APL/JHU CF-2647
May 23, 1957

Copy No. 77

A GROUND FLUTTER SIMULATOR

by

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Operating under Contract NOrd 7386 with the Bureau of Ordnance, Department of the Navy

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CF- 2647

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I. General Discussion

Between engineering analysis and flight testing there are several tests which help to establish the flutter characteristics of the given configuration. The tests may be of a static or dynamic nature, and some systems are so complex that none of the tests define the structure well enough to show its flutter tendencies. It is not the purpose of this report to describe the various possibilities, but to propose an additional technique for ground testing by a simulation of flutter.

It has been learned that in certain cases the distributed aerodynamic forces on a surface may be represented by a single concentrated force acting 180 degrees out of phase with the angle of attack. The flutter of a two-dimensional supersonic tail heavy surface can be understood on this basis, and it is implied that the structurally three-dimensional case can be handled equally well. A vibration test of the actual surface can be used to establish the magnitude of the "simulator speed". It is one step further to use a feed-back system to produce a vibration representative of the same flutter mode. It involves using an electrical signal representing the angle of attack to produce a shaker force 180 degrees out of phase with the signal. When the gain is raised to a certain critical level, a self-excitation is produced. The corresponding shaker force and the surface displacements can then be used for a direct computation of the flutter speed. The technique is currently being applied to a simple flat plate cantilever for which the flutter speed can be computed; the comparison of the simulator speed and the computed speed is needed to show that the device is operating satisfactorily.

More adequate simulation can be achieved by the use of more pickups and more shakers, so that the forces and moments produced will correspond more accurately to those produced by an oscillating airfoil.

II. Analysis Using Computed Data for Uncoupled Modes

It is well to begin with a simple three-dimensional surface having bending and rotational degrees of freedom, reacting to an aerodynamic force at the elastic axis, Figure 1.

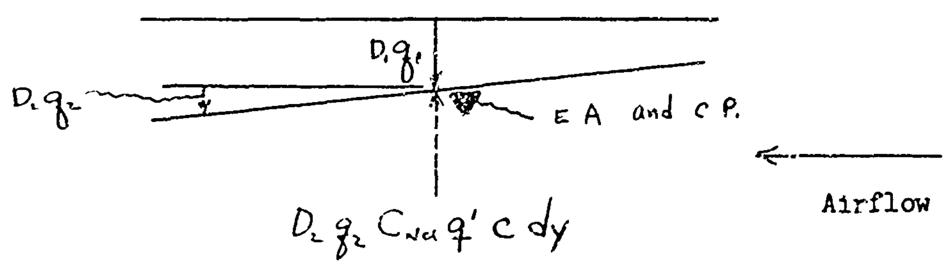

FIGURE 1

The application of Lagrange's Equations produces the two equations required for equilibrium:

(1) $$(k_h - M_e\,\omega^2)\,q_1 - S\,\omega^2 q_2 = \int_0^L p\,D_1\,c\,dy = -N\,q_2$$

(2) $$-S\,\omega^2 q_1 + (k_\alpha - I\,\omega^2)\,q_2 = 0$$

For a distributed spring mass system these quantities have the nature of generalized forces, defined on page 10.

A condition of simulated flutter exists when the determinant is equal to zero.

(3) $$k_{h_o} k_{\alpha_o} - G_h G_\alpha - \omega^2[M\,k_{\alpha_o} + I\,k_{h_o}] + M\,I\,\omega^4$$
$$+ S\,\omega^2 L - S^2\,\omega^4 + i\,[G_\alpha][k_{h_o} - M\,\omega^2] + i\,G_h\,[k_{\alpha_o} - I\,\omega^2] = 0$$

The real and imaginary parts are separately set equal to zero:

(4) $$k_{\alpha_o} g_\alpha\,[k_{h_o} - M\,\omega^2] = -k_{h_o} g_h\,[k_{\alpha_o} - I\,\omega^2]$$

Assume:

$$g_h = g_\alpha$$

(5) $$\omega^2 = \frac{2\,k_{\alpha_o} k_{h_o}}{M\,k_{\alpha_o} + I\,k_{h_o}}$$

$$k_h = k_{h_o} + i\,G_h$$
$$k_\alpha = k_{\alpha_o} + i\,G_\alpha$$
$$G_h = k_{h_o}\,g_h$$
$$G_\alpha = k_{\alpha_o}\,g_\alpha$$

