TECHNICAL REPORT 490-3

THE DESIGN OF TANKS FOR USE IN AN ACTIVE TANK SYSTEM

By

William C. Webster

July 1967

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HYDRONAUTICS, incorporated
research in hydrodynamics

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NOTATION

τ	Tank angle, rad.
R	Distance to tank center from ship centerline, ft
ρ_t	Density of tank water, slug/ft^3
$A(s)$	Cross-section area as a function of s, ft^2
A_o	Cross-section area of tank vertical legs $= b_1 \times b_2$, ft^2
S'	Effective length of U-tube, $S' = \int_o^{L_t} \frac{A_o}{A(s)} ds$, ft.
S''	Effective coupled length, $S'' = \int_o^{L_t} \frac{t_t(s)}{R} ds$, ft.
ω_t	Tank natural frequency, $\omega_t^2 = 2g/S'$, rad/sec.
b_1	Width of each tank, $C_{b_1} = b_1/B$, ft.
b_2	Length of tanks (in x-direction), ft.
ℓ	Half length of horizontal crossover, ft.
C_t	Fraction of horizontal leg with taper
$C_{a\ell} = a/\ell$	
H	Height of water in tank, ft.
a	Distance of horizontal leg centerline below main tank bottom, $C_a = a/B$, ft.
e	Length of pump cylindrical section, $C_e = e/dp$, ft.

d_p	Diameter of pump section, ft.
W_T	Weight of water in tank system, lbs.
s	Curvalinear distance along centerline of tank, measured from water's surface, ft.
$\bar{\zeta}_t$	Equivalent tank damping ratio, $\left(\bar{\zeta}_t = \frac{\omega_t R C'_{\ell_{total}}}{8g} \left\|\tau\right\|_{ave.}\right)$
$C'_{\ell_{total}}$	Loss coefficient in heads of velocity $\frac{V^2}{2g}$

$$C_\ell' = \sum C'_{\ell_i} = \sum C_{\ell_i} \left(\frac{A_o}{A(s_i)}\right)^2$$

f	Friction factor
D	Characteristic diameter, ft.
V	Velocity, usually taken as a mean effective velocity in the conduit, ft/sec.
r	Hydraulic radius, ft.

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INTRODUCTION

The control of activated, anti-roll tanks was investigated in great detail in a recent report by Webster and Dogan (1). In this report, a detailed analysis is made of the equations of motion, the seaway forces exerted on the ship, the stability of the control system, etc. Because the design of the controller was of primary importance for that study, little consideration was given to the precise configuration of the tank. In the development of the equations of motion, it was shown that five parameters characterized the tank. These were:

1. η_t, the ratio of natural roll frequency of the ship, ω_s, to the natural frequency of pendulation of the tank, ω_t.

2. k_{st}, the fractional decrease in $\overline{GM}$ caused by the existence of the free surface of the tanks.

3. τ_{max}, the maximum tank angle which can be achieved before tank saturation occurs.

4. η_{st}^2 the ratio of the square of the ship's roll frequency to the square of the so-called ship-tank coupling frequency.

5. The tank damping. For the case of the linear studies, this resulted in an equivalent linear damping ratio of the tank, $\bar{\zeta}_t$.

As a result of the study of the control, it was determined that it is advantageous to make η_t less than unity, k_{st} as large as possible (consistent with static stability requirements) and τ_{max} sufficiently large so that significant tank saturation does not occur in frequently encountered sea states. Further, it was shown that it is advantageous to make the inherent tank damping, ζ_t, as small as possible. The relatively high value of apparent tank damping is then obtained due to the action of the pump. In this way, the pumping power required is minimized. The parameter η_{st}^2 is affected by the vertical location of the tank. The higher the tank, the more negative and the more advantageous is the value of η_{st}^2. However, for typical installations the value of this parameter is always near zero. The effect of the vertical location is secondary in importance and, as a result, the tank is usually located in a place amenable to the ships overall arrangement. It is anticipated that the actual values of these parameters will be selected on the basis of the control, as presented in (1). The purpose of this report is to present techniques for the selection of the physical tank dimensions necessary to match these parameters. The design material presented in the following sections is for U-tube tanks since they are especially amenable to the installation of the pump in the horizontal cross-over duct. Other tank types, such as the free surface tank (see, for instance Vasta et al, Reference 3) do not lend themselves to such installations.

ELEMENTS OF U-TUBE TANK FLUID DYNAMICS

Equations of Motion

The following review of U-tube tank fluid dynamics is presented in an abbreviated fashion and only in sufficient detail as is needed for the design of these tanks. The basic mechanics of these tanks is well known and is presented in a variety of sources (Chadwick (2), Vasta, et al (3), Blagoveschensky (4)).

We consider first the motion of fluid in a generalized U-tube shown in Figure 1. It will be assumed that the radii of curvature of the bends in the pipe are large enough so that the motion of the fluid in the tank can be considered to be one-dimensional and to act along the centerline of tube cross-section (i.e. to act everywhere in the local direction of S shown in the figure). If the tank is assumed symmetric about the ship's centerplane, then the equation of motion of the tank fluid, assuming no motion of the ship or pumping within the tank, is

$$\rho_t R S' \overset{''}{\tau} + \rho_t g h (R\overset{'}{\tau}) + 2\rho_t g R\tau = 0 \qquad [1]$$

where

τ is the tank angle measured as shown in Figure 1,

ρ_t is the density of fluid in the tank,

g is the acceleration of gravity,

$S' = \int_0^L \frac{A_o ds}{A(s)}$, the effective length of the U-tube ,

A_o is the cross-section area of each vertical leg,

$A(s)$ is the local tube cross-section at a distance S along the tube centerline,

L is the length of the tube centerline, and

$h(R\dot{\tau})$ is the total head loss function.

Several assumptions are implicit in Equation [1] which are worth noting. First it is assumed that the head loss through the U-tube is proportional to the velocity of the fluid in each vertical leg, given by $R\dot{\tau}$. As a result of continuity, the fluid velocity at any location, s, in the tank is proportional to this velocity. The assumption is then that the head losses are somehow related to the local velocities.

The second implicit assumption embodies in [1] is that the side legs are vertical. No essential difficulty is encountered with other configurations but, for ease in presentation only vertical legged tanks are considered. Further, it is assumed that these vertical legs have a top on them which prevents the tank fluid from spilling over during severe pendulations. As a result, there will exist a maximum tank angle, τ_{max}, which the water can assume before the water slams against the tank top. When this happens, no further motion occurs in the tank and Equation [1] does not apply.

From Equation [1], we see that the natural frequency of the pendulation of the fluid in the tank, ω_t, is given by

$$\omega_t = \sqrt{\frac{2g}{S'}} \qquad [2]$$

Because of the nonlinear character of the losses, the damping of the motion cannot be characterized by a single exponential decay factor. However, by using the process of equivalent linearization, [1] can be expressed with the use of [2] as

$$\frac{2\rho_t gR}{\omega_t^2} \left[\ddot{\tau} + 2\bar{\zeta}_t\omega_t\dot{\tau} + \omega_t^2\tau\right] = 0 \qquad [3]$$

where

$$\bar{\zeta}_t = \frac{\omega_t}{4}\left\{\left|\frac{h(R\dot{\tau})}{(R\dot{\tau})}\right|\right\}_{ave} \text{, the equivalent linear damping ratio} \qquad [3a]$$

In the above equation, we see that $\bar{\zeta}_t$ is proportional to the mean of the absolute value of the ratio $h(R\dot{\tau})$ to $R\dot{\tau}$. For typical cases, $h(R\dot{\tau})$ is proportional to $(R\dot{\tau})^2$ and this average value of the bracketed quantity varies linearly with the average value of $\dot{\tau}$. Thus for each amplitude of motion we can associate an equivalent linear damping ratio.

So far, we have discussed the pendulation of the tank when no external forces are acting upon it. However, the ship's motions and, in the case of the activated tanks considered here, the pump dynamics cause the tank fluid to be constantly in motion. The complete equations of motion for the coupled ship-tank-pump system are derived in detail by Webster and Dogan (1) and will not be reproduced here. In that report it was shown that two other tank parameters which arise as a result of coupling effects are of significance in addition to the tank frequency and damping. These parameters are:

$$k_{st} = \frac{2\rho_t g R^2 A_o}{\Delta\ \overline{GM}}$$, the fractional decrease in $\overline{GM}$ due to the existence of the tank,

$$\eta_{st}^2 = \frac{\omega_s^2}{2g} \int_0^L \frac{t_t}{R} ds,$$ the ship-tank coupled frequency parameter (see Figure 1),

where

Δ is the ship's displacement (in pounds), and

$\overline{GM}$ is the ship's metacentric height.

In the above mentioned report it was shown that it is advantageous to have k_{st} as large as possible, corresponding to as large a tank as possible. However, since $k_{st} = 1$ corresponds to a complete loss of static roll stability for the ship, this figure represents an upper bound on k_{st}. Typically, values of k_{st} from 0.2 to 0.4 are used. The exact choice of k_{st} most frequently depends on the maximum loss of $\overline{GM}$ permissible for a given design in order to meet safety requirements.

The parameter η_{st}^2 reflects the influence of the vertical location of the tank. For tanks located high in the ship η_{st}^2 is negative; for those low in the ship η_{st}^2 is positive. The principal effect characterized by η_{st}^2 is the high frequency response of the tank. Changes of performance of the order of 10 percent improvement for high tanks and 10 percent degradation for tanks located very low in the ship are typical. At any rate, the higher the location the better. It is most common, however, for the range of locations available to be quite limited.

In conclusion, then, the design of a tank suitable for activation involves the choice of the configuration which leads to a given tank natural frequency and damping, has an acceptably large decrease in $\overline{GM}$ and is located in as high a location in the ship as possible. For a particular ship, the exterior envelope of the tank is usually specified and its location is specified. The design then becomes one of developing a configuration within the available envelope which matches ω_t, $\bar{\zeta}_t$ and k_{st}.

The following sections outline in detail the considerations in such a design and present charts for use in the computations.

The Tank Proportions

The total weight of fluid within the tank is often an important consideration to the naval architect. This weight not only must be included in the ship's displacement, but since water is the typical working fluid it is also an indirect measure of the volume of the stabilizer. The weight of the tank fluid depends on a number of requirements for the tank: the required $\overline{GM}$ loss, the tank frequency, the arrangement of the cross-over duct, the maximum tank capacity, etc. Because of its importance, it is of interest to determine the tank of least fluid weight which meets all of the requirements in order to gain insight into the proper proportions of the tank. For this study, we consider the tank shown in Figure 1. Here, it is assumed that the vertical legs are rectangular in cross-section and that they are interconnected by a duct of constant cross-sectional area, A_c.

In order to carry out this process of minimization, we must enumerate the restrictions to be placed on the tank. These are:

a. The tank is to have a given value of k_{st} (corresponding to the fractional loss of $\overline{GM}$). Here,

$$k_{st} = \frac{\rho_t g\ b_1 b_2\ (B-b_1)^2}{2\Delta\ \overline{GM}} \qquad [4a]$$

b. The tank is to have a given natural frequency, ω_t. From [1a] we have

$$\frac{2g}{\omega_t^{\ 2}} = S' \simeq 2H + \frac{b_1 b_2\ (B-2b_1)}{A_c} \qquad [4b]$$

c. The tank is to have a certain static capacity. If τ_{max} is the largest value of tank angle τ which can be achieved, then the tank can roll the ship to an angle $k_{st}\ \tau_{max}$ if all of the water is pumped to one side of the tank. Thus, we require the tank to have a given value of τ_{max} where

$$\tau_{max} = \tan^{-1}\left(\frac{2H}{B-b_1}\right) \simeq \frac{2H}{B-b_1} \qquad [4c]$$

d. The tank is to be realizeable within the ship's envelope. In specific

$$0 < b_1 < B/2 \qquad [4d]$$

The weight of the tank fluid, W_T, is approximately given by

$$W_T = \rho_t g \; [2Hb_1 b_2 + (B-2b_1)\, A_c] \qquad [5]$$

If the above restrictions are inserted into the weight equation, one gets

$$\frac{W_T}{\Delta} = 2k_{st}\; \overline{GM} \left\{ \frac{\tau_{max}}{(B-b_1)} + \frac{(B-2b_1)^2}{\left[\frac{2g}{\omega_t^2} - \tau_{max}(B-b_1)\right] (B-b_1)^2} \right\} \qquad [6]$$

For typical cases, $\frac{2g}{\omega_t^2} >> \tau_{max}(B-b_1)$. Therefore [6] will be further simplified to

$$\frac{W_T}{\Delta} = 2k_{st}\; \overline{GM} \left\{ \frac{\tau_{max}}{(B-b_1)} + \frac{\omega_t^2}{2g} \left[\frac{(B-2b_1)^2}{(B-b_1)^2}\right] \right\} \qquad [6a]$$

In order to minimize the ratio W_T/Δ, the above expression is differentiated and set equal to zero. The result is

$$C_{b_1} = \frac{b_1}{B} = \left\{ \frac{1-\tau_{max}\left(\frac{g}{\omega_t^2 B}\right)}{2-\tau_{max}\left(\frac{g}{\omega_t^2 B}\right)} \right\} \qquad [7]$$

For very high tank frequencies, we see that b_1 approaches B/2. That is, the distance between the two vertical legs shrinks to zero. At a frequency $\omega_t = \left(\frac{\tau_{max}\, g}{B}\right)^{\frac{1}{2}}$, C_{b_1} becomes zero. For this frequency and all lower frequencies the above analysis does not apply. The interpretation of this result is that at these low frequencies the system of least weight is one which the vertical legs are located as far outboard as possible.

It can be shown that for tank frequencies above $\left(\frac{\tau_{max} g}{B}\right)^{\frac{1}{2}}$, choice of C_{b_1} by [7] above leads to a tank of minimum weight, which is given by

$$\frac{W_T}{\Delta} = 4k_{st}\tau_{max}\frac{\overline{GM}}{B}\ \sigma \qquad [8]$$

where

$$\sigma = \left[1 - \frac{\tau_{max}}{4}\left(\frac{g}{\omega_t^{\ 2}B}\right)\right] \qquad [8a]$$

Figure 2 shows the variation in the values of C_{b_1} and σ as a function of $\tau_{max}\left(\frac{g}{\omega_t^{\ 2}B}\right)$.

It is appropriate to start the design of the tank by choosing a good value of C_{b_1} from figure by entering with the parameter $\tau_{max}\left(\frac{g}{\omega^2 B}\right)$. For cases where the restrictions on the tank location or too high a value of the above parameter prevents the use of the optimum value of C_{b_1}, then the vertical legs should be situated as far apart as possible, consistent with the obtaining of the given value of k_{st} and τ_{max}. The tank length, b_2, can be determined by means of [4a]. Here

$$b_2 = \frac{2\Delta \overline{GM}}{\rho_t g\, b_1 (B-b_1)^2} k_{st} \qquad [9]$$

Although this analysis is based on the assumption of a relatively uncomplicated tank configuration, experience has shown that the optimum C_{b_1} so computed is close to those for the more complicated configurations discussed in the next section.

The Natural Frequency of the Tank

It was seen in the first section that the natural frequency of the tanks depends only on the effective length of the U tube, S'. For a good active tank system, it has been shown in (1) that the tank should have a natural frequency significantly higher than the natural frequency of the ship in roll. As a result, we must obtain tanks with values of S' rather lower than typical for passive tanks. The tank configurations considered here must be amenable to the installation of the pumping means within the tank. In general the size of the vertical legs of the tank are governed by the required loss of $\overline{GM}$, as discussed in the previous section.

The cross-over duct then becomes a convenient element to vary in order to adjust the tank natural frequency. The lower leg also provides an ideal location for the installation of the pump since it is, in general, the lowest portion of the tank and provides the pump with the highest static head. It is clear that the cross-section area in way of the pump should be a circular cylinder. Also, a sufficiently smooth entrance should be given to the pump to prevent undesirable flow separation. If e is the total length of the pump cylinder and d_p the pump diameter, then the ratio of e/d_p should probably be kept greater than unity.

The integral for S', [1a], can be approximated by a finite sum

$$S' = \int_0^L \frac{A_o}{A(s)} ds \simeq \sum_{i=1}^{n} \frac{A_o}{A(s_i)} (\Delta s_i) \qquad [10]$$

Because of the stretching factor $\frac{A_o}{A(s)}$, small cross-section areas A(s) can contribute significant lengths in the above summation. Hence, careful thought must be given to the arrangement of the horizontal leg in order to regulate the integrated influence of A(s).

By means of [10], five different U-tube configurations were considered in detail in order to provide the designer with a set of reasonable choices. These configurations, numbered I through V are sketched in Figures 3, 4a and 4b. Figure 3 shows the basic configuration. Here the two vertical legs are joined by a straight, constant diameter round pipe of the same diameter as the pump, d_p. It is assumed that the tank spans the full beam of the ship, B. If the vertical legs are rectangular, each with an athwartships width of b_1 and length of b_2 then the value for S' is approximately given by

$$S' \cong 2H + \frac{2A_o \ell}{A(s)} \qquad [10a]$$

or

$$S' \simeq 2H + \frac{4b_1 b_2 \ell}{\pi d_p^2} \qquad [10b]$$

where

ℓ is the length of the cross-over pipe, and

H is the height of the fluid in the vertical legs.

The tanks of configurations II through V can be treated using [10a]. In these cases the d_p in [10a] is to be replaced by d_{eff}, the formulas for which are shown in Figures 4a and 4b. The tank types shown in Figure 4b are particularly important for activation in that the "underslung" cross-over gives greater flexibility in the arrangement of the pump.

From the definition of ω_t and [10a], we have the following approximate relation

$$\omega_t^2 = \left\{ \frac{g}{2H + \frac{4b_1 b_2 \ell}{\pi d_{eff}^2}} \right\} \qquad [11]$$

or in non-dimensional terms

$$\frac{\omega_t^2 B}{g} = \left\{ \frac{1}{C_H + \frac{2C_{b_1}}{\pi}(1-2C_{b_1})\frac{Bb_2}{d^2_{eff}}} \right\} \qquad [11a]$$

where

$C_H = H/B,$

$C_{b_1} = b_1/B$, as before, and

$d_{eff} = d_p$ for tank configuration I.

For design purposes, numerous plots of $\left(\frac{\omega_t^2 B}{g}\right)$ and $\left(\frac{d_{eff}}{d_p}\right)$ are presented. These plots embrace a wide range of likely values of tank dimensions. Figures 5-10 are curves of $(\omega_t^2 B/g)$ versus (Bb_2/d^2_{eff}) with contours of $C_H = H/B$. Each plot is for a particular value of the tank width coefficient $C_{b_1} = b_1/B$. Here, one sees that for low values of (b_2B/d^2_{eff}) that the exact value of C_H is crucial. However, for larger values of (b_2B/d^2_{eff}) the tank frequency is almost independent of C_H.

Plots of (d_{eff}/d_p) for each of the tank types (II through V) were developed. Figure 11 shows the variation of (d_{eff}/d_p) for tank type II for various values of the contraction ratio D/d_p and contraction length ratios from C_t = 0.1 to 0.9. Figures 12a and 12b show the variation of (d_{eff}/d_p) for various values of the contraction ratio b_2'/d_p and C_t. Figure 12a is useful for low values of the contraction ratio and 12b for larger values. Figures 13a - 13k show the variation of (d/d_{eff}) for various contraction ratios, C_t's and values of $C_{a\ell}$. Figure 14 shows the variation of (d_{eff}/d_p) for various values of $C_{a\ell}$.

The computation technique using these charts is straightforward. First the frequency desired for the tank is determined and the values of b_1 and b_2 chosen as discussed in the previous section. Then, the appropriate chart for C_{b_1} (Figures 5-10) is entered to determine the required d_{eff}. From the chosen pump diameter, d_p, and d_{eff}, the tank type charts (Figures 11-14) are used to determine the other parameters which characterize the tank.

The Damping in U-Tube Tanks

There are several causes of the damping in a conduit such as a U-tube. All of the causes are due to the viscosity of the fluid. The direct effect of the viscosity is one of the friction drag on the wetted surfaces of the conduit. There are several other losses in the conduit which are caused in a less direct way by viscosity. These effects are due to restrictions in the flow and may exhibit themselves as losses due to a contraction, an enlargement, an entrance, an exit, an obstruction, a bend, etc. Most of the material concerning these phenomena are empirical in nature and a large body of engineering data exists in a form which is convenient for the computation of damping.

In a hydraulic system consisting of many individual devices or distinguishable flow sections, the total loss is normally taken as the sum of the losses of each element, i.e.,

$$h = \sum_{i=1}^{n} \text{loss}_i = \sum_{i=1}^{n} C_{\ell_i} \frac{V_i^2}{2g} \qquad [12]$$

However, the velocity V_i in Equation [12] is the local velocity associated with the element in question. It is more convenient to work simultaneously with the local velocities at many individual elements by referring them all to a single velocity at some reference section of the flow path. For roll tanks the reference section is taken as the rectangular side tank area A_o. The principle of continuity for incompressible flows states

$$A_o V_o = A(s_i)\, V_i$$

where

V_i, V_o are the velocities at points i and o respectively, and

$A(s_i)$, A_o are the cross-section areas at i and o.

