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DIRECTIONAL WAVE SPECTRA MEASUREMENTS
OF THE CHESAPEAKE BAY
BENEATH THE BAY BRIDGE

by

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ABSTRACT

A program of experiments was carried out at the Naval Research Laboratories Chesapeake Bay Bridge Laboratory to assess the directional resolving capabilities of a particular array of five wave elevation detectors. It is desired to be able to distinguish waves both by their wave lengths and elevations and by their directions of propagation as part of an NADC sponsored study on surface wave modifications.

The collected data were analyzed and rather good success was had in deriving directional wave spectra. Recommendations are made to further improve the experimental procedure.

KEYWORDS

Wave Spectra
Ocean Waves
Directional Wave Spectra

INTRODUCTION

The manner in which the energy in sea waves is distributed with regard to the direction of travel is of great interest to wave forecasters. Thus, it is desired to be able to distinguish waves both by their wave lengths or numbers and by their directions of propagation when they are present simultaneously in a random, short crested, sea. Not only is this information of interest to wave forecasters, but it also may be used to cast light on the processes of wave generation, decay and modification by disturbing influences. In connection with surface wave modification studies being conducted by NADC, an experimental evaluation was made of the directional spectra of waves under the Chesapeake Bay Bridge.

This report describes work carried out to assess the directional resolving qualities of a particular array of five wave elevation detectors designed to measure waves in the test site. A diagram showing the layout of the wave sensors is shown in Figure 1. A description of the reasons for choosing these particular dimensions for locating the sensors is given in the section on Analysis and Results. Details of the physical suspension, support and orientation of the array of sensors, as well as a description of the principle of operation of the sensors, is given in the section on Instrumentation. Finally, the collected data and derived directional wave spectra are analyzed and recommendations made to further improve the experimental procedure.

SITE OF TESTS

Measurements were made of water surface elevation in the Chesapeake Bay. The wave sensors were suspended beneath the Chesapeake Bay Bridge, directly under the Naval Research Laboratory's (NRL) Chesapeake Bay Bridge Laboratory (CBBL) gondola. The chart in Figure 2 shows the geographical location and orientation of the bridge and the gondola. Also shown is some information on water depths of the Bay in the near vicinity.

The bridge is about seventy feet above the water level at this location and the gondola hangs about ten feet below the level of the bridge structure. The total work and equipment storage area on the gondola is approximately 600 square feet.

According to the "Tidal Current Tables"¹⁰ for 1966 the predicted tidal currents for the Baltimore Harbor approach off Sandy Point Maryland ranges from about 1.3 knots maximum at flood to about 1.1 knots maximum at ebb during the months of July, August and September. The changes in mean water level in this area due to tides are about one foot difference between mean high water level and mean low water level. The water in this part of the bay is mildly brackish.

Wind speeds and directions are variable, there being no significant prevailing wind pattern. Wind speeds as high as 60 miles per hour have been measured at the CBBL during a severe storm. The length of fetch, over which the wind may blow to generate waves, depends on the coastal boundaries and wind direction as well as on meteorological conditions. The indication of the coastal boundaries on the east, near the test site, is shown in Figure 2. More complete charts of the area are available in publications of the U.S. Coast and Geodetic Survey.¹¹

INSTRUMENTATION

The surface elevation detectors were provided by Block Associates, Incorporated, who acted as subcontractors to assist D.L. in all phases of the experimental set-up. The sensor consists of a one-eighth inch diameter copper wire coated with polyethylene so that the outside diameter was approximately three-eighths of an inch. It was caused to hang vertically, partially immersed in the water by means of a lead weight attached to its submerged end. The capacitance change between the copper wire and the sea water, which act as the plates of a condenser, due to water height changes

is detected by suitable electronic equipment. The sensors are five feet long with the upper end attached to a water-tight box which contains the electronic circuitry.

In principle, the transducer works as follows: the sensor is part of the circuit of an oscillator whose frequency is altered by changes in capacity due to water immersion changes. Another oscillator provides a fixed reference frequency. The difference frequency, obtained by comparing the outputs of the two oscillators, is transformed into a d.c. voltage representing the wave height. The circuit is set up so that ± 2.5 ft. change in water elevation corresponds to ± 5 volts output signal. Input to the transducer is 15 volts d.c.

The output of the transducer is amplified and recorded in two ways: on a magnetic tape recorder and on paper tape light beam oscillograph. A schematic diagram of the apparatus used is shown in Figure 3. The transducers and the power distribution and signal collection box are mounted on an aluminum tubular structure which is suspended from the bridge at a level of approximately three feet above the water. All other equipment is located on the CBBL gondola with a single electrical cable lead connecting the transducers, through the distribution box, to the power source and the amplifying and recording equipment. One channel of the tape recorder was used for voice recording of comments and observations made during data collecting periods.

A photograph of the tubular support structure on which the wave height transducers are mounted is shown in Figure 4. In this photo the apparatus is suspended at working level. Four $3/8$ inch diameter dacron, non-stretch, lines are connected from the gondola, through pulleys which are fixed in the main bridge girders, to rope yokes at the corners of the planar frame. Height adjustment and leveling was accomplished by paying out or taking in lines on the gondola. The material of the frame is $1-1/2$ inch diameter aluminum tubing connected by "quick-connect" fittings. The cross braces at the corners are aluminum angle bars. Overall dimensions are 21 ft. square on tube centers with each of the inside tubular members

spaced 7 feet from the side to which it is parallel. A motor driven winch was used to raise and lower the support structure. The support structure and sensors were raised out of the way of navigation and stowed beneath the roadway of the bridge when not in use. A sketch of the array suspended in position for wave measurements is shown in Figure 5.

Also shown in Figure 4 are the warning flag for navigation, the current meter near the upper right hand corner of the frame and the thermistor chain above the upper left hand corner of the frame. These devices were supplied by the CBBL. The current meter was of the Savonius rotor type where the rate of revolutions is a measure of the current speed. A freely pivoted vane is used to indicate the water current direction relative to magnetic North, which is correctable to true compass direction. The thermistor chain has eight thermistor beads set up to measure water temperature at levels two feet apart. In addition, CBBL provided a wind speed and direction indicator. This propeller-type indicator was mounted on the top of the bridge structure, in relatively undisturbed air, about 100 feet above the water surface.

The possibility of bridge motions being transmitted to the wave sensors suspended beneath the bridge was considered. According to R. Moulton¹² of NRL, who conducted a survey of the gondola vibrations, vertical oscillations occur in the frequency range of 10 to 17 cycles per second, with apparent discrete frequencies observed at 13, 15 and 17 cps. These oscillations occurred when trucks pass over the gondola and a maximum acceleration of 0.17 g's was recorded. This motion is very small and probably would not be transmitted to the sensor support frame in any case because the rope support system is quite "soft" and would act as a vibration isolator. The gondola vibrations may, however, influence the electronic recording apparatus. Since the water wave length corresponding to the measured frequencies is less than .05 ft., energy at this frequency is not of interest and would not be present in the sea spectrum. Any recorded signals at these higher frequencies would be caused by the intermittent passage of vehicles on the bridge and would constitute transient noise which would require special treatment in processing the data records.

TEST PROCEDURES AND PROGRAMS

The normal working day at the CBBL begins at 9:00 a.m. and ends at 3:00 p.m. All of the work on the present program was accomplished within these limitations on time.

On the 27th and 28th of July 1966, a preliminary expedition was made to survey the bridge in order to plan the installation of the array. At this time measurements of wave elevations were made with a single surface elevation detector suspended by a wire rope from the gondola. These data runs, numbers 1, 2 and 3, were recorded on magnetic tape and analyzed later to obtain point spectra which might be of use in finalizing the design of the array.

During a three day work period from 30 August through 1 September 1966 the array support structure was assembled, transported to the CBBL and secured to the bridge by the supporting lines. The assistance of a tug boat made available by the Annapolis Navy yard was essential for the accomplishment of this task. The wave sensors were secured to the array support structure in the desired positions and the electronic recording apparatus connected and tuned. Some of the sensors had to be adjusted slightly by means of a variable capacitor in the water-tight box to which the coated wire sensor is attached. No data records were obtained during this work period.

Data for obtaining directional wave spectra were obtained during a work session from 13 September through 15 September 1966. One day was spent in tuning the apparatus and assuring that the array could be suspended properly. Two of the sensors required additional minor adjustments of the variable capacitor. An improved lifting arrangement was provided at this time for raising and lowering the array support structure. On the remaining two days a total of five data records, runs number 4 through 8, were obtained.

It had been intended to further pursue data collection with this apparatus during the subsequent weeks. However, because of re-directing

of our efforts under the present contract, this work was not completed. Among the projects which had been envisioned were the use of an alternate scheme for arraying the sensors and a modification of the suspension system for the array to provide greater stability and less response to winds and water currents.

The following table records some features of time, weather, etc., for each of the data collection periods.

TABLE

<u>run no</u>	<u>date</u>	<u>time</u>	<u>wind</u>	<u>current</u>	<u>comments</u>
1	July 27, 66	2:00-2:30 pm	not recorded	not recorded	single point measurement
2	July 28, 66	10:26-11:27 am	"	"	"
3	"	2:00 -2:30 pm	"	"	"
4	Sept. 14, 66	12:42-1:05 pm	18 mph at 100°	0.4 knqts at 180°	five sensors
5	Sept. 14, 66	1:15 -1:34 pm	19 mph at 120- 150°	0.25 knqts at 180°	"
6	Sept. 14, 66	1:55 -2:15 pm	11 mph at 100°	0.1-0.15 at 180°	"
7	Sept. 15, 66	1:15 -1:23 pm	not recorded	not recorded	noise signal appeared on visicorder record proved not to be on tape
8	Sept. 15, 66	1:30 -2:05 pm	12 mph at 300° to 18 mph at 330°	"	five sensors

It will be noted from the table that wind and current data were obtained for some of the times of wave recording. The current meter direction indicator failed on 15 September, consequently these measurements were not available for runs 7 and 8. Visual observations of the effect of

the current on the wave sensor indicated that the current was substantially faster than on the previous day, however. Water temperature profiles were obtained with the thermistor chain and recorded over the period of the working days but they did not indicate unusual or particularly interesting thermal structure in the bay at this time and place.

ANALYSIS AND RESULTS

Current Status of Directional Wave Sensor Instrumentation

Some of the other techniques which have been used to obtain directional wave spectra include: analysis of stereo photographs of large areas of the ocean's surface, measurements of the heaving, pitching and rolling motion of a floating buoy, measurements of wave elevations with a linear array of wave sensors, and measurements of wave elevations with a two-dimensional array of three or four sensors. The present investigation is thus an extension of the use of two-dimensional arrays to allow for five sensors.

The stereophotographic method has been applied by Chase, et al,¹ in project SWOP (stereo wave observation project). An ambiguity is possible in directional wave spectrum determinations derived from this method since an instantaneous stereophotograph does not, in principle, distinguish between opposite directions. This difficulty was resolved by Chase, et al, by assuming that the waves could not be coming from a direction that is greater than 90 degrees from the wind direction. Analysis of such stereophotographic records is complicated and tedious, entailing much manual labor or sophisticated electronic photo-reading equipment, just to obtain the data which is to be processed and interpreted.

While the stereophotographic technique provides information on the water surface elevations over a wide area at an instant of time, the other techniques obtain information on water surface elevations, and sometimes slopes, at particular fixed positions in the sea over a long period

of time. The motions of a floating buoy, in particular the vertical accelerations and the two angles of pitching and rolling, have been used by Longuet-Higgins, et al,² to determine directional wave spectrum. This buoy had been calibrated in a wave tank to determine its response to known wave systems. The five numerical quantities determined, heave, pitch, roll and phases of two of these relative to the third, are sufficient to obtain the first five Fourier coefficients of the angular distribution of wave energy. If the geophysical situation is such that one might expect a very broad angular distribution of energy, these five coefficients should be sufficient but if the waves are expected to be coming from a well-defined direction these few harmonics are probably not sufficient to resolve the direction adequately.

Use of a linear array of wave elevation sensors has been made by Stevens³ to obtain directional wave spectra in Buzzards Bay, Massachusetts. Six stainless steel wire resistance-type surface elevation sensors were arranged in a straight-line array and suspended from a fixed wave observing platform. The spacings of the sensors used are integral multiples of 1.5 ft and the total array length is 19.5 ft so that the directions of waves between 3 ft and 38 ft in length can be resolved. The linear array of wave sensors is limited by the fact that it is unable to discern the direction of wave propagation unambiguously since waves coming from, say, the left side of the line of the array cannot be discriminated from those which come from the right side at the same angle from the array centerline. This is not always a major drawback, since it may be possible to infer this information in some other way. All of the techniques which utilize wave measurements at a finite number of discrete points for a long period of time make use of a theoretical dispersion relation between wave length and velocity of propagation in correlating signals obtained by the various sensors.

If it is desirable or necessary to resolve wave direction unambiguously with an array of surface elevation sensors two-dimensional arrays may be used. Various configurations may be conceived for various numbers

of sensors. Barber⁴ has considered the "efficiency" of some arrangements of sensors, consisting of three probes and four probes, and their abilities to resolve the directions of the waves. A very complete discussion of measurements of ocean swell is given by Munk, et al,⁵ who present a very useful treatment of the theory of arrays and of methods for treating and interpreting the data obtained. In their experimental work, Munk, et al, used three very sensitive and reliable pressure gages installed in a triangular array on the seaward side of San Clemente Island, California. The instruments were located about 10,000 ft from shore, with the sides of the triangle being about 900 ft. The water depth of approximately 330 ft causes the ocean to act as a sort of hydrodynamic low-pass filter, since the pressure fluctuations due to short, high frequency waves attenuate very quickly with distance below the surface. A theoretical relation between pressure signal, frequency and wave elevation must be used to obtain the energy spectrum of the waves. Many data records were obtained during the period from May to October 1959 and significant success was obtained in associating observed swells with distant storm sources. More recent work by Rank, et al,⁶ made use of four probes located in about 40 ft of water about 1/4-mile south of Block Island, R.I. The pressure transducers were located in an array which was composed of three outer gages, with the distance between being about 400 ft, and a fourth gage in the middle. Macovsky and Mechlin,⁷ and Pierson⁸ have proposed mounting sonic transducers on the decks of submarines which are progressing in the forward direction. Although measurements of power spectra have been made from data obtained with transducers mounted on submarines,⁹ this approach for obtaining directional wave spectra has not yet been implemented.

The present investigation utilized a five sensor array in an effort to improve directional resolving capabilities of two-dimensional arrays. The directional resolving capabilities of the wave detectors are particularly important for studies of surface wave modifications.

Theory of Directional Wave Spectra:

The following theoretical development essentially follows that of Reference 4 and 5. Amplified where reasons for clarity.

Consider a pure wave η_0 , with amplitude a_0 , frequency f_0 , and wave numbers or space frequencies, l_0 and m_0 , then η_0 is represented as

$$\eta_0(x, y, t) = a_0 \cos (2\pi(l_0 x + m_0 y + f_0 t) + \gamma_0)$$

Expressing l_0 and m_0 in polar co-ordinates k_0, α_0 ,

$$l_0 = k_0 \cos \alpha_0$$

$$m_0 = k_0 \sin \alpha_0,$$

it is seen that the wave, represented by η , is propagating from the direction determined by the angle α_0 . Hence knowledge of l_0 and m_0 determines the directions of propagation of the wave.

A real wave train η can be thought of as a linear combination of such pure waves. Using complex notation for convenience, η is given by

$$\eta(x, y, t) = \sum_n a_n (l_n, m_n, f_n) \cdot \exp (i2\pi(l_n x + m_n y + f_n t + \gamma_n)) . \quad (1)$$

It is the purpose of this investigation to compute the dominant directions of the waves, this will be accomplished by determining those values of a_n which contribute most to the expansion of η . The wave numbers l_n and m_n associated with these a_n 's will then give the dominant directions.

Forming the conjugate products $\eta^*(x, y, t) \cdot \eta(x+X, y+Y, t+T)$ and averaging over values of x, y , and t yields

$$\text{ave } \eta^*(x, y, t) \cdot \eta(x+X, y+Y, t+T) = \sum_n a_n^2 (l_n, m_n, f_n) \exp(i2\pi(l_n X + m_n Y + f_n T)) \quad (2)$$

Here we assume that the cross terms vanish; i.e. terms of the form

$$\lim_{\substack{t_b \rightarrow \infty \\ x_b \rightarrow \infty \\ y_b \rightarrow \infty}} \frac{1}{8t_b x_b y_b} \int_{-t_b}^{t_b} \int_{-x_b}^{x_b} \int_{-y_b}^{y_b} \exp \left[(i2\pi(l_n - l_k) x + (m_n - m_k) y + (f_n - f_k) t + (\alpha_n - \alpha_k)) \right] dx dy dt$$

are zero.

Note that this average just depends upon the difference in the arguments of η^* and η , that is, it only depends upon the space lags X and Y and the time lag T and not on x, y and t .

This average product is called the correlogram or lag correlogram of the wave and will be denoted by $\rho(X, Y, T)$.

The term a_n^2 is related to the energy of a wave and is called the wave power.

Writing (2) in a continuous form, and introducing a power density $E(l, m, f)$, yields

$$\rho(X, Y, T) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(l, m, f) \exp(i2\pi(lX + mY + fT)) dl dm df \quad (3)$$

$E(l, m, f)$ is known as the power spectrum.

Defining

$$C(X, Y, f) - iQ(X, Y, f) = \int_{-\infty}^{\infty} \rho(X, Y, T) e^{-i2\pi T f} dT \quad (4)$$

where C and Q are real, we have

$$C(X, Y, f) - iQ(X, Y, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(l, m, f) e^{i2\pi(lX + mY)} dl dm \quad (5)$$

This follows, since by (3) the right hand side is just the Fourier transform of $\rho(X,Y,T)$ which is the definition of $C-iQ$.

Setting $X=Y=0$ in (5) and noting that $E(\theta,m,f)$ is real it is seen that

$$C(0,0,f)+iQ(0,0,f) = C(0,0,f) = C(0,0,f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(\theta,m,f) d\theta dm$$

which is proportional to the total energy of a wave of frequency f . This is known as the point spectrum. $C(X,Y,f)-iQ(X,Y,f)$ is known as the cross spectrum.

Taking the two space transforms of (5), one has

$$E(\theta,m,f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [C(X,Y,f)-iQ(X,Y,f)] \exp(-i2\pi(\theta x+mY)) dx dy.$$

Hence by taking three transforms of ρ , $E(\theta,m,f)$ is obtained. Finding the maxima of $E(\theta,m,f)$ would then give the desired directional information.

The above formulation, while theoretically pleasing, cannot be used without modification, for any practical application. For it is tacitly assumed that ρ is known for all time and all space. This, of course, is absurd. Furthermore, the data obtained in any experimental situation is not known for continuous values of the independent variables.

The first step in the modification is to assume that the waves have an ergodic property, that is, it is sufficient to average over one of the three variables, x,y , or t , to obtain $\rho(X,Y,T)$. Since we have fixed probes, time is chosen as this variable.

The fact that data is known at discrete intervals of time and discrete points in space leads to the problem of aliasing. This problem is seen most readily when considering the time-frequency aspect. If data is known at every Δt seconds, it is impossible to resolve waves of frequencies $\frac{1}{2\Delta t} + f$ and $\frac{1}{2\Delta t} - f$; this is illustrated in Figure 6. On

the other hand, if it is known that there is a small probability that the wave is above a frequency of $\frac{1}{2\Delta t}$, then there would be little ambiguity in determining the proper frequency. The frequency $\frac{1}{2\Delta t}$ is known as the Nyquist or folding frequency. This ambiguity is carried over into the point spectrum and if any energy occurs above the folding frequency, let's say at $\frac{1}{2\Delta t} + \bar{f}$, it is added on to the value of the spectrum at $\frac{1}{2\Delta t} - \bar{f}$. The point spectrum obtained from such discrete data is an even function of f , as are all point spectra, and is periodic, with period $\frac{1}{\Delta t}$. Hence all the information possible is obtained by computing the point spectrum in a frequency range of 0 to $\frac{1}{2\Delta t}$. This range of the spectrum is called the principal part. These considerations are illustrated in Figure 7. Analogous problems occur when dealing with the space variables. The above facts play a major role in the design of any experiment when space or time frequency resolution is required.

