FTD-MT-65-567

AD 662757

FOREIGN TECHNOLOGY DIVISION

PRINCIPLES OF THE THEORY OF ROCKET FLIGHT

By

A. A. Dmitriyevskiy and V. N. Koshevoy

50 YEARS
1917 1967
GOLDEN ANNIVERSARY
FOREIGN TECHNOLOGY DIVISION

251

EDITED MACHINE TRANSLATION

PRINCIPLES OF THE THEORY OF ROCKET FLIGHT

By: A. A. Dmitriyevskiy and V. N. Koshevoy

English Pages: 240

TM6500097

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.	PREPARED BY: TRANSLATION DIVISION FOREIGN TECHNOLOGY DIVISION WP-AFB, OHIO.

Date 5 Sept. 1967

This document is a machine translation of Russian text which has been processed by the AN/GSQ-16(XW-2) Machine Translator, owned and operated by the United States Air Force. The machine output has been post-edited to correct for major ambiguities of meaning, words missing from the machine's dictionary, and words out of the context of meaning. The sentence word order has been partially rearranged for readability. The content of this translation does not indicate editorial accuracy, nor does it indicate USAF approval or disapproval of the material translated.

A. A. Dmitriyevskiy, V. N. Koshevoy

OSNOVY TEORII POLETA RAKET

Voyennoye Izdatel'stvo
Ministerstva Oborony SSSR

Moskva - 1964

Page 1 - 311

0848

1690

CIRC ABSTRACT WORK SHEET

(01)Acc Nr.	(65)SIS Acc Nr.	(40) Country of Info	(41)Translation Nr.
TM6500097	AM5014768	UR	MT6500567

(42)Author	Priority / Distribution
DMITRIYEVSKIY, A. A.; KOSHEVOY, V. N.	Priority II; Distribution STD

(43)Source

OSNOVY TEORII POLETA RAKET

(02)Ctry	(03)Ref	(04)Yr	(05)Vol	(06)Iss	(07)B. Pg	(45)E. Pg	(73)Date	(47)Subject Code
UR	0000	64	000	000	0001	0311	NONE	16, 19

Language			
RUSS	N/A	MOSKVA	VOYENIZDAT

(39)Topic Tags solid rocket, aircraft unguided rocket, rocket firing, rocket guidance, rocket thrust, rocket trajectory, rocket technology, rocket stability, optimum trajectory

(66)Foreign Title

SEE SOURCE

(09)English Title

PRINCIPLES OF THE THEORY OF ROCKET FLIGHT

	(97) Header Clas	(63) Clas	(64)Rel	(60) Release Expansion
	0	00	0	

ABSTRACT: This book discusses basic questions of the theory of the flight of controlled and unguided rockets of different assignment and the influence of their design and construction on conditions of flight. We consider the following forces and moments having an effect on the rocket in flight: tractive force (thrust), force of weight, control forces and moments, aerodynamic drag. The theoretical and experimental determination of aerodynamic coefficients is described, as well as the character of their change in the process of flight depending upon altitude and meteorological factors. The phenomena accompanying the flight at great heights and the phenomenon of aerodynamic heating are discussed. The basic positions of the theory of rocket flight are examined. Fundamental information is given on the formulation of equations of motion and the calculation of characteristics of trajectories of the flight of controlled and unguided rockets of different assignment. The concept on the establishment of optimum conditions of motion and the calculation of trajectories according to data of radar observations are discussed. Different methods of stabilizing rockets in flight and factors affecting the deviation of the rocket from the calculated trajectory are investigated. The causes of dispersion during firing by the rockets and the methods of decreasing the dispersion are explained. The material of the book, explanations, and formulas are illustrated by examples, diagram, drawings, and graphs. Chapters II, III and

A

IV were written by V. N. Koshevoy and the remaining by V. A. Dmitriyevskiy. They also edited the book. This book is intended for officers and students of military educational institutions and students of civil educational institutions. It can be useful to readers interested in rocket technology. English translation; 240 pages.

TABLE OF CONTENTS

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

---

rot	curl
lg	log

PRINCIPLES OF THE THEORY OF ROCKEY FLIGHT

Andrey Aleksandrovich Dmitriyevskiy
Vsevolod Nikolayevich Koshevoy

Basic questions are discussed in the book of the theory of the flight of controlled and unguided rockets of different assignment and the influence of their design and construction on conditions of flight.

We consider the following forces and moments having an efffect on the rocket in flight: tractive force (thrust), force of weight control forces and moments, aerodynamic drag.

The theoretical and experimental determination of aerodynamic coefficients is described as well as the character of their change in the process of flight depending upon altitude and meteorological factors. The phenomena accompanying the flight at great heights and the phenomenon of aerodynamic heating are discussed.

The basic positions of the theory of rocket flight are examined. Fundamental information is given on the formulation of equations of motion and the calculation of characteristics of trajectories of the flight of controlled and unguided rockets of different assignment. The concept on the establishment of optimum conditions of motion and the calculation of trajectories according to data of radar observations.

Different methods of stabilizing rockets in flight and factors affecting the deviation of the rocket from the calculated trajectory are investigated. The causes of dispersion during firing by the rockets and the methods of decreasing the dispersion are explained.

The material of the book, explanations, and formulas are illustrated by examples, diagram, drawings, and graphs.

Chapters II, III and IV were written by V. N. Koshevoy and the remaining by V. A. Dmitriyevskiy. They also edited the book.

Book is intended for officers and students of military educational institutions and students of civil educational institutions. It can be useful to readers interested in rocket technology.

INTRODUCTION

The theory of flight is a further development and practical application of the dynamics of a solid body. It considers the flight of different vehicles: aircraft, rockets, earth satellites, artillery missiles, aerial bombs, and others.

In this book, considering its purpose, the principles of the flight of rockets moving in the field of terrestrial gravitation are discussed. Space flight to other planets is not considered.

The theory of flight is studied by the solution of two basic problems. The first problem consists in the calculation of characteristics of the motion of rockets by data known beforehand and the second, in the detecting of optimum (best) conditions of motion and trajectories of the flight.

The importance of the first problem cannot be overestimated. If we were not able to solve it or solved it insufficiently accurate, then our rockets would not fall on the assigned target and our satellites would not get into the required orbit. For the solution of the set problems it is first of all necessary to determine correctly what forces act on the rocket in flight and to know what their magnitude will be at each moment of time. Further one should formulate differential equations of the motion of the rocket taking into account all forces effecting the rocket. As a result of solving the differential equations all the characteristics of motion are obtained: speed of the rocket, acceleration, time of flight, and coordinates of the rocket on which can be plotted the trajectory. Even a brief acquaintance with the first problem shows that it is very complicated.

The number of forces affecting the rocket in motion, the character of their change in the process of the flight, and also correspondingly the number of equations describing the motion and their form depend on the assignment of rocket, its construction, the method of stabilization in flight, and the proposed trajectory of the motion.

It is most complicated to calculate trajectories of guided rockets intended for combat with fast moving targets. Such trajectories are complicated spatial curves. A somewhat simpler calculation of trajectories of rockets is the "surface-to-surface" class.

Depending upon concrete conditions the problem of the calculation of trajectories of the motion of the rocket is solved, as a rule, with certain assumptions. Used more frequently than others is the assumption that the motion of the center of mass of the rocket and its oscillation around the center of mass do not influence one another and are considered separately. At first we calculate the motion of the rocket as a material point located in the center of mass of the rocket. Then by knowing the trajectory we determine the motion of the rocket around its center of mass. The rocket is considered as a solid body. Deformation of the body and the motion of fuel and gas inside the rocket are usually not considered. The change of position of the center of gravity in process of motion in practical calculations is also not considered.

With the solution of problems of the theory of rocket flight a large number of effective factors with a smaller number of assumptions can be considered during calculation on electronic computers. The method of numerical integration is sufficiently universal but very laborious, and only a small class of comparatively simple problems can be solved by tabular and analytic methods.

Design calculations of trajectories are conducted, as a rule, for an ideally realized rocket proceeding from normal meteorological conditions. However, in reality a number of factors appears which affect the deviation of the rocket in flight from the calculated trajectory. The dispersion of trajectories of separate shots can depend both on the construction and technology of manufacture of the rocket (for instance, the eccentricity of the tractive force) and on the deviation of conditions of flight from the calculation (for instance, the change of meteorological factors, the nonuniform climax of the critical section of the nozzle and gas rudders, the imperfection of the control system, and others). The study of factors affecting the dispersion of trajectories of rockets and the consideration of methods of the decrease in dispersion are also part of the theory of the flight.

The science of the flight of rockets and missiles is sometimes called external ballistics. Word "ballistics" is from the Greek word meaning "throw"; therefore the name "external ballistics of rockets" is just as widespread as the "theory of rocket flight."

The theory of flight is closely connected with mathematics, physics, theoretical mechanics, aerodynamics, meteorology, and geodesy, with help of which it obtains the necessary information for further investigations. In turn the theory of flight gives data for the design of rockets and control systems, for the development of rules and tables of firing, and others.

Let us examine further the basic stages of the history of development of the theory of rocket flight.

All progressive humanity is delighted by successes of the Soviet people in subduing interplanetary space. In 1957, for the first time in history, the launching of artificial earth satellites was begun. In January of 1959 the nose cone of a space rocket launched in the direction of the Moon surmounted terrestrial gravity. The nose cone of the rocket launched in September of 1959 delivered to the Moon a Soviet pennant and with the help of the third space rocket the side of the Moon never seen was photographed. Beginning in May of 1960, in accordance with the plan of space investigations, our country launched four satellite vehicles; with their help, equipment was developed and living beings were returned to Earth. Thereby, it became possible to send a person into space and to return him safely to Earth. Our country was the first to master interplanetary routes by successfully sending from a heavy earth Satellite in February of 1961 a space rocket on a trajectory to the planet Venus.

On 12 April 1961, on the satellite vehicle "Vostok," Major of the Soviet Army Yu. A. Gagarin, the first astronaut in the world, was successfully launched, flew around the Earth, and landed safely in an assigned region. This feat opened a new page in the history of the conquest of space. After Gagarin in August of 1961, on the satellite vehicle "Vostok-2," Major of the Soviet Army G. S. Titov flew around the Earth more than seventeen times, stayed in conditions of weightlessness about twenty-four hours, and thereby proved the possibility of prolonged space flights.

In August of 1962, a prolonged group flight around Earth was accomplished on satellite vehicles "Vostok-3" and "Vostok-4" by officers of the Soviet Army astronauts A. G. Nikolayev and P. R. Popovich, and in June of 1963 with brillant success of one more great space feat was accomplished. Soviet astronauts V. F. Bykovskiy and V. V. Tereshkova in the remarkable satellite vehicles "Vostok-5" and "Vostok-6" accomplished a many-day joint space flight and landed safely on the territory of our native land.

These flights into space considerably increased our knowledge about the Universe; they were the glorious triumph of labor, thought, and mind of Soviet man, discoverer of the space era, and a colossal scientific and technical success.

On 1 November 1963 in the Soviet Union there was launched the controlled maneuvering apparatus "Flight-1" equipped with special equipment and a propulsion system providing wide maneuvering in the circumterrestrial outer space.

Thus for the first time multiple wide maneuvering of spacecraft was carried out in conditions of flight.

The whole world sees that we stand on the threshold of still more surprising and magnificent discoveries. All these achievements were possible only with the use of the principle of jet propulsion.

The principle of jet propulsion is widely applied and in the creation of combat weapons: intercontinental ballistic rockets, global rockets, zenith guided missiles, winged missiles, controlled aerial bombs and torpedoes, antitank guided missiles, and many other types of armament.

The contemporary state of world jet engineering to a considerable degree is obliged to Russian science and to its remarkable traditions, and therefore noting the main stages of development of the theory of rocket flight we will deal mainly with native works.

The first informations on Russian rockets and missiles of barrel artillery can be found in the book, "Regulations of War, Gun, and Other Matters Concerning Military Science" written by Onisim Mikhaylov and published in 1620.

Up to the time of the appearance of the first scientific works in the field of rocket technology the theory of the flight of missiles of barrel artillery systems was at a sufficiently high level. The first theoretically founded work by calculation of the trajectory of a free projectile was written by G. Galileo

(1564-1642) and published in Bologna in 1638. From this work it became known that if the resisting force of air is not considered and the acceleration of gravity is taken as constant in magnitude and direction, then the trajectory described by the projectile will be a parabola. It is obvious that the parabolic theory can be used only when the assumptions accepted during its derivation essentially do not affect the results of calculations (in the first place we disregard the resisting force of air).

At the end of the 17th and beginning of the 18th Centuries the influence of the resisting force of air on the flight of fast flying bodies was studied with special interest. First works on the influence of a medium on body moving in it belong to English scientist I. Newton (1643-1727). The works of Newton pertained to the motion of bodies with low speeds and were not completely confirmed by later investigations.

Experiments connected with the measurement of the initial speed of a missile were conducted for the first time in Russia in 1727, and the first description of experiments on the determination of the drag of air by motion of spherical rifle bullets for considerable speeds in that time (520 m/sec) can be found in the book of the Englishman Robins, published in 1742.

The first solution to the problem on the motion of a missile in air, taking into account the drag was made in 1753 by a member of the Russian Academy of Sciences L. Euler (1707-1783). Later, in 1755, in his famous work, "General Principles of the Motion of Liquids," the beginning hydroaerodynamics was assumed.

The first scientific investigations in the field of rocketry belong to the Russian artillery general K. I. Konstantinov (1819-1871), who headed from 1847 the Petersburg Rocket Institution and had much to do with the improvement of the organization, production, and technology of the manufacture of rockets. Konstantinov expressed the physical essence of the motion of the rocket by the equality of increases of momentum of the rocket and the gas escaping from it. He also came to the important conclusion concerning the fact that the eccentricity of reaction is the basic cause of deflecting the rocket from the initially assigned trajectory of motion and showed the expediency of finned rockets for the improvement of the accuracy of firing.

Rocket artillery of that time attained the greatest development in the first ha.. of the 19th Century. Subsequently, the poor accuracy of the combat of rockets and the successes in this relation of the barrel firing weapon lead to the fact that prior to the First World War of 1914-1918 combat rockets were completely taken out of the armament of armies of all countries. Beginning from this time the theory of the flight is developed basically in reference to missiles of barrel artillery and later, from the end of the 19th Century, in reference to the demands of aviation which began to be developed vigorously.

In 1820 in Russia the Artillery School was opened and converted in 1855 into an Artillery Academy. The development of external ballistics, in many respects, is connected with these educational institutions. In particular, a professor of the Artillery School, V. A. Ankudovich, wrote the first textbook on external ballistics published in 1836. Lecturing in the Artillery Academy from 1855 to 1858 on external ballistics the well-known Russian mathematician M. V. Ostrogradskiy was the first to solve in general form the complicated problem on the motion of a spherical revolving missile in air.

From 1858 the school of Russian ballistics was headed by N. V. Mayyevskiy, (1823-1892), who contributed much to the development of Russian artillery. In particularity, his works on the study of the drag of air during high speeds of the motion of missiles and also the rotary motion of elongated missiles are very valuable. In his work "On the Influence of Rotary Motion on the Flight of Elongated Missiles in Air," printed in 1865, N. V. Mayyevskiy proved for the first time the existence of an oscillatory motion of the longitudinal axis of a missile during flight and investigated the property of this motion. N. V. Mayyevskiy was a talented scientist and designer. Under his leadership many native systems were created highly perfected for that time. The works of N. V. Mayyevskiy were continued by his pupil, the well-known scientist N. A. Zabudskiy (1853-1917).

The principles of contemporary dynamics of rockets were embodied in the works of Russian scientists I. V. Meshcherskiy and K. E. Tsiolkovskiy. Professor Ivan Vsevolodovich Meshcherskiy (1859-1935), outstanding teacher and scientist, in his works on theoretical mechanics derived equations of the motion of bodies of a variable mass to which the rocket should be related. He formulated the equation of the vertical motion of a rocket.

Konstantin Eduardovich Tsiolkovskiy (1857-1935) is rightfully considered the author of many basic ideas and theoretical positions embodied in the construction of contemporary space rockets and rocket missiles. In his early works K. E. Tsiolkovskiy descriptively explained the essence of rocket propulsion using as an example the displacement of a ship under the action of recoil force on the stern of a rapid fire continuously firing cannon. After the science-fiction narratives "On the Moon" and "Dream about Earth and Sky. Effects of Universal Gravitation" K. E. Tsiolkovskiy published in 1903 the work "Investigations of Outer Space by Rockets." Given in this work is the well-known formula determining the greatest speed of a rocket assuming the absence air resistance and gravity.

Expecting development of jet engineering and on the basis of his theoretical research, K. E. Tsiolkovskiy introduced a number of valuable propositions realized considerably later with the high level of development of science and technology. This pertains to his ideas of the use of liquid fuel for rocket engines of rockets during flights at great distances and the application of gas rudders of a rocket effective in a vacuum where air vanes have no effect. Tsiolkovskiy also proposed multistage rocket trains for the obtaining of high speeds. Without the application of this idea the flight of the rocket nose cone at great distances would now be inconceivable. Widely used in our time is the idea of automation of control of the motion of high-speed aircraft and rockets. The introduction of automatic control of the rockets permitted reaching a high accuracy of firing. As is known, nose cones of Soviet rockets tested in September 1961 with a firing range of 12,000 km deviated from the assigned point of fall by only 1 km.

Deeply understanding the difficulty connected with the flight of living beings in a rocket K. E. Tsiolkovskiy raised the question of experiments connected with taking into account the influence of inertial overloads on the human organism and the establishment of safety conditions for it. During the study of the influence of drag on a body moving in it K. E. Tsiolkovskiy examined the problem of the heating of bodies moving in air at high speeds, known now as the problem of aerodynamic heating, the solution of which is of great importance.

The idea of use of the rocket principle of motion was expressed in the death notes of revolutinary Nikolay Ivanovich Kibal'chich.

Much was contributed to the development of rocket technology (in the period of its onset) and the popularization of the principle of rocket propulsion by Russian researchers A. N. Fedorov, F. A. Tsander, and the talented inventor Yu. V. Kondratyuk.

It is natural that in conditions of tsarist Russia and furthermore with the weak development of technology the idea of K. E. Tsiolkovskiy and his students were not developed, and his works did not receive due recognition. Only during the

Soviet regime was it possible to obtain the results known to all the world.

In 1918, by the initiative of Vladimir Ilyich Lenin, the organization of the Central Aerohydrodynamic Institute (CAI) was created. The founder of the Central Aerohydrodynamic Institute was the most prominent Russian scientist-aerodynamicist Nikolay Yegorovich Zhukovskiy, who developed the principles of aerodynamic designs of aircraft and dynamics of flight.

The first experimental works in aerodynamics were conducted by N. Ye. Zhukovskiy at Moscow University and the Moscow Higher Technical School (MHTS). The world-renown scientists-academicians A. N. Tupolev, B. N. Yur'yev, A. A. Arkhangel'skiy, B. S. Stechkin, V. V. Golubev and many other were pupils of N. Ye. Zhukovskiy and some were members of the scientific student circle organized by N. Ye. Zhukovskiy at MHTS. On the initiative of N. Ye. Zhukovskiy in 1919 the Moscow Institute of Engineers of the Air Force was organized and converted in 1922 to the Air Force Academy now bearing his name.

In a special resolution of the Council of People's Commissars, Vladimir Ilyich Lenin called N. Ye. Zhukovskiy the father of Russian aviation.

Modern science on the dynamics of flight in many respects in indebted to the students of N. Ye. Zhukovskiy: Professors V. P. Vetchinkin, I. V. Ostoslavskiy, V. S. Vedrov, V. S. Pyshnov, Professor of the Air Force Academy D. A. Ventsel', Academician V. S. Pugachev, Professor G. F. Burago, and many others.

In 1918 by the decision of the Soviet government there was also created a continually active commission of special artillery experiments (KOSARTOP), which was assigned the solution of problems connected with the creation of artillery systems and with firing of them. In the commission Academicians N. Ye. Zhukovskiy, A. N. Krylov, S. A. Chaplygin and the most prominent scientists artillery men V. M. Trofimov, N. F. Drozdov, and G. P. Kisnemskiy worked diligently. Thus Academician A. N. Krylov during 1917-1918 for the first time developed a method of numerical integration of differential equations of the motion of missiles widespread at present in case of the determination of characteristics of the motion of rockets for a different purpose.

The founder of the new branch of aerodynamics, gas dynamics is rightfully considered to be Academician Sergey Alekseyevich Chaplygin (1869-1942), one of the most talented students of N. Ye. Zhukovskiy. Headed by S. A. Chaplygin for more than 10 years the Central Aerohydrodynamic Institute quickly exceeded the similar institutes of Europe and America in the scope of works, equipment, and their scientific significance. Until the end of his life S. A. Chaplygin remained the scientific lead of Soviet aero-gas dynamics. Many of its problems connected with the theory of flight are solved by scientists of the school of S. A. Chaplygin. Before World War II scientists N. I. Tikhomirov and V. A. Artem'yev designed the first solid-propellant rockets. Rockets of the Soviet Army during the years of World War II with success smashed the enemies of our country.

In the beginning of the Second World War when distances of firing small unguided rockets – rocket missiles were small and the design of them resembled a missile or mine, the system of equations describing the motion of the center of the mass of rocket hardly differed from the system of equations describing the motion of the missile of barrel artillery. To determine the resisting force of air it was sufficient to use model functions of drag or the so-called laws of drag.

The theory of the flight of unguided rockets is sufficiently represented in the book of F. R. Gantmakher and A. M. Levin published in 1959 (second edition).

With the increase of flying ranges and speeds respectively, with the introduction of control, and with the complication of design and form of the rockets the equations describing their motion were considerably complicated. The use on a wide scale of contemporary aero and gas dynamics, highly developed in the application to demands of aircraft building, was required. The solution of problems connected with the designing of rockets began to attract scientists-specialists in the field of dynamic stability, automatic control, pilotless control, and the guiding to the target.

The theory of flight of controlled ballistic rockets intended for the firing at very great distances, earth satellites, and interplanetary ships requires the most extensive participation also of astronomers, specialists in the field of celestial mechanics. The elliptic theory of the motion of planets, which began to be developed from the times of I. Kepler (1571-1640) and I. Newton and our time has been considerably improved, is used with success for the calculation of the motion of satellites and space routes. A significant role in the creation of the general theory of the motion of rockets is played by Soviet scientists.

Great difficulties have been encountered in solving the so-called inverse problems in searching for the most advantageous optimum conditions of the motion of rockets. Fundamental works in this direction are also published by our scientists.

The creation of contemporary multistage rockets, the technical basis of space flights, demanded enormous creative forces of many collectives of scientists, designing offices, and plants.

Achievements in the field of rocket technology are so manifold and great that they can be reflected only in separate major works. This work deals with the schemes of three basic classes of aircraft, the fundamental peculiarities of which are directly connected with the following statement. These are the guided rocket with liquid fuel intended for flight at relatively short distances guided rockets with solid fuel for aerial battle, and the unguided rocket (missile) on solid fuel for firing at short distances.

A rocket with liquid fuel (pump feed of fuel) is represented in Fig. 1.

As is known, the reaction force appears during the rejecting of the mass of some substance called the working medium. In rockets the tractive force of jet engine is formed during expiration from the nozzle of the motor of products of the burning of fuel, being in this case the working medium.

The liquid propellant of rocket engines, as a rule, consists of two substances, the fuel and oxidizer, called the components of the propellant. A fuel reserve is placed in tanks. Not enumerating all requirements of rocket fuels and their properties, let us note that in the quality of fuel a substance is selected which is able in mixing with the oxidizer during reaction of burning to liberate a great quantity of heat and gas.

The air and gas rudders control the rocket in flight.

Figure 2 shows the main parts of a rocket with two solid-fuel engines designed for aerial combat. The booster after burnout is separated.

Depending upon the assignment and design of the engine charge of it has a different form. The charge of the booster designed for fast acceleration of the rocket should have a small burning time and give a great tractive force. In these cases charges are used consisting of several tubular trains of burning both on the outside and from within. The charge of the sustainer ensures a smaller tractive force with a prolonged work of the engine.

The charge burns from the end, since the lateral surface of grain is covered by a special composition protecting it from burning along the length of the charge. Such a form of the charge ensures a long period of operation of the engine.

Figure 3 shows typical diagram of an unguided rocket using solid fuel.

Subsequently we will not distinguish between terms "rocket" and "rocket missile," since we consider the theory of the flight of these bodies. By the term "rocket" we mean all forms of rockets; however, in those cases when the term "rocket missile" in application to a specific sample is established, it is left without change.

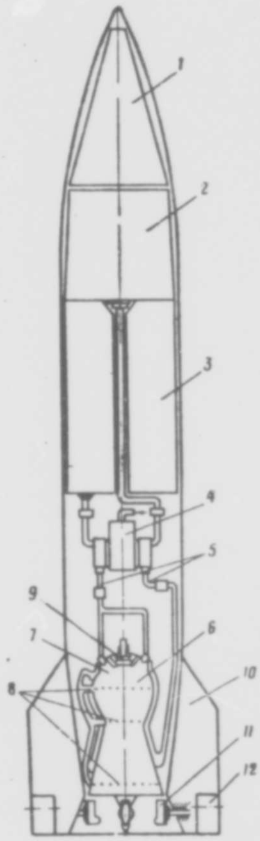


Fig. 1.

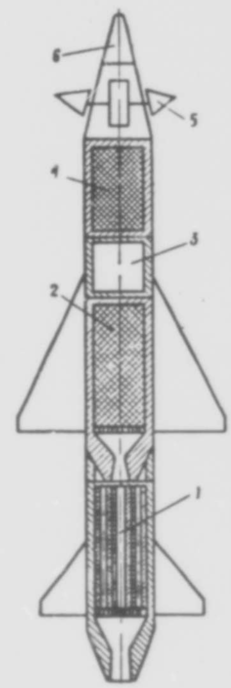


Fig. 2

Fig. 1. Diagram of a rocket with liquid-fuel engine (pump feed of the fuel): 1) nose cone; 2) fuel tank; 3) oxidizer tank; 4) turbopump assembly; 5) pipelines; 6) combustion chamber with nozzle; 7) precombustion chamber; 8) fuel injector; 9) alcohol value; 10) fin; 11) gas rudders; 12) air vanes.

Fig. 2. Diagram of an air combat rocket with a solid fuel engine: 1) booster; 2) sustainer; 3) control section ; 4) combat unit; 5) air vanes; 6) homing device.

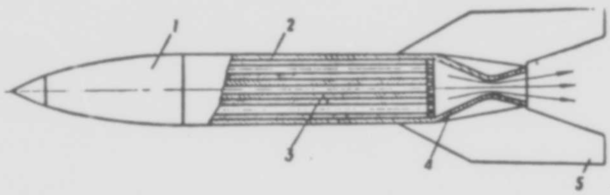


Fig. 3. Diagram of an unguided rocket with a solid-fuel engine:

1) combat unit; 2) engine chamber; 3) solid fuel; 4) nozzle; 5) fin.

C H A P T E R   I

GENERAL CHARACTERISTIC OF BASIC FORCES AND MOMENTS ACTING ON THE ROCKET IN FLIGHT

§ 1. System of Coordinates and Angles Determining the Position of the Rocket in Space

The tractive force of the main engine, air resistance (drag), gravity, and control forces act on rocket in flight. Since the vector of the total component of all forces usually does not pass through the center of the mass of the rocket, then in examining the motion of the rocket it is necessary to consider not only the acting forces, but also their moments with respect to the center of the mass.

The tractive force of the basic motor acts in a direction of the longitudinal axis of the rocket or close to it. The direction of the aerodynamic forces depends on the angle between the velocity vector of the motion of the rocket and its longitudinal axis. The direction of the action of gravity, as a rule, does not coincide with the two preceding directions. Moreover, the purposes of ballistic calculations are different. Therefore, to determine the position of the rocket in space it is impossible to be limited by any one system of coordinates. In general, for the calculation of trajectories and the solution of aerodynamic problems and questions of strength, stabilization and determination of characteristics of the dispersion with firing five systems of coordinates can be used: terrestrial (in rectangular or polar coordinates), connected, high-speed (continuous), semiconnected and semihigh-speed.

For the beginning of the terrestrial system of coordinates the point of the launch is accepted or another fixed point with respect to the Earth. The axis of the ordinates is directed along the radius of the Earth, the axis of abscissas coincides with the direction on the target, and axis OZ is directed to the right, if one were to look in the direction of axis OX, and it is perpendicular to the first two. This is the right rectangular system of coordinates. On drawings and diagrams axis of ordinates is usually located vertically, and the axis of abscissas, horizontally. With the calculation of elements of trajectories of ballistic long-range rockets and earth satellites we use the terrestrial system of polar coordaintes. The position of the center of the mass of the rocket is determined by the radius-vector and polar angles counted off in vertical and horizontal planes. Besides the terrestrial system of coordinates we use the spherical system accepted in astronomy. The celestial sphere is called the imaginary spherical surface of the determined radius. The position of the point of the center of mass of the satellite or rocket on the celestial sphere is determined, besides the radius, by two angles: the angle lying in the plane of the fundamental circle passing through the center of the sphere and the angle lying in the plane perpendicular to the fundamental circle. If for the basic plane we take the plane of the mathematical horizon, then the system of spherical coordinates is called horizontal. If, however, for the basic plane we take the plane of the celestial equator, then we will have an equatorial system of spherical coordinates (Fig. 4). In Fig. 4 the position of M in the equatorial system is determined by the angle of declination λ and hour angles η and $180^\circ + \eta$ plotted from the southern meridian westward. It is obviously that with the motion of the rocket or satellite the variables will be not only the angle of declination and hour angle, but also the radius of the sphere equal to the distance from the selected center of the sphere to the center of mass of the moving body.

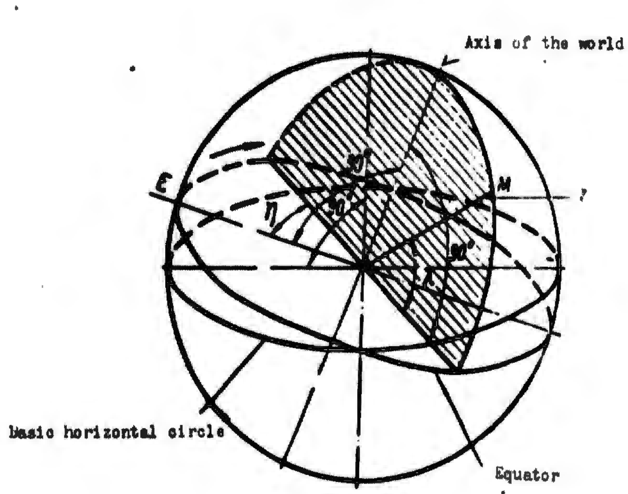


Fig. 4. Outer view of celestial sphere from the east side:
λ - angle of declination of point M,
η - hour angle.

The connected or, as it is sometimes called, moving coordinate system is rigidly united with the rocket and shifts together with it. The origin of the coordinates is usually located in the center of mass of the rocket. One of the axes of the coordinates is directed along the longitudinal axis of the rocket, and the other two are perpendicular to the longitudinal axis of the rocket and to each other.

If the rocket is constructed in an aircraft design, then one of the axes of the connected system of coordinates is directed along the chord of the profile of the wing, and the other is perpendicular to it in the plane of symmetry. In the semiconnected system of coordinates one of the axes coincides with the projection of the velocity vector on the plane of symmetry of the aircraft; the other two lie in a plane perpendicular to the projection of the velocity vector on the plane of symmetry (one is directed along the line of the intersecting planes, the other supplements the system up to the right).

In the continuous (high-speed) system of coordinates one of the axes coincides with the direction of the velocity vector of the flight of the center of mass of the rocket, and the other, perpendicular to it, lies in the plane of symmetry of the aircraft. As with the preceding, the continuous system of coordinates is a right rectangular system and is usually used during the study of phenomena flowing around bodies. The continuous system is connected with the velocity vector and shifts together with it during the motion of the rocket. During the study of phenomena flowing around in a reverse motion, when the model is motionless and the flow moves, the continuous system of coordinates is motionless and one of its axes is directed along the velocity vector of the undisturbed flow. In the semihigh-speed system of coordinates one of the axes, just as in the high-speed system, coincides with the velocity vector and other is directed perpendicular to it and lies in a vertical plane, and the third axis is horizontal.

For a connection between the different systems of coordinates a system of angles is taken (Fig. 5). The connection between the terrestrial and mobile systems of coordinates is carried out with the help of angles of pitch, bank, and yawing. Figure 5 (convenient for analysis of the motion of rockets of the "surface-to-surface" class) shows the general case when the velocity vector of the center of mass does not coincide with the axis of the rocket and is not located in a vertical plane coinciding with the direction of firing. The beginning of all systems of coordinates is placed in the center of mass of the rocket. Here the terrestrial coordinates are designated X, Y, Z; coordinates of the connected system are X_1, Y_1, Z_1; and the continuous coordinates are X_0, Y_0, Z_0. Plane A in which axis OX and OZ lie is horizontal. Plane B passing through the axis OX is vertical. Plane C is also vertical but passes through the longitudinal axis of the rocket. The angle lying

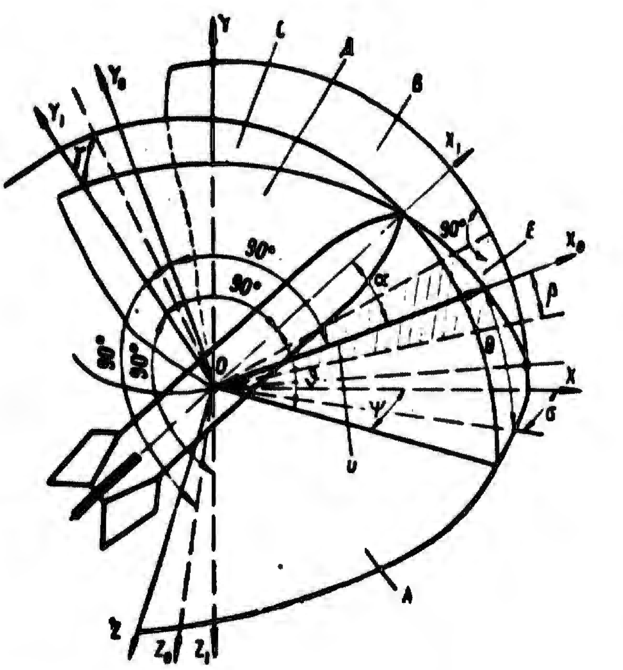


Fig. 5. Diagram of the mutual location of terrestrial, connected, and continuous system of coordinates:
ϑ - pitch angle; ψ - angle of yawing;
γ - angle of roll; α - angle of incidence;
β - angle of slip.

in plane C between the longitudinal axis of the rocket and its projection on the horizontal plane A is called the pitch angle and is designated by the letter ϑ. We distinguish the pitch angle with respect to the starting (horizontal) plane and local pitch angle measured with respect to the flowing horizontal plane occurring at a given moment under the rocket. It is expedient to consider the latter separation with the determination of characterisitcs of motion of rockets intended for firing at very great distances. The angle between the projection of the longitudinal axis of the rocket on horizontal plane A and terrestrial coordinate OX is called the yaw angle and is designated by ψ. The turn of the rocket about the longitudinal axis is determined by the roll angle γ, i.e., the angle between the body axis of coordinate OY_1 and the vertical plane C passing through the longitudinal axis of the rocket. It should be noted that axis of OY_1 must not necessarily lie in plane Д, as this can be represented in Fig. 5. For winged rockets constructed in aircraft design the angle of roll can be defined as the angle between the plane of symmetry and vertical plane passing through the axis of OX_1.

The connection between mobile and continuous systems of coordinates is carried out with the help of the angle of incidence and angle of slip. The angle between the velocity vector and longitudinal axis of the rocket is called the angle of incidence α and lies in plane Д called the drag plane. The angle between the velocity vector $\bar{V}$ and projection of the longitudinal axis of the rocket on plane E passing through the velocity vector and perpendicular to vertical axis is called the slip angle β. For rockets of aircraft design and aircraft the angle of incidence is called the angle between the projection of the velocity vector on the plane of symmetry of the aircraft and the chord of profile of the wing, and the angle of slip is the angle between the velocity vector and plane of symmetry of the aircraft. Figure 5 also shows angles θ and σ. Angle θ is called the angle of inclination to the horizon of the tangent to the trajectory (angle between the velocity vector and horizontal plane); angle σ is the angle of rotation of the trajectory.

If $\psi = \sigma = 0$, we obtain motion in one plane during which $\vartheta = \theta + \alpha$.

§ 2. Equation of Motion of the Rocket (in General Form) Thrust

In general we will consider motion to be the simultaneous separation and connection of particles to the basic variable mass of the body. A characteristic example of this model can be the motion of an aircraft with an airbreathing-jet engine, through the intake diffuser of which proceeds a counter flow of air necessary for the operation of the engine. Simultaneously with the intake of air the products of the combustion of fuel with great speed back from the nozzle of the engine creating thrust. In the process of the connection and separation of particles the mass of the body changes continuously. Let us assume that the mass of the body is variable and the speed of the connection and separation of particles do not depend on the speed of the body. For simplicity we will consider the case of linear motion.

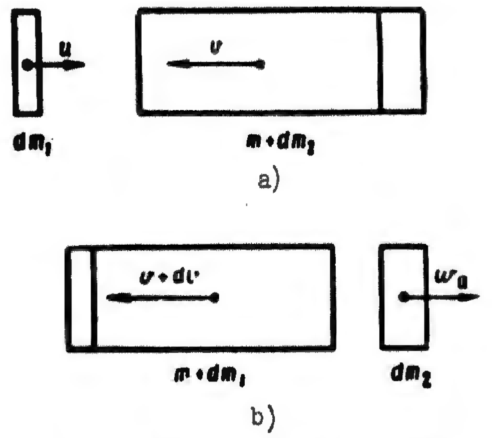


Fig. 6. Diagram of the change in mass: a) composition of mass before the connection and separation of particles; b) composition of mass after the connection and separation of particles.

Let us assume that in the considered moment of time t the body has mass $m + dm_2$ moving with speed v. For the interval of time dt the mass of the body will be changed due to the connection of the elementary mass dm_1 and simultaneous separation of mass dm_2 (Fig. 6). According to hypothesis established in the method of I. V. Meshcherskiy, the connection and separation of particles occurs during the infinitesimal interval of time similar to a shock. After connection the particle moves with a speed of the basic mass of the body, and the separated particle obtaining speed immediately loses interaction with the basic mass of the body. On the considered system of three masses forces act the resultant which is ΣF. As a result of the interaction among themselves masses m, dm_1, and dm_2 and under the action of forces ΣF, the speed of united mass $m + dm_1$ will be equal to $v + dv$. The speed of the mass dm_1 before the connection is designated u and the speed of mass dm_2 after separation – w_a.

Let us find the change of momentum of the system of masses m, dm_1 and dm_2 for the time interval dt and equate it to the pulse of external forces

$$m(v+dv)-mv+dm_1[(v+dv)-u]+$$
$$+dm_2(w_a-v)=\sum_i Fdt. \quad (1.1)$$

Carrying out the transformation, disregarding the $dm_1 \cdot dv$ and dividing both sides of equation (1.1) by dt, we obtain the equation of the motion of a body of a variable mass in general

$$m\cdot\frac{dv}{dt}+\frac{dm_1}{dt}(v-u)+\frac{dm_2}{dt}(w_a-v)-\sum_i F=0. \quad (1.2)$$

The equation similar to the one obtained was derived for the first time by I. V. Meshcherskiy and was named after him at the proposal of A. A. Kosmodem'yanskiy.

Considering the motion of the point of the variable mass in projections on the axis of coordinates X, Y, Z, and I. V. Meshcherskiy obtained the following equations:

$$m\ddot{x}+\frac{dm_1}{dt}(\dot{x}-u_x)-\frac{dm_2}{dt}(\dot{x}-w_{ax})-\sum F_x=0;$$

$$m\ddot{y}+\frac{dm_1}{dt}(\dot{y}-u_y)-\frac{dm_2}{dt}(\dot{y}-w_{ay})-\sum F_y=0;$$

$$m\ddot{z}+\frac{dm_1}{dt}(\dot{z}-u_z)-\frac{dm_2}{dt}(\dot{z}-w_{az})-\sum F_z=0, \quad (1.3)$$

where $\dot{x}=\frac{dx}{dt}=v_x$ is the derivative of x with respect to t equal to the projection of the speed on axis OX; $\ddot{x}=\frac{d^2x}{dt^2}=\frac{dv_x}{dt}$ is the second derivative of x with respect to t, equal to the derivative of v_x with respect to t, i.e., projections of acceleration on axis OX, and so forth.

I. V. Meshcherskiy called products of the form $(dm/dt)(\dot{x} - w_{ax})$ "the projection on coordinates axes of additional force." Taking one of the particular cases considered by I. V. Meshcherskiy when $dm_1 = 0$ and $dm_2 = dm$, we obtain

$$m\cdot\frac{d^2x}{dt^2}=\sum F_x+\frac{dm}{dt}(\dot{x}-w_{ax}), \quad (1.4)$$

allowing the describing of the rectilinear motion of the rocket with a jet engine of the usual type. Thus I. V. Meshcherskiy showed that the equation of the motion of a body of variable mass (rocket) can be described just as the equation of the motion of a body of a constant mass, including in the number of acting forces the "additional" force $(dm/dt)(x - w_{ax})$.

For the rectilinear motion of a rocket vertically upwards I. V. Meshcherskiy wrote the equation[1]

$$m\ddot{x} = -mg + p - \frac{dm}{dt}\cdot w - R(\dot{x}), \qquad (1.5)$$

where m is the mass of the rocket; g the acceleration of gravity; p the pressure of gases; w the magnitude of the relative speed which the burning particles have at the time of their separation; $R(\dot{x})$ the air resistance.

From the given equations of I. V. Meshcherskiy the formula determining the thrust of a jet engine can be obtained.

By stand thrust we understand the resultant forces of air pressure and exhaust gases applied to the motionless rocket being in a motionless undisturbed atmosphere. The rocket and atmosphere are taken motionless so that the thrust does not include the air resistance appearing with the relative motion of the rocket and atmosphere.

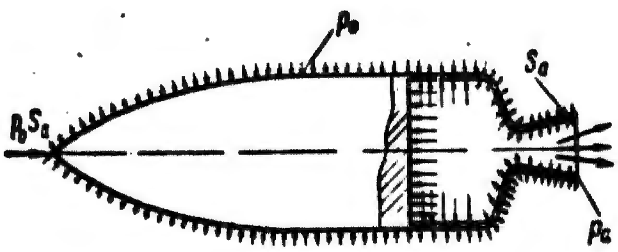


Fig. 7. Diagram of pressure distribution affecting a motionless rocket in a motionless atmosphere.

We use a particular case for a motionless rocket considered by I. V. Meshcherskiy when

$$\ddot{x} = \ddot{y} = \ddot{z} = 0.$$

Then the equation of motion (1.5) will be transformed into an equation of equilibrium:

$$mg - p + \frac{dm}{dt}\cdot w = 0. \qquad (1.6)$$

The component p + (dm/dt)w in equation (1.6) will be the thrust. Having obtained equation (1.5) I. V. Meshcherskiy did not give a detailed interpretation of component p, calling it "pressure of gases."

Let us reveal the meaning of component p. If in the determination of thrust the rocket and atmosphere are motionless, then there is no air resistance but only external pressure. Figure 7 gives a diagram of the pressure distribution effecting" the motionless rocket with an operating engine. Forces of atmospheric pressure p_0 act on the external surface. They are equal in magnitude to the produce of pressure on the area and are directed perpendicular to that site on which they act. All the forces having an effect on the lateral surfaces of a rocket balance one another. Since with an operating engine the atmospheric pressure does not act on

[1] In this case I. V. Meshcherskiy designated the vertical axis OX.

the output section of the nozzle through which the gases flow then there will pear the resultant force p_0S_a directed in the direction of the expiration of gases (S_a is the area of the output section of the nozzle). In the output section of the nozzle the directed force p_aS_a acts oppositely, where p_a is the pressure of exhaust gases in this section.

Thus in equation (1.6) we should replace p by $S_a(p_0 - p_a)$:

$$mg - S_a(p_0 - p_a) + \frac{dm}{dt} \cdot w = 0. \qquad (1.7)$$

As a result we obtain the equation for thrust in the form

$$P = \frac{dm}{dt} \cdot w - S_a(p_0 - p_a). \qquad (1.8)$$

Introducing into equation (1.8) the mass flow rate per second of gas

$$\left|\frac{dm}{dt}\right| = \frac{G_{сек}}{g},$$

we obtain the formula for thrust in contemporary writing

$$P = \frac{G_{сек}}{g} \cdot w + S_a(p_a - p_0). \qquad (1.9)$$

where $G_{сек}$ is the second flow rate of gas. The component $\frac{G_{сек}}{g} \cdot w$ is sometimes called reaction force.

The expression of thrust for a rocket located at the surface of Earth is

$$P_0 = \frac{G_{сек}}{g} \cdot w + S_a(p_a - p_{0N}), \qquad (1.10)$$

where p_{0N} is the atmospheric pressure at the surface of the Earth for normal weather conditions.

Comparing formula (1.9) and (1.10) we obtain

$$P = P_0 + S_a p_{0N}\left(1 - \frac{p_0}{p_{0N}}\right), \qquad (1.11)$$

where p_0 is the pressure at a given altitude; p_{0N} is the pressure at the surface of the Earth for normal weather conditions.

Designating $\Pi(y) = \frac{p_0}{p_{0N}}$, we get

$$P = P_0 + S_a p_{0N}[1 - \Pi(y)]. \qquad (1.12)$$

Factoring $\frac{G_{сек}}{g}$, from the right side of formula (1.9) we obtain the formula for the thrust:

$$P = \frac{G_{сек}}{g} \cdot w_e \qquad (1.13)$$

where

$$w_e = w + \frac{S_a g}{G_{сек}}(p_a - p_0); \qquad (1.14)$$

w_e is the magnitude called by the well-known French scientist P. Langevin effective exhaust velocity. Calculations show that in formula (1.14) the second component, as compared to the first, is small and usually is not more than 10-15%; therefore the effective exhaust velocity is determined basically by the velocity of gas at the output section of the nozzle, w. If we refer thrust to the flow rate per second we will obtain the formula determining the specific thrust or the so-called unit pulse:

$$P_{уд} = \frac{P}{G_{сек}} = \frac{w_e}{g} \,|\, \text{kgf}\cdot\text{sec/kgf} \,|. \tag{1.15}$$

The expression for the unit pulse can also be obtained from the common expression determining the thrust impulse:

$$J = \int_0^t P dt = \frac{w_e}{g} \int_0^t G_{сек} \cdot dt.$$

The full pulse of the thrust during the whole period of the engine's operation is

$$I = \frac{w_e}{g} \int_0^{t_k} G_{сек} \cdot dt = \frac{w_e \cdot \omega}{g},$$

where ω is the complete fuel consumption for the whole period of the engines operation.

Transferring I to ω we get

$$P_{уд} = \frac{w_e}{g} \,|\, \text{kgf}\cdot\text{sec/kgf} \,|.$$

With ballistic calculations the rocket is usually assumed to be a soild undeformed body. From mechanics it is known that characteristics of the motion of a solid body can be determined through the forward motion of the center of mass of a body and the rotation around the center of mass. Using the conclusions of I. V. Meshcherskiy we will write the equation of the forward motion of a rocket in vector form:

$$m\ddot{\bar{r}} = \bar{P} - \bar{F}, \tag{1.16}$$

where m is the mass of the rocket; $\ddot{\bar{r}}$ – the vector of instantaneous acceleration of the center of mass of the rocket; $\bar{P}$ – the thrust vector; $\bar{F}$ – the total vector of all the resisting forces acting on the rocket.

The equation of the rotation of a rocket will have the form

$$J \cdot \frac{d\bar{\omega}}{dt} = \bar{M}_P + \bar{M}_F, \tag{1.17}$$

where J is the moment of inertia of the rocket about the instantaneous axis of rotation; $\bar{\omega}$ – the vector of the instantaneous angular velocity of the rolling of the rocket; $\bar{M}_P$ – the vector of the moment of thrust; $\bar{M}_F$ – the vector of the moment of aerodynamic forces (including controlling moments).

The position of a solid body in space is determined by six independent magnitudes called degrees of freedom. In the case considered three of them are coordinates of virtual displacement of the center of mass of a rocket, and three are angles of a possible turn of the longitudinal axis of a rocket with respect to the center of mass. In accordance with this the equations of motion (1.16) and (1.17) written in common form can be developed into six differential equations. However, the unknowns are considerably greater, and the six equations appear to be insufficient.

It is necessary to know the time dependence: coordinates of the center of mass of the rocket (x, y, z); projections of the speed of the center of mass on coordinate axes (v_x, v_y, v_z); projections of the vector of instantaneous angular velocity of the rotation of the longitudinal axis of the rocket on coordinate axes (ω_x, ω_y, ω_z) and of angles of pitch, yawing, and bank (ϑ, ψ, γ).

Since there appears to be twelve unknowns, then to the six equations of the dynamics of a solid body should be added six more kinematic equations.

Let us write equation (1.16) in projections on the Earth's axis of coordinates x, y, z:

$$\left.\begin{aligned} m\cdot\frac{dv_x}{dt} &= P_x - F_x; \\ m\cdot\frac{dv_y}{dt} &= P_y - F_y; \\ m\cdot\frac{dv_z}{dt} &= P_z - F_z \end{aligned}\right\} \quad (1.18)$$

To these we will add the three standard kinematic ratios:

$$\left.\begin{aligned} \frac{dx}{dt} &= v_x; \\ \frac{dy}{dt} &= v_y; \\ \frac{dz}{dt} &= v_z. \end{aligned}\right\} \quad (1.19)$$

Equations of the rotation of rockets and aircraft are usually written in projections on combined coordinate axes. Any other system of coordinates not connected with the rocket shifts with respect to the rocket, and this leads to necessity to use variables of moments of inertia in the investigation of motion of the rocket, which introduces a complication. Let us develop equation (1.17) in projections on the combined coordinate axes X, Y, Z:

$$\left.\begin{aligned} J_{x_1}\cdot\frac{d\omega_{x_1}}{dt} + (J_{z_1} - J_{y_1})\omega_{y_1}\omega_{z_1} &= \sum M_{x_1}; \\ J_{y_1}\cdot\frac{d\omega_{y_1}}{dt} + (J_{x_1} - J_{z_1})\omega_{x_1}\omega_{z_1} &= \sum M_{y_1}; \\ J_{z_1}\cdot\frac{d\omega_{z_1}}{dt} + (J_{y_1} - J_{x_1})\omega_{x_1}\omega_{y_1} &= \sum M_{z_1}, \end{aligned}\right\} \quad (1.20)$$

where $\sum M_{x_1}, \sum M_{y_1}, \sum M_{z_1}$ are sums of projections of moments of all the external forces on combined coordinate axes; $J_{x_1}, J_{y_1}, J_{z_1}$ moments of inertia of the rocket with respect to axes of the coordinates; $\omega_{x_1}, \omega_{y_1}, \omega_{z_1}$ are projections of the angular velocity on the connected axes.

Time derivatives from angles ϑ, ψ, and γ are determined by the known equations:

$$\left.\begin{aligned}\frac{d\vartheta}{dt}&=\omega_{y_1}\sin\gamma+\omega_{z_1}\cos\gamma;\\ \frac{d\psi}{dt}&=\frac{\omega_{y_1}\cos\gamma-\omega_{z_1}\sin\gamma}{\cos\vartheta};\\ \frac{d\gamma}{dt}&=\omega_{x_1}-\mathrm{tg}\,\vartheta\,(\omega_{y_1}\cos\gamma-\omega_{z_1}\sin\gamma).\end{aligned}\right\} \quad (1.21)$$

Thus for finding the twelve unknowns we have twelve equations, and the problem of the determination of the characteristics of motion of the rocket can be solved.

Depending upon the formulation of the problem the influence of control on the motion of the rocket can be established, in two ways. If the forces and moments, including control forces and moments are known, then the unknowns will enumerate the twelve characteristics of motion which are determined in solving the systems of equations.

In the second case part of the characteristics of motion can be assigned: magnitudes of coordinates, velocities, or angles. The characteristics of motion of the rocket are set in the function of time or other magnitudes, for instance, $y = f_1(x)$ or $\vartheta = f_2(t)$, etc. These functions are called program equations. For a full investigation of controlled flight and stabilization of the aircraft, we must add to the written equations of control describing the operation of the system of control. Depending upon concrete conditions the equations of control can be different. In this case the unknowns will be the control forces and moments which should ensure beforehand the assigned program of the motion. In equations (1.18) and (1.20) the control forces and moments as yet have not been separated by us.

Additional complexities not reflected in the written equations consist in the fact that forces and moments are interconnected with characterisitcs of motion.

Let us discuss the magnitudes entering the written equations and the connections between them. The mass of the rocket is variable, and it depends on time and is equal to

$$m = m_0 - \int_0^t G_{сек}\,dt. \quad (1.22)$$

where m_0 is the initial mass of the rocket.

The flow rate per second of the operating substance of $G_{сек}$ in general is variable, since can be changed with the regulation of thrust.

Many forces and moments are interconnected with the characteristics of motion.

For instance, aerodynamic and control forces depend on the speed and altitude of the flight. Thrust is a variable depending on the position of the rocket above the surface of the Earth (formula 1.11).

Moments of inertia of rocket J_{x_1}, J_{y_1}, and J_{z_1} characterize the inertness of the rocket during its rotation around the corresponding axis. Let us recall that the moment of inertia is equal to the sum of the products from elementary particles of mass dm by the square of the distance up to the axis about which the moment of inertia is taken, for instance:

$$J_{x_1} = \int_{(w)} h_x^2 \, dm.$$

Sign (w) shows that the integral is taken for the whole volume. The moment of inertia, determined with respect to coordinate planes YOZ and XOZ,

$$J_{xy} = \int_{(w)} xydm$$

is called the product of inertia. If through the center of the mass of the rocket three mutually perpendicular axes are thus conducted so that J_{xy}, J_{yz}, and J_{zx} will be equal to zero, then such axes will be called the main central axes of inertia.

If the body axes of coordinate X_1, Y_1, and Z_1 are combined with the main central axes of inertia, then the equations of orbital motion will take the form of equations in the form of Euler (1.20). With the operating engine the moments of inertia will be variables due to the change of the mass of the body because of fuel consumption. The numerical values of J_{x_1}, J_{y_1}, and J_{z_1} for the rocket are determined just as for any composite body. We conditionally divide the rocket into separate parts having a simple geometric form and identical density of substance and determine the moment of inertia for each part separately. Further summation is carried out according to the rules of mechanics. Thus for the calculation of moments of inertia it is necessary to have detailed drawings of the rocket and to know the law of the change of its mass in flight. Work on the calculation determination of moments of inertia and their change is very tedious and is laborious. Therefore, during approximate ballistic calculations they are taken constant. With more exact calculations connected with the stability and controllability, it is necessary to consider the variability of the moments of inertia. Experimental methods of the determination of moments of inertia of composite bodies giving more exact results incomparison with the calculation methods are developed in sufficient detail in mechanics and here we will not be concerned with them. We will further consider more thoroughly the influence of Earth and aerodynamic forces on the flight of rockets.

CHAPTER II

INFLUENCE OF THE FIELD OF GRAVITATION OF THE EARTH AND ITS ATMOSPHERE ON THE FLIGHT OF ROCKETS

§ 1. The Acceleration of the Gravity

During the flight of rockets in space in the vicinity of the Earth they are attracted to Earth, or, as it is said, they undergo experience the influence of the field of gravitation of the Earth. The gravitational force of the rocket to Earth is one of basic forces affecting the rocket in flight. This force considerably affects the characteristics of the movement of the rocket (speed, acceleration, direction of flight, and so forth), and therefore we will dwell on this in greater detail.

According to law of universal gravitation any two bodies with masses m_1 and m_2 are attracted to one another with a force F directly proportional to the product of these masses and inversely proportional to the square of the distance r between them, i.e.,

$$F = \gamma \cdot \frac{m_1 m_2}{r^2}, \tag{2.1}$$

where F is the gravitational force in kgf; γ – proportionality factor called the gravitational constant or gravity constant; m_1 and m_2 – mass of bodies in $kgf \cdot sec^2/m$; r – the distance between the centers of masses of the bodies is equal in meters.

Gravitational constant γ is numerically equal to the force with which two bodies with masses at 1 $kgf \cdot sec^2/m$ are attracted to one another if the distance between the centers of these bodies is equal to 1 m. Experiments showed that

$$\gamma = 65{,}4 \cdot 10^{-11} \frac{m^4}{kgf \cdot sec^4}.$$

We apply the law of universal gravitation (2.1) to the case of the attraction of the rocket to Earth. For this it is necessary to know the form of the Earth and the distribution of its mass. Measurements showed that the form of the Earth can be sufficiently accurately considered an ellipsoid of revolution. The semiaxis of rotation is equal to 6,356,911 meters and is located in a north-south direction and the equatorial semiaxis of the Earth is equal to 6,378,388 meters. The mass of the substance of which the Earth consists conducted from the center of the Earth in concentric spheres is distributed not strictly evenly. All this leads to the fact that a rocket with a constant mass in flight above different sections of the surface of the Earth even at the same distance from the center of the Earth will be attracted to it with a different force. However, the change in attractive force will be small and it in most cases in calculations of trajectories of rockets is not considered, and the Earth is considered a sphere with a radius $R = 6.371 \cdot 10^6$ meters and with an equal distribution of masses in concentric spheres inside this sphere.

If we designate the mass of the Earth by the letter M and the mass of the rocket by the letter m, then the gravitational force of the rocket to the Earth will be equal to

$$F = \gamma \cdot \frac{Mm}{r^2}, \tag{2.2}$$

where F is the Earth's gravity; r is the distance from the center of the Earth to the center of the mass of the rocket.

In calculations it is more convenient to deal not with the gravity but with the acceleration imparted to the rocket by this force. According to the second law of Newton the acceleration from gravity $g_T = \frac{F}{m}$, and therefore from formula (2.2) it follows that

$$g_T = \gamma \cdot \frac{M}{r^2}. \tag{2.3}$$

In a particular case on the surface of the Earth the acceleration of gravity is equal to

$$g_{T0} = \gamma \cdot \frac{M}{R^2}. \tag{2.4}$$

Knowing γ, R, and $M = 0.6092 \cdot 10^{24}$ kgf·sec^2/m we find $g_{T0} = 9.82$ m/sec^2.

§ 2. Influence of the Rotation of the Earth on the Flight of Rockets

In the determination of the acceleration of a rocket from the Earth's gravity the rotation of the Earth around its axis was not taken into account. From theoretical mechanics it is known that the rotation of the Earth requires the consideration of two more accelerations which rockets flying in space in the vicinity of the Earth will undergo. These are the acceleration of migratory motion $j_{пер}$ and Coriolis acceleration $j_{кор}$.

From where do these accelerations appear? We will consider the absolute motion of a rocket about a motionless Earth, i.e., with respect to the system of coordinates located in the center of the Earth but not revolving together with it and all the time remaining motionless. The movement of the rocket with respect to the surface of the revolving Earth will be called relative. The migratory motion will then be the rotation of the Earth about its axis. The acceleration of the rocket in the complicated absolute motion will be

$$\bar{j}_{абс} = \bar{j}_{отн} + \bar{j}_{пер} + \bar{j}_{кор}, \tag{2.5}$$

where $j_{абс}$ is the vector of the absolute acceleration of the rocket; j_{OTH} – vector of the relative acceleration of the rocket; $j_{пер}$ – vector of the translational acceleration of the rocket; $j_{кор}$ – vector of the rotating acceleration of the rocket (Coriolis acceleration).

The relative acceleration is determined by the formula

$$j_{отн} = \frac{dv_{отн}}{dt}, \tag{2.6}$$

where v_{OTH} is the speed of the rocket with respect to the surface of the revolving Earth; $\frac{dv_{OTH}}{dt}$ is the derivative from the relative speed in time equal to the instananeous value of relative acceleration.

Direction $\bar{j}_{OTH}$ coincides with the direction of velocity vector v_{OTH}.

The value $j_{пер}$ is found from formula

$$j_{пер} = r_0 \Omega^2, \tag{2.7}$$

where r_0 is the shortest distance (Fig. 8a), from the center of mass of the rocket to the axis of the rotation of the Earth; Ω is the angular velocity of the rotation of the Earth.

Vector $\bar{j}_{пер}$ is directed from the center of mass of the rocket to the axis of rotation of the Earth along the shortest distance between them, and therefore is sometimes called centripetal acceleration (Fig. 8b).