The total head loss described in Equation [12] can be expressed

$$h = \sum_{i=1}^{n} C_{\ell_i} \frac{V_i^2}{2g} = \sum_{i=1}^{n} C_{\ell_i} \left(\frac{A_o}{A(s_i)} \right)^2 \frac{V_o^2}{2g} \qquad [13]$$

We define an effective loss coefficient $C_{\ell_i}{}' = C_{\ell_i}\left(\frac{A_o}{A(s_i)}\right)^2$, associated with each restriction. If we make use of the fact that $V_o = R\tau'$, then

$$h(R\tau') = \left(\sum_{i=1}^{n} C_{\ell_i}{}'\right)\frac{(R\tau')^2}{2g} = C'_{total}\,\frac{(R\tau')^2}{2g} \qquad [14]$$

Here, $h(R\tau')$ is the head loss function of Equation [1]. The factor $[A_o/A(s_i)]^2$ is the necessary multiplier of individual loss coefficients C_{ℓ_i}. The sum of the modified coefficients is denoted by C'_{total}.

We first consider the frictional losses in the tank as a conduit which initially will be considered to be a circular pipe. The viscous damping depends, as shown above, in a viscous loss coefficient. The value of this coefficient and the nature of its dependence on $R\tau'$ is strongly influenced by the character of the flow in the tank. If the motion is very slow or the radius of the tube is small, or both, then the flow is likely to be laminar. For cases of large velocity or radius, the flow is likely to be turbulent.

The existence of laminar flow in the prototype is unlikely because of the large sizes and velocities encountered therein. However, the existence of laminar flow in a small model of the tank is possible. It can be shown that the viscous friction coefficient in the laminar flow regime varies inversely with velocity and thus the damping in [1] becomes a strictly linear function. When the flow is fully turbulent, however, the friction factor becomes nearly constant and is only a function of the internal roughness inside the ducts. In this case, the damping is pure quadratic. Because of the difference between the laminar and turbulent flow regimes and the change they cause in the character of the equation of motion, great care should be made to avoid laminar flow in model studies.

<u>Pipes</u> - The loss due to turbulent flow in a pipe is usually expressed in the following form.

$$H_{\ell_p} = f \frac{L}{D} \frac{V^2_{pipe}}{2g} \qquad [15]$$

where

- f is the friction coefficient,
- L is the pipe length, ft.,
- D is the pipe diameter, ft., and
- V_{pipe} is the mean velocity in the pipe.

For preliminary estimates of the damping, all the quantities on the right side of Equation [15] are known except the factor f. This factor is a function of both the roughness of the pipe and the Reynolds number, and plots of pipe flows can be found in numerous handbooks and textbooks. One such diagram published by Moody (5) is reproduced in Figure 15 for reference. When using such diagrams to determine f, one should bear in mind that the flow in a stabilization tank is not steady. Hence the Reynolds number is some function of time. Fortunately, for rough pipes at high Reynolds numbers, the curves of f are very flat. During a large portion of an oscillation cycle, the Reynolds number is probably high enough that f can be assumed constant. The unsteadiness of flow also affects the f factor in another way. The accelerated flow in the tank causes pressure gradients which are different from those of steady flow even though the instantaneous flow may correspond to that of the steady case. Only when the flow is slowly-varying and quasi-steady can the factor f above be used without error. In any case, the approximation seems adequate for the present purposes because of the relatively long roll periods of a ship.

As an estimate, the Reynolds number and f can be determined using the mean velocity over one half a cycle of the water pendulation. During this interval the water flows in one direction. The effective loss coefficient C_{ℓ_1}' referred to the side tank velocity is

$$C_{\ell_{pipe}}' = f \frac{L}{D} \left(\frac{A_o}{A_{pipe}} \right)^2 \qquad [16]$$

Conduits - Frictional losses in uniform conduits with non-circular cross-sections can be treated in a similar fashion to pipes. The head loss for a constant cross-section conduit is expressed

$$H_\ell = f \frac{L}{4r} \frac{V_c^2}{2g} \qquad [17]$$

where

- f is the friction factor as before,
- L is the length of section,
- r is the hydraulic radius = cross-section area/wetted perimeter, and
- V_c is the average velocity through conduit.

Values for the friction factor f can be selected in the same manner as with round pipes by using the Moody Diagram, (Figure 15) replacing the diameter D by four times the hydraulic radius, 4r.

In order to determine the frictional loss through the crossover, usually it must be broken into several pieces and each piece treated separately. The above description applies to the straight sections of the conduit. All other sections require special consideration typical components which fall into this category are treated separately below.

Valves - The loss coefficient of a valve is very difficult to pinpoint because it depends on the setting which may vary according to the condition of operation. Even at full opening, a globe valve offers many times the resistance of a gate valve. In practice a valve can be set to give a very wide range of C_ℓ, and some sample valves are given in the table below taken from Engineering Hydraulics (6).

TABLE 1

Loss Coefficients for Valves

Type	C_ℓ	Type	C_ℓ
Globe valve (open)	10.0	Plug valve with screw ends (open)	0.77
Angle valve (open)	5.0		
Swing check valve (open)	2.5	(99 percent open)	0.86
Gate valve (open)	0.19	(98 percent open)	0.95
(3/4 open)	1.15	(95 percent open)	1.45
(1/2 open)	5.6	(90 percent open)	2.86
(1/4 open)	24.0	(80 percent open)	9.6
Diaphragm valve (open)	2.3	(70 percent open)	28.0
(3/4 open)	2.6		
(1/2 open)	4.3		
(1/4 open)	21.0		
Plug globe valve (open)	4.0		
(3/4 open)	4.6		
(1/2 open)	6.4		
(1/4 open)	780.0		

<u>Bends</u> - For smoothly turning pipe bends without guide vanes, the loss coefficient depends on the bending angle and the ratio of the radius of the bend to the diameter of the pipe. The loss coefficient C_ℓ for right angle pipe bends is plotted in Figure 16 (see Reference 7).

The following rule can be used as a guide for estimating the C_ℓ for bends less than 90 degrees:

Angle of Bend	C_{ℓ_b} = % of C_ℓ for 90° Bend
45°	75%
22.5°	50%

[18]

In rectangular ducts, the loss depends on the width w, the depth d in the plane of curvature, and the curvature which is represented by the centerline radius r. The following table taken from Engineering Hydraulics (6) gives C_ℓ for 90 degree bends.

TABLE 2

Loss Coefficients for Rectangular Ducts

w/d \ r/d	2/3	1	5/3
3	0.55	0.22	0.15
6	0.38	0.16	0.09

For rectangular ducts with bends less than 90°, reduction of C_ℓ values according to the values of C_b in [18] seems to be a reasonable approximation. Recall that in every case $C_\ell' = C_\ell[A_o/A(s_1)]$.

Guide vanes in bends offer a sensible way to reduce losses in the turns of a system. Such vanes act as flow straighteners and reduce the tendency of the flow to adopt longitudinal swirling. Practice has shown that, particularly in large conduits, longitudinal vortex motion will cause higher losses than predicted because of the longer effective path of the fluid in addition to the energy losses in the vortex itself. The merit of a system of guide vanes depends a great deal on the design, but in any case the losses can be reduced substantially.

A simple set of guide vanes made of metal plates can reduce the loss coefficient for 90° bends to 0.2 - 0.24, which is small compared to a coefficient of 1.0 for the same bend angle without vanes (see Figure 17). A satisfactory vane arrangement can be made from the sketch of Figure 18 taken from Reference 6. Refinements in design will provide further reduction of C_ℓ, but the expense and degree of reduction may be such that refinement is not worthwhile. Two possibilities exist: (1) using airfoil sections such as standard wing profiles (loss coefficient ≅ 0.136 (11)), or (2) shapes similar to turbine turning vanes (Figure 19) which have thickened central portions with tapered forward and trailing edges.

For preliminary calculations, the loss coefficient can be taken as $C_\ell \cong 0.2 - 0.24$ regardless of the design.

Referring the loss to the side tank velocity, the coefficient for 90° bends with turning vanes becomes

$$C'_{\ell_{vanes}} \cong 0.2 \left(\frac{A_o}{A(s_1)}\right)^2 \qquad [19]$$

In bends of less than $\pi/2$ the guide vane loss coefficient can be reduced by the same factors C_b given in (18).

Expansions and Contractions - In a stabilization tank, the geometry is almost always symmetrical. A contraction on one side will be accompanied by an expansion on the other side. Also a contraction in one part of a cycle of oscillation will become an expansion in another part of a cycle when the flow reverses. Hence, transition sections can be treated interchangeably.

The design of a transition influences the damping of the system. If large damping is desirable a transition should be made abrupt so as to increase the loss. For active tanks actual fluid damping should probably be kept rather small (see Webster and Dogan, Reference 1). In the latter case transitions should be made smooth and gradual to avoid separation.

The design of a gradual transition is usually governed by its behavior during the part of the cycle which causes expansion. Because of the presence of a positive pressure gradient in an expansion, the flow will have a tendency to separate. When separation occurs there is a marked increase in the loss, along with irregular velocity distributions.

The phenomenon of separation in a diffuser is quite complicated and is governed by a number of factors: angle of divergence, length of transition, upstream velocity distributions, and the entrance conditions. Because of the number of factors involved, there is no really comprehensive information available. There is general agreement that C_ℓ depends a great deal on the divergence angle and the diameter ratios. More recent experiments show that the upstream velocity distributions affect the loss, although not in a drastic way.

Some results of losses in divergent flows are presented in Figure 20 (Ref. 6). It appears that C_ℓ is a minimum when the half angle of divergence is about 3^{o}, primarily because separation does not occur until beyond that angle.

In stabilization tanks the flow is clearly unsteady, and the results obtained in diffusors for steady flow may not be completely valid. In fact, in decelerating flow there is a higher tendency to separate. Hence a diffusor designed for steady flow may not perform as well in the unsteady case. There seems to be no easy rule dictating the design of a diffusor for unsteady flows, but certainly a good policy would be to keep the half angle of divergence somewhat below 3^{o}, otherwise separation is likely. Note that in case a transition is required with a large diameter or area ratio and short overall length, splitter plates may be installed inside. In this way the divergence angle of each separate channel can be made small, although the increased wetted area results in increased fluid friction against the plates.

When the angle of divergence is small and separation is not present, the flow is not very different from that through a uniform pipe, and the friction loss can be estimated in a similar fashion. The velocity variation along the length must be taken into account. An approximate formula for losses in round pipe transitions is derived in Appendix A.

$$H_{\ell} = f \frac{L}{D^2} \frac{V_2^2}{2g} \cdot \frac{1}{4}\left[\left(\frac{D_2}{D_1}\right) + \left(\frac{D_2}{D_1}\right)^2 + \left(\frac{D_2}{D_1}\right)^3 + \left(\frac{D_2}{D_1}\right)^4\right] \qquad [20]$$

or that

$$C_{\ell_t} = f \frac{L}{D_2} \cdot \frac{1}{4}\left[\left(\frac{D_2}{D_1}\right) + \left(\frac{D_2}{D_1}\right)^2 + \left(\frac{D_2}{D_1}\right)^3 + \left(\frac{D_2}{D_1}\right)^4\right] = f \frac{L}{D_2} g \left(\frac{D_2}{D_1}\right) \qquad [20a]$$

where

D_1, D_2 are the two iameters at the two ends,

V_2 is the mean velocity at section 2,

f is the pipe friction factor based on the Reynolds number at a mean section, and

L is the length of transition.

The result [20] assumes a fairly high Reynolds number to insure turbulent flow, a ratio D_1/D_2 not too far from 1.0 so that f is essentially constant over the length, and that some mean f is an adequate approximation. The value of $g(D_2/D_1)$ is shown in Figure 21.

For non-circular cross-sections the formula above can be modified to

$$H_{\ell} = f \frac{L}{4r_2} \frac{V_2^2}{2g} \cdot \frac{1}{4}\left[\sqrt{\frac{A_2}{A_1}} + \frac{A_2}{A_1} + \left(\sqrt{\frac{A_2}{A_1}}\right)^3 + \left(\frac{A_2}{A_1}\right)^2\right] \qquad [21]$$

or that

$$C_{\ell_t} = f \frac{L}{4r_2} \cdot \frac{1}{4}\left[\sqrt{\frac{A_2}{A_1}} + \frac{A_2}{A_1} + \left(\sqrt{\frac{A_2}{A_1}}\right)^3 + \left(\frac{A_2}{A_1}\right)^2\right] \qquad [21a]$$

where

A_1, A_2 are the cross-section areas at the two ends, and

r_2 is the hydraulic radius = A_2/wetted perimeter at section 2.

The loss coefficients of Equations [20a] and [21a] are written in terms of the velocity V_2 through section 2. As always, the final loss coefficient must be referred to the velocity V_o through area A_o,

$$C_{\ell_t}' = C_{\ell_t} \left(\frac{A_o}{A_2}\right)^2 \qquad [22]$$

For abrupt expansions from A_1 to A_2 the following table from Reference 6 gives the loss coefficients C_{ℓ} for the expression (based on V_1):

$$H_{\ell} = C_{\ell_e} \frac{V_1^2}{2g}$$

TABLE 3

Loss Coefficients for Sudden Expansions

Area Ratio $\frac{A_1}{A_2}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C_{ℓ_e} (expansion)	1.00	0.81	0.64	0.49	0.36	0.25	0.16	0.09	0.04	0.01	0

In a like fashion, abrupt contraction losses are computed from the formula (based on V_2):

$$H_\ell = C_{\ell_c} \frac{V_2^2}{2g}$$

for which the coefficients are given in Reference 8

TABLE 4

Loss Coefficients for Sudden Contractions

$\frac{A_2}{A_1}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C_{ℓ_c}	0.50	0.48	0.45	0.41	0.36	0.29	0.21	0.13	0.07	0.01	0

The computation of the damping can be carried out in a direct manner. First, the U-tube is decomposed into its hydraulic elements: the vertical legs, bends, entrances, exits, valves, transitions, etc. The loss coefficient for each is estimated by means of the information set forth above and these coefficients are summed to form C_{total} and, thus $h(R\dot{\tau})$ (see Equation [14]). From [3a] we see that this loss can be interpreted as the equivalent linear damping ratio,

$$\bar{\zeta}_t = \frac{\omega_t R}{8g} C_{\ell_{total}} \left|\dot{\tau}\right|_{ave} \qquad [23]$$

For non-linear analyses, Equation [14] would be used directly in Equation [1].

Tank Sloshing

Owing to the size of anti-roll tanks, fluid sloshing could give rise to undesirable, high amplitude motions which might interfere with the simple U-tube oscillations of a tank fluid (see for instance References 8 and 9). Prediction of the troublesome frequencies can be made in an elementary fashion.

Figure 22 shows a sketch of a rectangular side tank of an anti-roll system. The question is to determine the resonant frequency in the slosh mode for this simple single tank. For oscillations similar to a standing wave, the natural frequency is given by Lamb (10, pp. 440-445)

$$\omega_t^2 = gk \tanh kH \qquad [24]$$

where

g = accel. of gravity,

k = wave number = $2\pi/\lambda$,

λ = wavelength, taken as an integer multiple of the tank length dimension, and

H = depth of fluid in tank.

The lowest frequency occurs when $\lambda = 2b_1$, so that

$$\omega_{slosh} = \sqrt{\frac{g\pi}{b_1} \tanh \frac{\pi H}{b_1}} \qquad [25]$$

Korvin-Kroukovsky (11) briefly discusses standing waves in a rectangular tank and gives a more generalized formulation for the wave number k (sloshing in two directions):

$$k^2 = \pi^2 \left[\left(\frac{m}{b_1}\right)^2 + \left(\frac{n}{b_2}\right)^2\right] \qquad [26]$$

$$\omega_{slosh} = \sqrt{gk \tanh kH}$$

where

m, n = integers 0,1,2,3, the lowest mode given by $m = 1$, $n = 0$, $yb_1 > b_2$.

In actual design these formulas are useful to make quick estimates of those frequencies for which the sloshing mode could be excited. If, for some reason, significant excitation at or near any of the lower modes can be anticipated it may become necessary to supply internal baffling in the tank to prevent these motions from becoming severe.

APPENDIX A

Frictional Loss of a Slightly Divergent or Convergent Pipe

The derivation of an approximate formula to compute the loss of a slightly tapered transition pipe section is given below. In the derivation, the friction coefficient f is assumed to stay constant.

For a straight tapered pipe section the diameter d at any point x can be written as

$$d(x) = D_o + kx \tag{A.1}$$

where

D_o = diameter at x = 0, and

k = proportional factor.

The cross section area A and the mean V are

$$A(x) = \frac{\pi}{4} d^2(x) = \frac{\pi}{4} (D_o + kx)^2 \tag{A.2}$$

$$V(x) = \frac{Q}{A(x)} = \frac{V_o A_o}{\frac{\pi}{4}(D_o + kx)^2} = \frac{V_o D_o{}^2}{(D_o + kx)^2} \tag{A.3}$$

From V, the head loss, dh_ℓ in a section ds is found to be

$$dh_\ell = f\,\frac{ds}{D}\,\frac{V^2}{2g} = f\,\frac{ds}{(D_o + kx)}\,\frac{1}{2g}\left(\frac{V_o D_o^{\,2}}{(D_o + kx)^2}\right)^2 \qquad [A.4]$$

and the total head loss is

$$h_\ell = \frac{fV_o^{\,2}D_o^{\,4}}{2g}\int_o^L \frac{1}{(D_o + kx)^5}\,dx$$

$$= \frac{fV_o^{\,2}D_o^{\,4}}{2g}\,\frac{1}{4k}\left(\frac{1}{D_o^{\,4}} - \frac{1}{(D_o + kL)^4}\right)$$

$$= \frac{fV_o^{\,2}D_o^{\,4}}{2g}\,\frac{1}{4k}\left(\frac{1}{D_o^{\,4}} - \frac{1}{D_1^{\,4}}\right) \qquad [A.5]$$

where

D_1 = diameter at $x = L$

Substituting

$k = \dfrac{D_1 - D_o}{L}$ into [A.5], we obtain

$$h_\ell = \frac{fV_o^2}{2g}\,\frac{1}{4}\,\frac{L}{D_1 - D_o}\left(1 - \left(\frac{D_o}{D_1}\right)^4\right) \qquad [A.6]$$

or

$$h_\ell = \frac{fLV_o^2}{D_o 2g}\;\frac{1}{4}\left[\left(\frac{D_o}{D_1}\right) + \left(\frac{D_o}{D_1}\right)^2 + \left(\frac{D_o}{D_1}\right)^3 + \left(\frac{D_o}{D_1}\right)^4\right]$$

whence

C_ℓ becomes

$$C_\ell = f\,\frac{L}{D_o}\,\frac{1}{4}\left[\left(\frac{D_o}{D_1}\right) + \left(\frac{D_o}{D_1}\right)^2 + \left(\frac{D_o}{D_1}\right)^3 + \left(\frac{D_o}{D_1}\right)^4\right] \qquad [A.7]$$

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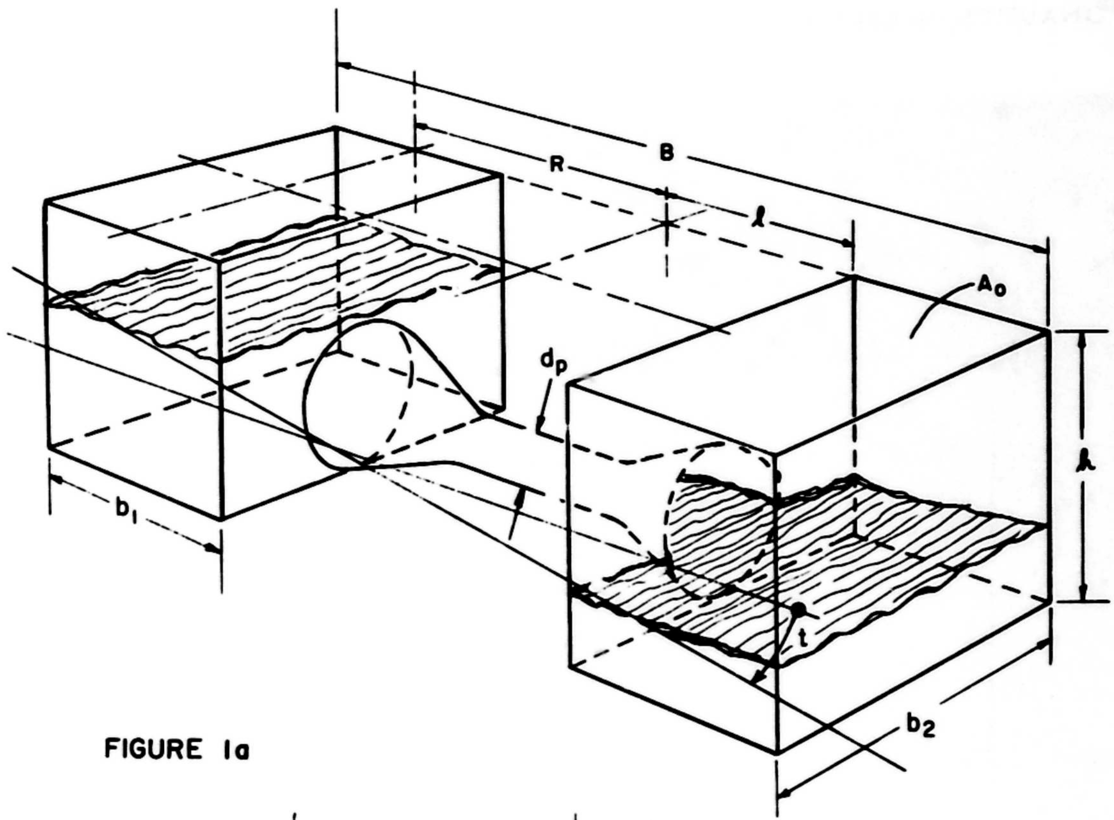