Data is also known only for a finite duration of time and a finite number of points. This introduces the concept of spectral windows. That is, if a wave had but one time frequency, it would theoretically appear as a spike in the point spectrum. Instead due to the finiteness of the data, it is smeared out, or the window distorts your "vision" of the frequency spectrum. An example of such a window is given in Figure 8. Again analogous problems arise with the space variables. The spectral window may also lead to negative values of the spectrum, while inherently the spectrum is positive.

It is convenient to extend the concept of a spectral window to include the problem of aliasing. That is the window we use to look at frequencies l_0, m_0, f_0 not only smears the spectrum at l_0, m_0, f_0 but gives spurious peaks at values other than l_0, m_0, f_0 . It is helpful in understanding the spectral window to consider our analysis as a black box, with the input a pure wave with parameters l_0, m_0, f_0 and the output some function of l, m, f . This function is nothing but the spectral window for the parameters l_0, m_0, f_0 . Since it is possible to compute these windows this point of view will be extremely helpful in interpreting the directional spectrum results.

If the depth of water is known, the frequency of a wave determines its wave length, hence the parameters ρ, m, f are dependent. For the depth and frequencies considered, we may use the deep water dispersion relationship

$$f^2 = \frac{g}{2\pi} k$$

where

$$\rho^2 + m^2 = k^2 \quad k > 0.$$

In computing the estimate for the directional spectrum, the three transforms of ρ will be taken. Fixing the frequency f , we will determine k and then let α vary where $\rho = k \cos \alpha$ and $m = k \sin \alpha$. The maxima of the spectrum for this fixed f will indicate the possible directions of propagation for the waves of this frequency. Due to the effect of the spectral window all peaks will not be indicative of a wave traveling in the corresponding direction. It will be assumed that the waves of a given frequency are unidirectional. Under this assumption and considering the window as a response to a black box we will determine this response for the frequency f and angle α corresponding to the different peaks. It will then be concluded that the α which best represents the other peaks will be the true direction. For low frequencies, where aliasing in the space frequencies is not a problem, α should be uniquely determined from the spectrum. But for higher frequencies aliasing in the space frequencies becomes prevalent. But we are further aided in the search for the correct α by the fact that the waves in a given frequency band tend to be unidirectional. Hence looking through a series of directional spectra for varying but close frequencies, the various spectra may all have a peak for some α , while the other peaks may occur at varying α 's.

Taking all these facts into consideration, the following estimates may be derived. For the derivation of these formulae and more detailed discussion of the previous theory, see reference 6.

It is assumed that n data points are available. A bar will be used to denote estimated quantities to distinguish them from their exact analogies.

$$\bar{\rho}(X_i, Y_i, r\Delta t) = \frac{1}{t_n - r\Delta t} \sum_{k=1}^{n-r} \rho(0, 0, t_k) \cdot \rho(X_i, Y_i, t_k + r\Delta t) \quad r \geq 0$$

$$\bar{\rho}(X_i, Y_i, -r\Delta t) = \frac{1}{t_n - r\Delta t} \sum_{k=1}^{n-r} \rho(0, 0, t_k + r\Delta t) \rho(X_i, Y_i, t_k)$$

$$U_K = \bar{\rho}(X_i, Y_i, 0) + 2 \sum_{r=1}^r \left[\frac{\bar{\rho}(X_i, Y_i, r\Delta t) + \bar{\rho}(X_i, Y_i, -r\Delta t)}{2} \right] \cdot \cos 2\pi r f \Delta t$$

$$V_K = 2 \sum_{r=1}^r \left[\frac{\bar{\rho}(X, Y, r\Delta t) - \bar{\rho}(X, Y, -r\Delta t)}{2} \right] \sin 2\pi r f \Delta t$$

$$\bar{C}(X, Y, 0) = (U_0 + U_1) / 2$$

$$\bar{C}(X, Y, r \max \Delta f) = (U_{K-1} + 2U_K + U_{K+1}) / 4 \quad k=1, 2, \dots, r_{\max} - 1$$

$$\bar{C}(X, Y, r_{\max} \Delta f) = (U_{r_{\max} - 1} + U_{r_{\max}}) / 2$$

$$\bar{Q}(X, Y, 0) = \bar{Q}(X, Y, r_{\max} \Delta f) = 0$$

$$\bar{Q}(X, Y, k \Delta f) = (V_{K-1} + 2V_K + V_{K+1}) / 4 \quad k=1, 2, \dots, r_{\max} - 1$$

$$\frac{\bar{E}(\ell, m, f)}{\bar{C}(0, 0, f)} = 1 + 2 \sum_i R_i(f) \cos \left[\theta_i(f) + 2\pi(\ell X_i + m Y_i) \right]$$

$$= \bar{D}(\alpha, f) = 1 + 2 \sum_i R_i(f) \cos \left[\theta_i(f) + 2\pi k d_i \cos(\alpha - \phi_i) \right]$$

$$DW(\alpha_0, f_0, \alpha) = 1 + 2 \sum_i R_i(f_0) \cos \left\{ 2\pi k_0 d_i \left[\cos(\alpha - \phi_i) - \cos(\alpha_0 - \phi_i) \right] \right\}$$

where the i summation is taken over the number of pairs of gauges.

r_{\max} = maximum number of lags

$$\Delta f = \frac{1}{2r_{\max} \Delta t}$$

X_i, Y_i = components of the vector connecting the i^{th} pair of spatial points

$$R_i(f) = \frac{\overline{C}(X_i, Y_i, f)^2 + \overline{Q}(X_i, Y_i, f)^2}{C(0, 0, f)^2}^{1/2}$$

$$\theta_i(f) = \arg(C(X_i, Y_i, f) + iQ(X_i, Y_i, f))$$

$$k = (\ell^2 + m^2)^{1/2}$$

$$\alpha = \arg(\ell + im)$$

$$d_i = (X_i^2 + Y_i^2)^{1/2}$$

$$\phi_i = \arg(X_i + iY_i)$$

$\overline{D}(\alpha, f)$ = estimated directional spectrum

$DW(\alpha_0, f_0, \alpha)$ = directional spectral window for a wave of frequency f_0 and direction α_0

Design of Array

It is seen by the definition of DW, that the spectrum will peak if the argument of the cosine is a multiple of 2π for all i . Therefore if we guarantee that there exists an i such that

$$k_o d_i (\cos(\alpha - \phi_i) - \cos(\alpha_o - \phi_i)) = 1 \quad (6)$$

for the values of k of interest and for all α and α_o then the aliasing problem will be minimized. Hence, if we require that one of the d_i 's is less than $\frac{1}{2k_{o \max}}$ this condition would be satisfied.

If we have at least three spatial points, two pairs of which are separated by equal distances and make an angle of 90° then condition (6) will be satisfied if

$$k_o d_i [\cos(\alpha - \phi_i) + \cos(\alpha - \phi_i - \pi/2) - \cos(\alpha_o - \phi_i) - \cos(\alpha_o - \phi_i - \pi/2)] = 2 \quad (7)$$

but

$$\begin{aligned} & k_o d_i \left| \cos(\alpha - \phi_i) + \cos(\alpha - \phi_i - \pi/2) - \cos(\alpha_o - \phi_i) - \cos(\alpha_o - \phi_i - \pi/2) \right| \\ &= k_o d_i \left| \cos(\alpha - \phi_i) + \sin(\alpha - \phi_i) - \cos(\alpha_o - \phi_i) - \sin(\alpha_o - \phi_i) \right| \\ &= k_o d_i \left\{ \left| \cos(\alpha - \phi_i) + \sin(\alpha - \phi_i) \right| + \left| \cos(\alpha_o - \phi_i) + \sin(\alpha_o - \phi_i) \right| \right\} \\ &= k_o d_i \cdot 2 \cdot \sqrt{2} \end{aligned}$$

Hence this common d may be chosen as

$$d = \frac{\sqrt{2}}{2k_{o \max}} \quad (8)$$

and then condition (6) will be satisfied.

In designing the array we chose $f = .6$ cycles per second to be the maximum frequency of interest. This checked out with our preliminary calculations and observations but turned out to be too low for the data reduced. For a wave of this frequency $\frac{1}{2k_0} \sim 7$ ft. Using 7 ft as the characteristic distance and making use of formula (8) the array previously indicated was chosen.

Results

Three runs of 1024 seconds were digitized every 5 seconds giving 2048 points. These data points were spectrally analyzed using the 1105 UNIVAC digital computer. The correlogram was computed with the aid of the Tukey-Cooley algorithm^() and a fifty lag analysis was carried out. With few exceptions satisfactory directional spectra were obtained and the directions agreed with the visual observations made at the time of the experiment. The three runs analyzed will be designated as runs 4, 6, and 8. The data of run 4 and 6 were obtained on September 14, 1966 and run 8 on September 15, 1966. Due to high frequency instrumentation noise, low pass analogue filters were used when the data was digitized. Since only four filters were available, the data from one probe was not able to be filtered. Since the filtering affected the phasing of the data the information from this probe could not be used in the calculation of the directional spectrum, although it would still give reliable point spectrum. Probe 4 was chosen as the one not to filter. Thus, although it was intended to use a five sensor array, the limitations of available data processing equipment restricted the final analysis to a four sensor configuration.

The data was taken about a zero mean, but no attempt to calibrate the probes was made. The calibration constants were not needed because the ultimate aim was to compute the directional spectrum and in computing the R_i 's the calibration constants would have cancelled. With this in mind the amplitude of the point spectrum cannot be given any physical units.

The co-ordinate system used had the support holding sensors 1 and 2 forming the x axis, increasing x in the direction from 2 to 1 (see Figure 1).

Run 4

Point Spectrum

All five sensors give roughly the same shaped spectrum (Fig. 9). The low frequency spike is caused by the increasing tide. The spike at $f = .12$ cycles per second is believed to be caused by the motion of the frame. This belief is substantiated by the varying amplitude from probe-to-probe which indicates that this motion was not caused by waves. Furthermore, the phasing between pairs of probes indicated that the frame was oscillating about a line roughly determined by probes 2, 5, and 4. That is, the motion causing the amplitude at this frequency was in phase for probes 1 and 2 but 180° out of phase for probes 1 and 3. If this hypothesis is correct then the motion of probes 1 and 3 should be greater than that at 2, 4 and 5. This is borne out by the size of the peaks relative to the peak at $f = .7$ cycles per second. The peak in the frequency band .25 - .45 is believed to be caused by two sources, a lower frequency contribution about .3 cycles per second caused by another mode of motion of the frame and a wave with center frequency about .38 cycles per second. This would explain the shoulders of the spectrum of probes 1, 2 and 3 at about .38 cycles per second, and the double peaked character of probe 5. Further evidence leading to this conclusion is obtained by studying the directional spectrum. The peak at .7 cycles per second is caused by wave motion.

Directional Spectra

As mentioned earlier, due to the spectral window some negative values of the spectrum may be obtained, but almost all negative values

would indicate that the phenomenon causing this could not be associated with the motion of the waves. This is the case when we look at the directional spectrum at $f = .12$ and $.3$ cycles per second (Fig. 10). Hence this gives further evidence that the energy at these frequencies was caused by mechanical motion of the array. As we leave the influence of the energy at $.3$ cycles per second a well defined peak is established at about 190° (Fig. 11). Superimposed upon these calculated spectra is the response of the spectral window with an input of a wave of the given frequency and angle equal to 190° . It is to be noted that the further we get away from $.3$ cycles per second, the better the agreement. This angle corresponded to the direction of a long period wave moving down the Chesapeake Bay observed not without difficulty, by most of the personnel present at the experimental site.

The directional spectra computed for the higher frequency peak is illustrated in Figures (12 - 14). While aliasing becomes extensive as the frequency is increased it is noted that each spectrum has a peak at about $\alpha = 60^\circ$. Superimposed upon these spectra is the response of the lag window for $\alpha_0 = 58^\circ$. The overall good agreement gives confidence that the direction of the wave was approximately 60° . Other angles were assumed and their responses were computed see Figures (15 - 17). In these cases there was not the uniform agreement with the computed directional spectrum. Hence the analysis leads one to conclude that the direction of propagation was about 60° . This again agreed with the visual observation by the personnel at the test site for the direction of the high frequency wind driven waves.

Run 6

The data from run 6 was taken about one hour after run 4. In this time the geophysical conditions did not change substantially and as one would expect the results of the spectral analysis did not vary. Band pass filters were used when digitizing the data hence the lack of the low

frequency spike on the point for sensors 1, 2, 3, and 5. The other general characteristics of the spectrum are conserved. It is to be noted that the peak at about .7 cycles per second has moved slightly to the lower frequencies; this is as one would expect as time goes on for wind driven waves.

The same comments for run 4 are pertinent here and will not be repeated. Figures 18 thru 24 give the corresponding plots.

Run 8

Again on run 8 band pass filters were used explaining the lack of a tidal peak for sensors 1, 2, 3, and 5. The geophysical conditions during run 8 were much different than during runs 4 and 6. The wind was blowing over a longer fetch and the waves were better developed. The movies of the second day indicate that the probes were oscillating to a much greater extent than on the previous day. The point spectra, Figure 25, on all five sensors give a well-developed spectra with the major peak at .44 cycles per second. This corresponds to a 25 ft wave.

The directional spectrum indicate directions about 350° for $f \sim .44$ cps, 20° for $f \sim .6$ and 230° for $f \sim .7$ cps (figures 26 - 31). The appropriate windows are also represented. In viewing the movies it is obvious that a wave train is coming from approximately 235° , superimposed upon this wave train is a lower frequency wave moving across it at roughly right angles or approximately 120° . No waves are discernible coming from 350° , although it is not beyond the realm of possibility that there actually was a wave train coming from that direction. It is also possible that the movement of the probes may have given rise to faulty phase information. The effect of such motion has not been analyzed.

DISCUSSION

Although only a limited amount of data was collected, and the originally planned test and development program was curtailed, the success that was obtained in analyzing and interpreting data for test runs 4, 6 and 8 indicates the effectiveness of the techniques applied. This success was obtained, moreover, in spite of some evident defects in the experimental set up, especially with regard to the array support system and the sensor motion, which were noticed during the tests and were set in relief by the analysis.

The use of arrays of wave elevation sensors and the use of buoy techniques^{2,14} are the two most promising methods for researching wave directional spectra. It is probable that each type of device has particular advantageous features which make its use preferable for certain applications, less desirable for others. Different data reduction and analysis methods may be applied as well, depending on the nature of the geophysical conditions, and some of these methods may be more effectively used for, say, buoys than for arrays. The present work on arrays of sensors must be considered incomplete, especially in view of the inability to use data from one of the sensors because of excessive high frequency "noise" on the signal.

For future development of the present experimental program certain modifications of the apparatus are called for. The means for suspending the tubular array support structure should be stiffened in order to avoid the low frequency oscillations which proved troublesome in the tests conducted to date. A sketch of a suspension system which should provide the improved stability is given in Figure 32. The suspension cables themselves should be significantly thinner than the three-eighth inch diameter dacron lines used, in order to minimize the wind loads. The wave sensors should be attached to the support structure in a rigid fashion so they do not respond to currents. The possibility of using thinner sensors should also

be investigated to reduce water current loads which tend to cause deflections and oscillations of the sensors. The design of this stiffened wave sensor arrangement should be tested in a towing tank to assure that it has sufficient rigidity.

CONCLUSIONS

1. The work conducted to date presents conclusive evidence of the ability of arrays of wave sensors to discern wave direction and of the effectiveness of the analysis techniques employed.
2. Further development work is required on the apparatus in order to permit a final assessment of the ultimate capabilities of these types of wave detecting arrays, particularly those with five sensors.

RECOMMENDATIONS

Further work should be undertaken to satisfactorily assess the merits of five sensor arrays for determining wave direction:

- 1) The support for the wave sensors should be made as nearly motionless as possible.
- 2) The wave probes themselves should be rigidly attached to the support to avoid pendulous motion under the action of wind and waves.
- 3) The spacing of the probes should be chosen in accordance with accurate information about the wave lengths to be expected.
- 4) Provision for calibration of the probes should be made in order to obtain wave energy spectra. On site calibrations are preferable, but laboratory calibration of the probe, together with electrical calibration during the tests may be acceptable.

Since data processing procedures are now well established for these arrays it should be possible to analyze and interpret the data which is obtained in a short period of time. This is important because the continuation of this program would still entail development and possible modification of the apparatus, as guided by the interim results.