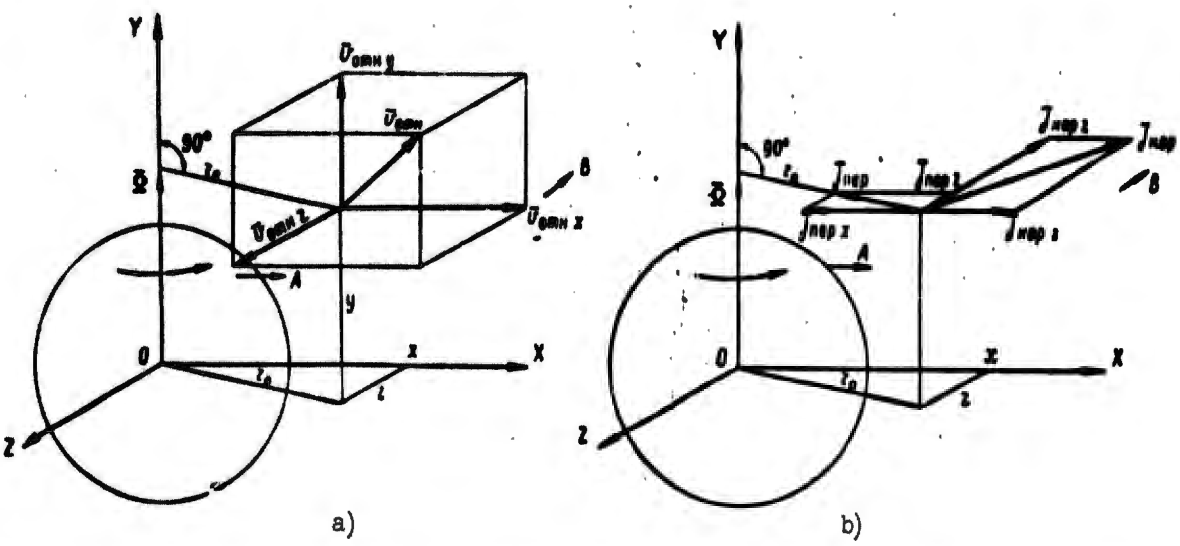


Fig. 8. Components speed and accelerations of the center of the mass of a rocket:
a) decomposition of the vector of relative speed of the rocket v_{OTH} along axis of the coordinates; b) components of rotary and Coriolis accelerations (in a horizontal plane).

The magnitude of Coriolis acceleration can be determined by following dependence:

$$j_{кор} = 2v_{отн}\Omega \sin(\bar{v}_{отн}, \bar{\Omega}), \qquad (2.8)$$

where $\sin(\bar{v}_{OTH}, \bar{\Omega})$ is the sine of the angle between vectors $\bar{v}_{OTH}$ and $\bar{\Omega}$.

The Coriolis acceleration is perpendicular to the plane conducted through $\bar{\Omega}$ and parallel to $\bar{v}_{OTH}$, and is directed always to the same side as the end of vector $\bar{v}_{OTH}$, if it revolves together with the Earth with an angular velocity Ω.

From formula (2.8) it can be established, that the Coriolis acceleration is equal to zero on condition when either $\Omega = 0$ (Earth does not revolve), $\bar{v}_{OTH} = 0$ (rocket is motionless with respect to Earth), or $\sin(\bar{v}_{OTH}, \bar{\Omega}) = 0$, i.e., $\bar{v}_{OTH}$ is in parallel with $\bar{\Omega}$ or OY, the axes of rotation of the Earth. Consequently, Coriolis acceleration will practically be equal to zero only in those moments of the flight when the speed $\bar{v}_{OTH}$ of the rocket with respect to the Earth is parallel to OY.

Equations of the motion of a rocket are formulated in projections on axes of coordinates. Let us show how the magnitudes of projections of relative, rotary and Coriolis accelerations will appear on the axes of a motionless system of coordinates X, Y, Z (Fig. 8b).

Projections of the vector of relative acceleration $\bar{j}_{OTH}$ will be equal to respectively

$$\left.\begin{aligned} j_{\text{отн}\,x} &= \frac{dv_{\text{отн}\,x}}{dt}; \\ j_{\text{отн}\,y} &= \frac{dv_{\text{отн}\,y}}{dt}; \\ j_{\text{отн}\,z} &= \frac{dv_{\text{отн}\,z}}{dt}, \end{aligned}\right\} \qquad (2.9)$$

time derivatives from projections of relative speed on axes of the coordinates.

Projections of the vector of the translational acceleration $\bar{j}_{\text{пер}}$ on axes of coordinates X, Y, Z can be determined from expressions

$$\left.\begin{aligned} j_{\text{пер}\,x} &= -x\Omega^2; \\ j_{\text{пер}\,y} &= 0; \\ j_{\text{пер}\,z} &= -z\Omega^2. \end{aligned}\right\} \qquad (2.10)$$

Here $j_{\text{пер}\,y} = 0$ because the projection of r_0 on axis OY is equal to zero, since r_0 is perpendicular to the axis of rotation of the Earth OY. Projections of r_0 on axes OX and OZ are equal respectively to x and z. Minus signs for $j_{\text{пер}\,x}$ $j_{\text{пер}\,z}$ were obtained because the corresponding vector components of the translational acceleration are directed opposite the positive direction of axes OX and OZ.

Projections of Coriolis acceleration are found by the formulas:

$$\left.\begin{aligned} j_{\text{кор}\,x} &= 2v_{\text{отн}\,z}\Omega; \\ j_{\text{кор}\,y} &= 0; \\ j_{\text{кор}\,z} &= -2v_{\text{отн}\,x}\Omega. \end{aligned}\right\} \qquad (2.11)$$

Let us explain formulas (2.11). Since the vector of Coriolis acceleration is perpendicular to the plane which passes through the vector $\bar{\Omega}$ (i.e., through the axis OY) and is parallel to $\bar{v}_{OTH}$, then its projection on axis OY is always equal to zero ($j_{Kop\ y} = 0$). The projection of Coriolis acceleration on axis OX (Fig. 8b), is perpendicular to the plane YOZ which passes through OY and is parallel to $\bar{v}_{OTH\ z}$. Therefore, on the basis of formula (2.8)

$$j_{\text{кор}\,x} = 2v_{\text{отн}\,z}\cdot\Omega\sin(\bar{v}_{\text{отн}\,z}, \bar{\Omega}) = 2v_{\text{отн}\,z}\cdot\Omega,$$

since $\sin(\bar{v}_{OTH\ z}, \bar{\Omega}) = 1$ since the angle between vectors $\bar{v}_{OTH\ z}$ and $\bar{\Omega}$ is equal to 90°. Projection $\bar{j}_{Kop\ x}$ is directed in a positive direction of the axis OX, i.e., positive since the end $\bar{v}_{OTH\ z}$ turns together with the Earth along arrow A (Fig. 8b).

The projection of Coriolis acceleration on axi OZ ($j_{Kop\ z}$) is perpendicular to the plane XOY, which passes through the axis OY and is parallel to the vector

$\bar{v}_{OTH\ x}$. Therefore

$$j_{\text{кор}\ z} = 2v_{\text{отн}\ x} \cdot \Omega \sin(\bar{v}_{\text{отн}\ x}, \bar{\Omega}) = -2v_{\text{отн}\ x} \cdot \Omega,$$

since $|\sin(\bar{v}_{OTH\ x}, \bar{\Omega})| = +1$ (angle between $\bar{v}_{OTH\ x}$ and $\bar{\Omega}$ is equal to 90°).

The minus sign for $j_{\text{кор}\ z}$ was obtained because the end of $\bar{v}_{OTH\ x}$ during the rotation of it together with the Earth turns along arrow B, i.e., opposite the positive direction of axis OZ.

Let us estimate the magnitudes of acceleration of migratory motion and Coriolis acceleration. The magnitude of the acceleration of migratory motion (2.7) depends on the distance of the rocket to the axis of rotation of Earth and on the angular velocity of the rotation of the Earth Ω which is equal to quotient of the division of 2π (angle in radians of a full turn of the Earth around its axis) by the time of this turn (23 hours 56 minutes and 4 seconds), i.e.,

$$\Omega = \frac{2\pi}{(23 \cdot 60 + 56)60 + 4} = 7{,}292 \cdot 10^{-5} \frac{1}{\text{sec}}.$$

Let us assume the rocket is in the plane of the equator at a distance of 2000 km from its surface, which is about 8370 km from the center of the Earth. Then the acceleration of migratory motion will be

$$j_{\text{пер}} = 8370 \cdot 10^{3} (7{,}292 \cdot 10^{-5})^{2} = 0{,}044\ \text{m/sec}^{2}.$$

From the formula (2.3) when r = 8370 km the acceleration of gravity is equal to $g_T = 5.70$ m/sec^2. As can be seen from the formula the translational acceleration is small since it comprises 0.77% of the magnitude of g_T, the acceleration of gravity. Therefore the acceleration of migratory motion in calculations of trajectories of rockets of a near and even average range of action is often considered together with g_T (Fig. 9) calling their geometric difference $\bar{g} = \bar{g}_T - \bar{j}_{\text{пер}}$ the acceleration of gravity. The acceleration of gravity is directed in general not to the center of the Earth but at an angle Λ called the geographic latitude of the site of point A. Because of the smallness of the magnitude $j_{\text{пер}}$ in calculations of trajectories it is possible to consider that $\bar{g}$ is directed to the center of the Earth at an angle λ, the geocentric latitude of the site of point A.

Let us look how g will be changed in the trajectory of the rocket. From triangle ABC (Fig. 9)

$$g = \sqrt{g_T^2 + j_{\text{пер}}^2 - 2g_T j_{\text{пер}} \cos\lambda},$$

or

$$\frac{g}{g_T} = \sqrt{1+\left(\frac{j_{пер}}{g_T}\right)^2 - 2\left(\frac{j_{пер}}{g_T}\right)\cos\lambda} \qquad (2.12)$$

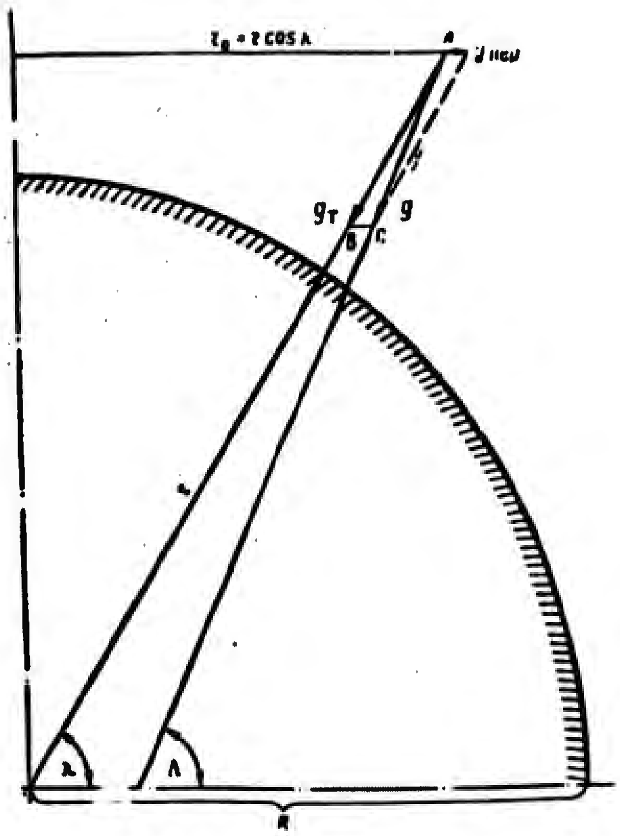


Fig. 9. Joint calculation of acceleration of gravity g_T and acceleration of the migratory motion $j_{пер}$ with the help of the acceleration of gravity g (the rocket is at point A above the surface of the Earth).

Since, as we were convinced, $j_{пер}$ is considerably less than g_T, then the square of their relation as compared to unity can be disregarded and instead of formula (2.12) we obtain

$$\frac{g}{g_T} = \left(1 - 2\cdot\frac{j_{пер}}{g_T}\cdot\cos\lambda\right)^{\frac{1}{2}}. \qquad (2.13)$$

Let us become acquainted with a more simple formula than (2.13); it is also an approximate formula for the determination of g, which we will obtain from formula (2.13) by applying the widespread method of expansion by the binomial theorem. The formula of the binomial expansion has the form:

$$(a+b)^n = a^n + \frac{n}{1}\cdot a^{n-1}b + \frac{n(n-1)}{1\cdot 2}\cdot a^{n-2}b^2 + \ldots + b^n. \qquad (2.14)$$

Let us expand expression (2.13) by the formula (2.14):

$$\frac{g}{g_T} = 1 - \left(\frac{j_{пер}}{g_T}\right)\cos\lambda + \frac{1}{4}\left(\frac{j_{пер}}{g_T}\right)^2\cos^2\lambda + \ldots$$

Disregarding members containing $\left(\frac{j_{пер}}{g_T}\right)$ in the second power and above we obtain

$$\frac{g}{g_T} = 1 - \left(\frac{j_{пер}}{g_T}\right)\cos\lambda, \qquad (2.15)$$

where $j_{пер} = r_0\Omega^2$, or $j_{пер} = \Omega^2 r \cos\lambda$ (Fig. 9).

Dividing the value of formula (2.3) by formula (2.4) we get

$$g_T = g_{T0}\cdot\frac{R^2}{r^2}, \qquad (2.16)$$

and then placing g_T from formula (2.16) and $j_{пер}$ into formula (2.15) we find

$$\frac{g}{g_T} = 1 - \frac{\Omega^2 r^3 \cos^2\lambda}{g_{T0}R^2}. \qquad (2.17)$$

Formula (2.17) shows that the ratio of the acceleration of gravity to the acceleration of terrestrial gravity depends on the distance r of the rocket to the center of the Earth and on the geocentric latitude λ and permits considering the change of g on the trajectory. In Table 1 results are given of calculations by this formula for different r and λ.

From Table 1 one can see that what with flights of rockets in space in the vicinity on the Earth at a distance of $r = 1.2R$ from the center of the Earth with an accuracy of about 0.5%, it is possible to consider the acceleration of gravity equal to the acceleration of terrestrial gravity. The nearer the pole ($\lambda = 90^{\circ}$) the less the difference between g and g_T.

Table 1

λ \ r	R	1,1 R	1,2 R	1,5 R	2,0 R
0°	0,9966	0,9954	0,9940	0,9884	0,9724
45°	0,9983	0,9977	0,9970	0,9942	0,9862
90°	1,0000	1,0000	1,0000	1,0000	1,0000

Formula (2.16) permits establishing the change of g_T with altitude. Thus, for instance, at an altitude of 100 km above the surface of the Earth $g_T = 9.52$ m/sec^2, i.e., approximately 3% less than at the surface of the Earth. If, however, the altitude of the ascent reaches 500 km g_T will decrease in magnitude about 14%. Hence it is clear why in calculations of trajectories of rockets intended for firing at comparatively short distances the acceleration of terrestrial gravity and consequently the acceleration of gravity are taken constant for the whole trajectory. In calculations of trajectories of rockets for firing at great distances it is necessary to consider the variability of g_T with altitude and to determine its magnitude by expression (2.16). If the calculation is approximate then we assume $g = g_T$; with the necessity to obtain magnitude g_T with possibly greater accuracy at each point of trajectory one should determine it by the formula (2.17) or even the formula (2.12).

The direction of the acceleration of gravity can be considered constant (along radius of the Earth at the launch point) only in calculations of trajectories of the flight of rockets at short distances; in other cases one should consider that it on the whole trajectory is directed to the center of the Earth. Only in the most responsible calculations of trajectories of intercontinental rockets is it necessary to consider that vector $\bar{g}$ is directed at angle λ to the equatorial plane of the Earth.

Let us estimate the magnitdue of Coriolis acceleration. Formulas (2.11) permit concluding that Coriolis acceleration is directly proportional to v_{xoz} the projection of the speed of the rocket on plane X, O, Z, since

$$j_{кор} = \sqrt{j^2_{кор\,x} + j^2_{кор\,y} + j^2_{кор\,z}} = 2\Omega\sqrt{v^2_{отн\,x} + v^2_{отн\,z}} = 2\Omega v_{xoz}. \tag{2.18}$$

Calculations by formula (2.18) show that when v_{xoz} = 1000 m/sec, $j_{кор}$ = 0.15 m/sec^2, i.e., it comprises approximately 1.5% of the acceleration of gravity. If v_{xoz} = 200 m/sec a neglect Coriolis acceleration will lead to an error of 0.3%. It should be noted that v_{xoz} changes in the trajectory, and therefore the magnitude of the error changes.

With comparatively small flying ranges the Coriolis acceleration is considered by means of the introduction of corrections for the full range of firing.

By fulfilling calculations of trajectories with a range of more than 50 km we should consider the curvature of Earth (Fig. 10). Instead at point C', as this is obtained in the assumption that the Earth is flat, the rocket will fall at point C.

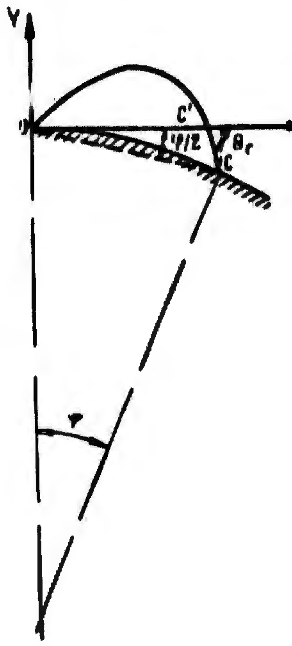


Fig. 10. Diagram illustrating the influence of the curvature of the Earth on the complete flying range of the rocket.

The difference between the distance along the arc of the circumference of the Earth OC = L and along the straight line OC' = X can be calculated by the approximate formula:

$$\delta = \frac{L-X}{X} = \frac{\varphi}{2\sin\varphi/2\left(\cos\varphi/2 - \frac{\sin\varphi/2}{\operatorname{tg}\theta_c}\right)} - 1, \tag{2.19}$$

obtained in the assumption that the segment of trajectory C'C is a straight line. The error from this assumption is small, since the magnitude of segment C'C is small. From formula (2.19) one can see that correction for the range of firing considering the curvature of the Earth depends on the angular distance φ and angle θ_c of inclination tangent to the trajectory at the point of fall.

Figure 11 gives curves for determining the correction for the range of firing considering the curvature of the Earth. These curves are obtained by calculations by the formula (2.19) and show, for instance, that for trajectories with θ_c = 60° and with the firing range X = 200 km the calculation of the curvature of the Earth increases calculation range by

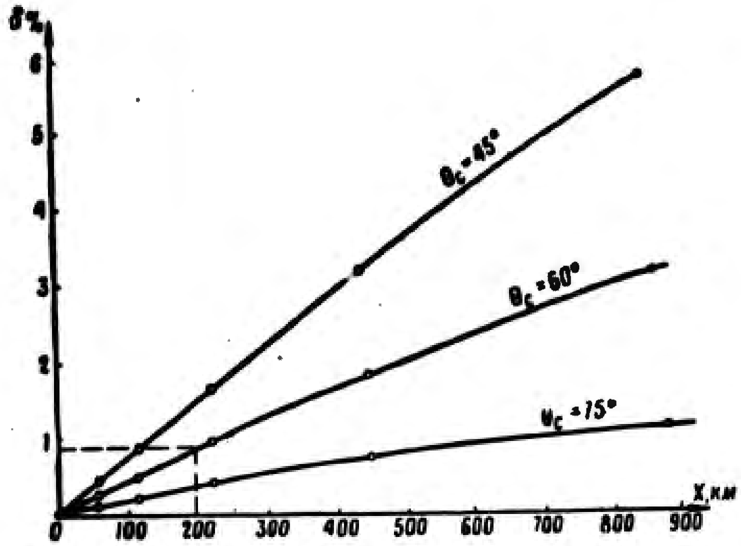


Fig. 10. Curves of the dependence of the error in the determination of range owing to the disregard of the curvature of the Earth δ on the magnitude of range X and angle of inclination tangent to the horizon at the point of fall of the rocket θ_c.

0.9%. The error in the flying range of rockets from disregarding the curvature of the Earth when X = 50 km and $\theta_c = 60^\circ$ will be approximately 0.2%, and it is sometimes disregarded.

It should be remembered that in calculations of trajectories of rockets the solution to the problem of the calculation or disregard of the curvature of the Earth and its rotation depends on the purpose of the calculation and requirements for its accuracy.

§ 3. The Earth's Atmosphere and Its Properties

The atmosphere is the air envelope of the Earth. The flight of rockets in space in the vicinity of the Earth occurs within limits of more or less dense layers of the atmosphere, which definitely affects the characteristics of the movement of the rocket. For instance, a ballistic rocket with a flying range of about 5000 km ascends above the surface of Earth to an altitude of about 1000 km. In order to consider the influence of the atmosphere on the flight of rockets it follows first of all to know the structure of the atmosphere and its properties.

Long term investigations of the atmosphere at first with the application of aerostats, aircraft, sounding balloons, and in recent years with help of meteorological and geophysical rockets, earth satellites, and spaceships with a person on board, allowing the penetration into its highest layers, accumulated much data explaining the structure of the Earth's atmosphere and its properties. Although everything is still not clear the process of investigating the atmosphere continues, and for the solution of problems of the theory of flight definite conclusions can be made about the struture and properties of the atmosphere of the Earth.

For a study of the laws of the distribution of meteorological elements at altitudes up to 25 km the method of sounding of the atmosphere with the help of radiosondes fastened to a balloon filled with hydrogen found wide application. Radiosonde is transducer of temperature, pressure, and sometimes humidity. Signals of temperature, pressure, and humidity are transmitted by code by means of a radio transmitter. After deciphering these signal we obtain experimental dependences for temperature, pressure, and humidity.

The law of the change of pressure with altitude, determined by the barometric formula (2.29), agree with experimental data with sufficient accuracy.

In connection with this we are often limited to the measurement of the temperature at different altitudes, and the pressure and air density are calculated by the theoretical dependences (2.29) and (2.31). Partial experimental data can be obtained by measurements of temperature and pressure with help of the aircraft on which the necessary instruments are installed. But this method does not have a wide application, since much time is required for measurement, the altitude of the flight is limited, and other reasons.

A second widely applied method is the method of sounding of the atmosphere by means of meteorological rockets.

The temperature of the atmosphere is measured by resistance thermometers and pressure by manometers of the corresponding range of pressures. At an assigned altitude the nose cone with the measuring equipment is separated from the rocket and descends by parachutes. In the descent of the nose cone transducers record the change in pressure and temperature, and the equipment transmits the obtained data to Earth. Then the influence of the interaction of the nose cone with the atmosphere is considered. Using the equation of the state of gas, barometric formulas, and also laws of the dynamics of gases, by the formulas of time $[T(t), p(t)]$ we find the dependences for temperature and pressure in the function of altitude.

Upper-atmosphere research requires the measurement of coordinates of the nose cone for different moments of time so that the function of time $[T(t), p(t)]$ can be transferred to the function of altitude $[T(y), p(y)]$.

With this method the stratosphere to an altitude of approximately 80 km is studied.

The third method is based on the use of geophysical rockets and artificial earth satellites.

This method is used for the investigation of the ionosphere (for altitudes greater than 80-100 km). The study of the ionosphere is carried out by different methods. To measure pressure and density of the atmosphere manometers are used allowing to determine atmospheric pressure up to 10^{-9} mm Hg.

With the investigation of the atmosphere at altitudes of about 450 km, the method of analysis of the diffusion of sodium vapor released from the rocket is also applied.

The atmosphere consists basically of dry air and water. The dry air is a mixture of nitrogen (~78%), oxygen (~21%), argon (~0.94%) and other gases (hydrogen, helium, carbon dioxide, and others).

The whole atmosphere can be conditionally divided into a number of layers with properties different from each other. The lower part of the atmosphere is called the troposphere extends from the surface of the Earth to such altitudes where the air temperature ceases to drop with altitude. The drop in temperature with altitude can be explained in the following way. Dry air is almost transparent for solar rays and is heated basically due to the heat of the Earth; therefore, the further from the surface of the Earth the less the air temperature. At the boundary of the troposphere thermal terrestrial radiation ceases, and the temperature no longer drops. The boundary of the troposphere depends on the latitude and longitude and on the time of the year. Thus the average annual altitude of the troposphere above the equator is approximately 17 km and above the pole where the Earth's surface is colder about 8 km. In the summer when the Earth returns air of greater heat the altitude of troposphere is higher and in winter lower. At latitudes of about 45° the altitude of the troposphere on the average is 11-12 km.

However, sometimes the air temperature with a rise from the surface of the Earth is at first increased and then starts to decrease. This phenomenon is called temperature inversion and occurs in the winter in the morning, or even in the summer after a cold night when the Earth during the night is cooled so much that it does not return the heat but on the contrary obtains it from layers of air adjacent to its surface and cools them.

In the troposphere a large part of the mass of air is found (up to altitudes of 10 km this part is approximately 75%) and almost all the moisture of the atmosphere. Therefore, in the troposphere clouds are formed, precipatation falls, there occurs an intense agitation of the air, and winds blow both in a horizontal and vertical direction (ascending and descending air streams). This leads to a great instability of the properties of the troposphere not only in the time of the year but also in small time intervals.

Directly above the troposphere there is the layer of tropopause (substratosphere), a transition layer between the troposphere and stratosphere; its thickness is 1-2 km.

Further there is the layer of air called the stratosphere extending to altitudes of the order of 80 km. The stratosphere is characterized by the almost complete absence of humidity, and the winds blow from east to west, i.e., opposite to the rotation of the Earth and are almost constant in direction. Temperature in stratosphere varies intensely only in the layer at an altitude from 35 to 50 km, where there is ozone which absorbs part of the shortwave solar radiation. From this the air temperature increases sometimes to a magnitude of +100°C, whereas on the average in the stratosphere it is -85°C at the equator and -45°C at the poles.

Above the stratosphere the ionosphere is located. It contains only about 0.5% of all the air mass of the atmosphere. In the ionosphere the air is greatly rarefied. In such a state the air absorbs most of the shortwave solar radiation and its temperature reaches 1500° and more by day, but at night drops considerably. Such fluctuations of temperature are accompanied by great vertical shifts of air. In the ionosphere night air flow and polar auroral occur. For instance at an altitude of the order 370 km a swayed curtain of beams is observed. These phenomena are electromagnetic in character, since from the interaction with solar radiation the air is ionized. The nature of them up to the end has not been clarified.

The influence of the ionosphere on the flight of rockets is small. The resistance of air is only several grams per square meter of cross section of the rocket. Therefore, in calculations of trajectories of long-range rockets, starting from altitudes of the order 120-150 km (and sometimes smaller), the air resistance is not considered. But even a low air density during a prolonged flight of a body can noticeably lower its speed. For instance, a multiturn flight of satellites in the upper layers of the atmosphere leads to such a decrease in their speed at which the satellites enter into the dense layers of the atmosphere and burn if measures of protection are not taken.

The heating of the rocket in the ionosphere from the interaction with air is small. In spite of the high temperature of the air the quantity of it in the ionosphere is so little that the influence of molecules on the surface of the rocket does not cause considerable heating.

The upper layers of the ionosphere located higher than 800 km are called the sphere of dispersion (dissipation). The sphere of dispersion completes the atmosphere. The upper boundary of the atmosphere is difficult to determine exactly. At present it is considered to be at an altitude of about 3000 km. Within the sphere of dispersion the rarefaction is so great that before the collision with each other the air molecules fly very great distances. In other words the length of the free path of the moleucles is very great and attains several hundreds of meters and more. With this the speed of the separate molecules can be so great that these molecules will abandon the field of terrestrial gravity and depart into outer space.

Air, as also any other gas, is characterized by pressure, density, and temperature. Moreover, since in the atmosphere there is moisture (water vapor), then for the charcteristics of properties of humid air it is necessary still to add the content of water vapor per unit volume of dry air.

All these parameters characterizing the properties of the atmosphere change both with altitude and time. The study of regularities of the change in atmosphere and forecasts of its state is the concern of the science called meteorology. The theory of flight without data of meteorology will not allow an accurate calculation of the trajectory of the rocket nor a prediction of the dispersion of the trajectories during the firing by many identical rockets.

In meteorology the air pressure is meas red in millimeters of mercury (mm Hg) and is called barometric pressure, and its magnitude is designated by the letter h. The connection between barometric pressure h in mm Hg and pressure p is in kgf/m^2, is given by the following formula

$$p=\frac{10333}{760}\cdot h=13{,}6h. \qquad (2.20)$$

In gases pressure, temperature, and density cannot be changed arbitrarily and satisfy the equation of state. For dry air within the atmosphere the following equation of state is valid:

$$\frac{p_1}{\Pi_1}=RT, \qquad (2.21)$$

where Π_1 is the mass density of dry air (specific gravity) in kgf/m^3; T is the temperature of dry air in absolute degrees ($^{\circ}$K); R is the gas constant of dry air equal to 29.27 kgf·m/kgf·degree.

Replacing in formula (2.21) value p_1 by barometric h_1 and using formula (2.20) we obtain

$$\Pi_1 = 13{,}6 \cdot \frac{h_1}{RT}. \tag{2.22}$$

The pressure under which humid air is found (mixture of dry air and water vapor), according to the law of Dalton, is equal to the sum of partial pressures of dry air and water vapor (partial pressure is called the pressure which would be in a volume occupied by a mixture of gases if it had been occupied by only one gas of the mixture).

If we designate the partial pressure of water vapor by e, then the barometric pressure in humid air h will be

$$h = h_1 + e. \tag{2.23}$$

The equation of state for steam water vapor can be thus written:

$$\Pi_2 = 13{,}6 \frac{e}{R_2 T}, \tag{2.24}$$

where Π_2 is the specific gravity of water vapor in kgf/m^3; R^2 is the gas constant of water vapor.

Temperatures of water vapor and dry air will be identical, since they are mixed forming humid air.

The specific gravity of humid air Π is composed of the specific gravity of dry air and water vapor, i.e.,

$$\Pi = \Pi_1 + \Pi_2.$$

Substituting values of Π_1 and Π_2 from formulas (2.22) and (2.24) and considering formula (2.23) and also the fact that $R_2 \cong (8/5)R$, we obtain

$$\Pi = \frac{13{,}6h}{RT}\left(1 - \frac{3}{8}\frac{e}{h}\right). \tag{2.25}$$

For the convenience of the calculation of the influence of atmospheric humidity we introduce the concept of virtual temperature into the calculations:

$$\tau = \frac{T}{1 - \frac{3}{8}\frac{e}{h}}. \tag{2.26}$$

This is temperature of dry air for which h and Π are the same as for humid air. Equality (2.25) with an introduction of virtual temperature is simplified:

$$\Pi = 13{,}6 \cdot \frac{h}{R\tau}. \tag{2.27}$$

If one were to consider that meteorologists give immediately the value of virtual temperature taking into consideration atmospheric humidity, then the convenience of introducing the concept of virtual temperature will become clear: it not necessary in calculations of trajectories to consider directly atmospheric humidity. The atmospheric state of the air is completely determined by barometric pressure h, specific gravity Π, virtual temperature τ, and also the speed and direction of the wind.

§ 4. Averaging Meteorological Factors and Standard Atmospheres

Parameters of the atmosphere (or, as it is said, meteorological factors) vary with time. These changes occur not only with the time of year, not only with the time of year, not only with the exchange of day and night, but also in shorter time intervals measured in hours and minutes. Wind is especially variable. Gusts of wind in the lower layers of the troposphere change every 1-2 seconds both the speed and direction of the air streams. In the upper layers of the troposphere jet streams of air are observed, and the rocket entering into these will move in conditions differing from the preceding.

How can we consider the influence of the atmosphere on the flight of rocket if its parameters change all the time? First of all it is necessary to find at each altitude above the surface of the Earth the mean values of parameters of the atmosphere, pressure, temperature, density, and so forth near which their changes occur and then to replace all variables of the values of parameters of the atmosphere by their mean constants. Such an idealized atmosphere with definite meteorological factors of its properties for each altitude will not change with time. This permits comparing data of any calculations of trajectories conducted for such an idealized atmosphere and revealing the influence of certain changes on the flight of the rocket. Regarding, however, the results of these calculations, they will be the average most likely. The influence of the de deviation of weather conditions from the standard is considered in the theory of correction.

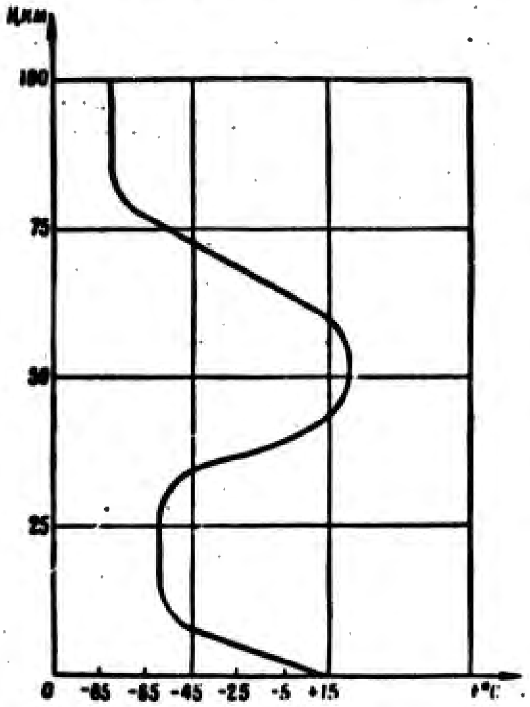


Fig. 12. Curve of the change in virtual temperature with altitude of the terrestrial atmosphere.

In the calculations the normal artillery atmosphere, international standard atmosphere, and temporary standard atmosphere are used. Let us consider their properties and peculiarities.

The normal artillery atmosphere (NAA)

considers the atmospheric humidity, which is especially important with the firing of unguided rockets (missiles) at comparatively short distances.

At sea level in the normal artillery atmosphere the following values are accepted: h_{ON} = mm Hg; Π_{ON} = 1.206 kgf/m^3; e_{ON} = 6.35 mm Hg (50% relative humidity); τ_{ON} = 288.9°K (or +15°C).

Here index "O" signifies that all magnitudes are measured at sea level, and index "N" indicates that they belong to the normal artillery atmosphere (NAA).

In NAA, as in other standard atmospheres, it is considered that through out the air is motionless, i.e., there is no wind.

The change in determining the parameter (virtual temperature) with altitude is shown in Fig. 12 and corresponds to the approximate average annual distribution of it throughout the atmosphere. The normal artillery atmosphere was developed up to altitudes of 30 km. Since at altitudes of more 30 km there is little atmospheric humidity, the value of the virtual temperature and standard temperature practically coincide. In Fig. 12, at altitudes above 30 km data of later investigations are given. Virtual temperature is called the determining parameter because it permits from conditions of vertical equilibrium of the atmosphere finding the change in barometric pressure with altitude and then with the help of the equation of state, the law of the change in the specific gravity of air. Let us show how this is done.

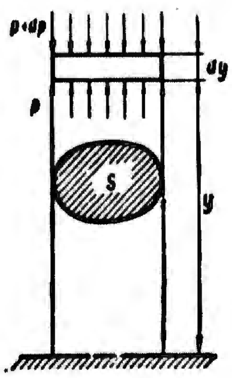


Fig. 13. Diagram of the conclusion of the condition of vertical equilibrium of the atmosphere.

Let us consider first of all the condition of vertical equilibrium of the atmosphere. For this at an altitude of y above sea level we will separate the infinitesimal element of air by the thickness dy (differential of y) with the area of base S (Fig. 13). From below pressure p acts on this element, and from above p + dp, where dp is infinitestimal change of pressure occurring with a change in altitude at an infinitesimal distance dy.

The condition of vertical equilibrium is the equality of weight of separated element of air of the difference of forces of pressure on it from below and from above, i.e.,

$$\Pi S dy = pS - (p + dp) S.$$

Reducing S and reducing similar members we obtain

$$\Pi\,dy = -dp. \tag{2.28}$$

Substituting into formula (2.28) the value of Π and p from expressions (2.27) and (2.20) we get

$$13{,}6\cdot\frac{h\,dy}{R\tau} = -13{,}6\,dh,$$

i.e.,

$$\frac{dh}{h} = -\frac{dy}{R\tau}.$$

This is the simplest differential equation with dividing variables. Solving this equation we obtain

$$\frac{h}{h_{0N}} = e^{-\int_0^y \frac{dy}{R\tau}}. \tag{2.29}$$

Knowing τ(y) from the curve in Fig. 12 we will be able to find by the formula (2.29) $h/h_{ON} = \Pi(y)$. The curve of function Π(y) is shown in Fig. 14. From the curve one can see that the barometric pressure continuously decreases with altitude.

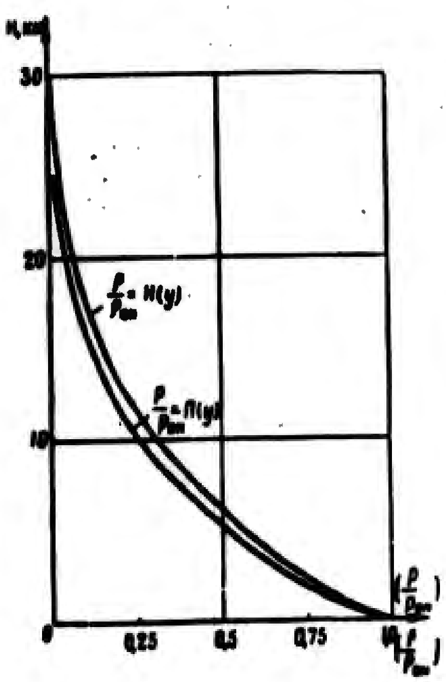


Fig. 14. Curve of functions Π(y) and H(y) infinite pressure and density of altitude of terrestrial atmosphere above sea level.

Then with the help of the equation of state (2.27) we will obtain the law of the change of the specific gravity of air throughout the atmosphere. Indeed, by knowing how τ is changed and also h with altitude from formula (2.27) we will obtain (when y = 0)

$$\Pi_{0N} = \frac{13{,}6\,h_{0N}}{R\tau_{0N}}. \tag{2.30}$$

The division of formula (2.27) by formula (2.30) gives

$$\frac{\Pi}{\Pi_{0N}} = \frac{h}{h_{0N}}\cdot\frac{\tau_{0N}}{\tau}. \tag{2.31}$$

This function is designated $H(y) = \frac{\Pi}{\Pi_{0N}}$. Its curve, obtained by calculation from formula (2.31) is also shown in Fig. 14.

Let us turn to the consideration of peculiarities of the international standard atmosphere (MCA) [ISA]. It differs from the NAA by the disregard of atmospheric humidity and also by certain values of parameters of the air at sea level. In ISA these are accepted: $T_0 = 288°K$; $\rho_0 = 0.125$ kgf·sec^2/m^4 (this corresponds to $\Pi_0 = 1.255$ kgf/m^3); $p_0 = 1.0333$ kgf/cm^2 (or 760 mm Hg).

At an altitude of 11 km above sea level the curve T_y is ISA has an angular point, which is inconvenient in the fulfillment of the calculation of trajectories by the numerical method. In other respects the character of the change of temperature and also the pressure and air density with altitude remains the same as in NAA, and therefore there is no need to cite the curves of these functions.

In fulfilling calculations of trajectories of rockets it is necessary to use tables for T(y), H(y), Π(y), which are given in books [13] and [26] and in other literature.

The temporary standard atmosphere [TSA-60](BCA-60) contains a height distribution mean values of basic thermodynamic parameters and other physical characteristics of the atmosphere for altitudes up to 200 km. TSA-60 differs from NAA and ISA basically by the calculation of thermodynamic parameters and physical characteristics up to altitudes of 200 km.

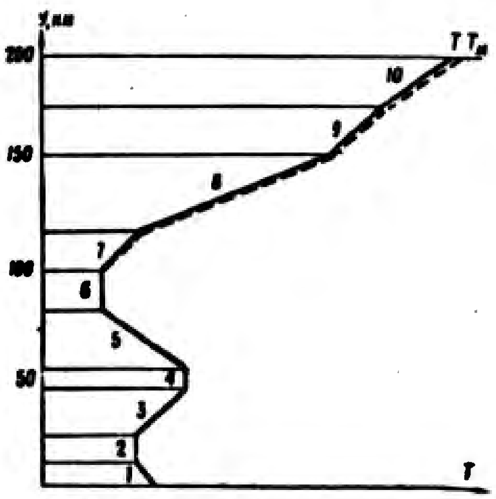


Fig. 15. Law of the change in temperature with altitude.

Tables of TSA-60 contain values of temperature, barometric pressure, density, function of pressure and density, speed of sound, dynamic and kinematic coefficients of viscosity, acceleration of gravity, and the mean path of the molecules.

The input value in the tables is altitude. The initial dependences are the laws of the change in temperature and molecular weight of the air depending upon altitude.

According to the character of the change in temperature with altitude, the atmosphere is divided into 10 layers (Fig. 15), in each of which a linear change is set about of the "molecular" temperature by the so-called geopotential altitude Φ, determined from the equation

$$\frac{d\Phi}{dy}=\frac{g_y}{g_{y0}}. \qquad (2.32)$$

The difference is in the magnitude of the temperature gradient

$$a_M=\frac{dT_M}{d\Phi}\ \text{(deg/m)}.$$

The kinetic temperature is connected with the molecular dependence

$$T=T_M\cdot\frac{M}{M_0},$$

where M_0 and M are the molecular weight of air at sea level and at altitude y.

For y = 0-95 km it is assumed M = const = 28.966 g/mole = M_0 (therefore, up to 95 km $T = T_M$). At altitudes above 95 km, due to the dissociation of oxygen, the moleuclar weight is changed according to the law

$$M = 23 + \frac{5{,}966}{145000} \sqrt{145000^2 - (y - 95000)^2}$$

(when y = 200 km, M = 27.114 g/mole).

At these altitudes T_M is considered by the formula

$$T_M = T_{M_*} + a_M (\Phi - \Phi_*),$$

where Φ_* and T_{M*} correspond to the lower boundary of the layer within which the value of gradient a_M is constant.

A change in pressure with altitude is determined by the equation of vertical equilibrium of the atmosphere. Considering air an ideal gas and considering the change in g_T with altitude we will have

$$\frac{dh}{h} = \frac{dp}{p} = -\frac{g_T}{g_{T0} RT} \cdot dy. \qquad (2.33)$$

But since the acceleration of g_T depends on altitude (2.16) we introduce function Φ.

Substituting into formula (2.32) the value g_T from the expression (2.16) and integrating from 0 to y, we obtain the formula for the scaling of y on Φ:

$$\Phi = \frac{R_3 y}{R_3 + y}, \qquad (2.34)$$

where R_3 is the mean radius of the earth.

From formula (2.33), taking into account expression (2.32), we will have

$$\frac{dp}{p} = -\frac{1}{RT} d\Phi. \qquad (2.35)$$

Integrating expression (2.35) we obtain formulas for pressure:

– in an isothermal layer (the same as (2.4, etc) in Fig. 15):

$$\lg p = \lg p_* - 0{,}434294 \frac{1}{RT} (\Phi - \Phi_*);$$

– in a layer with the temperature variable according to the linear law (1.3, etc.):

$$\lg p = \lg p_* - \frac{1}{a_M R} \lg \frac{T_{M_*} + a_M (\Phi - \Phi_*)}{T_{M_*}},$$

where p_* is the pressure on the lower boundary of the layer.

Formulas for densities are analogous.

The dynamic coefficient of viscosity is determined for altitudes up to 90 km by the Sutherland formula

$$\mu = \mu_0 \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + 110{,}4}{T + 110{,}4},$$

where $T_0 = 273.16^\circ K$;

μ_0 when $T = T_0$.

The speed of sound is found from the expression

$$a = \sqrt{kgRT} = 20{,}0463\sqrt{T} \text{ m/sec.}$$

Initial values of characteristics of the atmosphere (at sea level are determined on the basis of experiments and theoretical dependences: $p_0 = 1.03323$ kgf/cm^2 = = 760 mm Hg at latitude $45^\circ 32'40''$ at a temperature $T = 273.16^\circ K$ and specific gravity 13.595 g/cm^3; $T_0 = 288.16^\circ K$; $M_0 = 28.966$ g/mole; $\mu_0 = 175 \cdot 10^{-6}$ kgf·sec/m^4; $g_0 = 9.80665$ m/sec^2; $R = 29.27$ kgf-m/kgf·deg.

§ 5. Influence of the Change in Meteorological Factors on the Flight of Rockets

Let us consider the qualitiative influence of the change in meteorological factors at different points of the trajectory of the flight of the rocket.

First of all let us dwell on the influence of air density ρ (or its specific gravity Π) on the flight of rockets. An increase in air density leads to an increase in the value of all aerodynamic forces and moments (including the resisting force to the flight of the rocket). The increase in the resisting force to the flight of rocket will cause a deceleration of the flight and, consequently, range. With an increase in air density, moreover, osciallations of the rocket around the center of the mass caused by some factors (shock during launch, gust of wind, and so forth) will attenuate more quickly.

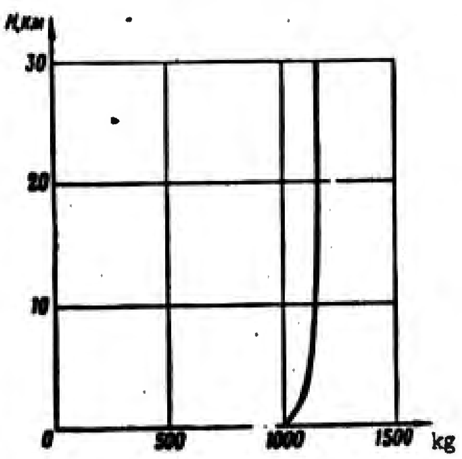


Fig. 16. Curve of the change in tractive force with altitude.

Let us now investigate the influence of the change in air pressure on the flight of rockets. Air pressure directly affects the magnitude of the tractive force.[1] A decrease in air pressure increases the static componenet of the thrust i.e., the second component in the expression (1.12). If we remember that with altitude air pressure quickly decreases, then it will become clear why tractive force increases in proportion to the ascent of the rocket upwards (Fig. 16). The comparatively small increase in tractive force is explained by the fact that the second components in expression (1.12), depending on air

[1]Tractive force = thrust [Tr. Ed. note].

pressure, does not exceed 10-15% the magnitude of tractive force.

The influence of the change in air pressure on the flight of rockets occurs also through air density. From the equation of the state of air (2.25) one can see that with an increase in h (with a constant temperature T) there occurs an increase in the specific gravity of air Π (or $\rho = \Pi/g$). Therefore, an increase in h, for instance, will decrease the flying range of the rocket, and, inversely, if the atmospheric pressure decreases then the air density decreases and the flying range increases.

Let us see how the temperature of the air influences the flight of rockets. This influence is carried out in two ways. The first is through air density. A temperature rise (with a constant pressure), as the equation of state (2.25) shows, leads to a decrease in air density, which increases the flying range of rockets. Secondly the influence of temperature is carried out in the following way. If the temperature increases, the value of speed of sound in air increases, and since from physics it is known that the speed of sound is equal to

$$a = \sqrt{\kappa g R T},$$

where κ is the adiabatic index (for air κ is equal to 1.405).

The relation $\frac{v}{a}$ depends on the speed of sound, where v is the speed of flight of the rocket. An increase in a will decrease this relation. In turn the air resistance to the flight of the rocket depends on $\frac{v}{a}$. With low speeds of the flight a decrease in the relation $\frac{v}{a}$ will lead to a decrease in the air resistance and an increase in flying range of the rocket, and with high speeds of the flight (as a rule, supersonic), on the other hand, a decrease in the relation $\frac{v}{a}$ increases the air resistance, and the flying range decreases.

As we see the total influence of temperature on the flying range is complicated and requires for manifestation in each concrete case numerical calculations.

The next meteorological factor, atmosphereic humidity, also influences the flight of rockets. Formula (2.26) shows that the value of virtual temperature depends on humidity. With an increase in humidity the virtual temperature is increased. We have just now considered what occurs with an increase in temperature. Let us add only the following. Atmospheric humidity is changed in rather wide range (from 0 to 1.2% content of water vapor per volume), but this changes the virtual temperature insignificantly, less than 1%. If we remember that humidity basically is concentrated within the troposphere, it will become clear why it follows to consider the influence of humidity only with the firing of rockets at short distances when a great part of the trajectory is within the troposphere.

Wind changes the direction and flying range of a rocket owing to the change in magnitude and direction of the relative speed of its movement in air.

The action of the wind on the rocket results in the turning of the axis of the rocket around the center of mass. On the powered flight trajectory when the rocket engine is operating, with the turn of the axis the direction of the tractive force will also turn. Consequently, there will appear an additional lateral component of thrust which will deflect the rocket to the side of the calculated trajectory.

Thus on a powered-flight trajectory the rocket will have two additional lateral forces applied to it: aerodynamic, created directly by the wind, and gas-dynamic, being part of the tractive force and caused by the turn of the axis of the rocket from the wind. Let us consider the influence of a cross wind on the flight of the rocket on the powered flight trajectoy and on the segment of free flight in the example of a rocket whose center of mass A is nearer to the nose cone than point B of the application of resultant aerodynamic forces (Fig. 17).

On the powered flight trajectory the direct influence of the cross wind with a speed $W_{бок}$ on the rocket creates an additional aerodynamic force $R_{бок}$ applied at point B. This force is equivalent to force $R_{бок}$ applied in the center of mass A of the rocket and to the force couple forming the moment $M_{бок}$ (reduction of force $R_{бок}$ to point A). This moment will cause the turn of the axis of the rocket at a certain angle $\Delta\varphi$ with reference to the tangent to the trajectory. At the same angle the line of force of thrust P turns. Let us expand force P on two components: $P_{кас}$, in the direction of the tangent to the trajectory, and $P_{бок}$, perpendicular to this tangent.

From Fig. 17 one can see that forces $R_{бок}$ and $P_{бок}$ are applied at point A and directed to various sides. In order that the rocket be accelerated the tractive force should be greater than the aerodynamic resisting forces. Therefore, it is obtained that $P_{бок} > R_{бок}$ and from action of wind the rocket will deviate to the side whence the wind blows, i.e., windward.

There is a different picture on the segment of free flight: where there is no tractive force there is no $P_{бок}$, and therefore the rocket will deviate under

Fig. 17. Diagram of the power load of a rocket from action of a cross wind with speed $W_{бок}$.

the action of force $R_{бок}$ to the side which the wind blows, i.e., leeward. Accurate calculations show that with an identical cross wind on the entire trajectory the drift of rocket due to the action of the wind on a powered flight trajectory can be even several times more than on the segment of free flight, and although the deflection occurs to various sides the impact point of the rocket all the same will be deflected from the plane of firing to the side which the wind blows.

Besides the lateral component of the wind, a longitudinal component acts on the rocket directed along the tangent to the trajectory of the rocket flight or against the wind. The influence of this longitudinal component of the flight of the rocket is directed with the rocket motion and decreases them if it acts opposite the direction of the flight of rocket.

It should be stipulated that, representing any wind by two components, lateral and longitudinal, we consider that the lateral component can be in any plane (horizontal, slanted) provided it is perpendicular to the tangent to the trajectory of the rocket. Such a decomposition of wind into two componenets is convenient for the manifestation of the qualitative influence of it on the flight of the rocket.

Having considered influence of the Earth's gravitational field and its atmosphere on the flight of rockets, let us turn to a detailed study of one of the basic forces applied to the rocket, air resistance which very considerably changes the parameters of its motion in the trajectory.

C H A P T E R III

THE INFLUENCE OF AIR RESISTANCE ON THE FLIGHT OF ROCKETS

§ 1. Aerodynamic Forces and Moments Affecting a Rocket in Flight and Their Coefficients

In the first chapter briefly all forces having an effect on rocket, among them air resistance, are characterized. Let us now consider this force in greater detail and the influence which it has on the flight of rockets.

The force of aerodynamic drag R is composed of forces of air pressure directed along the normals to the surface of the rocket and forces of air friction about this surface and tangent to it. This resultant force R is applied to the rocket at a point which is called the center of pressure (point D in Fig. 18). Usually the center of pressure does not coincide with the center of mass of a rocket. If one were to lead the force of aerodynamic drag to the center of mass of a rocket, as this was done in the second chapter adding to the center of mass A two forces R' and R'' equal in magnitude to R, parallel to it, and directed to the opposite sides, then the action of R on the rocket will be the same as the action of the force $R' = R$ and moment $\bar{M}_a$ formed by the force couple R-R''. Force $\bar{R}'$ applied in the center of mass of the rocket is called the main vector of aerodynamic forces, and moment $\bar{M}_a$, the main moment of aerodynamic forces. The basic action of the main vector of aerodynamic forces

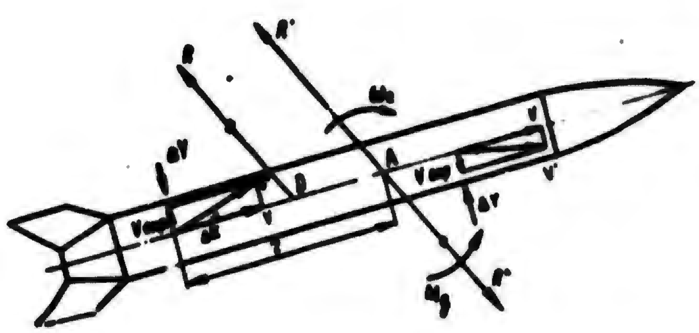


Fig. 18. Diagram of the reduction of air resistance to the center of mass of the rocket A.

consists in the deceleration of the flight of the rocket. Under the action of the main moment of aerodynamic forces the rotation of the rocket around the center of its mass occurs.

How do we determine the magnitude, direction, and point of application of R? The direct summation of forces of friction and pressure continuously distributed on the surface of the rocket are very complicated and laborious, since for this it is necessary to know the law of distribution of all these forces and to produce the complicated calculations.

First of all let us find the common formula for air resistance. From physical considerations it was established that air resistance depends on the form and dimensions of the rocket, the speed of its flight and rotation around the center of mass, on air density and its viscosity, on the speed of sound in air determined by its temperature and influencing the distribution of perturbations in the air, and also on the position of the rocket in the trajectory, i.e.,

$$R = f(v, d, \rho, \mu, a, \omega, \alpha, \beta), \tag{3.1}$$

where v is the speed of the flight of the rocket; d — characteristic dimension of rocket (for example, caliber); ρ — air density; μ — coefficient of viscosity of air; a — speed of sound in air; ω — instantaneous angular velocity of rotation of the rocket around the center of mass; α — angle of attack; β — angle of slip.

In formula (3.1) we considered the process of the flowing around the rocket by stable air and did not consider the influence on R of heat radiation from the air to the surface and certain other factors insignificantly changing the magnitude of the air resistance. The form of the rocket determines specific form of formula (3.1).

Experiments confirm that there is such a functional dependence (3.1). However, it is inconvenient since it has too many independent variables. Is it possible to decrease the number of parameters on which R depends? Yes, according to the theory of dimension and similarity. This theory is called to give preliminary information for a qualitatively-theoretical analysis of complicated physical phenomena for which either there is no clear scheme described by mathematical dependences, or if there is such a scheme the solution of mathematical dependences describing it is encountered by insoluable difficulties. With the help of the theory of dimension and similarity it was possible not only to convert the form of the functional dependence (3.1) so that the independent variables would be dimensionless, but also to reduce them by three. As a result we obtained the formula

$$R=\frac{\rho v^2}{2}\cdot\frac{\pi d^2}{4}\cdot\varphi\left(\frac{vd\rho}{\mu};\ \frac{v}{a};\ \frac{\omega d}{v};\ \alpha,\ \beta\right). \tag{3.2}$$

where φ is the sign of a certain functional dependence.

In general:

$$R=qSC_R \tag{3.3}$$

where $q=\frac{\rho v^2}{2}$ is the velocity pressure; $S=\frac{\pi d^2}{4}$ is the area of the middle section of the rocket; $\frac{v}{a}$ is the number of M; C_R is the dimensionless aerodynamic coefficient dependent on a number of factors (M, α, β, and others).

Experiments show that in separate cases factors $\frac{vd\rho}{\mu}$, $\frac{\omega d}{v}$, α, and β weakly affect R, and it is possible not to consider them. For instance, in the theory of the flight the following formula obtained in the assumption of the coincidence of the longitudinal axis of the rocket with a tangent to the trajectory is widespread;

$$R=\frac{\rho v^2}{2}\cdot SC_x\left(\frac{v}{a}\right), \tag{3.4}$$

where $C_x\left(\frac{v}{a}\right)$ or $C_x(M)$ is the coefficient (sometimes function) drag.

However, the theory of dimension and similarity is powerless to determine concretely the form of the functional dependence C_R or C_x; it is different for bodies of different form and can be determined either by experiments or theoretically according to the approximate scheme of physical phenomena with the flowing around the rocket by air (the latter is rarely successful).

Investigation with the help of the theory of dimension and similarity permitted obtaining the following formula for the magnitude of the main moment of aerodynamic forces $\bar{M}_a$:

$$M_a=qSml, \tag{3.5}$$

where, besides the earlier designated magnitudes, l is the characteristic dimension of the rocket (for instance, length); m is the dimensionless aerodynamic coefficient dependent on the form of the rocket, its position in the trajectory, the speed of rotation, and so forth.

Dependences (3.3) and (3.5) are useful and for the separate components $\bar{R}$ and $\bar{M}_a$.

Let us expand the main vector of aerodynamic forces on components along flow axes (X, Y, Z) and connected (X_1, Y_1, Z_1) systems of coordinates (Fig. 19). These components have the following names:

a) in the continuous system of coordinates: X – drag; Y – lift; Z – lateral force; b) in the connected system of coordinates: X_1 – longitudinal force; Y_1 – normal force; Z_1 – lateral force.

Components of the main moment of aerodynamic forces in continuous and connected systems have the following designations: M_x and M_{x_1} – moments of bank in continuous and connected systems of coordinates respectively; M_y and M_{y_1} – yawing moments; M_z and M_{z_1} – pitching moments (sometimes they are called longitudinal moments).

By knowing components $\overline{R}$ or $\overline{M}_a$ in any system of coordinates, it is easy find their values. For instance:

$$R=\sqrt{X^2+Y^2+Z^2}.$$

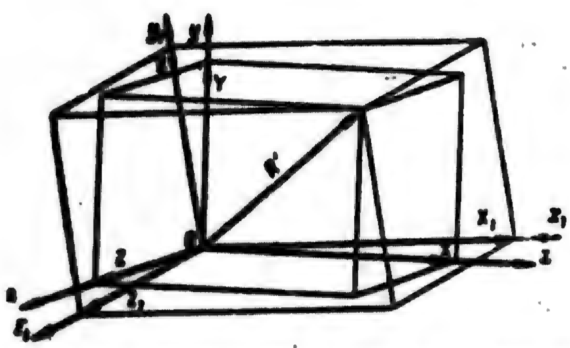


Fig. 19. Expansion of the main vector of aerodynamic forces $\overline{R}'$ on components in connected and coninuous systems of coordinates.

Directions of R and M are determined by three values of angles which can be calculated by the formulas:

$$\cos(x, R)=\frac{X}{R};$$

$$\cos(y, R)=\frac{Y}{R};$$

$$\cos(z, R)=\frac{Z}{R},$$

or

$$\cos(x_1, \overline{M})=\frac{M_{x_1}}{M_a}; \quad \cos(y_1, \overline{M})=\frac{M}{M_a}; \quad \cos(z_1, \overline{M})=\frac{M_{z_1}}{M_a}.$$

Inasmuch as the names of part of the components $\overline{R}$ and $\overline{M}$ coincide then it is certainly necessary to cite not only the magnitude of these components, but also to stipulate to what system of coordinates they belong.

In aerodynamic designs it is more convenient to deal not with the components of forces and moments, but with their coefficients. In accordance with formulas (3.3) and (3.5) we can assume that

$$\left.\begin{array}{l} X=C_x qS; \\ Y=C_y qS; \\ Z=C_z qS; \\ M_x=m_x qSl; \\ M_y=m_y qSl; \\ M_z=m_z qSl, \end{array}\right\} \quad (3.6)$$

where C_x, C_y, C_z, m_x, m_y, m_z are the corresponding dimensionless coefficients of forces and moments in the continuous system of coordinates.

In the connected system of coordinates components $\bar{R}$ and $\bar{M}_a$ are determined by formulas analogous to formula (3.6) but having the index 1 in designations of components of forces and moments and their coefficients.

Names of coefficients include names of those components of forces or moments which they characterize. For instance: C_x – drag coefficient; C_{x_1} – coefficient of longitudinal force; m_z – coefficient of pitching moment in the continuous system of coordinates; m_{z_1} – coefficient of the pitching moment in the connected system of coordinates.

From expression (3.6) one can see that values of coefficients of forces and moments differ from values of the very forces and moments by co-factors qS for forces and qSl for moments, which are reduced in both parts of the equality (3.7). Therefore, formulas connecting the forces and moments will be the same for the connection between their coefficients. For instance, from Fig. 19 the dependence is evident between components of force R in the continuous and connected systems of coordinates:

$$\begin{array}{l} X=X_1\cos(x_1, x)+Y_1\cos(y_1, x)+Z_1\cos(z_1, x); \\ Y=X_1\cos(x_1, y)+Y_1\cos(y_1, y)+Z_1\cos(z_1, y); \\ Z=X_1\cos(x_1, z)+Y_1\cos(y_1, z)+Z_1\cos(z_1, z). \end{array}$$

Consequently, coefficients of these forces will satisfy the following analogous dependences:

$$\left.\begin{array}{l} C_x=C_{x_1}\cos(x_1, x)+C_{y_1}\cos(y_1, x)+C_{z_1}\cos(z_1, x); \\ C_y=C_{x_1}\cos(x_1, y)+C_{y_1}\cos(y_1, y)+C_{z_1}\cos(z_1, y); \\ C_z=C_{x_1}\cos(x_1, z)+C_{y_1}\cos(y_1, z)+C_{z_1}\cos(z_1, z). \end{array}\right| \quad (3.7)$$

Formulas for the coefficients of moments will be analogous to (3.7), and in them it is necessary to substitute m_{x_1} instead of C_{x_1}.

Subsequently we will deal not with the forces and moments, but with their aerodynamic coefficients, remembering that magnitudes of the forces and moments themselves will easily be determined by the formula (3.6).

It is necessary to consider that for the convenience of calculation of trajectories of guided rockets in equations of motion (1.16) and (1.17) the control forces and moments are considered separately. Therefore, in such cases, determining the aerodynamic coefficients one should not take into account those surfaces of the rocket which participate in the creation of control forces.

For convenience of calculation, aerodynamic coefficients are divided into separate components.

