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STRENGTH AND USES OF FRESH AND SALT WATER ICE

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OF  
FRESH AND SALT WATER ICE

by

RALPH HANSEN  
AND  
HERBERT LINDST.

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STRENGTH AND USES

OF

FRESH AND SALT WATER ICE

by Ralph Hansen<sup>1</sup> and Kenneth Linnell<sup>2</sup>

I - Uses of Ice in the Arctic

The most important use of both sea ice and fresh water ice in the Arctic, in the present and in the past, has been in aid of transportation. The most famous journeys of such explorers as Peary, MacMillan and others were made over frozen seas, on foot or with sledge and dog team. In many Arctic areas frozen rivers, lakes and bays are regularly used for winter travel between land points, frequently shortening distances by many times over those required for overland travel. Ice makes possible the passage in winter of tractor-drawn supply trains over water bodies, swamps and bogs which in other times of the year would be impassable.

Even railroads have been constructed on floating ice. The Russians have been particularly persistent in developing this type of construction. A railroad line was, for example, constructed on the ice of Lake Ladoga in the siege of Leningrad in World War II, supplementing motor truck supply routes also laid out over ice.

The value of ice in assisting military operations by providing

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relatively easy means of crossing rivers, lakes, swamps and other water areas is obviously very large in Arctic areas.

The ice of the Arctic provides a surface to support floating weather and scientific stations, a notable example of which was the Russian Papanin Expedition which was established on the ice at the North Pole in May 1937 and for nearly a year gradually drifted southward, until removed from the ice at a point off the coast of East Greenland in February 1938.

Various possibilities for increasing the structural usefulness of ice remain to be explored. A start was made in this direction during World War II in an investigation carried out by Canadian and British investigators (7)\*. The object was to devise a means of constructing floating airdromes of ice, which would save critical materials required in other methods of creating landing facilities. The successful prosecution of the war in Europe resulted in abandonment of the project, but not before it had been found that by means of an admixture of about 14 per cent wood pulp the compressive strength of ice could be increased to 1100 psi and its tensile strength to 700 psi. Other means for using ice for various structural purposes have been suggested but have received little or no actual trial.

Ice serves as a water supply source in the Arctic. In some locations, surface water sources become so completely frozen in the winter that the only practical source of water is from the melting

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\* Figures in parentheses refer to the bibliography at the end of this article.

of ice or snow. In salt water areas, fresh water is obtainable by melting old sea ice; in summer, melting of thin ice forms pools of fresh water on the surface of the ice.

## II - Strength Properties of Ice

General Characteristics of Ice - Ice may be described as a highly viscous material having characteristics of a solid. Various observers have reported that the range of stress magnitudes within which ice is truly elastic is quite small. Although it fractures like a brittle material under breaking loads, ice will flow, or deform gradually, under non-failure loads, the rate of deformation under a given stress being higher at higher temperatures. Under favorable conditions for "creep", deformation may be quite rapid. Ice will also melt under pressure; the effect is most pronounced with ice temperature at  $32^{\circ}\text{F}$ . The pressure required to cause melting increases very rapidly with only a small drop in temperature below  $32^{\circ}\text{F}$ .

It will be apparent from these ice characteristics that it is not easy to express the action of ice under stress by one or two simple figures. For a material such as steel, working stresses may very easily and accurately be established in relation to its yield point. For ice the yield point is somewhat obscure, and is apparently below practical working stresses; as yet, the only really useful orientation point on the stress-strain curve is the

failure point. When a steel structure is designed, material of given characteristics is specified with assurance that the actual steel used will vary an insignificant amount from the assumed properties. With ice, however, the material which nature has formed must be used, and its strength will vary considerably, in accordance with the chance variations of the several strength-influencing factors involved. Even cakes of artificial ice manufactured apparently under identical conditions show wide spreads of results.

Published Test Results - Ranges of some ice strength results are shown in Table 1\*. These results represent tests under a wide variety of test methods and conditions on many types of ice (including artificial ice) and on specimens of a wide range of shapes and sizes. Most of the figures are individual maximum or minimum test values reported by the various investigators.

Table 1  
Ranges of Ice Strength Test Results

Type Test	Values Reported, psi.
Compressive Strength	70 to 1800
Tensile Strength	36 to 223
Flexure Strength	44 to 311
Shear Strength	68 to 353

Except for strengths of single crystals, the minimum values of the

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\* Principally from references (2), (4), (5) and (6) and from original data.

above ranges should actually approach zero, corresponding to the condition when ice is rotting or candied and is able to support virtually no load. Possibly the compressive strength values show the greatest range in part because more investigators have used this relatively easy test than the other types.

Effect of Intra-Crystal Structure of Ice - When an ice sheet is formed on the surface of a body of fresh water, the ice crystals form with their optic axes vertical; that is, with their basal planes horizontal, and grow downward with the added ice having its basal planes also parallel with the ice surface. This influences the over-all strength since, as reported by McConnell from his study of the way ice yields to stresses, the individual crystal behaves as if it consists of an infinite number of indefinitely thin sheets of paper, normal to the optic axis, fastened together with some viscous substance which allows one to slide over the next with great difficulty; the sheets "offer no resistance to bending, but utterly refuse to stretch except, of course, elastically" (10). Thus, on basis of the structure within the individual ice crystals themselves, and apart from other structural characteristics, the properties of ice determined from tests may be expected to vary according to the direction of orientation of the applied stresses between the horizontal and vertical, relative to the original surface of naturally formed ice.

