turn depends upon the testing time (9). Thus, it may be assumed that

$$\left.\begin{aligned} I_c &= I_{c_o}\phi_1(t) \\ r &= r_o\phi_2(t) \\ \text{and} \quad \eta &= \eta_o\phi_3(t) \end{aligned}\right\} \text{Probability functions controlling the statistical events.} \qquad [10]$$

Then the original assumptions given by Equations [4] and [5] may be used to derive a new probability function ϕ which is the resultant of the above three functions. It is suggested that this function, ϕ is the true erosion distribution as measured in an erosion test. Hence one of the purposes of an erosion tests it to determine the <u>erosion distribution</u>, ϕ.

PRACTICAL RESULTS OF THE EROSION THEORIES

Prediction of Cumulative Depths of Erosion

The erosion theories developed in the previous sections can be used to predict the cumulative depth of erosion taking into account the effect of time and this result is very useful for practical applications. From Equation [2],

$$\bar{I} = \frac{I_e}{I_{max}} = \frac{S_e}{I_{max}} \frac{dr}{dt} = \frac{S_e \cdot r_1}{I_{max} \; t_1} \frac{d(\frac{r}{r_1})}{d(\frac{t}{t_1})}$$

where r_1 is the depth of erosion at time t_1 corresponding to the peak intensity I_{max}. Hence,

$$\bar{I} = \frac{S_e r_1}{I_{max} \; t_1} \frac{d\bar{r}}{d\tau} \qquad [11]$$

where

$$\bar{r} = \frac{r}{r_1} .$$

As shown in the appendix

$$\bar{r} = \frac{\int_0^{\tau} \bar{I} \, d\tau}{\int_0^{1} \bar{I} \, d\tau} \qquad [12]$$

Figure 10 shows the relationship between the relative depth of erosion, $\bar{r}$ and the relative time, τ for various values of α in the Weibull type distributions shown in Figure 4. In fact the area under these curves in Figure 4 give the required integrals in Equation [12].

If the cumulative depth becomes large enough to affect the bulk characteristics of the erosion process, then the cumulative depth curves will change shape due to such effects. For example it is known for the case of rotating disk apparatus (1) that the damage rate increases after the steady state becomes the hole produced by cavitation damage acts as another source. In some experiments such as the damage on hydrofoils the damage rates tend to become zero since the cumulative depth of erosion reaches the thickness of the foil (Figure 11). These considerations point out the fact that the extrapolation of the test results to predict long term cumulative depths have to be done with caution taking into account the realistic interacting influences.

Determination of the Shape Parameter, α

In order to make use of the theories of erosion in engineering calculations and experimental correlations, it becomes essential to determine the shape parameter, α. The results shown in Figure 4 are replotted in Figure 12 where relative intensity $\bar{I}$ is plotted as a function of α for various values of τ. This figure will enable us to select the most suitable α to fit a given set of experimental data.

Determination of Erosion Rates as a Function of Time

It has been suggested earlier (4,5,6 and 7) that the steady state rate of erosion be used for correlating other properties of materials with erosion resistance. The reason was that the interacting influence of the testing time was not quantitatively understood and that it significantly affected correlations with other parameters.

However conducting each experiment up to steady state is quite time consuming particularly for strong materials. It may take as much as 30 to 40 hours for obtaining each steady state erosion rate. Furthermore, at lower intensities, the rates continue to decrease even after long test durations as shown in Figure 1. In such cases it will be difficult to accurately determine the steady state rate of erosion. From the present theoretical considerations, the above fact becomes obvious since the values of τ for the lowest amplitude curve in Figure 1 are only in the range of 1.5 to 2.

If the quantitative understanding of the relationship between the rate of erosion and time of erosion as suggested by the theory is fully established, then it is not necessary to test for very long durations. However, it would be still desirable to test beyond the peak rate and to present the data in the normalized form. For this reason, the peak rate of damage and the time at which the peak rate occurs become very significant measurements in an erosion test. Hobbs (10) as well as Plesset and Devine (11) suggested the use of peak rates for correlations although for a different reason. They thought that the peak was "flat" and independent of test time at least for some time before the rate of erosion started decreasing and in that sense the 'flat' peak was the steady state.

Determination of Erosion Strength of Metals

The erosion strength is related to the peak rate of erosion in the following manner (See Equation [A.22] of the appendix.

$$S_e = A^2 I_c \frac{M^2}{t_1^2 (\dot{r})^3_{max}} \qquad [13]$$

where

$$M = \frac{\alpha}{[e(e-1)]^{\frac{1}{2}}}$$

and

$$(\dot{r})_{max} = \frac{dr}{dt}\bigg|_{max} \text{ at } t = t_1$$

Following an earlier suggestion by the author (4), the strain energy may be assumed to be identically equal to the erosion strength for a few metals for which such correlations are verified (7) . Using one of these metals, erosion curves similar to Figure 1 may be obtained. From these erosion curves, the values of A, α and I_c may be obtained for any erosion test device and material. After such a "calibration" of the test apparatus, the erosion strengths of different metals may be determined for the use in design calculations. This procedure may also be used to quantitatively evaluate the environmental interactions on the erosion strength of metals. Much more experimental research needs to be done in this area.

Correlations With Liquid and Hydrodynamic Parameters

The values of A and I_c, thus evaluated from an erosion test may be used to correlate with different parameters affecting the erosion phenomenon. One such example is an attempt to formulate the scaling laws that govern the mechanism of cavitation damage (16). The steady state erosion rates were used in that case. Instead the peak values in terms of $(A/r_1)^2 I_c$ may also be used for such conditions.

MODIFICATIONS OF THE THEORY FOR BRITTLE AND VISCOELASTIC MATERIALS

The present theory of erosion can be extended to the cases of brittle materials such as rocks and concrete as well as to the viscoelastic materials such as elastomeric coatings. The major requirement for such an extension is the knowledge of the efficiency function η. Figure 13 show the variation of cavitation erosion rate of polished stone as a function of test duration (20). While this relationship is similar to that of metals, Figure 14 shows some test results for concrete (20). In this case, it appears that the rate of damage is a maximum at zero time or at least the value of t_1 is very small. Such results are also observed for the refractory metal, TZM at 400°F in liquid sodium (8).

The following treatment may be useful in dealing with some of the above results.

Assume η = constant (independent of time). Then the equation of erosion [6] becomes for n = 2

$$\frac{dI_e}{dt} + k\frac{I_e^{5/2}}{\eta^{\frac{1}{2}}} = 0$$

The solution is

$$\frac{1}{I_e^{3/2}} = \frac{\frac{3}{2}kt}{\eta^{\frac{1}{2}}} + C$$

If we assume $I_e = I_o$ at $t = 0$

$$C = \frac{1}{I_o^{3/2}}$$

$$\frac{1}{I_e^{3/2}} - \frac{1}{I_o^{3/2}} = \frac{\frac{3}{2}kt}{\eta^{\frac{1}{2}}}$$

$$\frac{I_o^{3/2}}{I_e^{3/2}} - 1 = \frac{\frac{3}{2}k I_o^{3/2}}{\eta^{\frac{1}{2}}} t \qquad [14]$$

Equation [20] may be used for the analysis of brittle materials which start with a maximum rate of damage.

Similarly, the theory for the viscoelastic materials can be developed from the knowledge of η as a function of time. The viscoelastic materials behave as brittle materials at high strain rates corresponding to high intensities of erosion. In such cases the theory for brittle materials may be used. At lower strain rates, additional problems such as adhesion failure and tearing will have to be considered. Further experimental research is needed in this area also.

CONCLUSIONS

The following significant conclusions may be drawn from the present investigations:

1. The elementary theory based on the assumptions of "accumulation" and "attenuation" of energy of bubble collapse or drop collision correctly predicts the shape of the erosion curves observed for metals.

2. In the elementary theory using Weibull type distributions, the value of the shape parameter, α, has a large effect on the erosion curves. However, the attenuation exponent does not affect the erosion curves significantly when $n = 2$, 3 or 4.

3. One set of experimental data for 304 stainless steel shows good correlation with theory when $\alpha = 3.57$ (corresponding to a nearly normal distribution for the efficiency).

4. The theory also predicts the cumulative depths of erosion as a function of erosion time.

5. A general theory of erosion can also be developed with a function called erosion distribution. It is suggested that the normalized erosion curve actually represents this erosion distribution.

6. Practical uses of these theories are:

   a. Prediction of cumulative depths,

   b. Understanding the influence of erosion time on erosion rate,

   c. Correlations with other liquid and flow parameters, and

   d. Determination of erosion strength.