FIGURE 1a

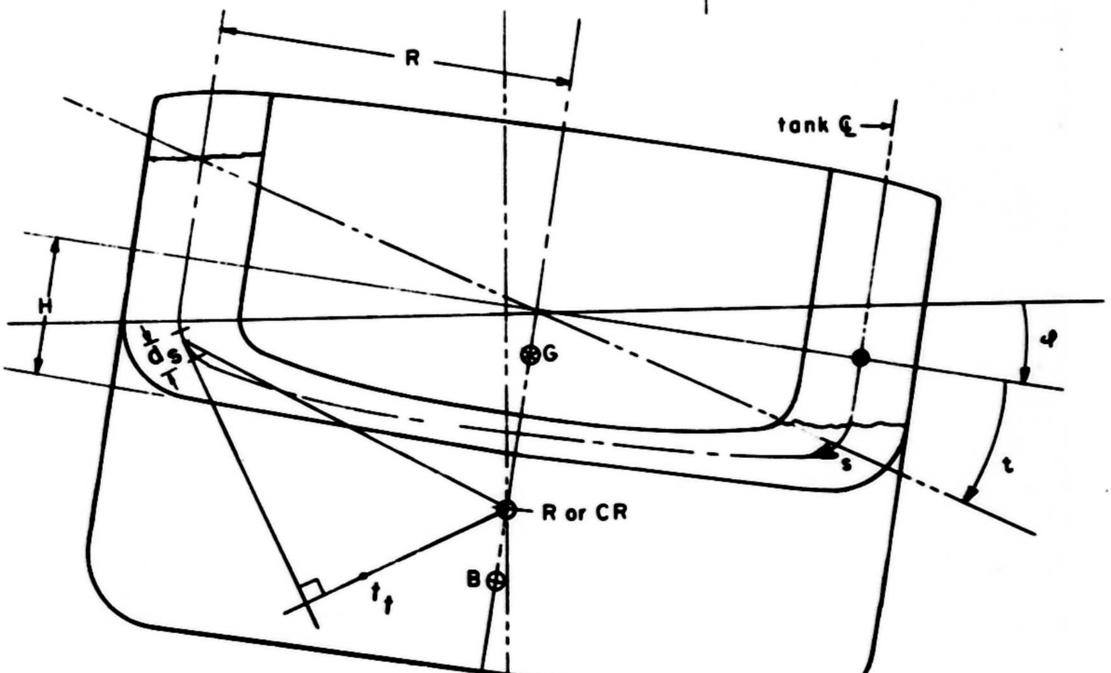


FIGURE 1b

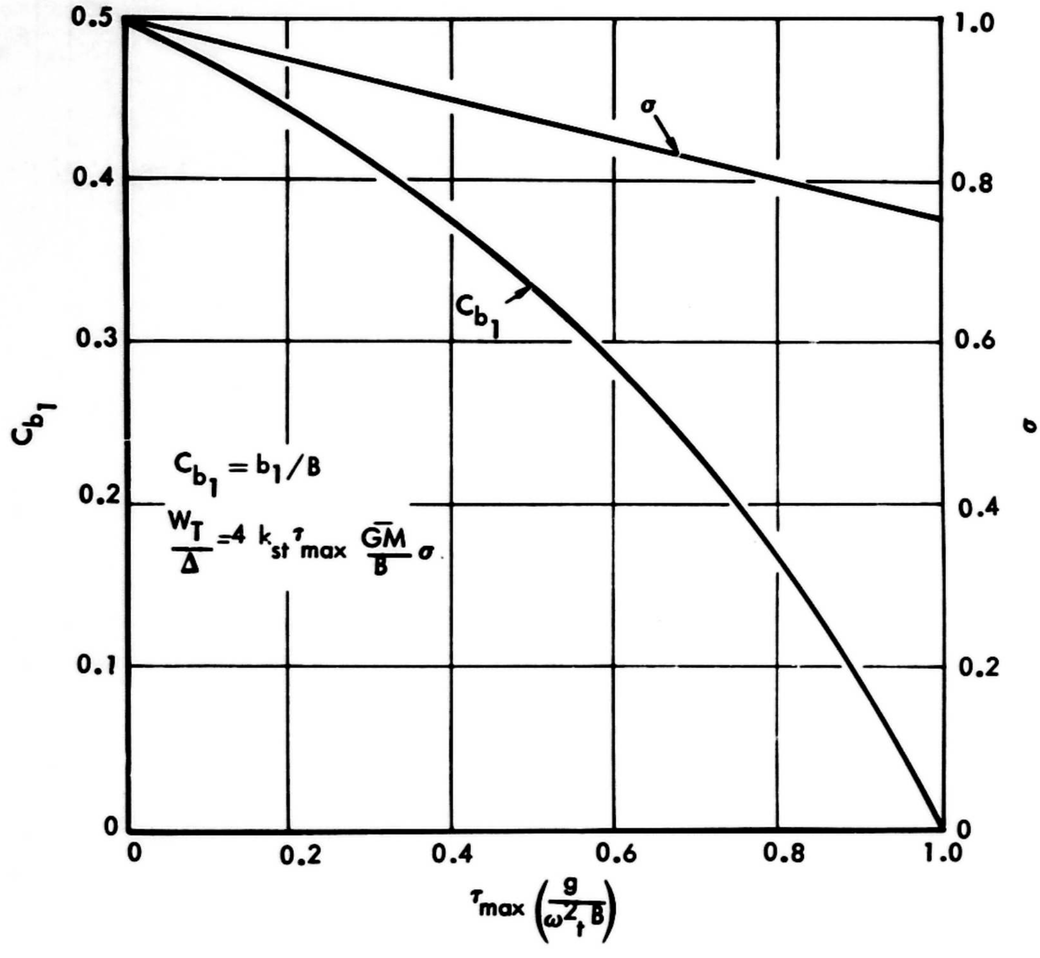


FIGURE 2 - THE VARIATION IN VALUES OF C_{b_1} AND σ WITH THE PARAMETER $\tau_{max}\left(\frac{g}{\omega_t^2 B}\right)$

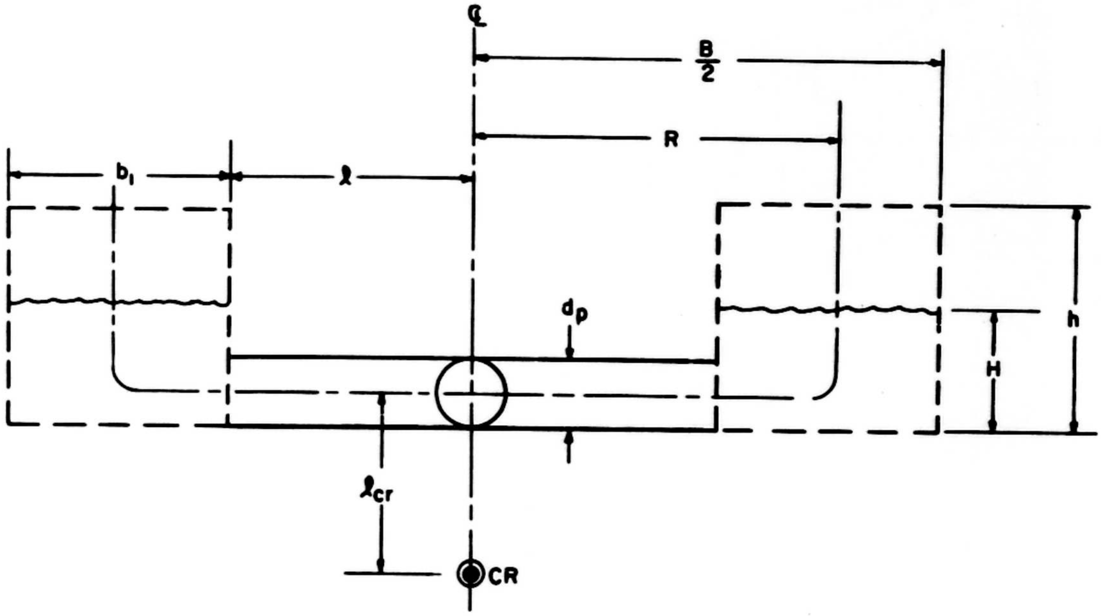


FIGURE 3 - TANK TYPE I

(EQUIVALENT TANK CONFIGURATION)

LET $H = C_H \cdot B$ — H = FLUID DEPTH

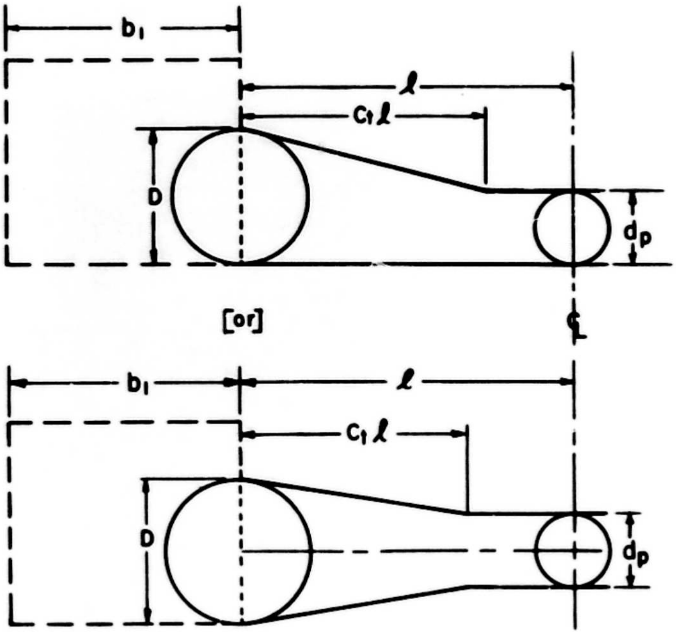
$b_1 = C_{b_1} \cdot B$ — b_1 = TANK WIDTH

$A_0 = b_1 b_2$ — b_2 = TANK LENGTH

$\ell = \frac{B}{2} - b_1$ — B = SHIP BEAM

$\ell_{cr} = C_{cr} \cdot B$ — CR = SHIP CENTER OF ROTATION

ℓ_{cr} = DISTANCE TO CR



$$\ell = \left(\frac{B}{2} - b_1\right)$$

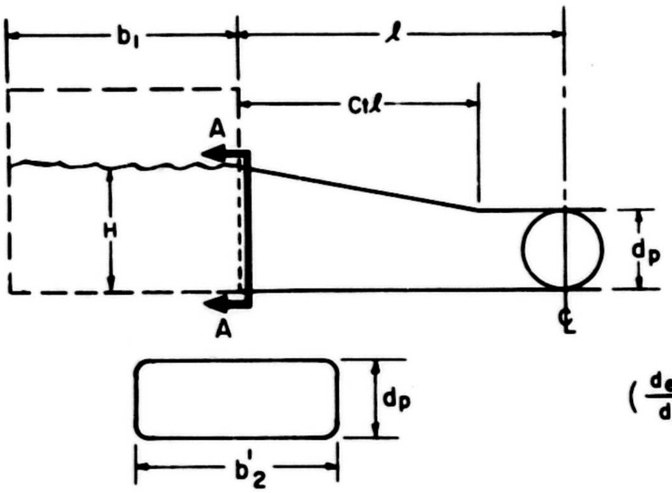
$$\left(\frac{d_{eff}}{d_p}\right) = \sqrt{\frac{1}{1-\left(1-\frac{d_p}{D}\right)C_t}}$$

TANK TYPE II

---

usually $b_2' = b_2$

$$\ell = \left(\frac{B}{2} - b_1\right)$$

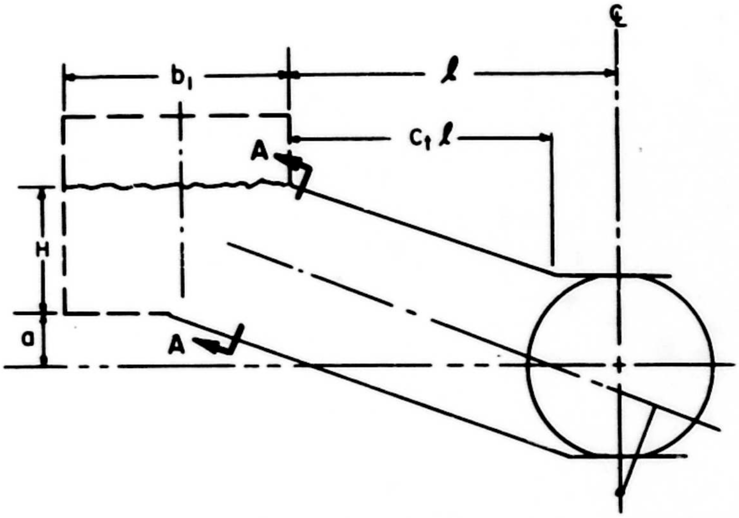


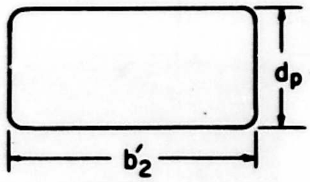
Section A–A

$$\left(\frac{d_{eff}}{d_p}\right) = \sqrt{\frac{1}{1+\left[\frac{1}{2}\sqrt{\frac{\pi d_p}{b_2'}}-1\right]C_t}}$$

TANK TYPE III

FIGURE 4a-FORMULAS FOR $\left(\frac{d_{eff}}{d_p}\right)$ FOR VARIOUS CONFIGURATIONS





SECTION A-A

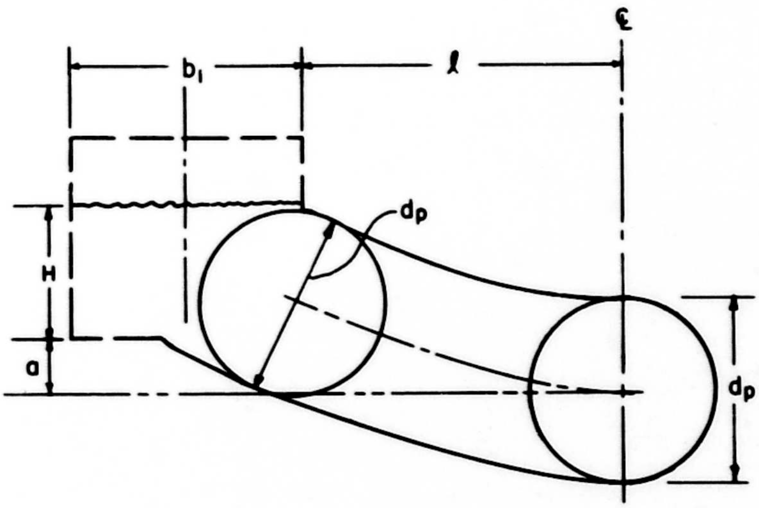
$$\left(\frac{d_{eff}}{d_p}\right)=\sqrt{\frac{1}{(1-C_t)+\frac{1}{2}\sqrt{\frac{\pi d_p\left(C_{a\ell}^2+C_t^2\right)}{b'_2}}}}$$

$a = C_{a\ell}\cdot\ell$

usually $b'_2 - b_2$

$\ell = \left(\frac{B}{2} - b_1\right)$

TANK TYPE IV



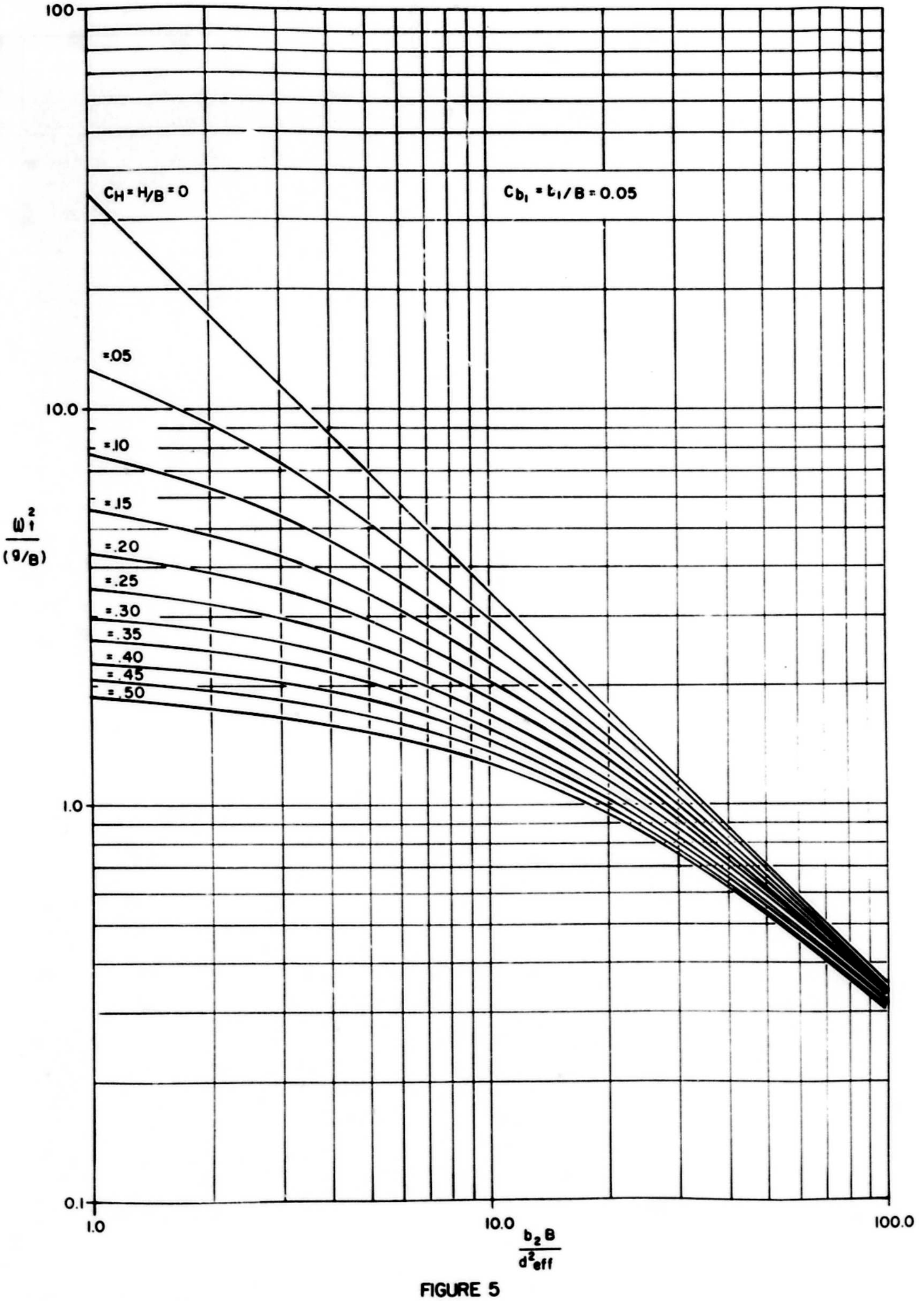
$a = C_{a\ell}\cdot\ell$

$\ell = \left(\frac{B}{2} - b_1\right)$

$$\left(\frac{d_{eff}}{d_p}\right)=\sqrt{\frac{1-C_{a\ell}^2}{1+C_{a\ell}^2}}$$

TANK TYPE V

FIGURE 4b- FORMULAS FOR $\left(\frac{d_{eff}}{d_p}\right)$ FOR VARIOUS CONFIGURATIONS



100
10.0
1.0
0.1
$C_H = H/B = 0$
$C_{b_1} = t_1/B = 0.05$
=.05
=.10
=.15
=.20
=.25
=.30
=.35
=.40
=.45
=.50
$\frac{\omega_t^2}{(g/B)}$
1.0
10.0
100.0
$\frac{b_2 B}{d^2_{eff}}$