Effect of Impurities in Ice - Presence of relatively small amounts of dissolved or suspended matter in water usually results in ice which is weaker and/or which deteriorates more readily under the effects of warmer temperatures and sunlight.\*

In any water body found in nature, there will be found some dissolved minerals and other impurities. Sea water at one extreme is an obvious example, but even the clearest natural fresh water body is not without impurities. When each ice crystal forms, it tends to reject these impurities, which then form a layer of concentrated impurities about the crystal. When adjoining crystals meet, the impurities form a layer between the crystals which has a lower melting point than that of the crystals themselves. This may greatly affect the physical properties of the ice even though the amount of the impurities is very small. Melting will always begin at these boundaries between the ice crystals and at a lower temperature than 32°F. Fresh water ice becomes rotten as thawing, starting at the inner crystal boundaries, separates the ice into separate needles or columns. This ice is said to be "candied". Thus, as a result of this crystal structure in bulk, the physical properties of ice may vary widely, especially when its temperature is near the melting point, depending on the amount of impurities in the ice, the temperature, the age of the ice, and the length of time that near-freezing temperatures have persisted.

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\* Additions of considerable amounts of such materials as wood pulp are reported, however, capable of increasing strength, as noted previously, and also may serve to slow melting (7).

In formation of sea ice, the effect of the salt content is, of course, very pronounced. Freezing of the water portions does not begin until a temperature of  $28.6^{\circ}\text{F}$ . is reached in normal sea water at a salinity of 35 parts per thousand. The structure produced is porous, containing pockets of brine from which solid salt crystals begin to precipitate when the ice cools to about  $17^{\circ}\text{F}$ . (9). The structure and, consequently, the strength properties of sea ice improve gradually, with time, as the concentrated brine drains and as the salinity of the ice consequently becomes gradually lower. Newly formed salt water ice of relatively high salinity is flexible and elastic, as compared to ice formed on fresh water bodies, (which is characteristically brittle), and it does not cackle. Salt water ice more than one year old is much tougher and stronger than young sea ice, but its surface is much more likely to be rough. It is reported that when old sea ice becomes sufficiently modified so that it approaches the composition of fresh water ice, it too will cackle.

From Russian data, young sea ice requires about  $1\frac{2}{3}$  times the thickness of old sea ice to carry the same load, but this rule is very generalized. Lake ice is usually assumed 2 to 3 times stronger than sea ice, although its brittleness does not permit it to stand as much bending without cracking. River ice is generally assumed not quite as strong as lake ice.

Any entrapped matter such as silt, clay or plankton (3)

tends to weaken ice in the warmer periods because it absorbs solar radiation and tends to cause internal melting because of the heat produced thereby.

Ice containing much entrapped air tends to be weaker than clear essentially air-free ice. Tests by the Soils, Foundation and Frost Effects Laboratory\* in 1946-47 on 6 x 6 x 18-inch beams of artificial ice showed about 12 per cent reduction in flexural strength on beams cut from cloudy ice manufactured from ordinary tap water, as compared with beams cut from clear air-free ice. Tests were performed at 10° to 13°Y.

Effects of Horizontal Stratification in Ice - The strength of ice may be affected by development of horizontal laminae of variable properties. One type of horizontal stratification development is that which occurs in coastal areas where river and stream inflows and tidal effects cause variations in the salinity of the water during the ice formation period. An influx of relatively fresh water under the ice surface will result in rapid freezing to this surface, which may have cooled to as low as 25.6°F. when normal salt water was present under the ice. The same effect occurs in the Polar Seas during the summer thaw periods when melt water from the surface sinks down through holes and freezes on to the cold under-surface of the ice pack. Stratification

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\* The Soils, Foundation and Frost Effects Laboratory of the New England Division, Corps of Engineers, U. S. Army, has carried out investigational work on snow and ice for the Airfields Branch, Engineering Division, Office of Chief of Engineers, Department of the Army, Washington, D. C.

in the lower part of the ice sheet, where critical tensile stresses occur when the ice is loaded, may be expected to have far greater effect on the over-all bearing capacity than laminations which develop in the upper half of the sheet.

In the polar ice pack such less regular horizontal structure layering occurs when pressure causes rafting and buckling of the ice sheet.

Effect of Vertical Cracks - Cracking of the ice sheet under the effect of temperature stresses results in a pattern of vertical defects or discontinuities distributed over the ice sheet. These do not ordinarily extend more than part way through the depth of the ice. Although they are conspicuous in the upper surface of the sheet at low temperatures and may be readily visualized as a possible cause of weakness, static load tests by the Soils, Foundation and Frost Effects Laboratory in 1947 registered essentially no observable effect of such cracks on the bearing capacity or failure action of the ice under a load. Cracks formed by the load showed very little tendency to follow the existing partial cracks. However, these tests involved only single load applications, and it is reported that under heavy traffic temperature cracks do have an effect to reduce the load bearing capacity of the ice sheet. Cracks which form from the surface downward will tend to become progressively deeper under the effect of traffic. Cracks which pass all the way through the ice sheet seriously lower the load capacity and are especially hazardous if they occur under a

snow blanket, as the snow may delay their refreezing and hide them from view. Cracks which form on the lower surface of the ice sheet due to loading tend to be self-healing if the ice temperature is below freezing, as they become instantly filled with water provided, of course, the load is not so great as to cause break-through.

Temperature during Ice Formation - When ice forms very rapidly at extremely low temperatures there is less opportunity for dispersion of the impurities which are rejected by the ice crystals. Young sea ice formed at  $14^{\circ}\text{F}$ . has entrapped about 5 parts per 1000 salt, but that formed at  $-40^{\circ}\text{F}$ . has 10 to 15 parts per 1000. Differences in structure (and consequently of strength) of the ice would be expected to result from such differences in actual volume of included matter and the manner of its inclusion. It may be noted that the more concentrated brine surrounding the freezing crystals during fast freezing would tend to cause formation of the individual ice crystals at still lower temperatures than in ordinary sea water, with possible effects upon the ice properties.

Residual Stresses - Internal stresses in ice specimens due to unadjusted temperature effects, to stresses introduced in obtaining and preparing test specimens, or to external loadings of the ice sheet are suspected to be in part responsible for variability of reported test results.

Effect of Temperature during Test - Temperature has a very pronounced effect upon the strength properties of ice, as shown in Table 2.