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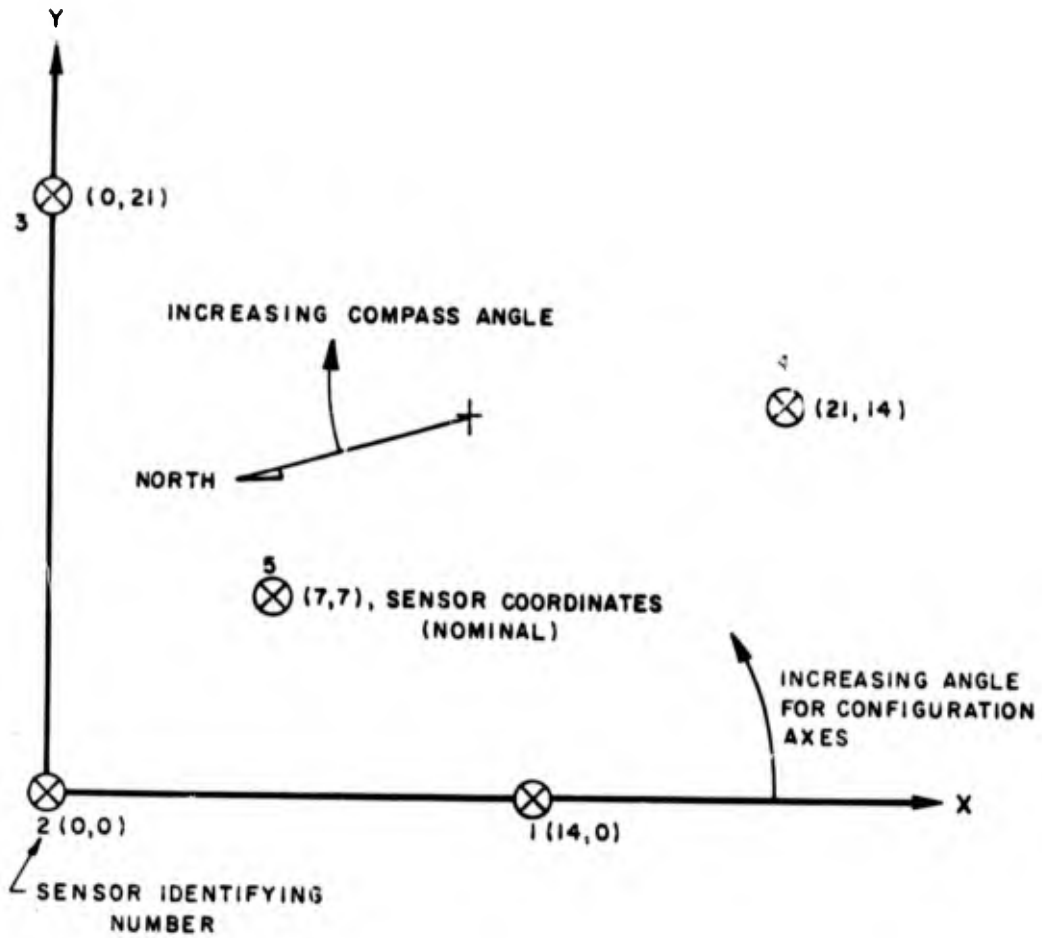