FIGURE 5

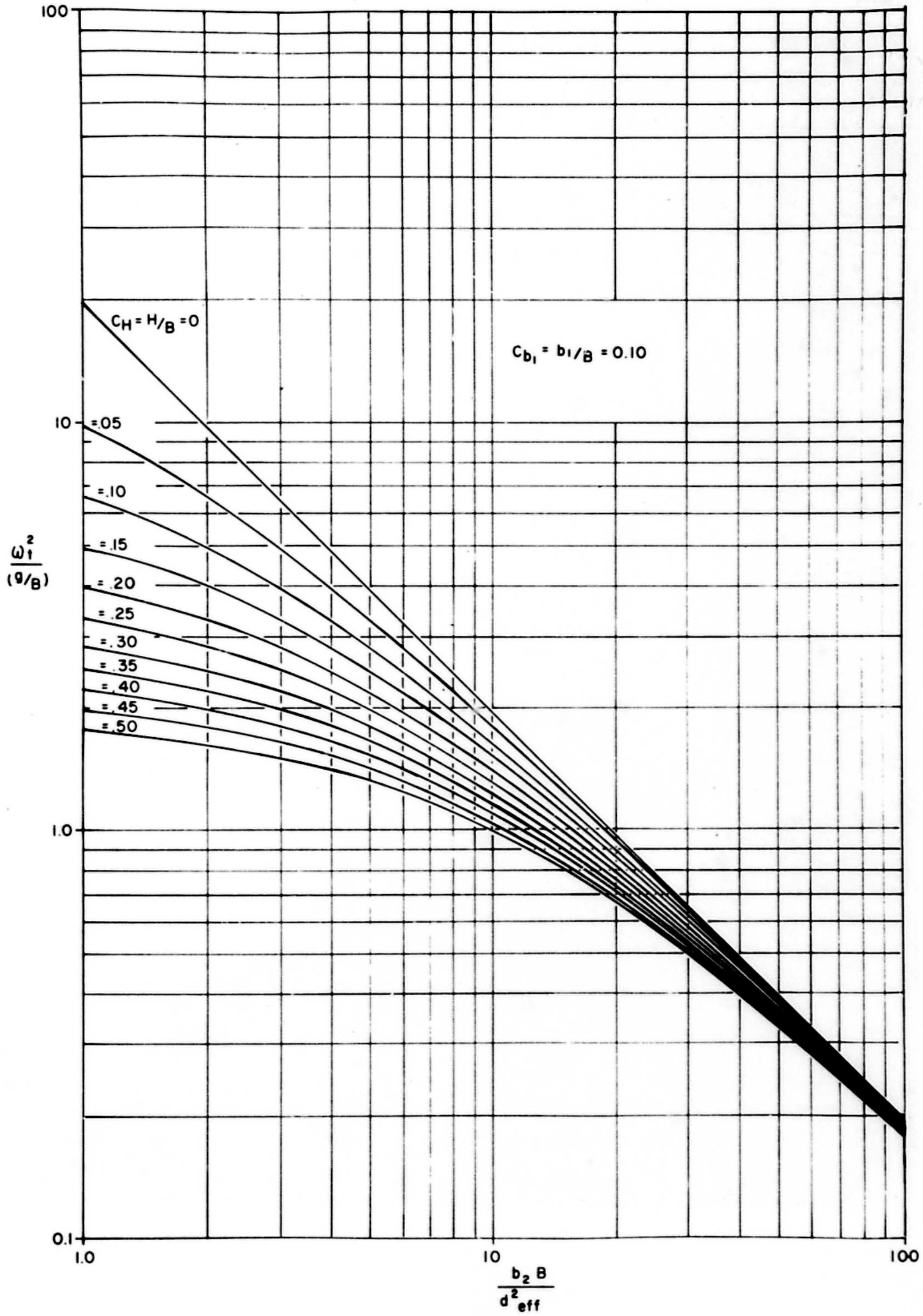

100
10
1.0
0.1
$C_H = H/B = 0$
= .05
= .10
= .15
= .20
= .25
= .30
= .35
= .40
= .45
= .50
$C_{b_1} = b_1/B = 0.10$
$\frac{\omega_t^2}{(g/B)}$
1.0
10
100
$\frac{b_2 B}{d^2_{eff}}$

FIGURE 6

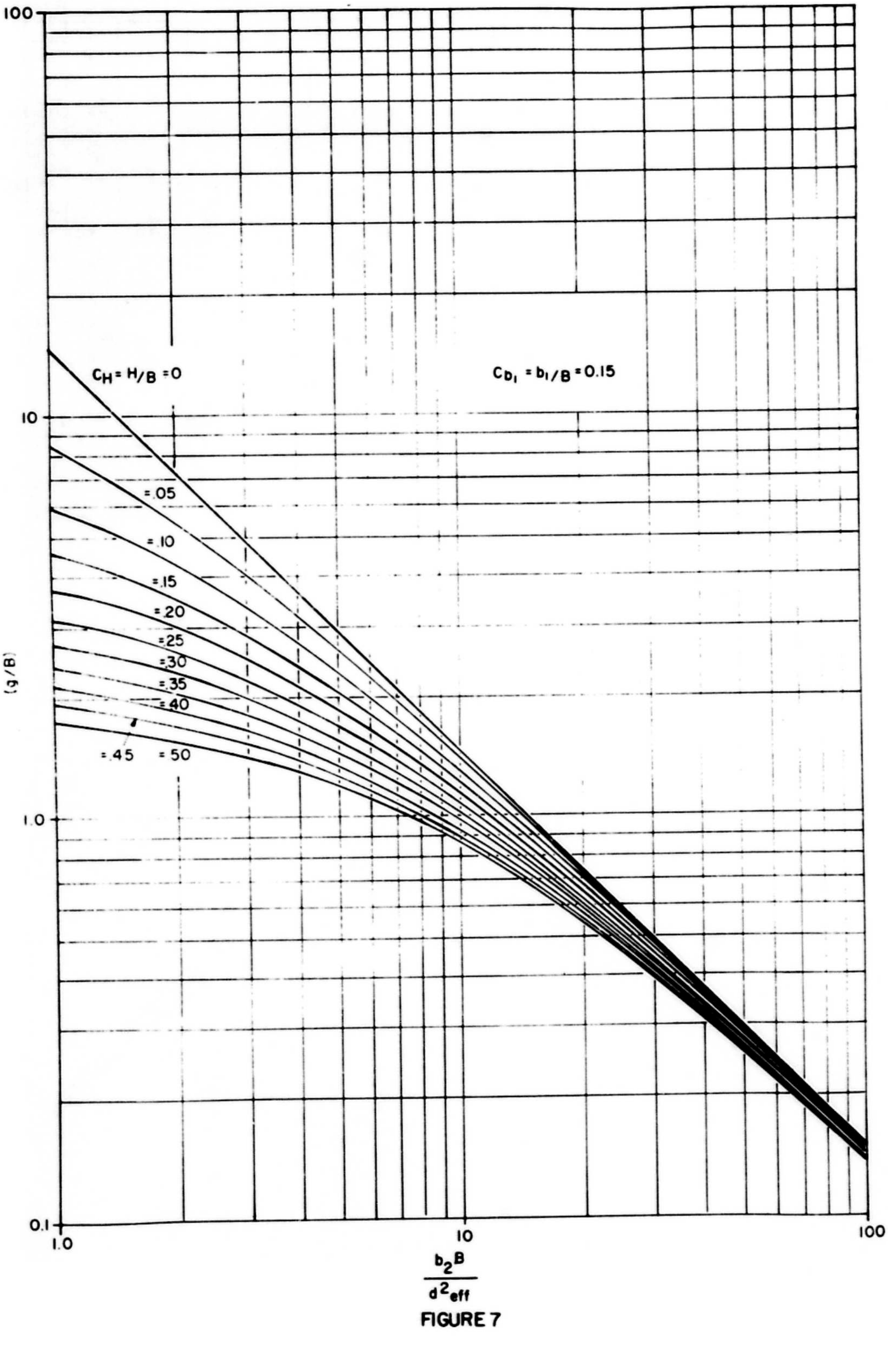
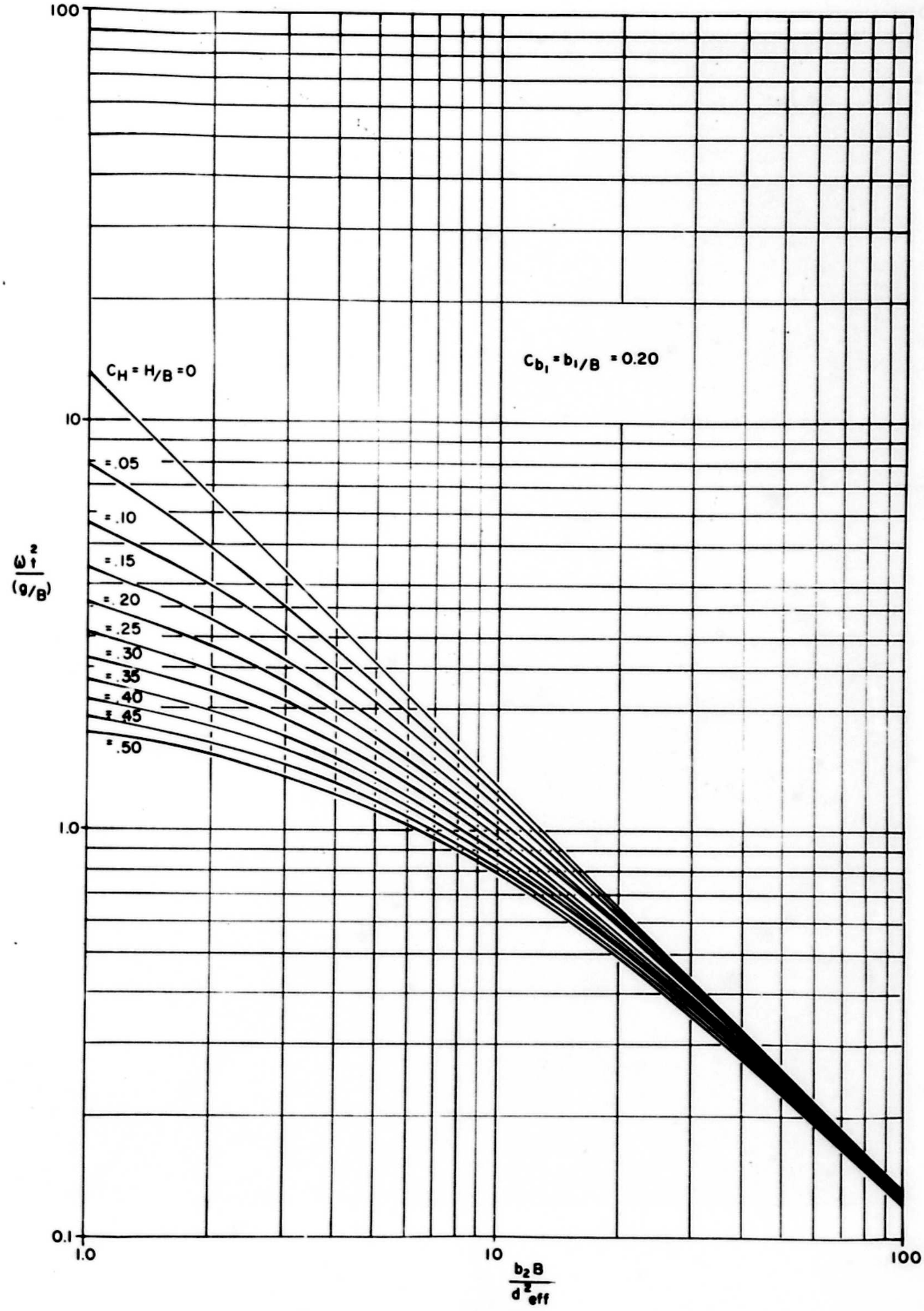

100
10
1.0
0.1
1.0
10
100
$C_H = H/B = 0$
$C_{D_1} = b_1/B = 0.15$
= .05
= .10
= .15
= .20
= .25
= .30
= .35
= .40
= .45
= 50
$\frac{\omega t^2}{(g/B)}$
$\frac{b_2 B}{d^2_{eff}}$

FIGURE 7



100
10
1.0
0.1
1.0
10
100
$C_H = H/B = 0$
$= .05$
$= .10$
$= .15$
$= .20$
$= .25$
$= .30$
$= .35$
$= .40$
$= .45$
$= .50$
$C_{b_1} = b_1/B = 0.20$
$\frac{\omega_t^2}{(g/B)}$
$\frac{b_2 B}{d^2_{eff}}$

FIGURE 8

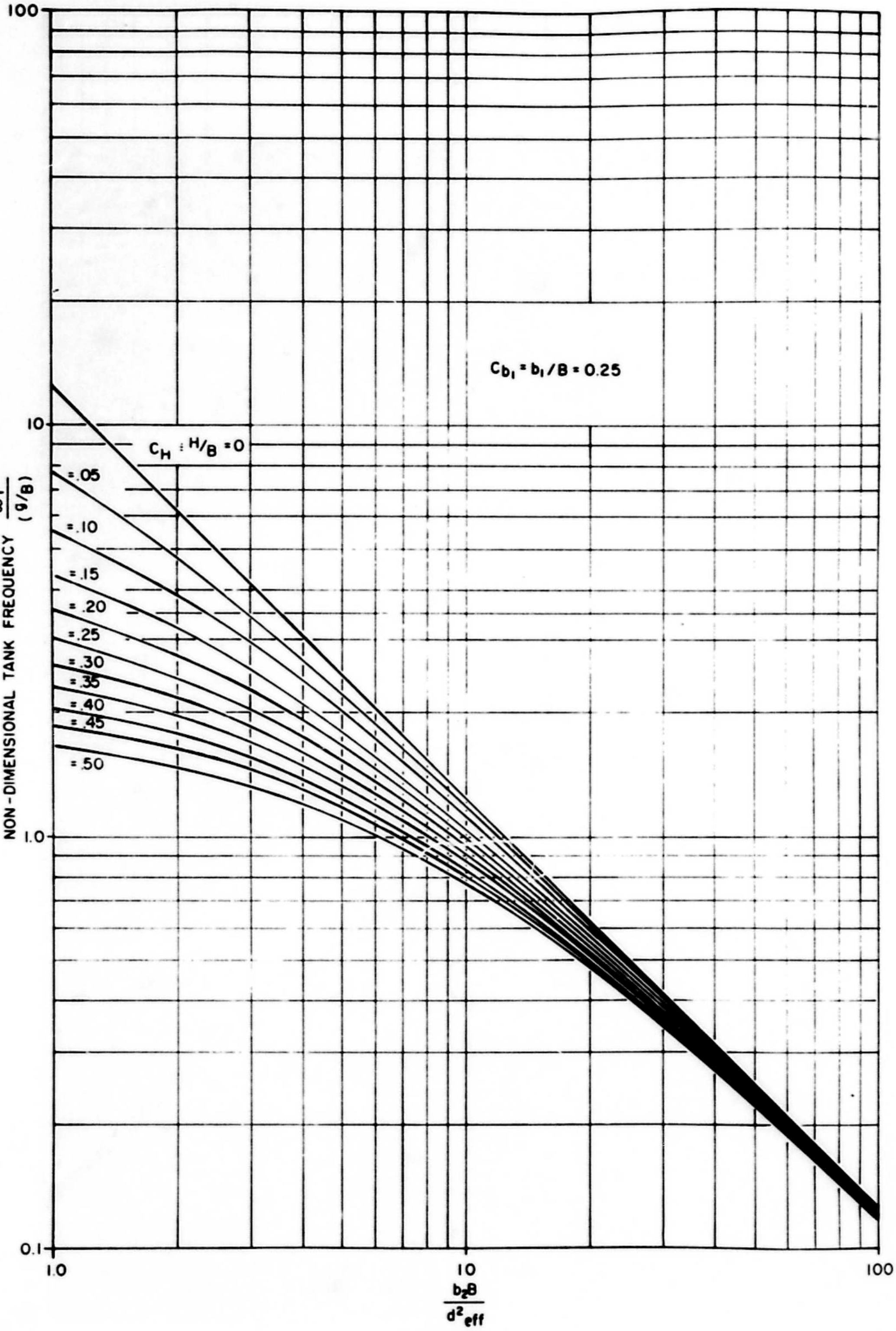

100
10
1.0
0.1
1.0
10
100
NON-DIMENSIONAL TANK FREQUENCY $\frac{\omega_t^2}{(g/B)}$
$\frac{b_2 B}{d^2_{eff}}$
$C_{b_1} = b_1/B = 0.25$
$C_H = H/B = 0$
=.05
=.10
=.15
=.20
=.25
=.30
=.35
=.40
=.45
=.50

FIGURE 9

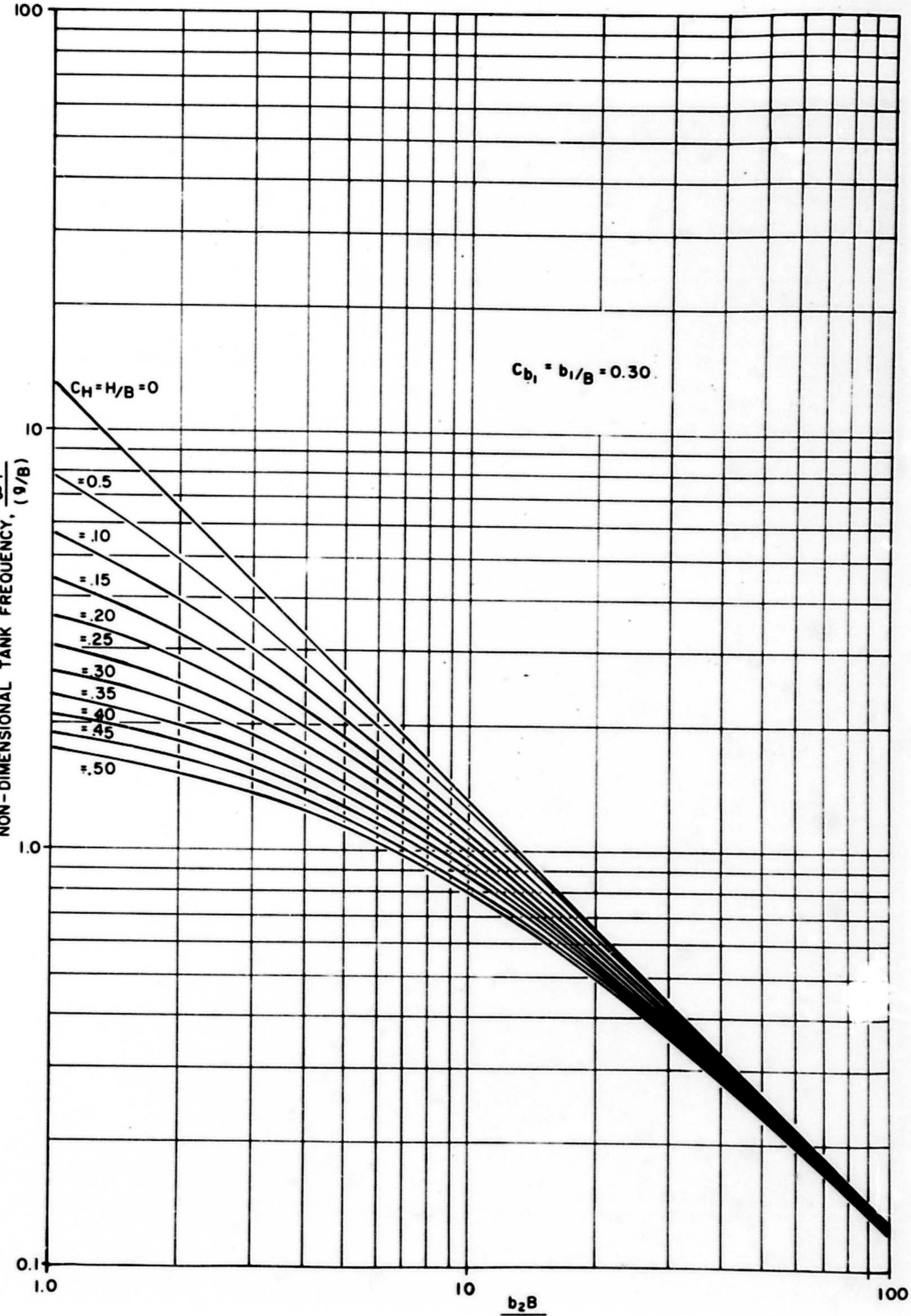

100
10
1.0
0.1
1.0
10
100
NON-DIMENSIONAL TANK FREQUENCY, $\frac{\omega_t^2}{(g/B)}$
$\frac{b_2 B}{d^2_{eff}}$
$C_{b_1} = b_1/B = 0.30$
$C_H = H/B = 0$
=0.5
=.10
=.15
=.20
=.25
=.30
=.35
=.40
=.45
=.50