Table 2

## Effect of Temperature on Strength of Fresh Water Ice

Temperature Test, °F.	Strength Test Results, psi			Direction of Loading in Relation to Crystal Optic Axes	Remarks	Investigator
	Compression	Flexure	Shear			
28 14 2	300 693 797			Perpendicular	3" to 5" square specimens of St. Lawrence River ice 5" high. Stress increased at approximate rates of 20 to 60 psi per second.	E. Brown (4)
28 to 30 14 to 16		156 240		Perpendicular *	3" wide x 2" deep beams of St. Lawrence River ice, 41" span, loads applied at 14" from supports. Stress increased in increments at various rates averaging between 0.175 and 1.4 psi/sec.	E. Brown (4)
28 to 30 14 to 16		184 214		Parallel *		
25 13 -8		196 203 214		Essentially Parallel * **	6" x 6" beams of air-free artificial ice, 18" span, third point loading, stress increased at constant rates between 1.9 and 2.6 psi/sec.	Soils, Fdn. & Frost Effects Laboratory
32 2 -9 -10		180 - 256 -	94 90 - 115	Parallel *	Artificial ice Flexure specimens 2 1/2" to 6" deep, 2 1/2" to 5" wide, 15" to 30" long. Shear specimens 2 1/2" sq. and 6" long. Specimens failed in one to three minutes.	Wilson and Horeth (15)
23 14 5 -4 -13 -22 -31 -40 -46 -51 -67 -76	220 252 354 394 488 506 581 615 665 694 725 779			Parallel. (Normal to the natural ice surface).	5 cm. cubes of fresh water ice. Lower platten moved at constant rate of 0.1 mm/sec. Yield of upper load-measuring platten unknown.	Vitman and Shandrikov (12)
-10° to 30° (temperature effect found negligible).			98 114	Parallel Perpendicular	3" x 3" square cross-section blocks of river ice loaded in double shear in direction of a diagonal. No numerical value quoted for loading rate.	Finlayson (6)

\* In flexure tests, directions of tensile stresses from which failure develops are actually at right angles to direction of loading.

\*\* Some curvature of crystal structure existed because ice was frozen in containers from outside toward center, but this was minimized so far as possible in cutting out the test beams.

Vitman and Shandrikov conclude from their test results, shown in Table 2, that the average compressive strength of ice increases more than four times as the temperature decreases to  $-76^{\circ}\text{F}$ . (12). The data on change of flexural strength with temperature also shows an increase with drop in temperature, though not as rapid as for compressive strength. Minlayson found that temperature has very little if any effect on the strength of ice in shear (6), which is in agreement with the results of Hereth and Wilson given in Table 2 above. Under a given rate of increase of stress, the rate of deformation of the specimen rises (while the ultimate strength drops) with an increase in ice temperature; thus, ice, acting like a highly viscous material, flows more readily at higher temperatures.

Effect of Rate of Loading on Strength Tests - There appears to have been relatively little practical investigation to determine the proper rates of load application for use in correlating test values with the load supporting capacity of ice. Some investigators have found ice strength results to be negligibly dependent on the rate of loading; others have found a marked effect. In some cases the differences may be due to the fact that the same ranges of rate of loading were not investigated.

Effect of Orientation of Test Specimen - Since the bending stresses which act in an ice sheet under load and which control in part the bearing capacity of the ice (the other factor being buoyancy) act in a direction parallel to the ice surface, analyses of bearing capacity should, theoretically at least, depend on tests

in which specimens are stressed with their crystal axes so as to take this into account. Practically, this offers difficulties, and if the bearing capacity of thick ice is to be determined from tests on cores recovered from borings into the ice, it will probably be necessary to establish an empirical relationship between strengths parallel to and at right angles to the surface of the ice sheet.

Size and Proportions of Test Specimens - As in testing concrete or soils, the size and proportions of test specimens may be expected to influence the results of tests on ice. Certainly a test on a small portion of a single "needle" would not give the same results as a test on a collection of full length "needles". What the minimum desirable size of specimen should be would be expected to vary with the type and condition of ice. However, there seems to have been little productive study given to this phase of ice testing as yet.

### III - Load Bearing Capacity of Ice

Records of Actual Ice Loadings - Records of the innumerable, and in many areas regular, actual ice crossings which have been made with motor vehicles, tractors, sleds, railroad trains, etc. appear to be a rather meager source of data on the supporting capacity of ice, for several reasons. First, when such crossings have been made, there appears to have only rarely been any attempt to record the

actual conditions of the crossing, such as, ice thickness and quality, weight of machine, temperature, size and shape of loaded area, presence of snow cover, etc. Secondly, when such observations have been made, they have been incomplete since all the factors involved may not have been recognized. Third, there seems to have been little interest in making such records generally available in publications. Empirical rules have been developed on basis of experience with crossings, but it is difficult to trace these back to original records.

Historical Military Rules - There are a number of old rules relating to the load bearing capacity of ice which mostly have their origin in military engineering and are the result of observation and experience in the movement of troops, equipment and supplies across frozen bodies of fresh water. The data in Table 3 are a modernized and improved version of these rules:

Table 3

Load Capacity of Ice

Load	Thickness of Ice	Minimum Interval
	Inches	Feet
Single rifleman on skis or snowshoes .....	1-1/2	16
Infantry in single file, 2-pace distance ....	3	23
Infantry columns, single horses, motorcycles, unloaded sleds or motor toboggans .....	4	33
Single light artillery piece, 1/4-ton truck, 4 x 4 .....	6	49
Light artillery, passenger cars, medium 1-1/2-ton trucks with total load of 3-1/2 tons .....	8	65
2-1/2-ton trucks, light loads .....	10	82
Closed columns of all arms except armored forces and heavy artillery .....	12	96
Armored scout cars, light tanks .....	14	115
20-ton vehicles .....	16	131
45-ton vehicles .....	24	164

Source: "Operations in Snow and Extreme Cold", U. S. Army Basic  
Field Manual FM 70-15, Reference (1).

The above rules should be considered to apply to fresh water ice  
of good color and soundness, away from unsupported edges. The thick-  
ness measurements should disregard any soft or poor quality ice on  
the top or bottom of the ice sheet.