Define:

$$\omega_\alpha^2 = \frac{k_{\alpha_o}}{I}$$

$$\omega_h^2 = \frac{k_{h_o}}{M}$$

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(6)
$$\omega^2 = \frac{2\,\omega_\alpha^2\,\omega_h^2}{\omega_\alpha^2 + \omega_h^2}$$

This value of the frequency is then used in the real part of the equation:

(7)
$$k_h\,k_\alpha\,[1 - g_h\,g_\alpha] - \frac{2\,\omega^2\,\omega_\alpha}{\omega_h^2 + \omega_\alpha^2}\,[M\,k_\alpha + I\,k_h] + M\,I\,\frac{(4)\,\omega_\alpha^4\,\omega_h^4}{(\omega_\alpha^2 + \omega_h^2)^2}$$

$$+ 2\,S\,\frac{\omega_h^2\,\omega_\alpha^2}{\omega_h^2 + \omega_\alpha^2}\,N - 4\,S^2\,\frac{\omega_h^4\,\omega_\alpha^4}{(\omega_h^2 + \omega_\alpha^2)^2} = 0$$

The equation ultimately simplifies to:

(8)
$$\frac{N\,S}{M\,I\,\omega_\alpha^2} = g^2\,\frac{[1 + (\frac{\omega_h}{\omega_\alpha})^2]}{2} + \frac{[1 - (\frac{\omega_h}{\omega_\alpha})^2]^2 + 4\,\frac{S^2}{MI}\,(\frac{\omega_h}{\omega_\alpha})^2}{2\,[1 + (\frac{\omega_h}{\omega_\alpha})^2]}$$

The equation can be put into a form suitable for correlating results with NACA calculations by using the definitions on page 4 and the ultimate result is:

(9)
$$\frac{V_s}{b\,\omega_\alpha} = \sqrt{\frac{\mu\,r_g^2\,\sqrt{M^2 - 1}}{\bar{x}\,b}\;\frac{[1 - (\frac{\omega_h}{\omega_\alpha})^2]^2 + 4\,(\frac{x}{r})^2\,(\frac{\omega_h}{\omega_\alpha})^2}{2\,[1 + (\frac{\omega_h}{\omega_\alpha})^2]}}$$

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TABLE 1

M_n	$\frac{\omega_h}{\omega_\alpha}$	$\frac{\bar{x}}{c}$	$\frac{\bar{r}}{c}$	μ	V_{NACA}	V_{Sim}	V_{sim}/V_{NACA}
2	1.00	.10	.25	7.854	1.80	1.65	.916
	1.00	.20	.25	7.854	2.20	2.34	1.065
	1.00	.25	.25	7.854	2.30	2.62	1.14
	.707	.10	.25	7.854	1.90	1.79	.94
	.707	.20	.25	7.854	2.01	2.08	1.04
	.707	.25	.25	7.854	2.20	2.26	1.03
5	1.00	.10	.25	7.854	2.80	2.77	.989
	1.00	.20	.25	7.854	3.90	3.93	1.01
	1.00	.25	.25	7.854	4.30	4.40	1.02
	.707	.10	.25	7.854	3.20	3.02	.94
	.707	.20	.25	7.854	3.70	3.51	.95
	.707	.25	.25	7.854	3.90	3.81	.98

ω_h = uncoupled bending frequency

ω_α = uncoupled torsion frequency

b = surface semi-chord

c = chord

$\bar{x}$ = distance of center of gravity aft of the elastic axis

$\bar{r}$ = radius of gyration about elastic axis

μ = $m/4\, b^2 \rho$

m = mass per unit span

ρ = density of air

M_n = Mach number

Note: The elastic axis is at mid-chord in this example.

III. Analysis Based Upon Measured Data from Forced Vibration

It has been shown that the two dimensional simulator speed has a correlation with the NACA data. The next step is to show that experimental data can be obtained from a distributed spring-mass system which can show that the real system is in a condition of simulated flutter. This involves the application of a single shaker force at some point on the elastic axis of the surface, with the assumption that the elastic axis and the centers of pressure coincide. The angular response of the system can then be monitored to find the condition wherein the angular response is 180 degrees out of phase with the applied force.

If the system vibrates in this particular shape at some supersonic Mach number, the distributed aerodynamic forces produce a generalized aerodynamic force. And if the force thus developed is equal to the generalized force produced by the single shaker, it is considered that such forces may effectively replace the shaker as a source of the observed vibration. In order to verify this point, one can compute these forces, equate them, and establish that their equality is true if the original determinant is equal to zero. The following equations show this result.