APPENDIX-A

DERIVATION OF THE DIFFERENTIAL EQUATION OF EROSION

Assumptions

1. Attenuation of the intensity of impact

$$I_i = \frac{A^n I_c}{(r+r_c)^n} \qquad \text{[A1]}$$

2. Efficiency of energy absorption

$$I_e = \eta \, I_i \qquad \text{[A2]}$$

$$\eta = \eta(t) \qquad \text{[A3]}$$

Definition of the Intensity of Erosion

$$I_e = S_e \frac{dr}{dt} \qquad \text{[A4]}$$

Differentiating Equation [A2], we get

$$\frac{dI_e}{dt} = \frac{d\eta}{dt} I_i + \eta \frac{dI_i}{dt} \qquad \text{[A5]}$$

Differentiating Equation [A1], we get

$$\frac{dI_i}{dt} = A^n I_c \frac{(-n)}{(r+r_c)^{n+1}} \frac{dr}{dt} \qquad [A6]$$

Furthermore from Equation [A1]

$$(r+r_c)^n = \frac{A^n I_c}{I_i} \; ;$$

$$(r+r_c)^{n+1} = \left(\frac{A^n I_c}{I_i}\right)^{\frac{n+1}{n}} \qquad [A7]$$

From Equation [A4]

$$\frac{dr}{dt} = \frac{I_e}{S_e} \qquad [A8]$$

Hence Equation [A6] becomes

$$\frac{dI_i}{dt} = (-n) \frac{A^n I_c}{\left(\frac{A^n I_c}{I_i}\right)^{(n+1)/n}} \cdot \frac{I_e}{S_e}$$

$$= \frac{-n \; A^n I_c}{\left(A^n I_c\right)^{\frac{n+1}{n}}} \cdot \left(\frac{I_e}{\eta}\right)^{(n+1)/n} \cdot \frac{I_e}{S_e} \qquad [A9]$$

Since $I_1 = \frac{I_e}{\eta}$ from Equation [A2]. Substituting Equation [A9] in Equation [A5]

$$\frac{dI_e}{dt} = \frac{I_e}{\eta}\frac{d\eta}{dt} + \eta \frac{(-n)\,A^n I_c}{\left(A^n I_c\right)^{\frac{n+1}{n}}} \left(\frac{I_e}{\eta}\right)^{(n+1)/n} \frac{I_e}{S_e}$$

$$= \frac{I_e}{\eta}\frac{d\eta}{dt} - \frac{kI_e^{(2n+1)/n}}{\eta^{1/n}}$$

where

$$k = \frac{n}{\left(A^n I_c\right)^{1/n} S_e} \quad .$$

Hence the differential equation of erosion is given by

$$\frac{dI_e}{dt} + \frac{kI_e^{(2n+1)/n}}{\eta^{1/n}} - \frac{I_e}{\eta}\frac{d\eta}{dt} = 0 \qquad \text{[A10]}$$

NORMALIZED DIFFERENTIAL EQUATION

The Equation [A10] can be normalized with respect to the parameters corresponding to the maximum intensity of damage observed in experiments. At

$$t = t_1$$

$$I_e = I_{max}$$

$$\frac{dI_e}{dt} = 0$$

$$\eta = \eta_1$$

Then

$$\tau = \frac{t}{t_1} \quad ; \quad d\tau = \frac{dt}{t_1}$$

$$\bar{I} = \frac{I_e}{I_{max}} \quad ; \quad d\bar{I} = \frac{dI_e}{I_{max}}$$

$$\bar{\eta} = \frac{\eta}{\eta_1} \quad ; \quad d\bar{\eta} = \frac{d\eta}{\eta_1}$$

Since

$$\frac{dI_e}{dt} = 0 \text{ at } t = t_1$$

Equation [A10] becomes

$$\frac{kI_{max}^{\frac{2n+1}{n}}}{\eta_1^{1/n}} = \frac{I_{max}}{\eta_1}\left.\frac{d\eta}{dt}\right|_{t=t_1}$$

$$k = \frac{I_{max}^{-\left(\frac{n+1}{n}\right)}}{\eta_1^{\frac{n-1}{n}}}\left.\frac{d\eta}{dt}\right|_{t_1} \qquad [A11]$$

Now Equation [A10] may be written as

$$\frac{I_{max}}{t_1}\frac{d\bar{I}}{d\tau} + \frac{kI_{max}^{\frac{2n+1}{n}}}{\eta_1^{1/n}}\frac{\bar{I}^{\frac{2n+1}{n}}}{\bar{\eta}^{1/n}} - \frac{I_{max}}{\eta_1}\frac{\bar{I}}{\bar{\eta}}\frac{\eta_1}{t_1}\frac{d\bar{\eta}}{d\tau} = 0 .$$

Hence,

$$\frac{d\bar{I}}{d\tau} + \frac{\bar{k}\,\bar{I}^{\frac{2n+1}{n}}}{\bar{\eta}^{1/n}} - \frac{\bar{I}}{\bar{\eta}}\frac{d\bar{\eta}}{d\tau} = 0 \qquad [A12]$$

where

$$\bar{k} = \frac{k\,I_{max}^{\frac{n+1}{n}}\,t_1}{\eta_1^{1/n}}$$

(from Equation [A11])

$$= \frac{I_{max}^{-\left(\frac{n+1}{n}\right)}}{\eta_1^{\frac{n-1}{n}}} \; \frac{I_{max}^{\frac{n+1}{n}} t_1}{\eta_1^{1/n}} \left.\frac{d\eta}{dt}\right|_{t_1}$$

$$= \frac{t_1}{\eta_1} \left.\frac{d\eta}{dt}\right|_{t_1} .$$

Hence,

$$\bar{k} = \left.\frac{d\bar{\eta}}{d\tau}\right|_{\tau=1} \qquad [A13]$$

GENERAL SOLUTION OF THE NORMALIZED DIFFERENTIAL EQUATION

Let

$$z = \bar{I}^{-\left(\frac{n+1}{n}\right)}$$

$$dz = -\left(\frac{n+1}{n}\right) \bar{I}^{\frac{-2n-1}{n}} d\bar{I}$$

Equation [A12] becomes

$$\exp[-\int f_1(\bar{\eta})d\tau] = \exp[-\ln(\bar{\eta})^{(n+1)/n}]$$

$$= (\bar{\eta})^{-\frac{n+1}{n}}$$

$$\exp[\int f_1(\bar{\eta})d\tau] = (\bar{\eta})^{(n+1)/n}$$

$$F(\bar{\eta}) = \int \bar{\eta}^{\frac{n+1}{n}} \cdot \frac{n+1}{n} \frac{\bar{k}}{\bar{\eta}^{1/n}} d\tau = \bar{k} \frac{n+1}{n} \int \bar{\eta}\, d\tau$$

Hence the general solution becomes

$$z = (\bar{\eta})^{-\frac{n+1}{n}} \left[\bar{k} \frac{n+1}{n} \int \bar{\eta} d\tau + C\right]$$

$$\bar{I} = \frac{\bar{\eta}}{\left[C + \frac{n+1}{n} \bar{k} \int \bar{\eta} d\tau\right]^{\frac{n}{n+1}}} \qquad [A14]$$

Boundary conditions:

At $\tau = 1$; $\bar{I} = 1$; $\bar{\eta} = 1$.

$$-\frac{n}{n+1}\,\bar{I}^{\frac{2n+1}{n}}\frac{dz}{d\tau}+\frac{\bar{k}\,\bar{I}^{\frac{2n+1}{n}}}{\bar{\eta}^{1/n}}-\frac{\bar{I}}{\bar{\eta}}\frac{d\bar{\eta}}{d\tau}=0$$

$$\frac{dz}{d\tau}+\frac{n+1}{n}\frac{1}{\bar{\eta}}\frac{d\bar{\eta}}{d\tau}z=\frac{n+1}{n}\frac{\bar{k}}{\bar{\eta}^{1/n}}$$

This is of the form

$$\frac{dz}{d\tau}+f_1(\bar{\eta})\,z=f_2(\bar{\eta})$$

General solution of this equation is

$$z=\exp[-\int f_1(\bar{\eta})d\tau]\int[\exp\int f_1(\bar{\eta})d\tau]f_2(\bar{\eta})d\tau$$

$$+\,C\exp[-\int f_1(\bar{\eta})d\tau]=\exp[-\int f_1(\bar{\eta})d\tau][F(\bar{\eta})+C]$$

$$\int f_1(\eta)d\tau=\int\frac{n+1}{n}\frac{1}{\bar{\eta}}\frac{d\bar{\eta}}{d\tau}d\tau$$

$$=\frac{n+1}{n}\ln\bar{\eta}$$

$$=\ln(\bar{\eta})^{\frac{n+1}{n}}$$

Hence

$$C + \frac{n+1}{n}\bar{k}\int_{o}^{1}\bar{\eta}d\tau = 1$$

$$C = 1 - \frac{n+1}{n}\bar{k}\int_{o}^{1}\bar{\eta}d\tau$$

Hence [A14] becomes

$$\bar{I} = \frac{\bar{\eta}}{\left[1 + \left(\frac{n+1}{n}\right)\bar{k}\left\{\int_{o}^{\tau}\bar{\eta}d\tau - \int_{o}^{1}\bar{\eta}d\tau\right\}\right]^{\frac{n}{n+1}}}$$

$$\bar{I} = \frac{\bar{\eta}}{\left[1 + \frac{n+1}{n}\bar{k}\int_{1}^{\tau}\bar{\eta}d\tau\right]^{\frac{n}{n+1}}} \qquad \text{[A15]}$$