FIGURE 1. ARRANGEMENT OF WAVE DETECTORS IN DIRECTION SENSING ARRAY

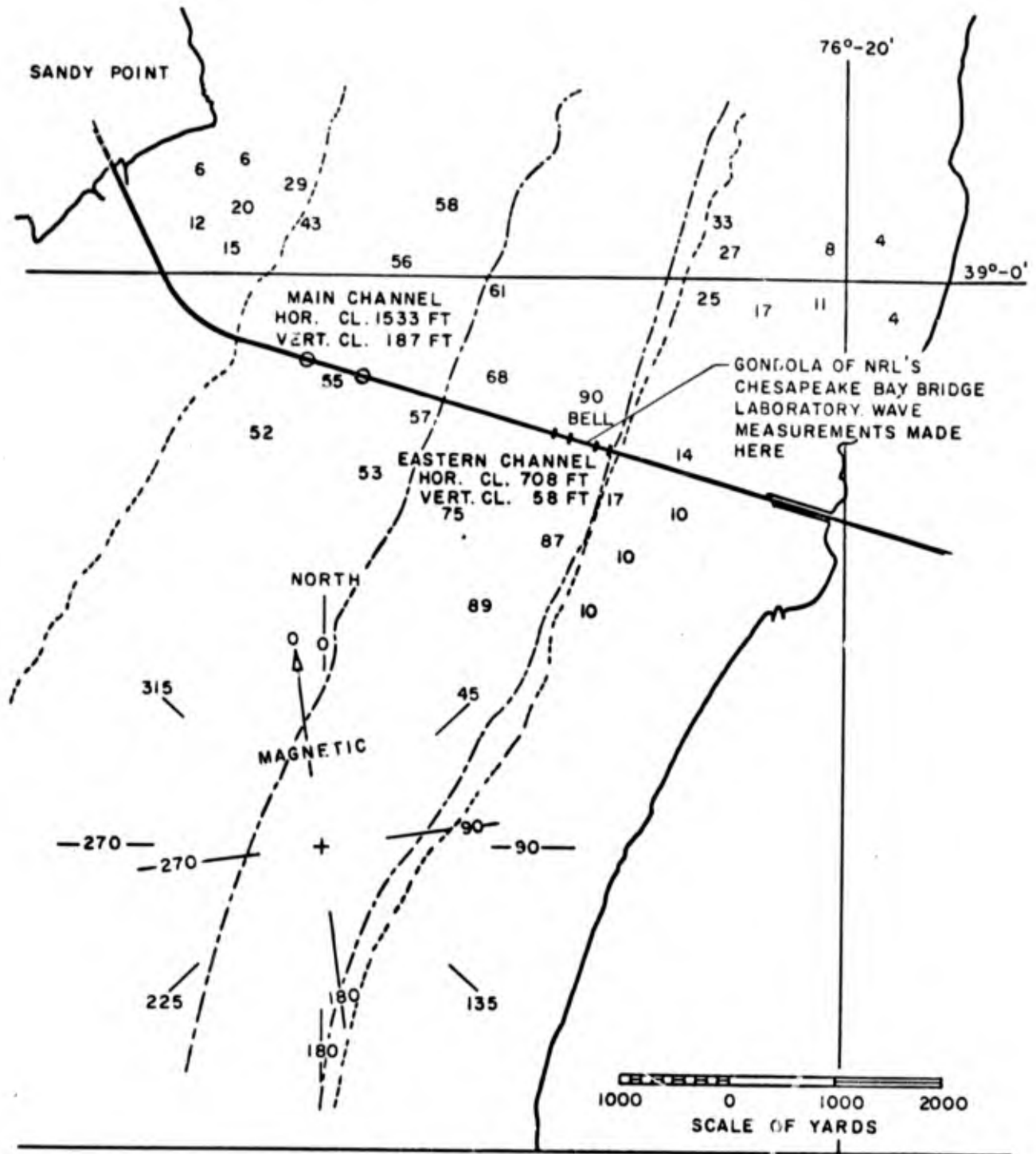


FIGURE 2. LOCATION AND ORIENTATION OF SITE OF WAVE MEASUREMENT TESTS IN CHESAPEAKE BAY

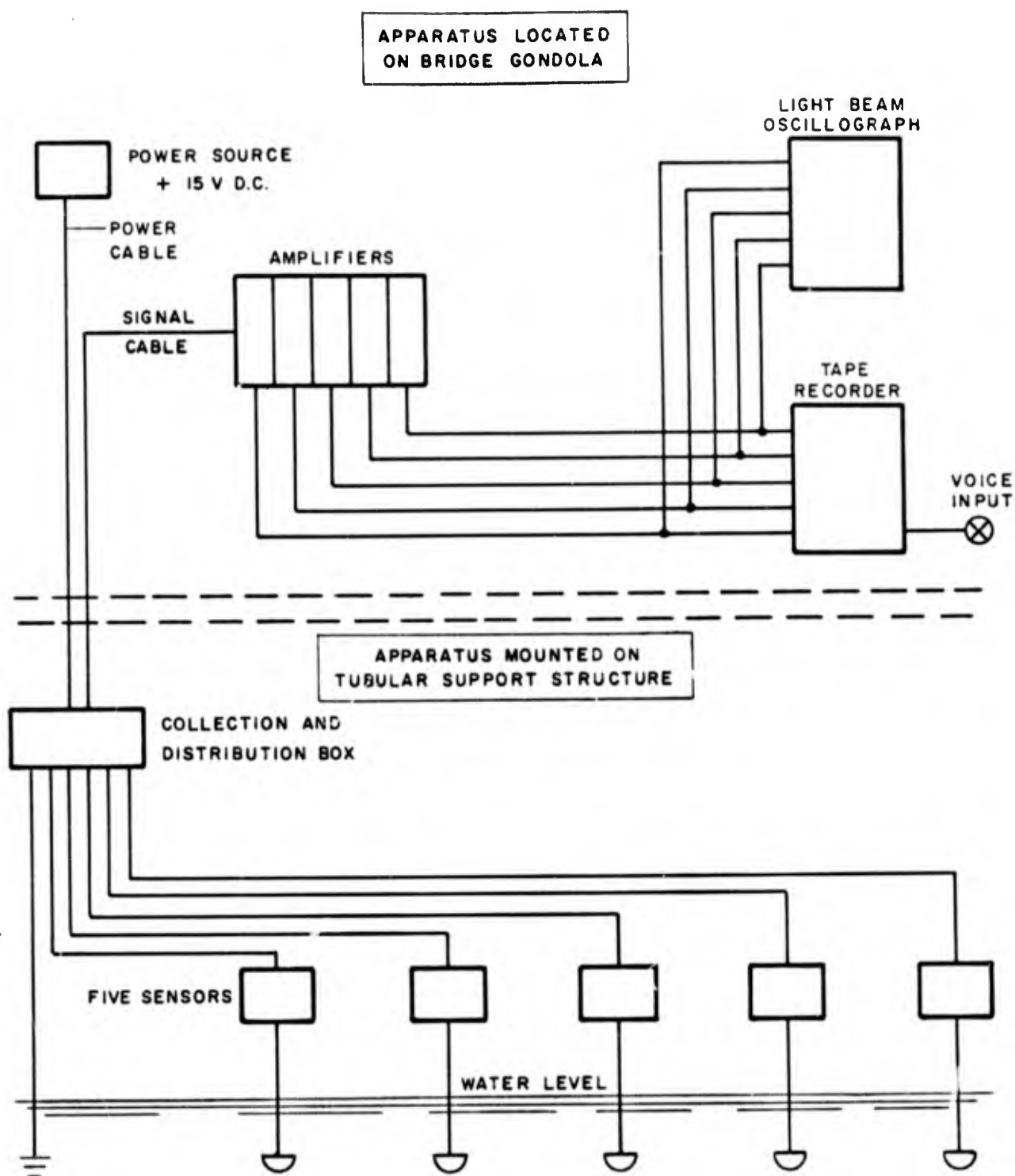


FIGURE 3. SCHEMATIC^o DIAGRAM OF DIRECTIONAL WAVE SPECTRA APPARATUS

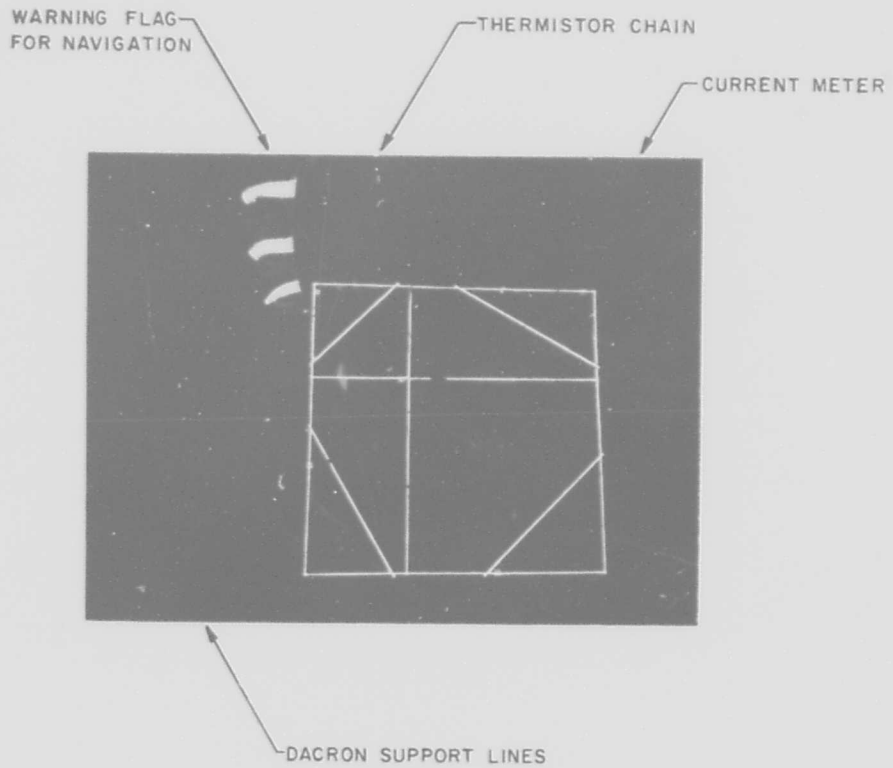


FIGURE 4. PHOTOGRAPH OF APPARATUS SUPPORT STRUCTURE
SUSPENDED IN DATA-COLLECTING POSITION

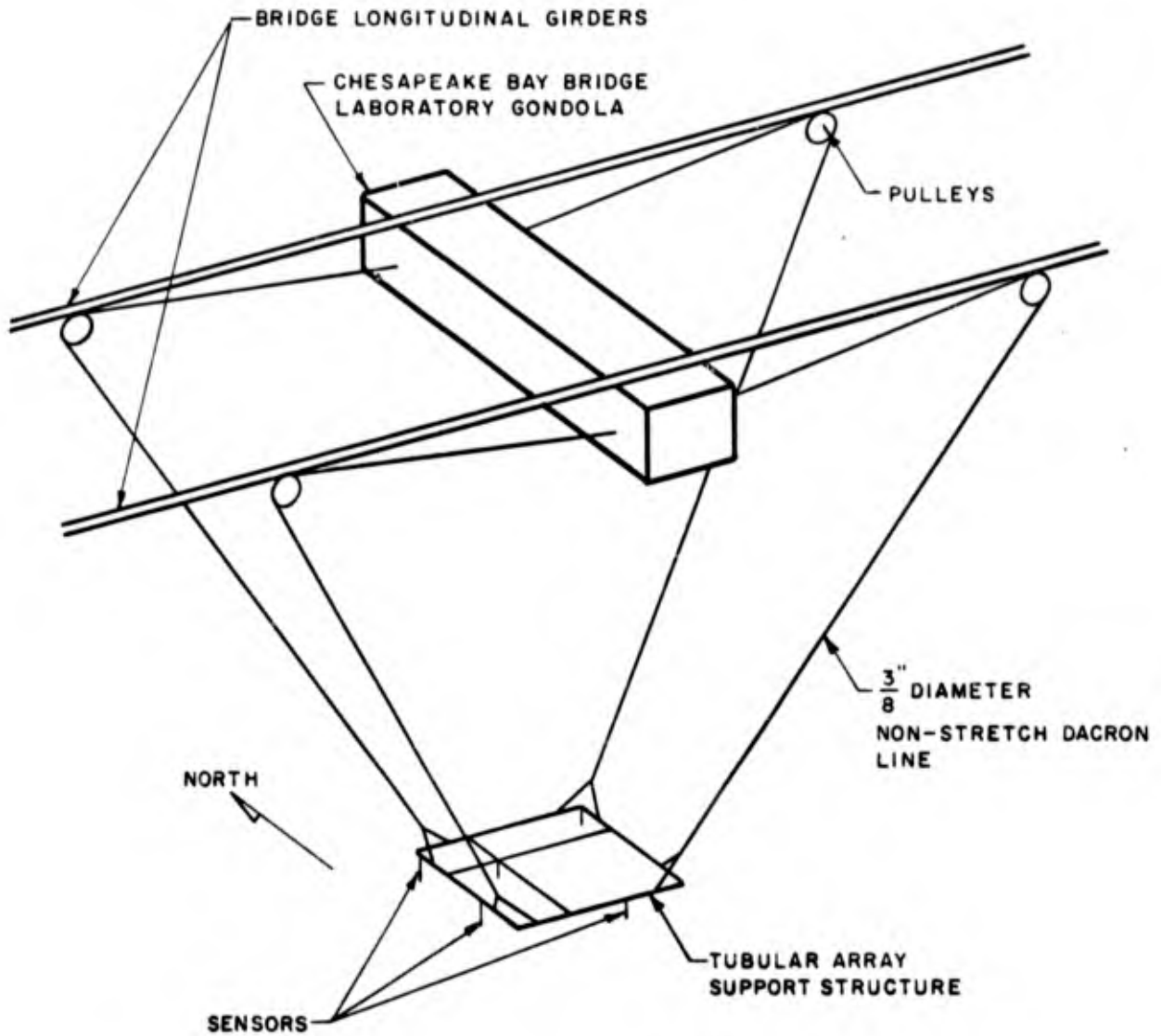


FIGURE 5. SUSPENSION OF ARRAY AND SUPPORT STRUCTURE UNDER BRIDGE

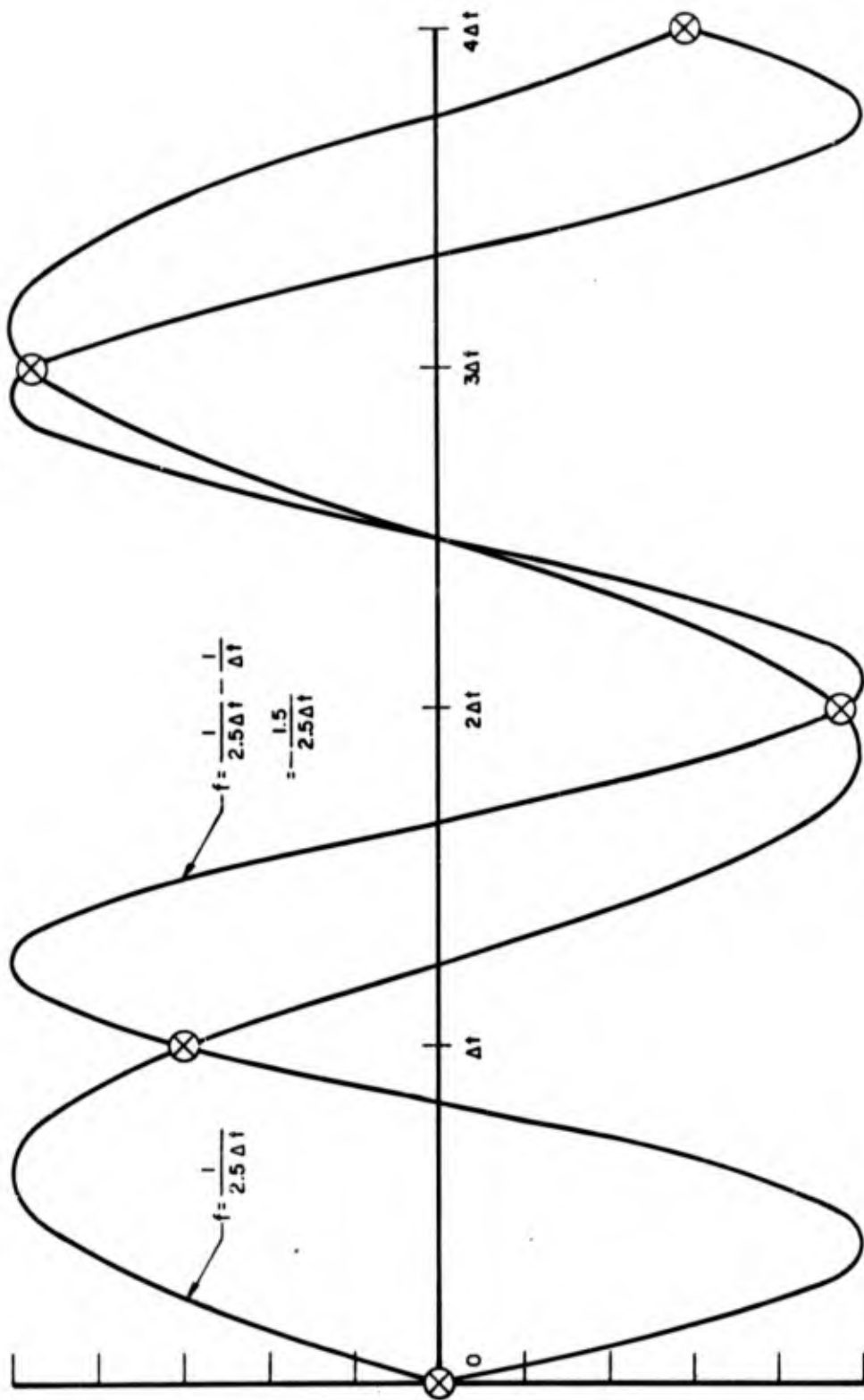


FIGURE 6. ALIASING FIVE EQUALLY SPACED DATA POINTS ARE FITTED BY TWO SINE WAVES DIFFERING BY $1/\Delta t$

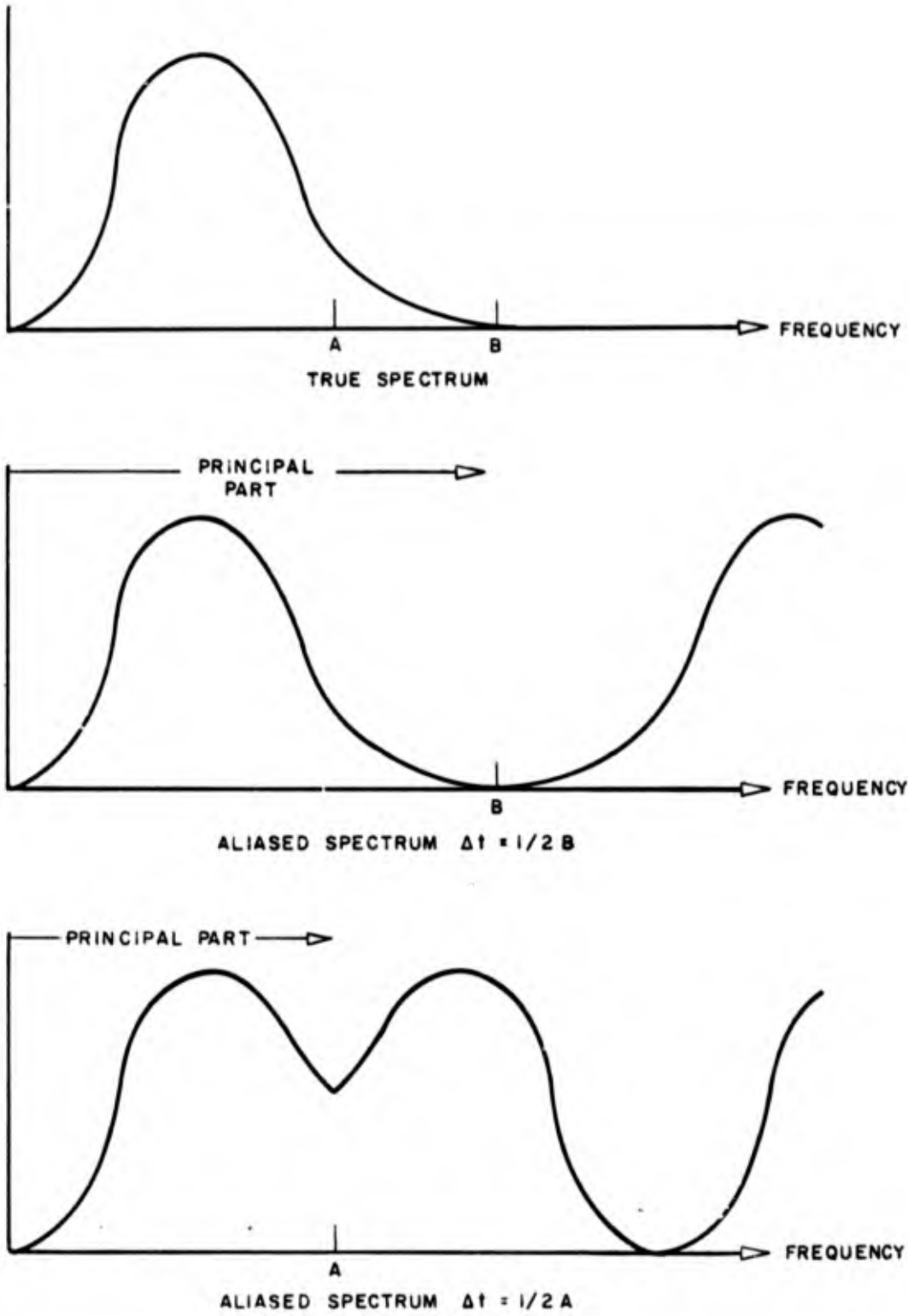


FIGURE 7. THE EFFECT OF THE SAMPLING INTERVAL ON THE ALIASED SPECTRUM

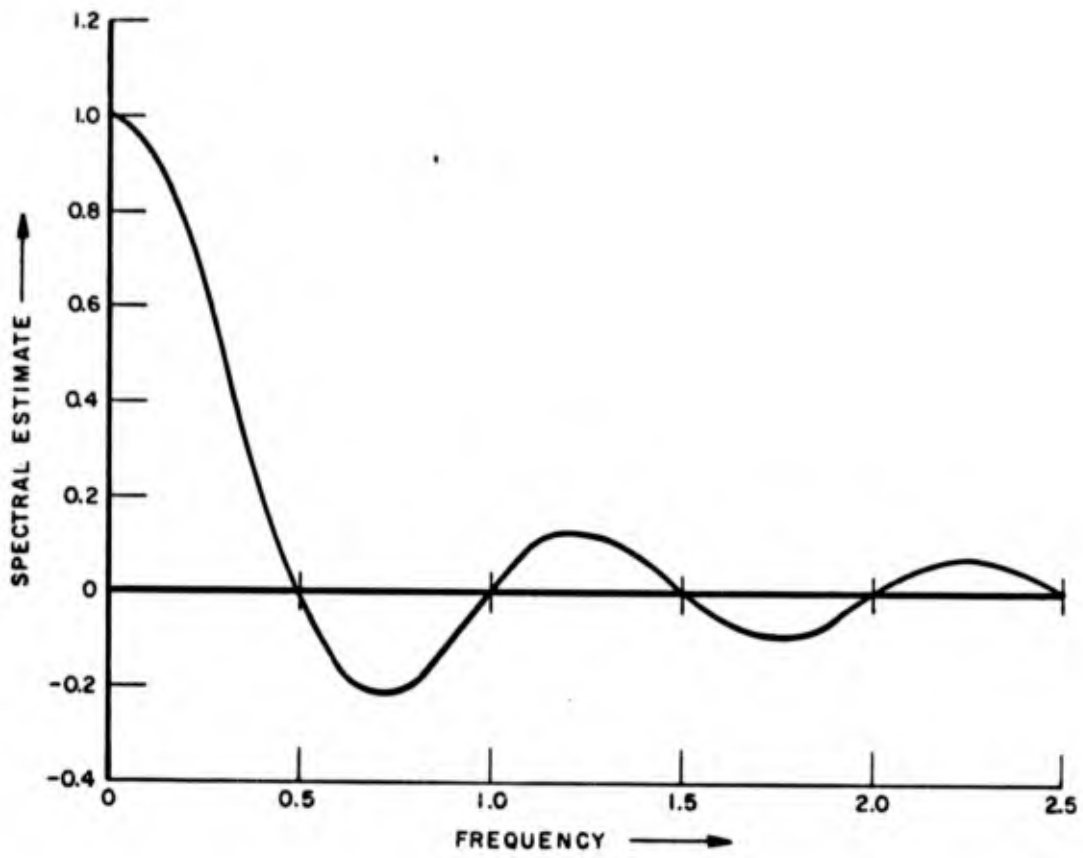


FIGURE 8. EXAMPLE OF SPECTRAL WINDOW

R-1204

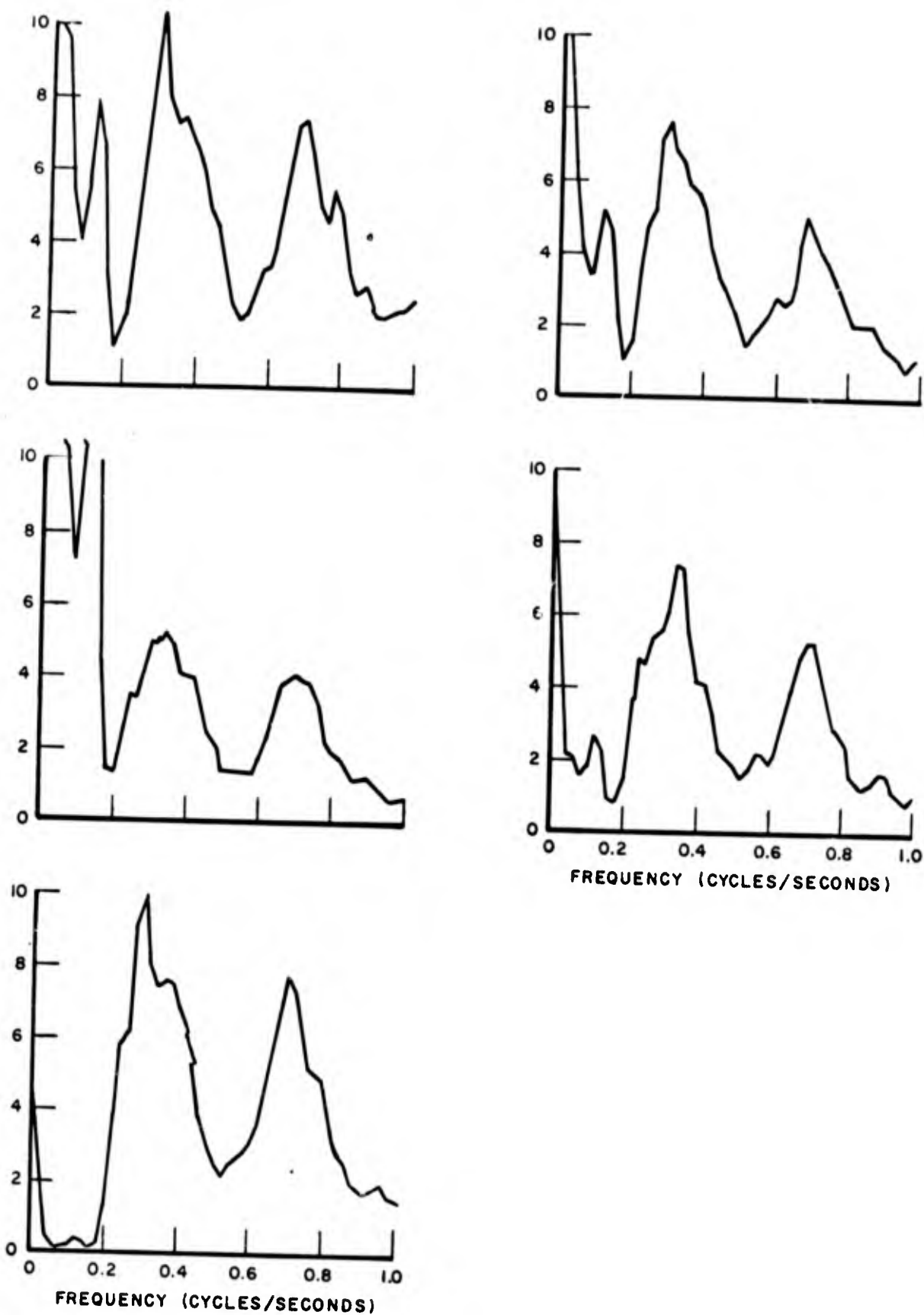


FIGURE 9. POINT SPECTRAL ESTIMATES RUN 4.