FIGURE 10

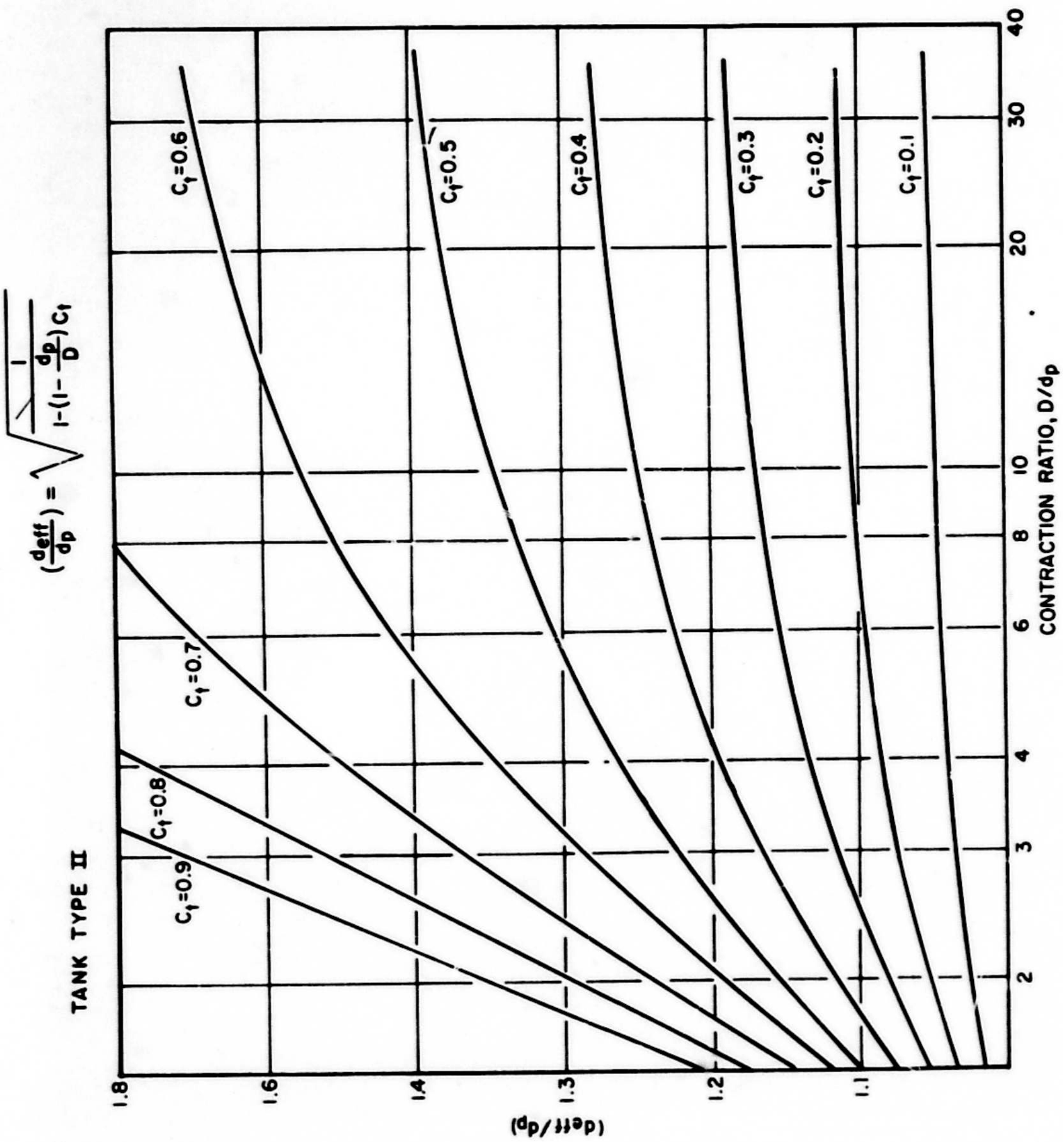

TANK TYPE II
$(\frac{d_{eff}}{d_p}) = \sqrt{\frac{1}{1-(1-\frac{d_p}{D})C_t}}$
C_t=0.9
C_t=0.8
C_t=0.7
C_t=0.6
C_t=0.5
C_t=0.4
C_t=0.3
C_t=0.2
C_t=0.1
1.8
1.6
1.4
1.3
1.2
1.1
(d_{eff}/d_p)
2
3
4
6
8
10
20
30
40
CONTRACTION RATIO, D/d_p

FIGURE 11

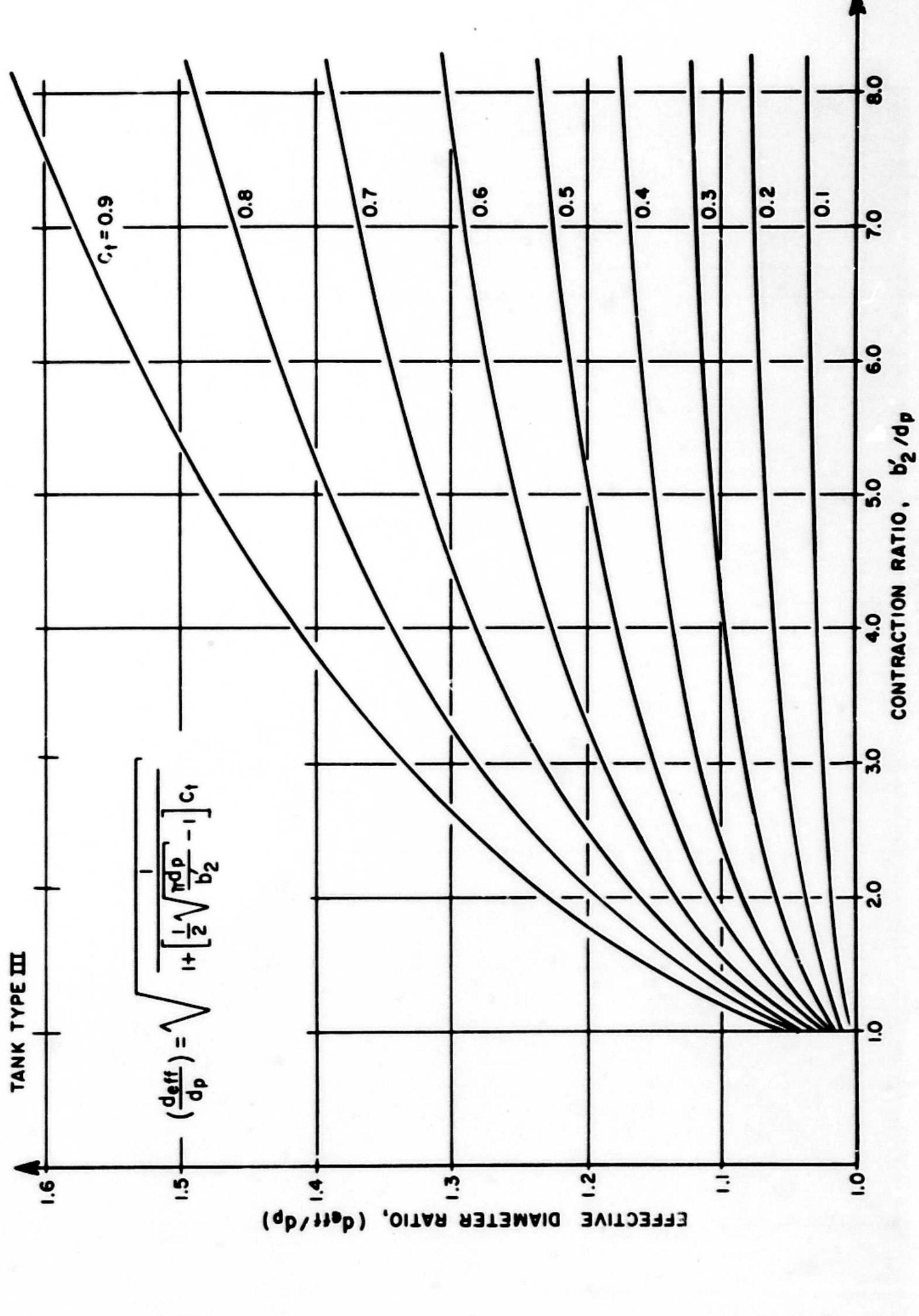

TANK TYPE III
$\left(\frac{d_{eff}}{d_p}\right) = \sqrt{\frac{1}{1+\left[\frac{1}{2}\sqrt{\frac{\pi d_p}{b_2'}}-1\right]C_t}}$
$C_t = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
EFFECTIVE DIAMETER RATIO, (d_{eff}/d_p)
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
CONTRACTION RATIO, b_2'/d_p

FIGURE 12

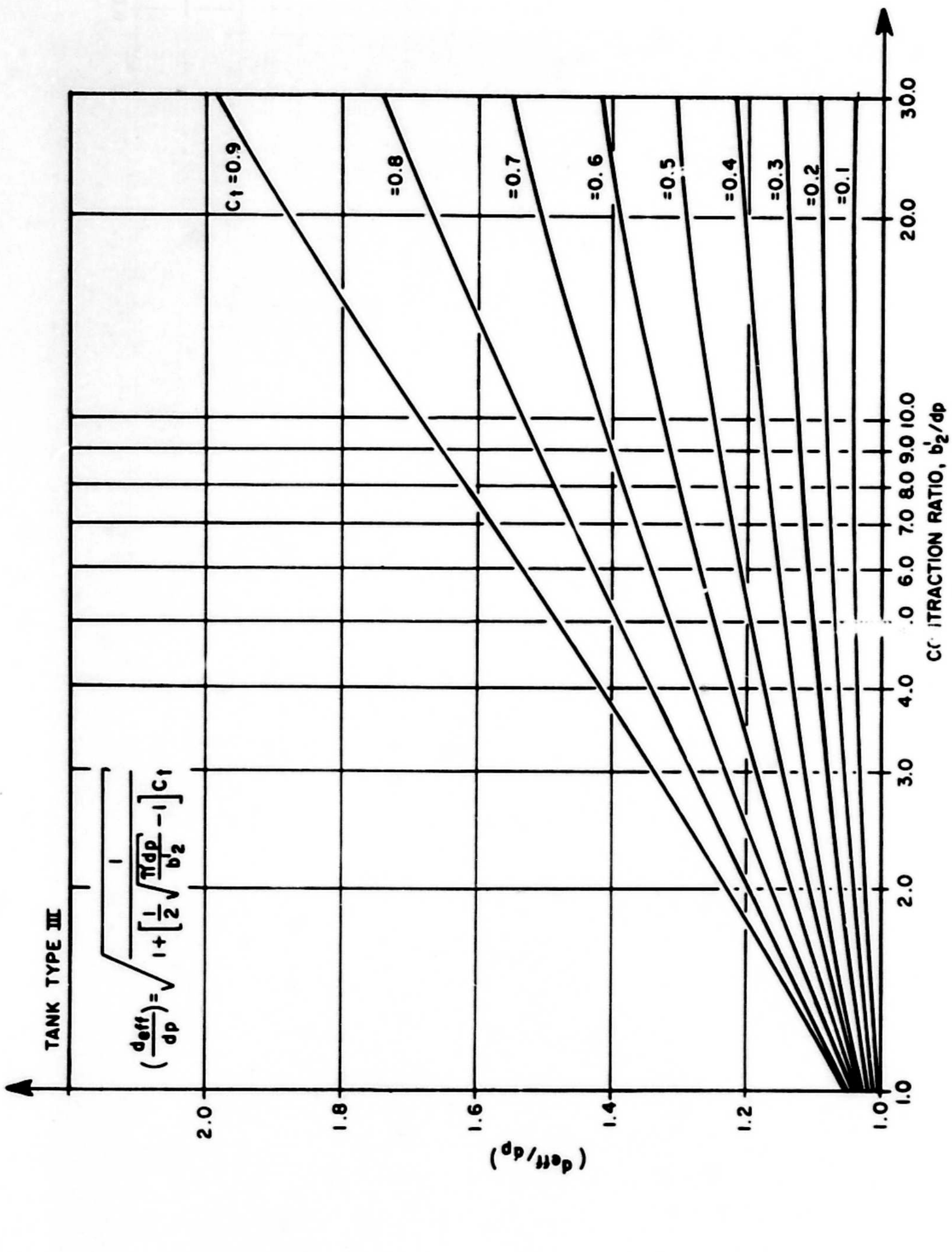

TANK TYPE III
$\left(\frac{d_{eff}}{dp}\right) = \sqrt{\frac{1}{1+\left[\frac{1}{2}\sqrt{\frac{\pi dp}{b_2'}}-1\right]C_t}}$
$C_t = 0.9$
$= 0.8$
$= 0.7$
$= 0.6$
$= 0.5$
$= 0.4$
$= 0.3$
$= 0.2$
$= 0.1$
2.0
1.8
1.6
1.4
1.2
1.0
(d_{eff}/d_p)
1.0
2.0
3.0
4.0
6.0
7.0
8.0
9.0
10.0
20.0
30.0
CONTRACTION RATIO, b_2'/dp

FIGURE 12b

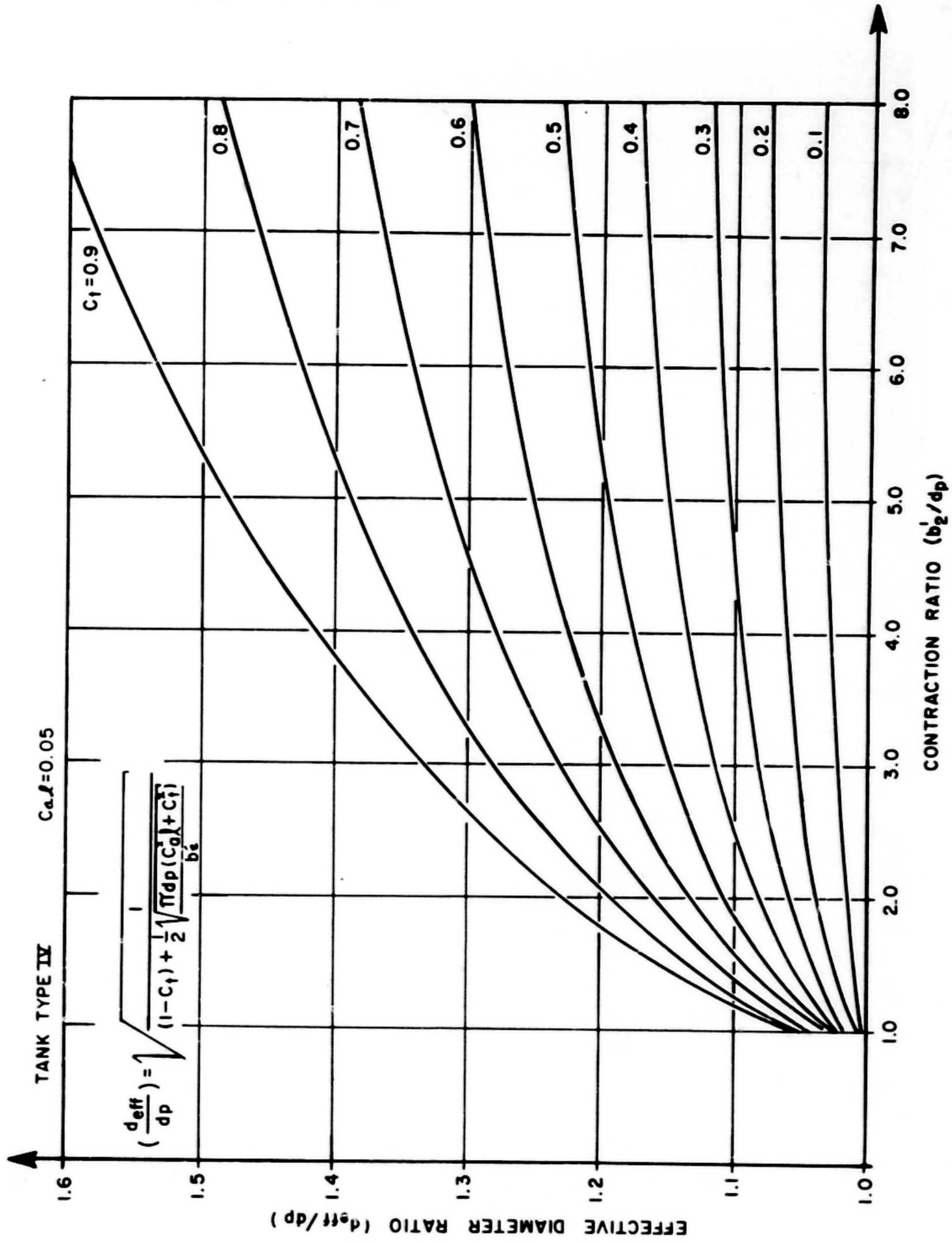

TANK TYPE IV
$C_{a\ell} = 0.05$
$\left(\frac{d_{eff}}{d_p}\right) = \sqrt{\frac{1}{(1 - C_t) + \frac{1}{2}\sqrt{\frac{\pi d_p (C_{a\ell} + C_t)}{b_2'}}}}$
$C_t = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.0
1.1
1.2
1.3
1.4
1.5
1.6
2.0
3.0
4.0
5.0
6.0
7.0
8.0
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)
CONTRACTION RATIO (b_2'/d_p)

FIGURE 13a

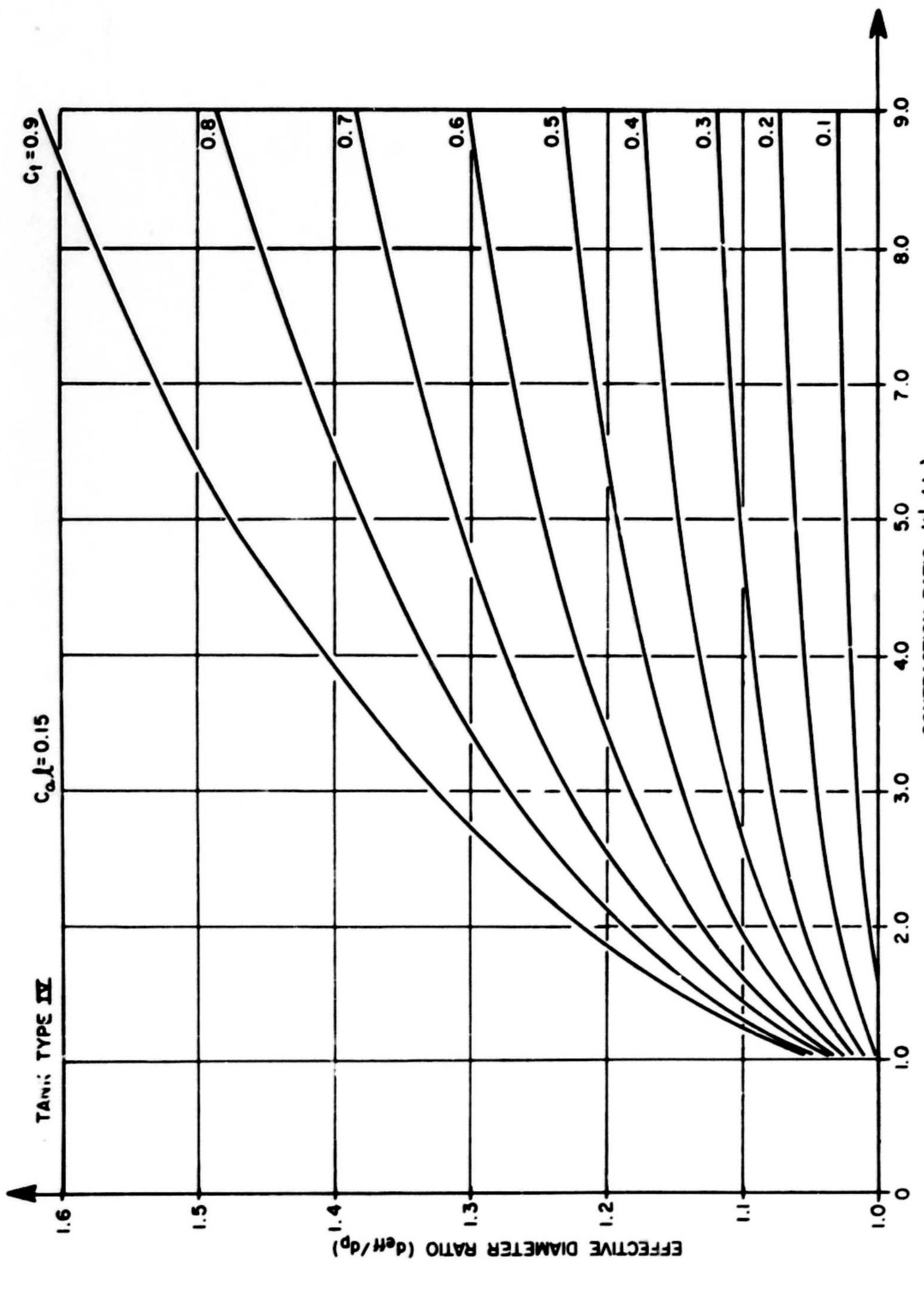

TANK TYPE IV
$C_a l = 0.15$
$C_t = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1.0
2.0
3.0
4.0
5.0
7.0
8.0
9.0
CONTRACTION RATIO (b_2'/d_p)
1.0
1.1
1.2
1.3
1.4
1.5
1.6
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)