The drag coefficient of a rocket is

$$C_x = C_{x_f} + C_{x_p} + C_{x_i} + C_{x_Д}, \qquad (3.8)$$

where C_{x_f} is the coefficient of friction determining the portion of forces of friction in the drag of the rocket; C_{x_p} is the drag coefficient from pressure along the normal to the surface of the rocket; in case of the flight of a rocket with supersonic speed $C_{x_p} = C_{x_в}$, wave drag coefficient, thus called because of the appearance of shock waves; C_{x_i} is the coefficient of induced drag; $C_{x_Д}$ is the coefficient of base drag.

Let us explain the physical meaning of coefficients of base and induced drags. During the flying rocket in its bottom part a region of lowered pressure will be formed in comparison with the pressure of ambient air. This occurs because of the flow separation from the surface of the narrowing part of the rocket which is accompanied by energy eddy formation.

The existence in the tail part of the rocket of a zone of reduced pressure increases the general resistance of the air, since in the determination of aerodynamic forces (and, consequently, their coefficients) the difference in pressures in perturbed and unperturbed air is considered, i.e., the so-called excess pressure. In the case of the flow separation in the tail part of the rocket excess pressure negative (a vacuum is formed and the air resistance to the movement of the rocket increases. The pressure of unperturbed air is considered in the formula of the tractive force of a jet engine (1.10).

If a rocket has a wing or fin there noticeably appears a lift effect on the drag force, which leads to the creation of induced drag. This occurs because the vector of lift of the rocket Y turns opposite the direction of the flight at a certain angle and gives the component X_1, induced drag (Fig. 20c). The turn is caused by the additional speed of directed (induced) eddy "moustaches" descending from the wing under the action of the incident flow (Fig. 20a). These eddies, N. Ye. Zhukovskiy and N. A. Chaplygin showed, appear with the flowing around of the wing by the air flow. The induced speed is directed downwards from the surface of the wing and being added with the approach stream velocity slopes it downwards (Fig. 20b). This changes the angle of attack and leads to the turn of the lift vector and creation of induced drag.

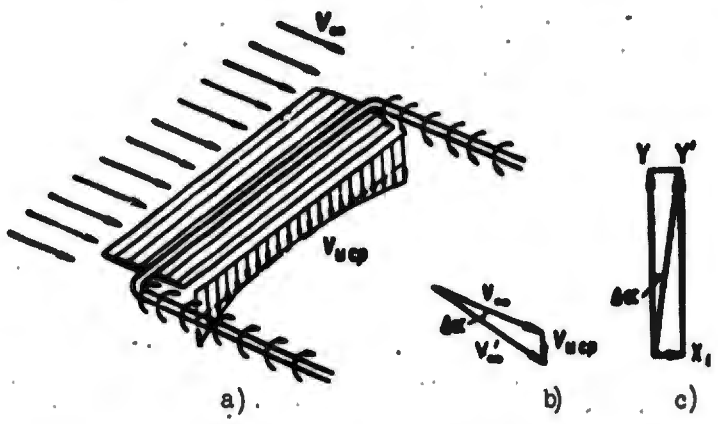


Fig. 20. Diagram of the formation of induced drag of the wing:

a) eddy "moustaches" formed with the flowing around of the wing induces speed $V_{\text{иcp}}$;

b) $V_{\text{иcp}}$ slopes the flow at angle $\Delta\alpha$;

c) drift of flow turns the vector of lift at $\Delta\alpha$ leads to the creation of the force of induced drag X_1.

The coefficient of force of induced drag, as shown calculations and experiments, confirm can be determined thusly:

$$C_{x_i} = \frac{C_y^2}{\pi\lambda}, \qquad (3.9)$$

where $\lambda = \frac{l^2}{S}$ is the wing aspect ratio; l is the wing span; S is the wing area in the plan.

From formula (3.9) one can see that the greater C_y and the smaller λ, the bigger the coefficient of the force of induced drag.

With the calculation of coefficients of lifting and lateral forces and all moments we usually divide them only into two components, taking into consideration frictional forces and forces of pressure on the normals to the surface of the rocket. For instance, the coefficient of lift is

$$C_y = C_{y_f} + C_{y_p}$$

where C_{y_f} is the coefficient dependent on friction; and C_{y_p} is the coefficient dependent on pressure.

Furthermore, in calculations of rotary (oscillatory) motion of rocket around the center of its mass, it is more convenient to separate from the vector of the main moment of aerodynamic forces with help of the theory of dimension and similarity, that part of it which causes damping of the rotation of the rocket. This part is the moment of damping $M_Д$. The nature of this moment involves the air resistance to the oscillatory motion of the rocket in the trajectory.

Let us assume that, for instance, the rocket revolves around its center of mass in a vertical plane with angular velocity ω_z (Fig. 18). Then separate points of the surface of the rocket will have speed v' consisting of the geometric sum of the speed of forward motion of the rocket v and the peripheral velocity $v_{окр}$, i.e., $\overline{v}' = \overline{v} + \overline{v}_{окр}$. This will lead to the change of the local angle of attack at magnitude $\Delta\alpha$, which is determined from the triangle of speed (Fig. 18) in the following way: $\Delta\alpha \approx \mathrm{arctg}\,\frac{v_{окр}}{v}$ or, because of the smallnesses of the quantity $\frac{v_{окр}}{v} = \frac{\omega_z r}{v}$, a more simple formula $\Delta\alpha \cong \frac{\omega_z r}{v}$, where r is the radius of rotation of the considered point. With the change of the local angle of attack lift also changes at magnitude ΔY proportional to $\Delta\alpha$. The reduction to the center of mass of the rocket of all forces of ΔY applied to separate points of the surface of the rocket will replace them by certain additional lift $Y_Д$ and moment of damping $M_Д$.

From Fig. 18 one can see that forces of ΔY having an effect on the left part of the rocket are directed downwards and on the right part, upwards. Therefore, the magnitude of force $Y_Д$ appears small and with calculations is usually not considered. Therefore, with the derivation of dependence (3.4) the small effect of angular velocity of rotation ω was indicated on the magnitude of the main vector of a aerodynamic forces.

It is another matter with the moment of damping. Forces of ΔY on the left and on the right of the center of the mass of the rocket tends to revolve it to one side, and the moment of damping proves to be considerable. Calculations show that the magnitude of the moment of damping is ~10% of the whole moment affecting the rocket. The moment of damping is always directed opposite the rotation of the rocket and tends to brake and die out.

The theory of dimension and similarity gives such an expression for the moment of damping:

$$M_z = -|m_z'| qSl^2 \cdot \frac{\omega}{v}.$$

where $m_Д$ is a certain coefficient dependent on the form of the rocket, number of M, and other magnitudes.

If we consider what $q = \frac{\rho v^2}{2}$, then we can write

$$M_z = -S\rho v l^2 |m_z| \omega, \tag{3.10}$$

where $m_Д$ is the coefficient of the moment of damping diffe. ing from $m'_Д$ by the co-factor 1/2.

If the rocket flies on the rectilinear section of the trajectory and oscillates around the center of mass in the vertical plane with a speed $\frac{d\alpha}{dt} = \dot{\alpha}$, formula (3.10) permits writing

$$M_z = -S\rho v l^2 |m_z| \dot{\alpha} \tag{3.11}$$

Thus, if one were to consider the moment of damping separately, the vector of the main moment of all aerodynamic forces can be represented by the sum $\bar{M}_a = \bar{M}_{CT} + \bar{M}_Д$, where $\bar{M}_{CT}$ is the stabilizing or tilting moment.

The names "stabilizing" (sometimes "restoring") and "tilting" indicate direction of action of the vector $\bar{M}_{CT}$. If, for instance, the pitching moment M_z is directed so that it tends to decrease the angle of attack of the rocket α obtained by it by some causes, then it is called stabilizing and the rocket itself, statically stable. If, however; M_z tends to increase α it is called tilting and the rocket, statically unstable. For a statically stable rocket the center of pressure is located after the center of mass (Fig. 18) and for a statically unstable rocket inversely. Let us recall that the moment created by the controls is considered separately and does not enter into $\bar{M}_{CT}$. Therefore, the concept of static stability does not reflect the actual stability of guided rockets in trajectory. Even the statically unstable rocket can be fully stable on the trajectory due to the correct operation of the control system. Then it is called dynamically stable. The stability of the flight of rockets is related in greater detail in the sixth chapter.

Let us study one more force separated from the main vector of aerodynamic forces. Figure 21 shows the mechanism of appearance of this force called the Magnus force. Figure 21a depicts a rocket streamlined by the counterflow of air with speed V_∞, revolving with angular velocity ω_{x_1} around its axis and being at an angle of attack α to the direction of the counterflow. The transverse part of the incident flow with speed $V_{\infty поп}$ interacts with particles of air dragged into rotation by the surface of the ro. .et (Fig. 21b). At the place where the flows meet there will be an increased pressure, and at the place where they are directed equally, a decreased pressure. As a result of such redistribution of pressure, in comparison with the case of the absence of rotation of the rocket, force R_{Ma} is obtained (Fig. 21b) which is perpendicular to the plane of resistance and revolves together with this plane.

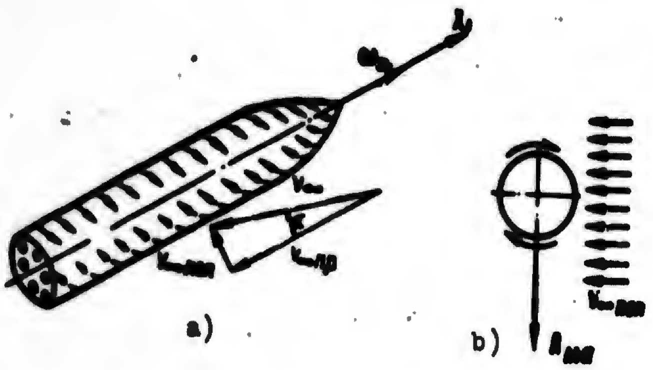


Fig. 21. Diagram of formation of the Magnus force:
a) revolving around its longitudinal axis the rocket is in the flow of air at angle of attack α; b) diagram of the flowing around of a rocket of a transverse component of incident air flow.

The theory of dimension and similarity permits finding the following form of the formula for Magnus force:

$$R_{Ma} = C_{R_{Ma}} qS \cdot \frac{\omega_{x_1} l}{v} \cdot \alpha, \qquad (3.12)$$

where $C_{R_{Ma}}$ is the coefficient of Magnus force; ω_{x_1} is the angular velocity of rotation of the rocket around the longitudinal axis symmetry (OX_1).

Formula (3.12) shows that the Magnus force is proportional to ω_{x_1}, and therefore the largest value will be for rockets and missiles stabilized in the trajectory of rotation around its axis with a great angular velocity (for instance, turbojet missiles). But also in this case the specific influence of this force is comparatively small. In the calculation of trajectories aerodynamic coefficients obtained without taking into account the influence of rotation of the rocket around its axis are used. The Magnus force is applied at the point not coinciding with the center of the mass of the rocket and therfore gives the moment with respect to the center of mass, which is not sufficiently studied at present.

Thus far we have considered the flows of air circumfluent the rocket to be stable. However, in practice we must study the unsteady flows of air and their influence on the aerodynamic coefficients of rockets. Where do we find the characteristic unsteady flowing around of the rocket? In the first place with sharp maneuvers of the rocket when the flow rate of air near the separate places of the surface of the rocket are changed by a considerable mangitude for a short interval of time. The same is obtained during the influence on the rocket of wind gust.

Furthermore, the oscillatory motion of the rocket around the center of mass and also the vibracion of insufficiently rigid sections of the body wing, and fins also lead to the unsteady flow of air at the surface of the rocket.

The complexity of solving problems of unsteady flows forces us to seek approximate solutions. With this goal we apply assumptions simplifying the complicated phenomenon of the unsteady flow. Thus, for instance, we consider that the aerodynamic forces and moments effective in the plane XOY (Fig. 19) is not influenced by the motion of the rocket in other planes although strictly speaking this is not true. Also very frequently the applied assumption about the fact that the magnitudes of lift and pitching moment are proportional to the angle of attack α turns out to be correct only for limited values of angles of attack. However, practically rockets are always rather well stabilized in the trajectory and their angular displacements are comparatively small, and therefore the above-mentioned assumptions turn out to be fully acceptable.

Considerable simplifications are introduced into the calculation by the hypothesis of stationarity, which assumes that aerodynamic forces and moments during the unsteady flow depend only on values of the speed V of forward and ω of rotational motions of rocket and on values of angles of attack α and slip β at each given moment of time. This assumption can be applied with the sufficiently smooth change of speed of forward and rotational motion of the rocket. One of the causes making the hypothesis of stationarity inapplicable is the delay of the downwash.

Let us consider this phenomenon in the example of a winged rocket with the fins located behind the wing (Fig. 64b). The flow encounters the wing at an angle of attack α, and passing it slopes at angle ε; therefore the fins are flowed around at an angle $\alpha-\varepsilon$. The larger α the larger ε. The time necessary for the flow to pass the distance from the wing to the fins will be designated Δt. Then during the change in the flight of the angle of attack of the wing α with speed $\frac{d\alpha}{dt} = \dot{\alpha}$ at any moment of time the fins will be flowed around at an angle of attack which had to appear earlier for the time Δt. If, for instance, α is increased, then as compared to the steady flow (in the case of a flight with α = const) the fins, because of the delay of downwash, will be flowed around at a smaller angle of attack and will create actually a smaller lift and smaller pitching moment. Therefore, from values of lift and pitching moment, determined by the hypothesis of stationarity, one should subtract corresponding magnitudes. These correction magnitudes are proportional to the change of angle $\Delta\alpha$, which the larger it is the greater the angle α is changed. If one were to consider that the change of α during the time Δt is equal to $\dot{\alpha}\cdot\Delta t$, then it is possible to understand why the corrections introduced into values of aerodynamic coefficients of lift C_y and pitching moment m_z are directly proportional to $\dot{\alpha}$.

Thus, taking into account the assumptions made with the unsteady character of the flowing around of the rocket by the air flow, the following can be assumed:

$$C_y = C_{y_0} + C_y^{\alpha}\alpha + C_y^{\dot{\alpha}}\dot{\alpha}; \qquad (3.13)$$

$$m_z = m_{z_0} + m_z^{\alpha}\alpha + m_z^{\dot{\alpha}}\dot{\alpha}, \qquad (3.14)$$

where C_{y_0} and m_{z_0} are values of coefficients of lift and pitching moment respectively when $\alpha = \dot{\alpha} = 0$; C_y^{α} and m_z^{α} are static derivatives of coefficients of lift and pitching moment respectively at the angle of attack; $C_y^{\dot{\alpha}}$ and $m_z^{\dot{\alpha}}$ are rotary derivatives of coefficients of lift and pitching moment at $\dot{\alpha}$.

The rotary derivative (at $\dot{\alpha}$) is also called magnitude $m_Д$ in expression (3.10) for the moment of damping.

The influence of the unsteady character flowing around the rocket by air at the magnitude of drag coefficient C_x is very complicated and as yet is insufficiently developed.

From expressions (3.6), (3.13), and (3.14) for symmetric rockets for which $C_{y_0} = m_{z_0} = 0$, with the condition of applicability of the hypothesis of stationarity, the following formulas are obtained:

$$Y = \frac{\rho v^2}{2}\cdot S C_y^{\alpha}\alpha; \qquad (3.15)$$

$$M_z = \frac{\rho v^2}{2}\cdot S l m_z^{\alpha}\alpha. \qquad (3.16)$$

In conclusion we give typical curves of the most widespread aerodynamic coefficients as functions of the angle of attack (Fig. 22) and the M (Mach) number (Figs. 23 and 24).

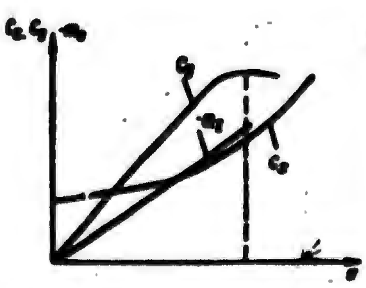


Fig. 22. Curve of dependences C_x, C_y, and m_z on the angle of attack α.

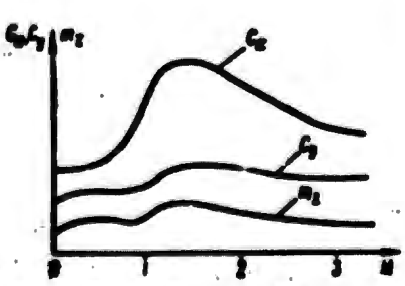


Fig. 23. Curve of dependences C_x, C_y, and m_z on the M number.

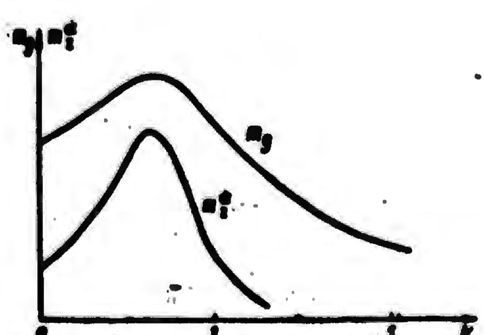


Fig. 24. Curve of dependences $m_Д$ and $m_z^{\dot{\alpha}}$ on the Mach number.

§ 2. Theoretical Bases of the Determination of Aerodynamic Forces and Moments

For the calculation of the trajectory of a rocket and its oscillatory motion it is necessary to know the magnitude of all the components of aerodynamic forces and moments at any moment of the time of flight. By what means will we solve this problem?

In the first place the obtaining of aerodynamic properties of a rocket is stipulated by calculation either from tables or graphs, or by empirical formulas reflecting the results of earlier conducted experiments, or by formulas obtained theoretically. The advantages of these means are evident: it is not necessary to conduct any experiments. Results are obtained relatively quickly even when there is only a sketch of the rocket, i.e., in the first stage of its design. The usually low accuracy, small interval of application of empirical formulas, and great labor-consumingness of calculations according to exact theoretical formulas are insufficient.

In second place, it is possible to make a model of a rocket in reduced scale and to subject it to tests in wind tunnels simulating the interaction of the rocket with the atmosphere during its flight. But an experiment in wind tunnels does not simulate the flight of a rocket accurately enough, and therefore, the aerodynamic characteristics are determined with errors which in separate cases can be very great. Besides wind tunnels, for carrying out the experiments reaction trolleys fixed on rails are used. A rocket or its nose cone is put on the trolley. Models of a rocket or its nose cone are also tested. The rocket motor acclerates the trolley with the studied object to the required speed, and transducers installed on it give the necessary information as to the speed, acceleration, course, forces and others. By processing the obtained data it is possible to calculate the forces effective on the rocket and their coefficients.

Tests for the purpose of the determination of aerodynamic coefficients are also conducted on aeroballistic tracks by means of firing a model from a special gun or artillery piece. The flight of model on trajectory is recorded by different methods (optical, magnetic, etc.) or is photographed.

In the third place, if a rocket already is made and can be launched, then with the help of different transducers and telemetric equipment transmitting readings of the transducers to Earth and also by conducting observations of the rocket flight with the help of different recording equipment set up on Earth it is possible to obtain the necessary data and to calculate the magnitude of certain components of aerodynamic forces and moments in different time intervals of the flight. As we see this way proposes the carrying out of flying tests of a rocket. Flying tests are usually conducted during the final adjustment of the rocket in the last stage of its creation.

Each of the methods of testing has its advantages and deficiencies; during the adjustment of the model they complement one another.

As aerodynamic characteristics of the rocket in wind tunnels are obtained during flying tests and also by calculation they will be related later, since at first it is necessary to study the theoretical bases of the determination of aerodynamic characteristics.

The science which permits the determining of aerodynamic forces and moments is called by aerodynamics. Without a knowledge of the bases of aerodynamics it is impossible to determine not only by theoretical means the aerodynamic characteristics of a rocket but also to find them by experimental data.

In aerodynamics the method of converting motion is often applied, which consists in the following. The flight of the rocket with velocity v with respect to the atmosphere (Fig. 25a), is replaced by the flowing around of the motionless rocket by air with the same velocity V = v (Fig. 25b). This turns out to be more convenient for the theoretical solution of the problem of the interaction on the rocket with air and is immediately realized in wind tunnels where flow of air on the motionless model of the rocket is forced with the necessary velocity . The correctness of the method of the conversion of motion is confirmed by the many years of practical application.

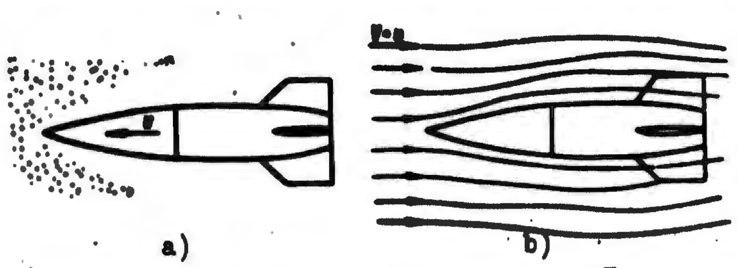


Fig. 25. Diagram illustrating the method of the conversion of motion: a) flight of the rocket in a motionless atmosphere with the speed v; b) airflow around the motionless rocket with the speed V = v.

Practice shows that, first of all, one should study the flow of air in the layer directly adjacent to the streamlined surface of the rocket. Really the flow in this shell determines the power influence of air on the rocket (frictional force and force of pressure) as well as the laws of transmission of heat from air to the surface of the rocket and inversely. It appears that in order to recognize how the air in this shell flows it is necessary to know how it flows and in the more distant layers from the rocket. In other words, it is necessary to determine the pattern of air flow near the rocket as a whole. The study of the flow of air around the rocket permitted establishing that the flow at various points has a difference resulting in the manifestation of forces of the internal friction in air (viscosity). Experiments show that the faster the speed of the flow of air changes the greater the forces of internal friction in it and inversely. Let us give formula of Newton well-known from physics and well confirmed by experiment:

$$\tau = \mu \frac{dV}{dy},$$

where τ is the shearing stress from forces of friction in air in kgf/m^2 (frictional force occurring on 1 m^2 of surface); $\frac{dV}{dy}$ is the velocity gradient in $\frac{1}{\sec}$, shows how fast the airspeed changes along the coordinate y directed along the normals to the direction of the flow of liquid; μ is the dynamic coefficient of the viscosity of liquid at kgf·sec/m^2.

For air $\mu \approx 2 \cdot 10^{-6}$ kgf·sec/m^2 (small). Therefore the forces of viscosity noticeable appear only in those places of the flow where the gradients of speed are great, for instance, in direct proximity to the streamlined surface of the rocket. The point is that particles of air touching the surface of the rocket as if they adhere to it and have a speed equal to zero. Owing to the internal friction of the particles of air located further from the streamlined surface will have a greater speed than the particles located nearer to the surface of the rocket. The speed of the air particles will increase in proportion to the distance from the surface of the rocket. However, this increase is not infinite and in a comparatively short distance the speed attains such a value which would be at this point of the flow if we did not consider the influence of the internal friction, i.e., forces viscosity in air. This is the boundary layer.

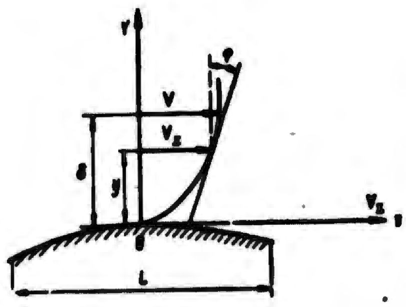


Fig. 26. Change in the speed of flow V_x according to the thickness of the boundary layer.

The boundary layer is called the region of flow directly adjoining the streamlined surface of the body in which the influence of forces of viscosity of liquid appears (air and, in general, gases) on its motion. The remaining part of the flow called free flow. In order better to understand this let us consider the typical picture of the change of speed in the boundary layer (Fig. 26). At point "0" of the surface streamlined by air we place the rectangular system of coordinates in such a manner that the Y axis goes along the external normal to the surface of the rocket. With an increase in y the horizontal projection of the speed V_x is increased, and the angle φ of the slope tangent to the curve $V_x(y)$ decreases. Mathematically this is expressed by a decrease in the gradient of speed $\frac{dV_x}{dy}$ in proportion to the distance from the streamlined surface. The place should be where $\frac{dV_x}{dy} = 0$. With this $y = \delta$. The magnitude δ is called the thickness of the boundary layer. It is usually small in comparison with dimensions of the streamlined body. For instance, with the airflow around the wing δ consists of several hundredths or even thousandths of a chord of the profile of the wing.

With the approximation of y to δ the speed V_x is changed so slowly that it is practically difficult to determine the boundary separating the boundary layer from the remaining part of the flow. Therefore, δ is found from the condition $V_x = 0.99 V$, i.e., the thickness of the boundary layer is considered to that place where the speed in the boundary layer V_x differs from the speed in the free flow V by a magnitude of approximately 1%.

Outside the boundary layer the viscosity of the air practically does not affect on its flow. Therefore, it can be considered non-viscous or, as is said, an ideal liquid.

Thus the whole flow is conditionally divided into two parts: boundary layer and free flow. In each of these parts properties of the air idealized, and in calculations we deal not with the real air but with its model. As a rule models of air are allotted different properties. In the free flow the air can be considered an ideal compressible liquid, but within the boundary layer it is not possible to consider the compressibility i.e., to consider the air as a viscous incompressible liquid.

Such a division of the flow into two parts and the application of simplified models of air in each of these parts of flow permits with sufficient accuracy for practice the describing of the interaction of the flying rocket with air by equations and the solving of these equations.

Let us allot these models the property of continuity. This means that however small the volume of air is considered the properties of air in this volume will remain constant. In this case it is possible to use differential calculus, i.e., to operate with such a volume of air the dimensions of which are differentials of coordinates (infinitesimal).

With common conditions at the surface of the Earth 1 mm^3 of air contains $27 \cdot 10^{15}$ molecules. Such a huge quantity of molecules in a small volume allows considering the air as a solid (continuous) medium. For a great discharging (in high layers of the atmosphere) air can no longer be considered continuous.

Peculiarities of aerodynamics of the flight of rockets in the high layers of the atmosphere will consider separately.

Let us agree further that the models of air are electrically neutral. In this case it is possible not to consider the presence in air of a certain quantity of electrically charged particles (electrons and ions). This will allow not to take into account the force of interaction of the air with electrical and magnetic fields.

Moreover, we will consider that in air no physical and chemical transformations occur with the release or absorption of energy.

With the made assumptions at any point of the flow the air will be characterized by the following parameters: pressure p (kgf/cm^2), mass density ρ ($kgf \cdot sec^2/m^4$), temperature in absolute scale T($^{\circ}K$), and velocity vector of the motion $\bar{V}$ of the given infinitesimal volume of air in the vicinity of the considered point.

All these parameters are continuous functions of coordinates of the given point x, y, z and time t, i.e.,

$$p(x, y, z, t),\ \rho(x, y, z, t),\ T(x, y, z, t) \text{ and } \bar{V}(x, y, z, t). \qquad (3.17)$$

These functions can have regions of break in the case of the appearance of so-called compression shocks, the theory of which will be considered at the end of the chapter.

Flows of air described by functions (3.17) are called by spatial unsteady (nonstationary). This is the most common form of flows but also the most complicated. Among the elementary functions it is almost impossible to detect suitable ones for the expression of p, ρ, T and $\bar{V}$. Therefore, in practice we try the flows by simpler ones but with similar properties to replace. So we usually consider the steady (stationary) flows, the parameters of which do not depend on time t.

The still simpler flows appear to be the flat or axisymmetric flows for which p, ρ T and V depend only on the two coordinates. The simples flow is the one-dimensional steady flow. For such a flow p(x), ρ(x), T(x), and V(x) are functions of only one coordinate.

§ 3. Basic Laws of Aerodynamics and Their Application in the Theory of Rocket Flights

To solve the problem of aerodynamics for any point (x, y, z) of the flow of air around the rocket at any moment of time we must find p, ρ, T, and V. This will be sufficient for the determination of forces of influence of the air on a rocket of any form, since it will be possible to simulate all forces from pressure along the surface of the rocket, normal and tangent the components which determine the desired force.

To detect the four unknown functions, p, ρ, T, and V, it is necessary to have four initial equations. These equations are called the principal equations of aerodynamics and reflect the most common properties of the flow of air satisfying the laws of mechanics. The principal equations are: equation of state, equation of discontinuity, equation of motion, and equation of energy. Inasmuch as these equations represent the principal interest, we will consider them in detail.

Equation of State

Experiments show that with the change of any two parameters from three (p, ρ, and T) the third parameter quite definitely changes. This occurs in both moving and motionless air. Consequently the parameters p, ρ, and T are connected by the dependence determined by the molecular structure of air and called the equation of state. Into the equation of state different constants of C_1 which characterize certain properties of air also enter. In common form the equation of state has the form

$$F(p, \rho, T, C_i) = 0.$$

For different gases and also for different conditions in which they are found many equations of state are known. The contemporary kinetic theory of gases permits determining the constants in the equation of state for any gas if for it the different characteristics (including those obtained by spectral analysis) are known.

The frequently applied equation in aerodynamics is

$$p = g\rho RT \text{ (R – gas constant)} \tag{3.18}$$

called in physics the equation of state of an ideal gas.

The equation of state is used for parameters of gas at a point. However, sometimes the parameters of gas in some volume are taken as the average for the whole volume, for instance, in the chamber of a jet engine, and then the equation of state is used for these average parameters.

For incompressible liquids the equation of state appears very simply: ρ = const.

Equation of Discontinuity

The equation of discontinuity is such an equation which expresses the law of conservation of mass: the mass of substance does not disappear and appear again. Therefore, there exists a quite definite relation between airspeed and air density:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z) = 0. \tag{3.19}$$

This is the equation of discontinuity.

For steady flows $\frac{\partial \rho}{\partial t} = 0$ the equation of discontinuity is simplified and has the form

$$\frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z) = 0. \tag{3.20}$$

For the incompressible liquid ρ = const the equation of discontinuity is still simpler:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0. \tag{3.21}$$

For the limited definite section of the one-dimensional steady flow the equation of discontinuity is obtained from the following considerations. A mass of air flowing in a unit of time through any section of the flow should be the same. Otherwise either the law of conservation of the mass will be disturbed or the flow will not be steady, since between some sections the mass of air will not be steady, since between some sections the mass of air will begin to be increased or decreased. But the magnitude of the mass of air flowing in a unit of time through the section of flow F is equal to ρVF. Therefore, the equation of discontinuity will have form

$$\rho VF = \text{const} \tag{3.22}$$

Equation of Motion

The equation of motion represents one of principal laws of mechanics: the produce of the mass of a body by its acceleration is equal to the force applied to the body. Let us apply it to the mass of air included in the elementary volume, and the equations of motion will formulate consecutively along all three axes of the coordinates. As a result we will obtain the following system of equations of motion in projections on the axis of coordinates in the form of Euler:

$$\left.\begin{aligned} V_x\cdot\frac{\partial V_x}{\partial x}+V_y\cdot\frac{\partial V_x}{\partial y}+V_z\cdot\frac{\partial V_x}{\partial z}=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial x};\\ V_x\cdot\frac{\partial V_y}{\partial x}+V_y\cdot\frac{\partial V_y}{\partial y}+V_z\cdot\frac{\partial V_y}{\partial z}=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial y};\\ V_x\cdot\frac{\partial V_z}{\partial x}+V_y\cdot\frac{\partial V_z}{\partial y}+V_z\cdot\frac{\partial V_z}{\partial z}=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial z}. \end{aligned}\right| \qquad (3.23)$$

With the derivative of formulas (3.18) mass forces (for instance, the force of weight) and frictional force were not considered, since it is established that for air they are small as compared to forces of pressure.

For one-dimensional steady flow the equation of motion will have the form

$$V\cdot\frac{dV}{dx}=-\frac{1}{\rho}\cdot\frac{dp}{dx},$$

or

$$VdV+\frac{dp}{\rho}=0. \qquad (3.24)$$

The integral of this equation has the form

$$\frac{V^2}{2}+\int\frac{dp}{\rho}=\mathrm{const} \qquad (3.25)$$

and is called the Bernoulli equation, which is correct for the elementary (infinitely thin) stream of air.

Equation of Energy

The equation of energy expresses the law of the conservation of energy in the flow of air. The law of the conservation of energy indicates that energy does not disappear and does not appear again and only passes from one form to another in equivalent quantities. What transitions of forms of energy occur in the air flows? From physics it is known that gas possesses internal (i.e., kinetic energy of molecules of gas in their chaotic body motion) and potential energy (i.e., energy of the position of molecules of gas). If the gas occurring in the flow moves, then it still has kinetic energy dependent on the speed of the flow. Moreover, in the flows of gas there occurs internal friction between the individual particles of gas and friction of gas against the surface circumfluous to it. Work of the forces of friction in irreversible form passes to heat, which by means of thermal conduction and radiation can be dispersed in the gas and to be transmitted to the objects surrounding it.

How are the forms of energy of gas expressed?

The internal energy of the unit of weight of gas (1 kgf) is equal to c_vT, where c_v is the specific heat capacity of gas in kcal/kgf·deg and T is the temperature of gas in degrees of absolute scale.

The potential energy of 1 kgf of gas can be determined by the work of the forces of pressure expended on the compression of it without friction, and heat supply and removal up to the given pressure p and density ρ. The magnitude of this work is equal to $\frac{p}{g\rho}$, and with the help of the equation of state $p = g\rho RT$ can be expressed in terms of temperature and will be RT (kgf-m/kgf). With the determination of potential energy the work forces of weight was not considered, which in the air flows circumfluent the rocket in flight is small in comparison with the work of forces of pressure in these flows.

The kinetic energy of 1 kgf gas moving with speed V is equal to $\frac{V^2}{2g}$ (kgf-m/kfg).

If one were to designate the work of forces of friction in 1 kgf air L_{Tp} and quantity of supplied (or removed from it) heat $\pm Q$ respectively, then the law of conservation of energy in the steady flow of air will be expressed by the following formula:

$$c_vT + ART + \frac{AV^2}{2g} + AL_{Tp} \mp Q = \text{const.} \qquad (3.26)$$

In this formula all forms of energy are determined in thermal units with the help of the thermal equivalent of mechanical work $A = \frac{1}{427} \frac{\text{kcal}}{\text{kgf-m}}$. We use equation (3.26) for the unit of weight (or mass) of gas moving in the flow in its trajectory.

In formula (3.26) it is convenient to consider the internal and potential energy of air together, since each of these forms of energy depends on the temperature T. Introducing the concept of enthalpy $i = c_vT + ART$, i.e., equal to the sum of the internal and potential energy and considering that $c_v + AR = c_p$, we obtain

$$i + \frac{AV^2}{2g} + AL_{Tp} \mp Q = \text{const.} \qquad (3.27)$$

where $t = c_pT$, and c_p is the specific heat capacity of air with the constant pressure in kcal/kgf·deg.

In aerodynamics we most frequently deal with adiabatic processes, i.e., such processes during which the air does not obtain heat externally and does not return heat. With this $Q = Q_{TP} = AL_{TP}$, i.e., the heat will be formed only owing to the work of forces of friction in air.

In this case equation (3.27) is simplified and takes the form

$$\frac{AV^2}{2g} + i = \text{const.} \tag{3.28}$$

We should say that it is possible to arrive at the same form of equation of energy by not only considering the process to be adiabatic, but not taking into account the force of friction ($L_{TP} = 0$) in air.

One more form of the recording of the equation of energy will be obtained after the following transformations of equation (3.28):

$$\frac{V^2}{2} + \frac{gi}{A} = \text{const},$$

where

$$\frac{gi}{A} = c_pT \cdot \frac{g}{A} = c_p \cdot \frac{p}{AR\rho} = \frac{c_p}{c_p - c_v} \cdot \frac{p}{\rho} = \frac{k}{k-1} \cdot \frac{p}{\rho};$$

since $gT = \frac{p}{R\rho}$ from formula (3.18) and $k = \frac{c_p}{c_v}$.

Thus, the equation of energy can be written thusly:

$$\frac{V^2}{2} + \frac{k}{k-1} \cdot \frac{p}{\rho} = \text{const.} \tag{3.29}$$

For the elementary stream of air in the flow around the rocket it is possible from the equation of energy (3.29) to obtain the equation of the adiabatic curve. For this let us use the equation of motion (3.24) and equation of state (3.18). Differentiating equation (3.29) and comparing it with equation (3.24) we get

$$\frac{k}{k-1} \cdot d\left(\frac{p}{\rho}\right) = \frac{dp}{\rho}.$$

After simple transformations we obtain ordinary first order differential equation with the separable variables:

$$\frac{dp}{p} = k \cdot \frac{d\rho}{\rho},$$

by integrating which from p_0 to p and from ρ_0 to ρ we obtain the equation of the adiabatic curve:

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k. \tag{3.30}$$

In those cases when there are no irreversible transitions of energy of the flow of air into heat and the entropy is constant, equation (3.30) is called also the equation entropy.

Thus we obtained the basic equations describing the motion of air as an ideal compressible liquid. These are the equations of state (3.18) discontinuity (3.20), motion (3.23), and energy (3.28). Together they form the system of equations which permits determining the unknowns p, ρ, T, V_x, V_y, V_z for any point of assigned flow with definite boundary conditions (for instance, the continuous flowing around of the surface of the rocket).

Let us show how the fundamental equations of aerodynamics are applied during rocket flight in the Earth's atmosphere by sufficiently simple but important examples for understanding the essence of the phenomena.

First of all let us turn to the one-dimensional is entropic steady flow of air. The formulas obtained for such a flow will be useful for the flow of air inside the thin (elementary) streams but are applied for such real flows which occur with the flow of gas through the nozzle, channels, etc.

Let us obtain the formulas by which knowing the airspeed in some section of the flow, it is possible to calculate the temperature pressure, and density of it in the same section. In the equation of energy (3.28) we determine the quantity on the right side for such a section of the flow where V = 0. The parameters of air in the section are called parameters of braking and are supplied by the index "0." We obtain

$$\frac{AV^2}{2g} + c_pT = c_pT_0$$

whence

$$\frac{T_0}{T} = 1 + \frac{AV^2}{2gc_pT}. \tag{3.31}$$

From the formula for the speed of sound, well-known from physics, $a = \sqrt{kgRT}$, we find $T = \frac{a^2}{kgR}$ and substitute it into expression (3.31).

Then

$$\frac{T_0}{T} = 1 + \frac{AkgR}{2gc_p} \cdot \frac{V^2}{a^2}. \tag{3.32}$$

Remembering that $AR = c_p - c_v$, and $k = \frac{c_p}{c_v}$, and designating $\frac{V}{a} = M$, from formula (3.32) we find

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2. \tag{3.33}$$

The Mach number is a dimensionless speed and applied widely in aerodynamics.

Applying the equation of the adiabatic curve (3.30) and equation of state $p_0 = g\rho_0RT_0$ for parameters of braking we get $\left(\frac{T_0}{T}\right)^{\frac{1}{k-1}} = \frac{\rho_0}{\rho}$ and $\left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} = \frac{p_0}{p}$, and therefore considering formula (3.33) will find

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{1}{k-1}}, \tag{3.34}$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}. \tag{3.35}$$

Dependences (3.33), (3.34), and (3.35) permit us by the M_∞ number characterizing the speed of rocket (or the speed of air incident on the motionless rocket) and also by air parameters p_∞, ρ_∞, and T_∞, determined at the altitude of the flight of the rocket, for instance, according to tables of the international standard atmosphere [26], to determine the parameters of braking of air p_0, ρ_0, and T_0. These magnitudes will in reality exist at the point of the rocket where the airspeed circumfluent its surface is equal to zero. Such a point is called the critical point of the rocket. Parameters of braking characterize either force (p_0) or thermal (T_0) influence of air on the rocket. They enter into empirical and theoretical formulas applied in the theory of the flight of rockets. Substituting into formulas (3.33), (3.34), and (3.35) M_∞, p_∞, ρ_∞, and T_∞, we find the desired dependences for the parameters of braking:

$$T_0 = T_\infty\left(1 + \frac{k-1}{2}\cdot M_\infty^2\right); \tag{3.36}$$

$$\rho_0 = \rho_\infty\left(1 + \frac{k-1}{2}\cdot M_\infty^2\right)^{\frac{1}{k-1}}; \tag{3.37}$$

$$p_0 = p_\infty\left(1 + \frac{k-1}{2}\cdot M_\infty^2\right)^{\frac{k}{k-1}}. \tag{3.38}$$

Now let us see how to find the speed of a one-dimensional flow in any section of it. For this we will turn to the equation of discontinuity (3.22), from which it is clear that first of all it is necessary to know for some section F_1 are values ρ_1 and V_1. Then, finding the value $\rho VF = \rho_1 V_1 F_1 = \text{const}$ of the right side of formula (3.22) we will be able, excluding ρ, to find the dependence $V(F)$ of the flow rate or M number from the area of its section. However, this dependence is complicated to obtain and in practice we use beforehand the composed tables. For instance, by the Mach number we can find $\frac{p}{p_0}$, $\frac{\rho}{\rho_0}$, $\frac{T}{T_0}$, $\frac{F}{F_*} = \frac{\rho_* V_*}{\rho V}$, where the parameters marked by the index * are determined at that point of flow where $V = a$ and are called critical parameters of the flow.

Further let us study the theory of shock waves.

With the interaction of bodies with supersonic flows of gas (or with the motion of bodies in motionless gases with supersonic speed) regions appear in which parameters of gas change extremely fast and intermittently. Such regions are called compression or shock (ballistic) waves. The name "shock wave" reflects a sharp intermittent increase in the density of gas at the front (on the surface) of this sudden increase.

Experience shows that it is otherwise impossible to change the supersonic flow of gas, to subsonic flow as transferring it through the shock wave. Therefore, during the flights of rockets within the atmosphere at such a speed when at least on one section adjacent to streamlined surface of the rocket there appears supersonic speed a shock wave will evidently appear. The minimum value of the speed of the rocket during which shock waves appear is called the critical speed of the rocket. The magnitude of this speed is less than the speed of sound, since the flow of air incident on the surface of the rocket is accelerated, and the local velocities of it can be considerably greater than the speed of the flight of the rocket.

With the appearance of shock waves near the flying rocket the phenomenon of shock stall approaches. The resistance of the flight of the rocket during a shock stall sharply increase, the lift decreases, and, moreover, vibrations can appear which are dangerous for the strength of the separate parts of the rocket (for instance, the wing or fin) vibration. Therefore, it is necessary to understand thoroughly the nature of shockwaves and to learn to calculate the parameters of gas during them. Without this it is impossible to determine the aerodynamic characteristics of a rocket flying at a speed exceeding its critical velocity and it is impossible to find measures of control with the harmful aftereffects of the shock stall and also by experimental means to determine by pressure measurements the magnitude of the supersonic speed.

The nature of the shock waves is connected with the specific (special) character of the propagation of weak perturbations (sound waves) during the motion of the source of perturbations with supersonic speed in a motionless gas. If the source of the perturbations in the form of a material point, the dimensions of which are negligible, will be motionless, the waves of the weak perturbation are evenly spread to all sides with the same speed a, the speed of sound. The surface of the wave of the weak perturbation is a sphere with the radius $r = at$, where t is the time which elapsed from the moment of the formation of this wave (Fig. 27a).

A different picture will be observed with the motion of the point source of

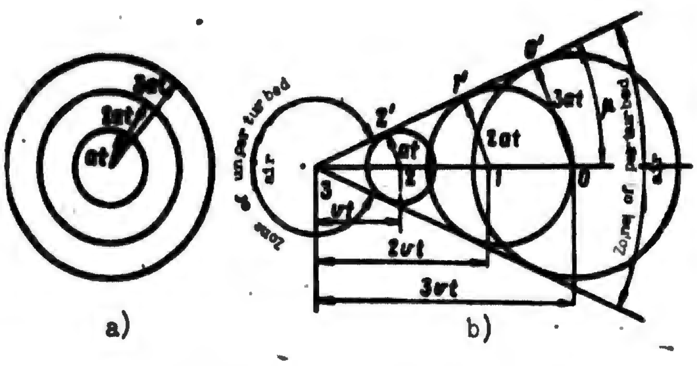


Fig. 27. Propagation of weak perturbations (sound waves):

a) with a stationary source, b) with the motion of this source in air with supersonic speed.

weak perturbations in motionless gas with speed V greater than the speed of sound a (Fig. 27b). Let us designate the position of the source of weak perturbations in terms of equal intervals of time by figures 0, 1, 2, and 3. At first it was at position 0, and at the considered moment upon the expiration of the interval of time 3t it shifted to position 3 passing 3Vt. During the time 3t the wave of weak perturbations, having formed at point 0, will pass 3at and occupy the position of the sphere with a radius 3at and with the center at point 0. The wave of perturbations, having formed at point 1, by the considered moment of time will be a sphere with the radius 2at and with the center at point 1, since from the moment of its formation only the time 2t elapsed. By this same cause the wave which was formed at the moment of the finding of the source of weak perturbations at point 2 will be a sphere with a radius at. Finally at the considered moment the source of weak pertubations is at point 3, and the wave only starts to be formed and the radius of the sphere of it is equal to zero.

It can be easily proved that all the spheres touch in one straight line emanating from point 3, i.e., they are inside the circular cone with the summit at this point. Indeed the line 3-2'-1'-0', touching all the spheres in the plane of Fig. 27b, is a striaght line, since triangles 3-2-2', 3-1-1', and 3-0-0' are similar as straight lines having an identical ratio of left legs to the hypotenuses:

$$\frac{2-2'}{3-2}=\frac{at}{Vt}=\frac{a}{V};$$

$$\frac{1-1'}{3-1}=\frac{2at}{2Vt}=\frac{a}{V};$$

$$\frac{0-0'}{3-0}=\frac{3at}{3Vt}=\frac{a}{V}.$$

Thus we see that with the motion of thepoint source of weak pertubations with supersonic sound all the pertubations spread only inside the cone with the angle at the summit μ = arc sin $\frac{a}{V}$, or μ = arc sin $\frac{1}{M}$. Forming this cone is the line of weak perturbations, since at any moment of time at each point one sphere of weak perturbations touches. This line of weak perturbations is called the Mach line or characteristic line. Let us recall that on the line of weak perturbations all the parameters of gas are changed to infinitesimals, and therefore the line of weak perturbations is not the shock wave.

How is it possible to explain the appearance of shock waves? They are formed during the motion with the supersonic speed of a body the surface of which can be considered consisting of an infinitely large quantity of material points, each of which is the source of weak perturbations. Therefore, in separate regions of space near the body a multiple summation of an infinitely large quantity of weak perturbations occurs; the parameters of gas in these regions are now changed not to infinitesimals but to finites. These regions are shock waves, zones of a sharp intermittent increase in density, pressure and temperature of gas.

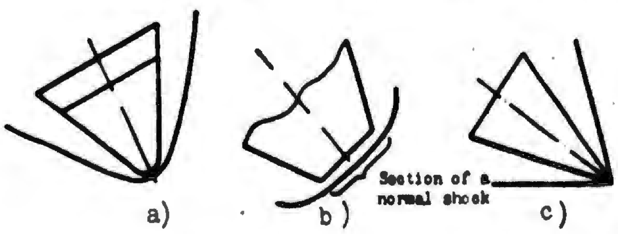


Fig. 28. Forms of shock waves:
a) curvilinear; b) linear; c) oblique

Shock waves can be complicated surfaces with curvilinear generating lines. With the entry into dense layers of the atmosphere of the head blunt part of the rocket with supersonic speed the shock wave (Fig. 28a), will be curvilinear and departing from the streamlined surface. If the blunting of the nose cone is made flat, then for the curvilinear shock wave there will be a brightly expressed section called the normal shock (Fig. 28b), the front of normal shock wave is perpendicular to the direction of flow. Frequently oblique shock waves are observed. These are rectilinear shocks, the front of which with the direction of the flow consists of an angle less than a right angle. Oblique shocks are formed (Fig. 28c), before the conical pointed nose cones of rockets and before wings and fins having a profile in the form of a wedge or rhomb and are characteristic in that they are adjoint to the streamlined surface, i.e., they touch it at least at one point.

What is the thickness of the shock waves? If the gas is non-viscous and do not conduct heat, the thickness of the shock wave is equal to zero. Only an approximate answer can be given to this question for real gas. For this it is necessary with the help of fundamental equations of aerodynamics to investigate with what greatest speed the parameters of a viscous gas can be changed in space taking into account the losses of heat to radiation and the thermal conduction and liberation of heat from the work of internal forces of friction in the gas. Such an investigation shows that the thickness of the shock wave is close to the mean value of the route of the free path of molecules in gas and has an order of magnitude of $\frac{\nu}{a}$, where ν is the kinematic modulus of viscosity of gas and a is the speed of sound in the gas. For air under normal conditions the thickness of the shock wave is close to 10^{-5} mm. Therefore, in calculations it is possible to consider the shock waves as infinitely thin and to depict them as surfaces and lines.

To calculate a shock wave we must know the parameters of gas before the shock and find the parameters of gas after the shock. Mathematical formulas depend on the state of gas after the shock. If the speed of the rocket is such that the Mach number attains 5-6, the properties of air after the shock practically do not change (the case of constant heat capacity). With Mach numbers from 5-6 to 7-8, in connection with the great increase in air temperature after the shock, the specific heat capacities prove to be not constant but dependent on temperature (the case of variable of heat capacity). Finally, with Mach numbers of more than 7-8 the temperature after the shock increases so much that air molecules begin to break and disintegrate into atoms, and then a thermal dissociation of molecules occurs. Properties of air after the shock are changed and depend on the relative number of disintegrating moleucles, i.e., on the degree of dissociation which depends on the temperature and to a lesser degree on pressure (the case of calculation of dissociation).

Let us consider the most complicated case of the calculation of dissociation in the example of the flowing around of a wedge-shaped profile by an undisturbed flow of air when an oblique shock wave appears (Fig. 29). The obtained calculation dependences will be useful for the curvilinear shock if it is divided into a sufficiently large quantity of oblique shocks (replacement of the curve by the broken line).

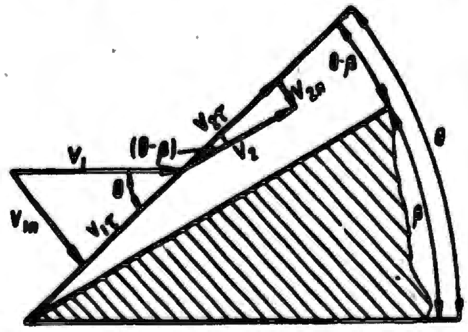


Fig. 29. Diagram of oblique shock wave with the flowing around of the wedge by a supersonic flow of air.

An experiment shows that advancing on the wedge-shaped profile with angle β, the undisturbed supersonic flow (with speed V_1) will turn also at angle β and will be flow along the surface of the wedge with speed $V_2 < V_1$, where a turn will be carried out with the transition of the stream of air through the front of the oblique shock wave. Thus after the shock wave there will be a uniform flow but with another direction and different velocity. With the transition through the shock wave all parameters of gas will be changed by the shock, but along the shock (both before it and after it) the parameters of air will remain the same. This explains why the tangent component of the airspeed before the shock $V_{1\tau}$ is not changed and is equal to the tangent component of the airspeed after the shock $V_{2\tau}$, i.e., $V_{1\tau} = V_{2\tau}$. Really if the component of speed changes its magnitude along the shock, then the cause of this could be only a change of pressure along the shock, which experimentally even for considerable lengths of the shock is not observed.

First of all let us examine in essence the physical phenomena with the transition of a particle of air through the front of the shock. We already know that the thickness of the shock wave is extremely small, and therefore the change of pressure on a particle of air transient through the shock will occur exceptionally fast. The particle will be as if subjected to a blow. During this blow, owing to the work of forces of internal friction, part of the energy of the particle will change irreversibly into heat, which after a short time of the process will not succeed in abandoning the particle neither by radiation nor thermal conduction and in addition will increase its temperature. From what has been said it is clear that the process of transition through the shock can be considered adiabatic, but it is impossible to consider it isentropic since the entropy of air S besides is sharply increased.

Let us compose the system of equations connecting the parameters of air before the shock with the parameters of air after the shock. First let us appropriate index 1, and secondly index 2.

According to the law of the conservation of mass, the mass of a particle of air which per unit of time will intersect the front of a shock wave through a unit of its surface on the normal to this surface should not be changed, therefore

$$\rho_1 V_{1n} = \rho_2 V_{2n} \qquad (3.39)$$

The change of momentum of the considered mass should be equal to the pulse having an effect on this mass of force, and consequently

$$\rho_1 V_{1n}(V_{1n} - V_{2n}) = p_2 - p_1, \qquad (3.40)$$

where $(p_2 - p_1)$ is the difference of pressures on the air partial intersecting the front of the shock wave. This difference is numerically equal to power applied to this particle, inasmuch as the time action is equal to unity, and the area on which the pressure acts is also equal to one.

The following will be the equation of the conservation of energy of air before and after the shock wave:

$$\frac{AV_1^2}{2g} + i_1 = \frac{AV_2^2}{2g} + i_2, \tag{3.41}$$

during the writing of which it was taken into account that the work of forces of friction L_{Tp} irreversibly transferred part of the energy of air into heat Q, which completely resembled an increase in the enthalpy of air after the shock wave. In other words, on the right side of the equation (3.41) two components Q and AL_{Tp} equal in magnitude are mutually destroyed.

For the air both before and after the shock the equation of state of ideal gas will be correct, i.e.,

$$p_1 = g\rho_1 \cdot \frac{R_0}{\mu_{1cp}} \cdot T_1 \quad \text{and} \quad p_2 = g\rho_2 \cdot \frac{R_0}{\mu_{2cp}} \cdot T_2,$$

where μ_{1cp}, μ_{2cp} are mean molecular weights of air respectively before and after the shock; R_0 is the universal gas constant.

Dividing these equations by each other we obtain

$$\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} \cdot \frac{T_2}{T_1} \cdot \frac{\mu_{1cp}}{\mu_{2cp}}. \tag{3.42}$$

Let us use the property of the tangent components of speed $V_{1\tau} = V_{2\tau}$, and then from triangles of expansion of speeds before and after the shock (Fig. 29) we get

$$\frac{V_{1n}}{\operatorname{tg}\theta} = V_{1\tau} = V_{2\tau} = \frac{V_{2n}}{\operatorname{tg}(\theta-\beta)},$$

i.e.,

$$\frac{V_{2n}}{V_{1n}} = \frac{\operatorname{tg}(\theta-\beta)}{\operatorname{tg}\beta}, \tag{3.43}$$

where θ is the angle between the front of the shock and direction of the incident flow (angle of shock).

The following two equations also are obtained from triangles of expansion of speeds:

$$V_{1n} = V_1 \sin\theta; \tag{3.44}$$

$$V_{2n} = V_2 \sin(\theta - \beta). \tag{3.45}$$

Earlier it was noted that with the dissociation the properties of air after the jump depend on the degree of dissociation and are determined by temperature T_2 and pressure p_2. Therefore the dependences for i_2, a_2, S_2, and μ_{2cp} are complicated and are obtained after thermodynamic calculations in the form of tables or graphs [14].

The system of equations is closed by the formula for the transition from the flow rate after shock waves to the Mach number:

$$M_2 = \frac{V_2}{a_2}. \tag{3.46}$$

In the obtained system of twelve equations (four of them are given by tables or graphs for i_2, a_2, S_2, and μ_{2cp}) there are the following twelve unknowns:

$$p_2,\ \rho_2,\ T_2,\ V_2,\ V_{2n},\ M_2,\ \theta,\ i_2,\ a_2,\ S_2,\ \mu_{2cp},\ V_{1n}$$

With this the following parameters of the gas before the shock should be known: c_{p_1}, k_1, p_1, ρ_1, T_1, V_1, M_1, i_1, a_1, S_1, μ_{1cp}, and β, which are given not arbitrarily since they are connected with each other by a number of relationships. For instance, $i_1 = c_{p_1}T_1$; $M_1 = \frac{V_1}{a_1}$, etc.

To solve the obtained system of equation comparatively simple methods have been developed. If we will deal with the normal shock these methods are simplified, since $\theta = \pi/2$; $\sin^2\theta = 1$; $V_{1n} = V_1$ and $V_{2n} = V_2$. Usually we determine the parameters of air by these methods in the case of variable heat capacities.

Methods for the case of constant heat capacities appear considerably simpler, i.e., when $M < 5\text{-}6$, since $\mu_{2cp} = \mu_1$, and instead of graphs or tables for t_2, a_2, S_2 simple formulas are useful. From results of calculations tables are compiled allowing an easy and quick determination for air from $k = 1.4$ to M_1 and β of the angle of the shock and also $\frac{p_2}{p_1}$; $\frac{\rho_2}{\rho_1}$; $\frac{T_2}{T_1}$ and the pressure ratio of deceleration after the shock to the impact pressure before the shock $\frac{p_{20}}{p_{10}}$.

For a practical application of the calculations of shock waves, for the case of a constant heat capacity it is convenient to use graphs of shock fields (Fig. 30). The shock polar is the geometric place of the ends of the velocity vector after the shock V_2, which is obtained with the assigned M_1 and k numbers of the incident flow if we change the angle of the wedge β and hold the beginning of vector V_2 all the time at one point (the pole). Essentially the shock polar is a hodograph of speed V_2 after the shock.

The shock polar permits finding the magnitude V_2 and also the angle of the shock θ. For this it follows from the pole O to conduct a line at an angle of the wedge β to the crossing with the shock polar plotted for the required M_1 (or V_1) and k at point p. The segment Op determines the magnitude V_2, and the angle between axis OV_x and perpendicular to the line Op will be the angle θ.

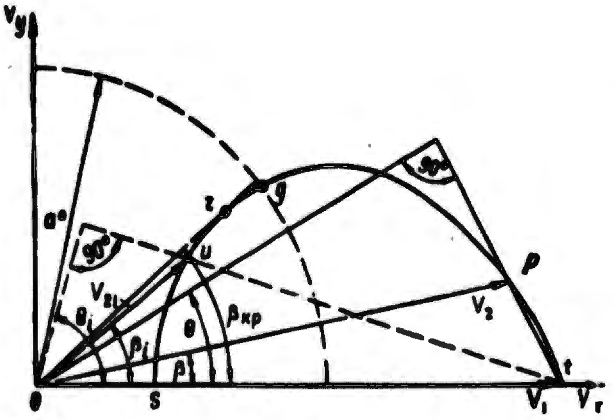


Fig. 30. Graph of a shock polar.

If the angle of wedge β approaches to zero, then with the help of the polar it is possible to establish that V_2 tends to V_1 (intensity of the shock drops, but the angle of the shock θ tends to the magnitude arc sin $\frac{1}{M_1}$, i.e., the shock turns into a line of weak perturbations which as it was earlier established exactly forms with the vector of the incident flow V_1 angle $\mu = \text{arc sin}\,\frac{1}{M_1}$.

If we increase angle β the end of vector V_2 will come to point g, and V_2 will be equal to the critical velocity a*. If angles of β are such that the end of V_2 is on the shock polar from point g to point r, then the speed after the oblique shock wave will be subsonic (smaller than a*), i.e., after the oblique shock wave the flow of gas can be both supersonic and subsonic. After the normal shock the flow rate will always be subsonic and equal to $V_2 = \frac{a^{*2}}{V_1}$. The normal shock on the shock polar corresponds to point S.

What will be with the shock wave if angle β continues to increase exceeding the value of $\beta_{кр}$ corresponding to the contact of line Or of the shock polar at point r? Consequently, for this case on the shock polar it is impossible to determine neither the magnitude V_2 or value θ for the oblique shock. This means that the oblique shock ceases to be rectilinear and when $\beta > \beta_{кр}$ is turned into a curvilinear departing shock. An experiment confirms such a recovery. The greater the departure and bend of the shock the greater the angle of wedge β. With an increase in the number of M_1 of the incident flow the departure of the shock from the point of wedge inversely decreases.

The segment of the shock polar from point S to point r is useful for calculating the departing curvilinear shock if it is considered consisting of a sufficiently large quantity of separate rectilinear shocks, i.e., the curvilinear front of the shock is replaced by the broken line. To determine the magnitude of velocity V_2 after some segment the curvilinear shock it is necessary to know the angle of inclination of this shock θ_1 to vector V_1. By plotting the line at this angle from the pole O and dropping a perpendicular on it from point t, we will find the line of crossing of this perpendicular and segment Sr of the shock the polar point u of the end of the desired speed V_{21}.

The form of the curvilinear departing shock, if it is possible to reproduce the experiment, can be obtained by means of photographing this shock. In the case of absence of experimental data the form of jump can be revealed only by means of a very complicated and bulky calculation producible on electronic digital computers. Some of these such calculations were made by P. I. Chushkin and N. P. Shulishina [15], who computed tables allowing to determine coordinates of the curvilinear departing shock during the flight of conical bodies in air blunted by the sphere or ellipsoid, with different Mach numbers. In these tables wave drag coefficients of such conical bodies are also given.

Let us note the interesting peculiarity of the flow of gas after the curvilinear shock: the irritational and isentropic flow past such a shock ceases to be both irrotational and isentropic. This occurs because the intensity of the shock at different places of it is different. In the average part of the curvilinear shock a segment of the normal shock can be found; according to the removal from this segment the angle of the shock decreases, and the intensity of the variation of parameters of gas past the shock also decreases. Particles of gas on adjacent trajectories will have a different speed and different value of entropy. This will lead to the fact that a transverse gradient of speed will be formed which will cause turbulence of the flow. In spite of the fact that the entropy particles of gas along trajectories after the shock will not noticeably be changed its value is different for different trajectories, and the flow cannot be considered isentropic (with identical entropy).