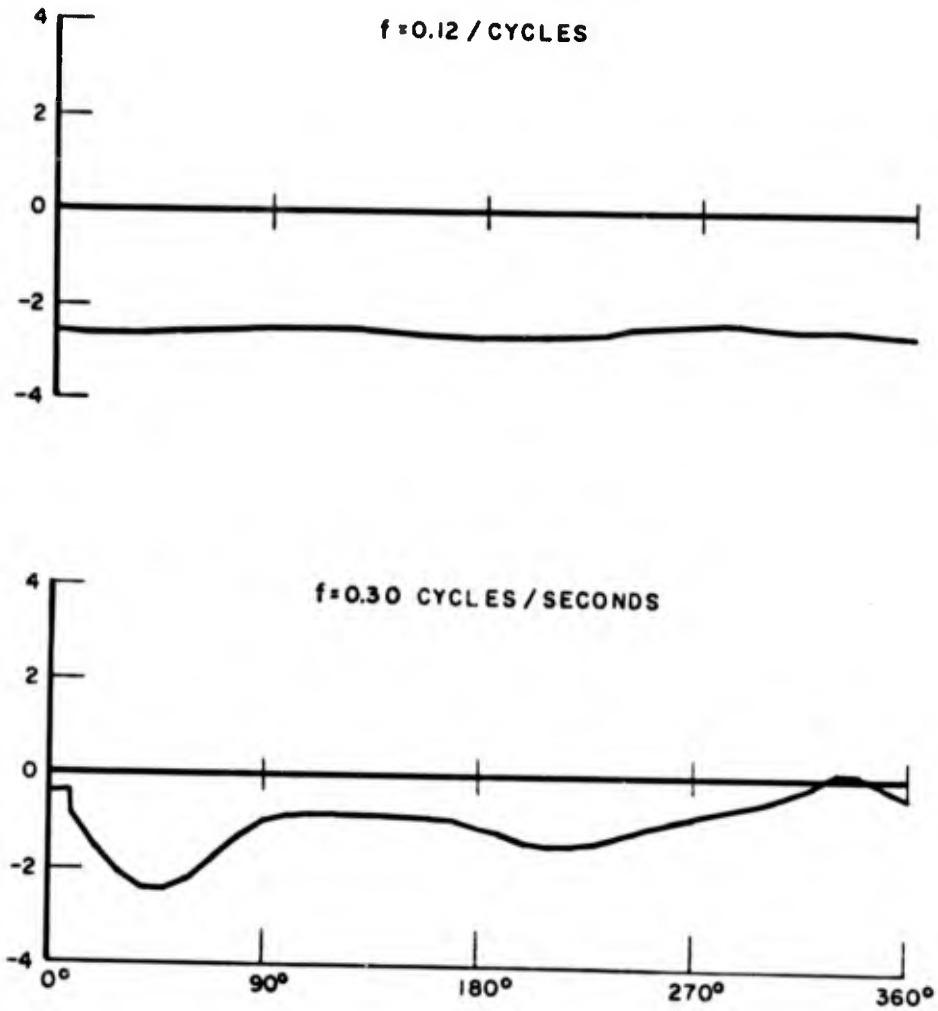


FIGURE 10. DIRECTIONAL SPECTRA RUN 4

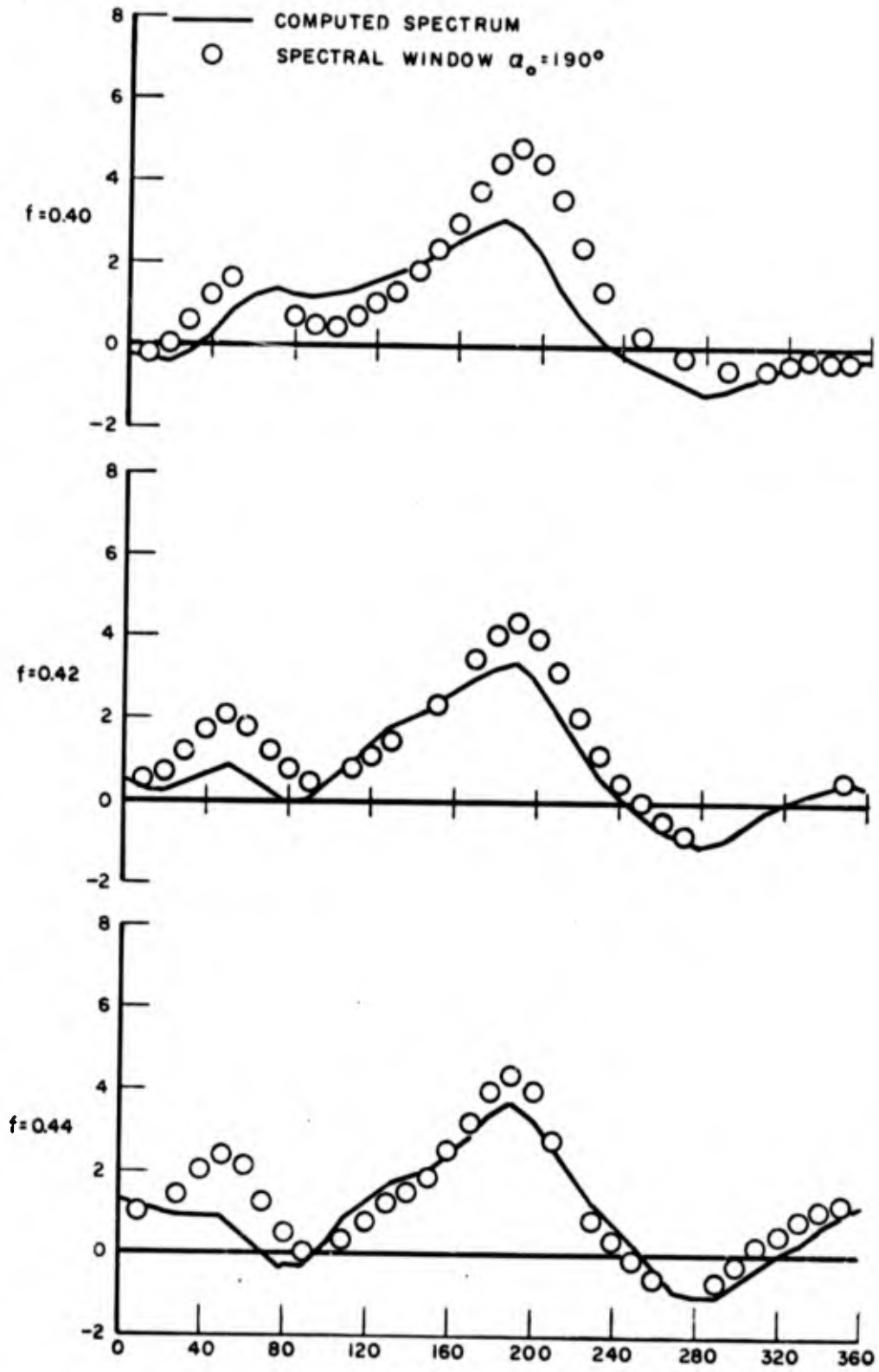


FIGURE 11. DIRECTIONAL SPECTRA RUN 4

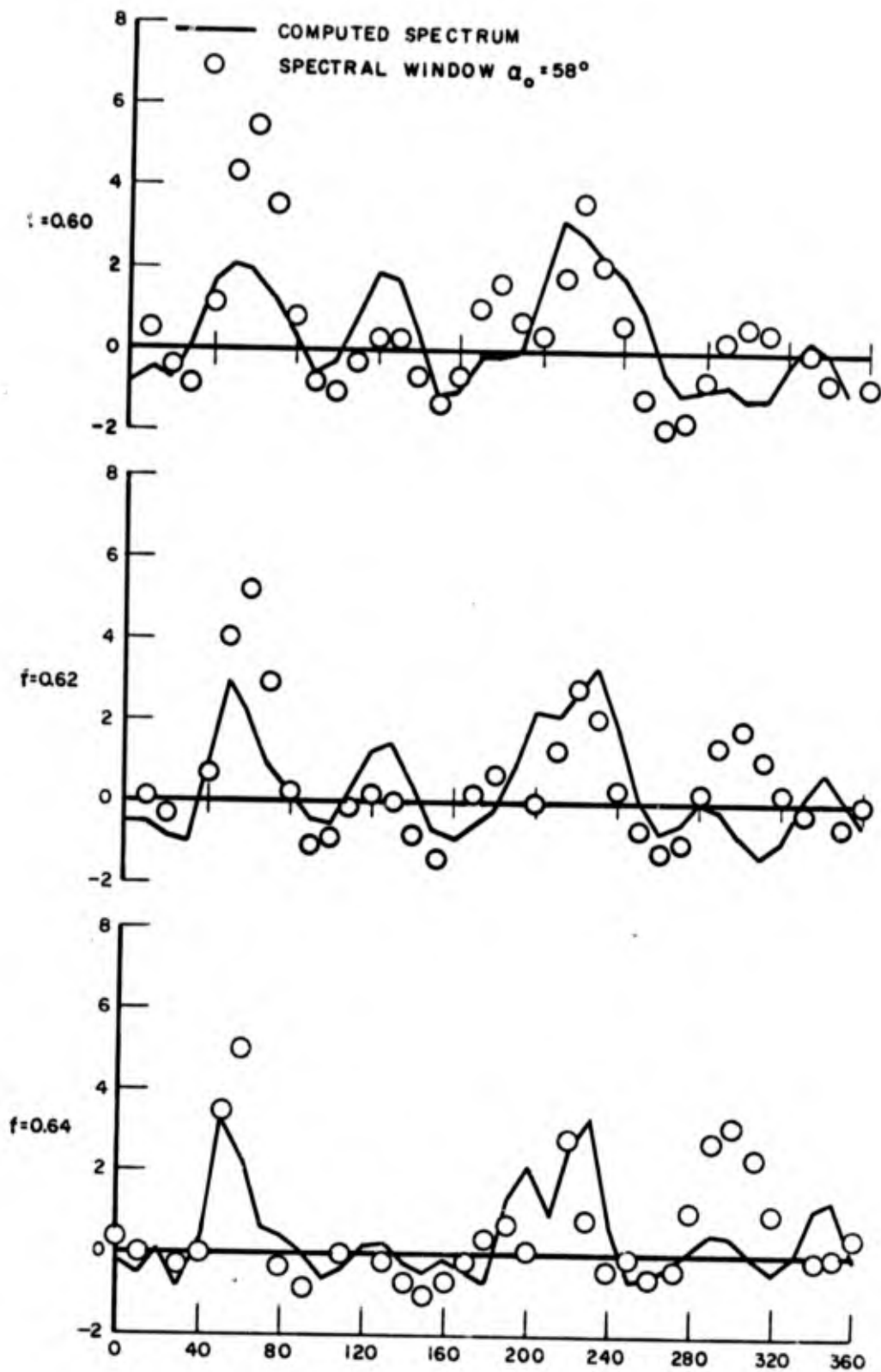


FIGURE 12. DIRECTIONAL SPECTRA RUN 4

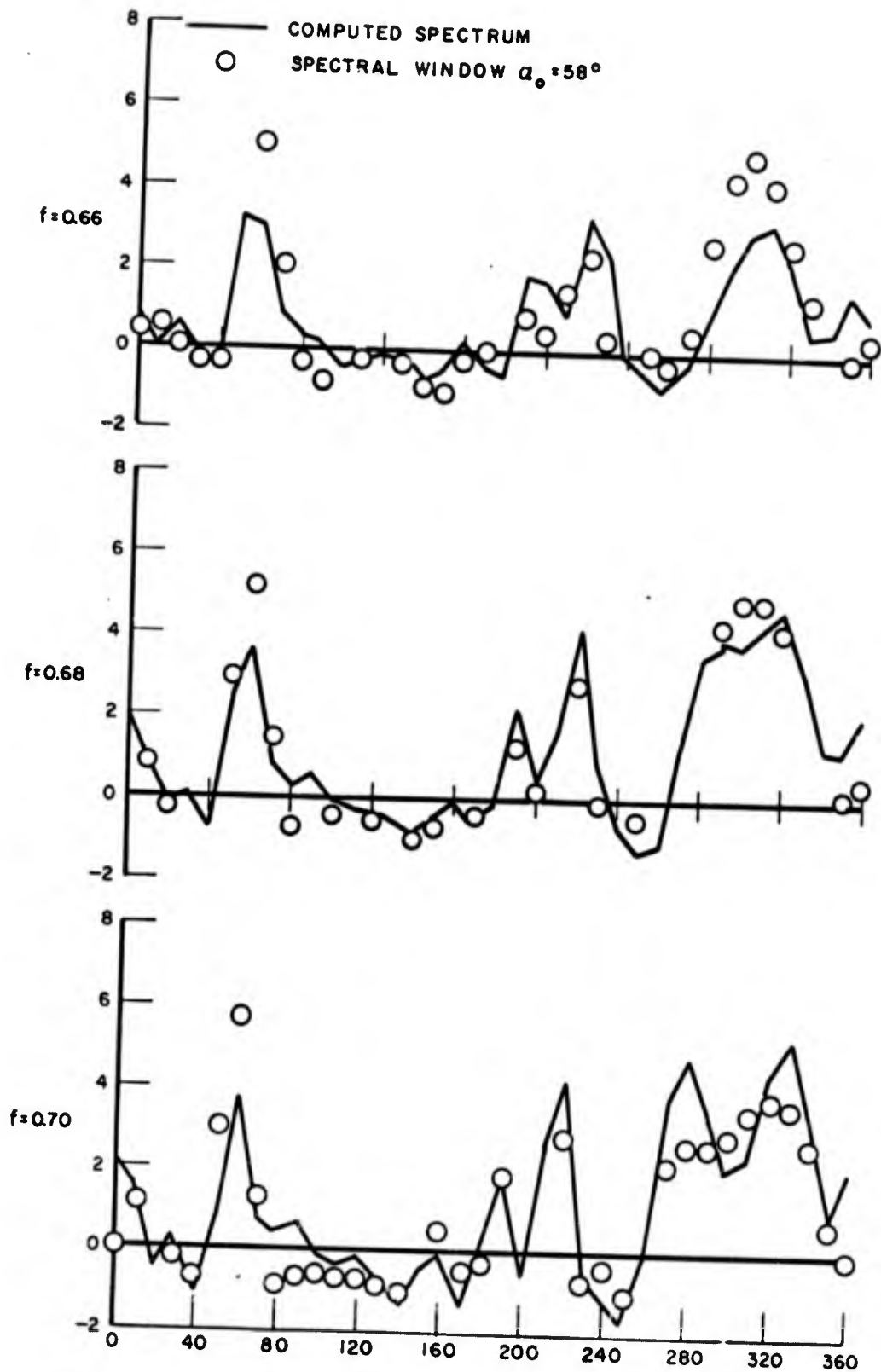


FIGURE 13. DIRECTIONAL SPECTRA RUN 4

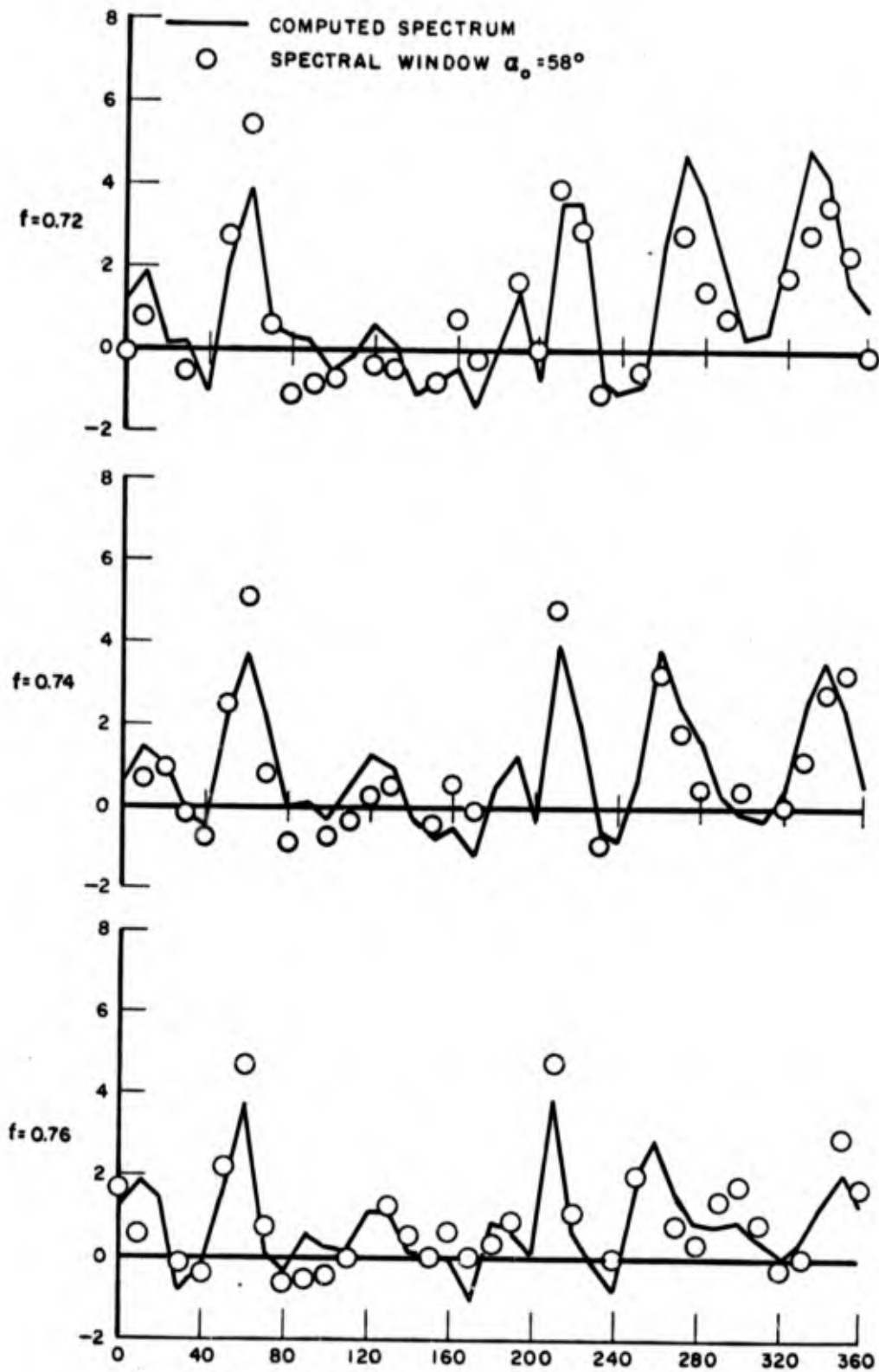


FIGURE 14. DIRECTIONAL SPECTRA RUN 4

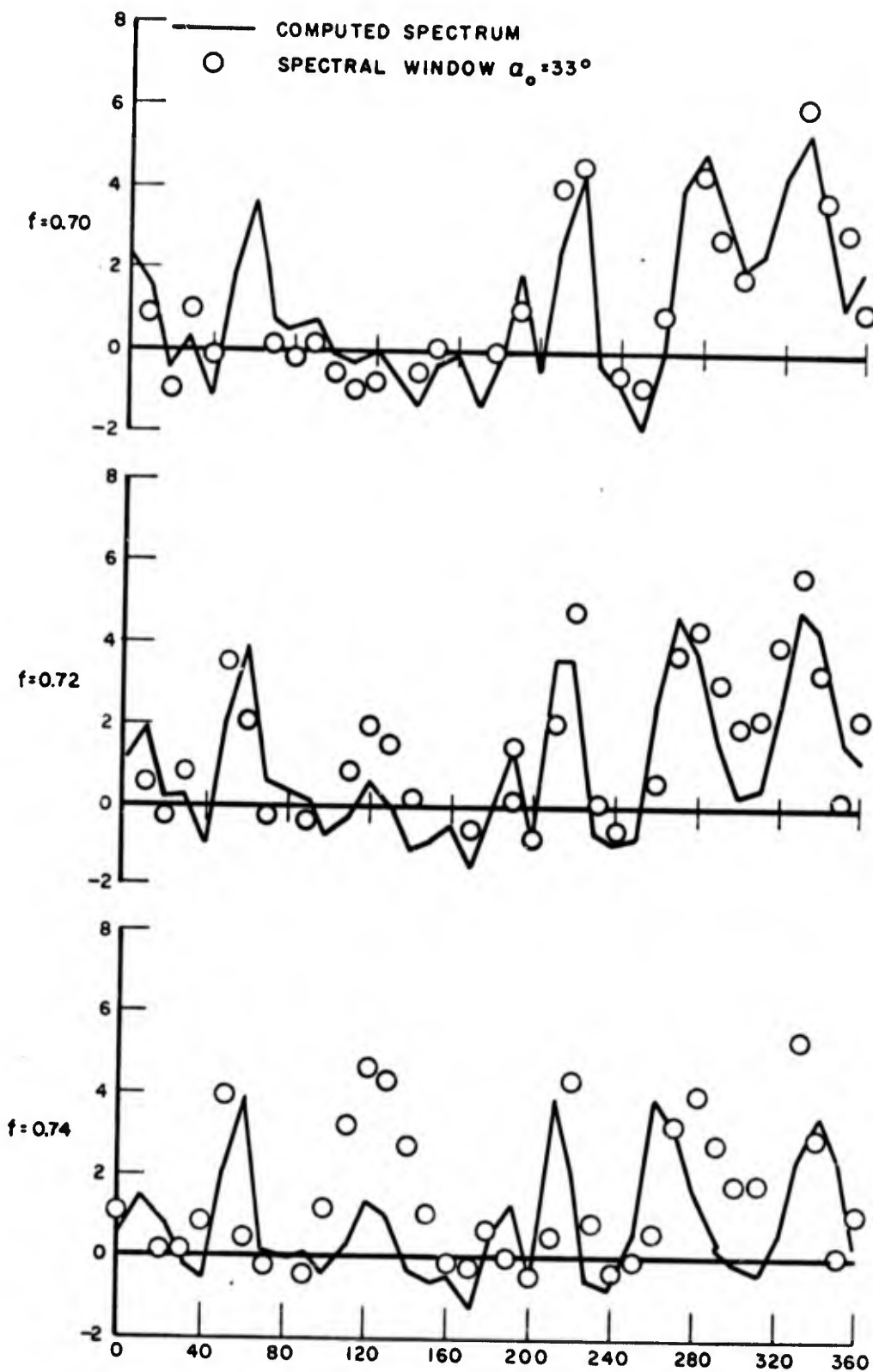


FIGURE 15. DIRECTIONAL SPECTRA RUN 4

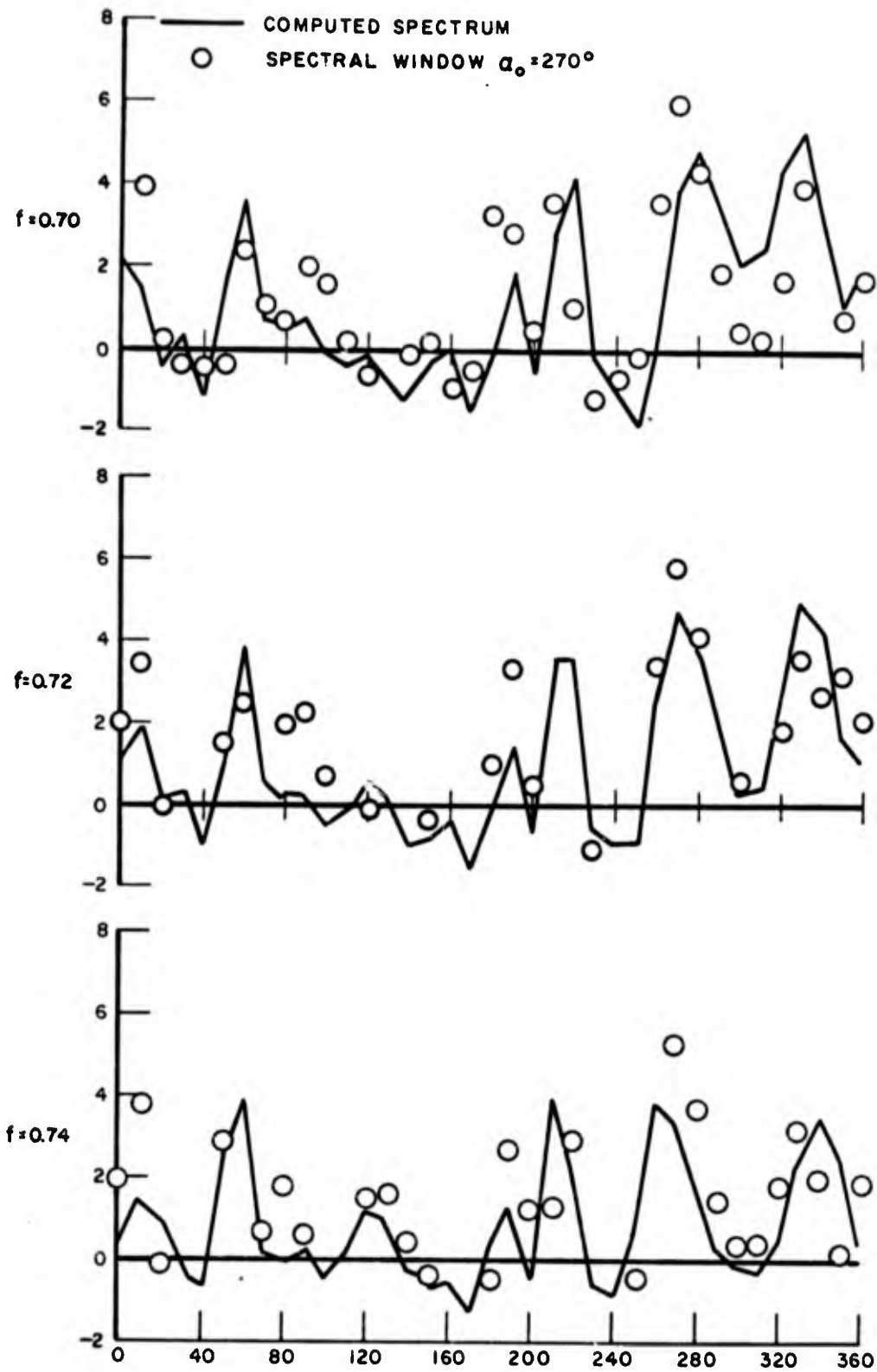