If n = 2

$$\bar{I} = \frac{\bar{\eta}}{\left[1 + \frac{3}{2}\bar{k}\int_{1}^{\tau}\bar{\eta}d\tau\right]^{2/3}}$$

When Weibull type distributions are used,

$$\eta = 1 - \exp(-\tau^{\alpha}) \qquad [A16]$$

At $\tau = 1$

$$\eta_1 = 1 - \frac{1}{e} = \frac{e-1}{e}$$

$$\bar{\eta} = \frac{e}{e-1}[1 - \exp(-\tau^{\alpha})]$$

$$\frac{d\bar{\eta}}{d\tau} = \frac{e-1}{e}[\alpha\tau^{\alpha-1} \exp(-\tau^{\alpha})]$$

At $\tau = 1$

$$\left.\frac{d\bar{\eta}}{d\tau}\right|_{\tau=1} = \frac{e}{e-1} \cdot \frac{\alpha}{e}$$

Hence

$$\bar{k} = \frac{\alpha}{e-1}$$

$$\bar{I} = \frac{\frac{e}{e-1}[1 - \exp(-\tau^{\alpha})]}{\left[1 + \frac{1.5\alpha}{e-1}\int_1^{\tau} \frac{e}{e-1}[1 - \exp(-\tau^{\alpha})d\tau\right]^{2/3}} \qquad [A17]$$

From Equation [A17], $\bar{I}$ is calculated as a function of τ for various values of α as shown in Figure 4. Figures 5 and 6 show the same relationships when n = 3 and 4 respectively. When Gumbel type distributions are used

$$\eta = \exp[-\beta \exp(-\tau)]$$

$$\eta_1 = \exp[-\frac{\beta}{e}]$$

$$\bar{\eta} = \frac{\exp[-\beta \exp(-\tau)]}{\exp\left[-\frac{\beta}{e}\right]}$$

$$\bar{k} = \left.\frac{d\bar{\eta}}{d\tau}\right|_{\tau=1} = \frac{1}{\exp\left[-\frac{\beta}{e}\right]} \frac{\beta}{e} \left[\exp\left(-\frac{\beta}{e}\right)\right]$$

$$\bar{k} = \frac{\beta}{e}$$

Hence

$$\bar{I} = \frac{\bar{\eta}}{\left[1 + \frac{3}{2}\frac{\beta}{e}\int_1^{\tau} \bar{\eta} d\tau\right]^{2/3}} \qquad [A18]$$

Figure 15 shows the variation of $\bar{I}$ with τ for three values of β.

CUMULATIVE DEPTH OF EROSION

$$\bar{I} = \frac{I_e}{I_{max}} = \frac{S_e}{I_{max}} \cdot \frac{dr}{dt} \quad ; \quad I_{max} = S_e \cdot \left(\frac{dr}{dt}\right)_{max}$$

Boundary conditions,

$$\text{at } t = t_1,\ r = r_1;\ \bar{r} = r/r_1;\ \tau = t/t_1\ .$$

Then

$$\bar{I} = \frac{S_e}{I_{max}} \frac{r_1}{t_1} \frac{d\bar{r}}{d\tau}$$

Hence

$$\int \frac{S_e r_1}{t_1 I_{max}}\, d\bar{r} = \int \bar{I}\, d\tau + C$$

$$\frac{S_e r_1}{t_1\ I_{max}}\, \bar{r} = \int_0^{\tau} \bar{I}\, d\tau + C \qquad [A19]$$

Boundary conditions,

$$t = 0\ ;\ r = 0\ ;\ \tau = 0\ ;\ \bar{I} = 0\ ;\ \bar{r} = 0\ .$$

Hence

$$C = 0.$$

Again at

$$t = t_1, \; \tau = 1, \; \bar{r} = 1$$

$$\frac{S_e r_1}{t_1 \, I_{max}} = \int_0^1 \bar{I} \, d\tau$$

Equation [A19] becomes

$$\bar{r} = \frac{\int_0^\tau \bar{I} \, d\tau}{\int_0^1 \bar{I} \, d\tau} \qquad [A20]$$

The variation of $\bar{r}$ with τ for various values of α when $n = 2$ is shown in Figure 10.

RELATIONSHIP BETWEEN THE MAXIMUM INTENSITY OF EROSION AND THE INTENSITY OF COLLAPSE

From Equation [A11]

$$\frac{n}{\left(A^n I_c\right)^{1/n} S_e} = \frac{I_{max}^{-\frac{(n+1)}{n}}}{\eta_1^{\frac{n-1}{n}}} \; \frac{\eta_1}{t_1} \frac{d\bar{\eta}}{d\tau}\bigg|_{\tau=1}$$

Using Weibull type distributions and the value n = 2

$$\frac{2}{(A^2 I_c)^{\frac{1}{2}} S_e} = \frac{I_{max}^{-3/2} \left(\frac{e-1}{e}\right)^{\frac{1}{2}} \alpha}{t_1 (e-1)}$$

$$I_{max}^{3/2} = \frac{\alpha}{2[e(e-1)]^{\frac{1}{2}}} \frac{I_c^{1/2} S_e A}{t_1} \qquad [A21]$$

RELATIONSHIP BETWEEN EROSION STRENGTH AND MAXIMUM RATE OF EROSION

From Equation [A21]

$$(\dot{r}_{max})^{3/2} S_e^{3/2} = M I_c^{1/2} \frac{A S_e}{t_1}$$

where

$$M = \frac{\alpha}{[e(e-1)]^{\frac{1}{2}}}$$

$$\dot{r}_{max} = \left.\frac{dr}{dt}\right|_{max} \quad \text{at } t = t_1$$

Hence

$$S_e = \frac{A^2 I_c M^2}{t_1^2 (\dot{r})^3_{max}} \qquad [A22]$$

REFERENCES

1. Thiruvengadam, A., "A Unified Theory of Cavitation Damage," Trans. ASME, Vol. 85, Jour. Basic Engineering, September 1963.

2. Thiruvengadam, A., "A Comparative Evaluation of Cavitation Damage Test Devices," HYDRONAUTICS, Incorporated Technical Report 233-2, November 1963 (See also Proc. Symp. on Cavitation Research Facilities and Techniques, ASME Publications, New York, May 1964).

3. Thiruvengadam, A., "Intensities of Cavitation Damage Encountered in Field Installations," HYDRONAUTICS, Incorporated Technical Report 233-7, February 1965. (See also Proc. Symp. on Cavitation in Fluid Machinery, ASME Winter Annual Meeting, ASME Publications, New York, November 1965).

4. Thiruvengadam, A., "The Concept of Erosion Strength," HYDRONAUTICS, Incorporated Technical Report 233-9, December 1965. (See Proc. Symp. on Erosion by Cavitation or Impingement, ASTM Annual Meeting 1966, ASTM Special Publication No. 408, Philadelphia, 1967).

5. Thiruvengadam, A. and Preiser, H. S., "On Testing Materials for Cavitation Damage Resistance," HYDRONAUTICS, Incorporated Technical Report 233-3, October 1963. (See also Jour. Ship Research, Vol. 8, No. 3, December 1964).

6. Waring, S., Preiser, H. S., and Thiruvengadam, A., "On the Role of Corrosion on Cavitation Damage," HYDRONAUTICS, Incorporated Technical Report 233-4, February 1964 (See also Jour. Ship Research, Vol. 9, No. 3, December 1965).

7. Thiruvengadam, A., and Waring, S., "Mechanical Properties of Metals and Their Cavitation Damage Resistance," HYDRONAUTICS, Incorporated Technical Report 233-5, June 1964. (See also Jour. Ship Research, Vol. 10, No. 1, March 1966).

8. Eisenberg, P., Preiser, H. S., and Thiruvengadam, A., "On The Mechanisms of Cavitation Damage and Methods of Protection," Trans. SNAME, Vol. 73, 1965.

9. Plesset, M. S., and Devine, R. E., "Effect of Exposure Time on Cavitation Damage," Trans. ASME, Jour. Basic Engr., December, 1966.

10. Hobbs, J. M., "Experience with a 20 kc/s Cavitation Erosion Test," Symp. on Erosion by Cavitation or Impingement, ASTM Annual Meeting, Atlantic City, June 1966.

11. Smith, A., Kent, R.P., and Armstrong, R. L., "Erosion of Steam Turbine Blade Shield Materials," Symp. on Erosion by Cavitation or Impingement, ASTM Annual Meeting, Atlantic City, June 1966 (See also Proc. Discussion on Deformation of Solids by the Impact of Liquids, Phil. Trans. Royal Society of London, Vol. 260, July 1966, p. 209).