The response of a two degree of freedom system to a force at the elastic axis is determined from LaGrange's Equations.

(10) $$(k_h - M_e \omega^2)\, q_1 - S\, \omega^2\, q_2 = P_a\, D_{1a}$$

(11) $$- S\, \omega^2\, q_1 + (k_\alpha - I\, \omega^2)\, q_2 = 0$$

The response of the bending coordinate, q_1, is:

(12) $$q_1 = \frac{P_a\, D_1 a}{\psi_1 - \frac{S^2\, \omega^2}{\psi_2}}$$

with:

$$\psi_1 = k_h - M_e\, \omega^2$$

$$\psi_2 = k_\alpha - I\, \omega^2$$

The displacement at the elastic axis is:

(13) $$h = D_1\, q_1$$

$$h = \frac{P_a D_{1a} D_1 \psi_2}{\psi_1 \psi_2 - S^2 \omega^4}$$

The angle at any spanwise station is:

(14) $$\alpha = \frac{P_a D_{1a} D_2 S \omega^2}{\psi_2 \psi_1 - S^2 \omega^4}$$

The generalized aerodynamic force is:

(15) $$Q_A = -\int_0^L C_{L\alpha} q^1 h \alpha C \, dy$$

(16) $$Q_A = -\frac{C_{L\alpha} q^1 P_a^2 D_{1a}^2 \psi_2 \quad S \omega^2 \int_0^L D_1 D_2 C \, dy}{[\psi_1 \psi_2 - S^2 \omega^4]^2}$$

The generalized shaker force is:

(17) $$Q_s = P_a h_a$$

(18) $$Q_s = \frac{P_a^2 D_{1a}^2 \psi_2}{\psi_1 \psi_2 - S^2 \omega^4}$$

A condition of flutter exists when these two forces are equal.

(19) $$-\frac{P^2 D_{1a}^2 \psi_2 S \omega^2 N}{(\psi_1 \psi_2 - S^2 \omega^4)^2} = \frac{P_a^2 D_{1a}^2 \psi_2}{(\psi_1 \psi_2 - S^2 \omega^4)}$$

with

(20) $$N = C_{L\alpha}\, q^{1} \int_{0}^{L} D_{1} D_{2}\, C\, dy$$

or:

(21) $$0 = S\,\omega^{2} N + \psi_{1} \psi_{2} - S^{2} \omega^{4}$$

This expression is to be compared with the result which is obtained by an expansion of the flutter determinant:

(22) $$\begin{vmatrix} \psi_{1} & N - S\,\omega^{2} \\ -S\,\omega^{2} & \psi_{2} \end{vmatrix} = 0$$

$$\psi_{1}\psi_{2} + S\,\omega^{2}\,[N - S\,\omega^{2}] = 0$$

$$S\,\omega^{2} N + \psi_{1}\psi_{2} - S^{2}\,\omega^{4} = 0$$

It is concluded that the condition for equality of the generalized shaker and aerodynamic forces ~~is equivalent to the statement that the flutter~~ determinant is zero.

If a frequency ω exists for which the determinant is zero even when ψ_1 and ψ_2 contain finite structural damping parameters, then a flutter condition exists. Insofar as the experimental condition is concerned, the possibility of flutter is recognized when the angle generated by the applied force is 180 degrees out of phase with it.

The above discussion applies to a restricted class of structures. The theorem might not work out so nicely when the axis of twist and the air-force centers of pressure do not coincide. An extension of these concepts to employ data from tests using shakers at many places on the surface is required to deal properly with complicated structures.

IV. Flutter Simulation

It is desirable to consider the use of a system of self-excitation, wherein an instrument converts angular motion to a voltage, and an amplifier converts this voltage into a voice coil current, and hence a shaker force. An adjustment of gain in such a system corresponds to an increase in shaker force per unit of surface angular motion. For a tail-heavy system, a gain will then be reached for which self-excitation is initiated, and the surface will vibrate in a simulator flutter mode exactly the same as the one reached by the forced vibration method. In this case, the end result is reached more quickly. A study of such a system is a necessary part in the development of more accurate flutter simulation by the use of many shakers and pickups.