If, however, the shock is rectilinear (normal or oblique) these phenomena do not occur, and after the shock the flow will remain irrotational and isentropic. When the curvature of the shock is small in the first approximation the flow after the shock can also be consider irrotational and isentropic.

§ 4. Theoretical Determination of Aerodynamic Coefficients

The theoretical determination of aerodynamic coefficients encounters great difficulties mainly mathematical. Considering volume and assignment of this book we limit ourselves only to the simplest examples.

We start from the determination of aerodynamic coefficients considering the frictional force of air against the surface of a rocket flying in it. For this purpose let us consider the plate in the flow of air and then we will extend the obtained data to the rocket. For a clarification of forces of friction of air against the streamlined surface it is necessary to consider regularities of flow of air in the boundary layer. We will consider the air within the boundary layer of the viscous incompressible liquid. It is possible to consider the influence of compressibility of air on its flow in the boundary layer by means of the introduction of correctives into formulas obtained for an incompressible liquid.

Let us consider for simplicity only the plane steady flow, i.e., that during all parameters of air are only functions of coordinates x and y (Fig. 26). In practice in very many cases the variations of parameters of air according to width of the boundary layer are insignificant, and therefore the results of the solution of the two-dimensional problem are widely applied.

Let us analyze the flow of air in the boundary layer. For this let us see how the fundamental equations of aerodynamics will appear for the considered case. The equation of state for an incompressible liquid will be the condition ρ = const. The continuity equation for the plane flow is obtained from the formula (3.21)

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}=0. \qquad (3.47)$$

From formula (3.25) when ρ = const we obtain the integral of the equation of motion (Bernoulli equation)

$$\frac{V^2}{2}+\frac{p}{\rho}=\mathrm{const.} \qquad (3.48)$$

The equation of motion for a viscous liquid from the equation of motion in the form of Euler (3.23) should differ by the presence of the components, considering the forces of internal friction which we, not making a complicated derivation, designate N_x and N_y:

$$V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+N_x; \qquad (3.49)$$

$$V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+N_y \qquad (3.50)$$

In such a form (taking into account the forces of viscosity) the given equations are called Navier-Stokes equations.

A direct solution to the system of the equation for the boundary layer is very complicated and succeeds only with the very simplest boundary conditions. For instance, the accurate solution of the motion of a viscous liquid near a thin plate belonging to academician N. Ye. Kochin (1944), is well-known.

Analyzing the order of components in equations (3.49) and (3.50), it is possible to conclude that

$$\frac{\partial p}{\partial y}=0. \qquad (3.51)$$

This is the most important result of the analysis of the boundary layer.

From formula (3.51) it follows that the pressure about the thickness of the boundary layer is constant and such that it is on the boundary between the free flow and boundary layer. In other words, the boundary layer does not change the pressure in a free flow, and therefore the hypothesis about the absence of a reverse boundary layer effect on the free flow is correct. The above mentioned permits calculating the pressure at the streamlined surface by formulas of an ideal compressible liquid in a free flow and, by measuring the pressure at the wall of the flow, comparing it with the calculation, checking thereby the accuracy of the mathematical formulas and the calculations themselves. The error connected with the disregard of thickness of the boundary layer is small, since the boundary layer is usually thin.

When does that which has been said about the constancy of pressure along the thickness of the boundary layer not take place? In the first place when δ is commensurable with the dimensions of the rocket, which occurs at the end of the long streamlined surfaces and, secondly, in the zone of interaction of the shock wave with the boundary layer where the pressure along the thickness of the boundary layer is variable.

Parameters of air in the boundary layer and, consequently, frictional forces against the streamlined surface essentially depend on the conditions of the flow, which can be laminar or turbulent. With the laminar flow particles of air move along smooth trajectories not being mixed; the flow itself has a laminar character. With the turbulent flow particles of air move along the complicated trajectories, shift in a transverse direction from layer to layer, and are mixed. The airspeed and other parameters at a given point of the turbulent flow are changed in time near the mean value. These variations of the parameters have a random character and are called pulsation. Strictly speaking, the turbulent flow is always transient. However, if were to apply the mean constant value of the parameters of air (p, ρ, T, and $\bar{V}$) then such an averaged turbulent flow will be steady, and for it the earlier obtained dependences will be correct.

Let us look at the typical streamline air flow of some body (Fig.31). In the nose section of the body there appears a laminar boundary layer a (Fig. 31).

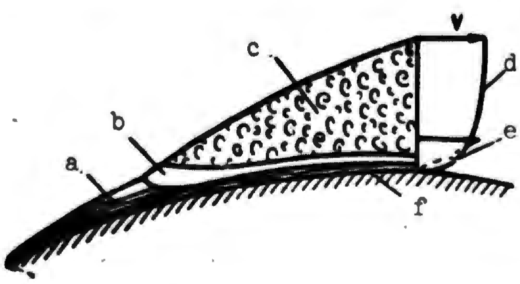


Fig. 31. Boundary layer with the flowing around of a body by an air flow: a - laminar boundary layer; b - transition layer; c- turbulent boundary layer; d - curve of speed in a turbulent layer; e - curve of speed in a laminar layer; f - laminar sublayer.

Further with the increase of the flow rate it passes to the turbulent boundary layer c in which, however always in direct proximity to the streamlined surface can separate the laminar sub-layer f. The transition from the laminar to turbulent conditions of flow occurs not at once but within the certain transition layer b.

The conditions of flow in the boundary layer can be recognize with the help of the dimensionless Reynolds number

$$Re = \frac{Vd\rho}{\mu},$$

where V is the flow rate; d is the characteristic dimension of the streamlined surface, ρ is the mass air density; μ isthe coefficient of viscosity of air.

The physical meaning of Re is that it is the standard of the relation of forces of inertia to forces of viscosity in air. The larger Re is the greater the share is apportioned to forces of inertia (ρV is the momentum of unit volume of air) and the smaller the forces of viscosity (μ determines the force of viscosity). With this, of course, distortions appear easier of trajectories of particles of air, whirling and mixing, since the forces of viscosity cannot oversome the action of different accidental pulses appearing in the air. The transition from laminar to turbulent conditions of flow is determined by $Re_{кр}$, a certain critical value of the number Re. If $Re < Re_{кр}$ then the laminar flow occurs; if, however, $Re > Re_{кр}$ then there is the turbulent flow.

Other things being equal the velocity curves in the turbulent boundary layer is more complete than in laminar, but the frictional forces are greater (d and e in Fig. 31). This is explained by the difference in the mechanism of friction in air. With the laminar flow friction is carried out owing to shifts in molecules of air in their oscillatory motion from a layer with one speed to a layer with another speed. With the turbulent flow such shifts carry out whole particles of air. Therefore, the equalizing of the speed along the thickness of the flow occurs faster.

Experiments showed that a change in the speed along the thickness of the boundary layer can be described by the following dependences $V_x(y)$;

$$V_x = a + by + cy^2 + dy^3 \quad \text{– for laminar conditions of the flow,} \tag{3.52}$$

where a, b, c and d – are coefficients determined from physical considerations;

$$V_x = V\left(\frac{y}{\delta}\right)^{1/7} \quad \text{– for turbulent conditions of the flow.} \tag{3.53}$$

Applying formulas (3.52) and (3.53) from equations (3.47), (3.48), (3.49), and (3.51) one can determine the characteristics magnitudes of the boundary layer at a plate with a length L located at an angle of attack $\alpha = 0$ in the undisturbed flow of air with the speed V_∞. Perturbations of the flow caused by such a plate are negligible, so that the speed on the boundary of the free flow and boundary layer is everywhere equal to V_∞, but the pressure is constant and equal to p_∞, the pressure in an undisturbed flow. Thus, for instance, for a laminar boundary layer the thickness of it δ at distance x from the beginning of the plate proves to be equal to

$$\delta = \frac{4{,}64x}{Re_x^{0{,}5}}, \tag{3.54}$$

where $Re_x = \frac{V_\infty x\rho}{\mu}$.

From formula (3.54) one can see that the thickness of the boundary layer is larger, the larger x and μ are and the smaller ρ and V_∞.

The coefficient of friction of the laminar boundary layer C_f, determining the frictional force of air against the plate $X_f = C_f qS$, is from the expression

$$C_f = \frac{1{,}3}{Re_L^{0{,}5}}, \tag{3.55}$$

where $Re_L = \frac{V_\infty L\rho}{\mu}$.

Analogous formulas for the turbulent boundary layer have the form

$$\delta = \frac{0{,}37x^{0{,}8}}{Re_x^{0{,}2}} \quad \text{and} \quad C_f = \frac{0{,}074}{Re_L^{0{,}2}}. \tag{3.56}$$

A comparison of the basic parameters of laminar and turbulent boundary layers leads to the following to conclusions.

For the turbulent boundary layer:

– V_x increases more quickly along the thickness of the boundary layer (curve of speeds is fuller);

– the thickness of the boundary layer along the length of the plate increases more quickly;

– the coefficient of frictional force is greater than for the laminar boundary layer, i.e., the turbulent friction is greater than the laminar;

– the larger the number Re, the more the parameters of the laminar and turbulent boundary layers differ.

Using these conclusions the designers of rockets try to make the profiles of wings and other surfaces in such a way that the boundary layer is laminar, and the force of aerodynamic drag to the flight smaller.

Formulas obtained for the plate are the basis of a widely applied method of approximation of the determination of coefficients of forces of friction of air against the surface of the rocket. This method is correct with the following assumptions:

– both in the laminar and turbulent boundary layer near rocket forces act which can be expressed by formulas obtained for the plate circumfluous by incompressible liquid;

– the transition from the laminar to turbulent boundary layer is carried out immediately without a transition zone;

– the distribution of parameters along the length of the turbulent boundary layer the same as if the turbulent layer started from the summit of the nose cone of the rocket (curve O-b-c in Fig. 32);

– a separation of the boundary layer from the lateral surface of the rocket does not occur.

The order of calculation of the resisting force of friction is the following.

1. By the altitude of flight ρ_∞ and μ_∞.

2. From the formula

$$Re_{кр} = \frac{V_\infty x_{лам} \rho_\infty}{\mu_\infty}$$

$x_{лам}$ is determined which is the laminar boundary layer (here $Re_{кр}$ is the critical Reynolds number).

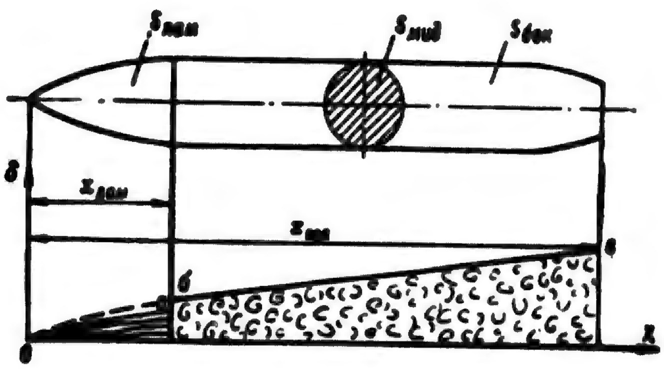


Fig. 32. Boundary layer of a solid of revolution.

As experiments showed it is possible to consider for subsonic flows $Re_{кр} = 4 \cdot 10^5$, and for supersonic flows $Re_{кр} = 6.5 \cdot 10^5$.

3. The coefficient of frictional force for the part of the rocket circumfluous the laminar boundary layer is calculated:

$$C_{f\,лам} = \frac{1.3}{Re_{кр}^{0.5}}.$$

4. The coefficient of frictional force for the part of the rocket circumfluous the laminar boundary layer is calculated:

$$C_{f\,тур} = \frac{0.074}{Re_{кр}^{0.20}}.$$

5. The Reynolds number for the entire length of the body $x_{пол}$ is found:

$$Re_{пол} = \frac{V_\infty x_{пол} \rho_\infty}{\mu_\infty}.$$

6. $C_{f\,тур}$ (coefficient of frictional force for the whole body) is determined on the condition that the turbulent boundary layer will be on the whole lateral surface $S_{бок}$. It is recommended to do this depending upon the magnitude $Re_{пол}$ by the formula (3.56) different empirical formulas ensuring best agreement with the experiment.

7. C_f (coefficient of frictional force for the whole rocket) is calculated taking into account the fact that the boundary layer acts on the surface $S_{бок}$, and the frictional force of the rocket is determined by the middle section (the biggest) $S_{мид}$;

$$C_f = C_{f\,лам} \cdot \frac{S_{лам}}{S_{мид}} + C_{f\,тур} \cdot \frac{S_{бок}}{S_{мид}} - C_{f\,тур} \cdot \frac{S_{лам}}{S_{мид}},$$

where $S_{лам}$ is the lateral surface of the rocket flowed around by the laminar boundary layer.

8. The resisting force of the rocket in the flow from friction against the lateral surface is determined:

$$X_f = C_f \cdot \frac{\rho_\infty V_\infty^2}{2} \cdot S_{мид}.$$

If the rocket has wings, planes and so forth, then for them frictional forces are similarly determined which are then added to the earlier found X_f.

Now let us examine the phenomenon of the separation of flow from the streamlined surface and see when the separation occurs and what are its results.

For this we will consider the boundary layer for the curvilinear surface АБ (Fig. 33). On segment АМ the sections of air streams decrease, since the flow encounters the convex surface; the speeds increase ($V_M > V_2 > V_1$) according to the equation of continuity, and the pressure in the flow and, consequently, in the boundary layer decrease (this is evident from the Bernoulli equation (3.48) where ρ = const).

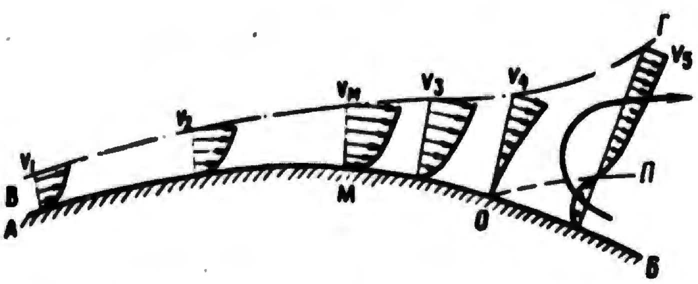


Fig. 33. Diagram of the formation of the separation of the boundary layer from the streamlined curvilinear surface:

АБ – streamlined curvilinear surface; ВГ – boundary between the free flow and boundary layer; М – point on the middle section of the streamlined body; О – separation point of the boundary layer; БОП – zone of reverse flow.

On segment МБ, on the other hand, the sections of air streams are increased, the speeds decrease ($V_5 < V_4 < V_3 < V_M$), and the pressures all over the section of the boundary layer are increased, which brakes the air streams over the whole section. Along the section of the boundary layer the air streams have a different kinetic energy. The less the energy the nearer the stream is located to the streamlined surface. This is clear, since the speed is less for these streams. Consequently, an identical increase in pressure along the section of the boundary layer will brake those streams more which are nearer to the streamlined surface and have a smaller kinetic energy. On the streamlined surface there can be the point O (Fig. 33) where $\left(\frac{\partial V_x}{\partial y}\right)_{y=0} = 0$ (tangent to the curve of speeds will be directed along the normal to the streamlined surface). Here near the surface the increase in pressure brakes the stream. On segment ОБ the pressure continues to increase and the air motion appears from places with great pressure to places with smaller pressure, i.e., opposite the flow in free flow. Along the line ОП a reverse motion interacts with the boundary layer and is carried away from the streamlined surface. A separation of the flow or separation of the boundary layer occur. Moreover, in the region of the line ОП vortexes are engendered. Point O is considered the separation point of the boundary layer. On section ОБ, owing to the separation of the boundary layer, the pressure sharply decreases, which increases the drag of the body to the flow and decreases the lift. From the energy viewpoint the increase in drag occurs due to the transformation of kinetic energy of forward motion of the flow (or the body if it moves in air) into energy of the rotation of air particles. The steeper the outlines of the afterbodies (tails) of the streamlined body change, the faster the boundary separates layer, since the cross section of streams will increase more quickly, the speed will increase faster, and the pressure rises faster, the more intensive are the streams of air in the boundary layer.

The above-mentioned shows that for practice it is important to be able to solve the problem of the boundary layer near the curvilinear surface and to determine the point of separation of the boundary layer. For this there are different methods the account of which emerges outside the present book.

On the magnitude of forces of friction a great influence is the roughness of the surface streamlined by air characterized by the altitude of prominence. The greater the altitude of prominence the greater the frictional force. It is possible to calculate the frictional force, taking into account the roughness of the surface by different empirical formulas and graphs obtained from data of experiments [12].

During high speeds of the flight of a rocket in the boundary layer a large quantity of heat is released, part of which can be diverted to the streamlined surface. Thermal processes in the boundary layer affect the magnitude of the frictional force. It appears that the earlier obtained formulas can be used when the parameters of gas (ρ, μ) are calculated with a certain mean temperature, called the characteristic temperature, with which we will examine somewhat later.

How are those aerodynamic coefficients which depend on the distribution of pressure along the surface of the rocket (with the exception of that part of its surface which is not circumfluous by the incident flow separated from it) determined theoretically? The magnitude of these coefficients depends on the magnitude and character of distribution of pressure along the surface of the rocket. Consequently, first of all it is necessary to determine theoretically the pressure at any point of the surface of the rocket of interest to us. This is a complicated theoretical problem, and it is solved comparatively easy only for bodies with simple forms of the surface (cone, plate, and so forth).

Let us consider the flowing around of a cone by a supersonic flow (Fig. 34) with an angle of incidence $\alpha = 0$, since it is the typical form of nose cones of many rockets. It appears that in the space between the surface of the cone and shock wave the flow of air has interesting properties: its parameters are not changed along any forming intermediate conical surface conducted from the vertex

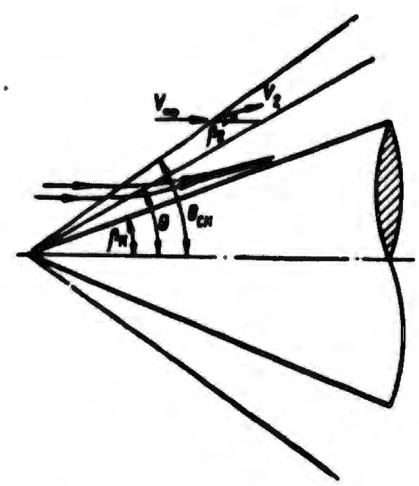


Fig. 34. Diagram of the flowing around of a cone by supersonic flow.

of the cone at an angle θ. With this $\beta_к \leq \theta \leq \theta_{ск}$, i.e., θ changes from the angle of the cone $\beta_к$ to the angle of shock $\theta_{ск}$. However, on various conical surfaces the air parameters with such properties are called <u>conical.</u>

The trajectory of a particle of air passing through the surface of a shock wave turns at the angle $\beta_2 < \beta_к$, and therefore with the further motion of the particle of air a gradual turn of its trajectory continues (Fig. 34). With this the speed decreases and the pressure rises.

The dependences between the parameters of air before and after the shock and the angle of turn of the flow β_2 with the flowing around of the cone by supersonic flow will be the same as with the flowing around of the wedge. This is obtained because the initial equations will be in both cases identical. It should only be rememuered, that the angle of rotation of the flow with the flowing around of the wedge (flat case) is equal to the angle of wedge, and with the flowing around of the cone less than the angle of the cone in which the spatiality of flow appears.

Figure 35 shows the shock polar 1 plotted for the definite flow rate of air $V_1 = V_\infty$. The theoretical solution to the problem of the flow around the cone

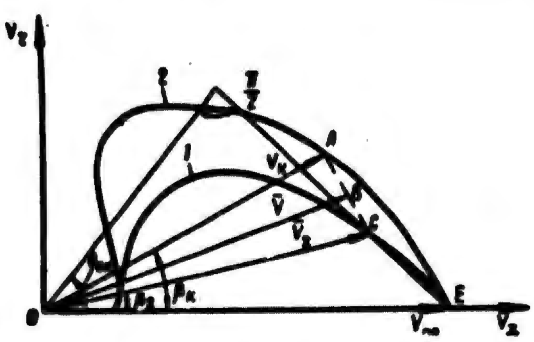


Fig. 35. Apple-shaped curve for cones circumflowed by a supersonic stream with velocity V_∞. 1 – shock polar; 2 – apple-shaped curve.

permitted plotting an apple-shaped curve 2 and whole network of dotted lines (ABC type), which are hodographs of the flow rate $\bar{V}$ between the shock wave and surface of the cone. In order to find the flow rate on the surface of cone $\bar{V}_к$ if follows from the beginning of 0 coordinates to draw a straight line at an angle $\beta_к$ of cone up to the crossing with the apple-shaped curve at point A. Segment OA will be the velocity vector $\bar{V}_к$ on the surface of the cone. If now from point A to point C we draw the dotted line of the hodograph of speed, then the segment OC will determine the magnitude of the velocity vector $\bar{V}_2$ of the flow after the shock and angle β_2 is the angle of rotation of flow after the shock wave. The angle of the shock θ_{CK} before the cone is determined between axis V_x and the perpendicular dropped from the beginning of the coordinates on the continuation of line EC. From tables we find breaking pressure after the shock p_{20}. Now according to dependence (3.38) we can find the pressure on the cone $p_K = p_{20}\left(1 + \frac{k-1}{2} \cdot M_K^2\right)^{-\frac{k}{k-1}}$ which, as we know from properties of the conical flow, will be identical over the entire surface of the cone. This makes it possible to calculate simply the force of drag of the cone from the pressure X_B (wave drag) and, consequently, C_{x_B}, the coefficient of this force.

X_B is equal to the product of excess pressure on the surface of the cone $(p_K - p_\infty)$ by the area of the projection of the surface of the cone on the plane perpendicular to its axis, i.e., by $S_K = \pi r_K^2$. Thus

$$X_B = (p_K - p_\infty)\pi r_K^2. \tag{3.57}$$

On the other hand, X_B can be expressed in terms of its aerodynamic coefficient:

$$X_B = C_{x_B} q S_K. \tag{3.58}$$

By comparing expressions (3.57) and (3.58) we obtain

$$C_{x_B} = \frac{p_K - p_\infty}{q} = \bar{p}_K, \tag{3.59}$$

i.e., the coefficient of wave drag of the cone when $\alpha = 0$ will be equal to the pressure coefficient on the surface of the cone.

For the cone there are tables allowing according to the number M_∞ of incident flow and angle of cone to find θ_{CK}, p_K, and even C_{x_B} [13].

Solutions for the cone serve as a basis for the method of approximation of the determination of pressure on the surface of a solid of revolution of arbitrary form (Fig. 36). Experiments showed that the pressure at the arbitrary point A is very close to the pressure which would be on the cone with angle β_K, equal to angle β_{K_a} of dip of the tangent to the axis of the solid of revolution drawn to surfaces of the body at point A. This method is called the method of local cones and permits determining the distribution of pressure on the surface of an arbitrary solid of revolution and to calculate the respective aerodynamic coefficients.

How are the aerodynamic coefficients of rockets of complicated form determined?

The most widespread are semiempirical methods which provide for the conditional separation of the rocket into a number of characteristic parts: nose, cylindrical and tail (after body) part of the body, separate consoles of the wing and fin, etc. Further, according to theoretical or empirical formulas for the given

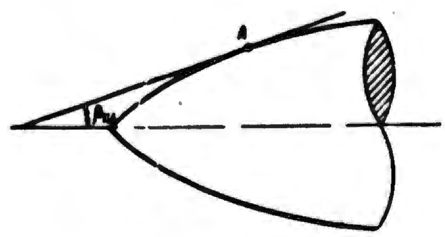


Fig. 36. Diagram illustrating the method of local cones.

Mach number, altitude of flight, and angles α and β, and according to geometric dimensions different aerodynamic coefficients of separate parts of the rocket are calculated. For instance, by the formula (3.9) the coefficient of induced drag of a wing; with the help of the method of local cones we find drag coefficient of the nose cone of the rocket, and so on. Aerodynamic coefficients of small protruding parts, as a rule, are estimated together from experimental data. Then, considering the correct principle of the independence of action of forces, which consists in the fact that aerodynamic forces of separate parts of the rocket (and this means their coefficients) do not influence on another, the analogous coefficients of all parts of the rocket are added. Thus aerodynamic coefficients of the whole rocket are obtained.

The principle of the independence of action of forces proves to be far from always correct, and therefore it is necessary to introduce a correction into the obtained aerodynamic coefficients of the rocket for the mutual influence on each other.

§ 5. Experimental Determination of Aerodynamic Coefficients

In the beginning of § 2 a brief characteristic of the methods of experimental determination of aerodynamic coefficients was given. Let us dwell in greater detail on the tests of models of rockets in wind tunnels and on flying tests.

The wind tunnel is a device in whose working part is created uniform flow of air which shifts with different speeds (or Mach numbers) and influences the investigated model, which is geometrically similar to a rocket and made in reduced scale.

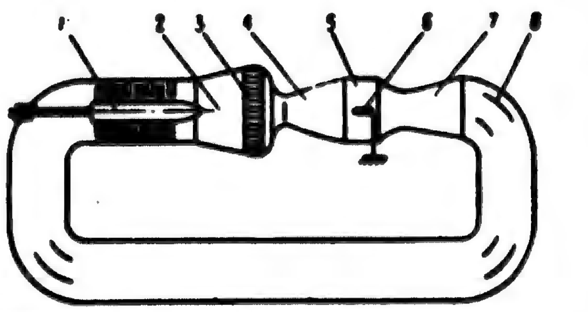


Fig. 37. Diagram of a wind tunnel with a closed contour:
1) compressor; 2) precombustion chamber; 3) straightening grid; 4) nozzle; 5) closed working part; 6) model of rocket; 7) diffuser; 8) turning blades.

There is large number of different forms of wind tunnels: those with a prolonged and short-lived action, with open and closed working parts, with a round or rectangular cross section of the working part, and others. According to the range of the flow rate the wind tunnels are divided into subsonic (M up to ~0.8), transonic (M from ~0.8 to 1.3), supersonic (M from ~1.3 to 4-5), and hypersonic (M above 4-5).

Let us examine the device frequently used for wind tunnels.

The wind tunnel with a closed contour (Fig. 37) through which circulates air is typical. In this case the power of the compressor 1 is consumed by the surmounting of all drags to the air movement inside the wind tunnel. To decrease the magnitude of these losses the internal walls of the tunnel are made smooth, and the turns are smooth. For this purpose there are turning blades 8 installed at places of the turn of the flow. In wind tunnels of the closed type the working part 5 with the model 6 can be both open and hermetically sealed. In the latter case there is no sucking of air from the space surrounding the tunnel, losses to air friction against the walls of the working part are less than losses to friction of air against air, and there is the possibility before the experiment to pump out part of the air from the tunnel in order to decrease the power of compressor necessary for the creation of a definite speed. Sometimes, on the contrary, a special heavy gas is pumped into the tunnel (for instance, freon), the speed of sound in which is small and the Mach number $M = \frac{V}{a}$ is therefore great with an identical flow rate. From the working part of the tunnel air enters into a special device (diffuser) which brakes the flow converting its kinetic energy into potential energy. By passing the route from the diffuser to the compressor with less speed the air loses less of its energy to the overcoming of the resistances.

Wind tunnels of the closed type can be not only subsonic but transonic and even supersonic. If the flow in the working part is supersonic, for its deceleration the diffuser should be at first tapered and then expanded. However, one should consider that for the creation of supersonic flow very high power of the compressor is necessary. For instance, the power of a compressor of a supersonic tunnel with the cross section of the working part 40 × 40 cm^2 attains 900 hp. Therefore, most frequently supersonic tunnels operate with brief action.

Figure 38 gives a diagram of a supersonic wind tunnel of brief action of the cylinder type with the preheating of air and ejection. The electric motor 1 puts into action the compressor 2, which during the time of preparation for the experiment should in the battery of cylinders 7 create the necessary reserve of air. To decrease the capacity of the cylinders the air in them is forced under great pressure. The air should satisfy the following requirements: it should not contain

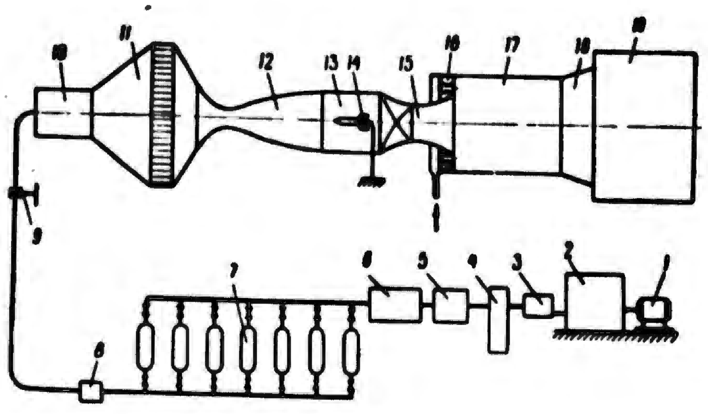


Fig. 38. Diagram of a supersonic wind tunnel of the cylinder type with the preheating of air and ejection:

1) electric motor; 2) compressor; 3) refrigerator; 4) oil sump; 5) receiver; 6) desiccant; 7) battery of cylinders; 8) reductor; 9) high speed value; 10) heater; 11) precombustion chamber; 12) nozzle; 13) closed working part of the tunnel; 14) model of the rocket; 15) diffuser; 16) ejector; 17) mixing chamber; 18) diffuser; 19) muffler.

impurities of oil which enter it in the compressor and should not have water vapor which at low temperature will be condensed in the working part of tunnel disturbing the homogeneity of the flow and covering the model with a layer of ice. Therefore, the air before entering the cylinders is subjected to thorough purification. First of all the air is cooled in the refrigerator 3 where vapors of oil are turned into a liquid which is separated from the air in the oil sump 4. Then the air enters the receiver 5 in which it is expanded, cooled, and loses most of the water contained in it. The rest of the water is removed from the air in the desiccant 6. Pressure in the precombustion chamber 11 of the tunnel usually does not exceed several atmospheres, and therefore the air passes through the reductor 8 which decreases pressure to the required magnitude. With the help of the high speed valve 9 the supply of air in the wind tunnel is ensured. The higher the flow rate of air in the working part of the tunnel 13 the lower its temperature. With numbers $M \cong 3.5$ in the open working part and about 5-6 in the hermetically sealed working part, the temperature of the air becomes so low that condensation of atmospheric oxygen begins. In order to avoid this and to have the possibility of increasing the airspeed in the tunnel, the air is heated in a special device 10. In the precombustion chamber 11 where the heated air enters it passes through a special grid which decreases the vorticity (turbulence) of the flow. The necessary flow rate is created by the detachable nozzle 12 which is profiled so that on the model of the rocket 14 placed in the working part of the tunnel 13 a uniform flow of air strikes air. To decrease the losses in the tunnel at the output of the working part the diffuser 15 is placed, in the narrowing part of which the air passing the system of oblique shocks is braked up to subsonic speed. The deceleration of flow continues then in the expanded part of the diffuser. The flow rate in the working part can be made greater the lower the pressure at the output of the diffuser. This pressure can be decreased with help of ejector. If the air under pressure passes through the nozzle unit of ejector 16 and accelerates to great speed, then in the chamber of mixing 17 at the output of the diffuser 15 will be formed a zone of reduced pressure. In other words, the ejector will ensure the sucking of air from the diffuser, increase the pressure drop at the entrance and exit of the working part of the tunnel, and therby increase the flow rate and M number of the tunnel. To decrease the noise produced by the wind tunnel, the airspeed is decreased in the diffuser 18 and the air is fed into the muffler 19, a chamber of large dimensions filled by a sound-absorbing material. Only by passing through the muffler the air with low speed is ejected into atmosphere.

Thus the contemporary supersonic wind tunnel is constitutes complicated complex of structures and equipment, but it permits obtaining uniform flows of air with a large M number.

Figure 39 gives a diagram of a device and principle of action of a typical hypersonic aerodynamic shock tube. In the section of high pressure 1 the compressor presses a light gas (for instance, hydrogen). With calculated pressure the diaphragm 2 breaks and a light gas with great speed flows into the sections of low pressure 3 where there was preliminarily created a high rarefaction of air. Through the air in section 3 there is at first the shock wave and then the hydrogen proceeding from section 1. Between them the air forms the so-called working plug which, passing through nozzle, obtains a uniform field of speeds and flows around the model of the rocket 4 placed in the working part of the tunnel 5.

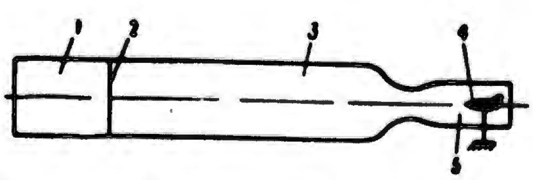


Fig. 39. Diagram of a shock wind tunnel: 1) section of high pressure; 2) diaphragm; 3) section of low pressure; 4) model of the rocket; 5) working part of the tunnel.

Shock wind tunnels allow the obtaining of the M number up to 20 and even more. However, the time during which the model is blown by the flow with uniform speed is very small and comprises only about 10^{-5} sec. For so small an interval of time the contemporary instruments permit measuring the pressure and temperature of the air circumfluent the model. Shock tubes are used mainly for determining properties of air during hypersonic speeds.

With the carrying out of experiments in wind tunnels the following methods are applied allowing to determine the qualitative picture of the streamline flow of the model of the rocket by the air flow and also to obtain a sufficiently accurate appraisal of magnitudes of forces and moments (or their coefficients) having an effect on the model of the rocket.

If one were to mount the model of the rocket with the help of a special holder on the wind-tunnel balance, it is possible to measure directly the components of the main vector of aerodynamic forces and to calculate the respective coefficients, for instance, C_x, C_y, m_z. Constructions of wind-tunnel balances are diverse (mechanical, strain balance, and so forth) and permit producing measurements of forces with great accuracy.

For the clarification of the distribution of pressure on separate surfaces of a model of a rocket or even over its whole surface so-called drain tests are conducted. For this at points of the surface of the model interesting to us we drill holes, and then with the help of metallic or rubber tubes the pressure of the air flow created by the wind tunnel from is transferred these holes to manometers and measured. By knowing the distribution of pressure on the surface of the model it is possible with the help of theoretical dependences to determine the coefficient coefficients aerodynamic forces and moments applied to the model of the rocket in the given experiment. Furthermore, it is possible to establish the presence or absence of the flow separation from the streamlined surface and the place where it happened.

At the disposal of the aerodynamicist-experimenter there are several optical methods of investigation Let us dwell on two of the most important and widely applied: schlieren and interference. The schlieren method permits obtaining a very graphic picture of the streamline flow, since in photography near the model zones of raised pressure and zones of rarefaction are visible and also are distinctly depicted in the form of dark bands of the shock wave. A diagram of the apparatus for the production of schlieren spectra of the streamline flow is given in Fig. 40. From the source 1

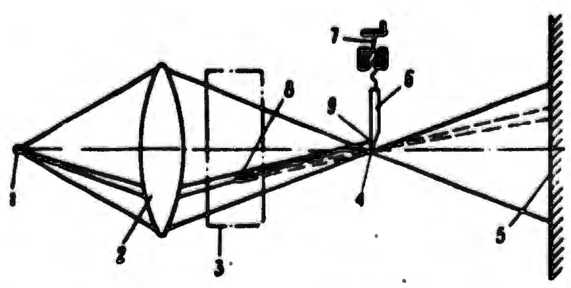


Fig. 40. Diagram of the apparatus for the production of schlieren photographs: 1) source of light; 2) lens; 3) working part of the wind tunnel; 4) focus; 5) screen; 6) knife diaphragm; 7) micrometric screw; 8) zone of air compression; 9) impact point of the refraction consolidation of the beam.

beams of light, with the help of lens 2, pass through the working part of the wind tunnel 3, converge at the focus 4, and are presented on the screen 5. The knife diaphragm 6 moves upwards or downwards by the micrometric screw 7 and before the beginning of the experiment is set so that its point is at the focus 4 and the screen has a uniform small illuminance. Let us assume that during the blowing of the model in the zone 8 of the working part of the wind tunnel a change in density of the air flow will occur. The air ceases to be an optically uniform medium and will refract the light beams in the direction of the place where the air density is greater. If in the region of the zone 8 the local condensation of the flow occurs the beams will be refracted, will pass higher than focus 4, and will be retained by the diaphragm at point 9, i.e., they will not reach the screen. Hence it becomes clear why shock waves on the screen (or on the photography of it) are obtained in the form of dark bands. On the other hand, zones of rarefaction will look lighter than the general background of the screen, since the light beams refracted by them go lower than the focus 4 and additionally illuminate the corresponding places on the screen. Figure 41 gives the typical schlieren photography of the flowing around of a model of an unfinned rocket by supersonic flow on which there is distinctly visible the form of shock waves and also the region of compression rarefaction, and vortex trail behind the bottom of the model.

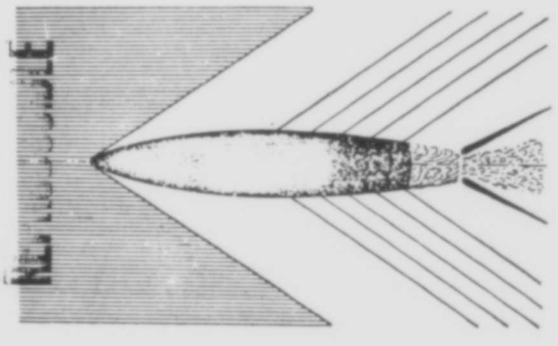


Fig. 41. Schlieren photography of the flowing around of a rocket model by supersonic flow.

The interference method is used for the experimental determination of the field of air density near the investigated model. By knowing the air density ρ at some point of the flow it is possible with the help of theoretical dependences obtained earlier to calculate at this point the pressure, speed, or temperature of the air. For instance, pressure is determined by the known density from the equation of the adiabatic curve (3.30). The speed is determined from the Bernoulli equation (3.29) according to known magnitudes ρ and p, and for the determination of temperature the equation of state of gas is applied (3.18).

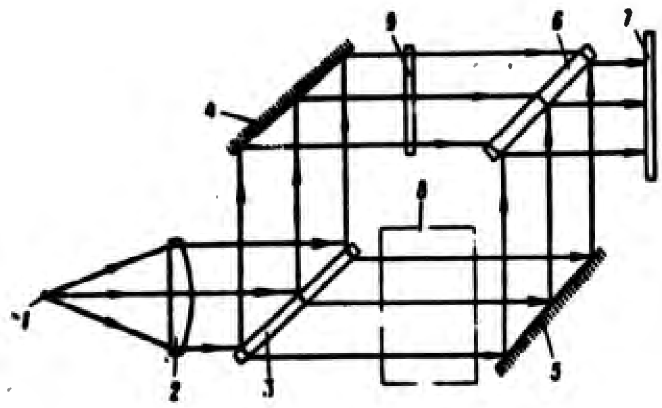


Fig. 42. Diagram of a four-mirror interferometer:
1) monochromatic source of light; 2) lens; 3 and 6) semitransparent mirrors; 4 and 5) mirror; 7) screen; 8) working part of the wind tunnel; 9) transparent glass.

How is the air density determined in the interference method? In order to answer this question one should remember that light constitutes electromagnetic oscillations of defined frequency. If two beams of light from one source pass an identical optical path, the phase of oscillation of them also will be identical. If, however, the optical path of these beams will be different coming to one point, they will have a different phase of oscillations. The phenomenon of optical interference consists in the interaction of beams of light with various phases of oscillation. With this on the screen where such beams drop there will be a uniform illuminance, but some picture with the alternating of light dark places will appear.

Let us examine the diagram of a four-mirror interferometer (Fig. 42). From the source of light 1 beam passes through lens 2 in parallel bundles on the semitransparent mirror 3 which reflects part of beams and passes part of them. With the help mirrors 4 and 5 the beams reflected and passed through mirror 3 fall on the semitransparent mirror 6 which gathers them together and feeds them to screen 7. The working part 8 of the wind tunnel is between mirrors 3 and 5.

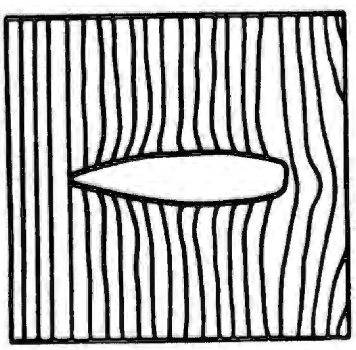


Fig. 43. Interference pattern of the streamline flow of a rocket model.

If the working part of the wind tunnel is closed, for the compensation of the change of magnitude of the optical path of light passing through the transparent glass in the working part of the tunnel between mirrors 4 and 6 the transparent glass 9 is placed. Before the experiment the semitransparent mirror turns at small angle, and therefore the beams reflected by mirrors 4 and 5 will not be combined on the mirror 6, and on the screen the interference picture in the form of parallel dark bands, which corresponds to the undisturbed flow with a uniform field of density, will appear. The model of the rocket in the air flow changes the air density. The optical path of light beams passing through sections of flow with different density is different. This distorts the initial interference picture, and the bands are distorted (Fig. 43). From physics it is known that the change in air density can be calculated if one knows the displacement of the interference band:

$$\rho = \rho_0\left[1 + \frac{m\lambda}{l(n-1)}\right],$$

where ρ is the air density at the point of flow where the displacement of the band is equal to m parts of the initial distance between the bands; ρ_0 is the density of the undisturbed flow; λ is the wavelength of the light sent by the source; l is the width of the working part of the wind tunnel; n is the index of a refraction of light by air.

Let us consider the flying tests. During the flying tests it is possible to trace the flight of the rocket from Earth, for instance, with the help of two phototheodolites located at the ends of a definitely selected base T_1, T_2 (Fig. 44) with the extent (length) known and to photograph the rocket through certain intervals of time. By knowing for each moment of time values of angles α_1, β_1 and α_2, β_2 (Fig. 44), one can determine the position of the rocket in space. Knowledge of the trajectory and the speed of the flight of the rocket permits checking the accuracy of the calculations and to make more accurate the separate calculated quantities.

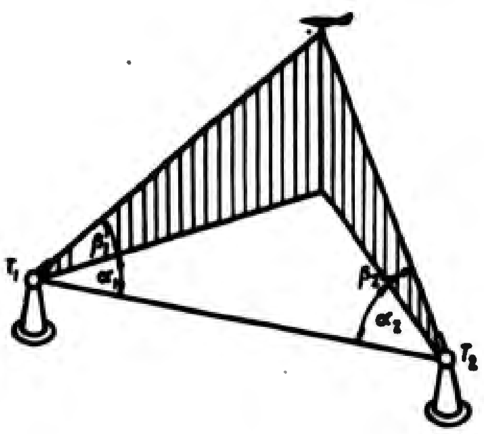


Fig. 44. Diagram of the determination of the rocket position in space with the help of two phototheodolites.

A deficiency in the application of of phototheodolites is the necessity to conduct flying tests in good clear weather and is eliminated by the replacing of the phototheodolites by two radar sets operating synchronously. Although the position of the rocket in space can be determined by one radar set the presence of the second permits increasing the reliability and accuracy of the given tests. The speed measuring method of the flight of the rocket, based on the Doppler effect can be used. The radar tracking the rocket sends to it a radio beam with a definite constant frequency of oscillations. The frequency of oscillations of the radio beam reflected from the

the rocket is continuously measured by a special device of the radar. By this frequency is determined the speed of the departure or approach of the rocket to the point where the radar is located. Thus at any moment of time we know the magnitude $\frac{dr}{dt}$, the derivative of the distance between the radar and rocket r by time t. By knowing $\frac{dr}{dt}$ and the trajectory of the rocket we can find the speed of it at any moment of time.

The limitedness of data obtained during the tracking of the flight of the rocket from Earth (trajectory and speed of flight) conditioned the fact that on board the rocket different data units, began to be placed and by means of telemetric equipment their data are transmitted to Earth. The information obtained from the rocket from transducers of acceleration is of great importance. To determine drag and lift it is necessary to know the angle of incidence of the rocket, which also is continuously measured by the trajectory. One of the methods of measuring the angle of incidence is the registration of the difference of pressures of the air flow circumfluent the cap placed on the nose cone of the rocket with holes from various sides. At the angle of attack $\alpha = 0$ the pressures at all holes of the cap are identical. If the angle of incidence $\alpha \neq 0$, the pressures will not be identical and by their difference one can determine the magnitude of the angle of incidence (for instance, calibrated cap beforehand in the wind tunnel). According to the magnitude of aerodynamic forces the respective aerodynamic coefficients are calculated. Integrating the data of the accelerometers we obtain values of respective speeds which are more accurate than with the photographing of the rocket in trajectory. Besides the above-mentioned, during the flying tests temperature of the surface of the rocket at different points, angles of bank, angles of displacement of the steering system, and so forth are measured. For rockets stabilized in trajectory by the rotation around its longitudinal axis it is important to know the change in the angular velocity of the natural rotation in trajectory. These data can be obtained from a radio detonator with a dipole antenna. During each turn of the rocket the axis of the dipole will be directed twice towards the receiver, and therefore the signal will pass through its least value. The angular velocity will be thus determined by half of the frequency modulating the radio signals taken from rocket.

Abroad the behavior of the axis of revolving rocket is determined with the help of solar camera having a slot through which on photographic film after each revolution of the rocket the image of the Sun falls in the form of a dash. Furthermore, there is another slot proceeding perpendicular to the longitudinal axis of the rocket and providing on the photographic film a reading line. The distance from the dash to the reading line depends on the angle between the axis of the rocket and direction toward the Sun. Thus the magnitude of the angle between the axis of the rocket and direction toward the Sun is recorded on the whole trajectory. The solar camera permits recognizing not only this angle, but also the number of turns of the rocket in a unit of time and the calculating of such coefficients how m_x, $C_{R_{Ma}}$ and others.

§ 6. Standard Functions of Air Resistance

One of the most important aerodynamic properties of rockets is the drag coefficient C_x. Therefore it is most fully studied both for missiles and for rockets of different assignment. It appears that for rockets (or missiles) close in aerodynamic form the curves of dependences $c_x\left(\frac{v}{a}\right)$ or $C_x(M)$ are such that for different values of Mach number their relation can be assumed approximately constant.

If one were to assume the values of $C_x(M)$ for some model rocket with the most frequently encountering aerodynamic shape to be the standard, designating them $C_{x\text{эт}}(M)$, then for any rocket with similar external outlines and with the dependence $C_x(M)$ the following condition will be observed:

$$i \approx \frac{C_x(M)}{C_{x\text{эт}}(M)}, \tag{3.60}$$

where i is the form factor of the given rocket with respect to the rocket accepted as standard.

The more similar the outlines of the rocket are to the standard, i.e., the nearer these rockets are in geometric similarity of their form, the more accurate the value of formula is observed (3.60). This makes it possible not to produce large and expensive works in the determination $C_x(M)$ for every newly designed rocket in the first stages of its creation but to be limited only to the determination of its form factor with respect to the rocket similar in aerodynamic form to the one examined. Then the necessary dependence $C_x(M)$ of the rocket will be obtained from the expression (3.60).

The value necessary in calculations of trajectories of the drag of the rocket (missile, mine, and others) with different values of the Mach number will be determined from the expression

$$X = S \cdot \frac{\rho v^2}{2} \cdot iC_{x\text{эт}}(M). \tag{3.61}$$

The numerical values of i depend on the form of rocket selected as the standard, and therefore reducing the value i for a certain rocket should always be indicated in reference to what model or standard form $C_{x\text{эт}}(M)$ the value i determined.

It is conveniently to use i for an agreement of the calculation by determination of the distance of firing with the experiment. In this case i will be considered not only the form of the rocket, but also all factors not reflected in the calculation (for instance, oscillations of the longitudinal axis of the rocket). This can explain the certain distinction in the numerical values of the coefficient of form determined as the coefficient of agreement with the experiment during the firing of identical rockets, but at different angles of departure and at different speeds at the end of work of the engine.

In calculations of trajectories it is convenient to deal not with the drag X, but with the acceleration J imparted to the rocket by the force. If we designate the mass of the rocket as a given moment by the letter m, on the basis of Newton's second law (mechanics)

$$J = \frac{X}{m}. \tag{3.62}$$

Let us reduce the formula (3.62) to a form more convenient in calculations with the help of formula (3.61) and also evident relationships $S = \frac{\pi d^2}{4}$; $\Pi = g\rho$, where d is the characteristic dimension of the rocket (for instance, the diameter of the middle section), and Π is the specific weight of air.

Taking into account what has been said we have

$$J = \frac{1}{m} \cdot \frac{\pi d^2}{4} \cdot \frac{\Pi}{g} \cdot \frac{v^2}{2} \cdot iC_{x\text{эт}}(M).$$

This equation can be written more briefly thusly:

$$J = CH(y)vG(v), \tag{3.63}$$

or

$$J = Ev, \tag{3.64}$$

where $C \frac{id^2 \cdot 10^3}{mg}$ is the ballistic coefficient; (3.65)

$$H(y) = \frac{\Pi}{\Pi_{0N}};$$

$$G(v) = \frac{\pi}{8} \cdot 10^{-3} \cdot \Pi_{0N} v C_{x\text{эт}}(M) =$$

$$= 4{,}74 \cdot 10^{-4} \cdot v C_{x\text{эт}}(M);$$

$$E = CH(y)G(v);$$

G(v) is the function of drag of the rocket accepted as the standard.

In the practice of the calculation of trajectories tables for standard functions of G(v) are widely applied which were compiled in reference to the so-called laws of drag, "Siacci," "1930" and "1943," and obtained by means of the processing of a large number of experimental firings of artillery missiles. With the application these laws one should remember that in the law Siacci the standard missile was a poorly streamlined missile of the time before the First World War, in the law of "1930" it was a missile with a very good aerodynamic form (long-range) but with a comparatively small quantity of explosive, and in the law of "1943" the standard missile was of the time of World War II with a good loading and, due to this a somewhat a worsened form.

In the theory of flight the function $F(v) = G(v) \cdot v$, is often applied. Taking into account this formula for the force of drag we will have the form

$$X = mCH(y)F(v). \qquad (3.66)$$

Figure 45 shows the curves of the model functions F(v) for different laws of drag. These curves permit making the following conclusions. In the first place the drag increases with an increase in the speed of the flight. Secondly, this increase is especially intensive in the region of transonic speeds of flight. And finally the greater the drag the worse the aerodynamic shape of the missiles accepted as standard. For instance, let us compare F(v) for the long-range missile (law of "1930") and the missile with little elongation of the nose cone (law of "Siacci").

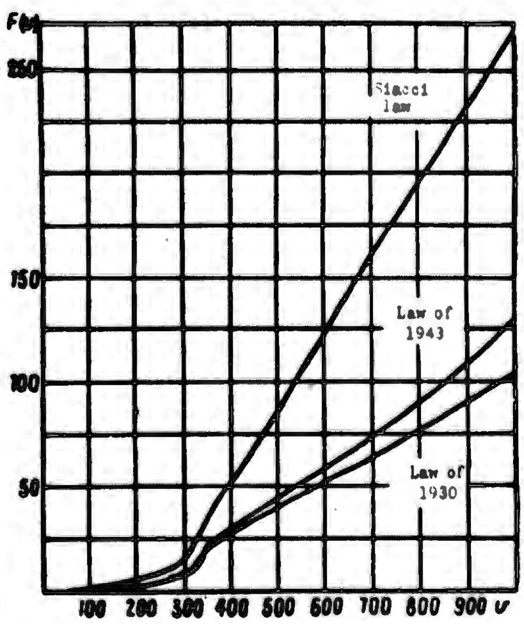


Fig. 45. Functions of F(v) for different laws of drag.

Model functions of drag are especially widespread with the calculation of trajectories of missiles (minutes) of barrel artillery.

In the case of the application of the individual dependence $C_x(M)$, obtained by calculation or by experiment for a given rocket, it 's expedient to calculate J from the following formula:

$$J = 0{,}474 \cdot \frac{d^2}{mg} \cdot v^2 C_x(M) H(y).$$

§ 7. Peculiarities of the Aerodynamics of the Flight of Rockets in High Layers of the Atmosphere and with Hypersonic Speeds

Such types of rockets as intercontinental, ballistic, and anti-rockets rise hundreds of kilometers above the surface of the Earth where the air density is insignificant. The average distance which molecules of air pass before collision with each other, called the length of free path of molecules, can be very great in high layers of atmosphere. And indeed, if at sea level under normal conditions the length of the free path of the air molecules is equal approximately to $6.4 \cdot 10^{-5}$ mm, then at an altitude of 80 km it is already more than 3 mm and at an altitude of 150 km reaches 40 m. The hypothesis accepted by us earlier that air is solid medium is inapplicable at great heights, since the length of the free path of air molecules λ is commensurable with the characteristic dimension of the rocket L, and not every molecule interacting with the surface of the rocket undergoes collision with other molecules. Consequently, the frequency of collisions of molecules is less than the frequency of collisions with the rocket. In dense layers of the atmosphere each molecule affects the surface of the rocket in close interaction with other molecules, since it collides continuously with them. The division of aerodynamics studying the properties of rarefied air flows is called superaero-dynamics.

Depending upon the magnitude of the ratio $\frac{\lambda}{L} = K$, called the Knudsen flow, the following forms of air flow are distinguished. If $K < 0.01$ the flow is solid and the regularities obtained by us for it are useful. If, however, $K > 10$ the flow is free-molecular, i.e., the molecules of air practically do not affect the flight of one another. Among these flows it is possible to separate the flow with a slip which approaches when $0.01 < K < 0.10$ and characteristically the slip of molecules near the surface of the rocket. In the region $0.10 < K < 10$ properties of the flow of air are very complex, and they are similar partially to properties of the free-molecular flow and partially to properties of the flow with slip. Such flows are related to transitional regimes of flow.

Let us examine properties of free-molecular flows of air. A rocket in a free-molecular flow does not affect its parameters, since the molecules reflected from its surface in conditions of great length of the free path practically do not influence flow because of extremely rare collisions. Therefore in front of the rocket shock waves do not appear even at great supersonic speeds. To determine resisting forces of the flight of a rocket we must know the magnitudes and directions of speeds of molecules before and after the shock against the surface of the rocket. The kinetic theory of gases established how speeds of molecules are distributed in the free-molecular flow before the shock against the surface of the rocket, but such a law of distribution after the shock is not known. Falling on the surface of the rocket the molecule is held there for some time (about 10^{-6} sec) and exchanges its energy and pulse with the material of the surface. The new direction and velocity of the molecule depend on the degree of this exchange the mechanism of which is insufficiently studied. It is therefore necessary to apply the simplified diagrams of reflection, such as, specular, diffusion, and Newtonian diagrams (Fig. 46). Specular reflection occurs with a constant tangential component of speed of the molecule to the surface of the rocket and with a change in the sign for the normal component of speed. Owing to this the angle of incidence of the molecule is equal to the angle of its reflection (Fig. 64a). The diffusion reflection assumes that the molecules will abandon the surface of the rocket at different angles and with different speeds (Fig. 46b). The law of distribution of these speeds will be the same as in the incident flow, but the mean velocity depends on the surface temperature of the rocket. The Newtonian diagram of reflection (Fig. 46c) assumes that air molecules reflected by the surface of the rocket will move along it, i.e., the normal component of speed $V_{\infty n}$ will be equal to zero after the reflection. In reality the diagram of reflection will be some intermediate reflection between the considered diagrams.

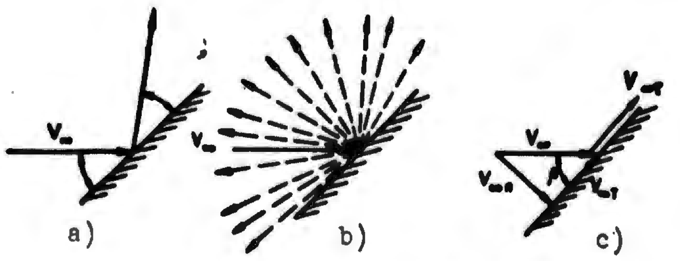


Fig. 46. Diagram of reflection of air molecules from the surface of the rocket; a) specular reflection; b) diffusion reflection; c) Newtonian reflection.

With the flow with the slip of properties rarefactions of air appear first of all in the boundary layer where, due to the small numbers of collisions with other molecules, it is possible the latter do not "adhere" to the streamlined surface but slip along it. The air temperature near the surface of the rocket will not be equal to the temperature of the material of this surface. Experimental investigations show that the thickness of the boundary layer in flows with slip is great, the friction of air against the surface is relatively great, and its viscosity greatly affects the pressure at the surface. The common dependences for the boundary layer become useless because of this.

Thus we see the complexity of the interaction of the rocket with rarefied air. This leads to the fact that the theoretical dependences describing this interaction are complicated and insufficiently reliable, and therefore for calculations experimental data are applied obtained by means of tests of models of rockets in wind tunnels, the firing of models in target ranges, and also the processing of flying tests of rockets in high layers of the atmosphere.

The criterion of similarity in superaerodynamics is considered the number $M = \frac{V}{c_1}$; where $c_1 = \sqrt{2gRT_{CT}}$ is average molecular speed, i.e., the most probable speed of the air molecules, and T_{CT} is the temperature of the surface of the wall.

Let us show how it is possible to calculate approximately the pressure coefficient if one were to accept the diagram of reflection of Newton (Fig. 46c). The change in the momentum of air reaching per unit of time per unit of surface of the rocket in the direction of the normal to it should be equal to the power impulse applied to this surface, i.e.,

$$\rho_\infty V_{\infty n}(V_{\infty n}-0)=(p-p_\infty)\cdot 1\cdot 1.$$

From Fig. 46c, one can see that $V_{\infty n} = V_\infty \sin\beta$, and therefore the pressure coefficient is

$$\bar{p}=\frac{p-p_\infty}{\frac{\rho_\infty V_\infty^2}{2}}=2\sin^2\beta. \qquad (3.67)$$

It appears that $\bar{p}$ depends only on angle β between the direction of motion and section of the surface. Calculating $\bar{p}$ for many points of the nose cone of the rocket with help of formula (3.67), one can determine the coefficient of its drag.

High speed is characteristic for the flight of many types of contemporary rockets. For instance the speed of the entrance into dense layers of the atmosphere of the nose cone of a ballistic rocket with distance of firing of about 5000 km reaches 6000 m/sec and the Mach number is $M \approx 18$. With such high speeds a number of problems studied in the division of aerodynamics of hypersonic speeds appear.

The most acute is the problem of the struggle with aerodynamic heating, since a considerable part of the kinetic energy of the nose cone of the long-range rocket, owing to friction against the air, turns into heat. Part of this heat by means of radiation and thermal conduction is dispersed in the air, and the remaining heat continues heating of the nose cone of the rocket. If one did not take special measures the nose cone could be greatly heated and even burn. To calculate the possible heating of the nose cone it is necessary to know the parameters of air in the boundary layer at its surface, which can be determined only in the process of calculation of the flow of air near the nose cone.

Let us virtually investigate the physical phenomena accompanying a flight with hypersonic speeds.

High speeds of a flight cause the appearance of intense shock waves in front of the nose cone. The pressure, density, and temperature of the air after the shock wave are increased so considerably that the thermal dissociation of molecules of gases of which air consists approaches. Energy is expended in the dissociation and this leads to the lowering of the temperature of the air. But in spite of this the air temperature after the shock wave can attain huge values. Thus, for instance, in a flight with the Mach number M = 24 at an alittude of 10,000 m after the linear segment the shock, the temperature of the air is equal to approximately 9400°K. An interesting peculiarity of the shock waves in the region of hypersonic speeds is their curvilinear form. Even around the sharp edges of the wing or near the sharpening of the nose cone the shock departs from the sharpening and is distorted. This is explained by the fact that practically sharpening is not ideal, and there is always a certain radius of blunting. Furthermore, with hypersonic speeds on a streamlined surface near the point a boundary layer of considerable thickness appears which makes the pointed part seem more blunt. The increase in the thickness of the boundary layer is explained by the decrease in the value of the number $Re = \frac{V_2 \rho_2 x}{\mu_2}$ after the shock.

In the theory of shock waves it is proven that with the increase in the Mach number of the incident flow the angle of inclination of the shock θ decreases. Therefore, in the region of hypersonic speeds shock waves are disposed very close to the surface of the streamlined body (Fig. 47). If it is remembered that with such high speeds the boundary layer is sufficiently thick, then the possibility of the interaction of the shock wave and boundary layer will become clear. This interaction appears most fully in the nose cone of the body near the sharpening (Fig. 47, 1), is called an intense interaction, and consists in the following. After the most distorted segment of the shock (where it is close to linear) the flow rate decreases considerably, but the pressure increases sharply. The region

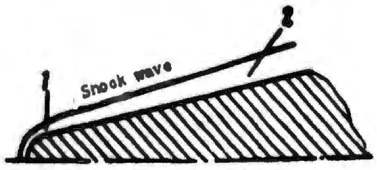


Fig. 47. Diagram of the flowing around of a body at hypersonic speeds:

1) region of intense interaction; 2) region of weak interaction.

of increased pressure penetrates into the boundary layer and brakes it, which can lead even to the local separation of the boundary layer from the streamlined surface. Friction in the region of the intense interaction decreases because of this.

Another picture will be in the so-called region of weak interaction (Fig. 47, 2), which is located on larger removal from the nose cone of the body than the region of the intense interaction. Here a smaller distortion of the shock characteristic. After the shock wave the flow rates are larger and the pressures smaller, and the influence on the boundary layer is not of the shock wave but the flow of air after it. This flow advancing on a sufficiently thick boundary layer, flows as if more thickened, and therefore in the flow there appears an additional induced pressure Δp which is transmitted to the boundary layer is so great that a noticeable deceleration of them due to the induced pressure is not obtained. Therefore, in the region of weak interaction the friction of air against the surface is great.