12. Baker, D.W.C., Jolliffe, K. H., and Pearson, D., "The Resistance of Materials to Impact Erosion Damage," Ibid,p.193.

13. Marriott, J. B., and Rowden, G., "The Erosion of a Cobalt-Chromium Alloy by Liquid Impact," Ibid p. 144.

14. Busch, H., Hoff, G. and Langbein, G., "Rain Erosion Properties of Materials," Ibid p. 168 (See also Proc. Symp. Erosion by Cavitation or Impingement, ASTM Special Publication 408, Philadelphia, 1967.

15. Heymann, F. J., "On the Time Dependance of the Rate of Erosion Due to Liquid Impact or Cavitation," Proc. Symp. Erosion by Cavitation or Impingement, ASTM Special Publication 408, Philadelphia, 1967.

16. Thiruvengadam, A., "On Modeling Cavitation Damage," HYDRONAUTICS, Incorporated Technical Report 233-10, August 1966.

17. Cole, R. H., "Under Water Explosions," Princeton University Press, Princeton, New Jersey, 1948.

18. Weibull, W., "Fatigue Testing and Analysis of Results," Pergamon Press, New York, 1961.

19. A Guide for Fatigue Testing and the Statistical Analysis of Fatigue Data (Supplement to Manual on Fatigue Testing, STP No. 91) ASTM Special Technical Publication No. 91-A (Second Edition, Philadelphia, 1963).

20. Thiruvengadam, A., Discussion of the paper by Donald Colgate on "Cavitation Damage of Roughened Concrete Surfaces," Jour. Hyd. Div. Proc. ASCE, Vol. 86, No. Hy.4, April 1960.

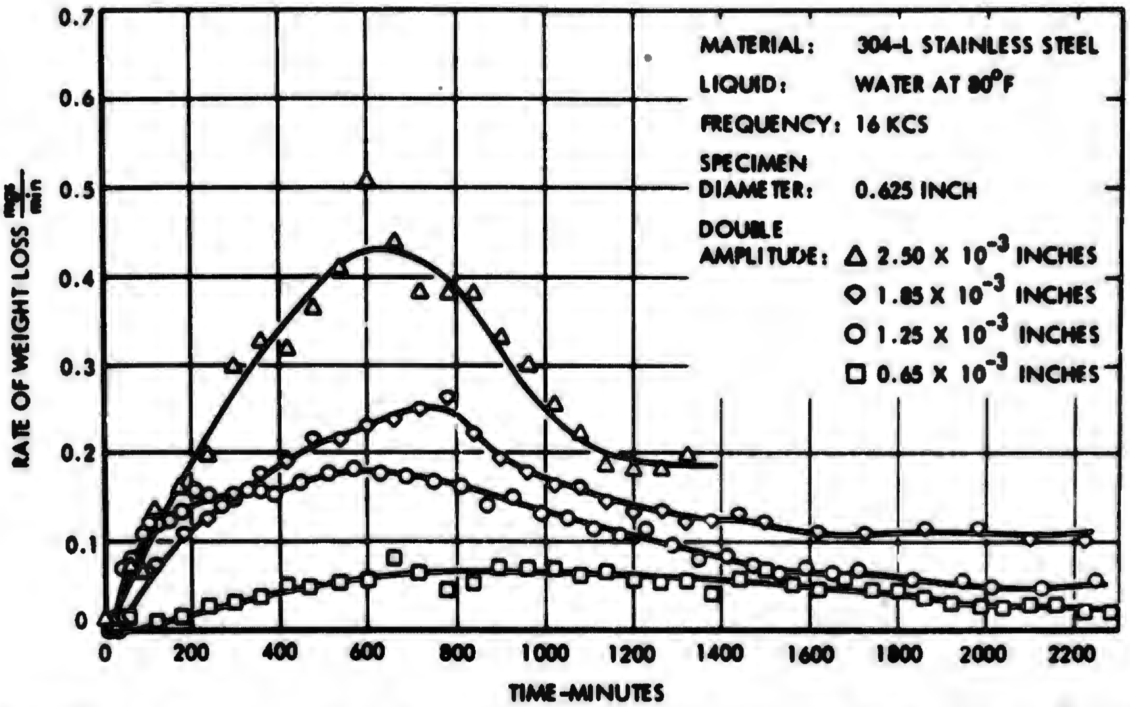

FIGURE 1 - EFFECT OF TIME ON CAVITATION DAMAGE RATE FOR VARIOUS AMPLITUDES (REFERENCE 5)

