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TECHNICAL REPORT

A RELATION BETWEEN SYSTEM VARIABLES RESULTING FROM PHYSICAL DIMENSIONS

by

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A Relation Between System Variables Resulting from Physical Dimensions

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ABSTRACT

A non-trivial relation between system parameters and system observables is developed which is based only on dimensional analysis. It finds application in the "BLP" test. This test is used to partially validate the numerical elements in least squares equations for geodetic parameters when the observables are Doppler data from earth satellites.

FOREWORD

This study was initiated under the Satellite Geophysics Project AIR 538001/2915/S4390001. The associated modification to the GEO computer program was made by Mrs. Carol Daniels. Extensive numerical studies were conducted by Mrs. Carol Ann Malyevac.

APPROVED FOR RELEASE:

/s/ BERNARD SMITH
Technical Director

INTRODUCTION

In the geodetic reduction of artificial satellite Doppler tracking data, we solve for the orbital parameters, the parameters of the gravitational field, and the geocentric coordinates of the tracking stations. The distance scale is established through the input value of the velocity of light. Changing the unit of length in which the velocity of light is presented changes the numerical values of the other parameters according to their length dimensions. The parameters, the computed Doppler data, and the partial derivatives of the latter are found to satisfy a non-trivial relation.

The dimensional considerations for more general equations are first presented. Following this is the "BLP" application to least squares equations for geodetic parameters based on Doppler tracking of earth satellites.

GENERAL RELATION RESULTING FROM PHYSICAL DIMENSIONS

Let u_k be the k^{th} dimensional standard unit such as 1 meter or 1 second.

Let u_k/s_k be a scaled dimensional unit such as .9 meters or 1.2 seconds.

Let the scaled unit of the j^{th} generalized system parameter be $\prod_k (u_k/s_k)^{N_{pjk}}$.

Let $P_j(s)$ be the dimensionless number of scaled units, and let P_j be the dimensionless number of standard units so that

$$P_j = P_j(s_k = 1, \text{ all } k) \quad .$$

We have then,

$$P_j(s) \prod_k (u_k/s_k)^{N_{pjk}} = P_j \prod_k u_k^{N_{pjk}}$$

or

$$P_j(s) = P_j \prod_k s_k^{N_{pjk}}$$

Further,

$$\left[\frac{\partial P_j(s)}{\partial s_k}\right]_{\text{all } s = 1} = N_{pjk} P_j$$

Now let the unit of the i^{th} observable D_i be $\prod_k (u_k/s_k)^{N_{Dik}}$. If there is a mathematical model of the system relating the observables to the parameters, say

$$D_i = D_i(P) \quad ,$$

we have

$$D_i[P(s)] \prod_k (u_k/s_k)^{N_{Dik}} = D_i(P) \prod_k u_k^{N_{Dik}}$$

or,

$$D_1[P(s)] = D_1(P) \prod_k s_k^{N_{D1k}} \quad .$$

Differentiating this equation with respect to s_k holding P fixed, evaluating at all s = 1, and substituting from above for $\frac{\partial P}{\partial s_k}$,

we obtain

$$\sum_j \frac{\partial D_1}{\partial P_j} N_{\rho jk} P_j = N_{D1k} D_1(P) \quad .$$

This is the basic relation resulting from physical dimensions.

As generalized system parameters for this analysis, we must of course include not only the parameters to be estimated by least squares, but also all other independent quantities entered into the system. The least squares parameters might be the initial trajectory coordinates and velocity components, the parameters of the gravitational potential, and the coordinates of the observation stations. The other quantities entered into the system might include the velocity of light, the angular velocity of the earth, the direction of the earth's axis of rotation, the coordinates of the sun and moon, the coordinates of the observation stations if they are not in the set of least squares parameters, the times of observation, etc.

In the next section we make a particular application of the identity resulting from dimension, but the identity has of course wider application.

APPLICATION TO PROCESSING OF OBSERVATIONS OF DOPPLER FREQUENCY

We now consider the system in which the observables are the Doppler frequency f_D received at stations fixed on the earth from an oscillator in an earth satellite. In this case, basically

$$D(P) = f_D(P) = - f_s \frac{\dot{|\bar{r}(P) - \bar{r}_R(P)|}}{c}$$

where

f_s = frequency of the oscillator

c = velocity of light

$\bar{r}$ = geometric coordinates of satellite in a non-rotating frame

$\bar{r}_R$ = coordinates of a receiver in the same frame

In a first model, the least squares parameters are the initial elements of the orbit, drag and radiation pressure coefficients, the earth fixed coordinates of the receiver stations, and the coefficients in the harmonic expansion of the earth's gravitational potential per unit mass, U.

We will focus on the unit of length so that $u_K = u_L$. We then have $N_{DiL} = 0$ and so

$$\Sigma \frac{\partial f_D}{\partial P_j} L_j P_j = 0$$

where L_j is the length dimension of P_j.