FIGURE 13b

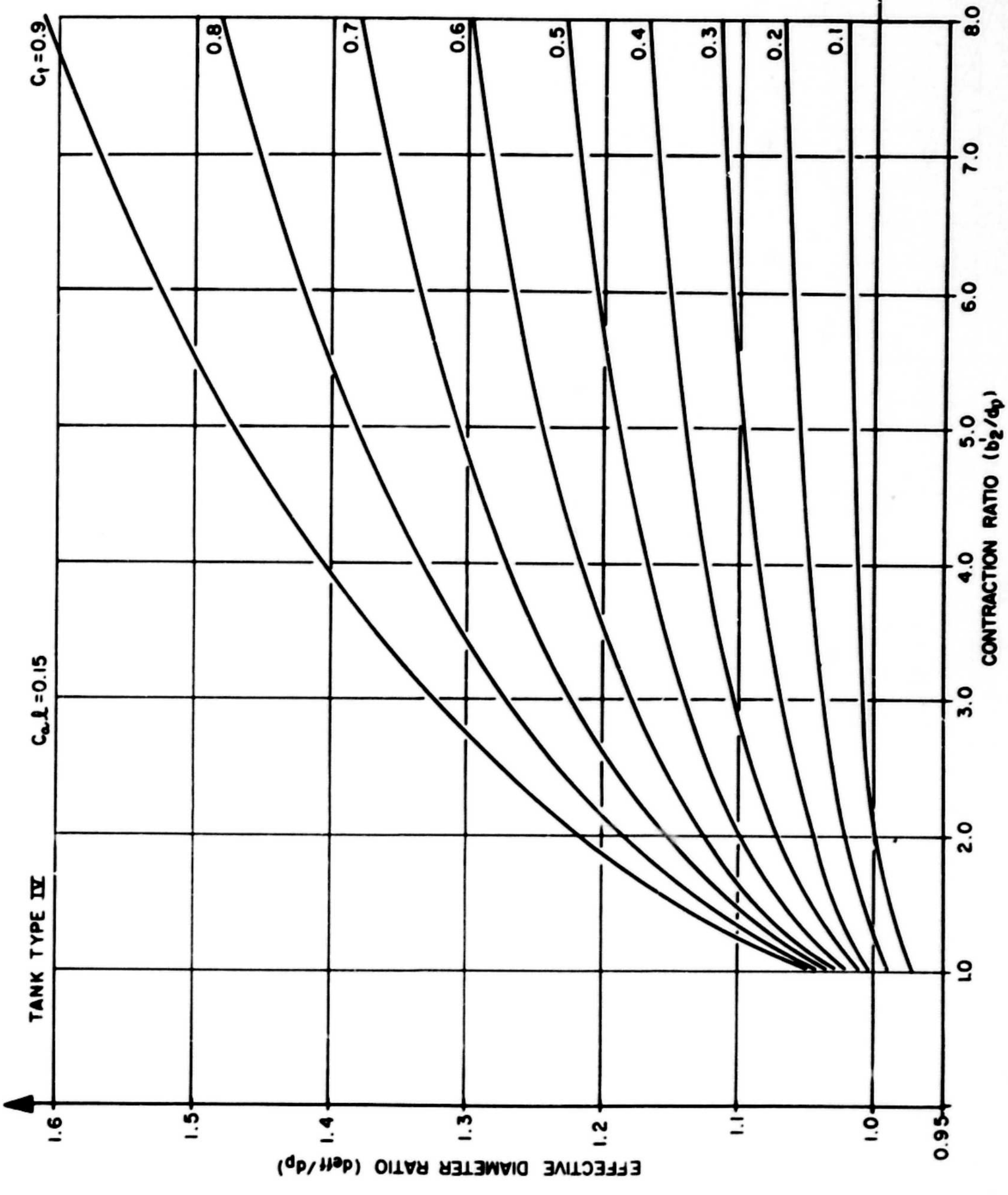

TANK TYPE IV
$C_a \ell = 0.15$
$C_t = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
CONTRACTION RATIO (b_2'/d_p)
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.95
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)

FIGURE 13c

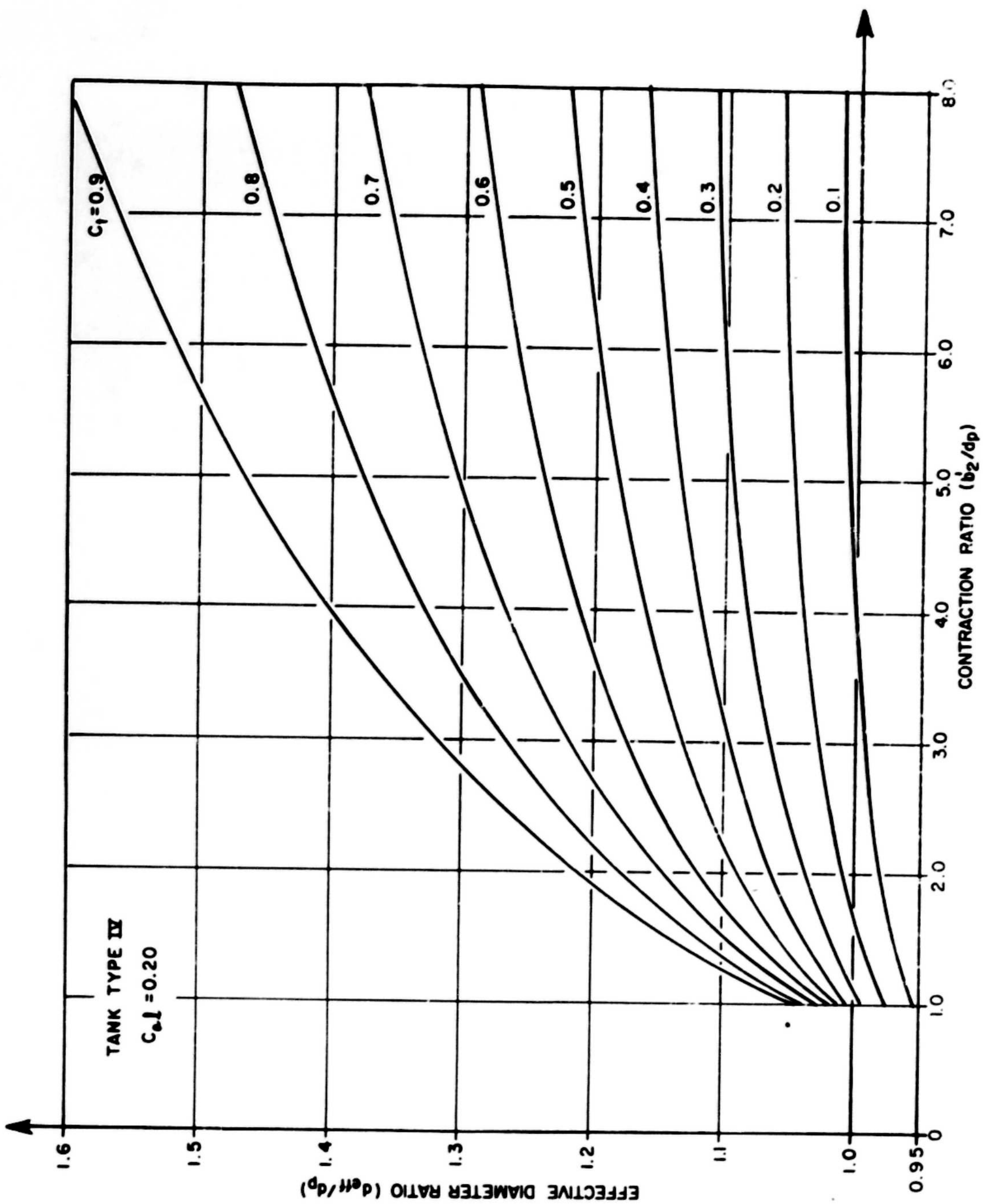

TANK TYPE IV
$C_a l$ = 0.20
C_l = 0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)
CONTRACTION RATIO (b_2'/d_p)
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.95
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0

FIGURE 13d

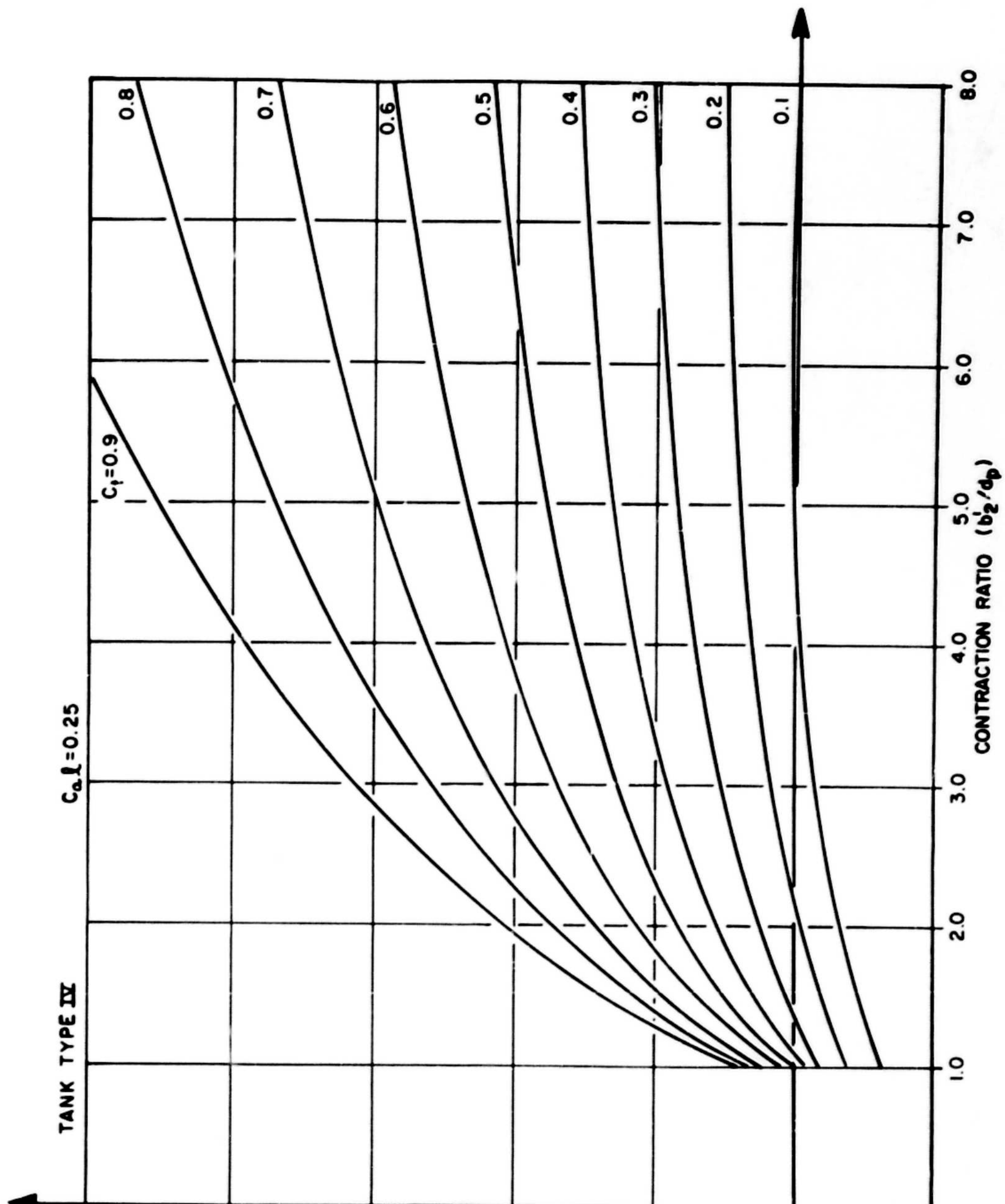
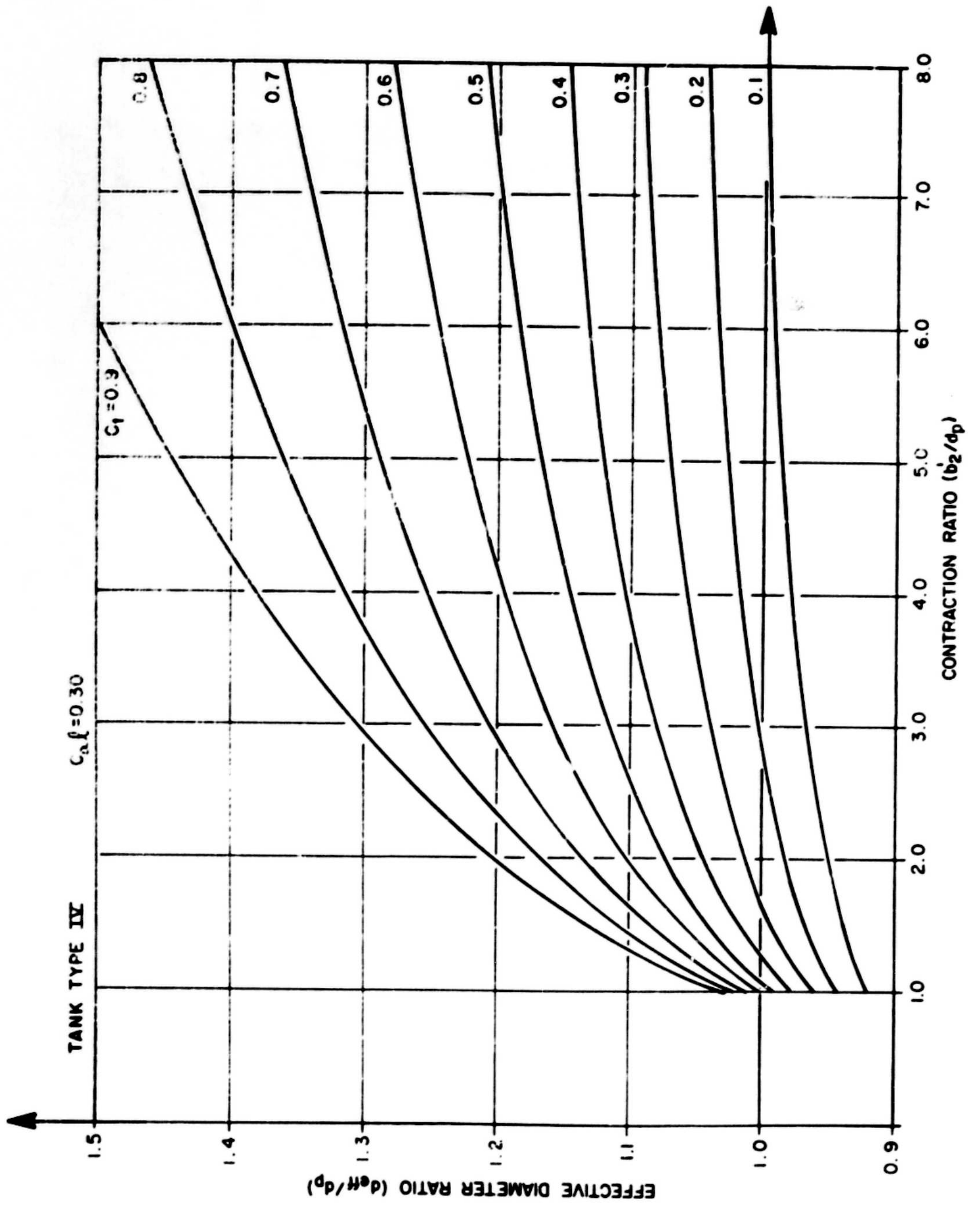

TANK TYPE IV
$C_a \ell = 0.25$
$C_l = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)
1.5
1.4
1.3
1.2
1.1
1.0
0.9
CONTRACTION RATIO (b_2'/d_p)
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0

FIGURE 13e



TANK TYPE IV
$C_a \ell = 0.30$
$C_1 = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.5
1.4
1.3
1.2
1.1
1.0
0.9
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
CONTRACTION RATIO (b_2'/d_p)

FIGURE 13f

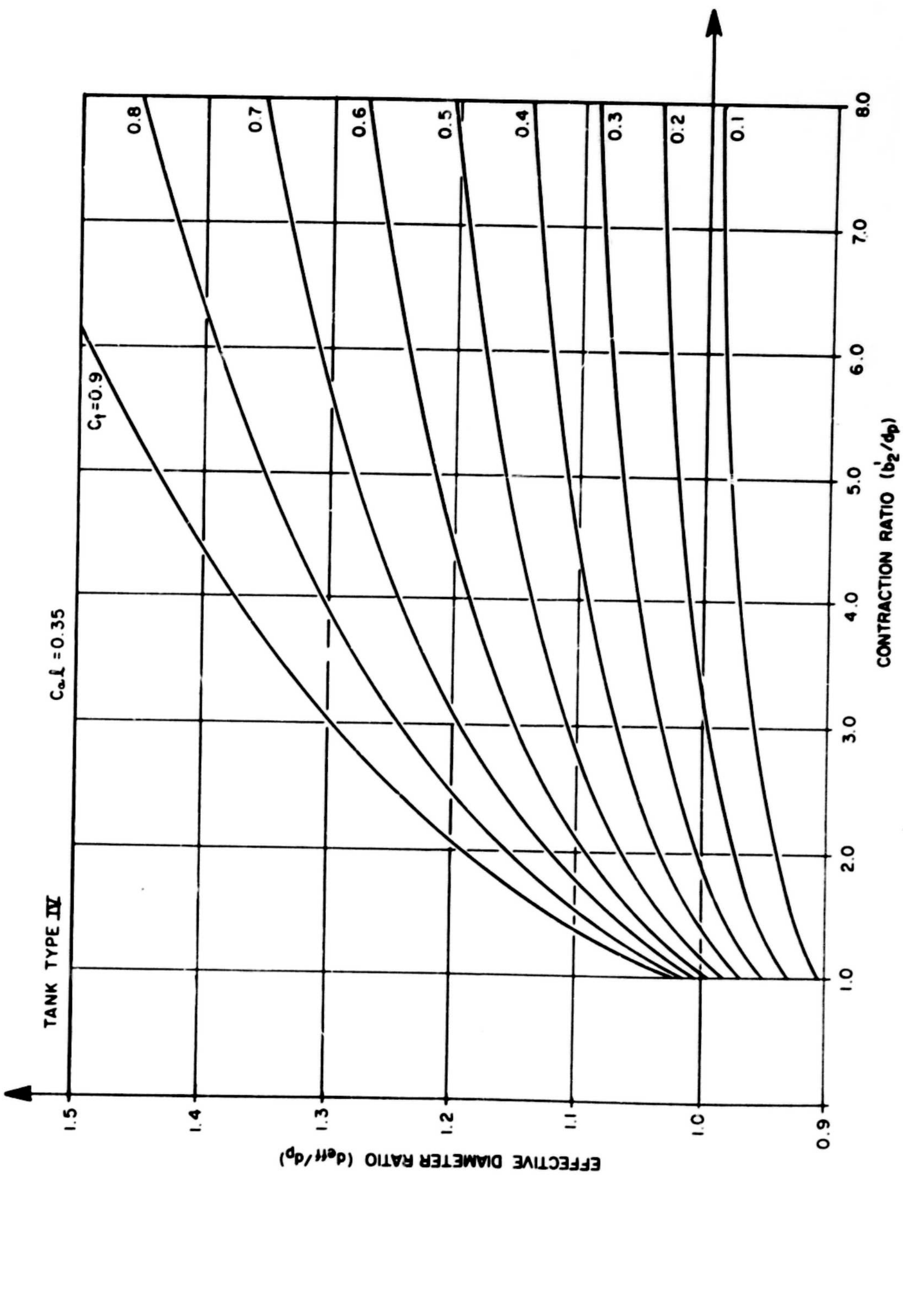

TANK TYPE IV
$C_a l = 0.35$
$C_f = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.5
1.4
1.3
1.2
1.1
1.0
0.9
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
CONTRACTION RATIO (b_2'/d_p)