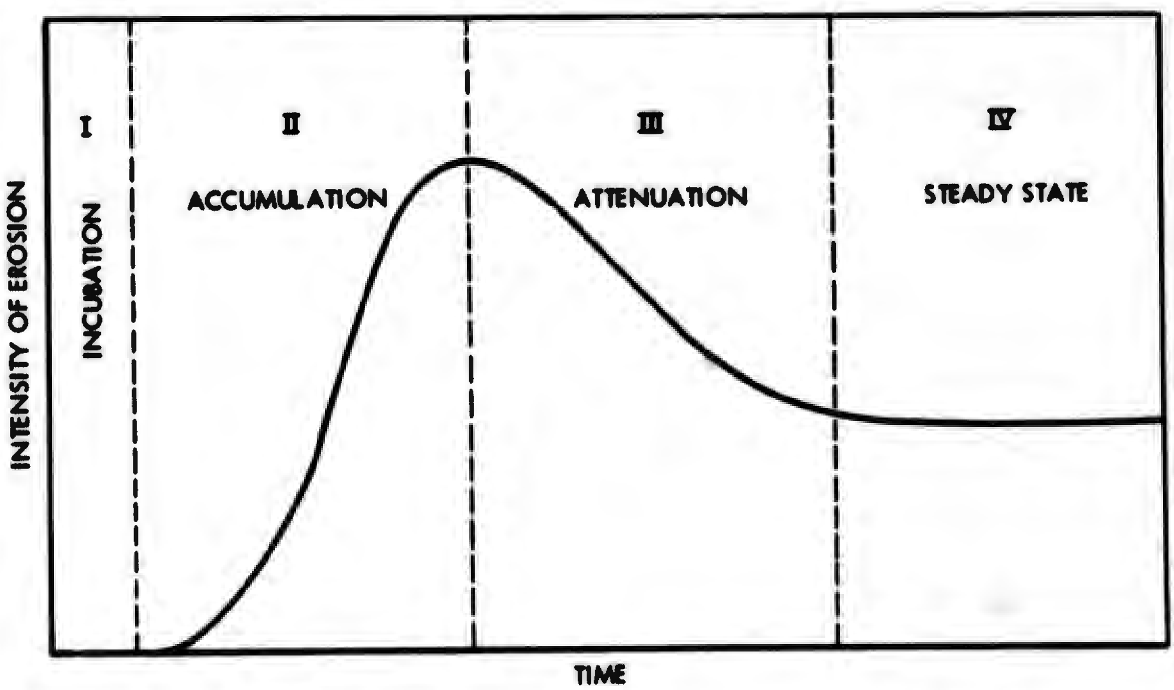

FIGURE 2 - EFFECT OF TIME ON INTENSITY OF EROSION

BLANK PAGE

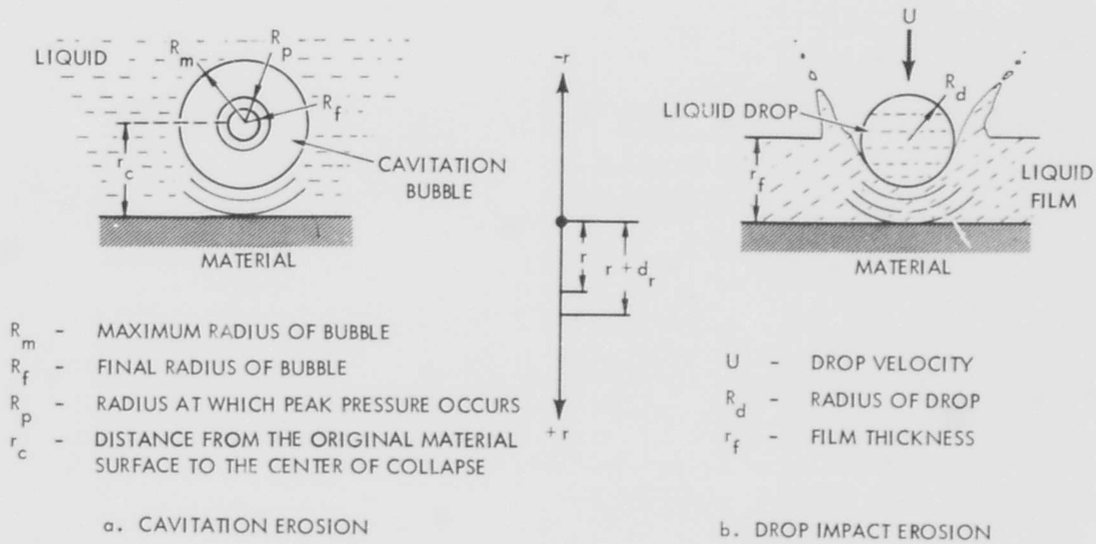

FIGURE 3 - DEFINITION SKETCH

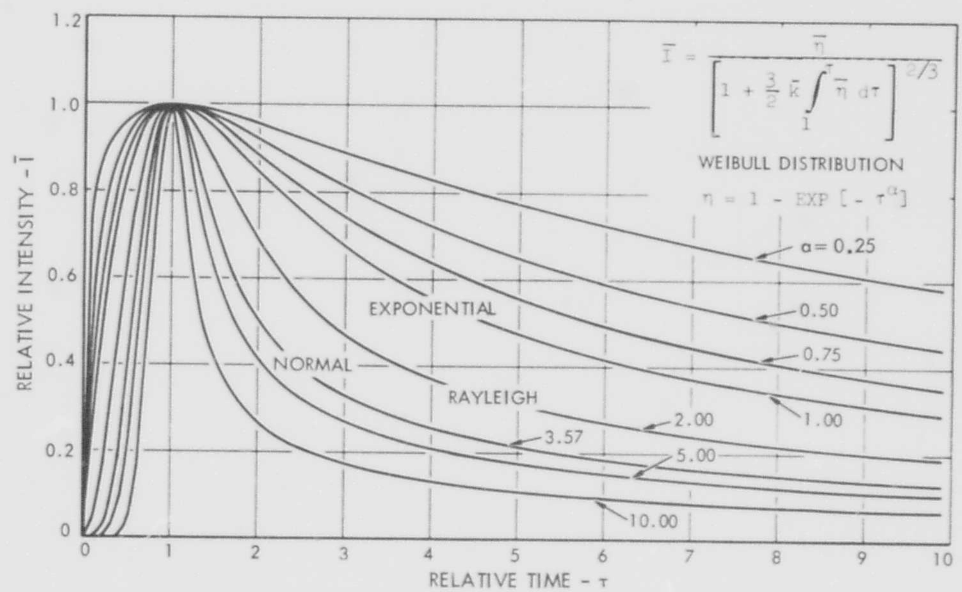

FIGURE 4 - THEORETICAL PREDICTION OF THE EFFECT OF TIME ON INTENSITY OF EROSION WHEN $n = 2$

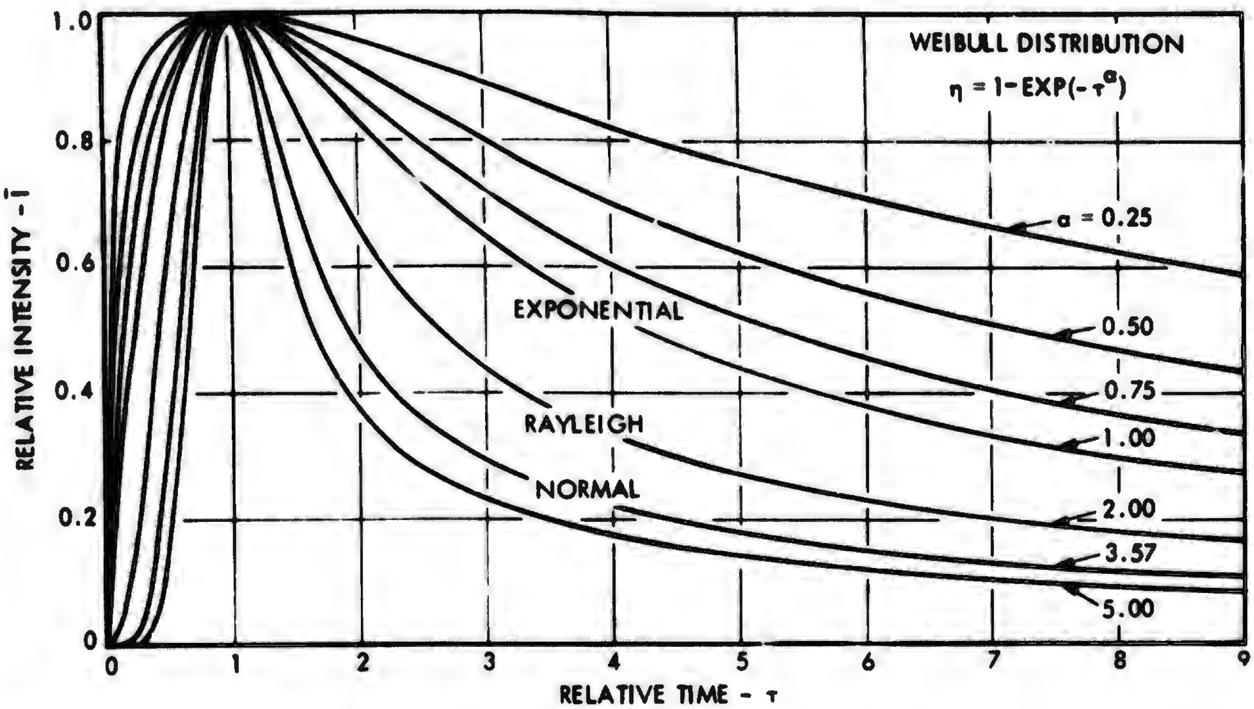

FIGURE 5 - THEORETICAL PREDICTION OF THE EFFECT OF TIME ON INTENSITY OF EROSION WHEN $n = 3$

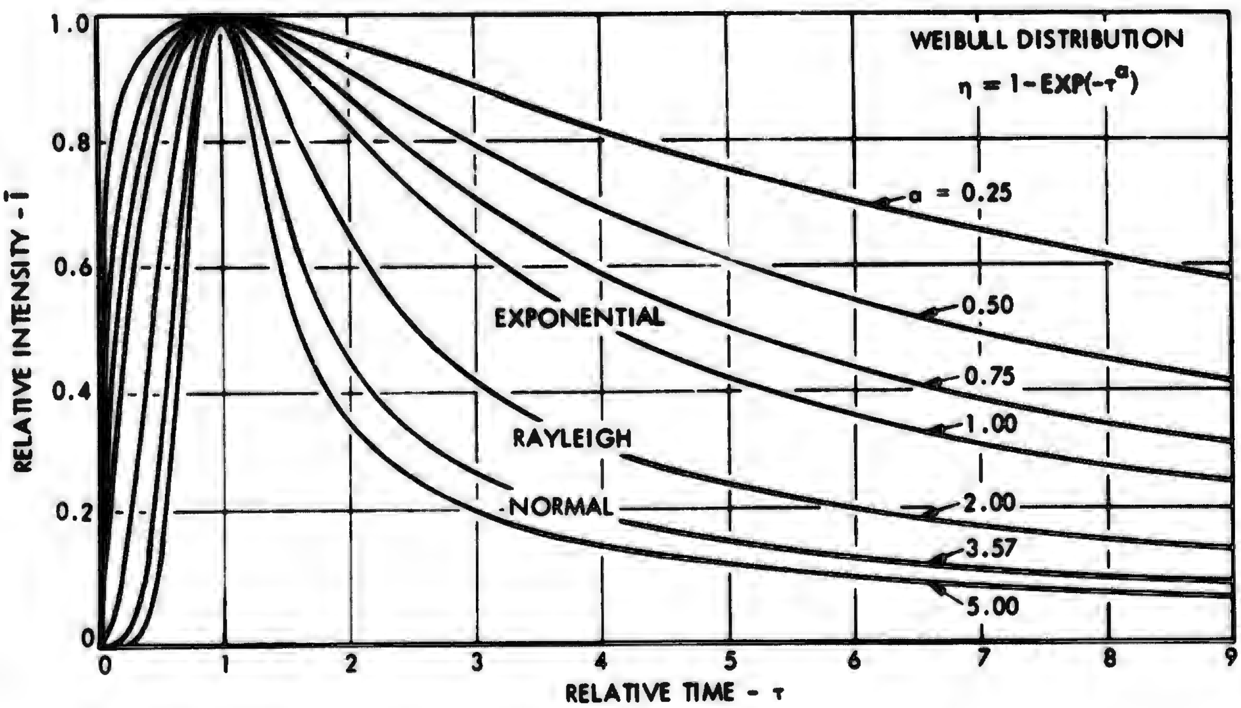

FIGURE 6 - THEORETICAL PREDICTION OF THE EFFECT OF TIME ON INTENSITY OF EROSION WHEN $n = 4$

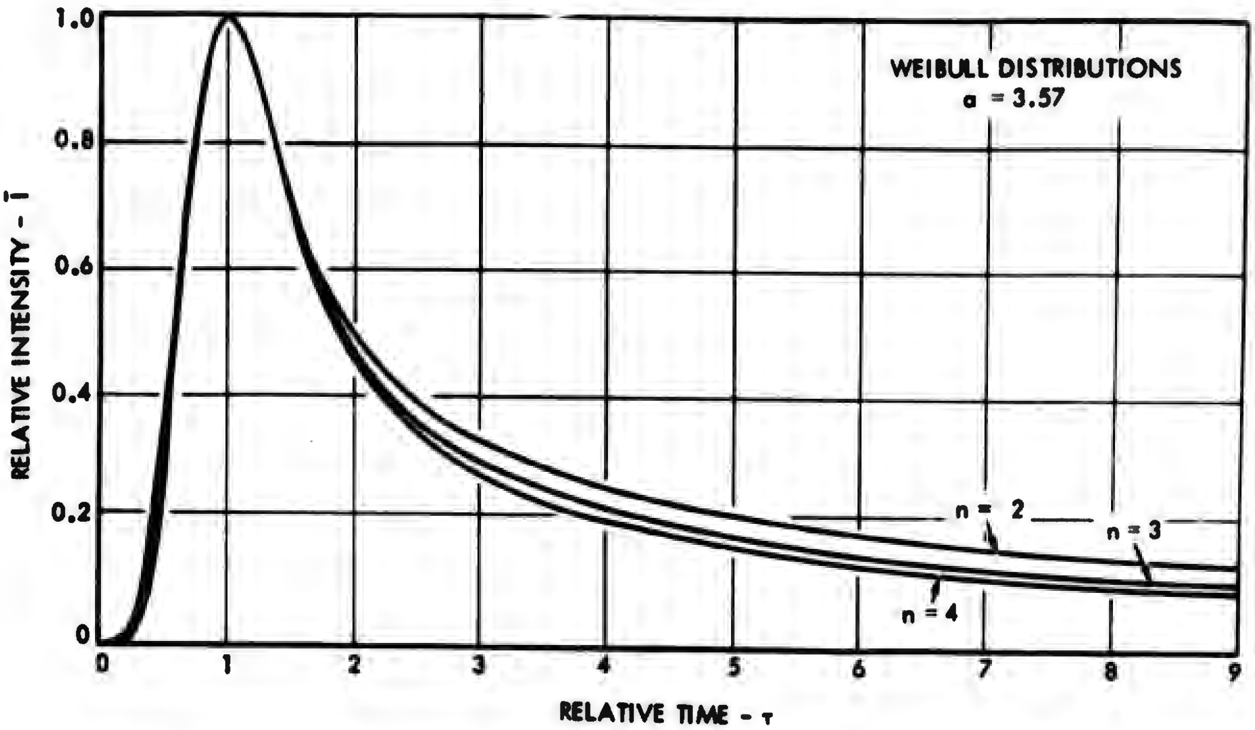

FIGURE 7 - COMPARISON OF THEORETICAL PREDICTIONS FOR THREE DIFFERENT ATTENUATION EXPONENTS

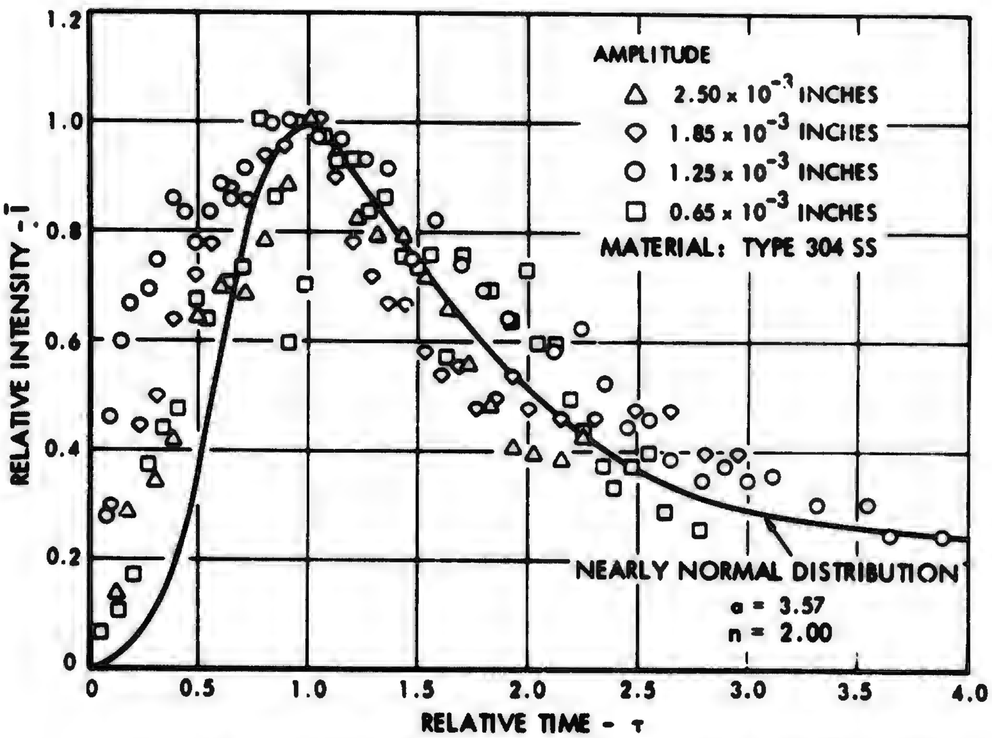

FIGURE 8 - COMPARISON OF THEORY AND EXPERIMENTS

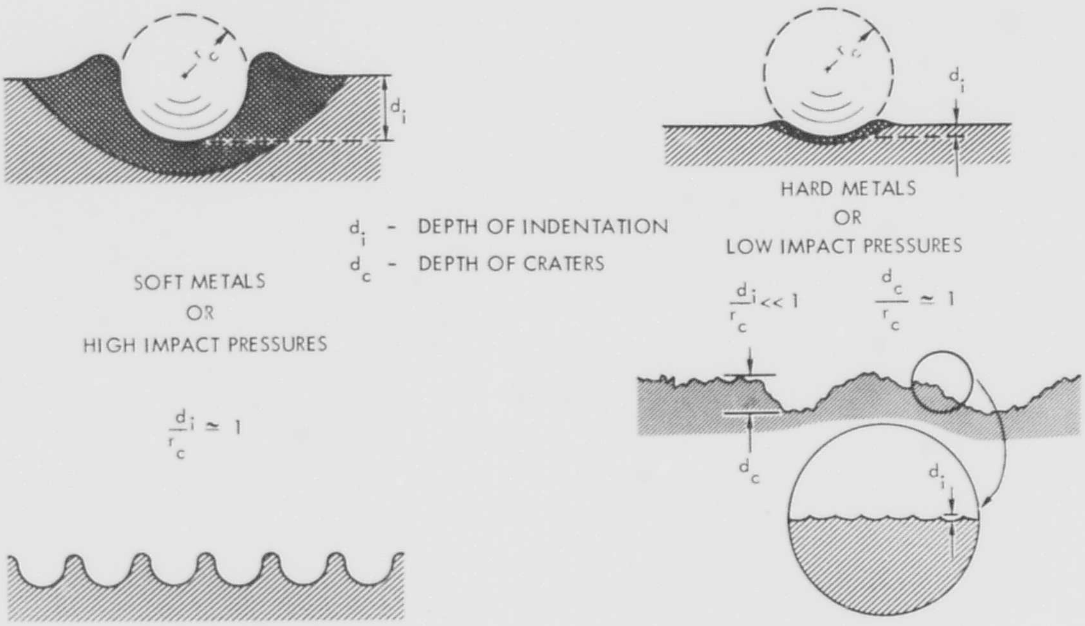

FIGURE 9 - POSSIBLE SURFACE ROUGHNESS PATTERNS IN METALS

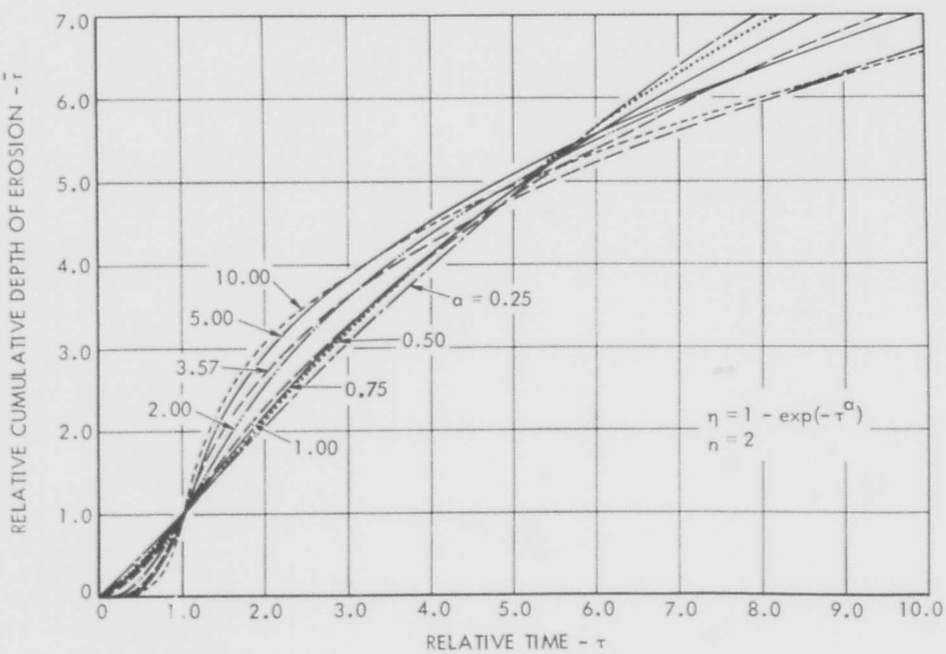