FIGURE 16. DIRECTIONAL SPECTRA RUN 4

R-1204

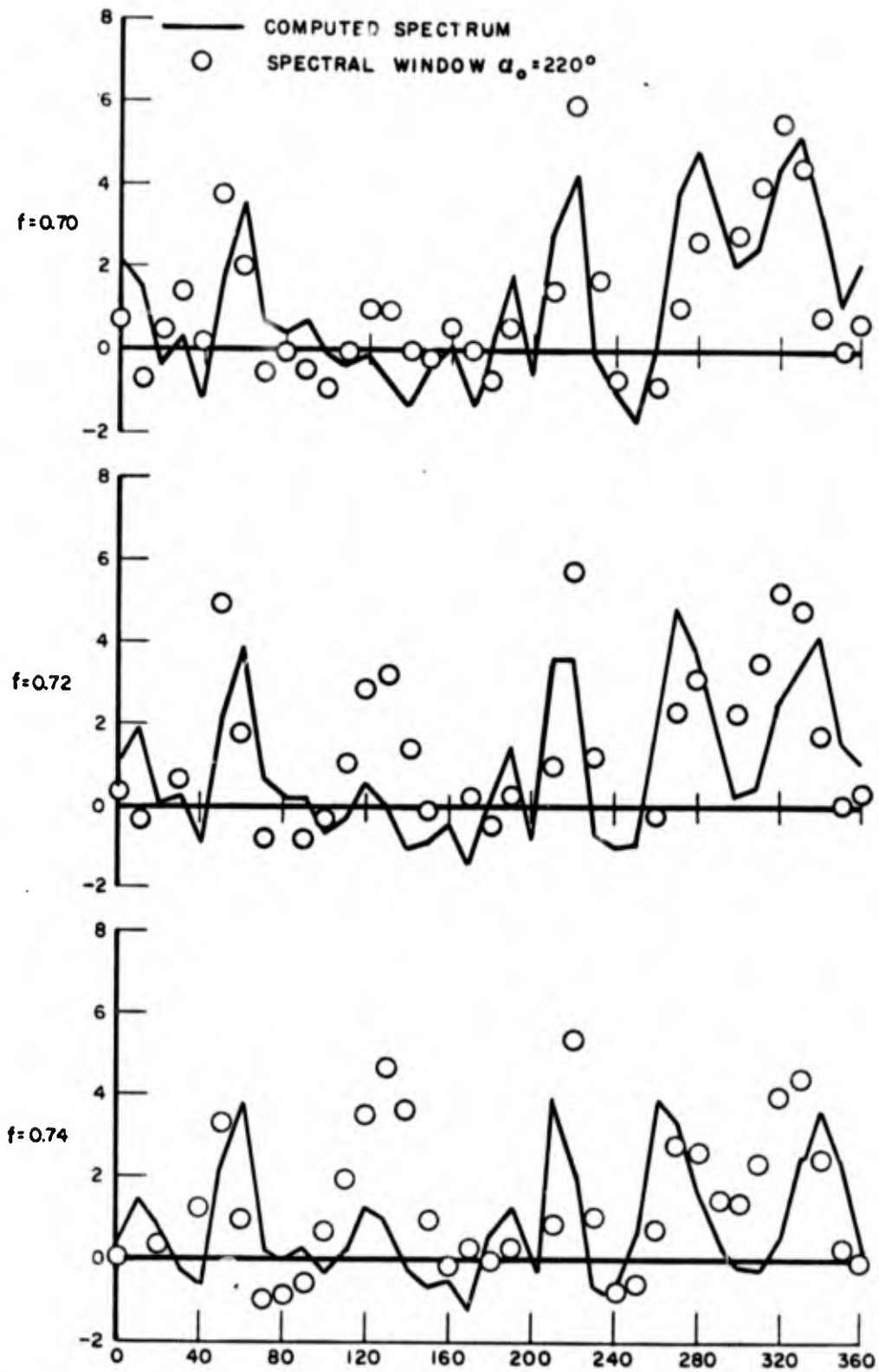


FIGURE 17. DIRECTIONAL SPECTRA RUN 4

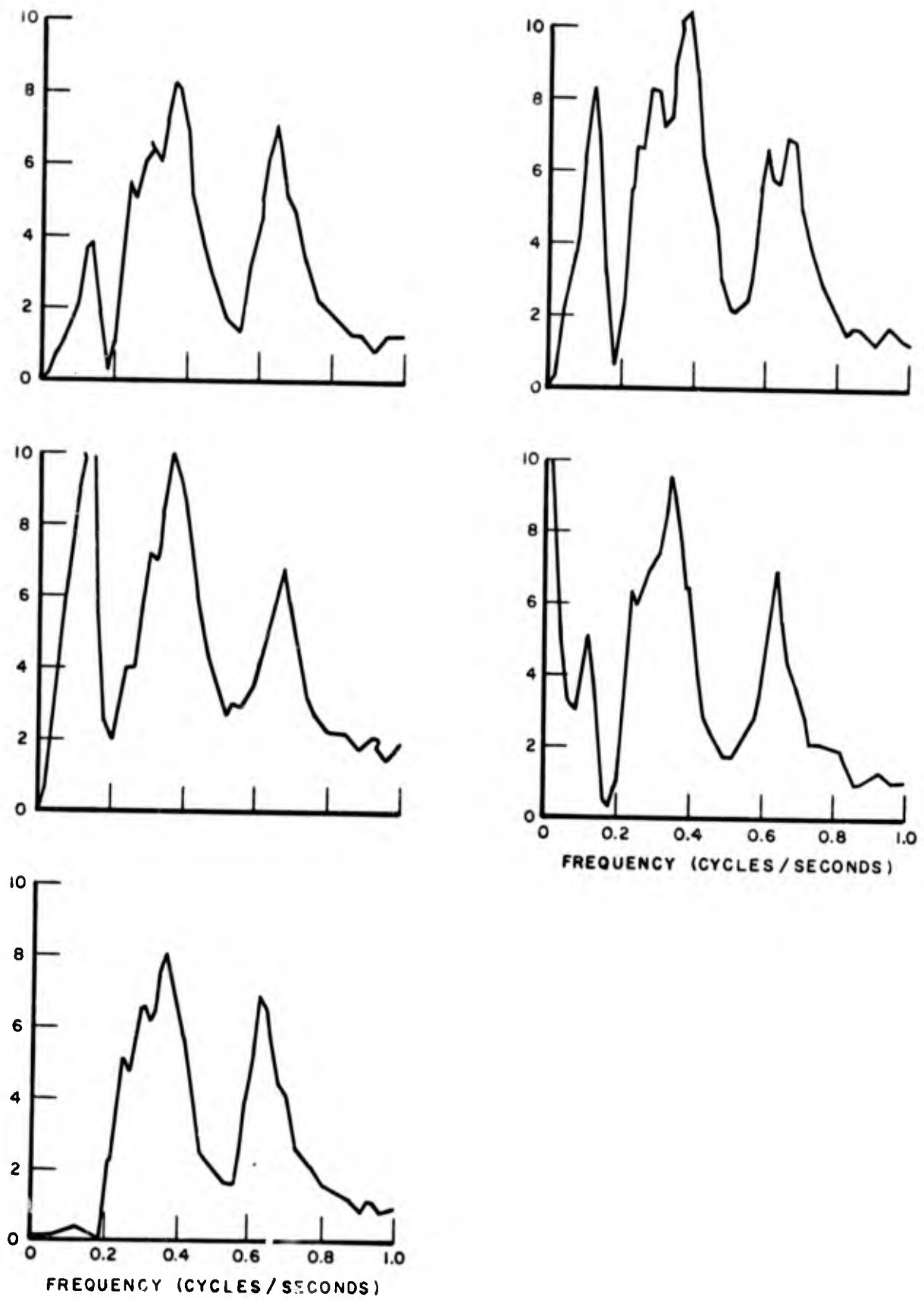


FIGURE 18. POINT SPECTRAL ESTIMATES RUN 6

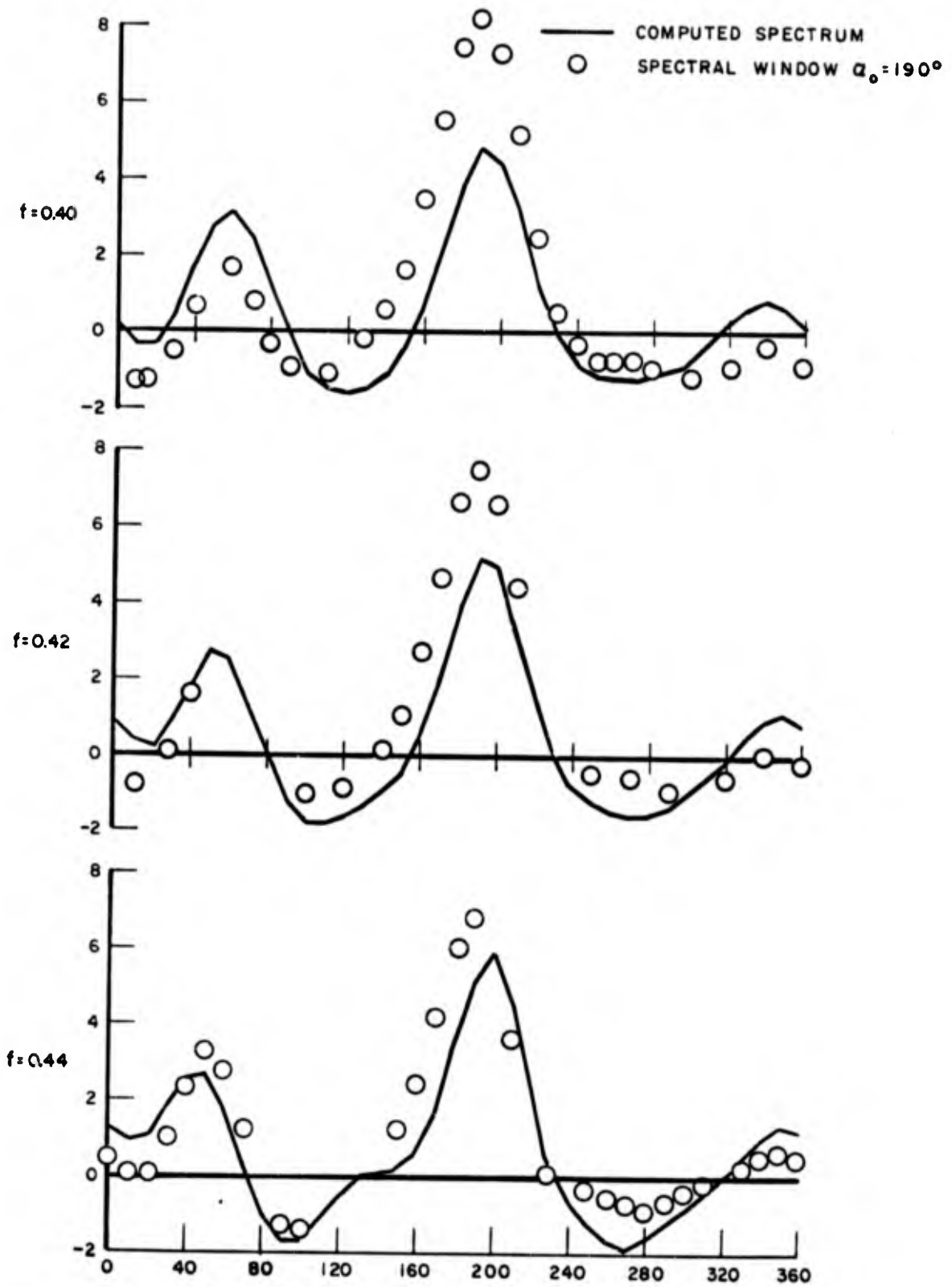


FIGURE 19. DIRECTIONAL SPECTRA RUN 6

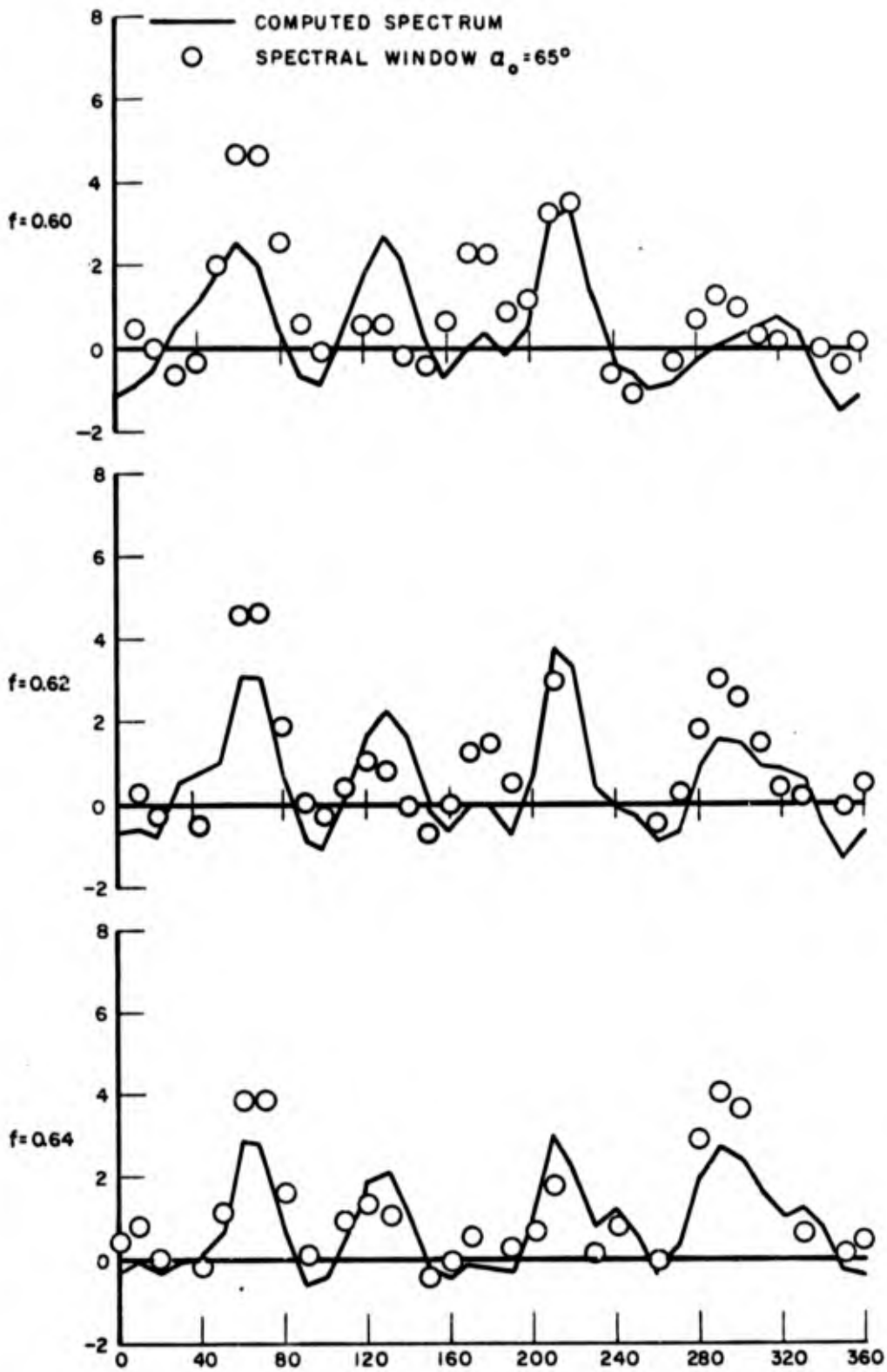


FIGURE 20. DIRECTIONAL SPECTRA RUN 6

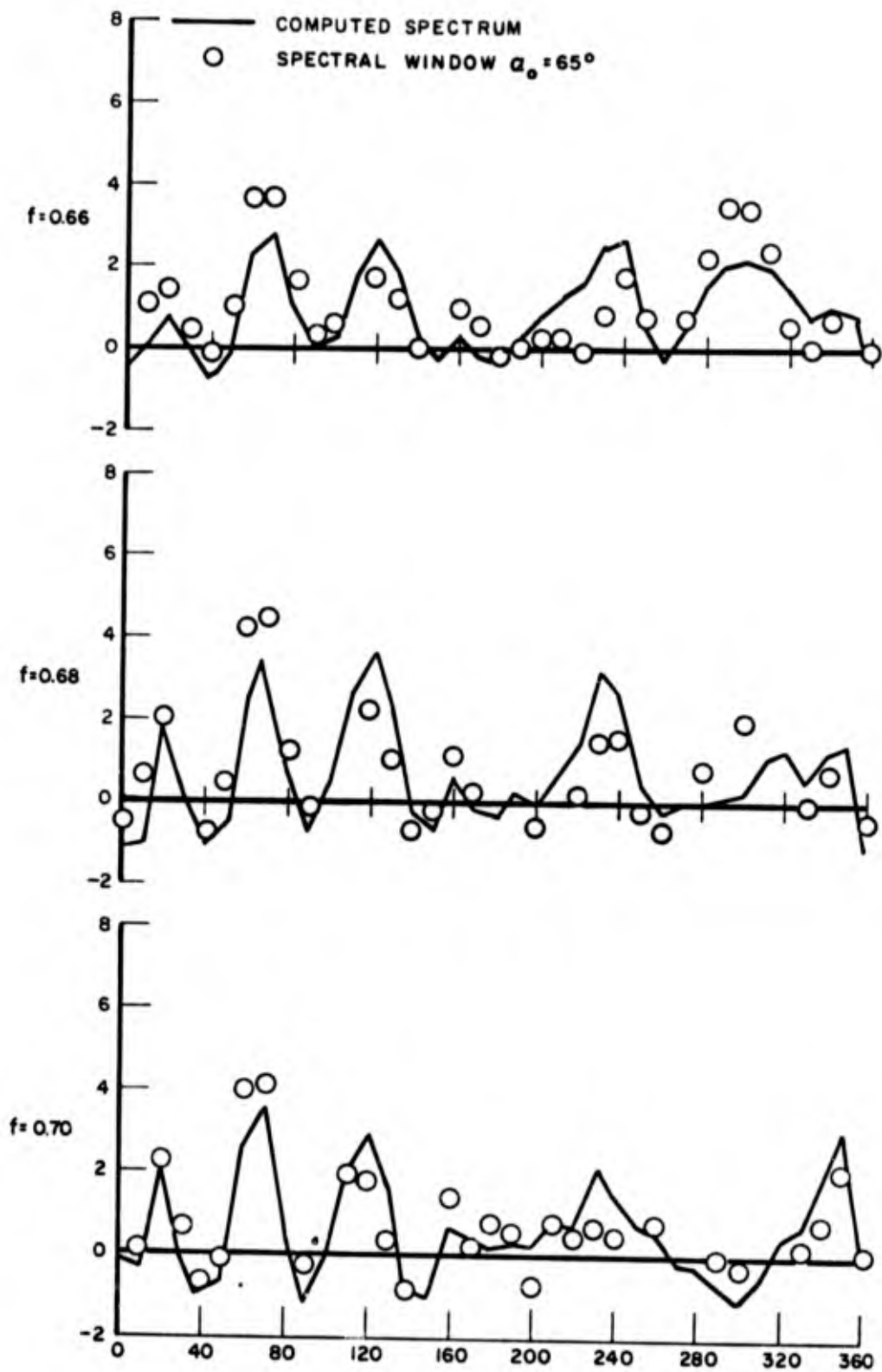


FIGURE 21. DIRECTIONAL SPECTRA RUN 6

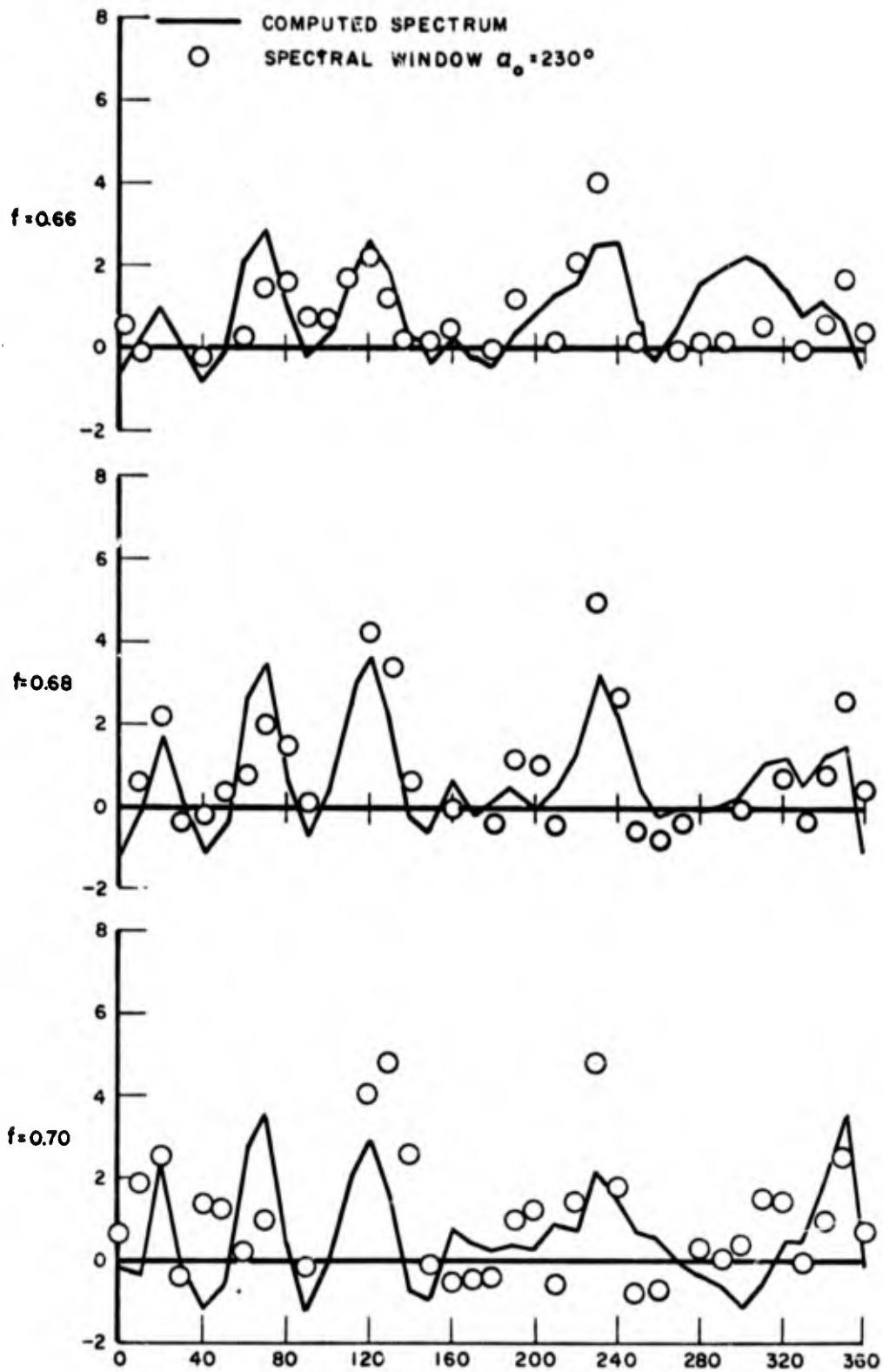


FIGURE 22. DIRECTIONAL SPECTRA RUN 6.