The complexity of the physical picture of the streamline flow is increased by eddy formation in streams of air which passed the bent shock wave. From turbulence there appears additional friction which changes the kinetic energy of the flow by irreversible means into heat. The entropy of air from this is increased, and since this increase is different for different streams, depending upon the angle of inclination of the shock, the flow after the shock cannot be considered isentropic hampers the presence of the subsonic zone of flow in the region of greatest distortions of the shock also hampers the calculations.

High air temperature in the boundary layer leads to an intense heating of the streamlined surface. There appears a heat flow from the boundary layer of air to the streamlined surface, which decreases the quantity of heat in the boundary layer and consequently lowers the air temperature in proportion according to the approach to the streamlined surface. The complicated law of the distribution of temperature in the thickness of the boundary layer is obtained. This in turn leads to the complicated change of all the characteristics of air dependent on temperature: μ, c_p, ρ, and otheres. The boundary layer with variable characteristics according to the thickness of it can be calculated only with the help of electronic digital computer machines. For engineering calculations a simple and convenient method is necessary, which proved to be possible to create only by means of averaging the air temperature according to the thickness of the boundary layer. Such an average temperature is the determining temperature, which by applying it is possible from earlier obtained formulas for an incompressible liquid to determine accurately enough the power influence of the boundary layer on the body and, furthermore, by calculating the heat transfer to calculate correctly the temperature of the streamline surface.

To calculate the magnitude of the determining temperature researchers proposed several empirical formulas. Readers who are interested in the problem of aerodynamic heating and the calculation of the thermal boundary layer should write to works [16] and [17].

Having investigated the nature of the resisting force of air to the flight of rockets in the basic laws of aerodynamics, let us now examine aerodynamic diagrams of rockets of different purpose.

C H A P T E R IV

AERODYNAMIC DESIGNS OF ROCKETS OF DIFFERENT PURPOSE

§ 1. Design Diagrams of Rockets

To guarantee the maximum range of the flight of a rocket, the stability of its flight in the trajectory and good controllability, and also for the execution of a number of other requirements the dimensions, external form, and controls of the rocket should be selected very carefully. In other words, the aerodynamic design of a rocket should be conducted.

By aerodynamic design is meant the rational selection of the external forms and mutual location of the body, wing, fins and controls of rocket for the purpose for best fulfilling its operational requirements.

The successful selection of the aerodynamic design diagram of the rocket governs its quality on the whole, and this question is purposely given much attention in the process of designing the rocket.

Let us divide all diagrams into two classes. To the first class will belong aerodynamic designs of such rockets for which the body does not have sharply protruding surfaces (unfinned rockets) and to the second class those with additional sharply protruding surface from the body (finned rockets), wings and fins.

Wings of a rocket are usually those surfaces which adjoin the body and serve for the creation of lift, and the fins are those surfaces which serve for the transfer of the center of pressure to a necessary place (for instance, for production of sufficient static stability of the rocket).

Aerodynamic designs of rockets of the first class can be divided into two groups: guided and unguided. An example of an aerodynamic design of an unfinned

GRAPHIC NOT REPRODUCIBLE

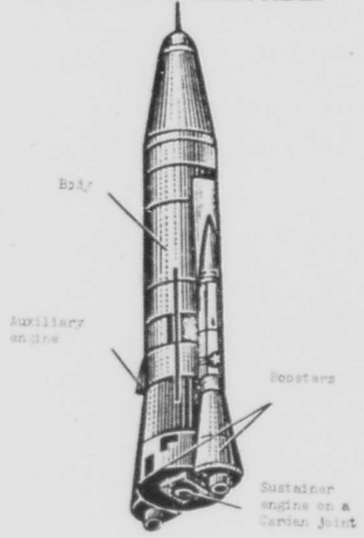


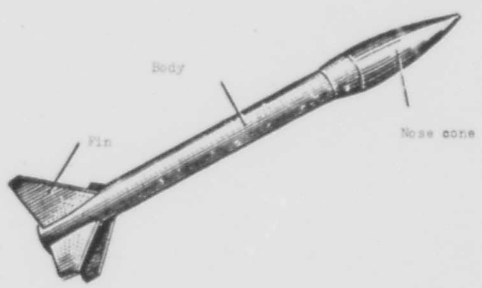
Fig. 48. Ballistic unfinned guided rocket.

guided rocket is the American ballistic rocket "Atlas" (Fig. 48) which has a body in the form of a solid revolution and is guided with the turn of engines secured on a Cardan suspension and also by special (control) motors.

Unfinned unguided rockets can be referred to as turbojet missiles (Fig. 103).

Aerodynamic designs of finned rockets (second class) can be divided into wingless and winged. Designs of wingless finned rockets, just as unfinned, are subdivided into unguided and guided. Examples of unguided finned wingless rockets are rockets of close range of operation, which "guard mortar" units shot during World War II, the American rocket "Honest John" (Fig. 49), and others.

A controlled finned wingless rocket is the American rocket "Corporal" (Fig. 50), which in a powered-flight trajectory is guided and stabilized with help of gas rudders, and for stability on the passive segment of the trajectory (in free flight) it has a fixed fin (stabilizer) in tail section due to which is statically stable.



GRAPHIC NOT REPRODUCIBLE

Fig. 49. Finned wingless unguided rocket.

Winged rockets as a rule, are guided (at least in a powered-flight trajectory), since the wing creating lift serves for flight control. With the classing of the rocket to a certain diagram it is sometimes difficult to distinguish the fin with a large surface from the wing with a small surface. For instance, the American air combat rocket "Genie" (Fig. 51) does not have a wing but has a fin, and its stabilizer is X-shaped and located in the tail section. This rocket is guided with the help of deflecting plates attached to the trailing edges of the fin. Because of the absence of a wing such a rocket cannot sharply change the form of trajectory, i.e., it possesses poor maneuverability.

Externally the Swiss antitank rocket "Cobra-4" (Fig. 52) is similar to the

"Genie" since it also has in the tail section X-shaped surfaces. However, they are sufficiently great, create great lift, and provide good maneuverability at a low speed of flight (~80 m/sec), and therefore the "Cobra-4" belongs to winged rockets.

GRAPHIC NOT REPRODUCIBLE

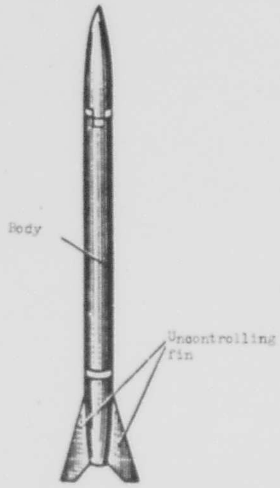


Fig. 50. Ballistic finned wingless guided rocket.

Winged rockets have many forms of aerodynamic designs. According to the presence and location of the controlling fin with respect to the wing (besides vertical) diagrams of aerodynamic designs of winged rockets are divided into three subgroups: normal, "canard" and "tailless."

The normal diagram of aerodynamic design assumes that the controlling fin (rudders) is located behind the wing in the tail section of the rocket. According to such diagram aircraft and many types of winged rockets, for instance, the American winged ground-to-air rocket "Bomarc" (Fig. 53), are usually constructed.

In the "canard" diagram the horizontal controlling fin is ahead of the center of gravity of the rocket in its nose cone. The American winged missile "Navajo" (Fig. 54) is constructed.

The diagram of a "tailless" rocket does not have a horizontal controlling fin. In particular, it is obtained with the use of a sweptback wing having a large sweepback angle. In this case the end part of the wing is so close to the tail part that the need for a horizontal controlling fin disappears. An example of the embodiment of such a diagram is the American winged rocket (winged missile) "Snark" (Fig. 55). Rockets constructed from the "tailless" diagram can have an uncontrolling fin (including the horizontal) located both ahead and behind the center of mass. The necessity of such a fin appears when there is tendency to improve the characteristic of the static stability and damping of the rocket. For instance, the American winged rocket "Falcon" of the air-to-air type (Fig. 56) has motionless surfaces ahead of the wing, which being cross-like, decrease

GRAPHIC NOT REPRODUCIBLE

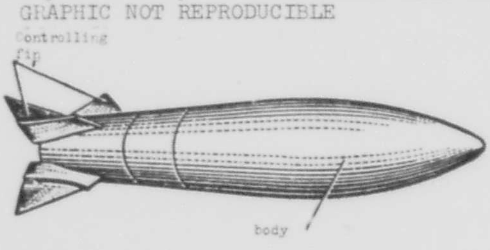


Fig. 51. Wingless guided air combat rocket.

the excessive static stability of the rocket. Such surfaces are called destabilizers. Destabilizers also improve the characteristic of damping.

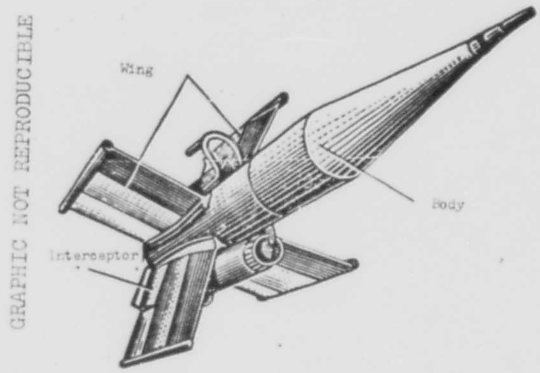


Fig. 52. Winged guided antitank rocket.

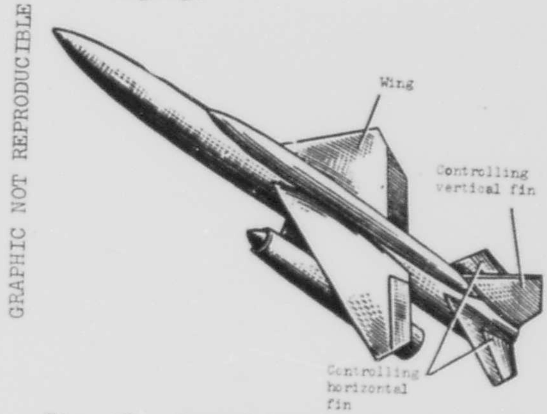


Fig. 53. Ground-to-air winged rocket.

An American rocket of the same class the "Sparrow-III" (Fig. 57), on the other hand has a cross-like motionless fin behind the wing. This fin ensures the static stability to the rocket and dampens it oscillation around the center of mass more quickly.

A variant of the "tailless" diagram is the diagram of the "flying wing". The body of such a rocket is almost completely inscribed in the outline of a wing.

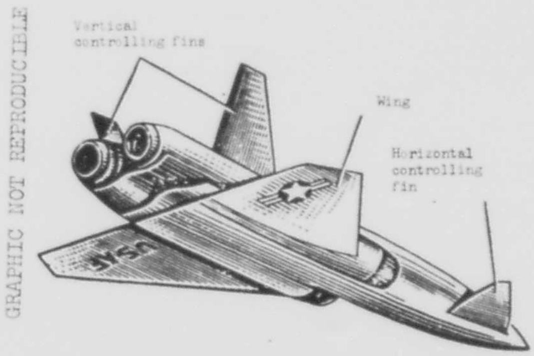


Fig. 54. Winged rocket (winged missile).

According to data of foreign source such a diagram will possible be used in rockets with an atomic engine.

Thus far we have examined aerodynamic designs of single-stage rockets. For multistage rockets the presence is characteristic of definite peculiarities in their aerodynamic design caused by the consecutive separation of the stages. The multistage rocket, as a rule, has great length and to give it stability the first stage frequently has a developed fin. Before the separation of the first stage the aerodynamic design of the rocket can be considered finned and wingless. After separation of the first stage, depending upon the assignment of the rocket, its aerodynamic diagram can also be finned wingless or unfinned (ballistic rockets) or winged (ground-to-air rockets and last gliding stage of the rocket).

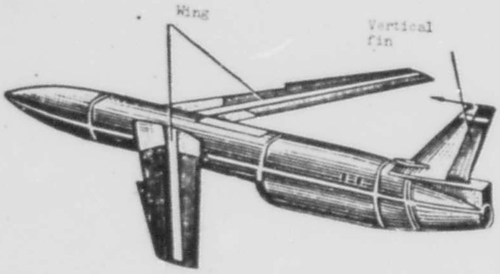


Fig. 55. Winged rocket (winged missile).

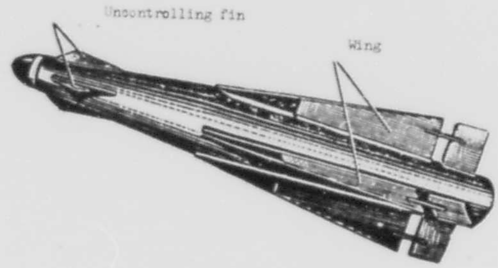


Fig. 56. Winged missile.

GRAPHICS NOT REPRODUCIBLE

Nose cones of ballistic long-range rockets are made, as a rule, detachable from the body. The reason for this is that body of the last stage of the rocket is destroyed, since it is not designed for great overloads which appear with the entrance with great speed into dense layers of the atmosphere. The accuracy of firing rockets with undetachable nose cone would be poor. To make the body of the last stage of the rocket sufficiently durable would be unprofitable because of the considerable increase in weight of the construction of the rocket.

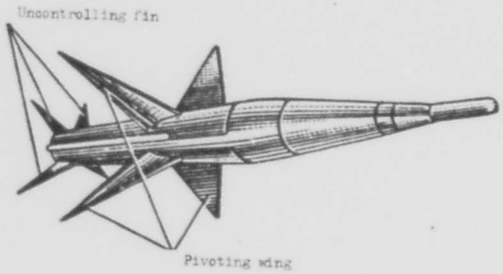


Fig. 57. Winged missile.

GRAPHIC NOT REPRODUCIBLE

For execution of its assignment the nose cone when entering the dense layers of the atmosphere should be definitely oriented with respect to the tangent to the trajectory. If the nose cone is statically stable, then such orientation will occur automatically. In sufficiently dense layers of the atmosphere the axis of a stable nose cone will accomplish a certain oscillatory motion around the tangent to the trajectory.

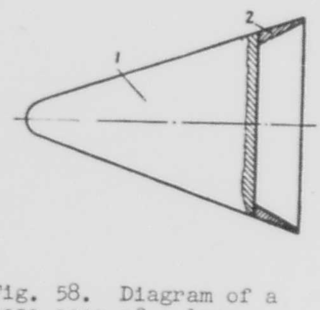


Fig. 58. Diagram of a nose cone of a long-range rocket:
1) warhead; 2) "shirt".

Nose cones of ballistic rockets are usually constructed in the form of a cone. For a uniform cone (with equal distribution of mass over the entire volume) the center of pressure is located nearer to the vertex than the center of mass. Therefore, to to impart static stability to the nose cone of the rocket it is necessary either to displace the center of gravity nearer to the vertex or to transfer the center of drag further away from the vertex. In practice a more conv nient alternate path is usually adopted supplying the conical nose cone a "skirt" (Fig. 58), which is a continuation of the conical surface.

§ 2. Controls of the Rocket Flight and Controlling Forces

An unguided rocket moves in a trajectory which is called natural or ballistic. For the natural trajectory it is characteristic that all forces applied to the rocket in flight are reduced to the resultant directed on the tangent to the trajectory. The guided rocket should have another trajectory different from the natural so that the rocket will successfully execute its task (for instance, hitting a moving target). The trajectory can cease to be natural if a force directed at an angle to the tangent to the trajectory will be applied to the rocket. Part of this force directed along the normal to the trajectory is called controlling. Device ensuring the necessary control force in a flight are called controls. Controls comprise a system of control of the motion of the rocket which are made up of a complex of equipment and devices ensuring measurement of deviations of the actual motion of rocket from the necessary direction of flight, the formation of the corresponding signal, and the creation of the control force.

Controls can be divided into three types: aerodynamic, gas-dynamic, and combined. This division is carried through depending upon the vector of what force applied to the rocket participates in the creation of the control force. Three kinds of forces act on the rocket in flight: gravity force (gravitational), aerodynamic, and tractive forces of the engines. As yet man has not been able to govern gravity forces.

Aerodynamic controls create a control force owing to the turn of the vector of aerodynamic forces. For instance, with help of elevator deflection there appears a control force displacing the rocket along the normal to the trajectory to a certain side.

Gas-dynamic controls for the creation of a controlling force deflect the vector of the tractive force (for instance, with jet vanes) from the direction of the tangent to the trajectory.

With the creation of controlling forces the combined controls use both aerodynamic forces and tractive forces.

Let us trace how the control force is created and how the deviation of the trajectory of the rocket occurs as illustrated by jet vanes. Jet vanes consist of four plates from a material which couteracts well the high-temperature stream of combustion products of fuel. Jet vanes are placed at the outlet of the nozzle and can turn at some angle to a certain side. Figure 59 shows how with the turn of one of the horizontal vanes at angle δ the force R_p appears, which applied to the vane (and, consequenlty, to the rocket as a whole). The component X_p of this force acts along the axis of the rocket, braking its flight. The other component Y_p of force R_p is a part of the control force. If one were to bring force Y_p to the center of mass of the rocket the action of it on the flight of the rocket will become clear. The moment created by the pair of forces $Y_p - Y''_p$, will turn the rocket around the center of mass A at a certain angle of incidence α owing to which the lift Y applied to the rocket in the center of mass will appear. The difference of forces $Y - Y'_p$ will constitute the magnitude of the control force under the action of which the center of mass of the rocket will begin to deviate in the direction of the control force.

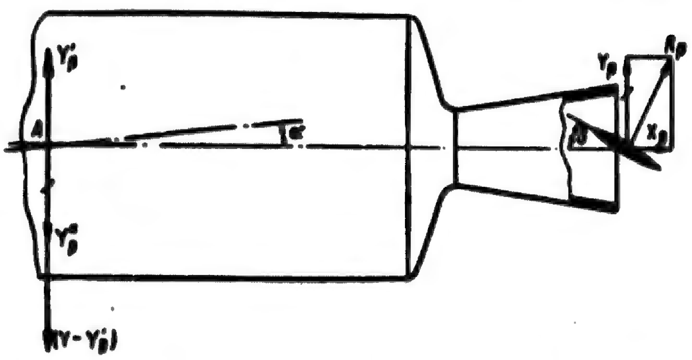


Fig. 59. Diagram of the creation of a control force of a rocket by means of a jet vane.

Aerodynamic controls are used for rockets flying in sufficiently dense layers of the atmosphere and with great speed, i.e., when the aerodynamic forces are sufficiently great. Aerodynamic controls are divided into the following groups: control surfaces, interceptors of air flow (spoilers), and turning wings.

Control surfaces, or simply rudders, can be placed at different places of rocket. In rockets of the "canard" diagram they frequently directly adjoin to the body of the rocket (Fig. 54) and are called <u>steering fins.</u> Very often the rudders are directly behind the wing (Fig. 55) or behind the fixed fin (Fig. 53).

Control surfaces in rockets serve as rudders elevators, ailerons, and elevons. The average position of the rudders is in a vertical plane, and a deflection of them to one side will cause a turn of the rocket to the right or left of the vertical plane in which the flight occurred. The elevators deviate from the horizontal plane in which the flight of rocket occurs. Operating the elevators permits the rocket to change the direction of flight in a vertical plane, i.e., to change the altitude of the flight. The combination of rudders and elevators makes it possible to control the rocket in two mutually perpendicular planes horizontal and vertical, i.e., to carry out practically any maneuver in space.

However, there is another control of the rocket in space. There is on the rocket, besides the elevators, rudders of bank or simply ailerons. Ailerons consist of two control surfaces located on the wing tips and deflecting to various side. If one aileron goes down the other goes up and inversely. If ailerons deflect the rocket inclines, banks, i.e., the angle of bank directed to the corresponding side appears. With this the elevator deviates not from vertical but from the inclined plane, and the horizontal component of the control force appears turning the rocket to the proper side (Fig. 60).

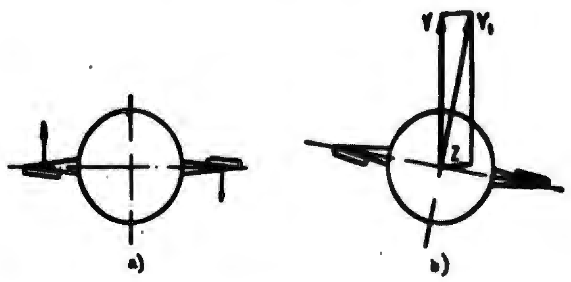


Fig. 60. Flight contol of the rocket by means of giving it bank:
a) deflection of ailerons begin banking the rocket to the right; b) turn of the normal component of force Y_1 led to the appearance of a lateral component control force Z shifting the rocket to the right.

Elevons differ from ailerons with the possibility to deviate from the surface of the wing to any side independently from each other. In rockets it may be necessary to turn the whole fin and not just part of it. This permits the controls to work more effectively at small angles of rotation.

Spoilers or interceptors of flow (Fig. 61) work on such a principle: from the surface of the wing a thin plate which brakes the flow is advanced. Pressure on the surface of the wing in front of the protruding plate is increased, and after plate a separation of flow occurs. This leads to a change in the magnitude of the lift. For convenience of control, with the help of spoilers the latter are brought into oscillatory motion, and therefore they are sometimes called vibrating spoilers. The amplitude and frequency of oscillations of the spoiler are not regulated. The magnitude of the control force is changed by means of

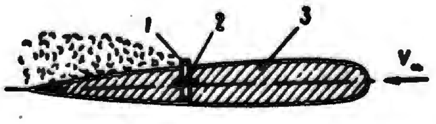


Fig. 61. Diagram of a spoiler device:
1) spoiler; 2) center of oscillation of the spoiler; 3) wing.

shifting the center of oscillation. The nearer the center is to the surface of the wing the greater the time during which the spoiler will be advanced and the wing lift will be changed greater, i.e., the magnitude of the control force will be larger. In thin wings the spoiler is alternately advanced first from one side then from the other side of the wing. The advantage of spoilers over rudders is the smaller weight of the controls (including the drive). Therefore, the spoiler control is used in antitank rockets where one of main requirements is a minimum takeoff weight (for instance, antitank rockets "SS-10" and "Cobra-4").

A deficiency of the spoiler control is that this group of controls not ensure the rocket of sharp maneuvering.

The pivoting wing is a common wing with the possibility to turn with respect to the body of the rocket around its longitudinal axis or perpendicularly to the longitudinal axis of the rocket. This turn of wing changes its angle of incidence leading to a change in the control force. The advantage of such a control is the good maneuverability of the rocket, since the control force is created owing to the turn of one wing, and the inertia is less and the turn can be made faster. The rocket should be turned with the pivoting wing when it is required to carry out a quick maneuver in the trajectory. For instance, the American rocket "Sparrow-III", intended for the combat of aircraft with air targets, has an X-shaped pivoting wing (Fig. 57).

The deficiencies of the pivoting wing are the large hinge moments necessary for turning the comparatively heavy wing. However, the power of the drive is not very great, the since angles of rotation are small and therefore the speed of turn may also be small. Another deficiency is a certain increase in the drag of the rocket due to gaps between the wing and the body. Furthermore, a shift in the center of gravity with burnout leads frequently to the appearance of a negative angle of incidence of the body and an increase in loads on the fin during a maneuver, which increases the bending moment applied to the body of the rocket.

Gas-dynamic controls can be applied with success in those cases when aerodynamic controls become little effective in rarefied layers of the atmosphere and also at low speeds of flight of the rocket (for instance, at its launch from the Earth).

Several methods of the turn of the vector of tractive force exist. We have already considered how the flight control of a rocket with the help of jet vanes occurs; however, in ensuring the rocket of sufficiently high maneuverability such vanes have two essential deficiencies: low stability, because of the fact that they are in a high-temperature medium moving with great speed, and also considerable drag. The first deficiency compels us to find specially stable materials for jet vanes, otherwise they can burn up toward the end of operation of the engine. Then the rocket will stop being controlled and will descend from the calculated trajectory. Because of the second deficiency part of the tractive force is lost to the surmounting of the drag of the jet vanes.

In ballistic rockets turning engines are widely applied. Thus, for instance, the American ballistic unfinned rocket "Atlas" is controlled on a powered-flight trajectory with the turn of a jet engine chamber. On this rocket there are special controlling engines ensuring lateral stability to the rocket and allowing the control of it on the initial section of free flight for the correction of the trajectory.

Attempts are known to control the rocket by means of a turn of the nozzle or a special cap placed immediately behind the nozzle and embracing the stream of gases. The control by redistribution of the tractive force in several nozzles is possible. For instance, on the English antitank rocket "Pye" there are a number of nozzles of the sustainer engine located about the circumference. The control system permits covering one or several nozzles and thereby creating the control force necessary in magnitude and direction. This rocket does not have aerodynamic controls.

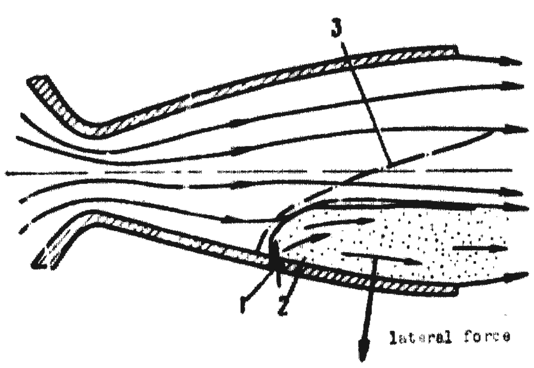


Fig. 62. Diagram of the creation of a lateral force by means of liquid injection into the supersonic part of the nozzle:
1) hole for the injection of the liquid;
2) region of flow of the injected liquid;
3) shock wave.

Recently considerable attention has been given abroad to the method of control by the vector of tractive force by menas of blowing (injection) liquid or gas into the nozzle of a jet engine [48]. The fluid flow injected into the supersonic part of the nozzle through the hole 1 (Fig. 62) interacts with the supersonic flow of gaseous products of combustion of the fuel and, deviating from the initial direction, flows into region 2.

The region of inject liquid 2 is a barrier for the supersonic flow in the nozzle, brakes this flow, and creates the shock wave 3. After the shock the speed of gas decreases, the pressure increases, and the flow of gas turns. With this the pressure on the walls of the nozzle after the shock wave turns out to be more than if there were no injection of the liquid. This leads to the appearance of the lateral force $Y_{бок}$ the magnitude of which is equal to the sum of all the forces from the increase in pressure as compared to the case of the absence of injection which have an effect on the wall of the nozzle after the shock wave.

By varying the quantity of liquid injected into the nozzle per unit of time, it is possible to control the magnitude of the lateral and, consequently, control force. The direction of the control force can be changed by injecting liquid through different holes located along the circumference of the cross section of the nozzle.

A certain decrease in the tractive force, owing to the irreversible losses of mechanical energy of the gas flow with the transition of it through the front of the shock wave, is approximately compensated by the increase of thrust from the expiration through the nozzle of the injected liquid.

Not enumerating other forms of gas-dynamic controls let us turn to the combined type of controls the principle of the action of which has been known for a long time and consists in the following. On trailing edge of wing or another fixed lifting surface (fin) the nozzle is made in the form of a turning slot through which gases pass with great speed. By regulating the turn of the slot it is possible to direct the stream of gases to a certain side of the lifting surface at a different angle. The control force will consist of the normal component of tractive force from the expiration of gases through the slot and of an additional normal component from forces of air pressure on the lifting surface, which appeared from the redistribution of pressure along the contour of the lifting surface with the interaction of the air flow circumfluent this surface with the stream of gases passing through the slot. At present there are attempts to use a jet spoiler. A stream of gases in a jet spoiler is guided through the slot in the wing upwards or downwards and considerably changes the wing lift. This change can be increased even more if one were to vary the exhaust velocity.

Deficiencies of combined controls are that they effectively operate only in sufficiently dense layers of the atmosphere and with a certain minimum of the speed

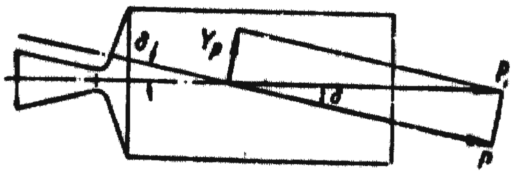


Fig. 63. Diagram of flight control of the rocket by the turn of an engine.

of flight, and turns of the slot-like nozzle are also needed in the reliable source of gases and mechanism.

Let us briefly examine the calculation of the magnitude of force appearing on the controls during their operation. It is most simple to determine it in the case of the control of turning the whole engine. From Fig. 63 one can see that $Y_p = P_1 \sin \delta$, where P_1 is the tractive force and δ the angle of rotation of the engine. The component of the tractive force along the tangent to the trajectory will be changed little, since it is equal to $P = P \cos \delta$, and δ is comparatively small and $\cos \delta$ differs little from unity.

If jet vans or air vanes in the form of deflecting surfaces are used, then for calculation the following formula are used:

$$Y_p = C_{yp}^{\delta} \delta S_p q = Y_{p0} \delta; \quad (4.1)$$

$$X_p = (C_{x0} + C_x^{\delta} \delta^2) S_p q, \quad (4.2)$$

where $Y_{p0} = C_{yp}^{\delta} S_p q$; S_p is the characteristic area of the vanes; q is the velocity head incident on the rocket of the air flow (or flow of gases in the nozzle of the engine for jet vanes); C_{yp}^{δ}, C_{x0}, C_x^{δ} are aerodynamic coefficients determined by calculation or according to experiments in wind tunnels. Formulas (4.1) and (4.2) show how magnitudes Y_p and X_p depend on the angle of rotation δ of the rudder surface. If for Y_p this dependence is linear (Y_p is proportional to δ), then that for X_p it is more complicated and approximately described by the parabola similar to coefficient C_x in Fig. 22. From formula (4.1) it is evident how according to the ascent of the rocket above the surface of the Earth Y_p created by aerodynamic controls decreases. Really since $q = \frac{\rho V^2}{2}$, then it proves to be that q is directly proportional to the air density ρ which quickly decreases with altitude.

Control forces created by spoilers, turning nozzles, caps, and so forth and also combined controls are determined mainly by experimental means because of the complexity and insufficient study of phenomena accompanying their operation.

§ 3. Aerodynamic Interference

Aerodynamic interference is called the mutual influence of bodies near one another in the flow of air on the flowing around of them by this flow. For instance, the flowing around by the air flow of a winged rocket consisting of a body and wing differs from the flowing around of the same bodies of the rocket and its wing but isolated from each other. The forms of streamlines of shock waves and the vortex trail will be different, and this will change the distribution of forces of pressure and forces of friction along streamlined surfaces. Aerodynamic interference leads to the fact that the sum of aerodynamic coefficients calculated separately for the body and wing of the rocket will not appear equal to the respective aerodynamic coefficient of the rocket on the whole having the same body and wing. Therefore, it is especially important to know the laws of aerodynamic interference. By knowing the laws it is possible not only to determine the aerodynamic coefficents of the rockets, but also by fulfilling the aerodynamic design of the rocket we can use reasonably the results of the interference in their interests. However, the phenomenon of aerodynamic interference is so complicated that up to the present time there have been no sufficiently simple and reliable theoretical solutions to this problem; and therefore by creating a rocket in calculations it is necessary to use either qualitative conclusions of the theory or empirical formulas.

Subsonic and supersonic air flows behave differently, and therefore let us examine the phenomenon of aerodynamic interference separately in conditions of the rocket flight first with subsonic and then and supersonic speeds.

The wing and body of the rocket at the place of their connection form a complicated surface remainding one of a dihedral angle (Fig. 64). As a rule this angle increases, and therefore the speed of the subsonic flow decreases and the pressure increases. Furthermore, the flow of air inside this angle undergoes a simultaneous deceleration from both walls (wing and body). All this leads to the deceleration of the flow and a sharp increase of the thickness of the boundary layer. The so-called diffuser pocket is formed. Inside the diffuser pocket even at small angles of incidence there occurs a breakdown of the boundary layer accompanied by a strong eddy formation. We already know that

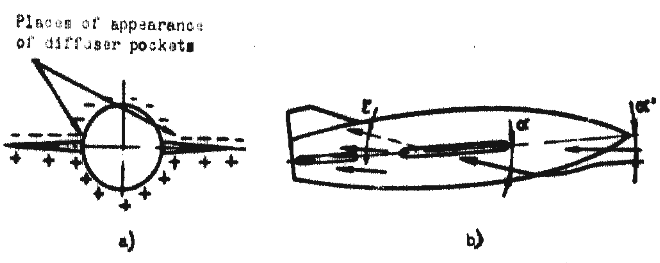


Fig. 64. Diagram of a winged finned rocket:
a) rarefaction above and compression of air under the wing is transmitted to the body increasing its lift; b) body of the rocket increases the angle of incidence of the wing by angle $\Delta\alpha = \alpha - \alpha'$, and the wing slopes the flow at angle ε.

this phenomenon leads to the increase of drag of the rocket and to the decrease of its lift.

The magnitude $\Delta C_{x\,инт}$, on which the drag of the rocket is increased, is determined by experimental data and is on the average 0.01-0.02. To decrease this harmful phenomenon (increase of drag of the rocket and decrease of its lift) the connection of the wing with the body is made smooth with the help of "fairings."

The presence of the body of the rocket disturbs the continuous distribution of the circulation about the wing (along the span), and this leads to the increase of induced drag of the wing. The influence of the body, and also the gondolas of motors if they are placed on the wing and are not extracted in its profile, is considered by the introduction into the formula of the coefficient of induced drag (3.9) instead of real wing aspect ratio λ of the effective extension

$$\lambda_e = k_\lambda \cdot \frac{\lambda}{1 + S_l/S},$$

where k_λ is the coefficient considering the form of the wing in the plan (its magnitude is 0.6-1.0); S_1 is the wing area overlapped by the body of the rocket; S is the wing area in the plan.

Thus, the variable part of the drag of the rocket dependent on the angle of incidence can be calculated by the formula

$$C_{x\,перем} = C_{xi} + \Delta C_{x\,инт}$$

Moreover, at subsonic speeds of flight the interference of the wing and body of the rocket increases the part of the drag coefficient of the wing $C_{x\,пост}$, not dependent on the angle of incidence. This increase is estimated by the coefficient of interference $k_{инт}$ entering the formula considering the decrease of $C_{x\,пост}$ owing to the covering of part of wing area by the body of the rocket. The drag coefficient of the wing, taking into account the interference, is determined from the formula

$$C_{x\,пост} = C_x\left(1 - k_{инт}\frac{S_l}{S}\right).$$

The wing and body mutually affect the lift of one another. This is explained in the following way. Let us assume the rocket is at a certain angle of incidence α' (Fig. 64b), then the body as if crushes under itself part of the flow, and therefore the streams of flow deviate under the body downwards and on each side of it upwards. This increases the angle of incidence of the wing by $\Delta\alpha$ and, consequently, its lift. The increase of pressure under the wing and decrease of pressure above the wing are transmitted to the body of the rocket (Fig. 64a) and also increase its lift. However, at subsonic speeds of the flight of rockets this increase in lift of the rocket almost is completely compensated by the decrease in lift due to the separation of the boundary flow in the region of the connection of the wing and body. Therefore, in the first approximation we consider the lift of the rocket equal to the sum of the lifts of the wing and body.

Now let us consider the phenomenon of aerodynamic interference of the wing and body of the rocket during the flight of it at supersonic speeds. At the connection place of the wing and body the expansion of supersonic flow increases its speed and decreases the pressure. This permits in most cases avoiding the separation of the boundary layer and, connected with this, the increase in drag and also decrease in lift of the rocket. If one were to consider that perturbations at supersonic speeds of the flight spread inside the cones with angle μ (Fig. 66), then it becomes clear that it is possible in the first approximation to disregard the change in the drag of the rocket owing to the change of induced drag of the wing with the connection of it with the body.

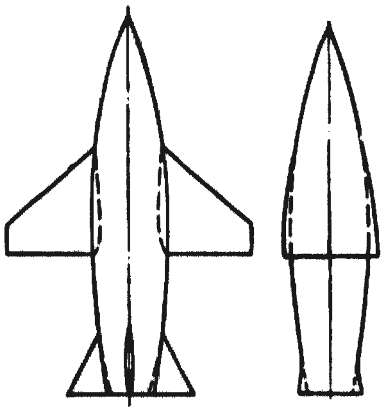


Fig. 65. Figure illustrating the application of the rule of "area":
a) diagram of rocket; b) equivalent solid of revolution.

The increase of the lift of the rocket from the mutual influence of the wing and body at supersonic speeds of the flight is considerable, and the coefficient of the lift of the rocket can be approximately calculated from empirical formulas.

The aerodynamic interference of the body and wing of the rocket change the magnitude of the pitching moment applied to the rocket and, consequently, the position of the center of pressure. The change of the moment from aerodynamic interference in the first approximation does not depend on the speed of the flight of the rocket (on M) and is determined from empirical formulas.

Everything said above on the aerodynamic interference of the body and wing can be related to the interference of the body and fin. However, with the presence on the rocket, besides the wing, of still another fin, this forces us to consider changes in the streamline flow of the rocket which are formed owing to the downwash caused ahead by the disposed wing and fin.

Let us consider the interference of the body of the rocket and fin which is in the zone of the flow perturbed by the wing of the rocket. The chief characteristic of this case of interference is the flowing around of the fin by the flow perturbed by the wing of the rocket. In the first approximation in the calculation we consider only the change of the angle of incidence of the fin owing to the downwash by the wing. The angle of incidence of the fin (Fig. 64b), is determined after the following formula:

$$\alpha_{on} = \alpha - \varepsilon,$$

where α is the angle of incidence of the rocket; ε is the mean value of the downwash angle behind the wing.

As it was noted above, because of the complexity of the phenomenon of aerodynamic interference, it is considered in calculations only approximately. It is especially complicated to consider it in the region of the transtransonic speed of flight when the system of local shock waves appears and is reconstructed prior to the shaping of the front and rear shocks when the local shock waves interact with the boundary layer. Therefore, in the practice of the construction of rockets the area rule is used, which permits reducing to a minimum the harmful phenomena of interference.

The area rule is based on the position well-checked by experiments about the fact that the drag of the rocket (Fig. 65a) is approximately equal to the drag of the solid of revolution having the same distribution of areas of the cross section along its axis as for the actual rocket (Fig. 65b).

Therefore, by connecting the body of the rocket with the wing and fin it follows thus to distribute the area of the cross section along the axis of the rocket as occurs for the solid of revolution of most advantageous form or close to such. The area rule is useful not only in the transonic speed range of flight, but also for supersonic speeds; however, it is necessary to fulfill the cross section not at a right angle but at an angle $\alpha = \text{arc sin } \frac{1}{M}$ to the axis of the rocket. Consequently, it is obtained that for each speed of supersonic flight the angle of the cross section will be its own, and therefore the form of the rocket providing the least drag is also its own. Rocket designers must design the rocket form so that it possesses the least drag at such speeds of flight during which the rocket must fly the greater part of the time. If one were to apply the area rule, the body of the rocket at connection places of the wing and fin becomes thinner (see dotted lines in (Fig. 65a), which considerably decreases the magnitude of the drag of the rocket.

In selecting the aerodynamic diagram of a rocket one should consider the rolling moments appearing in flight from different conditions, the sum of which is called the moment of oblique airflow. The cause of the appearance of the moment of oblique airflow is asymmetry of the flowing around of the rocket by air, for instance, during the carrying out of a maneuver in a horizontal plane with the angle of incidence. The asymmetry of the streamline flow is caused by the following factors:

1 — difference of the actual angles of sweepback of consoles of the wing;

2 — influence of downwash after the wing on the fin;

3 — blackout of one console of the wing by the body of the rocket;

4 — different places of the separation of flow from consoles of the wing, which are obtained because of different conditions of their streamline flow;

5 — different influence of wing tips on the streamline flow of the surface of the wing (end effect of the wing);

6 — different mutual influence of the wing and body of the rocket at places of their connection (root effect).

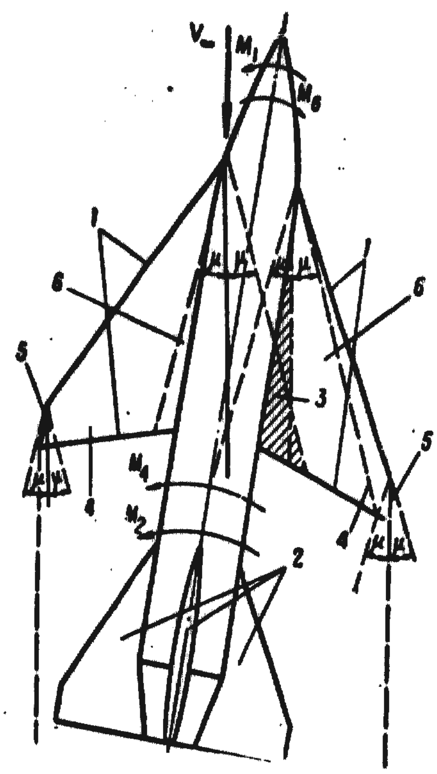


Fig. 66. Diagram of a rocket for the illustration of causes of the appearance of the moment of oblique airflow.

Figure 66 shows by respective numbers regions of manifestation of the asymmetry of the flowing around of a winged rocket and denotes by arrows the direction of the separate components of the moment of oblique airflow. There will be the same direction for the moments both with cruciform wings and the fins, but under the condition that the angle of incidence of the rocket is greater than the angle of slip. Designers of a rocket are thus obliged to design bank controls, for instance, ailerons, so that the magnitude of the rolling moment is created by them is larger than the moment of oblique airflow and the rocket after a maneuver can return quickly enough to its initial position.

§ 4. Ideas on the Most Advantageous Form and the Evaluation of Diagrams of Rockets of Different Assignment

In process of the aerodynamic designing of a rocket it is almost always necessary by all accessible means to strive to decrease the drag coefficient. This makes it possible to obtain an assigned speed of the rocket with a smaller propellant weight, i.e., this will decrease its launching weight; or with the assigned weight of the rocket this will increase its speed (and, consequently, the flying range); or with the assigned speed and launching weight this will increase the part necessary for a payload (warhead, container of instruments, etc).

It is practically difficult to make the outer form of a rocket so that it will have the least drag, since it is necessary to deviate from the most favorable form caused by the necessity to dispose the engine assembly, instruments, and armament and also by operating conditions, and others. The finite limitations on the form of the rocket are superimposed by the methods of its manufacture.

How can we decrease the drag coefficient C_x? In general it is determined by equation (3.8).

For a decrease in C_{xp} and C_{xB} the contour of the rocket should have a rational form. At subsonic speeds of the flight this form is teardrop-shaped. At supersonic speeds for a decrease in the wave drag one should use the pointed form of the body of a rocket and also profiles of the wing and fin. The greater the sharpening the less the angle of shock and the less the losses in it. Losses are especially great on sections of normal shock waves which appear before the blunted parts of the rocket and before the different flanges, and therefore we try to decrease them. In the region of supersonic (and especially transonic) speeds C_{xB} makes up the main part of C_x and to the decrease of magnitude C_{xB} the main attention of creators of a rocket is given.

To decrease C_{xB} in the region transtransonic speeds the leading edges of the wings and fin are made sweptback, and the actual wings and fin are made with small extensions. The relative thicknesses of the profile of the wing and fin should be as little as possible. These requirements satisfies well the triangular (in plan) wing finding wide distribution in rockets. The great advantage of such a wing is the stable position of the center of pressure at different speeds of flight, which facilitates the stabilization of the rocket.

Let us show how the sweepback of the leading edge decreases the drag of the wing and helps to overcome the shock stall. From Fig. 67 one can see that the profile of a sweptback wing is flowed around by flow with the speed of perpendicular to the leading edge and equal to $V_n = V_\infty \cos \varkappa$, where $\varkappa$ is the angle of the sweepback. The larger $\varkappa$ the less V_n with a constant V_∞. A decrease in V_n permits removing the shock stall into the region of larger values of V_∞ and considering lowering C_x in the range transtransonic speeds.

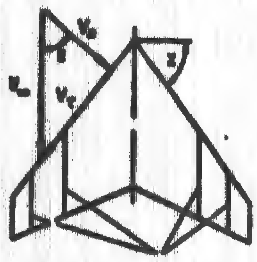


Fig. 67. Diagram of the flowing around of a swept-back wing.

In the region of subsonic and supersonic speeds it is desirable to have a rectangular wing, since it increases the effectiveness of the aerodynamic controls located gives to the rocket great lateral stability, and transfers the center of drag nearer to the nose cone of the rocket, which with the increase of the speed of the flight is usually displaced nearer to the tail section. For instance, on the American two-stage ballistic rocket "Pershing" the fin of the first stage is made triangular with a great sweepback, and the fin of the second stage flying with supersonic speed is rectangular.

The component of speed V_τ causes the overflowing of the boundary layer along the wing and the runoff of it from the wing tups which decreases the lift. For preventing this harmful phenomenon on the upper surface of the wing protruding surfaces, called aerodynamic ridges are made, however, excessive tapering of the form of the rocket can lead to undesirable consequences. The sharp nose of the nose cone and sharp edges of the wing and fin are quickly heated and can burn if the rocket flies within the terrestrial atmosphere at great speed: very sharp edges of the wings and fin with a relative small thickness cause separation of the flow from them with small values of the angle of incidence ($\alpha \cong 3$-5°). The sharp nose cone of the rocket has a great extension and poor damping, etc.

Others factors affecting the selection of form of the nose cone of the rocket should be remembered. Thus for rockets of tactical assignment, a considerable part of the trajectories of which is in dense layers of atmosphere and is subject to accidental influences of the wind and to changes of air density and its temperature, it is apparent expediently to make the nose cone as pointed as possible. This will produce the maximum decrease the drag of the rocket and time of its flight along the descending branch of the trajectory. The less this time is the less the trajectory of the nose cone of the rocket will deviate from the influence of the atmosphere (wind, air density, etc.), i.e., the accuracy of such a rocket hitting a target will be increased. The vulnerability from the action of antimissile means of the enemy also decreases. This is the path taken by the creators of the American rocket "Pershing" with a maximum firing range of 750 km. On the nose cone of this rocket there is a sharp tip with a fused covering resisting the influence of high temperatures of aerodynamic heating.

To decrease C_{xf} one should have a laminar boundary layer over the entire surface of the rocket; for this the surface of the rocket should be smooth and not have flanges and unevennesses, which can lead to turbulization of the boundary layer. It is necessary to remeber that a part of C_{xf} can be great not only at subsonic speeds of the rocket, but also at great supersonic speeds when C_{xB} decreases with the increase of speed. It is especially important to decrease C_{xf} for long rockets having a large lateral surface as compared to the area of the middle section and, consequently, the greater part of the frictional force in the total force of the drag.

To decrease C_{xi} one should take the wings with the largest possible lengthenings λ (see formula 3.9); the form of them should be near to elliptic (possessing the least induced drag). There are foreign rockets and aircraft having a conical twist of the wing when the nose cone of the profile of the wing is distorted downwards, and this distortion progressively increases to the wing tips. The wing of such a form turns the vector of lift nearer to the vertical and thereby decreases the force of induced drag (projection of lift on the direction of flight).

To decrease $C_{x\text{д}}$ the tail section of the rocket should be smoothly tapered so that the separation of the boundary layer is not premature. The earlier the separation of the boundary layer, the larger the area of the tail section of the rocket will be, subject to the influence of rarefaction, and the greater the force of the base drag will be.

Owing to the fact that on narrowing tail section the speed of the flow flowing around it increases, but the pressure decreases and the center of the drag of the rocket with the narrowing tail section shifts to the nose cone. This decreases the stability of the rocket. In order that this will not occur it is necessary to increase the tail fin area, which increases the drag of the rocket and decreases the gain from the creation of narrowing tail section.

It is interesting to note that presence of a stream of gaseous products of the combustion of fuel of passing from the nozzle to the tail part changes the picture of the streamline flow of this part of the rocket by air. Owing to the sucking (ejection) action of the stream, the separation of the boundary layer can occur later. Furthermore, the area of the action of the base drag decreases by the magnitude of the sum of the output sections of the nozzles of the jet engines. Thus on a powered-flight trajectory the base drag of the rockets decreases. Under favorable conditions of the location of the nozzles and form of the tail section of the rocket it is possible to obtain the full disappearance of the force of the base drag and even to create a "pushing" force.

Even a brief acquaintance with the idea of the "most advantageous form of rocket" convinces us that the process of aerodynamic design is very complicated. In the beginning through diverse variants are worked, and only after their thorough comparison is the best selected. With this one should strive to keep the aerodynamic diagram the simplest; every complication introduced into it should be justified. For instance, for the variant of the guided ballistic rocket with gas-dynamic controls it is necessary to make the fin only in that case if for stabilization in the trajectory (such rockets, as a rule, are unstable) too powerful controls will be demanded. In this case the presence of the fin in the tail section of the rocket will be justified, since it will transfer the center of drag nearer to the center of mass and will decrease the static instability of the rocket.

In conclusion of this chapter let us give a comparative appraisal of the most complicated but very widespread aerodynamic diagrams of winged rockets: normal, "canard," and "wingless". Advantages of the normal diagram are the following.

1. The wing is flowed around by an undisturbed flow, and therefore conditions of its operation are favorable.

2. The angle of incidence of the rudders α_p (Fig. 68) in flight is less than

the angle of incidence of the wing α_{K} which permits giving the rudders sufficiently large angles of incidence, ensuring a large control force and, consequently, a sharp maneuver of the rocket in flight. The separation of the flow from the surface of the rudders will begin only after it occurs on wing, and since this is not allowed because of the sharp decrease of the wing lift then it appears that the rudders will not limit the maneuverability of the rocket.

3. The bending moments applied to the body of the rocket are comparatively small, since of the small angles of attack of the rudders the loads on them are not very large.

4. The rudders can be used as ailerons for producing the bank of the rocket.

Deficiencies of the normal diagram consist of the following.

1. In the zone of the perturbed (tapered) flow the fin appears, which is subjected in this case to shaking from the influence of air impact loads of periodic character appearing from incidence on the fin of the eddying flow. This flow separates from the surface of the wing either at large angles of incidence (if the flight occurs with subsonic, more accurately subscritical speed) or vortexes are formed after the curvilinear shock wave (if the speed of flight is supersonic). In the first case the shaking of the fin is called buffeting and in the second high-speed buffeting. By avoiding buffeting the designers transfer the fin higher than the plane of the wing beyond perturbing flow, but the weight of the construction of rocket increases and the rigidity decreases, and there frequently appears a flutter-vibration of the fin with the possible breaking of it. The fin vibrates with a flutter owing to its low rigidity, since the deflections of a low-rigid fin periodically change the load on it on the side of the incident air flow.

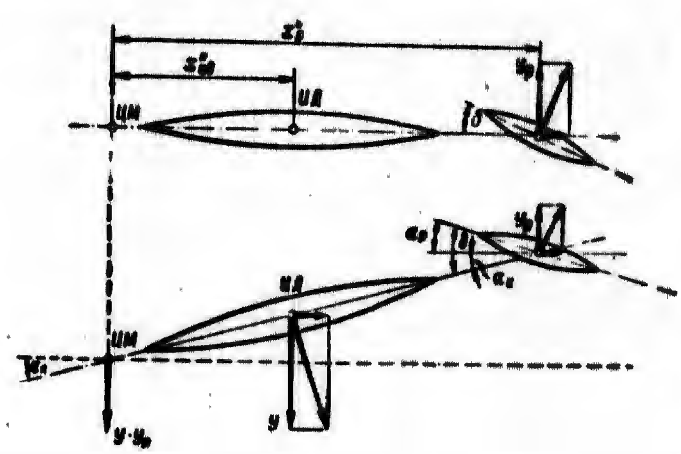


Fig. 68. Control of a rocket with a normal aerodynamic diagram:
a) position before the turn of the rocket;
b) position after the run of the rocket.

2. The lift of the rocket on the whole decreases with the deflection of the rudders, since the lift is directed to the opposite side than the wing lift and body of the rocket (Fig. 68).

3. The distance between the center of gravity and center of pressure

in rockets constructed from the normal diagram is less than for rockets made of the "canard" diagram with identical lifts Y and Y_p. Indeed from Fig. 68 and 69 one can see that from the condition of equilibrium during stabilization with the defined angle of incidence (equality of moments of forces Y and Y_p):

$$x^{н}_{ц.д} = \frac{Y_p x^{н}_p}{Y} \text{ and } x^{у}_{ц.д} = \frac{Y_p x^{у}_p}{Y}.$$

Usually $x^{у}_{p} > x^{н}_{p}$, and therefore $x^{н}_{ц.д} < x^{у}_{ц.д}$.

This worsens the stability of the rocket constructed of a normal diagram, and with an identical static stability and other equal conditions of oscillation rockets made of the diagram "canard" will dampen faster, since the moment of damping is obtained greater (further from the center of gravity the controlling fin is attributed owing to which it dampens the oscillation faster in a viscous medium, air).

Advantage of the "canard" diagram are:

1. The fin is in the undisturbed flow.

2. The lift of the rocket is increased owing to the lift of the rudders directed to the same side.

3. The characteristics of damping, as were mentioned above, for the "canard" diagram are better than for the normal.

4. The horizontal rudders have comparatively small dimensions.

5. The "canard" diagram permits easier distribution of the fuel tanks and propulsion system, inasmuch as the controls are in the nose cone of the rocket.

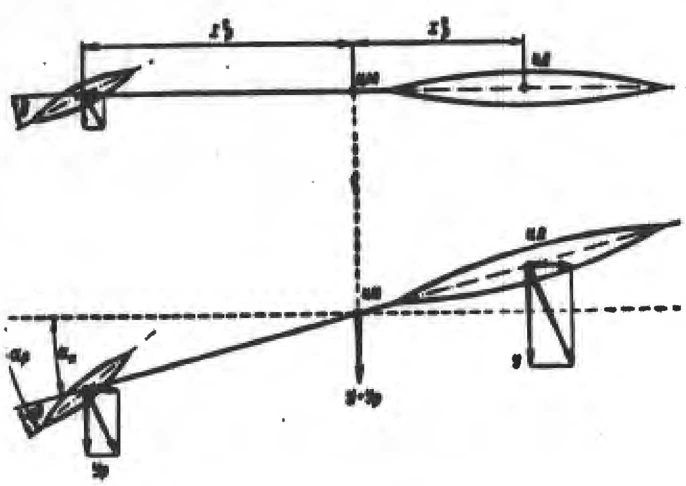


Fig. 69. Control of a rocket constructed of the diagram "canard":

a) position before the turn of the rocket; b) position after the turn of the rocket.

Thus the rocket carried of the "canard" type does not have deficiencies of peculiar to the normal diagram.

However, the "canard" diagram has its deficiencies which limit its practical application.

1. A flow tapered by rudders affects the wing, which leads to the appearance of a rolling moment effective about the longitudinal axis of the rocket. This larger moment the smaller the wing aspect ratio, since almost all its surface is subjected to the influence of tapered flow.

2. Because of the appearance of the rolling moment from the oblique airflow of the wing, the rudders cannot be used as ailerons since this moment of bank will be directed opposite to the rolling moment appearing from the action of the ailerons, and the work of the ailerons can be brought to zero.

3. Since the angle of incidence of the rudders (Fig. 69) is greater than the angle of attack of the wing by the magnitude of the angle of displacement of the rudder with respect to the body of the rocket, i.e.,

$$\alpha_p = \alpha_w + \delta,$$

the separation of flow can occur primarily from the rudders . With the separation longitudinal oscialLations of the rocket appear.

4. Due to narrow range in which the center of pressure should be with respect to the center of gravity, the design of the rocket completed in the "canard" diagram is very complicated.

5. Loads on the rudders and the moments bending the body of the rocket other things being equal, are greater than in the case of a normal diagram, since the angles of incidence of the rudders are great.

Besides those enumerated, to the deficiencies of the "canard" diagram one should add the difficulty of providing directional stability, since the long nose cone of the rocket creates great disturbing moments requiring a well-developed vertical fin with a large arm with respect to the center of gravity.

Advantages of the "tailless" diagram.

1. Downwash does not effect wing lift.

2. The absence of a horizontal controlling fin somewhat decreases the drag and lowers the weight of construction.

3. The rocket with a tailless configuration has small lifting surfaces and is compact.

Deficiencies of the "tailless" are the following.

1. It is not adapted for sharp maneuvers, since with rudders on the trailing edge of the wing it is difficult to accomplish a fast maneuver because of the small arm for the controlling moment (short distance from the axis of the turning rudders, to the center of gravity of the rocket). To increase the controlling moment it is necessary to increase the controlling lift, and this results either in the increase of the surface of the rudders or the increase in the angle of their deflection, which increase the power of the controls. Both of these increase the weight of the rocket.

2. It is difficult to obtain a sufficient static stability, since the distance between the center of mass and center of drag in a rocket of a "tailless" diagram is small.

3. Poor characteristics of damping. This is obtained because the surfaces of the rocket are located near the center of mass around which the oscillatory motion of the rocket occurs.

4. With the increase in the M number (speed of flight) the center of drag is quickly displaced, and therefore the reserve of static stability quickly increases. Thus, for instance, when M = 1.5 it is increased 2.5-3 times whereas for rockets of the "canard" diagram this increase is 1.3-1.5.

In estimating aerodynamic diagrams of rockets one should remember that each of them is good for its type of rockets. For instance, ground-to-air rockets, as a rule, are made with fins or wings, and so forth.

CHAPTER V

THEORY OF THE FLIGHT OF ROCKETS OF DIFFERENT ASSIGNMENT

§ 1. Flight of Long-Range Rockets to the "Earth-to-Earth" Class

Let us consider the flight of a well-stabilized guided rocket taking into account the rotation of the Earth and air resistance. Let us assume that the control at each moment of time combines the longitudinal axis of the rocket with the velocity vector tangent to the trajectory of the motion. With the well-working control it is possible not to consider the oscillation of the rocket with respect to the center of mass and to consider the motion of the rocket as the motion of the material point.

At first let us write the differential equation of motion of the center of mass of the rocket in vector form in absolute motion using formulas (1.16), (2.2), and (2.5):

$$m(\bar{J}_{\text{отн}} + \bar{J}_{\text{пер}} + \bar{J}_{\text{кор}}) = \bar{P} - \bar{R} - \bar{F}_{\text{т}} \qquad (5.1)$$

The place of launch and place of fall of the rocket of the class considered are on the surface of the Earth. The control of the rocket and the observation of its flight are carried out also with respect to points located on the Earth's surface. Therefore, the calculation of the trajectory of the rockets motion is usually conducted in the system of coordinates connected with the rotating Earth, i.e., the relative motion. Then equation (5.1) will have the form

$$m\bar{J}_{\text{отн}} = \bar{P} - \bar{R} - \bar{F}_{\text{т}} - m\bar{J}_{\text{пер}} - m\bar{J}_{\text{кор}} \qquad (5.2)$$

By using formula (5.2) we formulate a system of differential equations of the motion of the center of mass of the rocket in axes of coordinates X, Y, Z (Fig. 70)

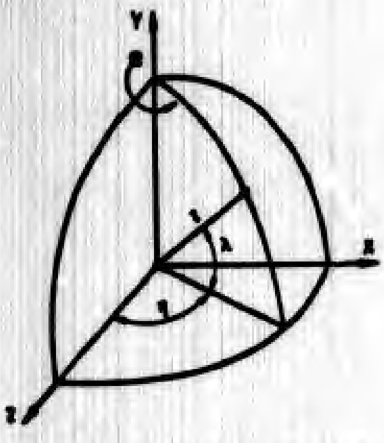


Fig. 70. Position of rocket with respect to the Earth: η – latitude; λ – longitude; r – radius vector.

connected with the Earth. The position of the rocket will be determined by coordinates x, y, z, or the geographical longitude η, latitude λ, and radius r. Preliminarily we will write the projections of the product of the magnitude of tractive force by the cosine of the angle between the direction of the vector of force and corresponding axis. Cosines of the angles will respectively be equal to:

$$\frac{v_{\text{отн}\,x}}{v};\ \frac{v_{\text{отн}\,y}}{v};\ \frac{v_{\text{отн}\,z}}{v},$$

where $v_{\text{OTH}\,x}$, $v_{\text{OTH}\,y}$ and $v_{\text{OTH}\,z}$ are projections of the speed of the center of mass of the rocket on the coordinate axes. Omitting subsequently the signs "отн" signs we obtain

$$v=\sqrt{v_x^2+v_y^2+v_z^2}$$

Projections of the tractive force are determined by the simple formulas:

$$P_x=P\cdot\frac{v_x}{v};\ P_y=P\cdot\frac{v_y}{v};\ P_z=P\cdot\frac{v_z}{v}. \qquad (5.3)$$

Air resistance depends on the speed of the rocket with respect to the atmosphere and is determined in the case considered by formula (3.3).

The projection of this force on the coordinate axes will also be equal to the magnitude of force multiplied by the cosine of the angle between the direction of the vector of the tractive force and the corresponding axis:

$$\left.\begin{aligned} R_x&=\rho\frac{v_x}{2}\cdot SC_R v;\\ R_y&=\rho\frac{v_y}{2}\cdot SC_R v;\\ R_z&=\rho\frac{v_z}{2}\cdot SC_R v.\end{aligned}\right\} \qquad (5.4)$$

We will define component forces of the gravitation of Earth using formula (2.16):

$$\left.\begin{aligned} F_{\tau x}&=mg_{0}\cdot\frac{R^2}{r^2}\cdot\frac{x}{r};\\ F_{\tau y}&=mg_{0}\cdot\frac{R^2}{r^2}\cdot\frac{y}{r};\\ F_{\tau z}&=mg_{0}\cdot\frac{R^2}{r^2}\cdot\frac{z}{r}.\end{aligned}\right\} \qquad (5.5)$$

where $\frac{x}{r}$, $\frac{y}{r}$, and $\frac{z}{r}$ are cosines of angles between the direction of the action of force F_T and the corresponding coordinate.