FIGURE 10 - THEORETICAL PREDICTIONS OF THE CUMULATIVE DEPTH OF EROSION

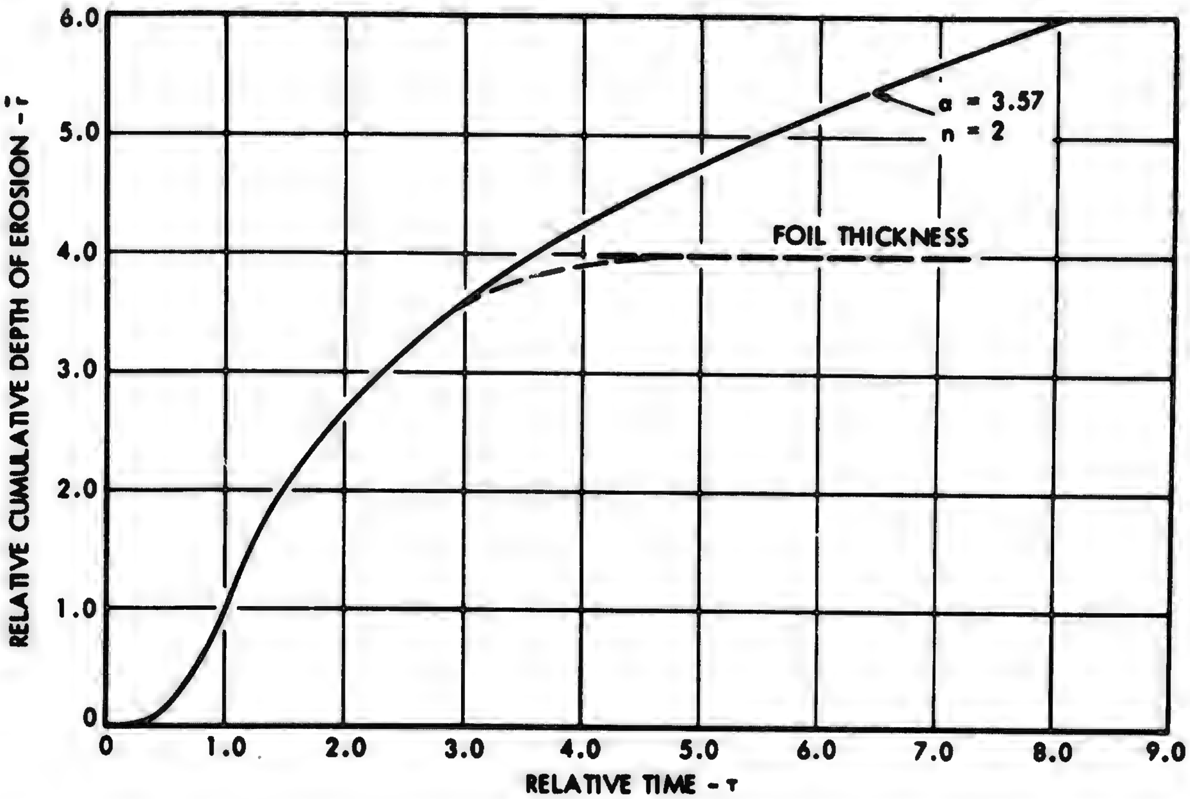

FIGURE 11 - ADDITIONAL CONSIDERATIONS SUCH AS FOIL THICKNESS MAY ALSO CONTROL THE CUMULATIVE DEPTH OF EROSION

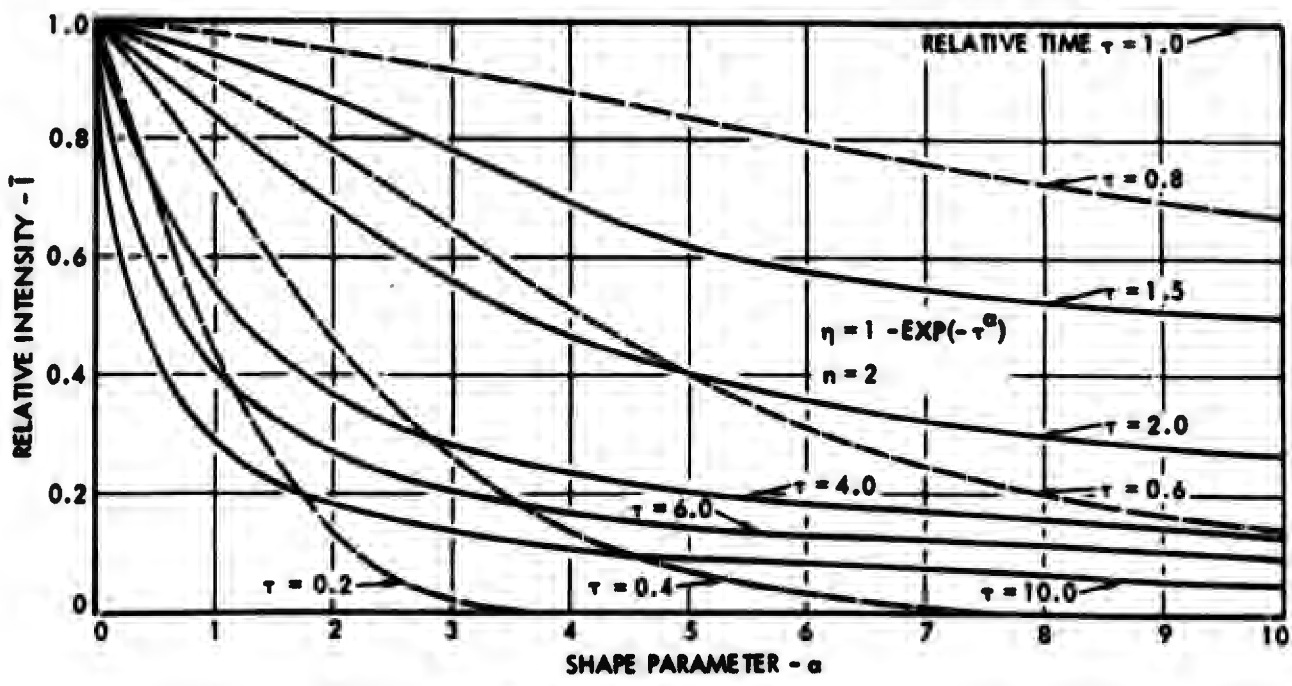

FIGURE 12 - CURVES TO DETERMINE THE SHAPE PARAMETER - α

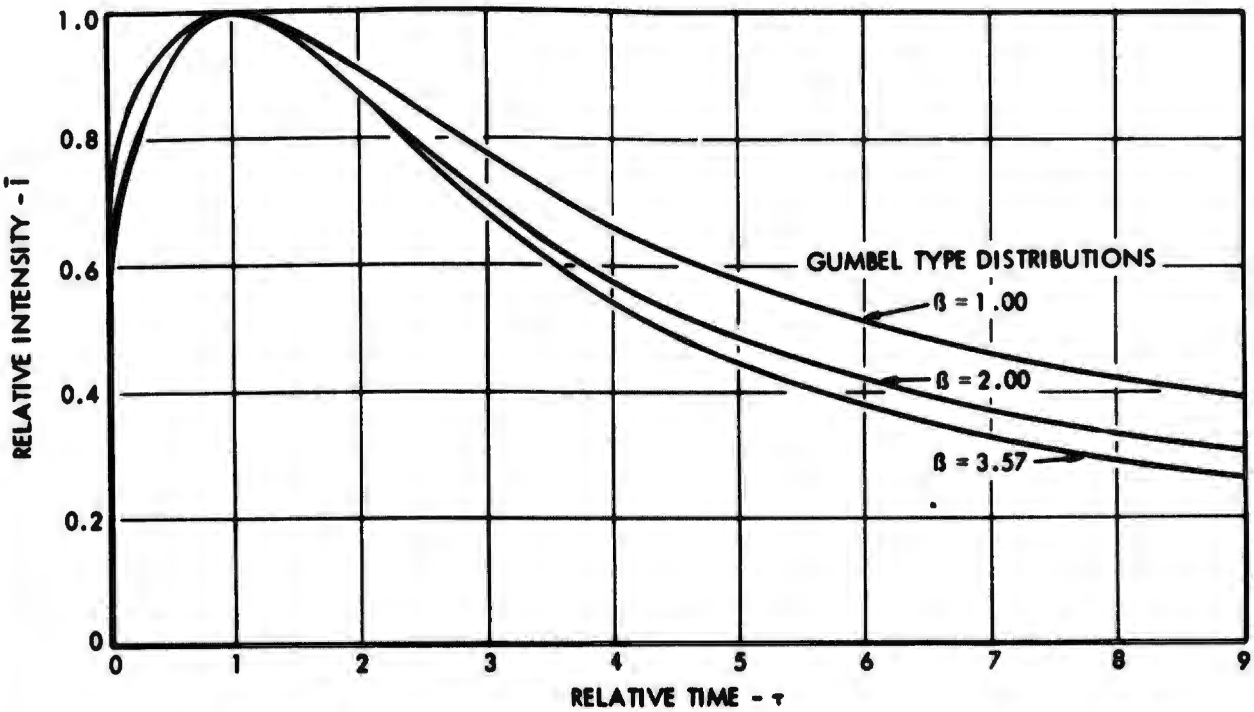

FIGURE 15 - THEORETICAL PREDICTIONS OF THE EROSION INTENSITY WHEN GUMBEL TYPE DISTRIBUTIONS ARE USED

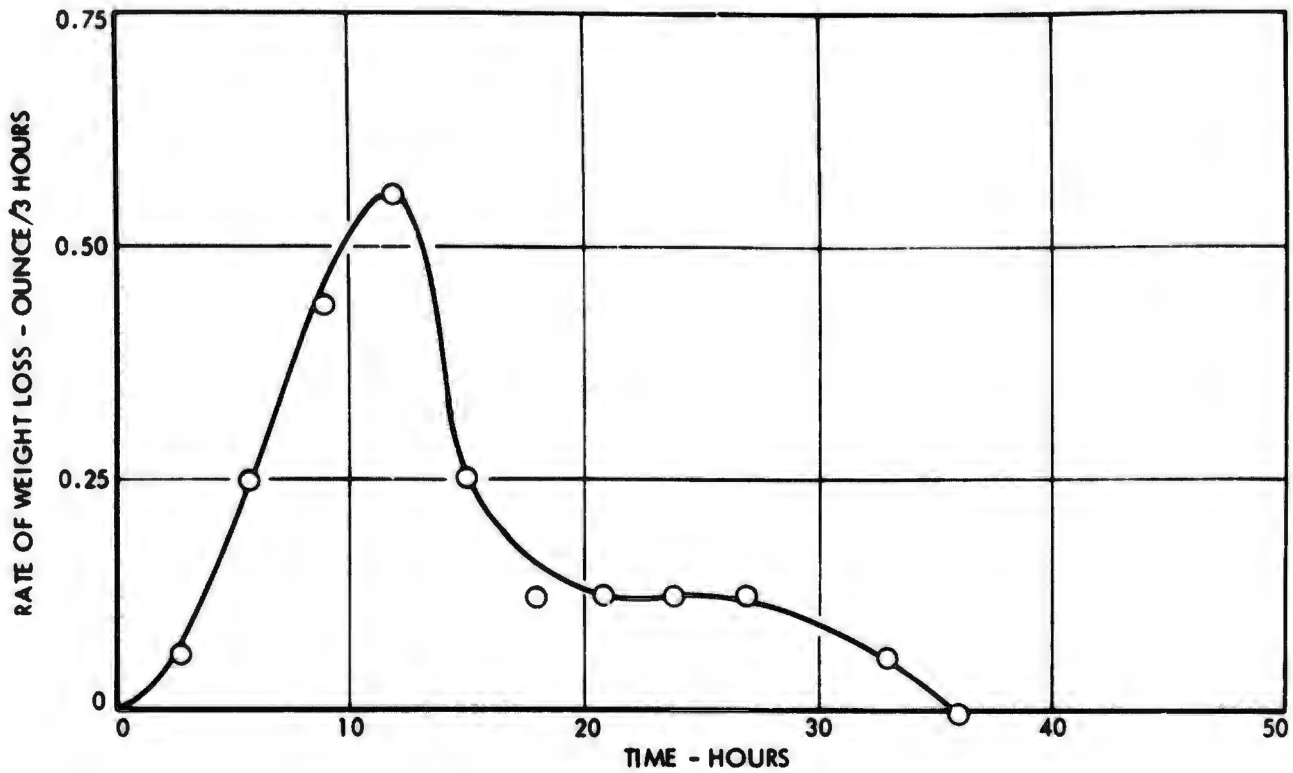

FIGURE 13 - TIME EFFECTS ON POLISHED ROCK (20)

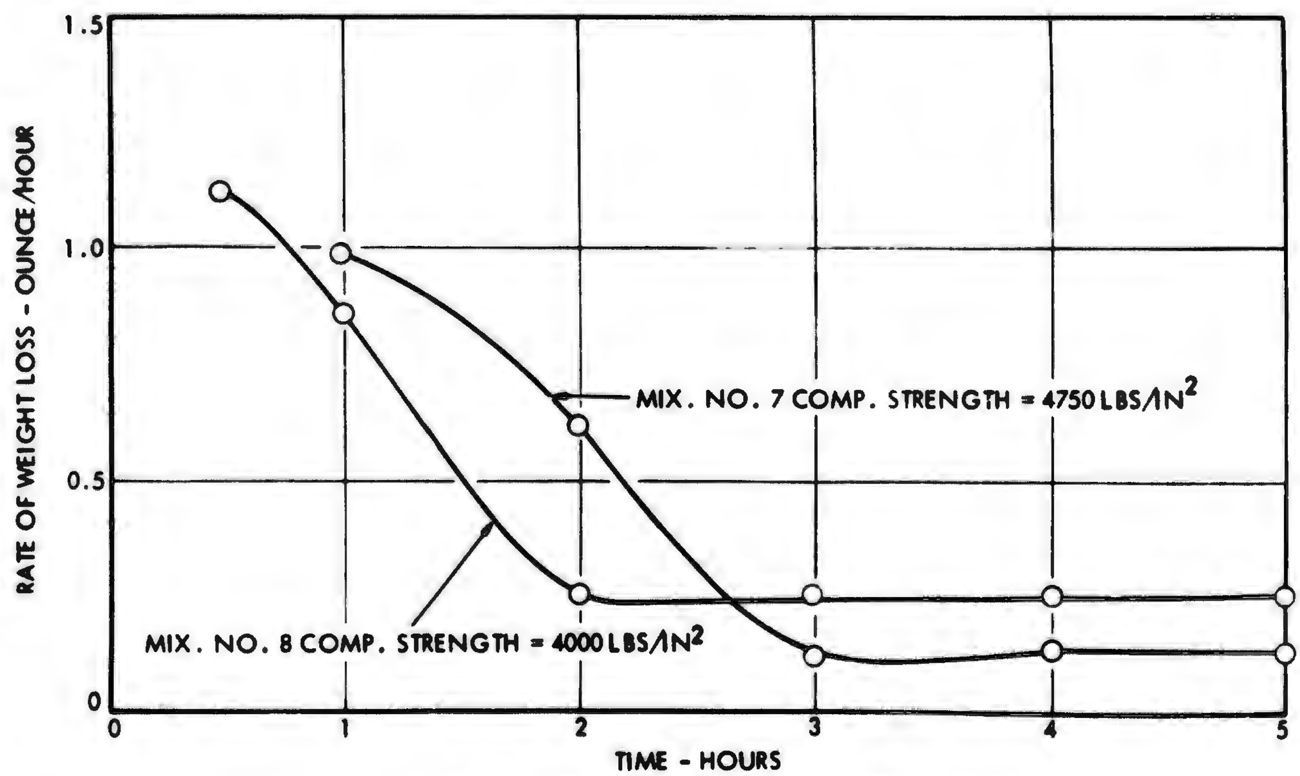

FIGURE 14 - TIME EFFECTS ON CONCRETE (20)

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)
HYDRONAUTICS, Incorporated
Pindell School Road, Howard County,
Laurel, Maryland

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE

THEORY OF EROSION

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Technical Report