FIGURE 13g

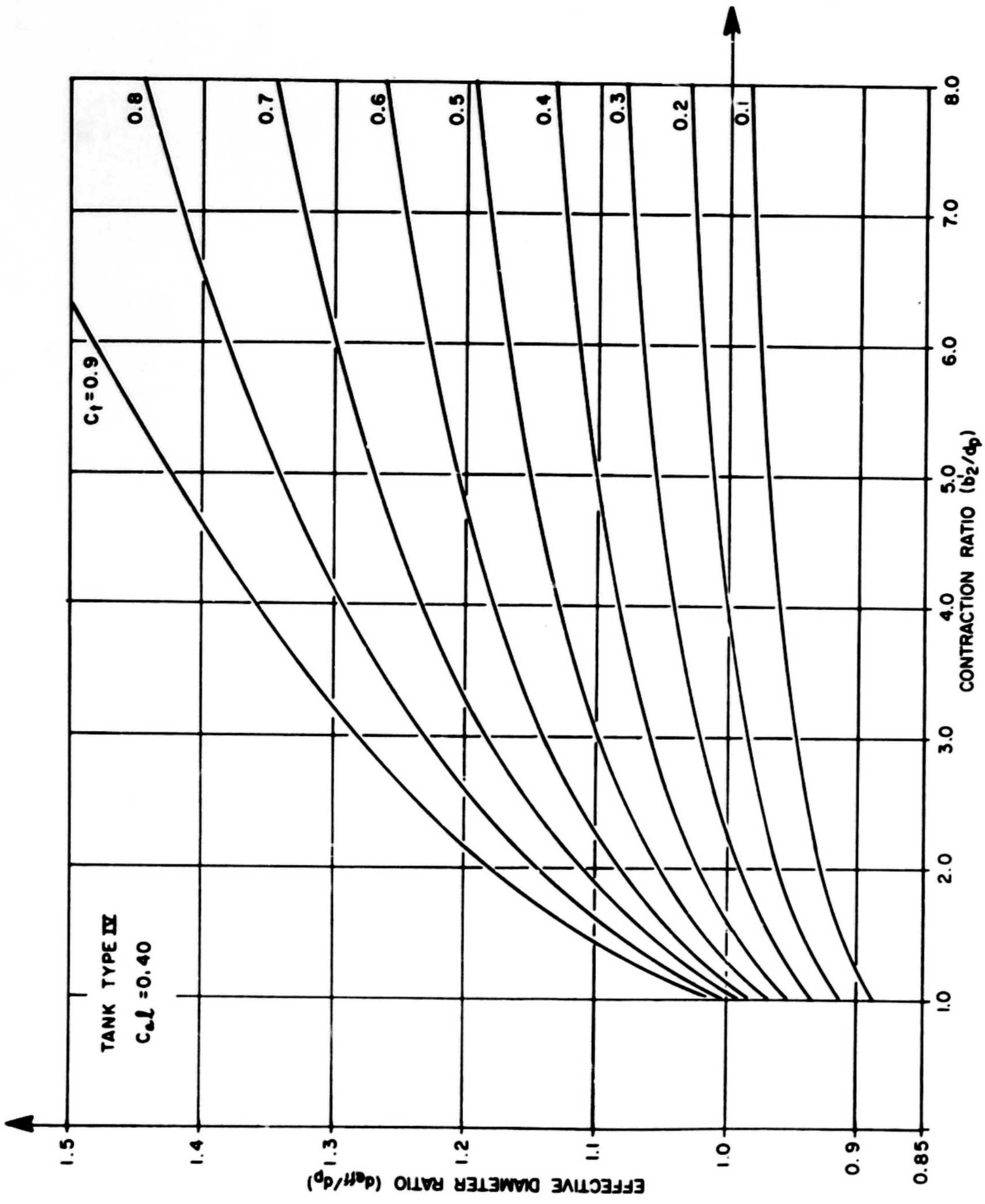

TANK TYPE IV
$C_a\ell$ = 0.40
C_t = 0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.85
CONTRACTION RATIO (b_2/d_p)
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0

FIGURE 13h-

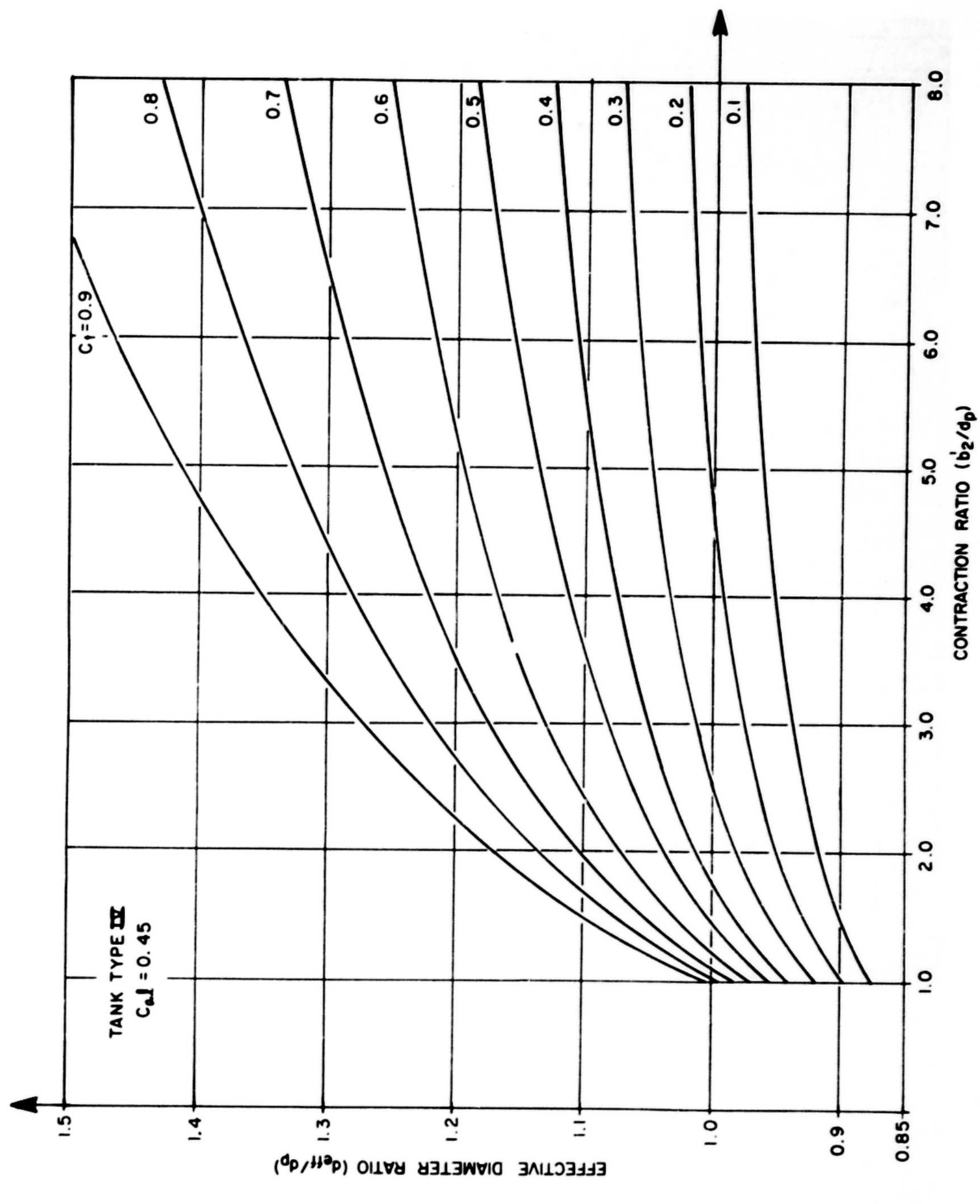

TANK TYPE III
$C_a l = 0.45$
$C_1' = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.85
CONTRACTION RATIO (b_2'/d_p)
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0

FIGURE 13i

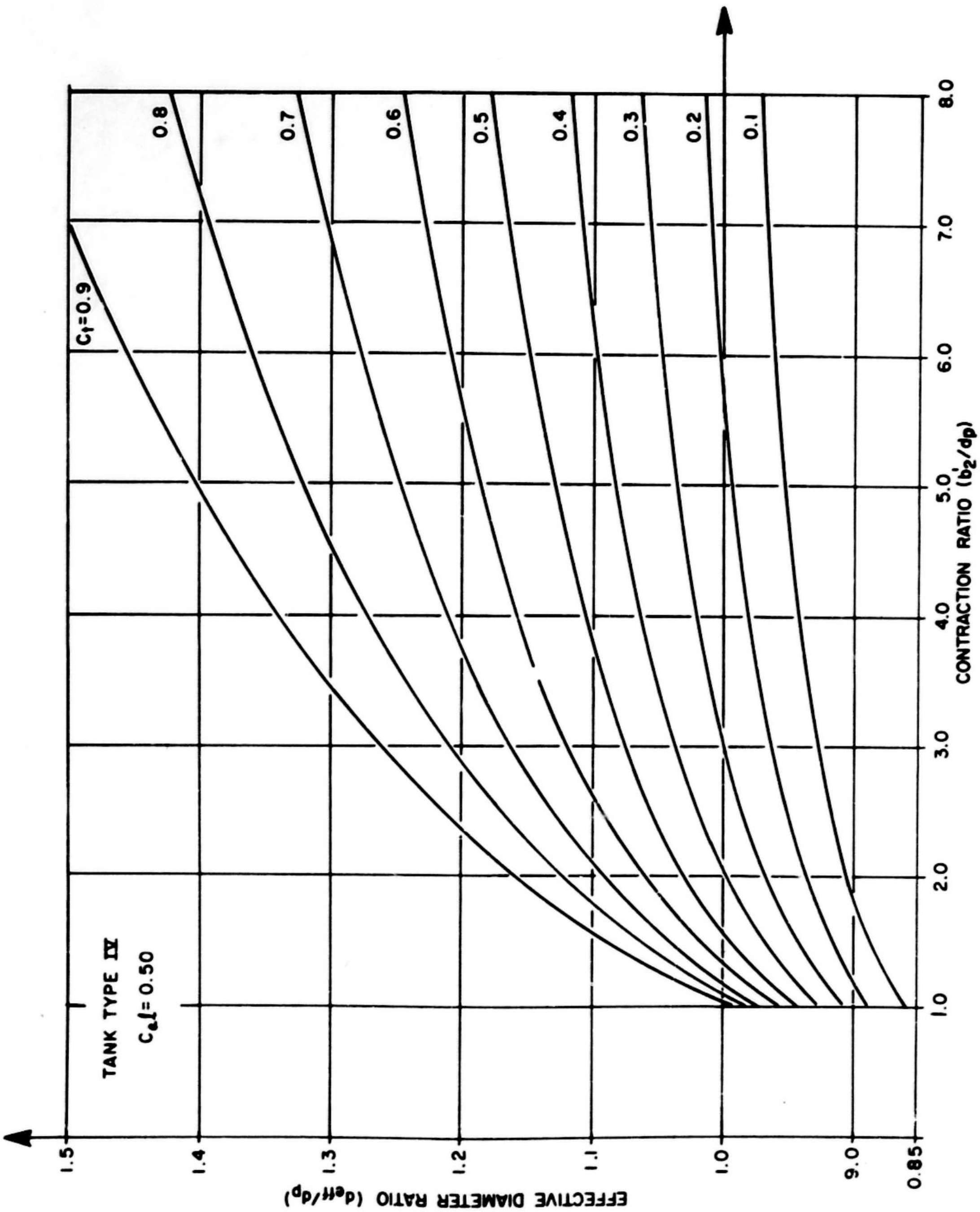

TANK TYPE IV
$C_{e}l = 0.50$
$C_t = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
CONTRACTION RATIO (b_2'/d_p)
1.5
1.4
1.3
1.2
1.1
1.0
9.0
0.85
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)

FIGURE 13j

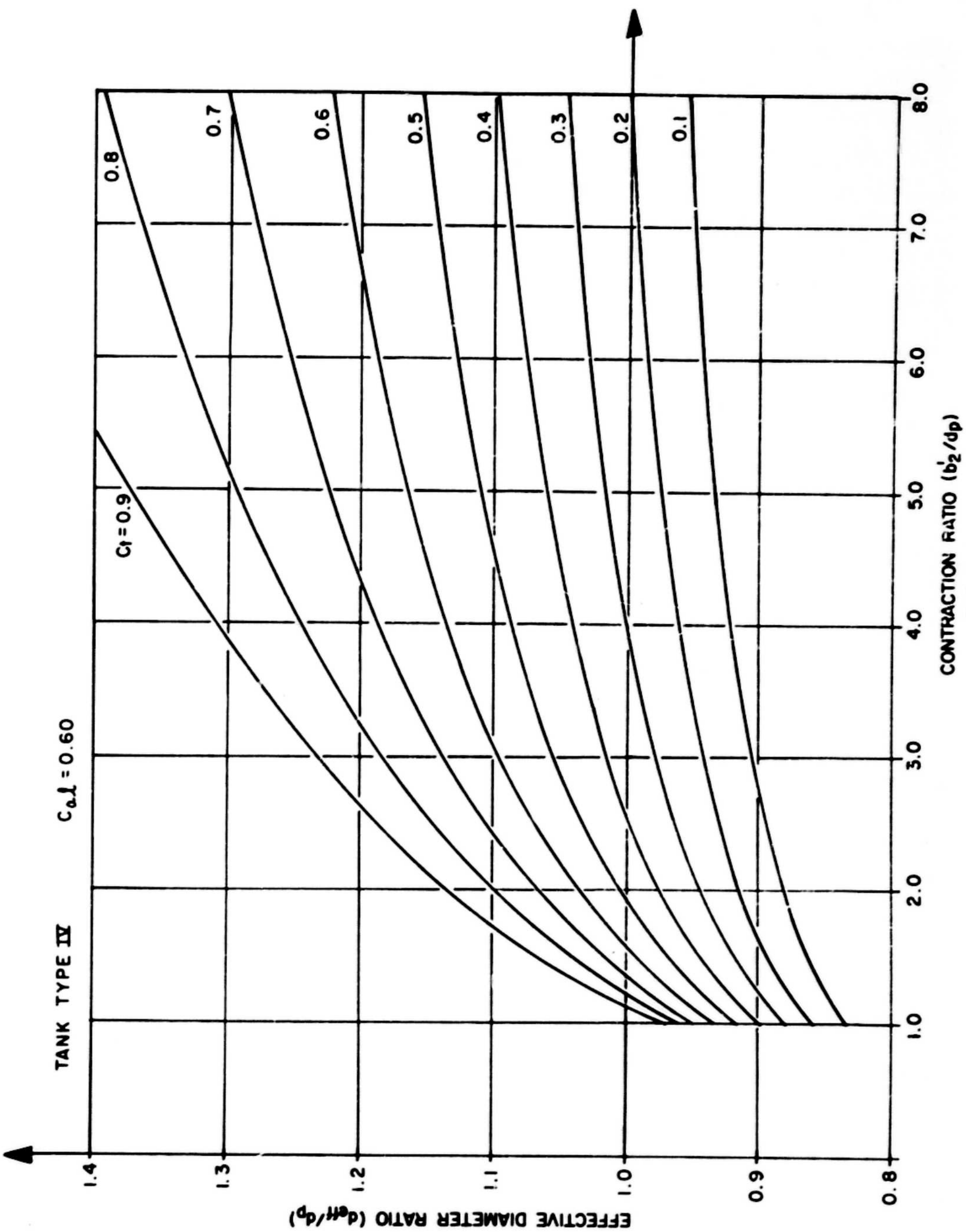

TANK TYPE IV
$C_a l = 0.60$
$C_t = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
EFFECTIVE DIAMETER RATIO (d_{eff}/d_p)
1.4
1.3
1.2
1.1
1.0
0.9
0.8
CONTRACTION RATIO (b_2'/d_p)
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0

FIGURE 13k

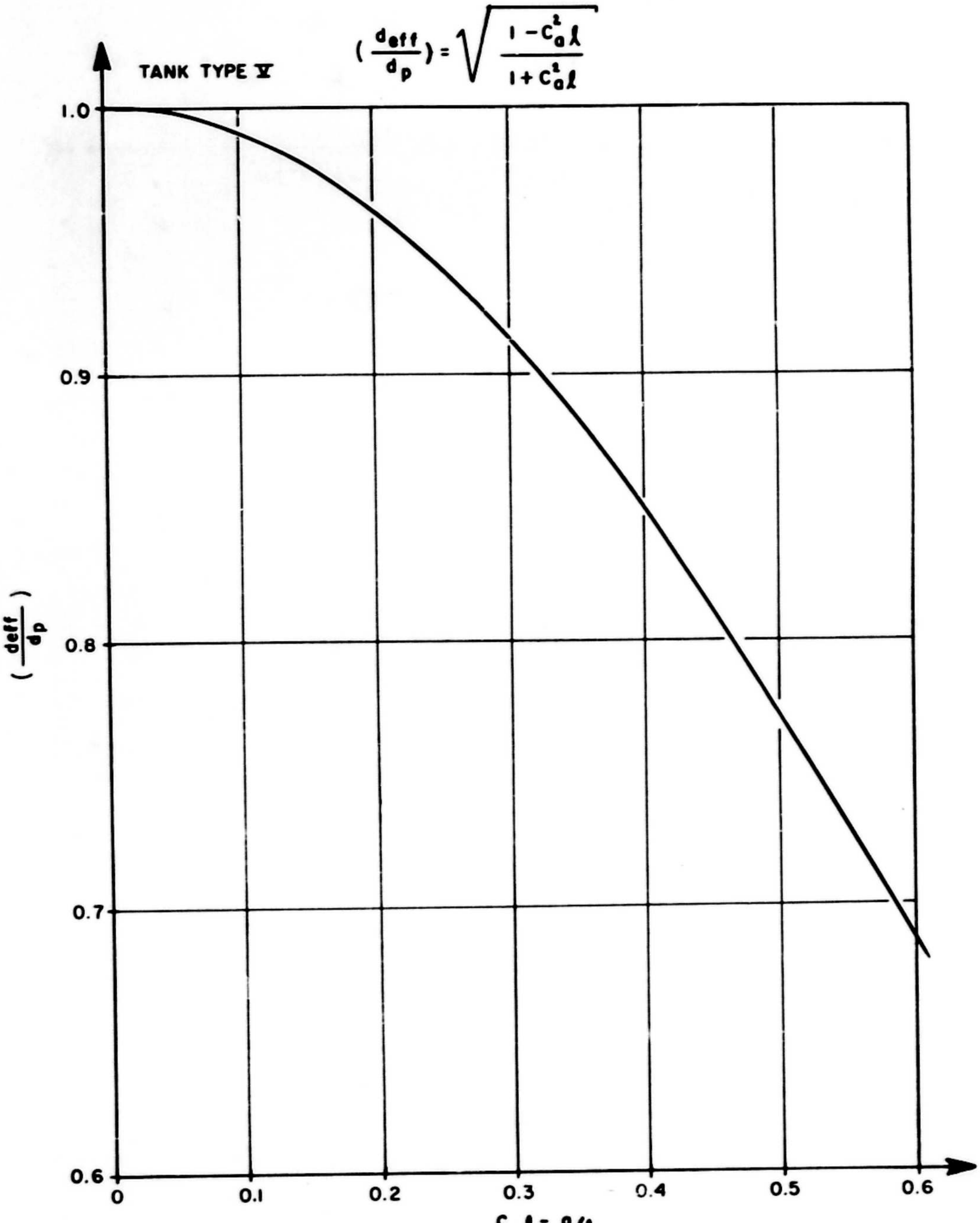


FIGURE 14 - EFFECTIVE DIAMETER RATIO d_{eff}/d_p AS A FUNCTION OF THE CURVED CONNECTING LEG.