Moskatov's Empirical Method - Moskatov (11) has given a  
procedure for determining required ice thicknesses for aircraft

on skis which he calls the "Method of Analogy". Formulas derived by this method are given in Table 4. The basic formulas for aircraft on skis have been adapted to the case of wheels by use of a factor of 20 per cent, which is used by the Russians.

Table 4  
Minimum Required Ice Thickness in Inches

Type of Ice	Aircraft on Skis	Aircraft on Wheels
Lake Ice	$t = \frac{27}{8} \sqrt{P}$	$t = \frac{81}{20} \sqrt{P}$
River Ice	$t = \frac{15}{4} \sqrt{P}$	$t = \frac{9}{2} \sqrt{P}$
Old Sea Ice	$t = \frac{27}{4} \sqrt{P}$	$t = \frac{81}{10} \sqrt{P}$
Young sea Ice	$t = \frac{81}{8} \sqrt{P}$	$t = \frac{243}{20} \sqrt{P}$

Source: "Airplane Landings on Ice", by K. A. Moskatov. Reference (11).

P is the gross weight of aircraft in tons and t is the ice thickness in inches. The formulas are said to apply only for ice formed and maintained at a temperature below 16°F. At higher temperatures 25 per cent greater thickness is required.

The above formulas are simple and they indicate that the carrying capacity of the ice varies as the square of the ice thickness; this is confirmed approximately by elastic theory analysis.

Design curves from the above formulas are shown on Plates

Nos. 1 and 2 where they are designated as "Russian (Moskatov) Empirical Analysis".

Elastic Theory Analysis - If a stationary load is placed upon the surface of a floating ice sheet which extends out to a large distance on all sides from the load, a deformation of the ice sheet occurs which is a maximum under the loaded area and which extends outward in all directions, decreasing in amplitude with distance from the center. When the deformation occurs, a buoyancy force is caused to act upward on the bottom of the ice sheet. This force is at any point directly proportional to the deflection at that point as long as the ice does not become submerged, with water flooding on top. At the same time, the bending strength within the ice sheet itself is mobilized. The buoyancy and bending resistance forces combine to provide the total supporting force for the applied load.\* Since under static loading the buoyant force of the underlying water at any point acts in exact proportion to the amount of deflection of the ice sheet at that point, the latter may be assumed to have a perfectly elastic support. If we may also assume the ice sheet has properties approximating those of an elastic slab, then the theories of elasticity may be applied to compute stresses and deformations in the ice sheet and to determine thicknesses required for given conditions of loading.

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\* When, as may occur along a shore line, the water drops away from under the ice leaving it suspended, the ice sheet will have at that location greatly reduced load supporting capacity.

As pointed out by Volkov (13), static loading of floating ice is more critical than impact loading. Under an impact force, the ice is simply compressed between the applied load and the underlying water; in order to develop bending stresses, a definite volume of water must be displaced from under the ice, near the area of load application, and under a sharply applied load the time required for this movement to occur is insufficient. Shearing action is usually not critical. Experience has shown that thin ice may be crossed by a moving load when a stationary load would break through and also that thin ice may sustain a stationary load for some time, then give away. Thus, load capacity studies may be based upon the static loading condition, except as it may be necessary to apply modifications for wave action under moving loads and for fatigue and weakening under repeated flexings.

For the condition of static loading, a comprehensive theoretical analysis of the bearing capacity of floating ice <sup>was</sup> ~~has~~ been developed <sup>in 1947</sup> by Mr. H. F. Shea of the Soils, Foundation and Frost Effects Laboratory by adapting to the case of an ice sheet formulas developed by Dr. H. M. Westergaard for determining the stresses at the interior portion of an elastically supported slab (14). In their general form, the formulas are not adapted to giving quick answers. However, the formulas permit analysis of a variety of combinations of loading patterns and intensities which could not otherwise be approached. These formulas are as follows:

a. Uniformly Loaded Circular Areas.

(1) Inside the area of loading:

$$\left. \begin{matrix} f_x \\ f_y \end{matrix} \right\} = \left( \frac{0.6517P}{t^2} \right) \left[ 1.6713 + 3/4 \log_e t - \log_e B + \frac{0.012B^2}{t^{3/2}} - \frac{r^2}{450B^2} \right] \quad \left( \begin{matrix} r \text{ as in} \\ \text{"roost"} \end{matrix} \right)$$

$$+ \left( \frac{0.1516P}{t^2} \right) \left[ \left( \frac{r^2}{450B^2} \right) (\cos 2\theta) \right] \quad (1)$$

The maximum stress due to an individual load will occur under the center of the loaded area, at  $r = 0$ ; for this case, and discarding the term  $\frac{0.012B^2}{t^{3/2}}$ , which is normally negligible, the above equation reduces to the following useful expression:

$$\left. \begin{matrix} f_x \\ f_y \end{matrix} \right\} = \left( \frac{0.6517P}{t^2} \right) \left[ 1.6713 + 3/4 \log_e t - \log_e B \right] \quad (1a)$$

(2) Outside the area of loading:

$$\left. \begin{matrix} f_x \\ f_y \end{matrix} \right\} = \left( \frac{0.6517P}{t^2} \right) \left[ 3.8794 + 3/4 \log_e t - \log_e r \right]$$

$$+ \left( \frac{0.1516P}{t^2} \right) \left[ \left( 1 - \frac{225B^2}{2r^2} \right) (\cos 2\theta) \right] \quad (2)$$

$$c = \sqrt{1.6d^2 + t^2} - 0.675t \text{ when } d < 1.724t$$

$$c = d \text{ when } d \geq 1.724t$$

b. Uniformly Loaded Elliptical Area - Where the loading

is uniformly distributed over an elliptical area (or where a uniformly loaded elliptical area may be reasonably assumed) the following stress equations apply:

(1) Inside the area:

$$\left. \begin{aligned} f_{x/y} \\ f_{y/x} \end{aligned} \right\} = \left( \frac{0.6517P}{t^2} \right) \left[ 5.0372 + 3/4 \log_e t - \log_e (a+b) - \frac{x^2}{a(a+b)} - \frac{y^2}{b(a+b)} \right] \\ \pm \left( \frac{0.1516P}{t^2} \right) \left[ \left( \frac{b-a}{a+b} \right) - \frac{2ab}{(a+b)^2} \left( \frac{x^2}{a^2} \pm \frac{y^2}{b^2} \right) \right] \quad (3)$$

(2) Outside the ellipse, for  $y = 0, x \geq a$ ,

$$\left. \begin{aligned} f_{x/y} \\ f_{y/x} \end{aligned} \right\} = \left( \frac{0.6517P}{t^2} \right) \left[ 5.0372 + 3/4 \log_e t - \log_e (x + \sqrt{x^2 - a^2 + b^2}) \right. \\ \left. - \frac{x}{(x + \sqrt{x^2 - a^2 + b^2})} \right] \pm \left( \frac{0.1516P}{t^2} \right) \left[ \left( \frac{a^2 + b^2}{b^2 - a^2} \right) \right. \\ \left. - \frac{\log_e^2 x}{(b^2 - a^2)(x + \sqrt{x^2 + a^2 - b^2})} \right] \quad (4)$$

(3) Outside the ellipse, for  $x = 0, y \geq b$ ,

$$\left. \begin{aligned} f_{x/y} \\ f_{y/x} \end{aligned} \right\} = \left( \frac{0.6517P}{t^2} \right) \left[ 5.0372 + 3/4 \log_e t - \log_e (y + \sqrt{y^2 + a^2 - b^2}) \right. \\ \left. - \frac{y}{(y + \sqrt{y^2 + a^2 - b^2})} \right] \pm \left( \frac{0.1516P}{t^2} \right) \left[ \left( \frac{a^2 + b^2}{b^2 - a^2} \right) \right. \\ \left. - \frac{\log_e^2 y}{(b^2 - a^2)(y + \sqrt{y^2 + a^2 - b^2})} \right] \quad (5)$$

In these formulas the notation is as follows:

$f_x, f_y$  = Bending stresses in p.s.i. at surface of ice at points  $x, y$  on co-ordinate axes.

$P$  = Total applied load in pounds.

$t$  = Thickness of ice in inches.

$e$  = Base of natural logarithms = 2.7182

$B$  = Ratio of radius of circular area to radius of standard bearing plate (15 inches) =  $\frac{c}{15}$

$r$  = Radial distance in inches from center of circular area to point of stress investigation.

$\theta$  = Angle between x-axis and line drawn between point of stress investigation and center of circle.

$a$  = Minor radius of elliptical area in inches.

$b$  = Major radius of elliptical area in inches.

$c$  = Substitute radius of loaded area in inches.

$d$  = Actual radius of loaded area in inches.

$a$  and  $b$  are further identified by the equation for an ellipse,  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

c. Example. To show the stresses induced by a 60,000-pound load transmitted to ice 50 inches thick by a B-29 main dual wheel unit assuming load is distributed equally over two circular areas.

(1) Given the following data:

Area (for one circular area) = 370 sq. inches

Radius of circle  $d$  = 10.9 inches

Distance between wheel centers = 37.3 inches

$$c = \sqrt{1.6d^2 + t^2} - 0.675t$$

$$P = 30,000 \text{ pounds}$$

$$t = 50 \text{ inches}$$

(2) Using formulas (1) and (2) for the case of the circular areas:

$$f = 37 \text{ p.s.i. when } x = 0 \text{ and } y = 0$$

$$f = 26 \text{ p.s.i. when } x = 37 \text{ p.s.i. and } y = 0$$

$$\text{total } f = 37 + 26 = 63 \text{ p.s.i.}$$

In the above formulas, the following constants were assumed, selected from published results, and intentionally chosen to give results on the conservative side:

$$\text{Poisson's Ratio for Ice} = 0.365$$

$$\text{Modulus of Elasticity for Ice} = 1,294,000 \text{ p.s.i.}$$

Observed values of Poisson's Ratio are reported variously from 0.25 to 0.38 and the value adopted above for the computation is, therefore, near the higher limit. Stress-strain moduli have been reported by various investigators from under 70,000 to over 1,500,000 p.s.i. Recent flexure tests by the Soils, Foundation and Frost Effects Laboratory showed a tangent modulus value of 610,000 p.s.i. on a single specimen of artificial ice at 10°F. up to a stress of about 25 p.s.i.; near the ultimate failure stress of 212 p.s.i. the secant modulus (related to the origin of the stress-strain curve) was approximately 190,000 p.s.i. A value of approximately 400,000 p.s.i. was also determined from the deflection of a naturally formed

fresh water ice sheet 5.4 inches thick in a 27-foot tank when loaded at approximately  $10^{\circ}\text{F}$ . Brown has also presented considerable data on effects of temperature, and of rate and range of loading, on stress-strain moduli (4), which indicate generally lower values than the one adopted above for computation of the equations. Reduction of assumed value for either Poisson's Ratio or Modulus of Elasticity in the equations will result in an indicated increase in load capacity.

Once the thickness of ice required for a given load, size of area and factor of safety has been determined, the thickness required for a different load on the same loaded area and with the same factor of safety may be computed from the relation that the thickness required varies directly with the square root of the load.

Preliminary Elastic Theory Ice Thickness Curves - Curves derived from the elastic theory equations as presented are shown on Plates Nos. 1 and 2 for aircraft with wheels and designated thereon by the phrase "Elastic Theory Analysis".

The curves of Plate Nos. 1 and 2 have been computed with the following ice strengths, assumed as conservative values for ice under the temperature conditions noted on the plates:

Hard, sound, fresh water lake ice = 150 p.s.i.

Old sea ice (over 1 year old) = 75 p.s.i.

Similar curves could be computed for other types of loadings, such as for tractors, sleds, aircraft with skis, and even structure

footings. Plates Nos. 1 and 2 show results of a few actual loadings by planes and in tests. The results are marked (o).