The amplifier necessary for the conversion of signal voltage to shaker current is not commercially available. Usually a phase shift is present between input and output which is a function both of load and frequency. Such a phase shift has been eliminated in a transistorized amplifier designed by George Bush of BTR/APL. The amplifier is approximately flat from D.C. to 1 KC, and preliminary tests have shown satisfactory phase shift characteristics. The development of the amplifier is the key to the whole problem of the simple simulator. For details on the circuit, reference (a)* should be consulted. The amplifier input voltage proportional to surface angular motion may be produced by a device which measures the fore and aft motion of the tip of an arm mounted perpendicular to the surface, Method A, Figure 2. This is one simple way of measuring the angle, but it is valid only when the plate is not deflecting fore and aft. A recent improvement, Method B, Figure (2), has been to measure the differential output from two pickups located at the leading and trailing edges. The pickup system is comprised of a Schaevitz linear differential transformer and a phase comparator. The comparator output voltage is used to drive the Bush Amplifier. The amplifier output voltage is used to drive a Goodman V-47 shaker as shown in Figure 3.

The operation of the simulator involves a slow increase in the gain of the system until the aerodynamic surface oscillates. The force acting may be found from the appropriate calibration factor and the shaker current. A measurement of the displacement at the shaking point then enables one to compute the generalized shaker input force.

$$Q_s = p_s h_a \tag{23}$$

The next step is to compute the generalized aerodynamic force that would be produced by the surface undergoing the same displacements at some chosen Mach number.

$$Q_A = C_{I_\alpha} q^1 \int_0^L h \alpha c \, dy \tag{24}$$

The expression for Q_A takes on an interesting form when the centers of pressure lie along the mid-chord, and the leading and trailing edge displacements h_f and h_r have been measured:

$$Q_A = C_{I_\alpha} q^1 \int_0^L \left(\frac{h_r^2 - h_f^2}{2}\right) dy \tag{25}$$

If the generalized shaker force is less than the generalized aerodynamic force a condition of flutter can be expected at the chosen Mach number.