5. AUTHOR(S) (Last name, first name, initial)

Thiruvengadam, A.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
April 1967	50	20

8a. CONTRACT OR GRANT NO.
Nonr 3755(00)(FBM) NR 062-293

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
Technical Report 233-11

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

[redacted]

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
Office of Naval Research
Department of the Navy

13. ABSTRACT

An elementary theory of erosion is derived based on the assumptions of "accumulation" and"attenuation" of the energies of impact causing erosion. This theory quantitatively predicts the relative intensity of erosion as a function of relative time and this prediction is in fair agreement with experimental observations. Since the intensity of collision, the distance of shock transmission and the material failure are all statistical events, a generalization of the elementary theory is suggested.

Some of the practical results of this theory are the predictions of the cumulative depth of erosion, the determination of erosion strength and the method of correlation with other parameters such as liquid properties and hydrodynamic factors. Modifications of this theory for brittle and viscoelastic materials are also suggested.

Note: An abstract of this report was published by the Jet Propulsion Laboratory, California Institute of Technology, Pasadena following a lecture on this subject on December 30, 1966 at the Turbine Erosion Conference organized by JPL.

BLANK PAGE

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Cavitation Erosion Turbine Erosion Rain Erosion Intensity of Erosion Duration of Erosion Erosion Strength Prediction of Erosion Cumulative depth of erosion Weibull distribution Gumbull distribution Metals, ceramics, polymers						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.