Comments on Elastic Theory Method and Thickness Curves -

Although as yet almost unproven, the elastic theory equations which have been given are considered the most accurate theoretical approach available. The constants used in the equations are subject to considerable modification as more is learned about the properties of ice. It may be that eventually more than one set of constants will have to be used.

For theoretical analysis purposes the ice sheet has conventionally been assumed to be in equilibrium under external forces and unstressed. This is likely to be far from the truth in the upper part of the slab. The stress pattern in the upper part of an unloaded ice sheet due to temperature and lateral forces must be very complex. That the stresses may be high is shown by the cracking which occurs due to temperature alone. Fortunately, however, elastic theory analysis shows that under an applied vertical load the critical stresses act in the lower portion of the slab where temperature stresses are at a minimum.

It should be emphasized that the curves apply at the interior of an ice sheet; near the edge of a crack or open water, the load supporting capacity can be expected to be much less.

It is also emphasized that the curves plotted on Plates Nos. 1 and 2 from elastic theory are preliminary and are necessarily

over-simplified. It is obviously not possible to adopt one strength value for fresh water ice and one for salt water and expect that all ice in all latitudes and locations will conform to either one or the other of these standards under all the wide variety of conditions of salinity, temperature, and other factors which will be encountered. Rather, the curves are guides to the approximate answer, based on conservative strength assumptions. From the small experience acquired to date, it is believed that the curves for factor of safety of 2.0 represent amply safe thickness values for individual loadings under the limitations stated on the curve sheets. Below the latter curves is a Zone of Uncertainty, extending to below the curves from the Russian (Moskatov) Empirical Analysis. It is this Zone of Uncertainty which future studies of the bearing capacity of ice sheets should explore. Eventually, there will probably be not just two but a considerable number of curve sheets to cover the many possible combinations of ice strength, temperature, salinity, intensity of use and other variables. With such curves and with the results of quick field tests of the characteristics of the ice in question, using test apparatus of types now beginning to be developed, accurate numerical evaluations should be possible of the load bearing capacity of ice. However, for the present, prudent procedure in critical cases is to weigh together the empirical rules and formulas, the elastic theory equations and the prompting of judgment and experience.

Effects of Moving Loads - Russian investigators have observed the possibility of danger from development of resonance in waves created in the ice sheet under moving loads and have actually encountered conditions in which several machines in motion and passing each other in both directions set up such wave action that the ice broke (8). They have reported that such occurrence becomes especially dangerous when the machines are travelling at speeds corresponding to those of the propagation of the ice waves and that in such a case, a single vehicle can produce sufficiently strong resonant vibrations to cause the ice to break. The Russians have prepared rules for speed and spacing of vehicles to avoid this effect. On ice about 24 inches thick, over water of the order of 16 to 33 feet deep, the Russians have found that at speeds between 3 and 9 miles per hour a depression of the ice under the vehicle moves at the same speed as the vehicle. At speeds above about 12 miles per hour, wave vibrations are formed which spread far to the sides of the path, at speeds which on this lake were found to be 19 to 25 miles per hour. They concluded that on this lake initial speeds of about either 15 or 25 miles per hour should be maintained and that speeds intermediate between these rates should be avoided. Overtaking of one machine by another should also be avoided, and spacings between vehicles varying from 200 feet to 650 feet and more were recommended, depending on the speed of movement and the strength of the ice. Particular caution was advised near shores where complex wave effects may occur.

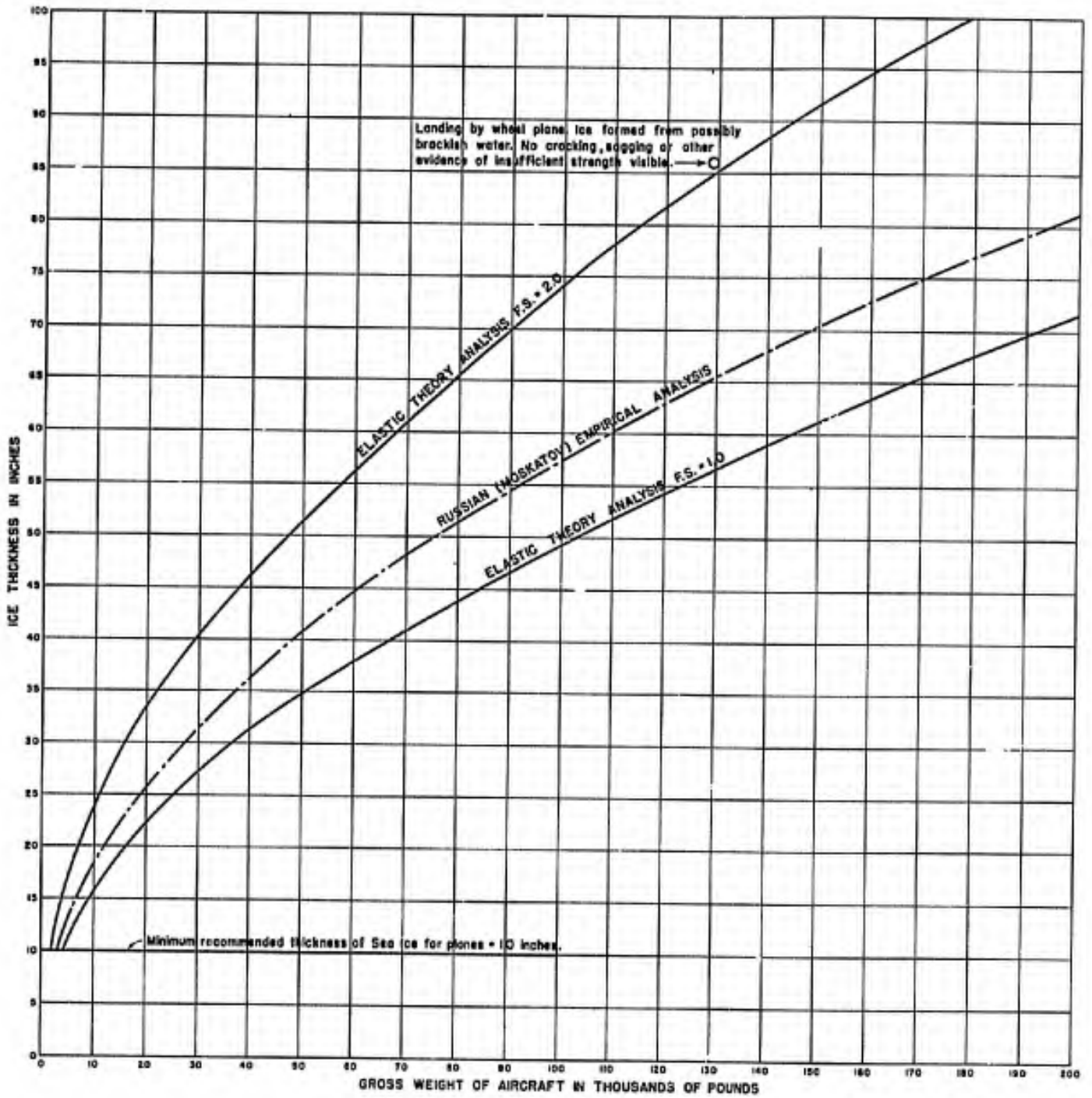
Experience indicates that repetition of loading under extensive operations causes ice to become weakened. Under such conditions, a desirable procedure is to provide sufficient area, or alternate routes, so as to permit frequent changes in traffic lanes. Travelling parallel to major cracks must be avoided at all times.