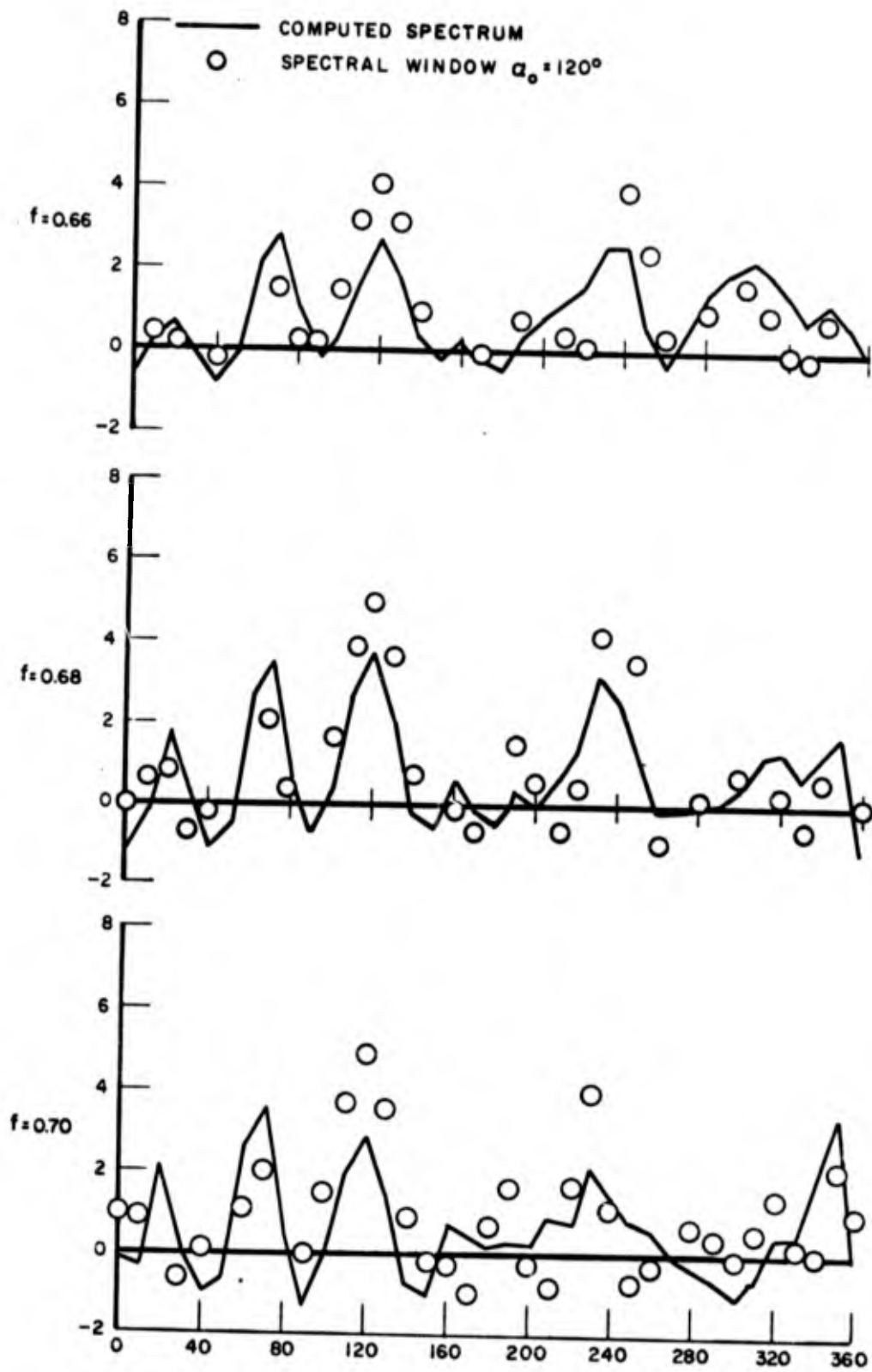


FIGURE 23. DIRECTIONAL SPECTRA RUN 6

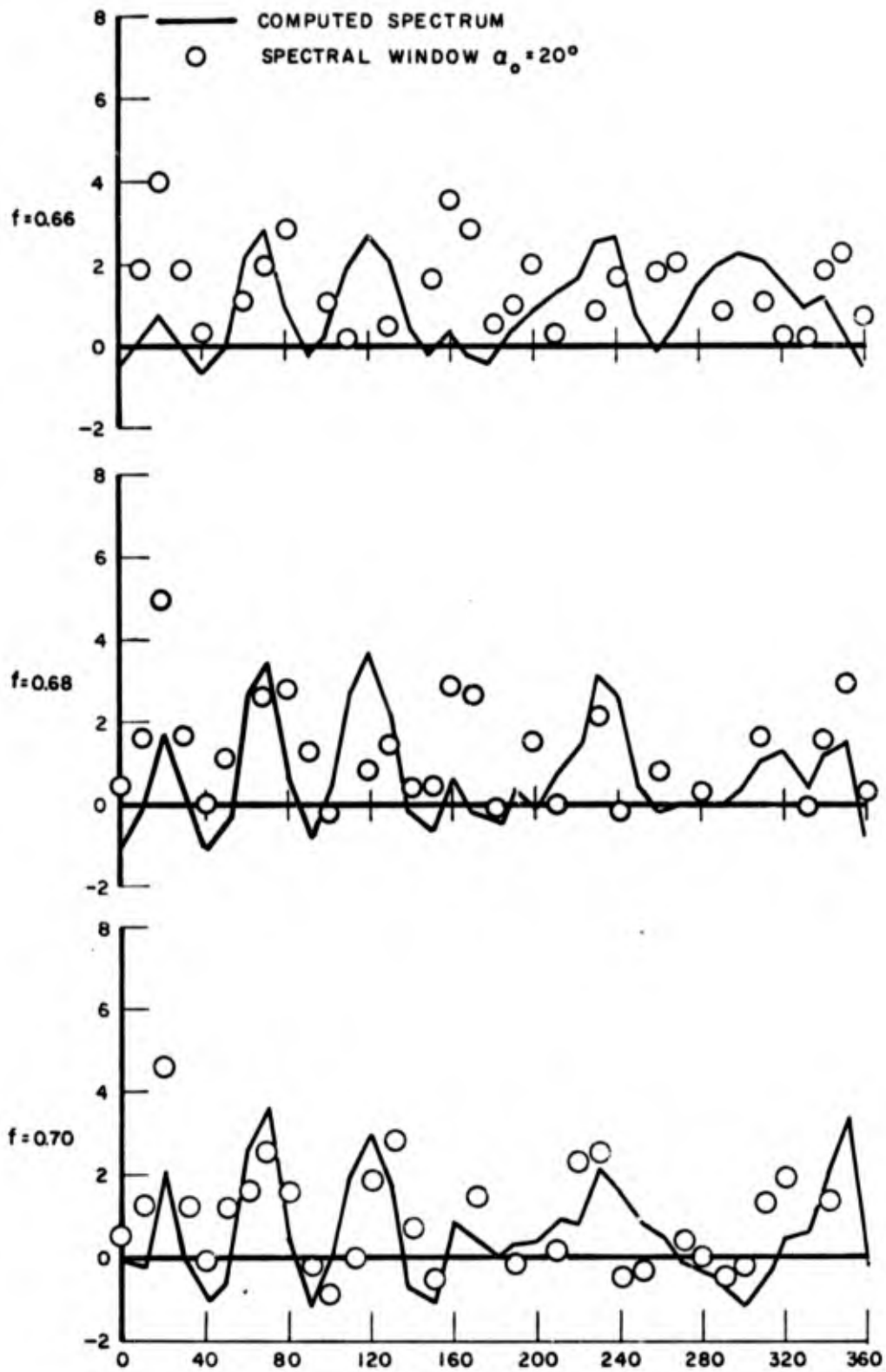


FIGURE 24. DIRECTIONAL SPECTRA RUN 6

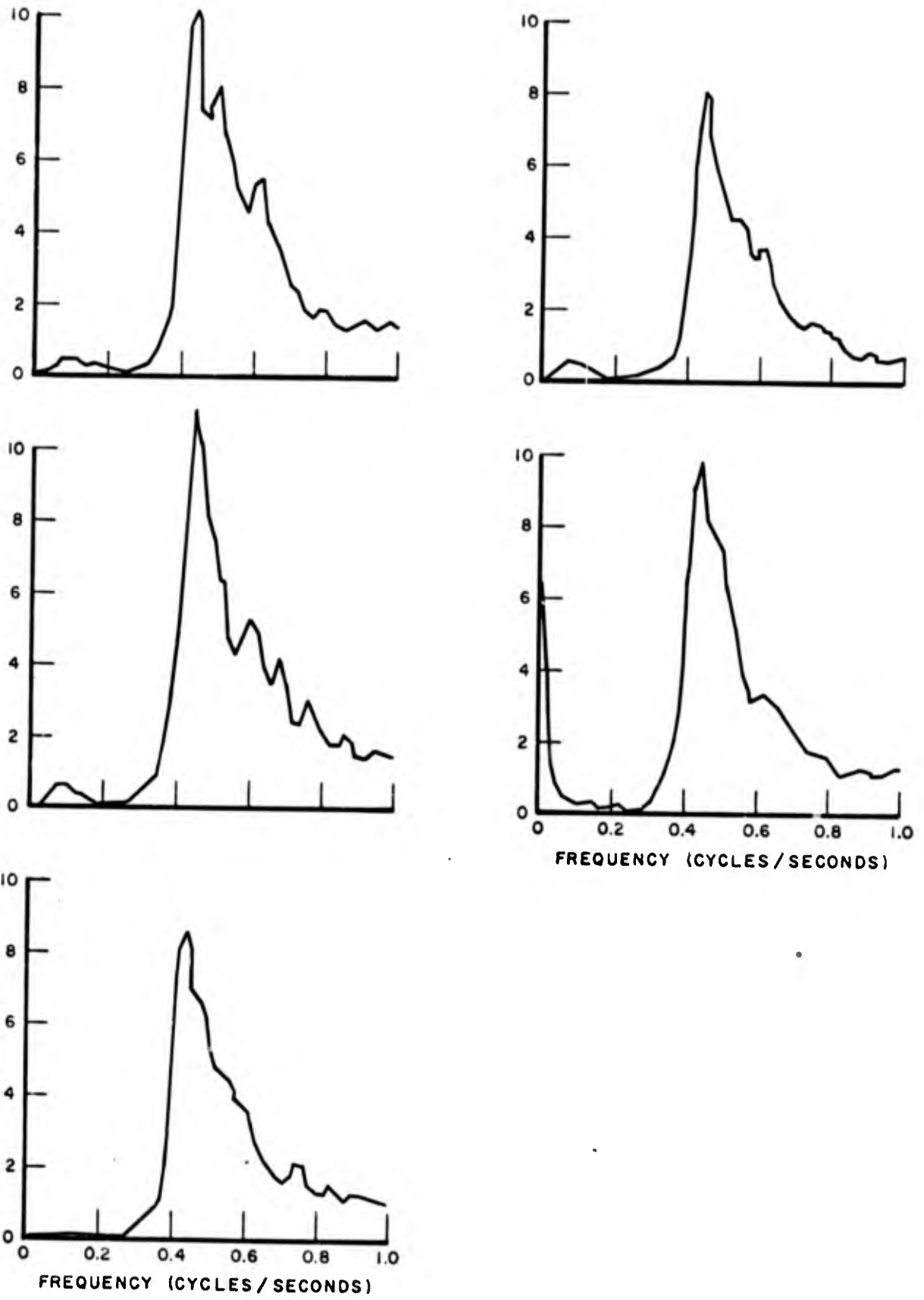


FIGURE 25. POINT SPECTRAL ESTIMATES RUN 8

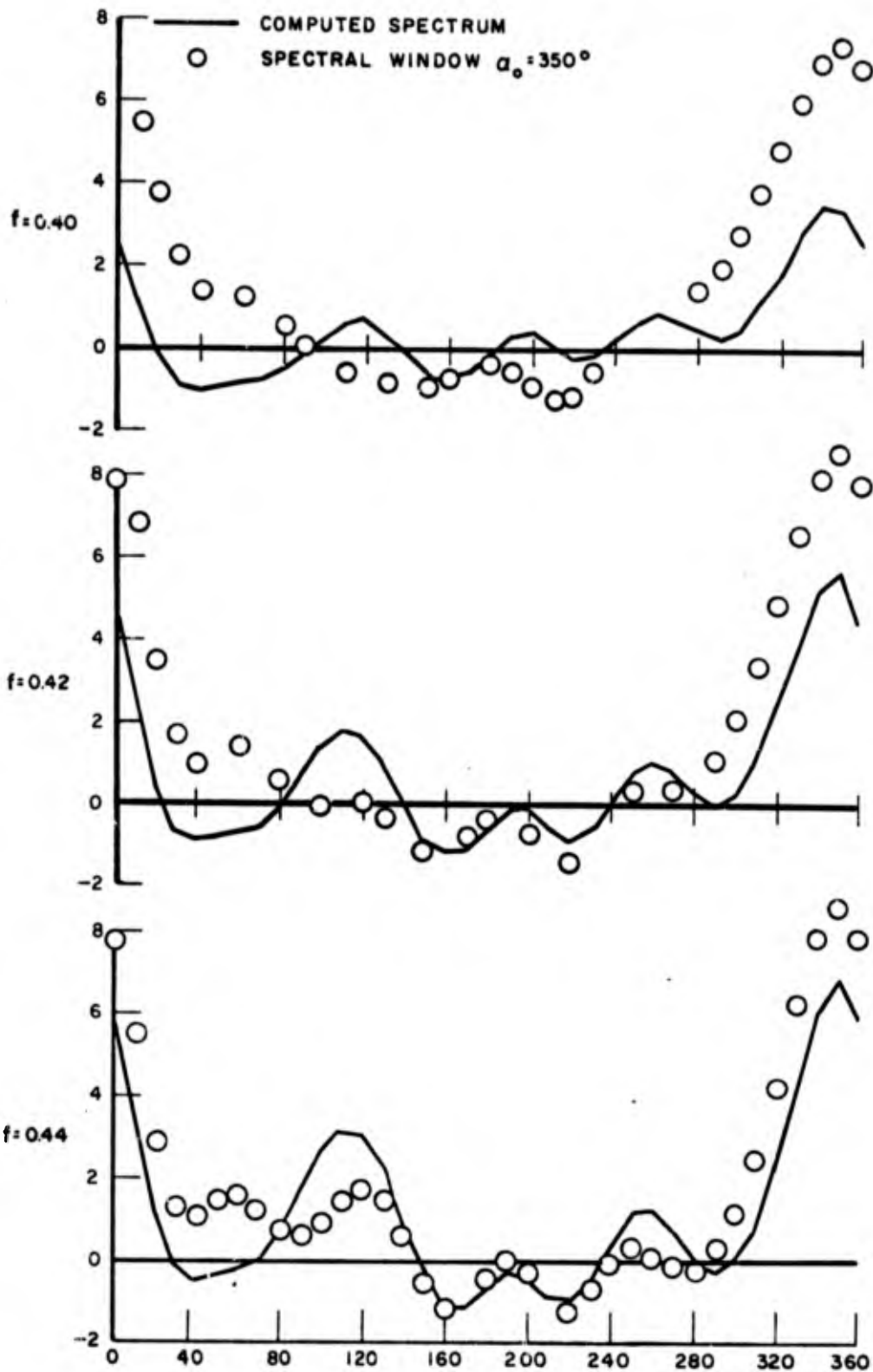


FIGURE 26. DIRECTIONAL SPECTRA RUN 8

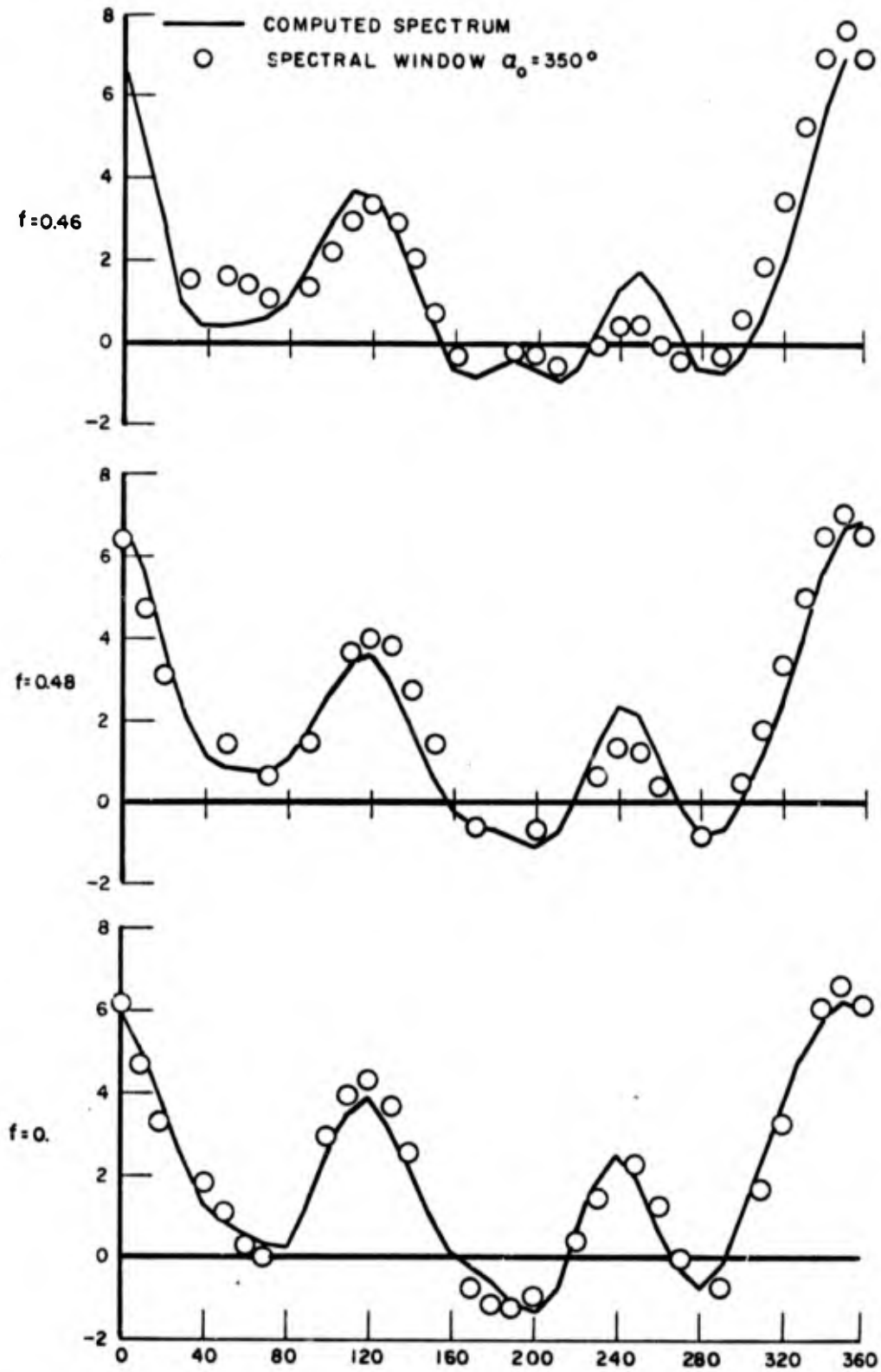


FIGURE 27. DIRECTIONAL SPECTRA RUN 8

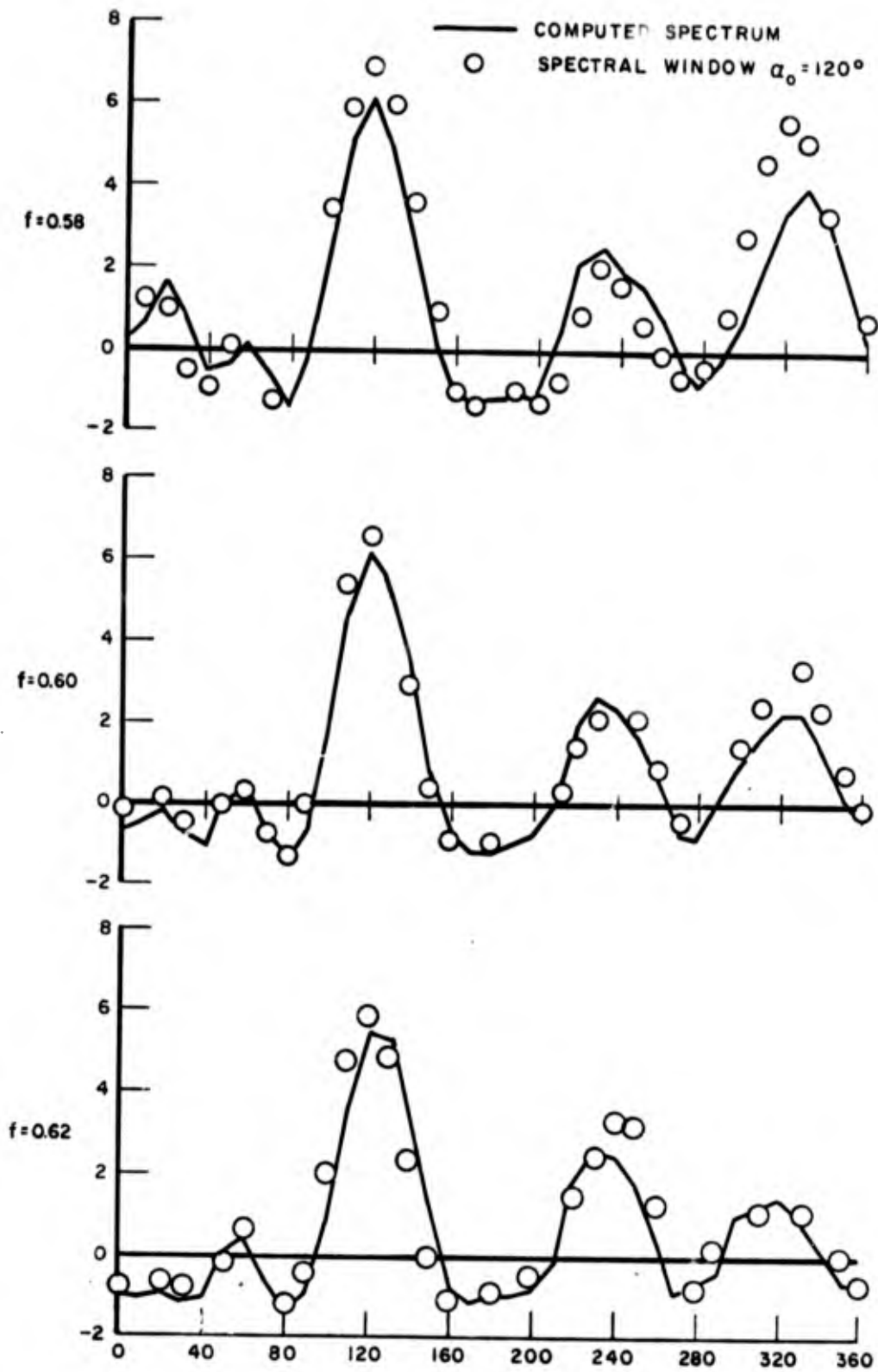


FIGURE 28. DIRECTIONAL SPECTRA RUN 8

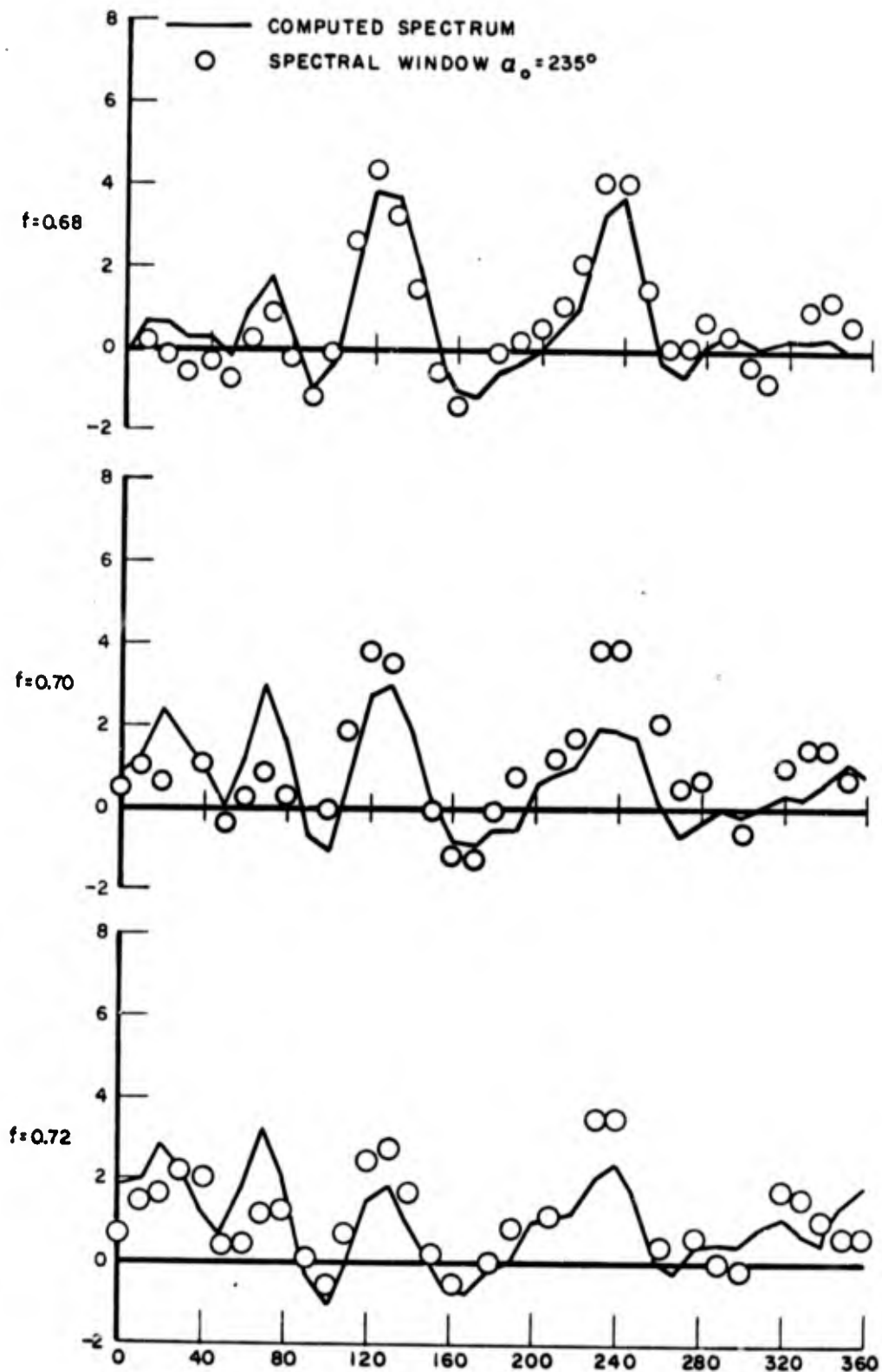


FIGURE 29. DIRECTIONAL SPECTRA RUN 8

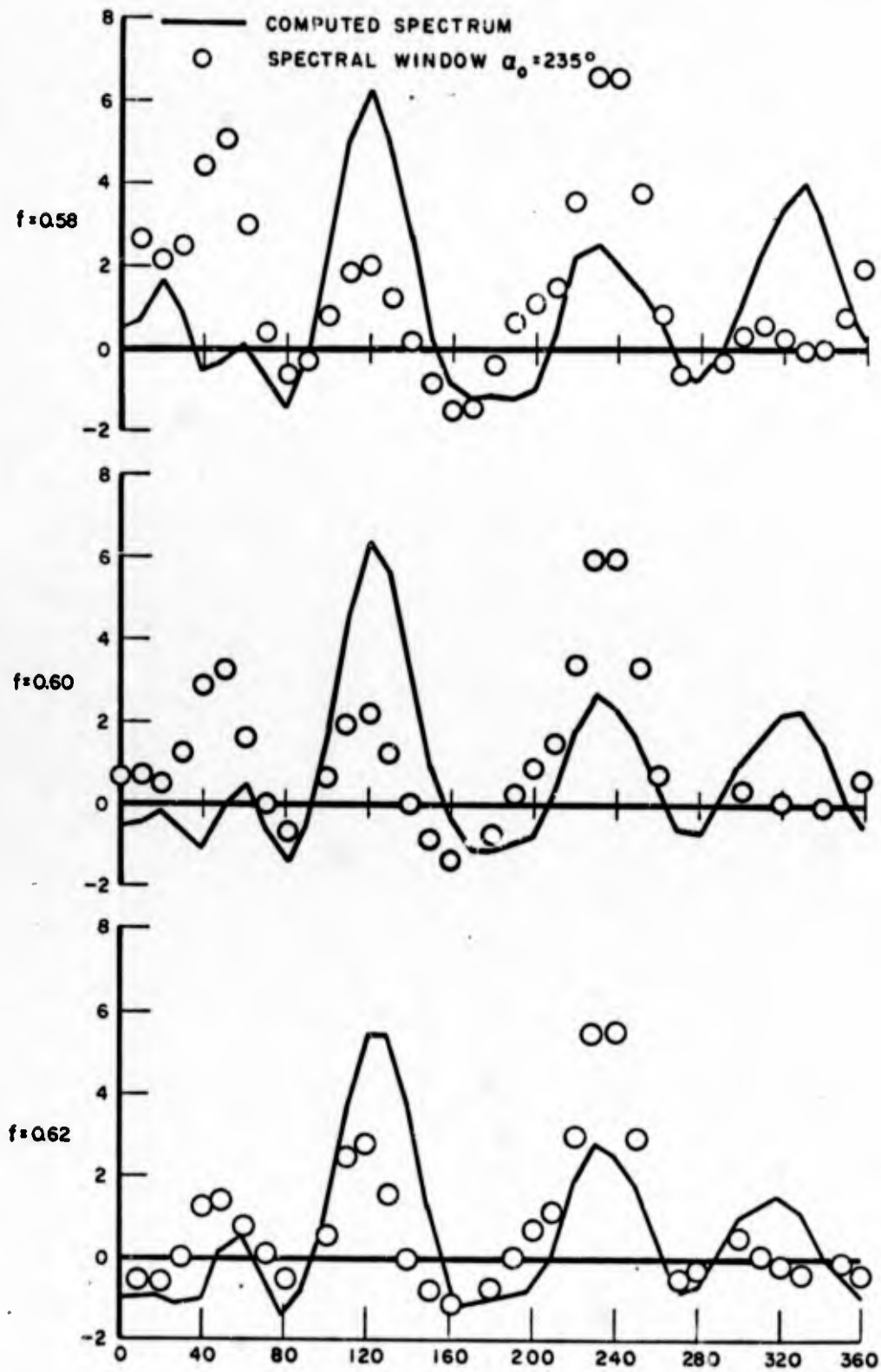


FIGURE 30. DIRECTIONAL SPECTRA RUN 8

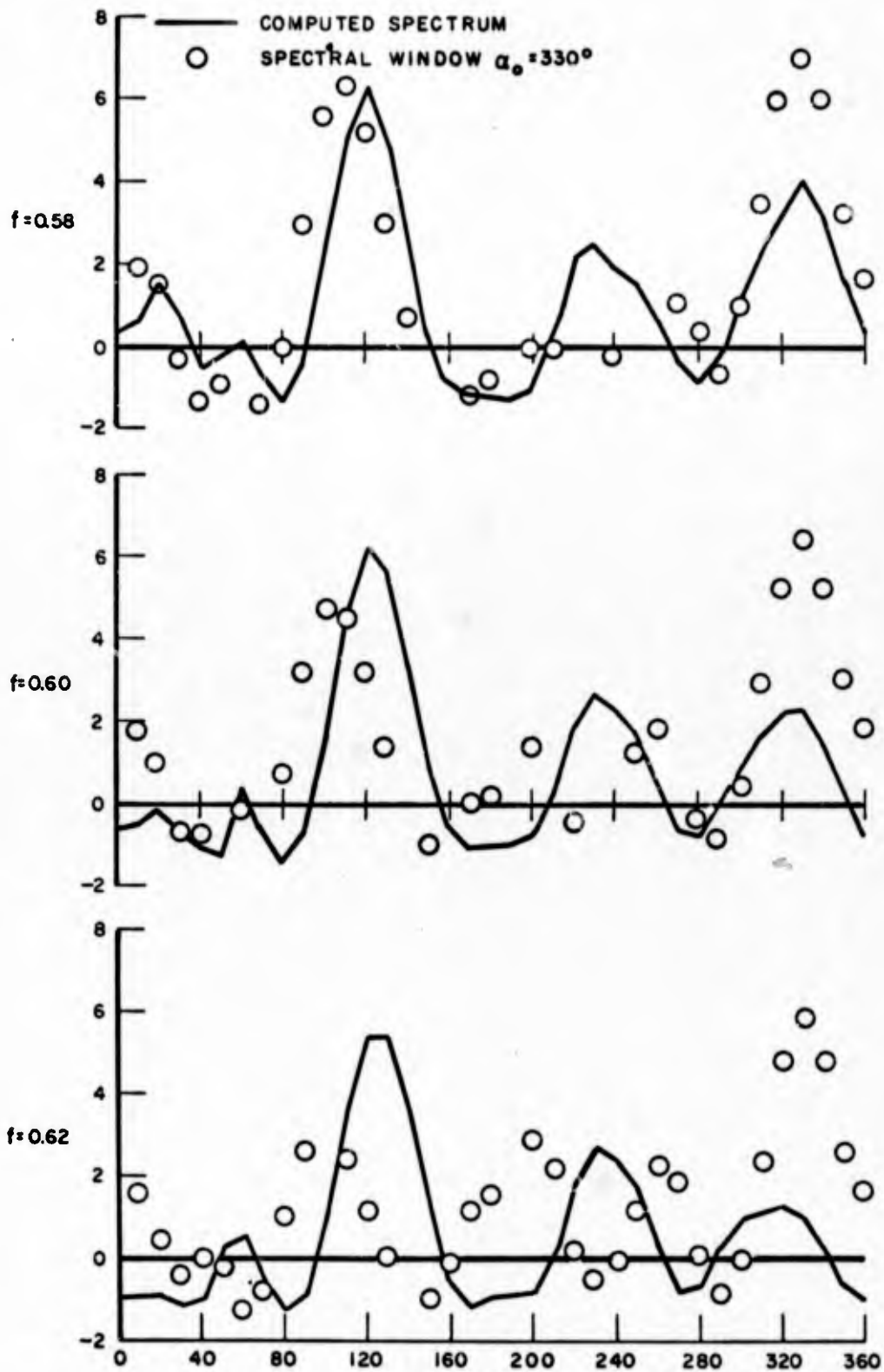


FIGURE 31. DIRECTIONAL SPECTRA RUN 8