If the gravitational acceleration potential, U, with dimension L^2/T^2, were represented by

$$U = \frac{1}{r} \sum_{n,m} \frac{1}{r^n} P_n^m \left(\frac{z}{r}\right) \left(A_{nm} \cos m\lambda + B_{nm} \sin m\lambda\right)$$

we would have

$$L_{A,nm} = L_{B,nm} = n + 3$$

If U were represented by

$$U = \frac{\mu}{r} \sum_{n,m} \left(\frac{R}{r}\right)^n P_n^m \left(\frac{z}{r}\right) \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda\right) ,$$

the redundant parameters μ and R render the gravitational parameters, C and S, dimensionless. However, we would have

$$\frac{\partial f_D}{\partial C_{nm}} C_{nm} = \frac{\partial f_D}{\partial A_{nm}} A_{nm} ,$$

so that if we hold μ and R fixed we must take

$$L_{C,nm} = L_{S,nm} = n + 3 .$$

In the first approximation we similarly take the length dimension of the drag coefficient to be

$$L_{Drag} = L\left(\frac{m\ddot{r}}{\dot{r}^2}\right) = -1$$

and of the radiation pressure coefficient to be

$$L_{Rad} = L\left(\frac{m\ddot{r}}{r_\odot / |r_\odot|}\right) = 1 \quad ,$$

where $r_\odot$ is the distance to the sun.

Therefore in the equation $\sum_j \frac{\partial f_D}{\partial P_j} L_j P_j = 0$ we use the following values of L_j

Generalized P_j	L_j
Gravit. Harmonic coef. C_{nm} , S_{nm}	n + 3
Initial coordinate of orbit	1
Initial velocity component of orbit	1
Geocentric radius of receiver station	1
Latitude of receiver station	0
Longitude of receiver station	0
Drag coefficient	- 1
Radiation pressure coefficient	1
Velocity of light	1

We disregard all other partial derivatives of f_D with respect to quantities involving length in the first approximation. The generalized parameter set, P, is then the least squares set of parameters augmented only to include the velocity of light, c. The column to be added to the matrix

$$A_{ij} = \frac{\partial f_{Di}}{\partial P_j}$$

in the equations of condition is

$$\frac{\partial f_D}{\partial c} = - \frac{f_D(P)}{c} \quad .$$

If allowance is made for transmission time which depends on c, we use

$$\frac{\partial f_D}{\partial c} = - \frac{f_D}{c} - \frac{f_s}{c^3} [\ddot{r}\cdot(r-r_R) + \dot{r}\cdot(\dot{r}-\dot{r}_R)] \quad .$$

Using the A matrix thus augmented, we have

$$ALP = 0,$$

L being the diagonal matrix with diagonal elements L_i, it follows that we also have

$$BLP = 0,$$

where B is the normal matrix A^TA augmented corresponding to the additional column of A.

This is the BLP relation. In practice, we compute a slightly simplified BLP in which $\frac{\partial f_D}{\partial c}$ is replaced by $- C \frac{\partial f_D}{\partial c}$ and P_c by - 1, and the modified BLP is still zero because the modification multiplies the c-row by -c and leaves the other rows unchanged.

DISCUSSION OF BLP TEST

The relation, BLP = 0, is non-trivial. It furnishes an extensive test of the numerical elements of the normal equations including much of the righthand side through the added column of A. It can detect logical errors, computer errors, manual errors, truncation errors in the integration of perturbation equations needed in A, etc., and it has had some success in these roles. It has even been useful in isolating error sources by operating on narrowed sets of D_i and P_j. The BLP = 0 is a necessary condition on B, but of course it is not sufficient to catch all errors. An error may have an effect on BLP too small to detect in some cases and will have no effect if the associated L_j is zero.

The Doppler BLP test is used in practice as described above. It suffers from small errors due to neglect of very small partial derivatives with respect to some other parameters. These include the following:

semi-major axis of orbits of sun and moon

parameters in the atmospheric density

parameters in the radiation pressure shadow

probably others in tropospheric refraction effects

Their neglect waters the test down somewhat.

In applying the test, almost the first question faced is how close numerically should BLP be to zero. After inspection of the terms $B_{kj}L_jP_j$ it was observed that, for fixed k the dominant term invariably was that for which $P_j = C_{oo}$ of the gravitational potential. Therefore a practical measure of how close the k^{th} component of BLP is to zero is the BLP ratio

$$\frac{(BLP)_k}{B_{k,oo}L_{oo}C_{oo}} = \frac{(BLP)_k}{3B_{k,oo}}$$

How close this ratio comes to zero under different conditions is described next.

A series of experiments was conducted using a normal matrix for a full gravitational field of more than 200 harmonic coefficients as well as for arc and station parameters. The observation span was however limited to two passes, one over each of two stations. The tests were made with various options exercised. In the simplest case, with no bias parameters of the passes, no refraction, no sun and moon, no transmission time, no drag, no radiation pressure, the BLP ratio, on the 14 decimal digit STRETCH computer, was $-.3 \times 10^{-11}$, a very good performance. Curiously, it was even better, $.9 \times 10^{-12}$, with bias parameters considered. It did less well with other options, as shown by the following table in which $(BLP)_k$ is $(BLP)_{oo}$.

Option on	BLP Ratio
None	$-.3 \times 10^{-11}$
Bias	$.9 \times 10^{-12}$
Sun and Moon	$.7 \times 10^{-9}$
Drag (const air density)	$.2 \times 10^{-9}$
Drag (density dependent on h)	$-.3 \times 10^{-7}$
Smaller integration time step	$-.5 \times 10^{-11}$
Transmission time	$-.2 \times 10^{-8}$
All on	$-.3 \times 10^{-7}$

The poor results with transmission time has led to the discovery of errors in the A matrix in allowing for transmission time. This is being repaired. The performance of other options such as drag with constant air density also requires further study.

For matrices generated from 8 day arcs instead of from 2 passes, the BLP ratio runs currently at something under 10^{-5}. The sensitivity to truncation error in the integration and to the transmission time errors in A still need evaluation.

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