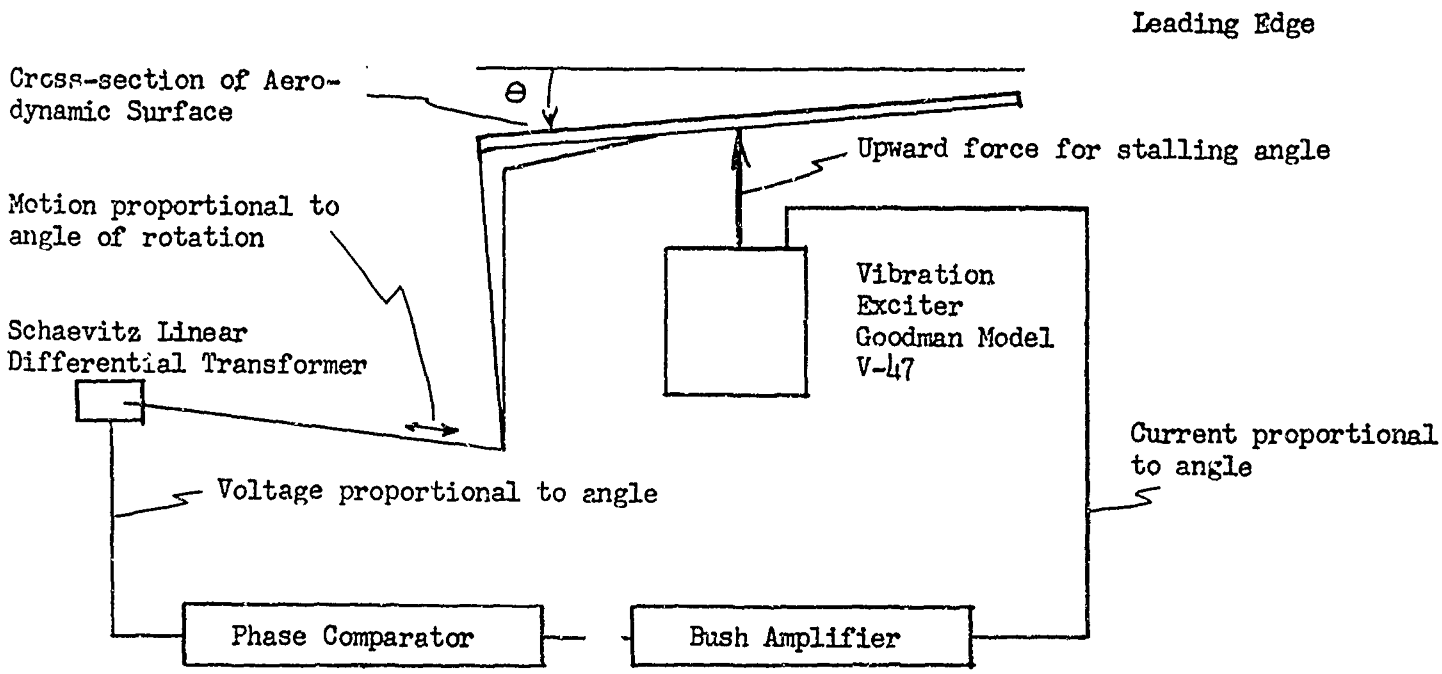
*Reference (a) APL/BBE Dwg. 57020

FIGURE 2

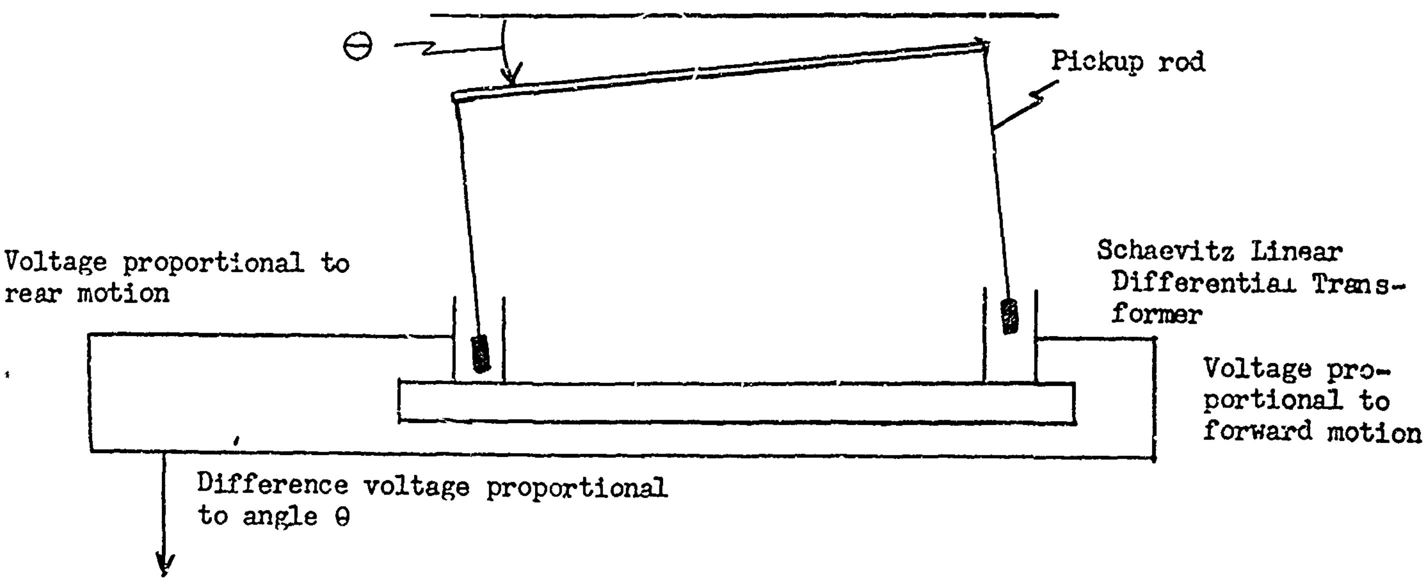
Ground Flutter Simulator

Schematic Diagram

Method (A) for Sensing Angular Motion



Method (B) for Sensing Angular Motion



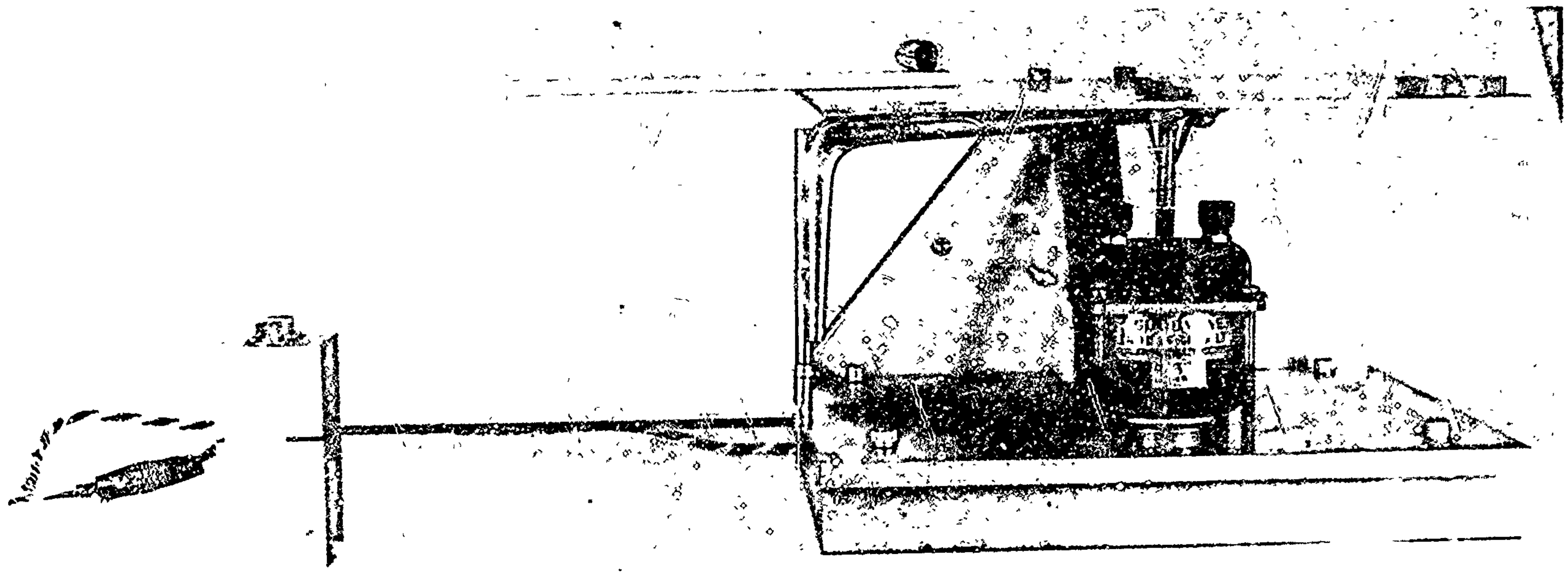


Fig. 3 A GROUND FLUTTER SIMULATOR

As the simple simulator now stands it is a tool for investigation of various simple surfaces to verify the ideas presented here. In order to detect flutter speeds of missile surfaces, the sensitivity has to be greatly increased by the use of an appropriate voltage amplifier. It should be pointed out that the simulator has a wider application to autopilot stability studies. One envisions two simulators attached to symmetrical wings activated with flight servo, gyro and allied electronics. The aeroelastic high frequency instabilities can thus be investigated.

It has been pointed out that the simple simulator is a tool for investigations, and that the results need careful evaluation in every case. In order to provide more adequate force and moment simulation, more shakers and pickups, together with appropriate circuitry for producing phase shifts, need to be added to the system.

It is believed that use of the simulator will be helpful in the determination of the flutter speeds of missile surfaces, and in evaluating the aeroelastic stability characteristics of assembled missiles.