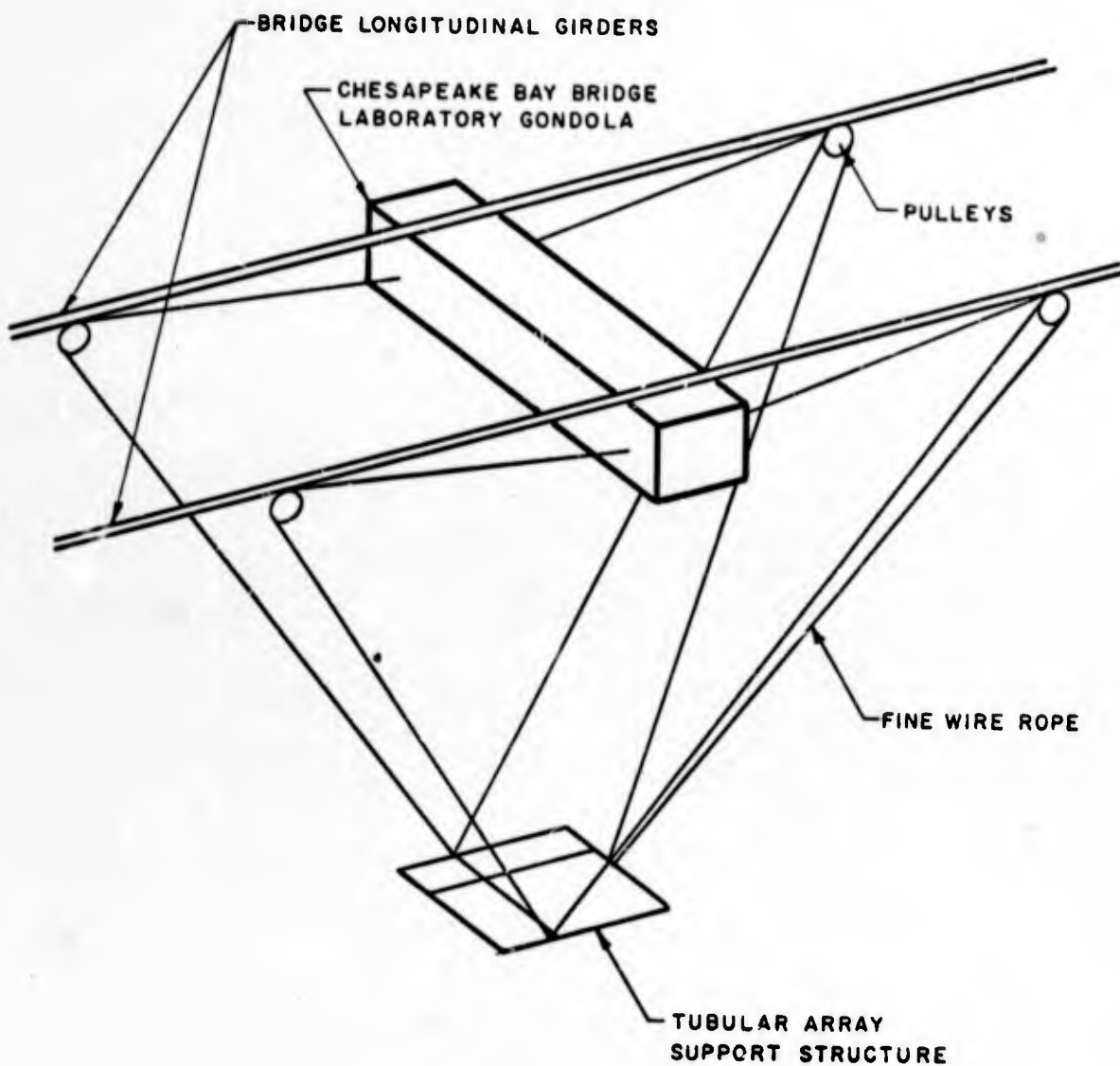


FIGURE 32. RECOMMENDED ALTERNATE ARRAY SUSPENSION SCHEME

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13. ABSTRACT <p>A program of experiments was carried out at the Naval Research Laboratories Chesapeake Bay Bridge Laboratory to assess the directional resolving capabilities of a particular array of five wave elevation detectors. It is desired to be able to distinguish waves both by their wave lengths and elevations and by their directions of propagation as part of an NADC sponsored study on surface wave modifications.</p> <p>The collected data were analyzed and rather good success was had in deriving directional wave spectra. Recommendations are made to further improve the experimental procedure.</p>			