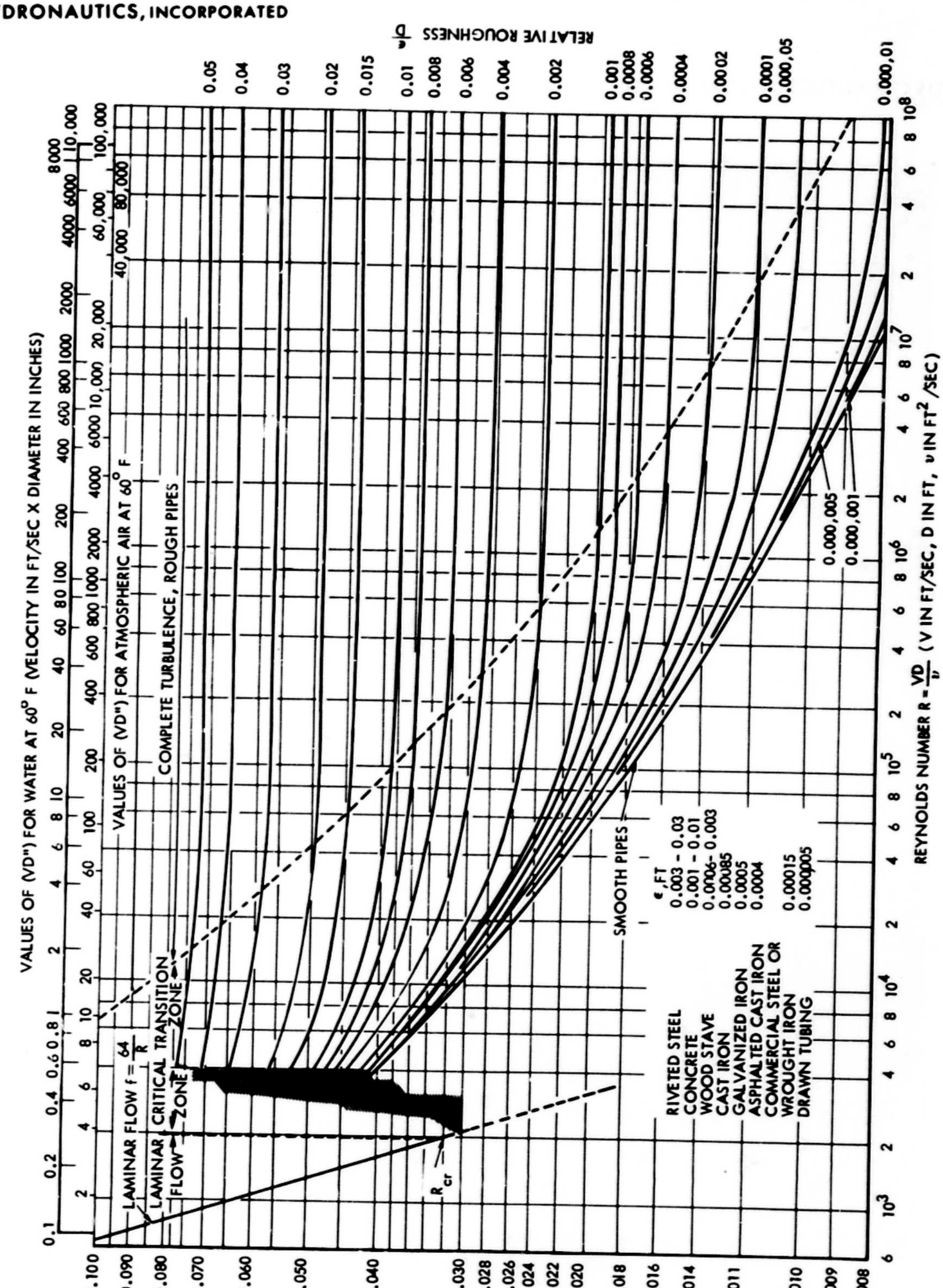


FIGURE 15 - MOODY DIAGRAM

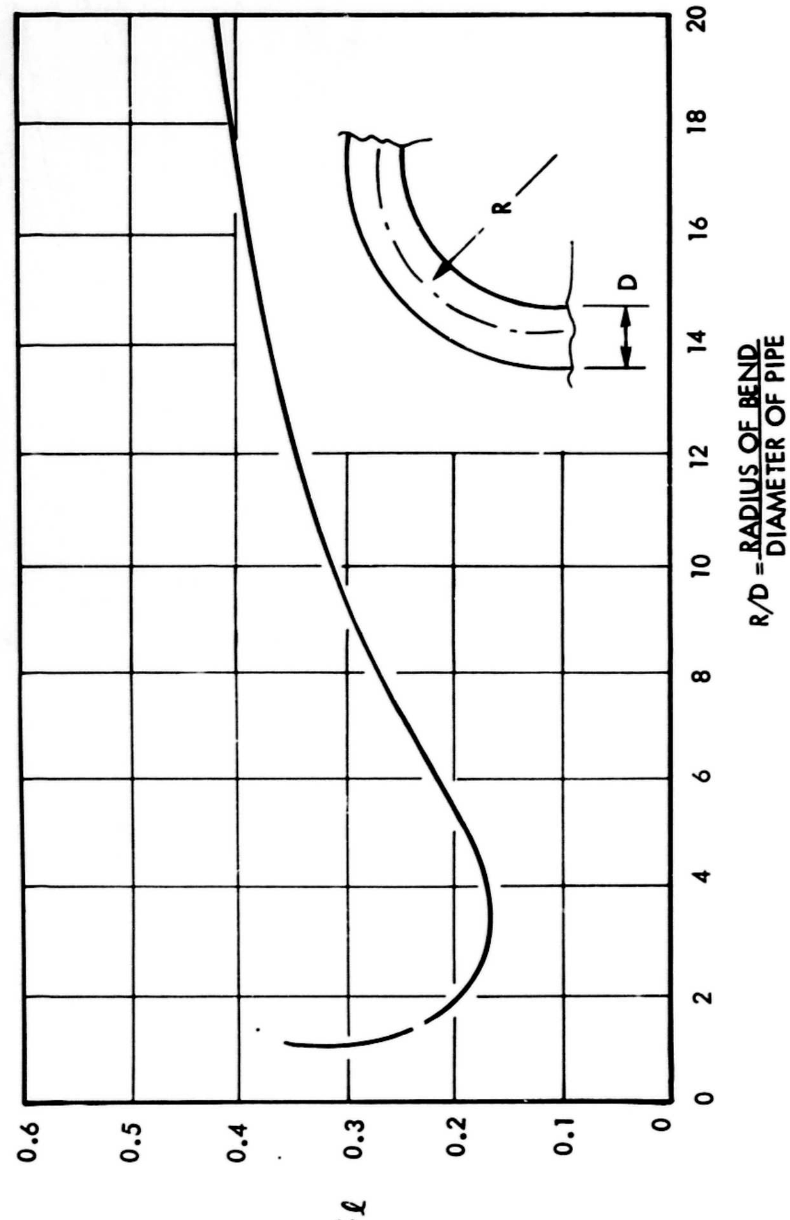
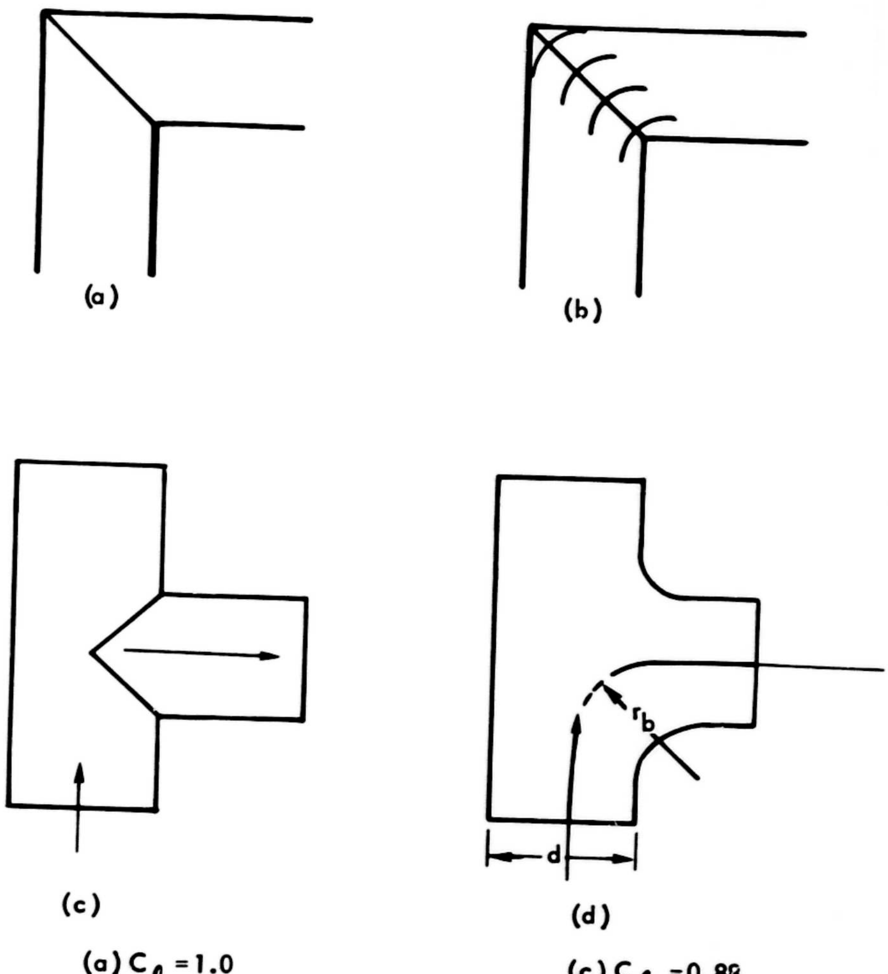


FIGURE 16 – LOSS COEFFICIENT C_ℓ FOR 90° BENDS IN ROUND PIPES



(a) $C_\ell = 1.0$

(b) $C_\ell = 0.136$ AIRFOIL BAFFLES
$= 0.24$ COMMERCIAL BAFFLES

(c) $C_\ell = 0.88$

(d) $C_\ell = 0.40$ WITH $r_b/d = 1.5$

FIGURE 17 - LOSS COEFFICIENTS IN PIPES

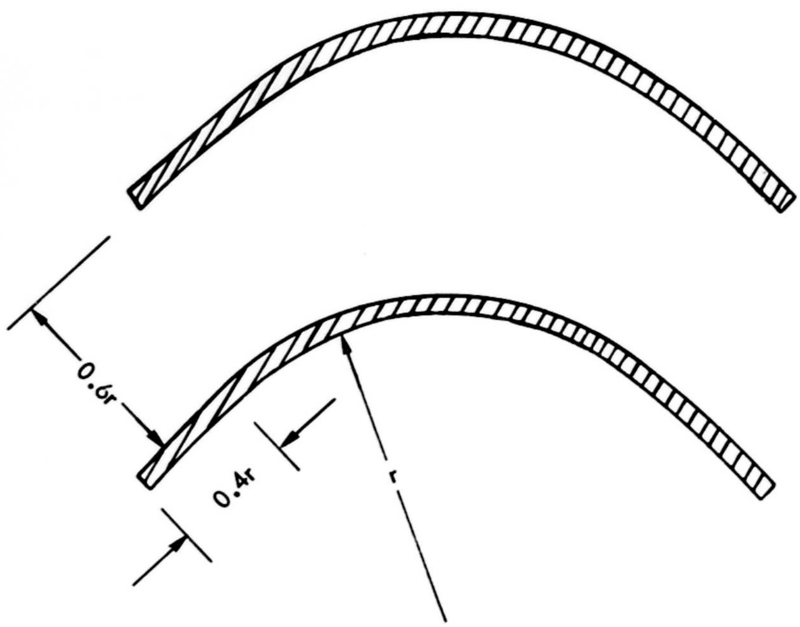


FIGURE 18 - COMMERCIAL TURNING VANES

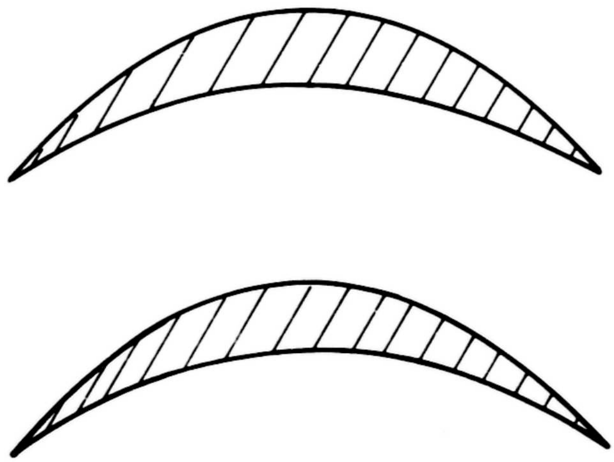
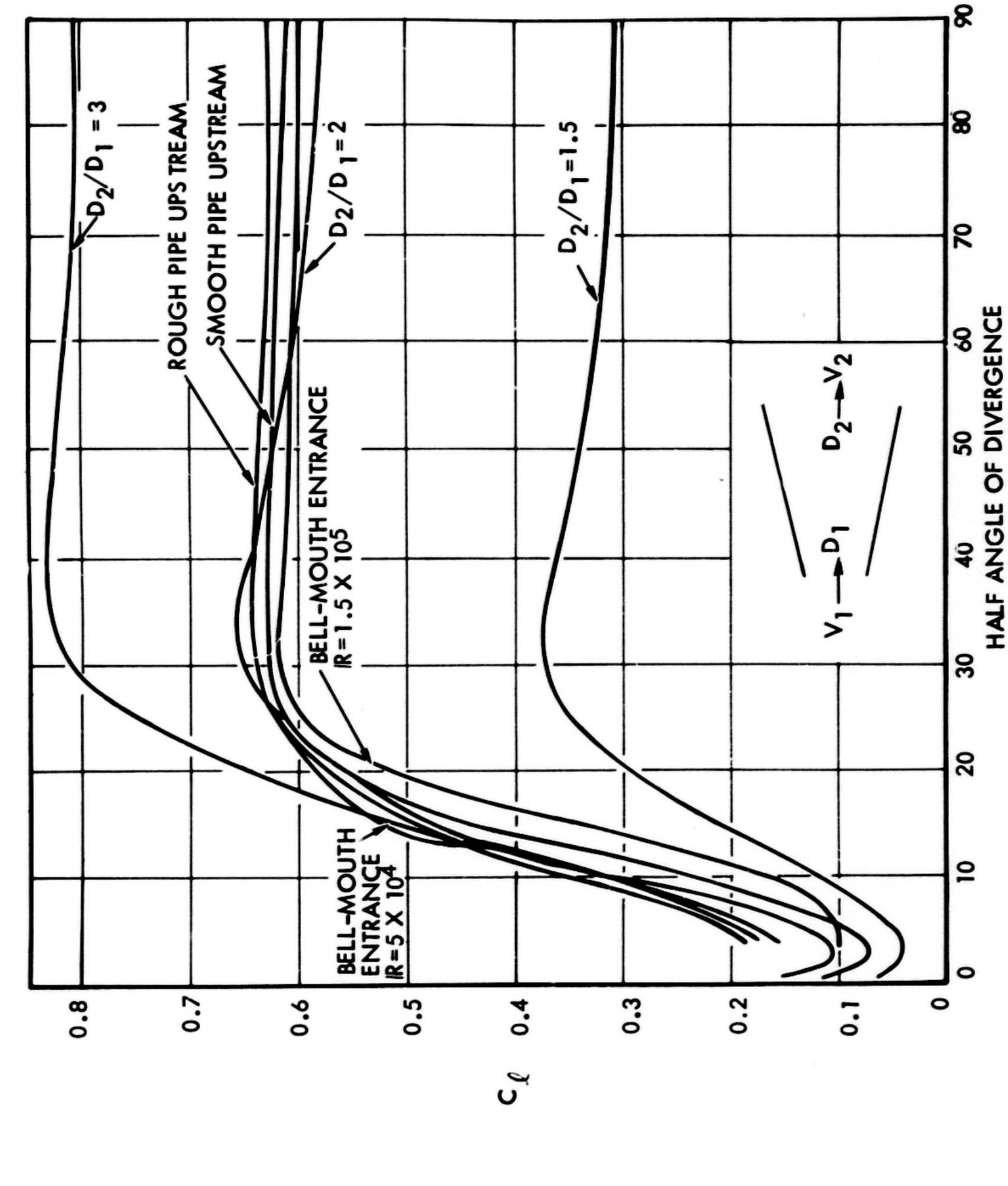


FIGURE 19 - TURBINE TYPE TURNING VANES



D2/D1 = 3
ROUGH PIPE UPSTREAM
SMOOTH PIPE UPSTREAM
D2/D1 = 2
D2/D1 = 1.5
BELL-MOUTH ENTRANCE
IR = 1.5 X 10^5
BELL-MOUTH
ENTRANCE
IR = 5 X 10^4
V1 → D1
D2 → V2
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Cℓ
0
10
20
30
40
50
60
70
80
90
HALF ANGLE OF DIVERGENCE

FIGURE 20

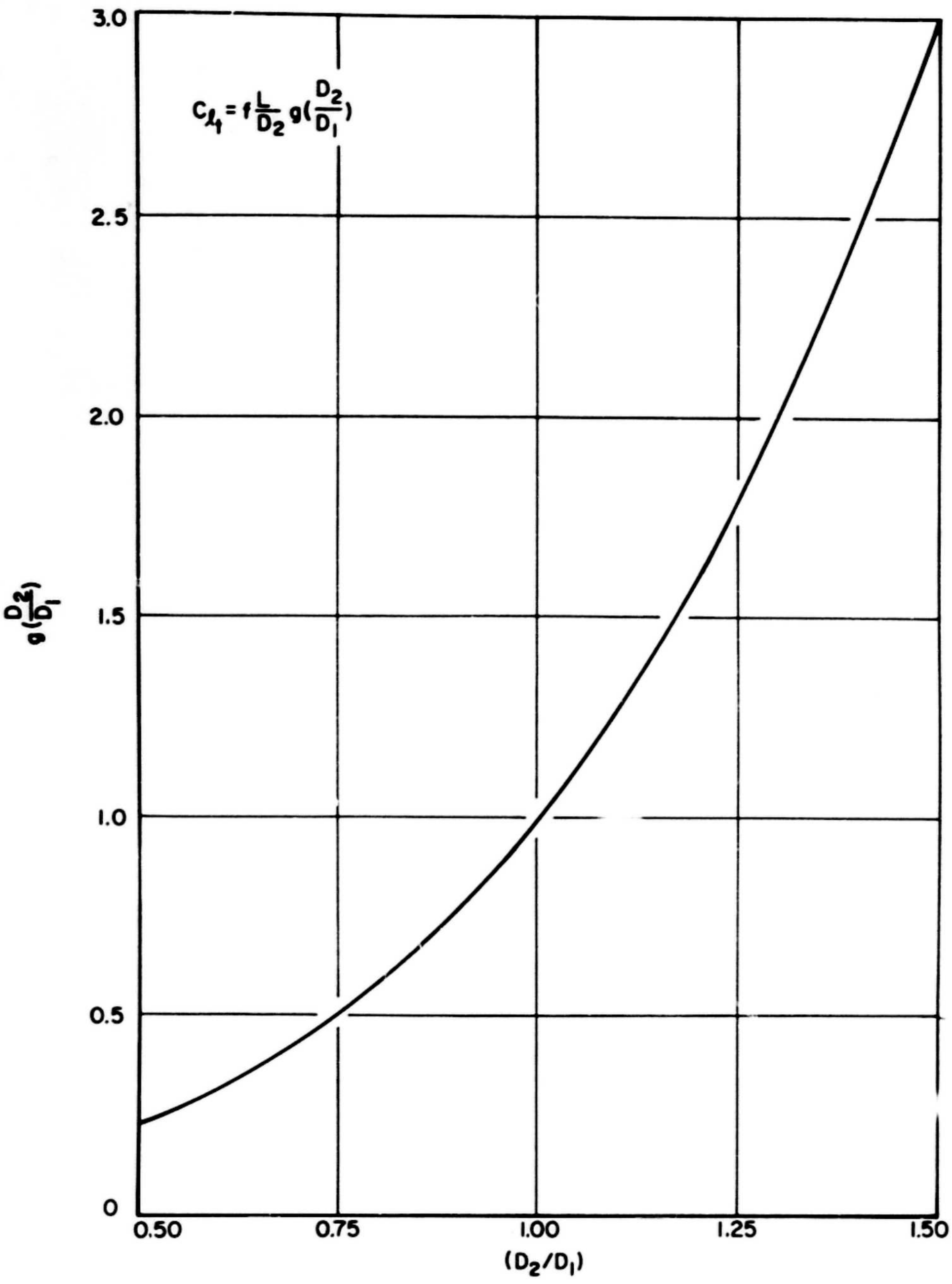


FIGURE 21- FUNCTIONS FOR COMPUTATION OF THE FRICTIONAL LOSS IN A SLIGHTLY DIVERGENT PIPE

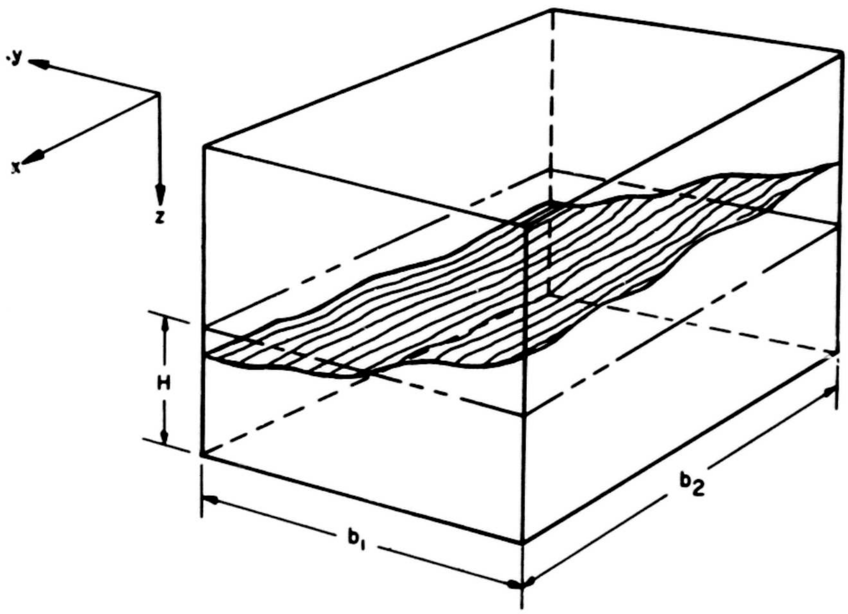


FIGURE 22– SLOSHING IN A SIDE TANK

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3. REPORT TITLE

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4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

July 1967

5. AUTHOR(S) (Last name, first name, initial)

Webster, William C.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
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13. ABSTRACT

Detailed consideration is given to the design of the tanks for activated anti-roll tank installations. The tanks considered here are of the U-tube type. Methods are presented for selecting the geometry of the tank to obtain the proper tank size frequency. The computation of the damping is made from standard, empirical relations used in hydraulics. In addition, the formulae for computing sloshing are also presented.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Anti-roll tanks						
U-tube						
Roll stabilization						

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