Using those written and also formulas (2.10) and (2.11) we obtain the system of differential equations describing the motion of the center of mass of the rocket:

$$\left.\begin{aligned}\dot{v}_x &= P\cdot\frac{v_x}{v}\cdot\frac{1}{m}-\rho\frac{v_x}{2}\cdot SC_R v\cdot\frac{1}{m}-\\ &\quad -g_0\cdot\frac{R^2}{r^3}\cdot x+\Omega^2 x-2\Omega v_z,\\ \dot{v}_y &= P\cdot\frac{v_y}{v}\cdot\frac{1}{m}-\rho\frac{v_y}{2}\cdot SC_R v\cdot\frac{1}{m}-g_0\cdot\frac{R^2}{r^3}\cdot y;\\ \dot{v}_z &= P\cdot\frac{v_z}{v}\cdot\frac{1}{m}-\rho\frac{v_z}{2}\cdot SC_R v\cdot\frac{1}{m}-\\ &\quad -g_0\cdot\frac{R^2}{r^3}\cdot z+\Omega^2 z+2\Omega v_x\end{aligned}\right| \tag{5.6}$$

Let us establish the connection between the position of the rocket in the selected rectangular system of coordinates and its geographic latitude and longitude. From Fig. (70) one can see that

$$\left.\begin{aligned}x &= r\cos\lambda\sin\psi;\\ y &= r\sin\lambda;\\ z &= r\cos\lambda\cos\psi.\end{aligned}\right\} \tag{5.7}$$

Differentiating twice in time will get:

$$v_x=\dot{x};\ v_y=\dot{y};\ v_z=\dot{z}\ \text{and}\ \ddot{x},\ \ddot{y}\ \text{and}\ \ddot{z}.$$

Substituting into formulas (5.6) results of the differentiation of formulas (5.7), we obtain the system of linear differential equations. Considering cumbersomeness of the system we will write it here in the form of functional dependences:

$$\left.\begin{aligned}\ddot{r} &= f_1(r,\psi,\lambda,\dot{r},\dot{\psi},\dot{\lambda},t);\\ \ddot{\psi} &= f_2(r,\psi,\lambda,\dot{r},\dot{\psi},\dot{\lambda},t);\\ \ddot{\lambda} &= f_3(r,\psi,\lambda,\dot{r},\dot{\psi},\dot{\lambda},t).\end{aligned}\right\} \tag{5.8}$$

The complexity of the solution of written equations is evident. However, difficulties are surmounted with the help of electric computers.

From the common system of equations it is easy to obtain a system describing the particular cases of motion. Characteristics of motion of the center of mass of the rocket with a nonoperating engine (on the passive section of the trajectory) can be determined with the use of equations (5.6) in which P must be equated to zero. With the calculation of motion at great heights in the circumterrestrial rarefied space, when it is possible not to consider the resisting force of air it is necessary in the formula (5.6) to omit the members taken from equation (5.4).

In the latter case it is easy to trace the influence of the rotation of the Earth on the flight of the rocket. Let us assume that point A in Fig. 71 and 72 is the projection on Earth's surface of a conditional point of the entrance of the rocket into rarefied layers of the atmosphere when with drag it is possible not to be considered. At first we will not consider the rotation of the Earth. The projection of the conditional point of the entrance of the rocket into dense layers of the atmosphere on the descending branch of the trajectory will be designated by B. Thus the projection of the planar trajectory on the surface of the Earth without taking into account its rotation will be AB. Axes of the coordinates are located in such a manner that OX is in the meridian plane passing through point A. The latitude λ_B and longitude η_B of point B can be easily determined by trigonometric functions of angles α, β, and 2ψ, shown in Fig. 71. Let us designate the time of the flight of the rocket in rarefied layers of the atmosphere by the letter t. During the time t the Earth will turn from west to east at the angle Ωt, and the longitude of the point of entrance of the rocket into the dense layers of the atmosphere will be

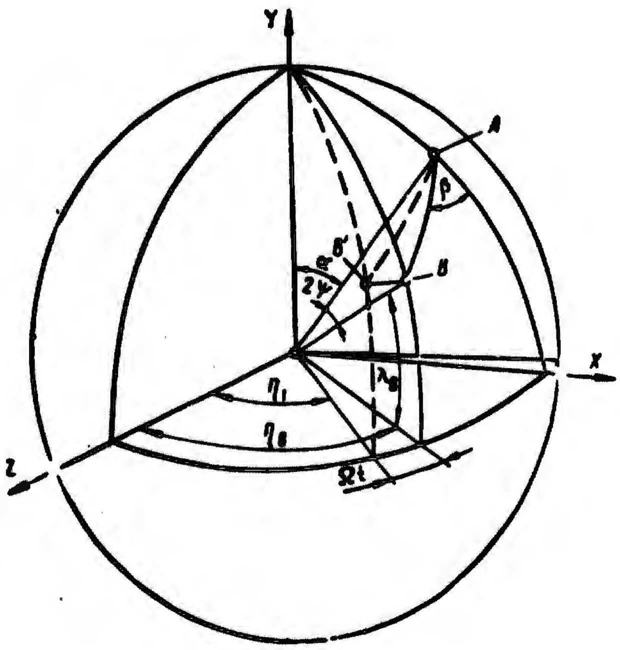


Fig. 71. Projection of the trajectory of the flight of a rocket over the Earth: AB – without taking into account the rotation of the Earth; AB' – taking into account the rotation of the Earth.

$$\eta_1 = \eta_B - \Omega t.$$

The latitude of the entrance point of the rocket into the dense layers of atmosphere will not be changed with the rotation of the Earth, and the latitude of the projection of this point on the surface of the Earth will not be changed.

The projection of the trajectory of the rocket over a revolving Earth will have the form of curve AB'. Correction for the rotation of the Earth BB' is plotted with any azimuth of the firing against its rotation, i.e., as it is shown in Fig. 71.

In the determination of time of motion of the rocket t and magnitude of section

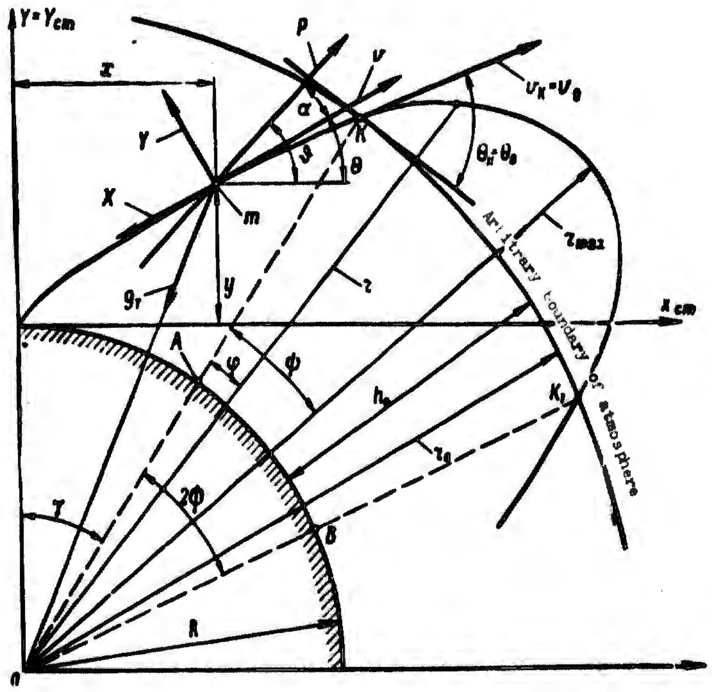


Fig. 72. Planar trajectory of a ballistic long-range rocket: section K - K_1 – elliptic part of the trajectory; $X_{CT}OY_{CT}$ – launching system of rectangular coordinates.

AB, it is necessary to use the absolute velocity of motion of the rocket, which is equal to

$$\bar{v}_a = \bar{v}_{\text{отн}} + \bar{v}_e$$

where $\bar{v}_e$ is the speed of the migratory motion of the launching point of the rocket.

Speed v_e depends on the latitude at which the rocket is launched and the azimuth of firing. The relative speed v_{OTH} should be determined at the selected point of entrance of the rocket into the rarefied layers.

The greatest mangitude of v_e, equal to the peripheral velocity of the Earth, is at the equator. With the launching of the rocket along the equator, in the direction of the rotation of Earth from west to east, to the speed v_{OTH} calculated with respect to the Earth must be added the speed equal approximately 465 m/sec. With the launching opposite the rotation of the Earth this magnitude must be subtracted from v_{OTH}.

With the launching of rockets putting into orbit our first artificial satellites to the value of v_{OTH} approximately 200 m/sec was added because of the rotation of Earth.

The common formulas (5.6) and (5.7) also permit obtaining a system of differential equations describing the flight of the ballistic rocket or its nose cone without taking into account the rotation of the Earth on the passive uncontrolled section of the flight outside the dense layers of the atmosphere.

If in the system of equations (5.6) v_z, $\dot{v}_z$, and Ω are equated to zero and members considering the tractive force and aerodynamic drag are omitted, then we will obtain the case considered corresponding to the planar trajectory of the motion. Remembering that angle $\lambda = 90 - \gamma$, and considering only the gravitation of the Earth, from formula (5.6) we obtain

$$\left.\begin{aligned}\dot{v}_x &= -g_0 \cdot \frac{R^2}{r^2} \cdot \sin\gamma;\\ \dot{v}_y &= -g_0 \cdot \frac{R^2}{r^2} \cdot \cos\gamma.\end{aligned}\right| \quad (5.9)$$

If $\eta = 90°$, then in formula (5.7)

$$\left.\begin{aligned}x &= r \cdot \sin\gamma;\\ y &= r \cdot \cos\gamma.\end{aligned}\right| \quad (5.10)$$

Twice differentiating the values of (5.10) and substituting results of the differentiation into formulas (5.9) after simple transformations we obtain:

$$\left.\begin{aligned}\ddot{r} - r\dot{\gamma}^2 &= -g_0 \frac{R^2}{r^2};\\ 2\dot{r}\dot{\gamma} + \ddot{\gamma} &= 0.\end{aligned}\right| \quad (5.11)$$

These are the known differential equations of the motion of the rocket on the unpowered section of the flight in the central gravitational field of the Earth without taking into account the drag and rotation of the Earth written in polar coordinates r and γ. The system (5. 11) is the basis of the so-called elliptic theory allowing approximately to determine the characteristic of motion of the ballistic rockets and earth satellites.

§ 2. Programmed Guided Flight

A programmed flight occurs with the observance of a change assigned beforehand of some characteristic of motion. For instance, for ballistic long-range rockets the program of the change of the pitch angle on a powered-flight trajectory is assigned ensuring the needed form of the trajectory.

Let us formulate an equation of motion describing controlled flight. For simplification we will not consider the influence of rotation of the Earth, and we will consider gravity constant in magnitude and direction. Let us assume that the roll and yawing control ensures the flight of the rocket in a vertical plane passing through the launching place and target. With the accepted assumptions it is possible to consider that forces acting on the rocket will lie in one plane, and the trajectory of the flight is a flat curve. The acceleration during curvilinear motion can be presented as the sum of tangential and normal accelerations. These accelerations are directed along the so-called natural axes of the coordinates. The tangential (tangent) acceleration is equal to $\dot{v}$ and is directed along the tangent to the trajectory. Normal acceleration, sometimes called centripetal, is directed along the normal to the trajectory center of curvature and is equal to $\frac{v^2}{\rho}$.

The curvature of the trajectory $\frac{1}{\rho}$ can be represented by the formula

$$\frac{1}{\rho} = \left|\frac{d\theta}{dS}\right|,$$

where $d\theta$ is the elementary change of the angle of inclination of the tangent to the trajectory;

dS is the elementary segment of the curve.

By substituting we will obtain the formula for the normal acceleration:

$$\frac{v^2}{\rho} = v\left|\frac{d\theta}{dt}\right|.$$

The sign of the derivative $\frac{d\theta}{dt}$ depends on the form of the trajectory. If θ decreases with the increase of the arc there will be a minus sign, if inversely then a plus sign.

As example we will consider the motion of a winged missile during launch with help of a booster. Let us compile a diagram of acting forces applied to the center of mass (Fig. 73). Projections of these forces on the tangent and normal for clarity are placed in Table 2.

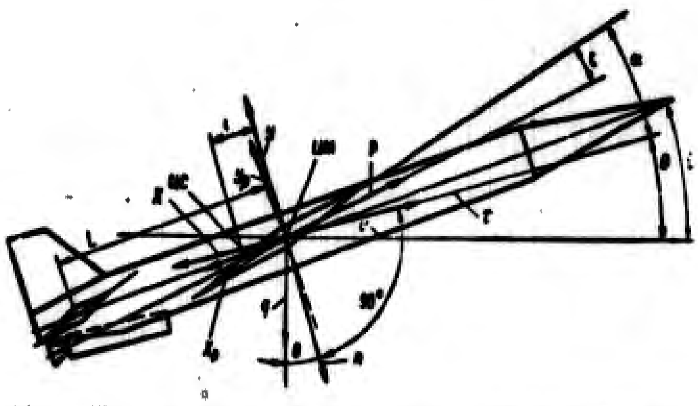


Fig. 73. Diagram of the action of forces with the flight of the winged missile under the action of a booster: τ – direction of the tangent to the trajectory; n – direction of the normal to the trajectory.

Designations of forces and angles are taken from Fig. 73. Control forces are brought to the center of mass [TsM] (ЦМ) and forces couples are not shown on the figure. Sign before the projection shows the direction of the component force. If the direction of componenet coincides with the direction of the axis there is a plus sign, if does not coincide, a minus sign.

Table 2.

Force	Tangent component (projection on axis τ)	Normal component (projection on axis n)
Tractive force	$P\cdot\cos(\alpha - \xi)$	$-P\cdot\sin(\alpha - \xi)$
Drag	X	—
Lift	—	$-Y$
Component of force of weight	$G\cdot\sin\Theta$	$G\cdot\cos\Theta$
Loss of thrust on the surface control	$-X_p\cdot\cos(\vartheta - \Theta)$	$+X_p\cdot\sin(\vartheta - \Theta)$
Control forces	$-Y_p\cdot\sin(\vartheta - \Theta)$	$-Y_p\cdot\cos(\vartheta - \Theta)$

In Table 2 by X_p and Y_p the sum of forces given by the monotypic control systems, is implied and by P the total thrust of all the engines.

Let us write the equation of the motion in projections on the tangent and normal:

$$\left.\begin{aligned} m\dot{v} &= P\cdot\cos(\alpha-\xi) - X - q\cdot\sin\theta - \\ &- X_p\cdot\cos(\vartheta-\theta) - Y_p\cdot\sin(\vartheta-\theta); \\ mv\dot{\theta} &= P\cdot\sin(\alpha-\xi) + Y - q\cdot\cos\theta - \\ &- X_p\cdot\sin(\vartheta-\theta) + Y_p\cdot\cos(\vartheta-\theta). \end{aligned}\right\} \quad (5.12)$$

On the right side of the latter equation the signs are changed to the opposite as compared to the table, because the derivative $\dot{\theta}$ with the decrease of angle θ has a minus sign.

In the equation of rotary motion we will consider the following moments.

Moment of aerodynamic forces M_a which is approximately equal to the product of $Y\cdot l$, where Y is the lift and l the distance between the center of mass TsM and center of drag [TsS] (ЦС) (Fig. 73). The moment of control forces is M_p, which is approximately equal to the product $Y_p\cdot L$, where L is the distance from the center of mass of the winged missile.

Certain rockets have several controls. For instance, the rocket of the "V-2" type (Fig. 1) has jet and air vanes. In this case in equations of motion, instead of the control forces and moments, there should be the sum of forces and moments: $\sum X_{pi}$, $\sum Y_{pi}$, $\sum M_{pi}$, etc. Also the tractive forces and moments from them for different simultaneoulsy operating engines should be summarized. The equation of rotary motion will have the form

$$J\ddot{\vartheta} = M_a + M_p \quad (5.13)$$

To the three written equations must be added two evident kinematic equations

$$\left.\begin{aligned} \dot{x} &= v\cdot\cos\theta; \\ \dot{y} &= v\cdot\sin\theta \end{aligned}\right\} \quad (5.14)$$

and equation determining the program of the flight.

Numerical values of projections of all forces on normal axes of the coordinates depend not only on the construction and dimensions of the aircraft, but also on the magnitude of angles α, ξ, θ and ϑ. Since the booster prior to the moment of its separation from the winged missile is rigidly secured, the angle ξ does not change during the flight; angles α and θ change and could determine the characterisitics of the motion, but a reliable measurement of them presents great difficulties.

The pitch angle ϑ is changed relatively simply and reliably enough in flight. Therefore it is selected as an element determining the program of the flight in a vertical plane.

The form of the program $\vartheta = f(t)$ for the pitch angle depends on the construction of the rocket, form of trajectory, and method of control. Figure 72 shows the trajectory of a ballistic rocket; Fig. 74 gives trajectories of the motion of a winged missile and winged gliding rocket. The winged missile, as a rule, starts from slanted directing then passes to a horizontal flight at a constant altitude up to the region of the target where it dives on the target. The altitude of the flight is set beforehand. Therefore the pitch angle ϑ during the climb changes smoothly from the initial value to $\vartheta \approx 0$. The change of the pitch angle depends on the barometric pressure of the ambient air changing with altitude.

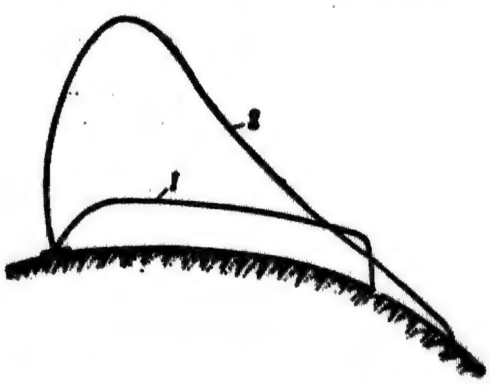


Fig. 74. Trajectory of the flight: 1 – of a winged missile; 2 – of a winged gliding rocket.

After the separation of the booster the horizontal flight of the winged missile is carried out under the action of a sustainer engine. Let us assume that the thrust vector of the sustainer engine is directed along the longitudinal axis of the body of the winged missile, then the angle $\xi = 0$ (Fig. 73). Furthermore, for the horizontal flight $\vartheta = \theta = 0$ and $\dot{\theta} = 0$ and from formula (5.12) we obtain the system of equations describing the horizontal flight. When $y = H = \text{const}$

$$\left.\begin{aligned} m\dot{v} &= P-(X+X_{\text{упр}});\\ Y \pm Y_{\text{упр}} &= q. \end{aligned}\right\} \qquad (5.15)$$

When $m = \text{const}$, if $P > X + X_{\text{упр}}$, the speed will be increased; if $P < X + X_{\text{упр}}$ the speed will decrease. For a flight with a constant speed equality $P = X + X_{\text{упр}}$ is necessary. Since during the flight m and q decrease owing to the burnout of fuel, then for the observance of the constancy of the altitude of the flight it is necessary to change due to the change of the angle of incidence of $Y_{\text{упр}}$ and Y connected among themselves.

The long-range ballistic rocket takes off vertically with a further gradual decrease of the pitch angle (Fig. 72). Toward the end of the controlled flight a small rectilinear inclined section is set on which the engine will be turned off when the rocket achieves the calculated speed. Turning off the engine on the rectilinear section decreases the influence of perturbations connected with the shutdown of the engine on the deflection of the rocket from the calculated trajectory. The angle of inclination of the rectilinear section to the horizon, together with the initial speed, determines the required distance of firing. An example of the calculation curve of the change of the pitch angle of the ballistic rocket is shown in Fig. 75.

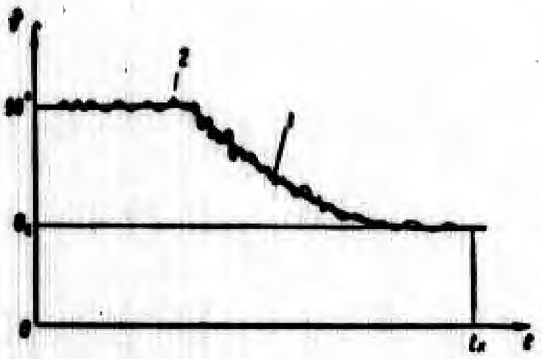


Fig. 75. Calculated program of the change of the pitch angle with placing of the ballistic long-range rocket on the uncontrolled part of the trajectory: 1 — ideal curve; 2 — example of recording of the real change of the pirch angle in the process of flight.

The equation of the curve $\vartheta_{np} = f(t)$ should ensure a smooth transition from $\vartheta = 90^{\circ}$ to $\vartheta = \vartheta_{K}$. Let us formulate a system of equations describing the controlled flight of a ballistic rocket on a powered-flight trajectory. Let us consider the two-dimensional problem, and we will not consider the influence of the rotation of the Earth. Control forces Y_p and X_p will not be considered separately but will be included in drag and lift. We will direct the tractive force along the longitudinal axis of the rocket. Let us formulate the system of equations in projections on the axis of rectangular starting system of the coordinates (Fig. 72). The projection of the velocity vector on the X axis will be designated by the letter u and the projection of it on the Y axis by the letter w. We denote projections of acceleration of terrestrial gravitation on the same axis g_{TX} and g_{TY} respectively:

$$g_{\tau x} = g_{\tau} \cdot \sin \gamma;$$
$$g_{\tau y} = g_{\tau} \cdot \cos \gamma.$$

Having written equation of motion in projections on the selected axes of the coordinates and having added the usual kinematic and trigonometric relationships evident from Fig. 72, we obtain the system of equations

$$\left.\begin{array}{l} \frac{du}{dt} = \frac{P \cdot \cos(\Theta + \alpha) - X \cdot \cos\Theta - Y \cdot \sin\Theta}{m} - g_{\tau} \cdot \sin\gamma; \\ \frac{dw}{dt} = \frac{P \cdot \sin(\Theta + \alpha) - X \cdot \sin\Theta + Y \cdot \cos\Theta}{m} - g_{\tau} \cdot \cos\gamma \\ \frac{dx}{dt} = u; \; \frac{dy}{dt} = w; \; \mathrm{tg}\,\Theta = \frac{w}{u}; \; \mathrm{tg}\,\gamma = \frac{x}{R+y}. \end{array}\right\} \quad (5.16)$$

If we add to the written equations an equation determining the program pitch angle $(\alpha + \theta) = \vartheta_{пр} = \vartheta(t)$, and an equation determing the change of the mass of the rocket $m(t)$, then the system can be solved by numerical integration.

For winged rockets, for instance, for the A-4B rocket of former German army, the initial section of the motion had a trajectory similar to the trajectory of the free flight such a winged rocket should pass into conditions of a gradual gliding descent (Fig.74). In this case the program should ensure takeoff with a gradual decrease of the pitch angle. With the achievement of an optimum altitude the change of the pitch angle should ensure a gradual transition to conditions of a gliding descent.

The system of equations describing the gliding descent of a rocket with a constant pitch angle can be obtained from the equation (5.12) and (5.14). Let us take, as also for the horizontal flight, $\xi = 0$ (Fig. 73), $-\vartheta = -\theta = \text{const}$ and $\dot{\theta} = 0$. The minus sign shows that ϑ and θ are read off from the horizontal downwards. Then

$$m\dot{v} = P - X + q\cdot\sin\theta - X_p;$$
$$Y \pm Y_p = q\cdot\cos\theta;$$
$$\dot{x} = v\cdot\cos\theta;\ \dot{y} = v\cdot\sin\theta.$$

The equations of the controlled flight with the engine turned off can easily be obtained from preceding equations if the thrust $P = 0$.

During the flight accidental perturbations and the work of the whole control system lead to oscillations of the longitudinal axis of the rocket and, consequently, to the deviation of the pitch angle from the program values (Fig. 75). During well working control the pitch angle insignificantly deviates from the calculated flight determined by program.

The program of the flight can be established not only by the pitch angle the form of trajectories, or altitude of the flight. The change of speed with time $v(t)$ the normal overloads or some other characteristics of motion can be assigned beforehand. Programming can be carried out not only in a vertical plane but also in a horizontal plane and also for space trajectories.

The selection of the program of flight, taking into account all the requirements given to the rocket, is one of serious stages of designing.

§ 3. Controlled Flight of Rockets Designed for Combat with Fast-Moving Targets

Trajectories of the flight of guided rockets designed for combat with fast-moving targets are complicated space curves. The form of the trajectory depends on the type of launch, character of the target, characteristics of its motion, method of guiding the rocket to the target, mutual location of the target and launcher, on the direction and magnitude of vectors of speeds of the rocket and target before encounter, and on other causes.

The ballistic calculation should give all the basic parameters of the trajectory proceeding from which it is possible to judge the necessary characteristics of the control system and rocket complex on the whole. Usually the calculation is carried out in several approximations. At first we establish the characteristics of the trajectory of the center of masses of the rocket, taking into account the fundamental equations of the dynamics of the flight and kinematic dependences ensuing from the assumed method of guiding. With this the expediency of the selected method of guiding to the target and the possibility of striking the target are established, and the curvature of the trajectory and the normal accelerations of rocket are determined. In the process of the designing and manufacture of a rocket ballistic calculations are repeated with the introduction into them of new data on the rocket and the system of its stabilization and control. For correctly stabilized and well-guided rockets the real trajectory is close to the calculated trajectory.

The carrying out ballistic calculations in complete volume is a complicated and laborious matter. It is possible, however, using kinematic methods of investigations of trajectories to find certain properties of them, allowing an initial judgement on the acceptibility of a certain method of guiding and the establishment of the boundaries of initial data within which it is necessary to carry out more exact solutions.

With purely kinematic methods of investigation, kinematic equations determining the direction of the velocity vector of the rocket are solved apart from equations describing the dynamics of motion. The necessity to have beforehand the dependence of the rocket on time $v_p(t)$ limits the applicability of such methods. For many types of rockets considering the changeability in the process of motion of the mass of the rocket, tractive force, and aerodynamic forces, it is impossible to obtain $v_p(t)$ without a solution to problems of dynamics of motion of the rocket. In most cases kinematic methods permit having analytic solutions utilized with investigation of the motion of homing rockets. The speed of the rocket is most frequently taken constant.

For the convenience of consideration of complicated trajectories it is expedient to divide them into three basic phases. The first will considered the launching phase on which the flight is usually uncontrolled. In the second phase the control system enters and the rocket is put into a trajectory corresponding to the selected method of guiding to the target. The third phase is the guiding phase and precedes the encounter of the rocket with the target.

Characteristics of motion on each of the phases depend on many above-mentioned factors. In the first (launching) phase the control, as a rule, does not work and the calculation of trajectory can be conducted from the system of equations describing the flight of unguided rockets.

The calculation of the trajectory of the second phase, the phase of the transition from free flight to controlled is the most complicated. However it effects the guidance path preceding the encounter of the rocket with a target very little . In this section we will consider the plotting of only the last phases of guidance paths corresponding to the assigned method of guiding.

For rockets striking fast moving targets command guidance, homing guidance, and combined methods of guidance control are used abroad. One of the characteristics peculiarities of every guidance system is the method inherent to it of the guiding to the target. Two methods are well-known: guiding to future position and guiding by the method of combination. The latter method is used only with command methods of guidance. Guiding to future position is used both with command methods and homing guidance.

The combination of different methods of guidance are known.

With future position guiding the following principles determing the magnitude of the lead angle are applied: guiding with a constant lead angle, consecutive lead, guiding with parallel and proportional approach, and guiding with a zero lead angle by the so-called linear curve.

Let us consider the guidance path of rockets corresponding to the named principles of guidance.

Future Position Guidance

Future position firing is the basic method of the firing of trunk artillery at fast-moving targets and is well developed. As it is known the position of the future is selected in such a manner that during the time of the motion of the missile the target also shifts to the future position (Fig. 76).

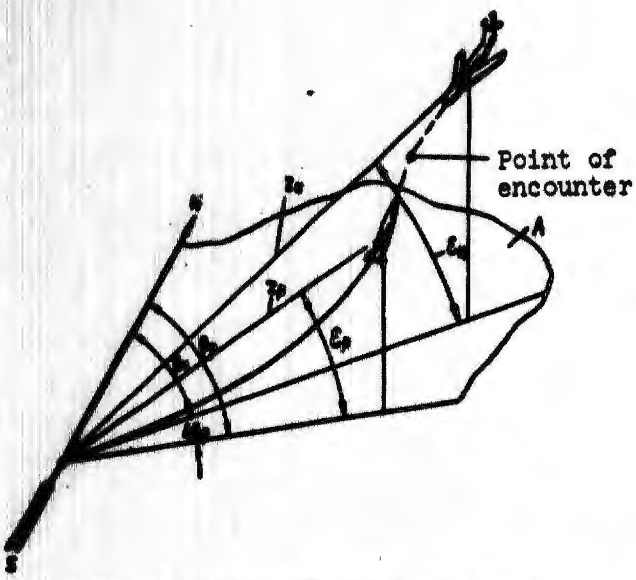


Fig. 76. Position of the rocket and target before encounter: A – horizontal plane; $\varepsilon_{\text{ц}}$ – angle of elevation of target; ε_p – angle of elevation of the rocket; $\beta_{\text{ц}}$ – azimuth of target; β_p – azimuth of rocket.

If with command guidance we designate by $\varepsilon_{\text{ц}}$ and ε_p angles of elevation of the target and rocket and by $\beta_{\text{ц}}$ and β_p their azimuths, then the connection between these angles will be determined from Fig. 76 by the simple equalities

$$\varepsilon_p = \varepsilon_{\text{ц}} + \Delta\varepsilon_p; \quad (5.17)$$

$$\beta_p = \beta_{\text{ц}} + \Delta\beta_p \quad (5.18)$$

In the process of guiding lead angles $\Delta\varepsilon_p$ and $\Delta\beta_p$ are variable. There are many formulas in which lead angles are given. In common form their functional dependences can be expressed thusly:

$$\Delta\varepsilon_p = f(r_{\text{ц}}, r_p, \dot{r}_{\text{ц}}, \dot{r}_p, \varepsilon_{\text{ц}}, \varepsilon_p, a, b \ldots); \quad (5.19)$$

$$\Delta\beta_p = f_1(r_{\text{ц}}, r_p, \dot{r}_{\text{ц}}, \dot{r}_p, \varepsilon_{\text{ц}}, \varepsilon_p, a_1, b_1 \ldots), \quad (5.20)$$

where $r_{\text{ц}}$, r_p are radii-vectors of the target and rocket; $\dot{r}_{\text{ц}}$, $\dot{r}_p$ are speeds of the change of radii-vectors in the process of guiding; a, b, a_1, b_1, etc., are constants characteristic for the given system of control.

The concrete form of the functional dependence is determined with the designing of a whole rocket complex. It is possible to indicate only two general requirements. The direction of the motion of the rocket should change smoothly with a possibly great approach of the trajectory to rectilinear. It is also natural that magnitudes of the current angular leads $\Delta\varepsilon_p$ and $\Delta\beta_p$ should decrease in the process of guiding and when $r_p = r_{\text{ц}}$ should be $\Delta\varepsilon_p = \Delta\beta_p = 0$; otherwise, the rocket will fly wide of the mark.

With a correct launch of the rocket and rectilinear motion of the target with a constant speed the ideal trajectory of the flight of a rocket is close to the trajectory of free flight. Therefore, in the first approximation characteristics of the motion of the rocket can be determined by a solution to the system of equations describing free flight with independent variable time. Having calculated the family of trajectories with different initial angles of departure θ_0 and by knowing the characteristics of motion of the target we can obtain the values of initial lead angles. In reality the trajectory of motion of the rocket will be somewhat distorted owing to the maneuver of the target and other causes, and the assumed instantaneous point of encounter will change its position. Such a very complicated calculation can be made only by having data on the whole rocket complex.

Let us now consider the kinematics of motion of the target and rocket on the last phase of the path before their encounter on the trajectory of future guiding position. We will assume that the target and rocket move in one plane.

The relative position of the rocket and target will be determined by the distance r (we will call it the radius-vector) and the two angles α and β between the instantaneous velocity vectors of the rocket and target and line of sight of coinciding with r (Fig. 77). Angle α determines the lead. For the orientation of the line of sight with respect to the motionless system of coordinates we introduce the angle γ. Let us designate as before the angle of inclination of the velocity vector of the rocket to the horizontal line in terms of

$$\theta = \gamma + \alpha.$$

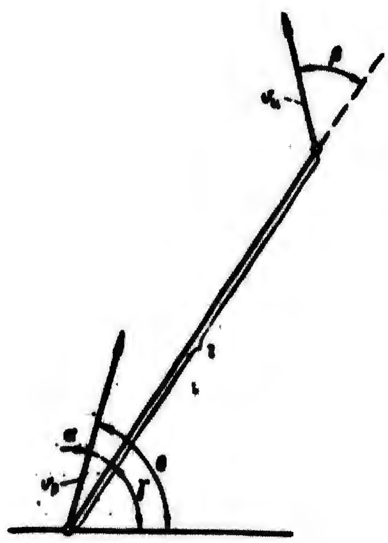


Fig. 77. Diagram of angles determining the relative position of the rocket and target.

The change in distance between the rocket and target during an infinitesimal interval of time is equal to the difference of projections of speed $v_{ц}$ and v_p toward the direction of the vector r:

$$\frac{dr}{dt} = v_{ц} \cdot \cos\beta - v_p \cdot \cos\alpha \tag{5.21}$$

The minus sign before the second component shows that with the increase of speed v_p the distance r decreases. The speed of the change of angle γ will be

$$\frac{d\gamma}{dt} = \frac{v_{ц} \cdot \sin\beta - v_p \cdot \sin\alpha}{r} \tag{5.22}$$

Normal accelerations of the target and rocket are determined by the product of the tangents and angular velocities:

$$a_{nц} = v_{ц}\left(\frac{d\beta}{dt} + \frac{d\gamma}{dt}\right); \tag{5.23}$$

$$a_{np} = v_p\left(\frac{d\alpha}{dt} + \frac{d\gamma}{dt}\right). \tag{5.24}$$

Let us consider the case of the constant lead angle $\alpha = \alpha_0$ = const and rectilinear motion of the target.

It is obvious that for these conditions $a_{n\text{ц}} = 0$, i.e.,

$$\frac{d\beta}{dt} + \frac{dq}{dt} = 0,$$

since $v_{\text{ц}} \neq 0$, and

$$\frac{d\beta}{dt} = \frac{v_{p} \cdot \sin\alpha_0 - v_{\text{ц}} \cdot \sin\beta}{r}. \qquad (5.25)$$

Factorying out $v_{\text{ц}}$ on the right sides and designating

$$p = \frac{v_p}{v_{\text{ц}}},$$

we obtain

$$\frac{d\beta}{dt} = \frac{v_{\text{ц}}(p \sin\alpha_0 - \sin\beta)}{r}. \qquad (5.26)$$

The condition of the ideal lead during which the encounter of the rocket with the target is ensured has the form

$$v_p \cdot \sin\alpha = v_{\text{ц}} \cdot \sin\beta. \qquad (5.27)$$

For the considered case $\alpha = \alpha_0 = \text{const}$ we will designate β in equality (5.27) in terms of β_B calling it the angle of obligatory encounter:

$$\sin\beta_B = p \cdot \sin\alpha_0. \qquad (5.28)$$

Then

$$\frac{d\beta}{dt} = v_{\text{ц}} \cdot \frac{(\sin\beta_B - \sin\beta)}{r}. \qquad (5.29)$$

Taking in expression (5.21) $\alpha = \alpha_0$ and dividing the expression (5.21) by the expression (5.29) we get

$$\frac{dr}{r} = \frac{\cos\beta - p \cdot \cos\alpha_0}{\sin\beta_B - \sin\beta} \cdot d\beta. \qquad (5.30)$$

Let us investigate the possible conditions of encounter of the rocket with the target with the accepted method of guiding. For guaranteeing the encounter r should decrease, and this means that

$$\frac{dr}{dt} < 0. \qquad (5.31)$$

If one were to proceed from the expression (5.21) then this inequality should be ensured

$$\cos\beta < p \cdot \cos\alpha_0.$$

Using the condition of the ideal lead (5.27) we obtain

$$p^2(1 - \sin^2\alpha_0) > 1 - p^2 \cdot \sin^2 z_0$$

Consequently, with the considered method of guiding, for a guarantee of the rocket hitting the target it is necessary to have $p > 1$, i.e., the speed of the rocket should be greater than the speed of target. Moreover, with definite p and α_0 for the guarantee of the equality (5.28) the value of β_B corresponding to it should be kept in the guidance process. It is obvious that in real conditions with the maneuver of target this is not observed, and consequently, the reliability of the encounter of the rocket with the target is not ensured.

The cases are of practical interest when

$$p > 1,\ \alpha_0 < 90^\circ \text{ and } \sin\alpha_0 < \frac{1}{p}.$$

For these conditions let us find the possible magnitudes of normal accelerations determining the overloads of the rocket with its approach to the target.

The normal acceleration of the rocket is determined by formula (5.24). Remembering that for the accepted conditions $\frac{d\alpha}{dt} = 0$, and $\frac{d\gamma}{dt}$ is found by the formula (5.22), we obtain

$$a_{np} = v_p v_a \left(\frac{\sin\beta - p\cdot\sin\alpha_0}{r}\right).$$

We determine the dependence r on β by integrating the expression (5.30):

$$\int_{r_0}^{r} \frac{dr}{r} = \int_{\beta_0}^{\beta} \frac{\cos\beta - p\cdot\cos\alpha_0}{\sin\beta_B - \sin\beta}\cdot d\beta.$$

Converting this expression after the integration we get

$$a_{np} = \frac{2v_p^2}{pr_0}\left(\sin\frac{\beta-\beta_B}{2}\right)^{2-k}\cdot\left(\cos\frac{\beta+\beta_B}{2}\right)^{2+k}\times$$
$$\times\left(\sin\frac{\beta_0-\beta_B}{2}\right)^{-1+k}\left(\cos\frac{\beta_0+\beta_B}{2}\right)^{-1-k}, \qquad (5.32)$$

where

$$k = p\cdot\frac{\cos\alpha_0}{\cos\beta_B}.$$

For a guarantee of the encounter of the rocket with the target β should tend to β_B. Then the limiting accelerations will be:

when $k < 2$ $\lim a_{np} = 0$;
when $k > 2$ $\lim a_{np} = \infty$;
when $k = 2$ $0 < \lim a_{np} < \infty$.

It is obvious that only the case $k \leq 2$ can be used practically.

An analysis of formula (5.32) gives the region of possible values of p as a function of α_0 (Fig. 78). As can be seen from the figure the region of theoretically possible conditions of encounter with homing guidance with a constant lead angle is extremely limited.

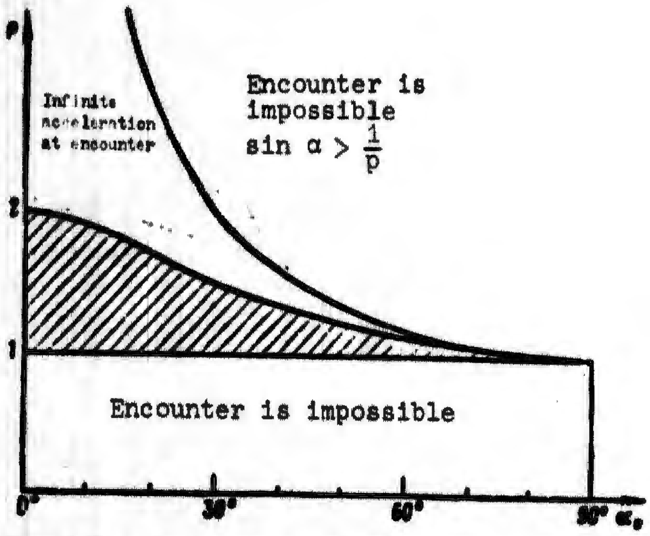


Fig. 78. Dependence of the relative speed of the rocket on constant lead angle. The shaded area corresponds to conditions of encounter of the rocket with the target with a finite magnitude of normal accelerations.

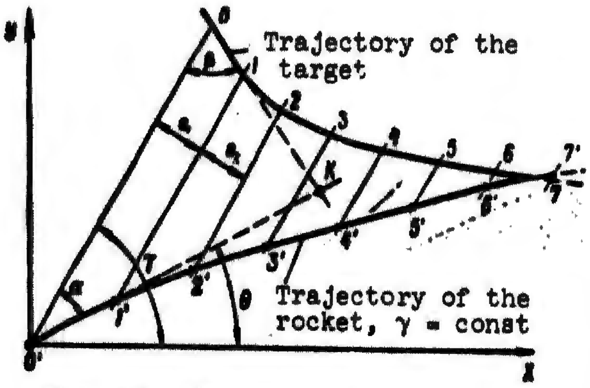


Fig. 79. Approximate plotting of the trajectory of a rocket guided by the method of consecutive leads (parallel approach): K — conditional point of encounter if the target and rocket maintain their direction of motion and speed which they had in first phase (in the interval between straight lines 0-0' and 1-1').

If we were to apply the condition of an ideal lead to the formula (5.22), then we will obtain $\frac{d\gamma}{dt} = 0$, and this means that in the guidance process the line of sight will automatically remain parallel. Such a method of lead guiding obtained the name of the method of parallel approach. With the rectilinear motion of the target the rocket will fly along a direct line.

With the more general case of the curvilinear motion of the target and parallel displacement of the line of sight the lead angle α cannot remain constant and should change depending upon the change of angle β. Considering the changeability of the angle such a method of guiding is sometimes called the method of consecutive leads. With the known characteristics of the trajectory of the motion of the target the approximate graphic plotting of the trajectory of the rocket causes no difficulties. The method of plotting is shown in Fig. 79. Plotted on the trajectory of the motion of the target are positions of it, 0, 1, 2, 3, ..., corresponding to the intervals of time Δt, and from every point straight lines are conducted parallel to the direction of the initial angle of sighting. Plotted on the initial line of sight 0 - 0' is the position of the center of mass of the rocket (point 0') and from it on the line of sight 1 - 1' an intersection is made by dividers. The opening of the divider is established proceeding from the condition of the ideal lead (5.27). Taking approximately for the first section

$$\sin\beta_{(0)} \approx \frac{\bar{a}_1}{0-1} \text{ and } \sin\alpha_{(0')} \approx \frac{\bar{a}_1}{0'-1'}.$$

we will obtain the length of the path of the rocket for the first segment

$$\overline{0'-1'}=\overline{0-1}\cdot\frac{v_{p1}}{v_{u1}}.$$

This dimension will be equal to the opening of the divider. From point 1' is made mark 2' on line 2 - 2' by the dimension

$$\overline{1'-2'}=\overline{1-2}\cdot\frac{v_{p2}}{v_{u2}}$$

etc. Plotting continues up to the crossing of trajectories of the target and rocket. The average speeds of the target and rocket on each of the segments can be defined as the semisum of speeds in the beginning and end of the considered segment. The necessity to have beforehand the dependence of the speed of the rocket on time $v_p(t)$ limits the applicability of the graphic method of plotting the trajectory.

Now we will formulate a system equations approximately describing the considered case of the motion of the rocket in a vertical plane. Like the preceding system, considering the rocket ideally controlled, i.e., where the axis of it coincides with the velocity vector we use the equation of motion

$$m\frac{dv_p}{dt}=P-X-mg\cdot\sin\theta.$$

From Fig. 79 one can see that $\theta = \gamma - \alpha$, where γ is the magnitude of the constant dependent on the initial position of the target and rocket, and α is determined under the condition of an ideal lead (5.27). Using the usual kinematic dependences

$$u=v_p\cdot\cos\theta \text{ and } w=v_p\cdot\sin\theta$$

and the condition of the ideal lead, we obtain the system consisting of three differential equations and one trigonometric:

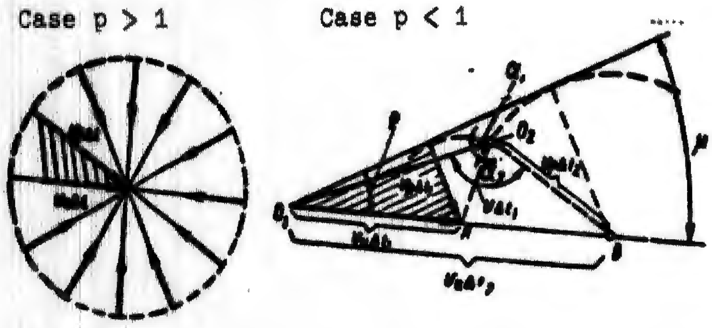
$$\left.\begin{aligned} m\cdot\frac{dv_p}{dt}&=P-X-mg\cdot\sin(\gamma-\alpha);\\ \frac{dx}{dt}&=u;\\ \frac{dy}{dt}&=w;\\ \sin\alpha&=\frac{v_u}{v_p}\cdot\sin\beta. \end{aligned}\right| \quad (5.33)$$

With the known characteristics of the motion of the target (v_u and β) the system is easily solved by numerical integration.

The method of calculating the trajectory during the approach of a rocket with a target in an inclined plane will be considered below.

To establish the possibilities of the considered method of guiding let us conduct kinematic investigation of conditions of encounter with constant speeds of

Case p > 1 Case p < 1



a b

Fig. 80. Diagram of the interception of a rectilinearly moving target when p = const and with the parallel method of approach.

the target and rocket. For the case p > 1 the rectilinearly moving target can be intercepted at any relative position of the rocket and target during the time Δt, inasmuch as on angles α and β only one limitation of the ideal lead is superimposed (Fig. 80a). In the case p < 1 the region of positions of the rocket at which the interception of the target is possible is considerably reduced. To guarantee the interception the equality $v_p \Delta t_2 = v_ц \Delta t_1$, should be kept as before; however, the rocket before the interception should be found in space bounded by a cone the angle of which (Fig. 80b) $\mu = \arcsin p$. If in the initial position the rocket is found on the surface of the cone with angle μ, then the only direction ensuring interception will be the direction of the motion perpendicular to forming cone. If the rocket is inside the cone different initial lead angles from α_1 to α_2 are possible, and extreme trajectories of the motion of the rocket $\overline{O_2A}$ and $\overline{O_2B}$ will be rectilinear. Trajectories of the motion of the rocket lying between O_2A and O_2B for the guarantee of encounter should be curvilinear. With the maneuvering of the target the region in which its interception is possible still decreases. This question required additional study.

Let us see how the normal accelerations of the rocket will change with the considered method of approach. It is obvious that with the rectilinear uniform motion of the target $a_{np} = 0$, and only cases of the maneuvering of the target are of interest. Since according to the basic condition of guidance $\frac{d\gamma}{dt} = 0$, that

$$a_{nц} = v_ц \cdot \frac{d\beta}{dt} \text{ and } a_{np} = v_p \cdot \frac{d\alpha}{dt}. \qquad (5.34)$$

Differentiating the condition of the ideal lead and substituting in it the value of (5.34), we obtain the connection between normal accelerations of the rocket and the target:

$$a_{np} = a_{nц} \cdot \frac{\cos\beta}{\cos\alpha}.$$

After simple transformations we get

$$a_{np} = a_{nn}\frac{\sqrt{1-\sin^2\beta}}{\sqrt{1-\frac{\sin^2\beta}{p^2}}}.$$

It is obvious that for the case $p > 1$ normal accelerations of the rocket will be less than the normal accelerations of the target.

For the case $p < 1$ if the rocket before the approach is on the boundary of the region of possible interception, on the generating cone with angle μ (Fig. 80b), then

$$\alpha = \frac{\pi}{2},\ \cos\alpha = 0,$$

$$\frac{a_{np}}{a_{nn}} = \infty.$$

Obviously such a case cannot be used. If we were to designate the practically acceptable relation of accelerations in terms of

$$k = \frac{a_{np}}{a_{nn}},$$

then the initial angle β_0 should be equal to

$$\beta_0 = \arcsin\sqrt{\frac{p^2(k^2-1)}{k^2-p^2}}.$$

It is obvious that $\beta_0 < \mu$, and this means that the region of rational positions of the rocket before its approach to a target is less than the region determined with the condition of the possibility of interception.

Let us consider further the method of guiding to a target with the alternate lead angle α but determined by some dependence assigned beforehand. Widely known is the use of the linear dependence between the angular velocity of rotation of the velocity vector of the center of mass of the rocket and the angular velocity of rotation of the line of sight when

$$\frac{d\theta}{dt} = a\cdot\frac{d\gamma}{dt}.$$

This method of guiding to a target received the name of proportional approach. Using equation (5.22) we obtain

$$\frac{d\theta}{dt} = a\cdot\frac{v_u\cdot\sin\beta - v_p\cdot\sin\alpha}{r}.$$

According to Fig. 77 $\theta = \alpha + \gamma$, then

$$\frac{d\alpha}{dt} = (a-1)\frac{v_u\cdot\sin\beta - v_p\cdot\sin\alpha}{r}.$$

This equation in final form is not integrated, and it is necessary to solve it numerically jointly with equation (5.21). We can investigate the possible variants of the encounter of a rocket with a target only after the calculation of the family of trajectories responding to the concrete assignment. For the simple case of the horizontal rectilinear motion of the target when a = 2 and p = const, the normal acceleration of the rocket remains final under the condition

$$p \cdot \cos \theta_0 \geq -1.$$

Future position guiding is widely used both with command methods of control of the rockets and with homing guidance. The selection of the lead method depends on the assignment of the rocket and is established with the designing of the entire complex as a whole.

Linear Curve Guidance

Let us consider as earlier the motion in a vertical plane of passing through the beginning of an ideal trajectory of guidance.

According to the principle of the plotting of a linear curve at every moment of time the velocity vector of the center of mass of the rocket should be directed towards the target, i.e., guidance with a zero lead angle is carried out. With the known characteristics of the trajectory of the motion of the target the graphic plotting of the trajectory of the rocket is also very simple (Fig. 81). By plotting on the trajectory of the target the positions of it, 1, 2, 3, ..., corresponding to the intervals of time Δt, and by plotting in the direction of the target consecutively vectors $v_{pi}\Delta t$, where v_{pi} is the averaged speed of the rocket for the given interval time, we obtain the trajectory of the motion of the rocket. The average speed v_{pi} can be determined as the semisum of speeds in the beginning and end of the considered segment. With the large scale of plotting and small intervals of time Δt, i.e., with a large number of points the curve can be plotted with an accuracy sufficient for solution of certain practical problems.

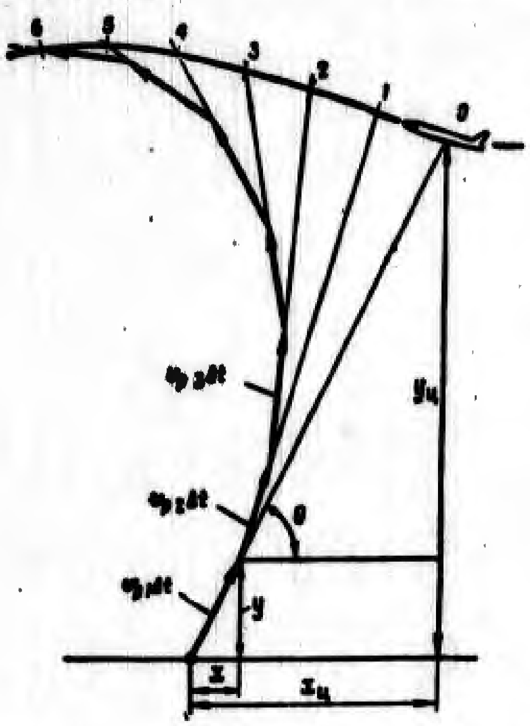


Fig. 81. Diagram of the interception of a target in a vertical plane by a linear curve.

Now we will formulate a system of equations approximately describing the considered case of motion of an ideally guided rocket. Similarly to the preceding we use the earlier written equation of motion. In this equation for the method of linear curve guiding the angle θ will be determined from the kinematic dependence

$$\operatorname{tg}\theta=\frac{y_{ц}-y}{x_{ц}-x}.$$

Differentiating this equation we get

$$\frac{d\theta}{dt}=\frac{\cos^2\theta}{(x_{ц}-x)^2}\left[(x_{ц}-x)(w_{ц}-w)-(y_{ц}-y)(u_{ц}-u)\right],$$

where $w_{ц}=\frac{dy_{ц}}{dt}$ is the vertical component of the speed of the target; $u_{ц}=\frac{dx_{ц}}{dt}$ is the horizontal component of the speed of the target: $w=\frac{dy}{dt}$ and $u=\frac{dx}{dt}$ are the vertical and horizontal components of the speed of the rocket respectively.

Using the usual kinematic dependences

$$u=v_p\cdot\cos\theta \text{ and } w=v_p\cdot\sin\theta,$$

we obtain the system from four equations:

$$\left.\begin{aligned} m\cdot\frac{dv_p}{dt}&=P-X-mg\cdot\sin\theta;\\ \frac{d\theta}{dt}&=\frac{\cos^2\theta}{(x_{ц}-x)^2}\left[(x_{ц}-x)(w_{ц}-w)-\right.\\ &\left.-(y_{ц}-y)(u_{ц}-u)\right];\\ \frac{dx}{dt}&=u;\\ \frac{dy}{dt}&=w. \end{aligned}\right\} \quad (5.35)$$

As usual

$$v_p=\sqrt{u^2+w^2}.$$

The system can be solved by methods of numerical integration. The system is universal and is useful for any initial angles of departure and speeds. The readings of angles θ should be done, as usual, counterclockwise. With the help of the proposed method the motion of the rocket in an inclined plane passing through the axis OX can be investigated. Considering that the study of the spatial motion of the rocket in coordinates X, Y, Z requires a complicated system of equations, we will assume that maneuver of the target and rocket is carried out in one plane. Then the fundamental equation of the motion in plane B (Fig. 82) will have the form

$$m\cdot\frac{dv_p}{dt}=P-X-mg\cdot\sin\varphi,$$

where φ is the angle between the velocity vector at the given point and horizontal plane.

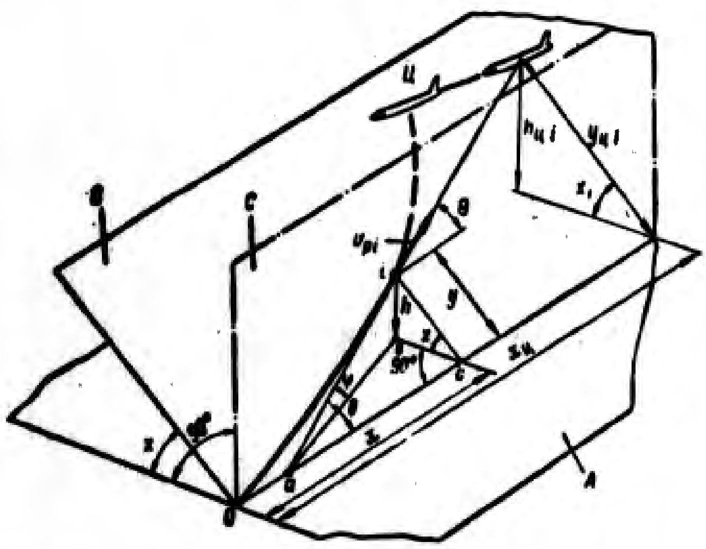


Fig. 82. Diagram of the interception of a target in an inclined plane by a linear curve.

Let us determine sin φ in terms of the angle θ variable in the process of motion, lying in the plane of interception, and the assigned angle χ determining the inclination of the actual plane of the intercention of the horizon.

From the mutually connected triangles abc, abi, and bci we obtain

$$\sin\varphi = \sin\theta \cdot \sin\chi.$$

It is natural that when $\chi = 90^\circ$ $\sin\varphi = \sin\theta$, and when $\chi = 0$ $\sin\varphi = 0$. The other three kinematic equations of the system (5.35) will remain without change. Thus for the calculation of trajectories of the guidance path of a rocket in a plane determined by angle χ, we obtain the system of equations differing from the system (5.35) in that in the first equation sin θ will be replaced by φ.

The numerical solution of such a system does cause no difficulties, since φ depends on θ and the constant χ. With the calculation of P and X one should consider that they depend not on y, as in the system (5.35), but on the real altitude $h = y \cdot \sin\chi$. This system can also be used for an approximate calculation of a spatial ideal trajectory of the guiding of the rocket with any maneuver of the target. For such a calculation it is necessary to assume additionally that at each step of integration the instantaneous plane of maneuver of the target is retained. The position of the plane is determined by angle χ_1 constant for the given step of integration.

Angle χ_1 is determined in terms of the ratio

$$\sin\chi_i = \frac{h_{ц i}}{y_{ц i}}, \qquad (5.36)$$

where $h_{ц i}$ and $y_{ц i}$ are targets coordinates determined for the given step of integration (Fig. 82).

As an example we will give results of the solution of the system (5.35) for initial conditions of the problem taken from the book [34].

Initial conditions.[1] The speed of the target flying horizontally toward the rocket is $v_ц = 200$ m/sec. The speed of the rocket in the beginning of the work of the guidance system is $v_p = 800$ m/sec and consequently, $p = 4$. The target flies at an altitude of 9150 m. The initial slant range is 36,900 m. The initial angle of the line of sight is

$$\gamma_0 = \arc\sin \frac{9150}{36900} = 14°20'$$

Results of the solution of this example are given in Table 3.

Let us consider the kinematics of motion of the rocket on the last phase of the path before its encounter with a target assuming p = const. We obtain the change in distance between the target and rocket and the speed of turn of the line of sight for the linear curve proceeding from formulas (5.21) and (5.22) assuming $\alpha = 0$ in them:

$$\frac{dr}{dt} = v_ц(\cos\beta - p); \quad \frac{d\gamma}{dt} = \frac{v_ц \cdot \sin\beta}{r}. \tag{5.37}$$

Normal accelerations of the target and rocket are determined from formulas (5.23) and (5.24):

$$a_{nц} = v_ц\left(\frac{d\beta}{dt} + \frac{v_ц \cdot \sin\beta}{r}\right); \tag{5.38}$$

$$a_{np} = \frac{v_p \cdot v_ц \cdot \sin\beta}{r}. \tag{5.39}$$

With the rectilinear motion of the target $a_{nц} = 0$ and

$$\frac{d\beta}{dt} = -\frac{v_ц \cdot \sin\beta}{r}. \tag{5.40}$$

Dividing the value of (5.37) by the values of (5.40) we obtain the important formula:

$$\frac{dr}{d\beta} = -r\frac{\cos\beta - p}{\sin\beta}. \tag{5.41}$$

Let us investigate condition of encounter of rocket with a target with linear curve guidance. From formula (5.37) one can see that for ensuring the encounter of the rocket with a target it is necessary to observe inequality $p > \cos\beta$, and with this $\frac{dr}{dt} < 0$ and magnitude r will decrease.

Since the greatest possible value is $\cos\beta = 1$, then for guaranteeing the encounter with any initial positions of the target and rocket it is necessary to

[1]Conditions of the problem are transferred to the metric system of units.

have $p > 1$, i.e., the speed of rocket should exceed the speed of the target. This condition clearly is illustrated by a particular case of attack in the tail section of the target when

$$\cos\beta = 1 \text{ and } \frac{dr}{dt} = v_{\text{ц}}(1 - p).$$

If $p < 1$ the rocket will not overtake the target.

In the particular case of a frontal approach p can be any value. However, this case should not be considered with the selection of the speed of the rocket v_p, since even a small deviation of the target from the rectilinear flight can lead to a miss.

Table 3

t	$x_{\text{ц}}$	x_p	y_p	v_p	θ°	$\frac{m}{m_0}$	Remarks
sec	m	m	m	m/sec	degree	--	
0	35752	0	0	800	14°20'	1,000	Beginning of guidance
2	35352	1560	400,6	810,4	14°29'	0,983	
4	34952	3140	811	821,4	14°40'	0,966	
6	34552	4740	1232	833,0	14°50'	0,949	
8	34152	6361	1665	845,3	15°02'	0,932	
10	33752	8005	2110	858,2	15°15'	0,915	
12	33352	9673	2569	871,7	15°30'	0,898	
14	32952	11365	3042	885,9	15°45'	0,881	
16	32552	13083	3532	900,9	16°03'	0,864	
18	32152	14828	4040	916,6	16°24'	0,847	
20	31752	16600	4568	933,1	16°47	0,830	
22	31352	18401	5119	950,4	17°15	0,813	
24	30952	20231	5697	968,3	17°49'	0,796	
26	30552	22089	6307	987,5	18°32'	0,779	
28	30152	23975	6956	1007,4	19°32'	0,762	
30	29752	25885	7659	1028,1	21°06'	0,742	
32	29352	27807	8446	1049,5	25°06'	0,728	
33,38	29076	29082	9150	1064,6	32°12'	0,722	Point of encounter

Let us define the limits of possible normal accelerations of the rocket with its approach with a target on a linear curve. We substitute into formula (5.39) dependence r on β obtained after the separation of the variables and integration of the equation (5.41):

$$\int_{r_0}^{r} \frac{dr}{r} = -\int_{\beta_0}^{\beta} \frac{(\cos\beta - p)\,d\beta}{\sin\beta}.$$

After substitution we will have

$$a_{n_p} = \frac{v_p^2 \cdot \sin\beta_0}{r_0 p} \left(\frac{\sin\frac{\beta}{2}}{\sin\frac{\beta_0}{2}}\right)^{2-p} \cdot \left(\frac{\cos\frac{\beta}{2}}{\cos\frac{\beta_0}{2}}\right)^{2+p}. \tag{5.42}$$

Encounter with departing target is reliably ensured when $\beta = 0$. In all cases except the encounter attack with the approach of the rocket with the target β tends to zero. Then from expression (5.42) the limiting accelerations will be when $2 > p > 1$ $\lim a_{np} = 0$, $p > 2$ $\lim a_{np} = \infty$. With the encounter attack $\cos \frac{\beta_0}{2} < 0$, and β tends to π and the accelerations will change the sign remaining when $p > 2$ infinitely large. Since the real rocket cannot move with an infinitely great normal acceleration, then as it approachs a target it will go out on the calculated trajectory and can fly wide of the mark. Thus for guaranteeing the full reliability of an encounter with ideal guiding on a linear curve one should have the condition

$$2 > p > 1.$$

If by rules of firing we exclude the possiblilty of sharp maneuvers of the rocket, then when $p > 2$ it is possible to strike the target with acceptable values of overloads.