It is also reported that ice will break up much more readily under vehicles with low speed motors than under those with high-speed motors, because of the differences in frequency and intensity of vibrations transmitted to the ice.

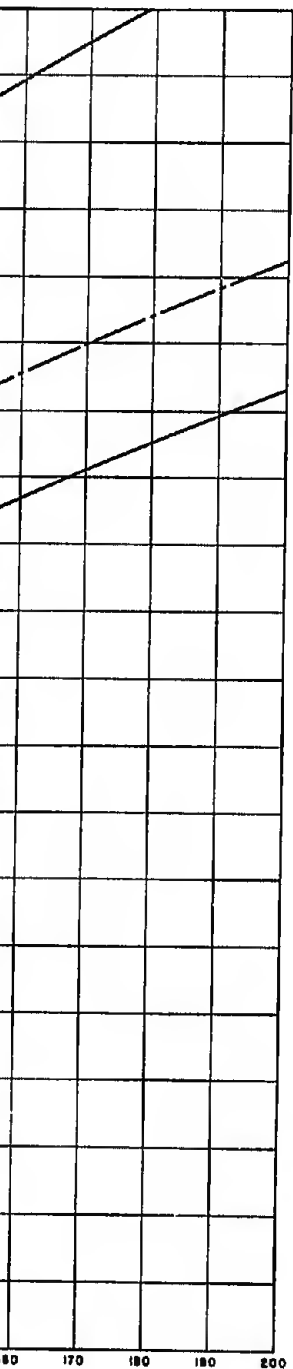
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A.



NOTES

The curves shown are for old sea ice at surface temperatures below about +16°F. At higher air temperatures, indicated thicknesses should be increased by approximately 25 per cent for equal factors of safety. At temperature near melting point curves should be considered inapplicable. For young sea ice, scanty Russian data indicates that increased thicknesses may be required up to about  $2/3$  times thicknesses specified for old ice.

Indicated factors of safety are for safety against ice cracking under static loads applied in the interior of an extensive ice sheet, at a considerable distance from an open crack (an unhealed crack passing completely through ice thickness) or from open water. Data is not at present available for sea ice on relation between load required to produce initial cracking and load required to produce complete break-through.

For factors of safety other than those given (1.0 and 2.0) it will be approximately correct to assume a design load equal to the actual load multiplied by the desired safety factor and then to use this modified design load to obtain required ice thickness from the curve for factor of safety = 1.0.

For wheel loads at edge of an open crack or at edge of open water the ultimate load capacity is estimated to be approximately  $1/3$  to  $4/10$  of the load value indicated by the curve for factor of safety = 1.0.

Design curves are based on U.S.A.F. wheel load data and average minimum spacing between wheels. For tire pressures higher than usual for weight of plane, and for wheel spacings unusually close, greater minimum thicknesses will be required.