SYMBOLS

k_h = generalized complex stiffness appropriate to the uncoupled bending mode.

k_h = $k_{ho} (1 + i\, g_h)$

k_{h_o} = $[\int_0^L m\, D_1^2\, dy]\ \omega_h^2$

g_h = damping parameter

k_α = same as above except that it applies to the uncoupled torsion mode.

k_{α_o} = $[\int_0^L i_p\, D_2^2\, dy]\ \omega_\alpha^2$

M_e = generalized inertia force for the uncoupled bending mode.

M_e = $\int_0^L m\, D_1^2\, dy$

S = generalized inertia coupling (measure of unbalance) between the two modes.

S = $\int_0^L m\, x\, D_1\, D_2\, dy$

I = generalized inertia element for the uncoupled torsion mode.

I = $\int_0^L i_p\, D_2^2\, dy$

q_1 = amplitude of time dependent function for the bending mode.

q_2 = same for the torsion mode.

P_a = amplitude of oscillating force at frequency applied at point "a".

D_{1a} = modal displacement at point "a" due to bending. (Note that "a" is on the axis of twist.)

D_1 = bending modal displacement at any point.

D_2 = uncoupled torsional modal displacement.

$C_{L\alpha}$ = supersonic lift coefficient, $\frac{4}{\sqrt{M^2 - 1}}$

q^1 = dynamic pressure, $\frac{1}{2} \rho\, v^2$

ρ = air density.

v = air speed.

C = total chord at a given station

y = spanwise coordinate.

p = pressure acting on the aerodynamic surface