Linear curve guidance is accomplished usually with the homing guidance of rockets designed for the striking of comparatively low-speed naval and ground targets. Guidance at high-speed targets requires special conditions of firing.

Guiding by the Method of Combination

With guiding by the method of combination the control system holds the rocket on a straight line connecting aiming point with a target. Due to this it is said that the rocket is sighted by the method of covering the target, and the trajectory of the guiding is called a <u>three-point curve.</u> A graphic plotting of a three-point curve is given in Fig. 83. On the trajectory of the target its positions are marked 0, 1, 2, 3..., corresponding to the small intervals of time Δt. With mobile point of aiming, for instance, with the guiding from a ship or aircraft, on the trajectory of the motion of the aiming point also marks $0', 1', 2', 3'$..., are plotted corre ding to the selected interval of time Δt. Marks on trajectories of the motion of the target and aiming points corresponding to identical moments of time are connected by straight lines. With a fixed aiming point we will have a pencil of straight lines convergent to it (Fig. 83). On one of the straight lines point is plotted corresponding to the beginning of trajectory of guidance. One should pay special attention, to the finding of this point, if the place of launch of the rocket is at considerable distance from the point of aiming, and it is impossible to assume them combined. From the beginning of the trajectory of guidance

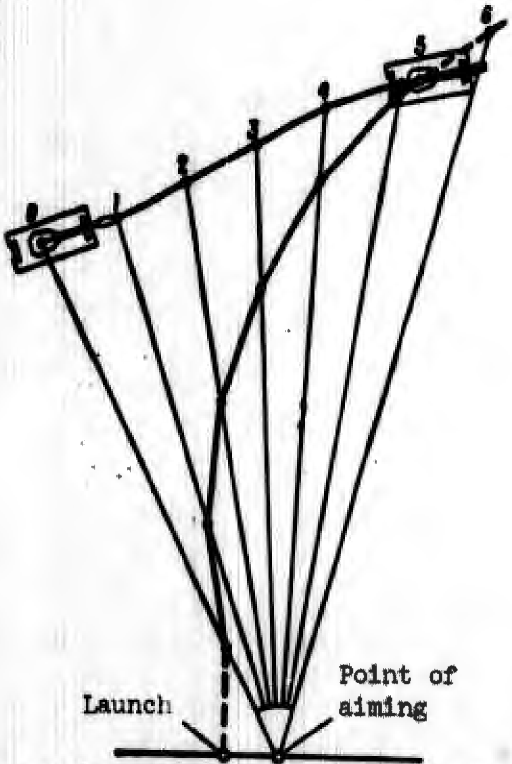


Fig. 83. Approximate plotting of the trajectory of a rocket guided by the method of combination.

consecutively on each of the straight lines the position of the rocket is plotted by dividers the opening of which is taken equal $\bar{v}_{pi} \cdot \Delta t$. Plotting continues to the crossing of the trajectory of the target and rocket. The average speed of the rocket on a segment can be determined, as

$$\bar{v}_{pi} = \frac{v_{pi} + v_{pi+1}}{2},$$

where v_{pi} and v_{pi+1} are the speed of the rocket in the beginning and end of the considered segment.

After the plotting of points of positions of the rocket smooth curve is drawn through them.

At every moment of time the velocity vector of the rocket is directed toward the expected point of encounter of the rocket with a target at a certain angle to the radius-vector passing through the aiming point, center of mass of the rocket, and the target. Guiding with a certain lead angle is obtained where the line of sight will form part of the radius of the vector revolving with the motion ot the target and rocket around the point of aiming. Let us determine the magnitude of the lead angle α. Let us assume during the infinitesimal interval of time dt the target will shift from poisition a to position b (Fig. 84). With this the radius vector will turn at angle $d\gamma$, and the rocket will shift from position a' to position b'. Proceeding from Fig. 84 with a high degree of accuracy it is possible to write that

$$\sin\beta = \frac{bc}{v_{ц}\,dt} \text{ and } \sin\alpha = \frac{b'c'}{v_p\,dt},$$

and the relation

$$\frac{b'c'}{bc} = \frac{\sqrt{x^2+y^2}}{\sqrt{x_ц^2+y_ц^2}} = \frac{r_p+dr_p}{r_ц-dr_ц} \approx \frac{r_p}{r_ц}.$$

Hence

$$\sin\alpha = \frac{1}{p} \cdot \frac{r_p}{r_ц} \sin\beta. \qquad (5.43)$$

The formula obtained illustrates well the connection of the method of guiding by three points with the earlier considered methods. In the beginning of the guiding when $r_p \approx 0$, $\alpha = 0$ and the trajectory of the motion of the rocket is close to a linear curve, at the end of the guiding when $r_p \approx r_ц \sin\alpha = \frac{1}{p}\sin\beta$ and the trajectories is close to the trajectorie of guiding plotted proceedings from the conditions of an ideal lead (5.27). On the whole the method of guiding by three points can be considered as a method of guiding with consecutive lead in which the lead angle is determined by the formula (5.43), and the angle γ is a function of target position data with respect to the aiming point: $\operatorname{tg}\gamma = \frac{y_ц}{x_ц}$.

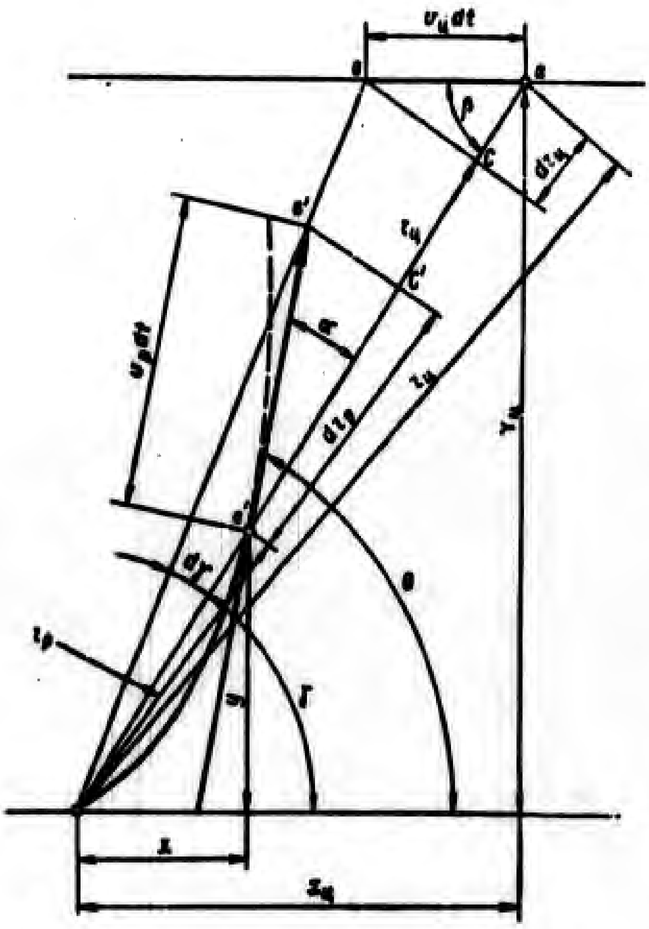


Fig. 84. Diagram of the plotting of angles for the combination curve.

Let us give the formula determining the angular velocity of the rotation of the velocity vector of the rocket $\frac{d\theta}{dt}$ necessary for formulation of a system of equations describing the motion of the rocket:

$$\frac{d\theta}{dt} = \frac{d\theta}{d\gamma}\cdot\frac{d\gamma}{dt}.$$

For the rectilinear horizontal motion of the target the derivative $\frac{d\gamma}{dt}$ is easily determined from Fig. 89 considering that $\beta = \gamma$:

$$\frac{d\gamma}{dt} = \frac{v_ц}{r_ц}\cdot\sin\gamma. \qquad (5.44)$$

For the determination of $\frac{d\theta}{d\gamma}$ we present that

$$\operatorname{tg}\theta = \frac{dy}{dx} = \frac{d\,(r_p\cdot\sin\gamma)}{d\,(r_p\cdot\cos\gamma)}.$$

Differentiating and dividing the numerator and denominator of the right side by dγ we obtain

$$\operatorname{tg}\theta = \frac{\sin\gamma\cdot\frac{dr_p}{d\gamma} + r_p\cdot\cos\gamma}{\cos\gamma\cdot\frac{dr_p}{d\gamma} - r_p\cdot\sin\gamma}. \qquad (5.45)$$

We will also find the derivative $\frac{dr}{d\gamma}$ proceeding from Fig. 84.

Equating $r_p + dr_p$ to r_p we get

$$(r_p d\gamma)^2 + (dr_p)^2 = (v_p dt)^2.$$

Dividing by $(d\gamma)^2$ and using the expression (5.44) we obtain

$$\frac{dr_p}{d\gamma} = \sqrt{\left(r_{\text{ц}} \cdot \frac{p}{\sin\gamma}\right)^2 - r_p^2}.$$

After differentiation of the formula (5.45) and transformations will have

$$\frac{d\Theta}{dt} = \frac{2v_{\text{ц}} \cdot \sin\gamma}{r_{\text{ц}}}\left(1 + \frac{r_p \cdot \operatorname{ctg}\gamma}{\sqrt{\left(r_{\text{ц}} \cdot \frac{p}{\sin\gamma}\right)^2 - r_p^2}}\right). \qquad (5.46)$$

Using the fundamental equation of dynamics and the common kinematic equations we obtain the system of equations describing the ideal trajectory of the guiding of a rocket by a three-point curve. The solution to this system, as the preceding solutions, should be carried out by the numerical method.

The normal acceleration of the rocket will be determined, as usually, by the formula

$$a_{np} = v_p \cdot \frac{d\Theta}{dt},$$

or

$$a_{np} = \frac{2pv_{\text{ц}}^2 \cdot \sin\gamma}{r_{\text{ц}}}\left[1 + \frac{r_p \cdot \operatorname{ctg}\gamma}{\sqrt{\left(r_{\text{ц}} \cdot \frac{p}{\sin\gamma}\right)^2 - r_p^2}}\right]. \qquad (5.47)$$

From this formula several general conclusions can be made. With the approach of the rocket to the target r_p tends to $r_{\text{ц}}$ and with an encounter $r_p = r_{\text{ц}}$; therefore,

$$\left(r_{\text{ц}} \cdot \frac{p}{\sin\gamma}\right)^2 - r_p^2 = r_{\text{ц}}^2\left(\frac{p^2}{\sin^2\gamma} - 1\right).$$

In the case $p < 1$ with a certain γ an infinitely great normal acceleration of the rocket is possible. Thus in the design of the rocket it is necessary that $p > 1$. At the same time a_{np} is directly proportional to the square of the speed of the target, increases with an increase in p, and with large values of $v_{\text{ц}}$ and p can appear impermissible. Furthermore, the normal acceleration of the rocket is inversely proportional to the distance up to the target $r_{\text{ц}}$ and with small distances of firing can also appear excessively large. This leads to the fact that the method of guiding by a three-point curve is used for rockets designed for combat with comparatively low-speed targets, for instance, antitank rockets (missiles) controlled from stationary and mobile command posts (ships, aircraft, tank) and designed for the striking of low-speed or stationary targets.

Guiding by a three-point curve is carried out usually by command control methods by beam or wires.

A tendency to use the best properties of either command methods of control or homing guidance led to the appearance of the combined methods. The trajectory of the motion of the rocket with a combined method of control consists of separate segments corresponding to accepted method of guidance on each of them. For instance, command guidance to future position can be combined with homing guidance with parallel approach. For the increase in accuracy of guiding a possibly smooth conjunction of trajectories of separate segments is desirable, not only by angle θ but also by the angular velocity $\frac{d\theta}{dt}$.

The selection of a certain method of control and method of guidance to the target is established in every concrete case with the designing of the rocket, proceeding from its assignment after thorough scientific, engineering, and economic investigations. With this it is a goal to make the system of control simpler and the weight of it and the rocket itself less.

§ 4. Uncontrolled Flight

Rockets with short range (rocket (missiles) of field artiller), as a rule, are uncontrolled; certain types of antitank rockets, ground-to-air, and others are also uncontrolled. The flight of such rockets is uncontrolled on the entire trajectory; the flight of guided rocket with the control turned off also becomes uncontrolled.

Stabilization of unguided rockets (rocket missiles) is carried out by the fin or rotation. With a stable flight of well stabilized rockets angles of incidence appear small, and the effect of lift and overturning moment is insignificant. Therefore, it is possible to consider separately the forward motion of the center of mass of a rocket and its motion with respect to the center of mass. Let us consider the forward motion of the center of mass. For this we will examine the free flight of rockets designed for firing at comparatively short distances. In many cases it is possible not to consider the rotation of the Earth and its curvature, and the gravity can be assumed constant in magnitude and direction.

The system of equations describing the free flight of the center of mass can be to considered as a particular case of the system describing controlled flight. Therefore, in the system of equations (5.12) we will omit the terms considering the influence of control. For the majority of unguided rockets the vector of tractive force coincides with the axis of the rocket and $\xi = 0$. If one were to consider that for small angles of attack $\cos\alpha \approx 1$, but $\sin\alpha \approx 0$ and $Y \approx 0$, and to add the common kinematic relationships, then we will obtain the system of equations describing the motion of the center of mass of unguided rockets:

$$\left.\begin{aligned} m\dot{v} &= P - X - q\cdot\sin\Theta; \\ mv\dot{\Theta} &= -q\cdot\cos\Theta; \\ \dot{x} &= v\cdot\cos\Theta; \\ \dot{y} &= v\cdot\sin\Theta. \end{aligned}\right\} \qquad (5.48)$$

Let us recall that the first two equations are written in the normal system of coordinates, i.e., in projections on the tangent and normal to the trajectory.

Frequently we use the system of equations of the motion of an unguided rocket with the independent variable

$$\lambda = \frac{\int_0^t G_{sec}\,dt}{G_0},$$

which is the relation of the weight of the burnt fuel to the initial weight of the rocket.

In the first equation of the system (5.48), on the basis of formulas (1.13) and (3.66), we replace P and X. Moreover, from the determination of λ it is easy to obtain

$$dt = \frac{G_0}{G_{sec}}\cdot d\lambda.$$

After simple transformations we obtain

$$\left.\begin{aligned} \frac{dv}{d\lambda} &= \frac{w_e}{1-\lambda} - \left[\frac{C_x H(y) F(v)}{1-\lambda} + g\cdot\sin\Theta\right]\frac{G_0}{G_{sec}}; \\ \frac{d\Theta}{d\lambda} &= -\frac{G_0}{G_{sec}}\cdot\frac{g\cdot\cos\Theta}{v}; \\ \frac{dx}{d\lambda} &= \frac{G_0}{G_{sec}}\cdot v\cdot\cos\Theta; \\ \frac{dy}{dt} &= \frac{G_0}{G_{sec}}\cdot v\cdot\sin\Theta. \end{aligned}\right\} \qquad (5.49)$$

The system describing free flight on the unpowered section of the trajectory after turning off the engine is easily obtained from equations (5.48) with the condition $P = 0$:

$$\begin{aligned} m\dot{v} &= -X - q\cdot\sin\Theta; \\ mv\dot{\Theta} &= -q\cdot\cos\Theta; \\ \dot{x} &= v\cdot\cos\Theta; \\ \dot{y} &= v\cdot\sin\Theta. \end{aligned}$$

It is obvious that in the latter system the mass of the flying rocket is constant, since the engine ceased operating. This system is used for the calculation of trajectories of missiles of trunk artillery. If drag sets through model function $g(v)$, then in the Cartesian system of coordinates equations of the flight of the unpowered section of the trajectory will have the form

$$\dot{u} = -CH(y)G(v)u;$$
$$\dot{w} = -CH(y)G(v)w - g;$$
$$\dot{x} = u;$$
$$\dot{y} = w.$$

Let us recall that u and w are the horizontal and vertical projections of speed v. Since on the unpowered section of trajectory there is no consumption of mass, then ballistic coefficient C = const. In the written system the change in the speed of sound with altitude is not considered.

The system of equations describing the free flight of the center of mass of a ballistic rocket (or its nose cone) on an unpowered section of the trajectory, taking into account the influence of drag, is easily obtained from the system (5.16) if in it the thrust P = 0:

$$\left.\begin{aligned} \frac{du}{dt} &= -\frac{X\cdot\cos\Theta + Y\cdot\sin\Theta}{m} - g_r\cdot\sin\gamma; \\ \frac{dw}{dt} &= -\frac{X\cdot\sin\Theta - Y\cdot\cos\Theta}{m} - g_r\cdot\cos\gamma; \\ \frac{dx}{dt} &= u;\ \frac{dy}{dt} = w;\ \mathrm{tg}\,\Theta = \frac{w}{u};\ \mathrm{tg}\,\gamma = \frac{x}{R+y}. \end{aligned}\right\} \tag{5.50}$$

In contrast to the simpler system (5.11), written in polar coordinates without taking into account the drag, the system (5.50) in final form is not solved. It must be solved by numerical methods.

For the approximate calculation of small trajectories of free flight passing in a vacuum, in that case when it is possible to disregard the rotation of the Earth and its curvature, equations are used which do not consider the drag. For small speeds of motion in air, approximately up to 50 m/sec, drag can be disregarded, and it can be considered that the body flies as if it were in a vacuum. In these cases the only acting force will be the force of weight. The system of equations is evident and is known from a course in physics:

$$\dot{x} = v_0\cdot\cos\Theta_0;\quad \dot{y}_0 = v_0\cdot\sin\Theta_0 - gt, \tag{5.51}$$

where v_0 and θ_0 are the initial speed and angle of departure.

§ 5. Analytic Solutions to Equations

The solution to equations of flight is necessary both in the designing and also in the use of rocket complexes. In order that a rocket will hit its target or an earth satellite will enter an assigned orbit, it is necessary to calculate correctly the trajectory of their motion.

With the guiding of a rocket at a mobile target characteristics of the motion of the target and rocket are determined by instruments tracking them. Equations of flight of target and rocket are solved in the guidance process, and to the control system of the rocket are sent corresponding signals changing the direction of motion of the rocket. A fast solution of the complicated systems of differential equations of flight is possible only on electronic computers. Analytically, without the application of computers, it is possible to solve comparatively a few relatively simple problems. Along with this analytic methods permit simply conducting preliminary investigations connected with the determination of the characteristics of motion of the rocket.

The solution in the case of the rectilinear motion of the rocket is most simple if we do not consider the drag and gravity. In this case the equation of motion will have the form

$$m\dot{v} = P. \tag{5.52}$$

Substituting values P and m from formulas (1.15) and (1.22), we obtain

$$dv = w_e \frac{G_{sec}\,dt}{q_0 - \int_0^t G_{sec}\,dt}.$$

Introducing the new variable $z = q_0 - \int_0^t G_{sec}\,dt$, integrating, and substituting, we obtain the speed of the rocket at the moment of time t:

$$v = w_e \ln \frac{q_0}{q_0 - \int_0^t G_{sec}\,dt}. \tag{5.53}$$

Designating the complete fuel consumption to the end of the engine's operation t_K by $\omega = \int_0^{t_K} G_{sec}\,dt$, we obtain the formula determining ideal greatest speed which the rocket can have without taking into account gravity and drag, i.e.,

$$v_{max} = w_e \ln \frac{q_0}{q_0 - \omega}, \qquad (5.54)$$

where $\ln \frac{q_0}{q_0 - \omega}$ is the natural logarithm of the fraction the numerator of which is the initial weight of the rocket and the denominator, the weight of the rocket after the fuel consumption.

The given formula was first derived by K. E. Tsiolkovskiy and named after him. Although the formula is obtained for ideal conditions as we will subsequently see it can be applied with success for theoretical research in the field of rocket technology.

We obtain formulas determining the free flight of a body of constant mass without taking into account the drag but taking into account the force of weight. Differential equations (5.51) are elementarily integrated with the initial conditions: $t = 0$, $\dot{x}_0 = v_0 \cdot \cos\theta_0$ and $\dot{y}_0 = v_0 \cdot \sin\theta_0$. After integration the dependences x and y on time t are obtained:

$$x = v_0 t \cdot \cos\theta_0;$$
$$y = v_0 t \cdot \sin\theta - \frac{gt^2}{2}.$$

Excluding t we obtain the equation of the trajectory in the form of the well-known equation of parabola:

$$y = x \cdot \operatorname{tg}\theta_0 - \frac{gx^2}{2v_0^2 \cdot \cos^2\theta_0}. \qquad (5.55)$$

The speed at any moment of time can be calculated by the formula

$$v = \sqrt{v_0^2 - 2gy}.$$

The altitude of the trajectory is equal to

$$Y = \frac{v_0^2 \cdot \sin^2\theta_0}{2g}. \qquad (5.56)$$

The maximum range in a horizontal direction is

$$X = \frac{v_0^2 \cdot \sin 2\theta_0}{g}. \qquad (5.57)$$

The time of the flight to the impact point is

$$T = \frac{2v_0 \cdot \sin\theta_0}{g}. \qquad (5.58)$$

Let us investigate the differential equations (5.11) determining the motion of a body of constant mass in the central gravitational field of the Earth without taking into account the thrust, drag, and rotation of the Earth. Let us consider the beginning of the trajectory to be point K and substitute accordingly γ by φ

(Fig. 72). Then as a result of the simple solution of the system (5.11) we can obtain the formula of the trajectory of motion of the center of mass of the rocket:

$$r=\frac{p}{1-e\cdot\cos(\psi-\varphi)}. \qquad (5.59)$$

The obtained dependence is the equation of the conic section in polar coordinates.

In analytic geometry conic sections are called lines of crossing of the lateral surface of a circular cone by different planes. The magnitude p is called the parameter of the curve and e the eccentricity of the curve.

For the considered case

$$p=\frac{v_0^2\cdot\cos^2\Theta_0}{g_{\tau 0}}\cdot\frac{r_0^2}{R^2}, \qquad (5.60)$$

where $r_0 = R + h_0$ determines the conditional boundary of the atmosphere starting from which it is possible to disregard the drag (Fig. 72).

$$e=\sqrt{1+\frac{r_0^2 v_0^2\cdot\cos^2\Theta_0}{g_{\tau 0}^2 R^4}\left(v_0^2-2g_{\tau 0}\cdot\frac{R^2}{r_0}\right)}. \qquad (5.61)$$

Frequently we are also not considered with influence of the atmosphere on the form of the initial and end surface sections of the trajectory and assume that the curve described by equation (5.59) starts directly from the surface of the Earth. In this case in formulas (5.60) and (5.61) it is necessary to take $r_0 = R$, then the formula for the eccentricity of the curve will have the form

$$e=\sqrt{1+\frac{v_0^2\cdot\cos^2\Theta_0\left(v_0^2-2g_{\tau 0}R\right)}{g_{\tau 0}^2 R^2}}. \qquad (5.62)$$

Depending upon the initial conditions of flight (v_0 and θ_0) the trajectory of the motion of the center of mass of the rocket determined by the formula (5.59) can be a circle, ellipse, parabola, and hyperbola.

If in formula (5.59) we assume e = 0, then r = p = const and the trajectory of the motion of the center of mass of the rocket will be a circle. For the obtaining of e = 0 it is necessary that the second component in the subradical expression (5.62) be equal to unity. This requirement can be fulfilled if one were to launch in a circular orbit, i.e., to assume $\theta_0 = 0$. Then it is easy to show from formula (5.62) that there should be the equality

$$v_0^2=g_{\tau 0}R.$$

The speed determined from this relationship is called the orbital velocity:

$$v_{0I} = \sqrt{g_{T0} R}.$$

For the conditional launching from the surface of the Earth when R = 6371 km $v_{0I} \approx 7900$ m/sec.

Since in reality the beginning point of the trajectory should be at a considerable distance from the surface of the Earth, then replacing R by r and using the formula (2.16) we obtain

$$v'_{0I} = \sqrt{g_{T0} \cdot \frac{R^2}{r_0}} < 7900 \text{ m/sec.}$$

When e = 1 the trajectory of the flight of the center of mass will be a parabola. If the starting point is on the surface of the Earth, then for the guarantee of e = 1 it is necessary by the formula (5.62) to have $v_0^2 = 2g_{T0}R$. The speed determined from the last relationship is called the escape velocity:

$$v_{0II} = \sqrt{2g_{T0} R} \approx 11200 \text{ m/sec.}$$

If the beginning trajectory calculated by the formula (5.59) is on the departure from the surface of Earth, then from formula (5.61) we find

$$v'_{0II} = \sqrt{2g_{T0} \cdot \frac{R^2}{r_0}} < 11200 \text{ m/sec.}$$

At launching with $\theta_0 \geq 0$ and $v_0^* \geq v_{0II}^*$ the rocket (or its nose cone) will overcome the Earth's gravity and will fly away into interplanetary space. If $v_0^* > v_{0II}^*$, then e > 1, and the trajectory of the flight will be a hyperbola.

Speeds v_{0I}^* and v_{0II}^* are speeds in absolute motion. For the calculation of velocity with respect to the Earth it is necessary, as it was shown earlier, to consider the speed of migratory motion dependent on the rotation of Earth.

At any value of e, within the limits of $-1 < e < 1$ (besides e = 0 and $\theta_0 = 0$) the trajectory turns into an ellipse. If $0 < e < 1$ and $\theta_0 = 0$, then the trajectory will be an ellipse for which the conditional center of the Earth will be the focal point of take-off (Fig. 85).

The nose cone of the rocket launched thus becomes an artificial earth satellite with an elliptic orbit. Therefore, the investigation of the motion of the material point in the central field of gravitation obtained the name of elliptic theory. The beginning of the elliptic theory was assumed I. Kepler in 1619. From Kepler's first law it follows that the planets move in ellipses at the focal point of which is the Sun. Frequently the motion satisfying equation (5.59) is called Kepler motion.

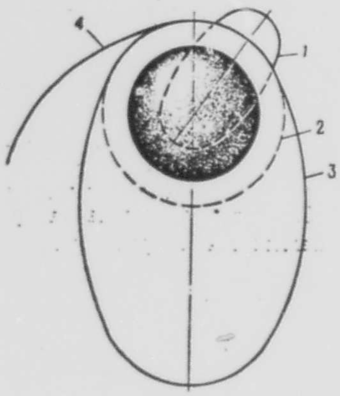


Fig. 85. Trajectories of space vechicles calculated from equations of conic sections: 1 — trajectory of a ballistic rocket; 2 — circular orbit of a satellite; 3 — elliptic orbit; 4 — parabolic trajectory.

For ballistic long-range rockets in most cases it suffices to have $v_0 < 7900$ m/sec which corresponds to $e < 0$. By selecting the corresponding conditions of the launch the required distance can be obtained (Fig. 72). Other characteristics of elliptic trajectory are obtained simply also from the given formulas. Assigning values of vectorial angles $\varphi_0, \varphi_1, \varphi_2, \ldots \varphi_n$, by the formula (5.59) we determine values $r_0, r_1, r_2, \ldots, r_n$ corresponding to them. The largest value r_{max} and the angle ψ corresponding to it are also determined from the formula (5.59): r_{max},, if one were to take $\varphi = \psi$, and ψ if one were to substitute $\varphi_0 = 0$ and $r = r_0$, then

$$r_{max} = \frac{p}{1-e}; \quad (5.63)$$

$$\psi = \arc\cos \frac{r_0 - p}{er_0}. \quad (5.64)$$

The flying range measured along the arc of the circle with a radius of r_0 is equal to $2\psi r_0$. The distance along the surface of the Earth

$$\breve{AB} = 2\psi R.$$

The speed along the elliptic orbit will be determined by the formula (Fig. 86)

$$v = \sqrt{v_r^2 + v_t^2}, \quad (5.65)$$

where v_r is the speed in a radial direction equal to $\dot{r}$; v_t is the tangential speed in a direction perpendicular to the vector r equal to $r\dot{\varphi}$.

Substituting $\dot{r}$ and $r\dot{\varphi}$ in formula (5.65) and expanding their value, we can obtain the dependence

$$v = f(r_0, v_0, \Theta_0, r).$$

The time of the motion is somewhat more complicated. If the second equation of system (5.11) is multiplied by r it cannot be presented in the form

$$\frac{d}{dt}(r^2\dot{\varphi}) = 0.$$

Integrating we obtain

$$\dot{\varphi} r^2 = C. \quad (5.66)$$

The arbitrary constant of integration C will be determined from the initial conditions when

$$t = 0;\ r = r_0 \text{ and } \dot{\varphi}_0 = \frac{v_0 \cdot \cos\Theta_0}{r_0};$$

$$C = r_0 v_0 \cdot \cos\Theta_0.$$

Substituting the value r into formula (5.66), transforming and integrating it we obtain the expression for time:

$$t = \frac{p^2}{r_0 v_0 \cdot \cos\Theta_0} \int_0^{\varphi} \frac{d\varphi}{[1 - e \cdot \cos(\psi - \varphi)]^2}. \qquad (5.67)$$

The integral is taken in the final form, but the formula is complicated.

Characteristics of the motion of the nose cone of the rocket put into orbit and becoming an artificial earth satellite are also determined approximately by the elliptic theory. For instance, the perigee, the point of minimum distance of elliptic orbit from the conditional center of the Earth, is equal to

$$r_{\pi} = \frac{p}{1+e}.$$

The apogee, the point of the greatest distance from the conditional center of Earth, is equal to

$$r_{\alpha} = \frac{p}{1-e}.$$

The semimajor axis of the ellipse of the orbit is

$$a = \frac{r_{\pi} + r_{\alpha}}{2} = \frac{p}{1-e^2}. \qquad (5.68)$$

The semiminor axis is connected with the semimajor through the parameter of ellipse p and is equal

$b = \sqrt{ap}$.

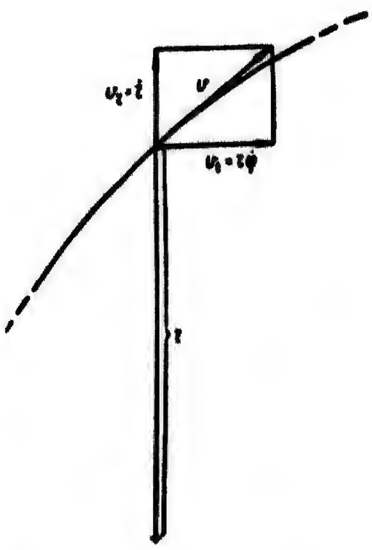


Fig. 86. Decomposition of the vector of speed on radial v_r and tangential v_t components.

A projection of the trajectory of the motion of an artificial satellite on the Earth is constructed approximately, proceeding from the assumption that the direction of the plane of orbit of the satellite is constant with respect to the stars. By knowing the speed of the satellite along the orbit and the angular velocity of the rotation of the Earth, it is simple to construct a diagram of projections of positions of the satellite on the Earth's surface. With an accurate calculation of the characteristics of the motion of long-range rockets and earth satellites and the construction of projections of their positions on the surface of the Earth, it is necessary additionally to consider a number of perturbing factors. We consider that the Earth is a spheroid formula (2.16) should be definitized and developed in projections on the axis of the coordinates. It is necessary to take into account the Coriolis force, the possible influence of the Moon, and others. Calculation of influence of the Moon on the orbit of the earth satellite will demand consideration of the problem called in celestial mechanics the three-body problem. As a result of the action of perturbing factors the trajectory of the motion of a long-range rocket or the orbit of an earth satellite will be somewhat deviated from the plane constantly directed with respect to the stars, but the characteristics of the motion will not correspond accurately to those obtained by the approximate elliptic theory. The calculation of influence of all perturbing factors is very complicated.

The solved differential equations of flight did not contain terms taking into account the drag. As was shown in the third chapter the change in aerodynamic coefficients depending upon the speed of the body in air cannot be exactly described by functions of simple form. Therefore, only certain particular cases of equations of flight in air are analytically solved with the introduction of additional assumptions.

The system of form (5.49) is used most frequently. Besides the assumption accepted in the obtaining of the actual system (5.49) we additionally consider that the flow rate per second of gases emerging through the nozzle is constant in time and equal to

$$G_{sec} = \frac{\omega}{\tau}, \qquad (5.69)$$

where ω is the weight of propellant expended during the time of operation of the rocket engine and τ is the period of duty of motor. A function of the change in air density with altitude is replaced by the constant $H(y_{cp})$.

After the corresponding transformations we can obtain this equation from the first equation (5.49)

$$\frac{dv}{d\lambda} = \frac{w_e}{1-\lambda} - \frac{aF(v)}{1-\lambda} - b \cdot \sin\theta, \qquad (5.70)$$

where

$$a = C_x H(y_{cp}) \frac{S\tau}{\omega};$$

$$b = \frac{g\tau}{\omega}.$$

The last equation can be integrated if one were to take angle θ for constant, then

$$v = w_e \ln\frac{1}{1-\lambda} - a\int_0^{\lambda} \frac{F(v)}{1-\lambda} \cdot d\lambda - b\lambda \cdot \sin\theta. \qquad (5.71)$$

The first component of the right side, constituting a modified formula of K. E. Tsiolkovskiy (5.54), determines the speed of the rocket without taking into account the influence of the drag and weight. The second component considers the influence of drag and the third influence of weight. Thus the remaining equations of the system (5.49) can be integrated in the same way.

The second component of the equation (5.71) contains under the integral the experimental function F(v) assigned by a table; for the calculation of the third component it is necessary to select the constant value of sin θ. There is difficulty in the solution of other equations of the system (5.49), therefore calculation is made with the application of special tables on separate sections of the trajectory in several approximations.

One of the methods of calculation is presented in work [29].

The equation of the free flight of a rocket on the unpowered section of trajectory can be also solved analytically. Methods applied for the solution of such equations are well-known from external ballistics of missiles of barrel systems. The system (5.50) can be easily converted so that independent variable will be the angle of inclination of the velocity vector to the horizon θ:

$$\left.\begin{aligned} \frac{du}{d\theta} &= \frac{C}{g}\cdot H(y)\,vF(v); \\ \frac{dt}{d\theta} &= -\frac{u}{g\cdot\cos^2\theta}; \\ \frac{dx}{d\theta} &= -\frac{u^2}{g\cdot\cos^2\theta}; \\ \frac{dy}{d\theta} &= -\frac{u^2\cdot\mathrm{tg}\,\theta}{g\cdot\cos^2\theta}. \end{aligned}\right\} \quad (5.72)$$

The first differential equation gives after the solution a dependence (in polar coordinates) v = f(θ), called the equation of the hodograph of speed. Equation in differential form before the solution is also called equation of the hodograph. This equation is convenient in that it is connected with other equations of the system (5.72) only through the function H(y), changing slightly with altitude. If one were to take H(y) constant, then in the equation the two variables u and θ remain, since

$$F(v) = F\left(\frac{u}{\cos\theta}\right).$$

In spite of this, the equation nevertheless cannot be integrated with the arbitrary form of function F(v). Additional simplifications are necessary. The first analytic solution was carried out by L. Euler with the quadratic law for the drag.

If we assume $F(v) = Bv^2$ and designate $b = CBH(y_{cp})$, that in the equation of the hodograph variables can be separated, and the equation is integrated

$$\int_{u_0}^{u} \frac{du}{u^3} = -\frac{b}{g}\int_{\theta_0}^{\theta} \frac{d\theta}{\cos^3\theta}. \tag{5.73}$$

Both integrals are solved simply.

Speed is determined from the expression $v = \frac{u}{\cos\theta}$. Although the substitution into the remaining equations of the system (5.72) of the dependence $u = f(\theta)$, obtained after the solution (5.73), permits dividing the variables of the equations in the final form are not solved. For the determination of x, y, and t it is necessary to apply the numerical method of L. Euler or special tables. With the use of the described method of the solution of equations of flight, it is necessary to consider that the square law of drag will agree with experimental data at comparatively low speeds (less than 250 m/sec).

The analytic method of the solution of equations of flight is also well known owing to the introduction of additional assumptions. In the equation of the hodograph the function $F(U)$, is introduced where U is the magnitude having the dimension of speed and is called pseudospeed. Value U is determined from Fig. 87:

$$U = \frac{v\cos\theta}{\cos\theta_0}.$$

The vector of the pseudospeed is parallel to the vector of the initial speed and has the same horizontal projection U, which is the actual v. Since the simple replacement of F(v) by F(U) gives considerable errors, then additional correction factors are introduced into the equation of the hodograph.

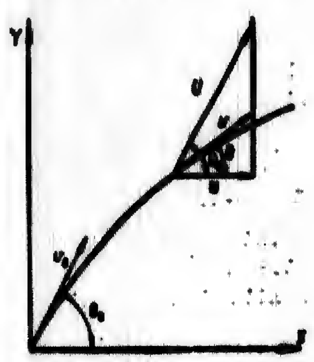


Fig. 87. Plotting the vector of pseudospeed U.

The substitution has the common form

$$H(y)F(v) \approx kF(U),$$

where k is the compensating error of the correction factor. Special investigations showed that for the flat firing with small θ_0 and large initial speeds the angle θ changes little along the trajectory, and $H(y) \approx 1$. In this case $k \approx \frac{1}{\cos\theta}$ is sufficient. For ground-to-surface firing with great initial speeds, although the cos θ changes little in the trajectory but $H(y) \neq 1$, the following replacement is necessary

$$H(y)F(v) \approx H(y_{cp})F(U)\frac{\cos\theta_0}{\cos\theta}.$$

i.e.,

$$h=H(y_{cp})\frac{\cos\theta_0}{\cos\theta}.$$

For average angles of departure and speeds substitution of the preceding is somewhat more complicated

$$H(y)F(v)\approx\beta F(U)\frac{\cos^2\theta_0}{\cos\theta},$$

i.e.,

$$h=\beta\frac{\cos^2\theta_0}{\cos\theta},$$

where β is the additional numerical correction factor.

The last replacement bears the name of "Siacci substitution." After the introduction of it into the equation of the hodograph and a separation of variables we can be obtain

$$\int_{\theta_0}^{\theta}\frac{d\theta}{\cos^2\theta}=-\frac{g}{c\beta\cdot\cos^2\theta_0}\int_{U_0}^{U}\frac{dU}{UF(U)}.$$

The integral of the left side is undertaken in the final form and is equal to $\left|\operatorname{tg}\theta\right|_{\theta_0}^{\theta}$. The integral of the right side, just as the integral of the remaining equations of the system (5.72), in final form is not undertaken. For the determination of tg θ, t, x, and y tables of special functions D(U), I(U), A(U), and T(U), called tables of "basic function," are applied.

The input value into these tables is the pseudospeed U. The standard working formulas have a very simple form. For example, the coordinate

$$x=\frac{1}{c\beta}\left[D(U)-D(v_0)\right].$$

The solution based on the use of tables of "auxiliary functions" for which the input values are v_0 and c'x, where c' = cβ is known.

For large angles of departure the method of pseudospeed gives considerable errors. Tables of numerical values of the coefficient β are compiled for angles $\theta_0 \leq 60^\circ$ (28). The application of the method of pseudospeed in principle is possible for the calculation of characteristics of the motion of a body of variable mass, i.e., for the calculation of powered-flight trajectories of unguided rockets.

The first equation of the system (5.49) can be replaced by equation

$$\frac{du}{dt}=\left[\frac{a}{1-\lambda}-\frac{id^2}{G_{\cos}}\cdot 10^3 H(y)\frac{F(v)}{1-\lambda}\right]\cos\theta. \qquad (5.74)$$

This is the projection of the equation of motion on the axis OX of the cartesian system of coordinates.

If one were now to assume, just as with the solution of the equation (5.70), that G_{cex} = const, $H(y) \approx H(y_{cp})$ and $\theta = \theta_{cp}$

$$\frac{d\lambda}{1-\lambda} = \frac{d\left(v \cdot \frac{\cos\Theta}{\cos\Theta_{cp}}\right)}{w_r\left[1-kF(v)\right]}, \tag{5.75}$$

where all the constants and the numerical correction factor are included in k.

If additionally one were to assume

$$v = \frac{u}{\cos\Theta} \approx \frac{u}{\cos\Theta_{cp}}$$

and to designate

$$U = \frac{v \cdot \cos\Theta}{\cos\Theta_{cp}},$$

then after integration we obtain

$$\ln\frac{1}{1-\lambda} = \frac{1}{w_r}\int_{U_1}^{U}\frac{d(U)}{1-kF(U)}, \tag{5.76}$$

where U_1 is the value U for the beginning of the section of integration.

Similarly in the remaining equations of the system (5.49) the variables can be divided, and the equations are integrated. The numerical solutions obviously require the application of special tables.

The given approximate analytic solutions are convenient in that they permit finding the characteristics of motion of the center of mass of the rocket for all sections of the trajectory of motion.

In the practice of ballistic calculations the so-called tabular methods of solution are applied, using which the elements of characteristic points of the trajectory can be found for instance, the peaks or impact points. Ballistic tables are widely used for the calculation of elements of trajectories of the flight of missiles of trunk artillery. These tables can be used for calculations of unpowered sections of trajectories of rockets. An investigation of the system of equations (5.72) shows that elements of the trajectory are determined by three parameters: v_0 – initial speed; C – ballistic coefficient; θ_0 – angle of departure. After a large number of calculations of trajectories, with the use of the system of equations (5.72), tables are compiled the inputs into which are v_0, C, and θ_0. Given in the tables usually are values of complete distance X, altitude of the trajectory Y, full time of the flight – T, speed at the point of impact – v_c, and angle of inclination of the tangent to the trajectory at the point of impact – θ_c.

Calculations by the tables lead to the simple interpolation according to v_0, C and θ_0.

For the calculation of trajectories of surface-to-air firing to the three input parameters a fourth is added, the time of motion.

Using the tables of surface-to-air firing it is possible for the trajectory determined by v_0, C, and θ_0 to find x_t, y_t, and v_t corresponding to different times of flight of the missile t_1, t_2, t_3, etc.

The beginning of the unpowered section of the flight of the rocket of the "surface-to-surface" class is sometimes at a considerable altitude, and therefore a direct use of ballistic tables can give considerable errors. In this case it is necessary to use the law of the similarity of trajectories of Langevin.

The French scientist, P. Langevin, established the dependence between the characteristics of two trajectories for which the temperature and pressure of ambient air correspond to the point of the beginning of the trajectories are different.

With the establishment of the dependence it was assumed that the change of temperature with altitude is the same as with the standard atmosphere. Let us designate the horizontal distance of the unpowered section of the trajectories by x_2, the elements of the beginning of the unpowered section by the index k, and the end of the section by the index C (Fig. 88).

From the law of similarity of trajectories we find

$$\left.\begin{aligned} x_2 &= \frac{\tau_k}{\tau_{0N}} \cdot \Phi_x(C^*, v_{kN}, \theta_k); \\ y_c &= \frac{\tau_k}{\tau_{0N}} \cdot \Phi_y(C^*, v_{kN}, \theta_k); \\ \theta_c &= \Phi_\theta(C^*, v_{kN}, \theta_k); \\ v_c &= \sqrt{\frac{\tau_k}{\tau_{0N}}} \cdot \Phi_v(C^*, v_{kN}, \theta_k); \\ T_c &= \sqrt{\frac{\tau_k}{\tau_{0N}}} \cdot \Phi_T(C^*, v_{kN}, \theta_k). \end{aligned}\right\} \quad (5.77)$$

where

$$C^* = C_k \cdot \frac{h_k}{\tau_{0N}};$$

$$v_{kN} = v_k \sqrt{\frac{\tau_{0N}}{\tau_k}}.$$

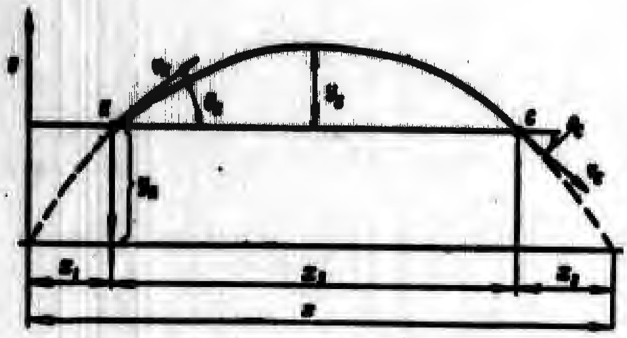


Fig. 88. Diagram of the division of the trajectory of a rocket into separate sections.

In these formulas the letters v_K, θ_K, h_K and τ_K designate correspondingly the speed, angle of departure, barometric pressure, and virtual temperature at the point of the beginning of the unpowered section of the trajectory.

The ballistic coefficient for the unpowered section (see formula 3.65) is equal to

$$C_{II} = \frac{id^2}{G_0 - \omega} \cdot 10^3.$$

Values of function Φ_x, Φ_y, Φ_θ, Φ_v, and Φ_T are taken from the common ballistic tables according to the input values C*, $v_{\tau K}$, and θ_K. Thus, for instance, $\Phi_x(C^*, v_{\tau K}, \theta_K)$ is equal to the distance determined by ballistic tables for numerical values of magnitudes shown in parentheses.

The complete flying range is equal to

$$X = x_1 + x_2 + x_3.$$

The first powered-flight trajectory is calculated by one of the methods about which was discussed above or is solved by one of the numerical methods.

The third section can also be calculated approximately by expanding in series the function y = f(x) near point C. This method is well-known in the ballistic of trunk systems.

The methods of approximation of the calculation of trajectories of rockets having a relatively short firing range can be reffered to as the so-called method of the equivalent artillery missile. This method assumes the detecting of such initial conditions of departure of the artillery missile (θ_0 and v_0), at which at the point of the end of operation of the engine of the trajectory of the artillery missile would have coincidence with the trajectory of the rocket in magnitude and direction of the velocity vector, i.e., at the point with coordinates x_K and y_K there would be the equality $v = v_K$ and $\theta = \theta_K$. It is natural that the equivalent artillery missile should have the same magnitude of ballistic coefficient as that of the rocket on the unpowered section of the trajectory, i.e., C_{II}. The calculated trajectory of the equivalent missile should coincide with the trajectory of the of the rocket on the unpowered section of the trajectory. Up to the point with corrdinates x_K and y_K the trajectories of the rocket and missile do not coincide. The initial conditions of the trajectory of the equivalent missile can be determined with the help of ballistic tables or by one of the analytic methods of the solution of problems of flight of artillery missiles, for instance, the method of pseudospeed.

By estimating the analytic and tabular methods of the solution of problems of the theory of flight one should still repeat that these methods are approximate and considered a comparatively small class of special problems. At the same time they permit comparatively simply the conducting of different ballistic investigations.

§ 6. Numerical Methods of Solving Equations of Flight and the Application of Electronic Computers

The complexity of the phenomenon of the controlled flight of rockets the requirement of fast and high accuracy of the calculation of characteristics of motion, and also the great volume of calculations lead to the necessity to conduct investigations in the field of the theory of flight with the use of electronic computers.

Problems connected with the theory of flight for the first time began to be solved with the use of mathematical machines and computers in instruments controlling artillery fire.

Contemporary electron computers by the principle of the solution of mathematical problems are divided into three classes: electron digital computers of discrete (continuous) analog computers of continuous action, analog computers of continuous action or the so-called analog computers, and computers of mixed types.

Machines of every class can be large and universal designed for the solution of various complicated mathematical problems and relatively small specialized designed for the solution of monotypic problems in a definite field of technology. In our country there have been a number of machines of classes which are used in the national economy and defense technology. The following universal digital computers are widely used [STRELA] (СТРЕЛА), [BESM] (БЭСМ), M-2, M-3, [URAL-2] (УРАЛ-2), and others.

Machine BESM, developed in 1953, for a long time remained the most high-speed computer in Europe. Among analog computers of continuous action these were serially produced: [IPT-5] (ИПТ-5), [MPT-9] (МПТ-9), [MPT-11] (МПТ-11), [MN-2] (МН-2), [MN-7] (МН-7), and many others.

Examples of specialized computers are the [PUAZO](ПУАЗО) (instruments of the control of artillery surface-to-air fire) and different computers of the system of antiaircraft defense. In forces of [PVO](ПВО) (antiaircraft defense) and [PRO](ПРО) (antimissile defense) radar instruments together with computers are used for the detection of targets and the guiding to them of guided rockets and aircraft-interceptors.

The numerical methods well developed in mathematics permit reducing the solution of different very complicated problems to the fulfillment of four arithmetic operations: addition, subtraction, multiplication, and division. This principle of numerical methods is assumed as the basis of the creation of digital computers. The carrying out of arithmetic operations in a defined sequence is determined by the program of work of the machine. The program of work can be set up for the solution of different problems, and therefore digital computers are universal. The great merit of them is the high accuracy of the solution. The main disadvantages are the complexity of the equipment, the laborious job of setting up a program and the complexity of the preparation of the machine for work.

In electronic analog computers mathematical quantities are depicted in the form of continuously changing values of voltages. All the actions are produced with voltages reflecting the given values in the corresponding scales. The great merit of such machines is the comparative simplicity of the equipment and the compactness. Deficiencies of the models can relate to the possibility of obtaining considerable errors reaching several percent.

Problems of theory of flight in principle can be solved both on digital and analog computers. Considering the peculiarities and possibilities of serial computers of a certain class, it is preferable to use digital computers. Computers of continuous action are irreplaceable during the analog formation of the process of oscillations and flight control of rockets. Let us consider the peculiarities of the solution of equations describing the motion of the center of mass of rockets on electronic digital computers. Specific diagrams and examples will be given in reference to the common Soviet computer [URAL-2] (УРАЛ-2).

For the automatic carrying out of calculations in accordance with the program the electronic numerical computer should have the following three main parts: the arithmetic unit intended for the execution of direct calculation; the work memory unit in which numbers and commands are stored the results of intermediate calculations are recorded, the needed numbers are taken and distributed; and the system for guaranteeing the automatic operation of the computer. The fundamental block diagram of the URAL-2 computer is given in Fig. 89. The arithmetic unit (AY) executes the arithmetical and logical operations above the numbers; in addition to that the AY has a device for the reception and issuing of information and also units of interaction with the control system of the computer and independent control of the AY. The memory unit consists of three storage units: the operational storage unit on ferrites for which the memory units are ferrite cores, the storage unit on a magnetic drum with control units for the recording and readout of information, and the storage unit on magnetic tape. The input into the computer of initial information consisting of numbers and commands is carried out through a special storage unit on puched tape.

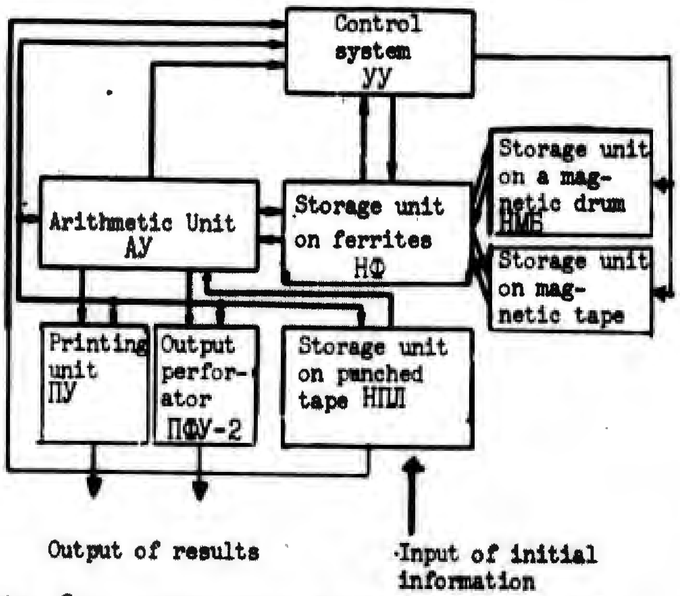


Fig. 89. Fundamental block diagram of the computer URAL-2.

Results of calculations of the computer can issue through the output perforating unit on punched tape and through the printing unit on paper tape. The control system ensures the automatic control of all parts of the computer in accordance with the program and manual control from the panel. Besides the enumerated parts of the computer interconnected by the assembling of circuits and in operation, there are still the so-called external units operating separately from the computer. This is the keyboard unit with the help of which on punched tape is put a set of commands and numbers. The punched tape of initial information is prepared separately in two copies for the purpose of a check. The mutual checking of tapes is conducted on the control-counting unit. During the check in the case of the noncoincidence of puncture on the tapes a corresponding signal is given.

The preparation of a problem for the solution of it on the computer machine consists of two basic stages.

The first stage is the selection of the numerical method of the solution and formulation of mathematical formulas.

The second stage is the programming.

Let us consider the basis of numerical methods of the solution of equations of flight. The method of numerical integration for the solution of equations of external ballistics was first proposed by A. N. Krylov in 1917. This method uses the theory of interpolation and finite differences.

In the preceding paragraph it was shown that basic difficulties with the integration of equations of the theory of flight consist in the functions determining the drag F(v) or $C_x(M)$, do not have a simple analytic form and are usually given by a table. Also given by a table is the function of the change in air density with altitude H(y). Methods of numerical integration permit calculating the definite integral from the function by the assigned table. In this case the so-called interpolating function is used which under the sign of integral replaces the real function whose analytic form is unknown. If such a replacement occurs on the small segment of the curve, that a sufficiently high accuracy of integration can be obtained.

Let us assume that the interpolating function will be y = f(x), then the increase of the definite integral from x_n to x_{n+1} is equal to

$$\Delta I_n = \int_{x_n}^{x_{n+1}} y\,dx. \qquad (5.78)$$

The simplest interpolating function is linear. The formula of linear interpolation is obtained from the rule of proportions:

$$\frac{y-y_n}{y_{n+1}-y_n} = \frac{x-x_n}{x_{n+1}-x_n}, \qquad (5.79)$$

where $y_{n+1} - y_n = \Delta y_n$ is the difference between the subsequent and preceding values of the function corresponding to the values of the argument; $x_{n+1} - x_n = h$ is the interval of change of the argument or the so-called step of the argument.

According to the formula (5.79) the unknown value of the function corresponding to the value of the argument x is equal to

$$y = y_n + \frac{x-x_n}{h}\cdot \Delta y_n.$$

With the linear interpolation the area under the curve will be separated into a number of trapezoids, the calculation of the total area of which is very simple (Fig. 90). At the same time the integration by the rule of trapezoids for the obtaining of high accuracy of the soltuion requires a small step of the argument h. The small step will lead to a large volume of the calculating work. Moreover, with practical calculations it is not always possible to have a table of experimental or calculated data with a small step of the argument.

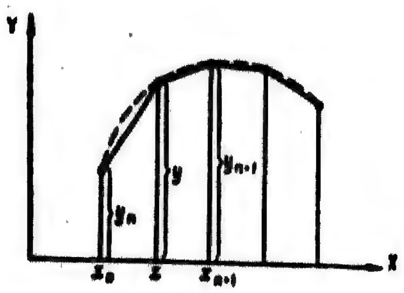


Fig. 90. Integration by the trapezoidal rule.

If with the numerical integration of equations of flight to special interpolating functions are used, the step of integration can be increased as compared to the integration by the rule of trapezoids. The interpolating function is comprised in the form of a polynomial of a degree per unit smaller than that of the number of assigned values of functions on the accepted segment of interpolation. Furthermore, the interpolating function should pass through all the assigned points of the interval.

In solving the many technical questions the interpolating function of Lagrange is widely used. In the numerical integration of equations of flight with the application of keyboard computers (electrical calculating machines) and tables of logarithms, interpolating formulas formulated with the use of tables of finite differences are applied.

If a certain function $y = f(x)$, for instance; $C_x(M) = f(M)$, is assigned by the table with a change in the argument through the constant step h, then the finite difference of the first order or first difference is called

$$\Delta y = \Delta f(x) = f(x+h) - f(x).$$

The second difference is

$$\Delta^2 y = \Delta^2 f(x) = \Delta[\Delta f(x)] \text{ etc.}$$

In order to compile the table of finite differences it is necessary to subtract from every value of the function the value preceding it and to record the obtained results in the column on the right in one line with the subtrahend (see the table).

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_{n-3}	y_{n-3}	Δy_{n-3}	$\Delta^2 y_{n-3}$	$\Delta^3 y_{n-3}$
x_{n-2}	y_{n-2}	Δy_{n-2}	$\Delta^2 y_{n-2}$	$\Delta^3 y_{n-2}$
x_{n-1}	y_{n-1}	Δy_{n-1}	$\Delta^2 y_{n-1}$	$\Delta^3 y_{n-1}$
x_n	y_n	Δy_n	$\Delta^2 y_n$	$\Delta^3 y_n$
x_{n+1}	y_{n+1}	Δy_{n+1}	$\Delta^2 y_{n+1}$	
x_{n+2}	y_{n+2}	Δy_{n+2}		
x_{n+3}	y_{n+3}			

If we use horizontal line of the table of differences it is possible by applying (5.78) to obtain the following formula determining the increase of definite integral from x_n to x_{n+1}:

$$\Delta y_n = h\left(y_n + \frac{1}{2}\Delta y_n - \frac{1}{12}\Delta^2 y_n + \frac{1}{24}\Delta^3 y_n\right).$$

If one were to use the slanted extrapolation line of the table of differences the formula will have the form

$$\Delta y_n = h\left(y_n + \frac{1}{2}\Delta y_{n-1} + \frac{5}{12}\Delta^2 y_{n-2} + \frac{3}{8}\Delta^3 y_{n-3}\right). \quad (5.80)$$

The latter formula is interesting in that it permits having Δy_n, on the basis of the preceding differences Δy_{n-1}, $\Delta^2 y_{n-2}$ and $\Delta^3 y_{n-3}$. Using this formula it is possible to integrate forward being based on the character of the change in functions. If one were to compile a table of finite differences of derived functions of y'_x, that it is possible using the given formulas to solve the ordinary first order differential equations to which the differential equations describing the flight of the rockets are reduced. For the slanted line by analogy with the formula (5.80)

$$\Delta y_n = h\left(y'_n + \frac{1}{2}\Delta y'_{n-1} + \frac{5}{12}\Delta^2 y'_{n-2} + \frac{3}{8}\Delta^3 y'_{n-3}\right). \quad (5.81)$$

Having y_n and adding Δy_n we obtain the following value of the function:

$$y_{n+1} = y_n + \Delta y_n. \quad (5.82)$$

The essence of numerical integration results in the application of the last two simple formulas. By gradually moving step by step and increasing the table of differences, one can determine all the values of the function $y = f(x)$ in the assigned limits.

The solution to the problems of the theory of flight is complicated by the fact that it is necessary to solve several equations jointly. It is desirable to select the system with the smaller number of equations solved jointly and with the argument most convenient for the given problem. For instance, for the calculation of the powered-flight trajectory of the unguided rocket it is convenient to take the system according to the argument t. For the numerical solution we will reconstruct the system of equations (5.48) on cartesian coordinates and replace in the first equation P and X from formulas (1.12) and (3.66). For the simplicity writing we usually designate tg θ = p. Then we obtain

$$\left.\begin{aligned} \frac{du}{dt} &= u\frac{m_0}{m}(D-E); \\ \frac{dp}{dt} &= -\frac{g}{u}; \\ \frac{dy}{dt} &= up; \\ \frac{dx}{dt} &= u, \end{aligned}\right\} \qquad (5.83)$$

where

$$D = \frac{1}{v}\left\{\frac{P_0}{m_0} + \frac{S_a p_{0N}}{m_0}\left[1 - \Pi(y)\right]\right\},$$

and $E = C_x H(y) G(v)$.

It is obvious that the first three equations of the system (5.83) should be solved jointly. For the constant increment of the table of finite differences it is necessary to have besides the initial values the first three values x, y, u, and p. These values are usually obtained by the method of successive approximations. Calculations are made with the multiple use of the formula (5.81), and the result is registered in a special form called the basic form (Table 4).

Lines in the calculation form are increased in the following order.

1. By the formula (5.81) with the replacement of $\dot{y}$ by $\dot{u}$ and corresponding differences Δu_n is calculated.

2. With the use of formula (5.82) the following is calculated:

$$u_{n+1} = u_n + \Delta u_n.$$

3. By second formula (5.83) $\dot{p}_{n+1}$ is calculated.

4. By the formula (5.81) with the corresponding replacement Δp_n is calculated.

5. By the formula (5.82)

$$p_{n+1} = p_n + \Delta p_n.$$

6. Knowing u_{n+1} and p_{n+1} with the third formula of the system (5.83) y_{n+1} is determined.

Table 4

				$\dot{y} = pu$					$\dot{p} = -\frac{g}{u}$					$\dot{x} = u$		$\dot{u} = (D-E)u\frac{m_0}{m}$							
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
№	t	m	$\frac{m_0}{m}$	y	Δy	$\dot{y}$	$\Delta\dot{y}$	$\Delta^2 y$	p	Δp	$\dot{p}$	$\Delta\dot{p}$	$\Delta^2\dot{p}$	x	Δx	u	Δu	$\Delta^2 u$	$\dot{u}$	$\Delta\dot{u}$	$\Delta^2\dot{u}$	$\Delta^3\dot{u}$	v
0																							
1																							
0																							
1																							
2																							
0																							
1																							
2																							
3																							
4																							

7. By y_n and the corresponding differences with the help of the formula of numerical integration formulated along the broken line of the table of differences Δy_n is calculated.