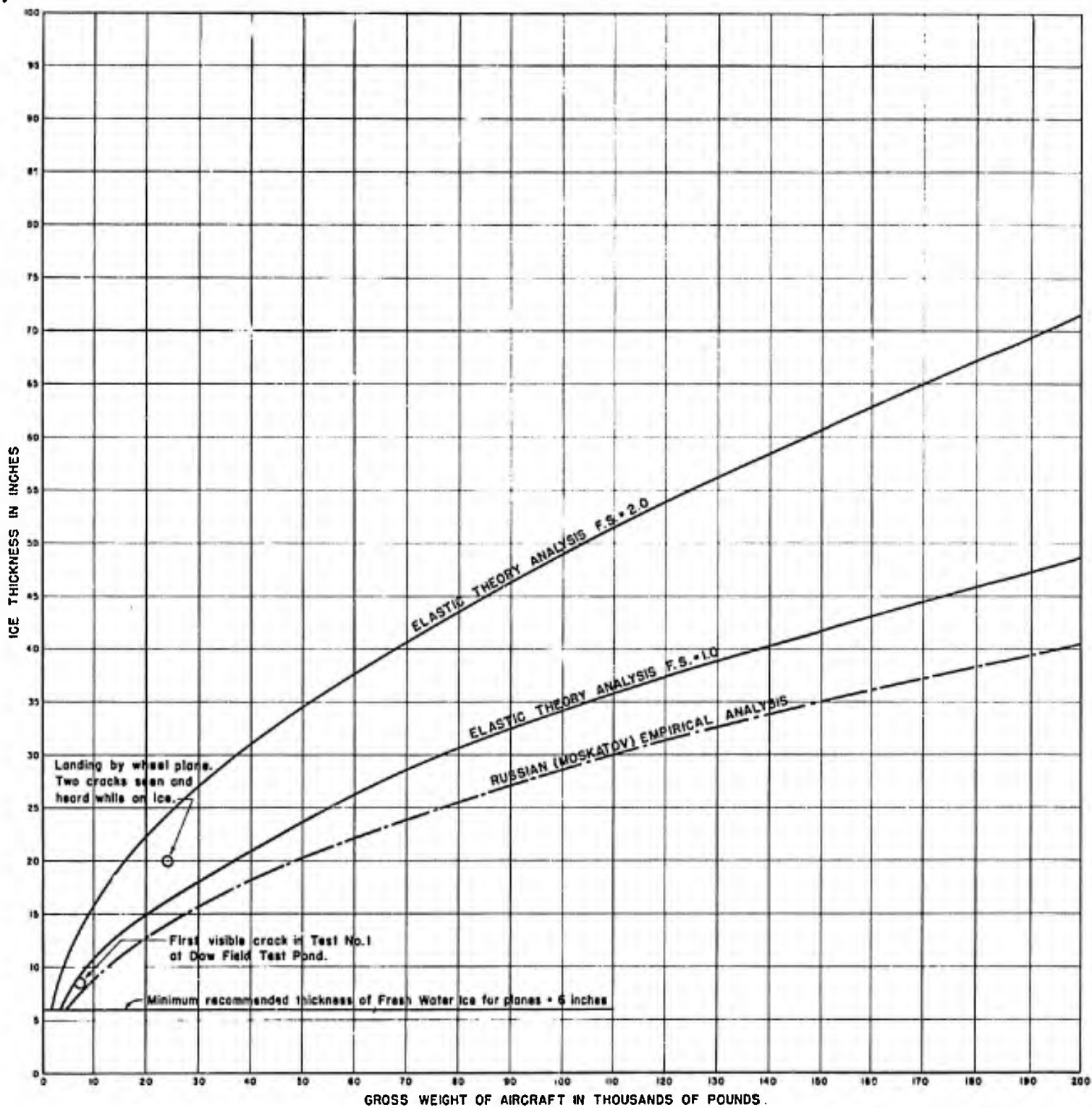
PRELIMINARY-SUBJECT TO VERIFICATION

BEARING CAPACITY OF OLD SEA ICE  
FOR  
AIRPLANES WITH WHEELS

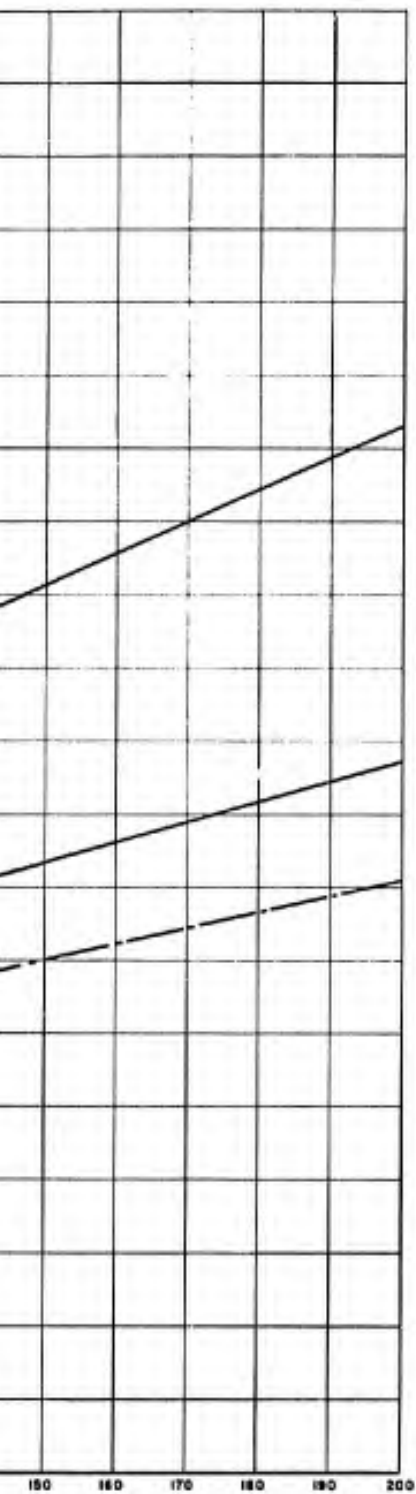
JANUARY-1949

PLATE NO. 1

B.



A.



B.

NOTES

The curves shown are for good quality fresh water lake ice at surface temperatures below about +16°F. At higher air temperatures, indicated thicknesses should be increased by approximately 25 per cent for equal factors of safety. At temperatures near melting point curves should be considered inapplicable. Fresh water ice should not be used after it has started to candle, that is, to separate into long vertical needles.

Approximately 12 per cent greater thickness should be required for river ice than for lake ice under otherwise identical conditions.

Indicated factors of safety are for safety against ice cracking under static loads applied in the interior of an extensive ice sheet, at a considerable distance from an open crack (a crack passing completely through ice thickness) or from open water. Ultimate load capacity for complete break-through under such interior loading condition may be found considerably greater than load required to produce initial cracking, if temperature is considerably below freezing.

For factors of safety other than those given (1.0 and 2.0) it will be approximately correct to assume a design load equal to the actual load multiplied by the desired safety factor and then to use this modified design load to obtain required ice thickness from the curve for factor of safety = 1.0.

For wheel loads at edge of an open crack or at edge of open water the ultimate load capacity is estimated to be approximately 1/3 to 4/10 of the load value indicated by the curve for factor of safety = 1.0.

Design curves are based on U.S.A.F. wheel load data and average minimum spacing between wheels. For tire pressures higher than usual for weight of plane, and for wheel spacings unusually close, greater minimum thicknesses will be required.

PRELIMINARY - SUBJECT TO VERIFICATION

BEARING CAPACITY OF  
FRESH WATER ICE FOR  
AIRPLANES WITH WHEELS

JANUARY-1949