8. By the formula (5.82) this is determined:

$$y_{n+1} = y_n + \Delta y_n.$$

9. Using values y_{n+1}, u_{n+1}, and p_{n+1} with the first formula of the system (5.83) $\dot{u}_{n+1}$ is calculated.

Since the formula (5.81) is formulated with the help of a slanted extra polation line, after the increment of the lines it is necessary to correct Δu_n by a more accurate formula of the broken line.

The described method of the calculation is used with the solution of problems of the theory of flight on keyboard computers and also with the application of tables of logarithms. It is used for calculations comparatively small in volume when the use of electronic computers considering the complexity of the program error detection and correction economically is not justified. It is expedient to use it for check computations with the program error detection and correction on the electron computer.

The peculiarity of the solution of systems of equations of the theory of flight on the computer URAL consists in the fact that the ordinary differential equations to which the majority of equations describing the flight belongs are solbed on the computer according to the method of Runge-Kutta. This is also true of the numerical method of integration, but it permits calculating without the repetition of calculation by means of approach. Assumed as a basis of the method if the decomposition of the desired function in a series in given powers of $(x - x_n)$ of the argument near the assigned known point (x, y) of function y(x):

$$y = y_n + (x - x_n) y'_n + \frac{(x - x_n)^2}{2!} \cdot y''_n + \ldots + \frac{(x - x_n)^m}{m!} \cdot y_n^{(m)} + \ldots$$

As a result of investigations the following standard-working formula is obtained:

$$\Delta y_n \approx \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4),$$

where

$$k_1 = h \cdot f(x_n; y_n),$$

$$k_2 = h \cdot f\left(x_n + \frac{h}{2};\ y_n + \frac{k_1}{2}\right),$$

$$k_3 = h \cdot f\left(x_n + \frac{h}{2};\ y_n + \frac{k_2}{2}\right),$$

$$k_4 = h \cdot f(x_n + h;\ y_n + k_3).$$

Applying the formula of Runge-Kutta we find the approximate value Δy_n, after which by the formula (5.82) we determine y_{n+1}. For the new initial point we take $x_{n+1} = x_n + h$ and $y_{n+1} = y(x_n + h)$. Repeating the calculations we obtain the table of the sougnt function in the assigned limits. As can be seen from the given formulas the integration is reduced to simple actions and can be carried out by the arithmetic unit of the electronic digital computer. Moreover, for the solution on computer it is necessary to present in the form of polynomials or tables the change in air density with altitude (function H(y) and the change of mass of the rocket with time m(t). If some complicated process of the control of thrust is assumed, it is necessary to assign by a polynomial or table the law of the change in tractive force with altitude or time. Special attention should be given to the assignment of the function determining the drag $C_x(M)$ or F(v). With the preparation of the solution for each calculated magnitude the accuracy of the solution or the number of significant digits are established. For instance, the coordinates x and y are calculated correct to 0.1 m or 1 m, and the time is calculated correct to 0.01 sec, etc. Simultaneously it is expedient to establish beforehand the step of integration, ensuring the necessary accuracy of the solution proceeding from peculiarities of the calculated trajectory. With the transition through the speed of sound curves $C_x(M)$ or $F(v)$ sharply change (see Figs. 23 and 45), and the great step of integration can not ensure the assigned accuracy. It is necessary to consider that the computer can operate with an automatic selection of the step. In the solving process the obtained accuracy is compared with the assigned. If the error on the selected step of integration at least for one of the equations does not satisfy the assigned accuracy, then the step of integration automatically decreases twice (halved). However, it is necessary to consider that similar work requires a very large program, and for its realization much computer time will be expended. Therefore, it is more preferable where this is allowed by the required accuracy of calculations to operate with a constant step of integration. Similar considerations should be considered with the desire to take into account the changeability of $C_x(M)$ with altitude. As is known from the third chapter, the function $C_x(M)$ is variable with altitude owing to the change in the Reynolds number. The obtaining of curves of $C_x(M)$ for different altitudes involves no great difficulty. However, the use of several curves will lead to the increase in the program of calculations and, consequently, to the increase in computer time.

With the preparation of the assignment to carry out the calculations, it is necessary to anticipate also the character and volume of the prepared information issued by the printing unit of the computer. With the program error detection and correction it is well to have for control checks the results of solutions obtained by some other method, for instance, the numerical integration with the application of keyboard computers and tables of logarithms.

In conclusion let us note that the possibilities of contemporary electronic digital computers are very great, but the composition of the programs is a sufficiently complicated and laborious matter. A poorly formed program lowers the productivity of the computer. With the preparation of the problem for solution on the computer, the composition of the program, and debugging of the machine, it is necessary to know well the system of the computer and its peculiarity. It is also necessary to have a clear idea about the problems of the subject to be solved. After the program error detection and correction the computer can replace dozens of people and execute work in periods inaccessible with manual methods of calculations.

§ 7. Concept on Optimum Solutions of the Theory of Flight

The establishment of the most advantageous (optimum) conditions of motion is one of the most important and complicated problems of the theory of flight. As an example of an optimum problem it is possible to cite the determination of the angle of departure at which can be obtained the greatest horizontal flying range with the assigned initial speed. The angle determined from these conditions is called the angle of maximum range $\theta_{0\,max}$. The angle of maximum range is determined most simply with the value g_T = const if one were not to consider drag. From formula (5.57) it follows that $X = X_{max}$ when $\sin 2\theta_0 = 1$, i.e., when $\theta_{0\,max} = 45^\circ$. A characteristic example is such a determination of the angle of maximum range within the elliptic theory when $g_T \neq$ const. From Fig. 72 one can see that the maximum range will correspond to the largest value of the angle ψ determined by the formula (5.64). From Kepler equations we can be obtain the formula connecting angle ψ and θ_0:

$$\operatorname{tg}\psi = \frac{1}{2}\frac{r_0 v_0^2 \cdot \sin 2\theta_0}{g_{T0}R^2 - r_0 v_0^2 \cdot \cos^2\theta_0}. \tag{5.84}$$

Using the known rule of detecting the maximum of the function, it is necessary to take the derivative $\frac{d}{d\theta_0}(\operatorname{tg}\psi)$ and to equate it to zero. After transformations we will obtain the formula determining the angle of the maximum range:

$$\sin\theta_{0\,max} = \sqrt{\frac{g_{T0}R^2 - r_0 v_0^2}{2g_{T0}R^2 - r_0 v_0^2}}. \tag{5.85}$$

Figure 91 gives the curve $\theta_{0\,max} = f(v_0)$, comprised for case $r_0 = R$ (Fig. 72). From the graph one can see that with small initial speeds the angle of maximum range is close to 45°.

With an increase in the speed the angle of maximum range decreases attaining zero at orbital velocity.

If one were to consider drag the problem of the determination of the angle of maximum range would not be solved in analytic form without additional simplifications. The most simple way is the consecutive carrying out of large quantity of calculations for different values of θ_0 and the plotting of curves $X = f(\theta_0)$. The value of $\theta_{0\,max}$ can be determined either by curves or methods of inverse interpolation. Even for the simple cases of the motion of a body of constant mass in air (missile of trunk artillery) the angle of maximum range depends not only on the initial speed but also on the caliber and form of the missile united by the formula of ballistic coefficient. From the curve in Fig. 92 one can see that in the dependence on caliber the angle of maximum range can change in wide limits from 30° to 60°.

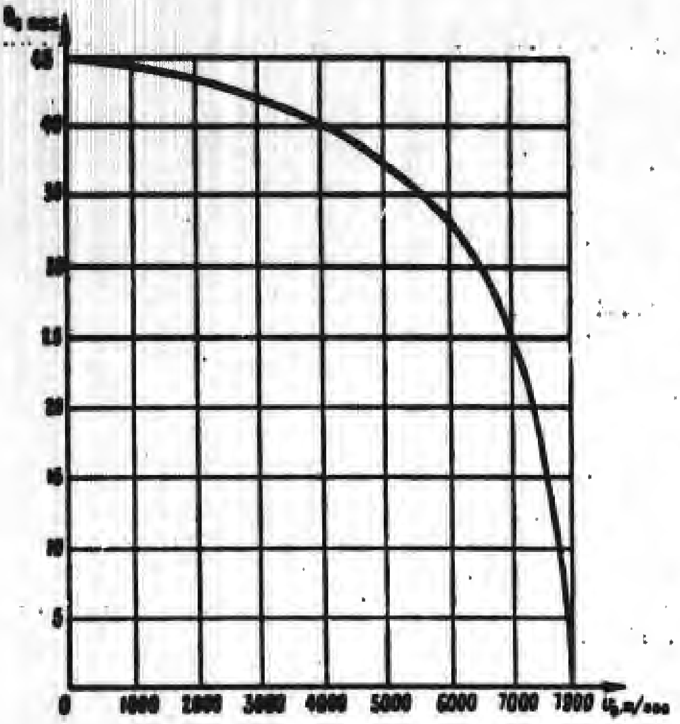


Fig. 91. Dependence of the angle of maximum range on speed v_0 by the elliptic theory.

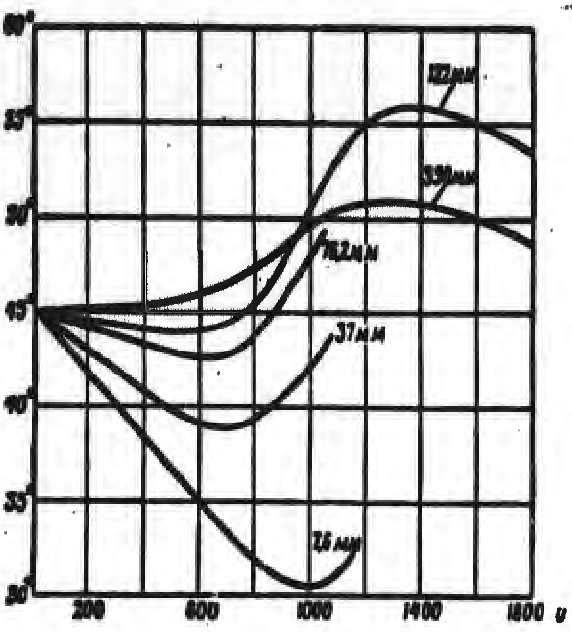


Fig. 92. Dependence of the angle of maximum range on the initial speed and caliber of the missile.

The solution of extreme problems in rocket technology began with the so-called second problem of Tsiolkovsky consisting in the determination of the law of the change of mass of a rocket and its speed depending upon the time at which can be expected the greatest altitude of the ascent of the rocket. In statement of the Tsiolkovsky problem it is solved for the vertical ascent of the rocket in a constant field of gravitation without taking into account the drag. As the investigation showed later a calculation of the drag is necessary.

The most validated selection of optimum variants of the solution to different problems in rocket technology is based on strict mathematical methods of calculus of variations. The object of calculus of variations is the investigations on the extremum (maximum or minimum) of the singular magnitudes called functions.

The functional is a variable quantity of the value which is determined by the selection of one or several functions. For instance, the area of a certain surface is a functional, since it is determined by the selection of a function entering into the equation of the surface $z = z(x, y)$. The drag of a medium to a body moving in it with a definite speed also is a functional, since it depends on the function determining the form of the surface of the moving body. One of the first problems solved by the method of calculus of variations was the problem on the curve of the fastest slope. In this problem it was required to determine the form of the flat curve connecting the two points located at a different altitude by which the body would be rolled in the shortest time. If one were not to consider the drag of the medium and the friction such a curve turns out to be a cycloid.

The calculus of variations started to be developed at the end of the 17th Century. The founder of it is rightfully considered the member of the Russian Academy of Sciences, L. Euler. Let us consider one of the schemes of the solution to the variational problem of rocket technology. Let us assume that it is required to find the function determining the fuel consumption (change of the mass of the rocket) at the time during the realization of which the altitude of vertical ascent of the rocket will be the greatest. Let us use the Meshcherskiy equation (1.5) for the vertical ascent of the rocket omitting in it for simplification of investigation the magnitude P and assume g = const. Replacing $\dot{x}$ and $\ddot{x}$ by v and dv/dt, we will have

$$m\cdot\frac{dv}{dt}=-mg-\frac{dm}{dt}\cdot w-R(v). \qquad (5.86)$$

Let us assume that the mass of the rocket changes by the dependence $m = m_0 f_t$, where f_t is a function characterizing the change in the mass of a rocket (fuel consumption) in the process of the operation of the engine. In the beginning of the motion $f_t(0) = 1$.

Substituting in the equation (5.86) value m by $m_0 f_t$ and dividing all the terms by the constant m_0 we obtain

$$f_t\cdot\frac{dv}{dt}=-f_t g-\frac{df_t}{dt}\cdot w-\frac{R(v)}{m_0}. \qquad (5.87)$$

Let us carry out the change of variables introducing the value of the elementary path of the rocket dS:

$$\frac{dv}{dt}=\frac{dv}{dS}\cdot\frac{dS}{dt}=v\cdot\frac{dv}{dS};$$

$$\frac{df_t}{dt}=\frac{df_t}{dv}\cdot\frac{dS}{dt}\cdot\frac{dv}{dS}=f_v v\cdot\frac{dv}{dS}.$$

Then

$$f_t v\cdot\frac{dv}{dS}=-f_t g-v f_v w\cdot\frac{dv}{dS}-\frac{R(v)}{m_0}.$$

After transformation and integration we obtain

$$S = \int_{v_0}^{v_K} \frac{(f_t + f'_v w)\, v\, dv}{f_t g + \frac{1}{m_0} R(v)}. \tag{5.88}$$

Then let us find such a function $f_t(v)$ at which the value of the integral will be the largest, i.e., let us determine such a law of the change of mass from speed at which the altitude of ascent of a rocket will be the greatest.

The finding of such a function is called the investigation of the integral S to extremum. The investigation should be conducted under definite boundary conditions: in the beginning of the investigated section the speed of the rocket is $v = v_0$ and $f_t = 1$, i.e., the mass $m = m_0$ and at the end of the section $v = v_K$ and $m = m_K$ respectively. These conditions are determined by the design features of the considered type of rockets. By omitting the actual investigation requiring a special mathematical preparation we will write the final result:

$$f_t(v) = \frac{\frac{R(v)}{m_0}(v - w) + \frac{1}{m_0}\frac{dR(v)}{dv} \cdot vw}{gw}. \tag{5.89}$$

The dependence obtained cannot be directly used for the solution to our problem, since the majority of the systems of equations describes the motion of rockets with the independent variable t. From formula (5.87) it is easy to obtain

$$f_t = \frac{dv}{dt} = -f_t g - f'_v \frac{dv}{dt} \cdot w - \frac{R(v)}{m_0},$$

whence directly

$$t = \int_{v_0}^{v_K} \frac{(f_t + f'_v w)\, dv}{f_t g + \frac{1}{m_0} R(v)}.$$

After the solution of the integral the dependence v(t) can be found. Having two dependences $f_t(v)$ and v(t) it is possible to determine the desired dependence, i.e., to find the function determining the change of mass of a rocket at the time when the path of the rocket will be the greatest.

Let us draw attention to the fact that the general consumption of the mass (fuel) remains constant and only the optimum (most favorable) variant of its expenditure during flight is detected.

Many variational problems of the theory of flight of rockets were solved by Professor Kosmodem'yanskiy [8]. A number of variational problems connected with the launching of ballistic long-range rockets and artificial earth satellites have been solved by Okhotsimskiy and Eneyev [10].

In practical work theoretical variational methods are always combined with constructive developments. Let us give several well-known examples. Earlier we described the construction and peculiarity of the flight of winged rockets. The winged rocket of the former German army, "A-4B" was a ballistic rocket of the "A-4" type supplied with wings 2.2 m^2 in area. The trajectory of its flight is shown in Fig. 74. The "A-4" rocket had a firing range of 320 km and, similar to it in weight, the winged rocket "A-4B" approximately 592 km. Thus owing to design changes it was possible to increase the distance approximately 1.8 times.

A determination of the most advantageous program of flight for these types of rockets, taking into account all the requirements, can be the object of investigation with help of calculus of variations. Let us give one more characteristic example concerning multistage rockets. Using the formula of K. E. Tsiolkovskiy we compare the ideal maximum velocity of the motion of the last stage of a three-stage rocket with the velocity of a one-stage rocket having the same initial weight and amount of fuel as a multistage rocket.

We take the initial launching weight of both rockets at 60 t and the effective exhaust velocity of the working fluid at 2500 m/sec. Let us assume that in each of the rockets there are 45 t of fuel completely depleted during flight. For a single-stage rocket the rates

$$\frac{G_0}{G_0 - \omega} = 4$$

approximately corresponds to the relation for the "A-4" rockets. The ideal maximum speed of a single-stage rocket by formula (5.54) is

$$v_{\text{и}} = 2500 \ln 4 = 2500 \cdot 1{,}386 = 3460 \text{ m/sec.}$$

Further let us assume that in the tanks of the first stage of the composite three-stage rocket there are 25 t of fuel, with the weight of the stage at 30 t; in the tanks of the second stage, 16 t of fuel, with the weight of the stage at 20 t; in the tanks of the last stage, 4 t of fuel, with the weight of the stage at 10 t.

The maximum ideal speed of the last stage will be

$$v_{\text{и}} = 2500\left(\ln\frac{60}{60-25} + \ln\frac{30}{30-16} + \ln\frac{10}{10-4}\right) = 4560 \text{ m/sec.}$$

Thus due to a design change (application of a three-stage rocket) the nose cone of it with payload will have a speed approximately 1.3 times greater than the speed of a single-stage rocket. The determination in the first approximation of the most advantageous number of stages and their weight relationships should be object of investigation with the help of calculus of variations.

The investigations of optimum conditions of the flight of rockets in interplanetary space are of enormous importance.

§ 8. Methods of Change in the Firing Range of Rockets

From Fig. 88 one can see that the complete distance of firing is composed of the distance x_1 corresponding to the section of flight of the rocket with the engine operating and the sum of distances $x_2 + x_3$ corresponding to the flight of the rocket on the unpowered section with a non-operating engine. Almost always for rockets of the surface-to-surface class the unpowered section of the trajectory is considerably larger than the powered and basically determines the complete firing range. The distance corresponding to the powered section of the trajectory depends on the parameters of the end of the powered-flight trajectory. A flight on the powered section of the trajectory can be controlled and uncontrolled. In both cases the characteristics of the powered flight are calculated in such a way as to ensure the most advantageous conditions of flight on the unpowered section of the trajectory. Considering that the complete range is determined by the unpowered section of the trajectory it is possible to write the functional dependence:

$$X = F(v_{\kappa}, \theta_{\kappa}, C_{\text{II}}, y_{\kappa}, x_{\kappa}, a_1, a_2, \ldots, a_n), \tag{5.90}$$

where v_{κ}, θ_{κ}, y_{κ}, and x_{κ}, are parameters of the end of the powered section of the trajectory; C_{II} is the ballistic coefficient on the unpowered section of the trajectory; α_1, α_2, ..., and α_n are secondary factors determining the distance (to them can be attributed, for instance, the influence of weather conditions).

The control of all factors affecting the distance, or, at the least the calculation of them in the process of flight, is a problem whose solution is associated with great difficulties. However, this is not always necessary. For instance, for ballistic long-range rockets a greater part of the trajectory passes in rarefied layers of the atmosphere, and therefore the influence meteorological factors turns out to be insignificant and they cannot be considered. For rockets of operational-tactical assignment, the distance of the firing of which is small relatively and the greater part of the trajectory passes in the dense layers of the atmosphere, the influence meteorological factors is considered an introduction of corrections into the intial conditions of firing. If one were not to consider the possibility of the change in aerodynamic properties of a rocket in the process of flight (i.e., the ballistic coefficient C_{II}), then the factors determining the firing range will be parameters of the end of the powered section of the trajectories: speed v_{κ}, angle θ_{κ}, and coordinate x_{κ}, y_{κ}.

To obtain the assigned distance it is necessary to turn off the engine, i.e., to ensure the necessary $v_к$ with definite values $\theta_к$, $y_к$, and $x_к$.

Let us assume that $v_к$, $\theta_к$, $y_к$, and $x_к$ are computed values of parameters of the end of the powered section determining the assigned distance, and $v_{кд}$, $\theta_{кд}$, $y_{кд}$, and $x_{кд}$ are real values of these parameters.

To compare the real parameters with the calculated it is necessary to equip the rocket with equipment which would allow the measuring during flight of real values of all parameters determining the distance and comparing them with the calculated values. This is a very complicated problem.

Only with a strict observing of all calculation characteristics of the powered flight is it possible to obtain at the same moment of time the equality

$$v_{кд}=v_к;\quad \theta_{кд}=\theta_к;$$
$$x_{кд}=x_к;\quad y_{кд}=y_к. \tag{5.91}$$

Apparently it is easiest to reach the rated conditions of motion with command methods of control with a constant sighting of rocket by radio facilities. Similar systems of control can be used in rockets of the "surface-to-air" and "air-to-surface" class. However, for rockets of the "surface-to-surface" these methods are not applied for several reasons. With self-contained guidance systems which can be supplied to rockets of the "surface-to-surface" strict observance of the rated conditions of motion on all parameters presents great difficulties, and consequently the observance of all equalities of (5.91) is also difficult to reach.

Inasmuch as the complete distance depends on the four parameters, the thought intrudes that a deviation of one of them from the computed value would have been possible to compensate by the corresponding change of another but in such a way as to obtain the calculated distance of the firing. For this it is necessary to compare during flight the real distance determined as a function of the combination of parameters $v_д$, $\theta_д$, $x_д$, and $y_д$ with the calculated distance. In this case, besides the measurement of all parameters in the process of flight it is still necessary to have on board the rocket a calculating mechanism which by obtained information would calculate the expected real distance, compare it with the calculated distance and send the corresponding commands to the controls. Coordinates of the position of the rocket in this case do not have to be determined with respect to the Earth.

For instance, with an astronavigation guidance system the coordinates of a rocket are determined with respect to some big star.

As was already said, for ballistic rockets the greater part of the trajectory is uncontrolled, and the complete distance should be determined by the combination of parameters on a powered-flight trajectory.

In order to consider the influence of each of the parameters it is necessary to determine the deviation of the distance from the calculated distance caused by the deviation from calculation of each of the parameters. Let us assume that

$$x = F(v_K, \theta_K, x_K, y_K). \qquad (5.92)$$

Let us take the total differential from this function. From mathematics it is known that the total differential is equal to the sum of the partial differentials, then

$$dx = \frac{\partial x}{\partial v_K}\cdot dv_K + \frac{\partial x}{\partial \theta_K}\cdot d\theta_K + \frac{\partial x}{\partial x_K}\cdot dx_K + \frac{\partial x}{\partial y_K}\cdot dy_K. \qquad (5.93)$$

Partial derivatives $\frac{\partial x}{\partial y_K}$, $\frac{\partial x}{\partial \theta_K}$, $\frac{\partial x}{\partial x_K}$, and $\frac{\partial x}{\partial y_K}$ determines the change of the complete distance with a change of each of the parameters separately on an infinitesimal.

It is possible with a certain error to replace the infinitesimal by the finitely small values and to take the linear dependence of the distance on each of the parameters.

Then from formula (5.93) we find:

$$\Delta x = \left(\frac{\Delta x_v}{\Delta v_K}\right)\cdot \Delta v_K + \left(\frac{\Delta x_\theta}{\Delta \theta_K}\right)\cdot \Delta\theta_K + \left(\frac{\Delta x_x}{\Delta x_K}\right)\cdot \Delta x_K + \left(\frac{\Delta x_y}{\Delta y_K}\right)\cdot \Delta y_K. \qquad (5.94)$$

We designate $\mu_v = \frac{\Delta x_v}{\Delta v_K}$, $\mu_\theta = \frac{\Delta x_\theta}{\Delta \theta_K}$, $\mu_x = \frac{\Delta x_x}{\Delta x_K}$ and $\mu_y = \frac{\Delta x_y}{\Delta y_K}$, and will present Δv_K, $\Delta\theta_K$, Δx_K, and Δy_K as differences between the calculated and their real current values:

$$\Delta v_K = v_д - v_K;$$
$$\Delta \theta_K = \theta_д - \theta_K;$$
$$\Delta x_K = x_д - x_K;$$
$$\Delta y_K = y_д - y_K.$$

Then from formula (5.94) we find:

$$\Delta x = \mu_v(v_д - v_K) + \mu_\theta(\theta_д - \theta_K) + \mu_x(x_д - x_K) + \mu_y(y_д - y_K). \qquad (5.95)$$

It is obviously that $\Delta x = 0$ when the real distance coincides with the calculated and

$$\mu_v v_к + \mu_\theta \theta_к + \mu_x x_к + \mu_y y_к = \mu_v v_д + \mu_\theta \theta_д + \\ + \mu_x x_д + \mu_y y_д. \qquad (5.96)$$

For a definite rocket proceeding from the assigned distance can be calculated the left side of the equality (5.96), which is called the linear four-term controlling function or linear four-term functional. This method of control requires measurements in the flight of $v_д$, $\theta_д$, $x_д$, and $y_д$ and a calculation of the right side of equality (5.96) and comparison of it with the precomputed value of the controlling function.

At the time of the approach of equality (5.96) the engine of rocket should be turned off. Since the magnitude and direction of the velocity vector at the time of the turning off of the engine can be determined not only in terms of $v_к$ and $\theta_к$ but also in terms of projections of the velocity vector on right-angle coordinate axes $u_к$ and $w_к$, then the controlling functional can be formulated proceeding from the functional dependence:

$$x = F(u_к, w_к, x_к, y_к).$$

In this case the rocket should be supplied by instruments measuring $u_д$, $w_д$, $x_д$, and $y_д$. The selection of a certain method of formulation of the controlling function depends on concrete conditions. Difficulties of realization of the system of control founded on the use of the four-term controlling functions are evident. At present for rockets of the "surface-to-surface" class having a great unpowered section of the trajectory the method of range control of firing is applied, which is based on the calculation of the unequality of the influence of the above-mentioned parameters on the firing range.

If one were to investigate the influence of small deviations of each of the parameters ($\Delta v_к$, $\Delta\theta_к$, $\Delta x_к$, and $\Delta y_к$) on the deviation of the complete distance, then it will appear that $\Delta x_к$ and $\Delta y_к$ affect Δx little. The influence of $\Delta\theta_к$ depends on the actual magnitude $\theta_к$. Figure 93 shows curves illustrating the dependence of the distance corresponding to the unpowered section of the trajectory on $v_к$ and $\theta_к$. From the graph one can see that the distance is less sensitive to a change in the angle of departure in region of angles close to the angle of maximum range. Therefore, if one were to turn off the engine with the program angle of pitch close to the angle of maximum range, i.e., to assume $\phi_к = \theta_{к\,max}$ (Fig. 75), then the influence of $\Delta\theta_к$ on Δx will be small. From formula (5.95) we will obtain with a

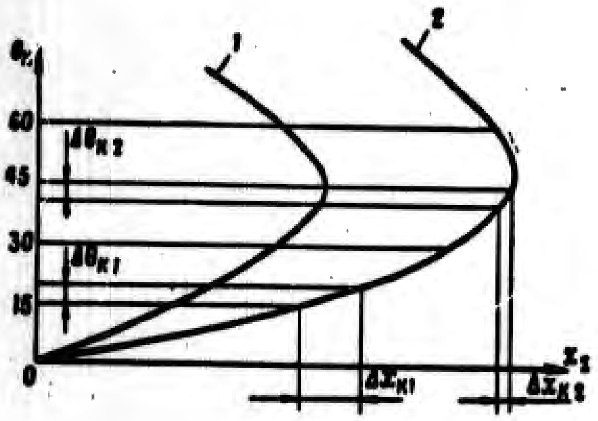


Fig. 93. Dependence of range on speed on v_{κ} and angle of departure θ_{κ}: 1 — for speed $v_{\kappa 1}$; 2 — for speed $v_{\kappa 2}$; $v_{\kappa 2} > v_{\kappa 1}$. When $\Delta\theta_{\kappa 1} = \Delta\theta_{\kappa 2}$ the deviation of range $\Delta x_{\kappa 1} > \Delta x_{\kappa 2}$.

certain error

$$\Delta x = \mu_v (v_\text{д} - v_\text{к}).$$

When $\Delta x = 0$ we will have the monomial linear functional $\mu_V v_\text{к} = \mu_V v_\text{д}$, which corresponds to the control speed. In this case during the flight only the speed is measured and with the achievement of the equality $v_\text{д} = v_\text{к}$ the engine will be turned off. The speed should be measured with as great an accuracy as possible. Contemporary airborne equipment of the measurement of speed, known by foreign sources, produces errors embodied in the very principles of the measurements. However, the accuracy of distance firing proves to be acceptable.

§ 9. Calculation of Trajectories of Ballistic Rockets from Data of Radar Measurements

Ballistic rockets are one of the most threatening forms of rocket weapons, and the development of effective methods of combat with them is one of the most important problems confronting contemporary military technology. For the timely interception and destruction of a ballistic rocket it is necessary as early as possible to know the characteristics of the trajectory of its motion and the possible point of impact. Therefore, in the system of antimissile defense, besides batteries of antimissile missiles there are radar stations of detection and tracking of ballistic rockets. Radar stations are divided by the location of them from the defended object on three: stations of distant detection, stations of accurate tracking, and stations operating in the zone of direct interception. For a reliable operation of the system of antimissile defense early-warning radar stations should be located at a distance of not less than 1600 km from the defended object. They provide approximate data for the calculation of characteristics of the trajectory of the motion of the ballistic rocket. Radar tracking stations are located not nearer than 500-800 km from the defended object, track the target [RDD] (РДД) for several tens of seconds, and continuously determine the coordinates of it with as high an accuracy as possible. Radar stations working directly in the zone of interception should ensure the tracking of the target and antimissile missile, determine their coordinates, and jointly with the guidance system direct the antimissile missile at the point of assumed encounter. Radar stations work in a complex with computers which according to data of radar measurements determine the characteristics of the motion of the rocket [47].

The crew of the trajectory of the ballistic rocket from data of radar measurements is a complicated problem. Let us give an idea of the so-called final methods of calculation determining the dependence between the measured magnitudes and sought parameters of the trajectory. Final methods of calculation are based on the use of laws of the theory of flight. To calculate the trajectories of ballistic rockets the theory of Kepler motions is used.

To calculate the trajectory means to determine the position of the plane of trajectory with respect to the Earth to calculate the dimensions of the trajectory and to determine the transit time of the ballistic rocket of characteristic points of the trajectory, its summit and point of impact.

We arrive at a solution on the assumption that the ballistic rocket moves without drag in a central gravitational field and without taking into account the influence of the rotation of the Earth. Then in accordance with the rocket discussed earlier we will describe elliptic trajectory.

The radar station, located at point C, determines the distance to the target ρ_1, its azimuth β_1, and the angle of elevation of the target α_1 (Fig. 94).

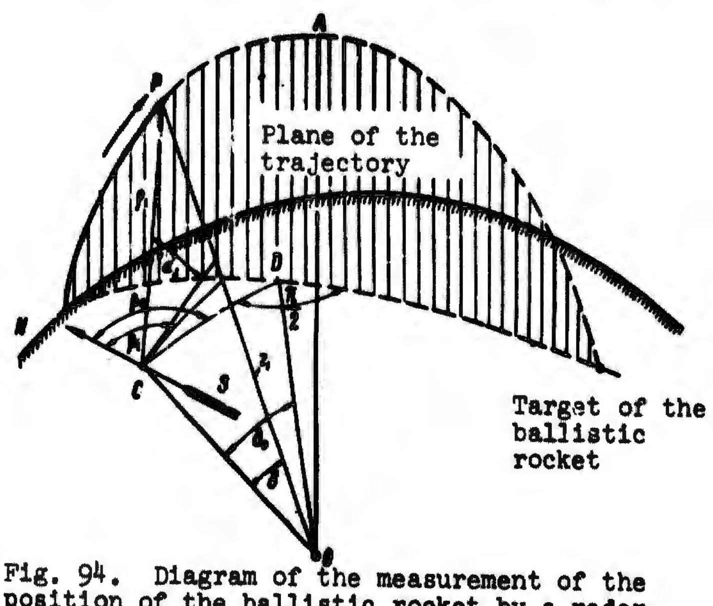


Fig. 94. Diagram of the measurement of the position of the ballistic rocket by a radar station at the time t_1.

In Fig. 94 CPO is the plane of the triangle passing through the location of the radar station (C), the instantaneous position of the ballistic rocket (P), and the conditional center of Earth (O).

To calculate the trajectory from the minimum of data with one radar station it is necessary to have two measurements of azimuths β_1 and β_2, two measurements of angles of elevation of the target α_1 and α_2, two measurement of the slant range ρ_1 and ρ_2 one measurement of the speed of the change in distance (doppler speed), and one recording of the time of observation.

The orientation of the plane of trajectory with respect to the Earth can be determined by two angles: angle δ_0 between the plane of trajectory of the rocket and the line connecting the point of location of the radar station with the center of the Earth (OC) and the azimuth β_0 determining the position of the plane of the triangle [COD](СОД) relative to the direction northward. The plane of the triangle COD is perpendicular to the plane of trajectory of the rocket. From the geometric considerations

$$\operatorname{tg}\delta = \frac{\rho\cdot\cos\alpha}{R+\rho\cdot\sin\alpha}.$$

Having two corresponding measurements ρ and α, it is possible to calculate the two values δ_1 and δ_2. Using the two azimuths β_1 and β_2 and calculated values of δ_1 and δ_2 one can determine β_0 by the formula from spherical trigonometry:

$$\operatorname{tg}\beta_0 = \left[\frac{\operatorname{tg}\delta_1\cdot\cos\beta_1 - \operatorname{tg}\delta_2\cdot\cos\beta_2}{\operatorname{tg}\delta_2\cdot\sin\beta_2 - \operatorname{tg}\delta_1\cdot\sin\beta_1}\right]. \quad (5.97)$$

After the angle of δ_0 will be determined from the equality:

$$\operatorname{tg}\delta_0 = \cos(\beta_0 - \beta_1)\operatorname{tg}\delta_1. \quad (5.98)$$

The major semiaxis and eccentricity of the elliptic trajectory will be determined with the help of angle θ_1, lying in the plane of the trajectory and counted off of the line [OD](ОД)(Fig. 95):

$$\Theta_i = \pi \pm \operatorname{arc\,tg}[\operatorname{tg}(\beta_0 - \beta_i)\sin\delta_0], \quad (5.99)$$

where β_1 is one of the measurements of azimuth of the target.

The minus sign pertains to the direction of the flight shown on the drawing (Fig. 95) and the plus sign to the flight in the opposite direction. On the basis of the measured magnitudes we can calculate also the product of $r^2\dot{\theta}$, the speed of the rocket v_0 and the semimajor axis of the ellipse

$$a = \frac{p}{1-e^2}. \quad (5.100)$$

The trajectory of the rocket with respect to the surface of the Earth can be oriented with help of angle ω_0, determining the position of the peak of the trajectory (Fig. 95). The time of passage of the rocket through the summit t_0 will be determined from the Kepler equation:

$$t_0 = t_1 - (u_i - e\cdot\sin u_i)\sqrt{a^3}, \quad (5.101)$$

where

$$u_i = \arcsin\left[\frac{\sqrt{1-e^2}\cdot\sin(\theta_i-\theta_0)}{1+e\cdot\cos(\theta_i-\theta_0)}\right],$$

where θ_i is the value of the angle corresponding to the moment of time of measurement of data by the radar station, t_1.

Given formulas are constructed taking into account the theory of Kepler motions and are correct for the calculation of elliptic sections of the trajectory. If the radar stations obtain information on rockets moving in the air, for instance, on a powered-flight trajectory, then the methods of calculation and the mathematical formulas should be different. Moreover, the final methods of calculation in pure form cannot be applied, since the radar stations obtain information on the moving target with various kinds of interferences. At the same time range of operation of the radar stations is such that they can give much information during a certain time. This permits, besides the dependences of the theory of flight, using methods of mathematical statistics, which increases the reliability of the calculations. For the obtaining of timely information at the command posts of antimissile defense and at launching sites of antimissile missiles the computers operating in complex with the radar stations should be especially high speed.

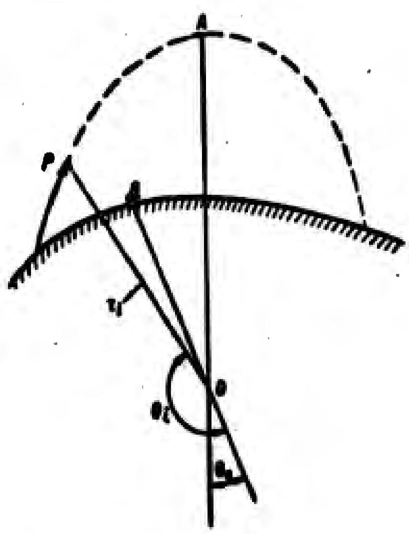


Fig. 95. Determination of the position of the ballistic rocket at the moment of time t_1 in the plane of the trajectory.

CHAPTER VI

STABILIZATION OF ROCKETS IN FLIGHT AND FIRING ERRORS

It is difficult to overestimate the importance of problem of stabilization of space vehicles. If space vehicles (rocket, aircraft) are not stable in flight, then it obviously will be impossible to expect that they will fly correctly in the assigned direction. Guarantee of stabilization flight is part of the general problem of the stability of motion. The problem of the stability of motion was first solved by the well-known Russian scientists: N. Ye. Zhukovskiy in his work "On the Stability of Motion," written in 1882, and A. M. Lyapunov in the work "General Problem on the Stability of Motion," published in 1892. At present the theory of the stability of motion is considerably improved. At the same time the complexity of the mathematical apparatus of the classical theory of the stability and insufficiency of initial information on the newly designed object frequently compels us to look for a less strict, but sufficiently reliable, means of calculating space vehicles on the stability of motion. In this chapter certain examples of similar calculations will be discussed.

Depending upon the assignment of the rocket (space vehicle) the method of the stabilization of it in flight is selected. The stabilization of unguided rockets is possible either by fins or rotation. For guided rockets stabilization is attained due to the correct selection of the aerodynamic shape of a rocket and operation of the flight controls. To guarantee a stable flight and the observance of the necessary characteristics of motion the rocket is supplied with instrument of flight stabilization.

§ 1. Concept on the Stability of Flight

The investigation of any space vehicle, including a rocket, on the stability of motion is carried out in two ways. First it is possible to formulate and solve the complete system of differential equations describing the flight, taking into account the action of all forces, including the disturbing forces, which may cause an incorrect flight of the aircraft. After the solution of the system of equations according to the characteristics of motion we judge the regularity of the flight. This way, although it is a theoretically strict one, is not always used in practice because of the impossibility to have exhausting data on all the perturbing factors, for instance, the action of wind gusts, eccentricity of the tractive force, and others. Secondly, an investigation of stability of motion can be conducted without considering the action of the perturbing factors and without solving the complete system of differential motion equations. Having formulated the differential equations of deviations of elements of the trajectory from the calculated and conducting an analysis of these equations, it is possible to judge the stability of the flight of the aircraft.

In the theory of the investigation of the stability of space vehicles the method of small perturbations has been used much. In this method the deviations of parameters of the perturbed motion from undisturbed motion are assumed to be so small that in the perturbation equations these parameters should be considered in the form of sums containing deviations of the parameter only in the first power. For instance, the speed, angle of incidence, and pitch angle in the perturbed motion can be represented thusly:

$$\left.\begin{aligned} v &= v_* + \Delta v; \\ \alpha &= \alpha_* + \Delta\alpha; \\ \vartheta &= \vartheta_* + \Delta\vartheta, \end{aligned}\right\} \qquad (6.1)$$

where v_*, α_*, and ϑ_* are the speed, angle of incidence, and pitch angle in the undisturbed motion the characteristics of which are determined without taking into account the action of the perturbing factors, and Δv, $\Delta\alpha$, and $\Delta\vartheta$ are deviations of elements of the trajectory after perturbation.

Thus the actual perturbing forces and the effect from their action are not considered. Only changes of the characteristics of motion after the action of perturbing forces are considered on the assumption that these changes are small. The method of small perturbations permits reducing the perturbation equations of the rocket motion to linear differential equations solved relatively simple.

Numerous investigations showed that for symmetric space vehicles to which the majority of rockets belongs it is possible to consider separately longitudinal and lateral stability. This considerably simplifies the calculations.

Let us consider a simpler case of investigation of the stability of motion of a winged missile in rectilinear flight when with the change of mass of the missile and density air ρ cannot be considered. Let us take simplifications of the system (5.12) already utilized by us earlier. We will consider that the vector of the tractive force coincides with the longitudinal axis of the rocket, i.e., $\xi = 0$ and will include X_p and Y_p correspondingly in drag and lift. Then from formulas (5.12) and (5.13) will obtain:

$$\left.\begin{aligned} m\cdot\frac{dv}{dt} &= P\cdot\cos\alpha - X - q\cdot\sin\Theta;\\ mv\cdot\frac{d\Theta}{dt} &= P\cdot\sin\alpha + Y - q\cdot\cos\Theta;\\ I_z\cdot\frac{d^2\vartheta}{dt^2} &= M_z \end{aligned}\right\} \qquad (6.2)$$

For linearization terms entering the right side of the equations of motion are decomposed in Taylor series. Let us recall the mathematical bases of the Taylor expansion assuming that we will further use only the first terms of decomposition. For simplicity we will also take the dependence of each of the terms of the system (6.2) only from two arguments. Similar simplifications, not distorting the essence of the question, permit considerably reducing the writing of the formulas. Taking into account what has been said, the formulas of decomposition can be thus written:

$$f(x+\Delta x, y+\Delta y) = f(x, y) + \left[\frac{\partial f(x,y)}{\partial x}\cdot\Delta x + \frac{\partial f(x,y)}{\partial y}\cdot\Delta y\right], \qquad (6.3)$$

where f(x, y) is the value of functions with constant values of the arguments.

Components included in the brackets are partial differentials of the considered function. From what is written one can see that an increase in the function with a change in arguments on Δx and Δy will be equal to

$$\Delta f(x, y) = f(x+\Delta x, y+\Delta y) - f(x, y). \qquad (6.4)$$

Let us assume that the aerodynamic components, were earlier agreed upon, will depend only on two magnitudes, speed of the flight v and angle of incidence α:

$$X = f_1(v, \alpha);$$
$$Y = f_2(v, \alpha);$$
$$M_z = f_3(v, \alpha).$$

Expanding the last dependence in a series according to formula (6.3), we obtain:

$$\left.\begin{aligned} X&=f_1(v_*, \alpha_*)+\left(\frac{\partial X}{\partial v}\right)_*\Delta v+\left(\frac{\partial X}{\partial \alpha}\right)_*\Delta\alpha; \\ Y&=f_2(v_*, \alpha_*)+\left(\frac{\partial Y}{\partial v}\right)_*\Delta v+\left(\frac{\partial Y}{\partial \alpha}\right)_*\Delta\alpha; \\ M_z&=f_3(v_*, \alpha_*)+\left(\frac{\partial M_z}{\partial v}\right)_*\Delta v+\left(\frac{\partial M_z}{\partial \alpha}\right)_*\Delta\alpha. \end{aligned}\right\} \quad (6.5)$$

The sign * shows that the given magnitude pertains to the undisturbed motion at the time corresponding to beginning of perturbation.

Introducing conventional designations we rewrite formula (6.5) in such a form:

$$\begin{aligned} X&=X_*+X^v\Delta v+X^\alpha\Delta\alpha; \\ Y&=Y_*+Y^v\Delta v+Y^\alpha\Delta\alpha; \\ M_z&=M_{z_*}+M_z^v\Delta v+M_z^\alpha\Delta\alpha. \end{aligned}$$

Similarly we expand terms containing the tractive force P:

$$\begin{aligned} (P\cdot\sin\alpha)&=P_*\cdot\sin\alpha_*+P^v\Delta v\cdot\sin\alpha_*+P_*\cdot\cos\alpha_*\Delta\alpha; \\ (P\cdot\cos\alpha)&=P_*\cdot\cos\alpha_*+P^v\Delta v\cdot\cos\alpha_*-P_*\cdot\sin\alpha_*\Delta\alpha. \end{aligned}$$

Considering the weight of the rocket q on the section of perturbation to be constant we will obtain by the formula of decomposition

$$\begin{aligned} (q\cdot\sin\Theta)&=q\cdot\sin\Theta_*+q\cdot\cos\Theta_*\Delta\Theta; \\ (q\cdot\cos\Theta)&=q\cdot\cos\Theta_*-q\cdot\sin\Theta_*\Delta\Theta. \end{aligned}$$

We linearize the first equation of the system (6.2) having preliminarily substituted on the left side of the equation

$$\frac{dv}{dt}=\frac{d(v_*+\Delta v)}{dt}=\frac{d\Delta v}{dt}.$$

Using the results of expansion in series we obtain

$$\begin{aligned} m\cdot\frac{d\Delta v}{dt}&=P_*\cdot\cos\alpha_*+P^v\Delta v\cdot\cos\alpha_*-P_*\sin\alpha_*\Delta\alpha- \\ &-X_*-X^v\Delta v-X^\alpha\Delta\alpha-q\cdot\sin\Theta_*-q\cdot\cos\Theta_*\Delta\Theta. \end{aligned}$$

Carrying out the corresponding substitutions we obtain for the second equation

$$\begin{aligned} m(v_*+\Delta v)\frac{d\Delta\Theta}{dt}&=P_*\cdot\sin\alpha_*+P^v\Delta v\cdot\sin\alpha_*+P_*\cos\alpha\Delta\alpha+ \\ &+Y_*+Y^v\Delta v+Y^\alpha\Delta\alpha-q\cdot\cos\Theta_*+q\cdot\sin\Theta_*\Delta\Theta. \end{aligned}$$

Linearization of third equation is evident.

Proceeding from geometric considerations we replace

$$\Delta\Theta=\Delta\vartheta-\Delta\alpha.$$

Because of the smallness of α angles we take $\sin \alpha_* \approx \alpha$ and $\cos \alpha_* \approx 1$. Let us omit in the second equation the term $m\Delta v \cdot \frac{d\Delta\theta}{dt}$, having the second order of smallness and use the equality which can be obtained from the system (6.2) for the undisturbed motion:

$$P_* \cdot \cos \alpha_* - X_* - q \cdot \sin \Theta_* = 0$$

$$P_* \cdot \sin \alpha_* + Y_* - q \cdot \cos \Theta_* = 0.$$

As a result of the transformations we obtain the system of equations:

$$\left.\begin{aligned} m\frac{d\Delta v}{dt} &= (P^v - X^v)\,\Delta v - (P_*\alpha_* + X^\alpha - \\ &\quad - q \cdot \cos \Theta_*)\,\Delta\alpha - q \cdot \cos \Theta_* \Delta\theta; \\ mv_*\left(\frac{d\Delta\theta}{dt} - \frac{d\Delta\alpha}{dt}\right) &= (P_*^v \alpha_* + Y^v)\,\Delta v + \\ &\quad + (Y^\alpha + P_* - q \cdot \sin\Theta_*)\Delta\alpha + q \cdot \sin \Theta_* \Delta\theta; \\ I_z \frac{d^2\Delta\theta}{dt^2} &= M_z^v \Delta v + M_z^\alpha \Delta\alpha. \end{aligned}\right\} \quad (6.6)$$

This system consists of linear uniform differential equations with constant coefficients. The method of integration of such equations of reduced to a dimensionless form is well-known. The common solution of the system of equations is the sum of particular solutions of the form $x = ke^{\lambda t}$

$$\left.\begin{aligned} \Delta v &= A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + A_3 e^{\lambda_3 t} + A_4 e^{\lambda_4 t}; \\ \Delta\alpha &= B_1 e^{\lambda_1 t} + B_2 e^{\lambda_2 t} + B_3 e^{\lambda_3 t} + B_4 e^{\lambda_4 t}; \\ \Delta\theta &= C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 e^{\lambda_3 t} + C_4 e^{\lambda_4 t}, \end{aligned}\right\} \quad (6.7)$$

where λ_1, λ_2, λ_3, and λ_4 are roots of the characteristic equation of the fourth power

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0. \quad (6.8)$$

This equation is formulated with the solution of the system of differential equations (6.6).

Coefficients of general solution A_1, B_1, and C_1 and coefficients of the characteristic equation a_1, being real magnitudes, are determined proceeding from coefficients of the basic system (6.6) and initial conditions for each concrete case separately.

Roots of the characteristic equation (6.8) can be real or complex conjugate, and therefore for the general judgement about characteristics of motion of a space vehicle after perturbation a more detailed analysis of equations is necessary (6.6). Conducted investigations showed that three forms of roots of the characteristic equation (6.8) is possible to which three types of the perturbed motion correspond.

If all four roots of the characteristic equations are real the solution of the equation (6.6) will be like the solution of equation (6.7). With this the change of each of the elements of the trajectory will be determined as a resultant four aperiodic functions. If two of four roots will be real, and two will be complex conjugate of the type $\lambda = \xi \pm i_\nu$, the solution of the equation (6.6) written by us for the purpose of reduction only for the change of the pitch angle will have the form

$$\Delta\theta = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + F e^{\mu t} \cdot \sin(\nu t + \gamma_3). \tag{6.9}$$

In this case the change in each of the elements will be determined by the summation of functions characterizing the process of oscillation with two functions characterizing the aperiodic motion. If roots of the characteristic equation are two pairs of complex roots the solution to the equation (6.6) will have the form

$$\Delta\theta = C_1 e^{\alpha t} \cdot \sin(\beta t + \psi_3) + F_1 e^{\mu t} \cdot \sin(\nu t + \gamma_3). \tag{6.10}$$

In this case the change in each of the elements will be determined by the two functions characterizing the process of oscillations.

The change in magnitudes of increases $\Delta\vartheta$, Δv, $\Delta\alpha$, and others is connected with the concept on the stability of the motion of a space vehicle. If increments increase with time the motion will be unstable if the increase does not change in time, then such a space vehicle can be called neutral. If the increase in decreasing approaches zero such an apparatus can be called stable.

Stability of motion is determined by the character of the exponential function $e^{\lambda t}$. If when $t \to \infty$ $\lambda > 0$, the value of the function increases indefinitely, and the motion is unstable. To guarantee the stability it is necessary to have when $t \to \infty$ the value $\lambda < 0$. Then the exponential function will approach zero, and the motion will be stable. The condition of longitudinal stability of the rectilinear established motion of a space vehicle, worked out taking into account the possible types of roots of the characteristic equation, is usually formulated in the following way: to guarantee the longitudinal stability it is necessary and sufficient that the real roots and real parts of the complex roots of the characteristic equation (6.8) be negative.

A qualitative answer to the question on the stability of a space vehicle can be obtained by not determining the roots of the characteristic equation (6.8). To obtain negative values of real roots and real parts of the complex root it is necessary and sufficient to fulfill the condition:

$$a_1 > 0;\ a_2 > 0;\ a_3 > 0;\ a_4 > 0;$$
$$a_1 a_2 a_3 - a_1^2 a_4 - a_3^2 > 0.$$

With the observance of this condition the motion will be stable.

Let us recall that the simple case of constants v and ρ. If v ≠ const and ρ ≠ const then it is necessary to check the stability of the motion for different points of the trajectory. It is especially complicated to solve the problem for rockets designed for combat with surface-to-air high-speed maneuvering targets. If is known beforehand the possibility of at least a brief action of large perturbing forces and moments, then the check according to the method of small perturbations does not give assurance in the stability of the space vehicle. In this case it is necessary to formulate and to solve differential equations of the motion taking into account the perturbing factors. A simple example of a similar approach is given in the following paragraph.

§ 2. The Use of Electronic Analog Computers for the Solution of Problems of the Theory of Flight

Electronic analog computers reproduce continuously in voltages the physical process interesting the researcher. This analogy is attained by the fact that the process occurring in the electrical circuit collected on the machine is described by the same differential equations as the one considered. A change in the different electrical magnitudes including the voltages is easily recorded and obtained in the form of curves. The deciphering of curves with an application of corresponding scales permits having not only a qualitative picture of the phenomenon, but also a numerical value of different characteristics of the process.

Let us consider the peculiarity of separate basic elements of analog computers. Operations of summation, of integration, of inversion of voltage (change of sign), and differentiation are carried out by the computer with the help of electronic amplifiers of direct current. Moreover, the computer contains factors, functionals, and recording elements. To solve a certain problem on the electronic computer it is necessary at first to compile a block diagram consisting of separate sections. For this it is important to known assignment of each of elements and its characteristics.

Let us consider the circuit of operation of a d-c amplifier as a summing element of voltages. Usually the three-stage amplifier used gives at the output a voltage proportional to the input with a reverse sign:

$$y = -kx,$$

where x and y are the input and output voltages, and k is the amplification factor equal approximately to 10^5-10^6.

The switching of the amplifier into the circuit of the computer by the circuit shown in Fig. 96 gives the summing link. With the formulation of the equation of the summing network it is assumed that the amplifier almost consumes no power at the input and that through the resistance feedback R passes the full current $I \cong y:R$, where y is the output voltage of the summing network.

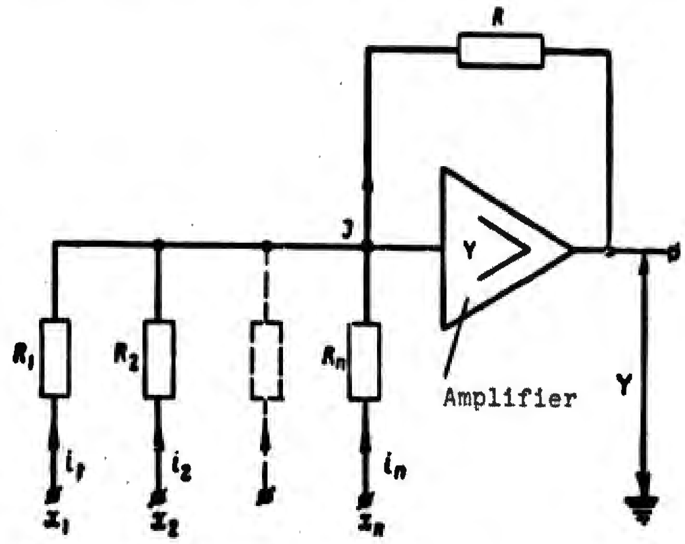


Fig. 96. Diagram of a summing network of an electronic computer.

By not setting a goal to determine the accuracy of the summing network and considering the sign of the output voltage it is possible using the summation rule of forces of currents with parallel inclusion to write

$$I = -\sum_{1}^{n} i_{\nu}$$

where i_B is the current intensity at the inputs to the summating networks.

Using Ohm's law we obtain the voltage at the output of the summating network

$$y \approx -R\sum_{1}^{n} \cdot \frac{x_i}{R_i}, \qquad (6.11)$$

where x_1 and R_1 are correspondingly the voltage and resistance at the inputs to the summing network.

In the case of the equality of all resistances $R_1 = R_2 = \ldots = R_n = R$, we obtain

$$y \approx -\sum_{1}^{n} x_{\nu}$$

and with the equality of resistances and input voltages we obtain at the output the product of the voltage of the input by the constant

$$y \approx -nx, \qquad (6.12)$$

where x is the input voltage, and n is the number of inputs. If one were to apply formula (6.11) to a circuit with one input having an input impedance of R_1 and voltage of x_1 the output voltage will be

$$y \approx -\frac{R}{R_1}\cdot x_1. \qquad (6.13)$$

This formula approximately describes the result given by the circuit of the scaling network.

The assignment of nonlinear functions in analog computers is possible by two methods. For any one model dependence, for instance, y = sin x special converters are used; functions of arbitrary form are carried out with the help of nonlinear converters assembled from separate cells consisting of the coupling of diode elements with the summing amplifier.

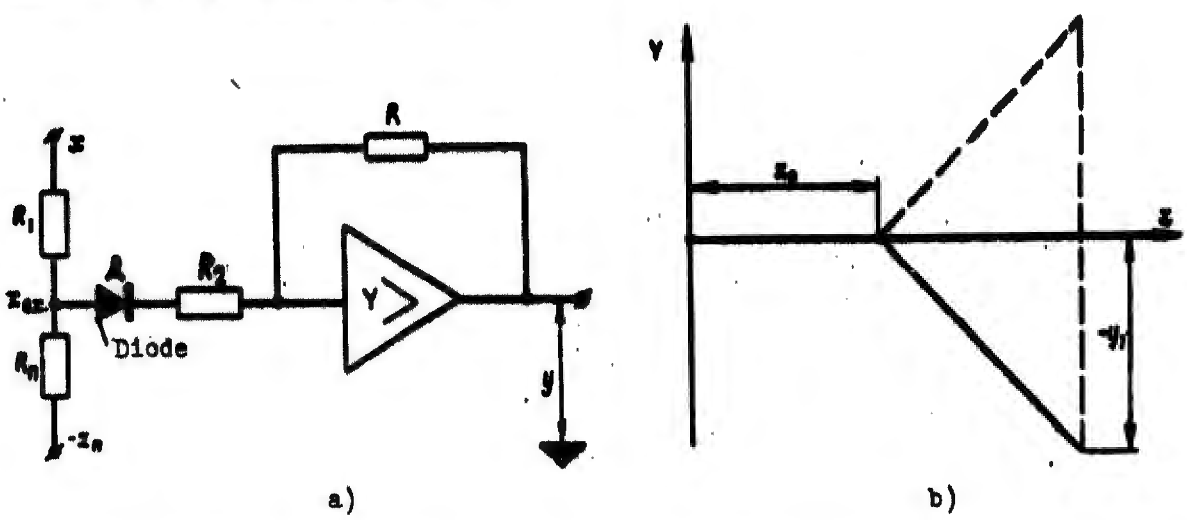


Fig. 97. Diode element of an electronic computer: a) circuit of an element with a diode; b) graph of the voltage of a diode element.

The magnitude of the voltage at which the current will go through the diode is established by the potentiometer proceeding from the formula

$$x_0 = -x_{\Pi}\cdot\frac{R}{R_{\Pi}}, \qquad (6.14)$$

where R_{Π}, and x_{Π} are the resistance and reference voltage (Fig. 97a). With the increase in the voltage x at the input up to $x = x_0$ the output voltage y remains equal to zero, since the diode does not pass a signal to the input of the amplifier (Fig. 97b). When x becomes larger than x_0 the voltage at the input to the amplifier x_{BX} will be positive, the diode will begin to pass the signal, and the voltage at the output of the amplifier will increase proportionally to $x - x_0$ (see the slanted straight line on the graph of Fig. 97b). During this time the output voltage will be determ..ned by the formula

$$y = -(x - x_0)\frac{RR_{\Pi}}{R_1R_2 + R_2R_{\Pi} + R_1R_{\Pi}}. \qquad (6.15)$$

The angle of inclination of the straight line will be determined by the magnitude of the resistances. A combination of several circuits similar to the one described permits with a certain approximation reproducing in voltages the function of arbitrary form assigned to the table containing abscissas and ordinates of points lying on the approximated curve. In Soviet computers of the [IPT](ИПТ), and [MPT](МПТ) types are applied blocks of variable coefficients realizing the step approximation of time-dependent functions (Fig. 98).

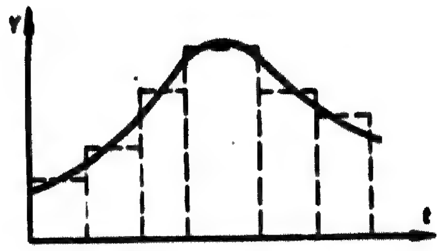


Fig. 98. Step approximation of a time-dependent function.

The integration of the functions is carried out with the help of the integrating network consisting of the amplifier into the feedback of which is switched a capacitor (Fig. 99). Considering as before the amplifier gain to be infinitely large and assuming that the amplifier does not consume power at the input, it is possible to write that charging current of the capacitor is equal to

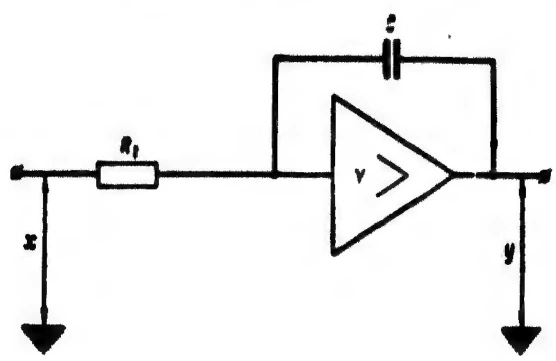


Fig. 99. Circuit of the integrating network.

$$i = \frac{x}{R_1}. \qquad (6.16)$$

Voltage on the terminals of the capacitor will increase with the speed:

$$\frac{dy}{dt} = -\frac{i}{C},$$

where C is the capacity of the capacitor.

Replacing i from the formula (6.16) we obtain the basic formula of the integrating network:

$$y_t = -\frac{1}{CR_1}\int_0^t x dt, \qquad (6.17)$$

where $\frac{1}{CR_1}$ is the constant factor before the integral.

When $CR_1 = 1$

$$y_t = -\int_0^t x dt.$$

Each analog computer consists of a whole series of basic elements and other complicated auxiliary devices described by us.

A solution to the problem on the spatial motion of a guided rocket in a complete formulation (with the smallest number of assumptions as possible) is a rather complicated problem on analog computers. In a number of cases it is impossible to solve the two-dimensional problem within the assumptions usually used in numerical integration. This is explained by the fact that the aerodynamic forces and moments nonlinearly depend on many variables, α, β, v, and others (see the third chapter). The known methods from the mathematical theory of electronic computers allowing the solution to the problem with functions of many variables are also very difficult with practical execution. Therefore, the solution of complicated problems of the theory of flight is more expedient on digital electronic computers. Solved on the computers are problems with additional assumptions determined by peculiarities of a specific computer outlined for use.

Considering what has been said let us consider the motion of the rocket with respect to the center of masses only in one vertical plane on the unpowered section of the trajectory. We consider the characteristics of the motion of the center of mass to be known and determined by some other method for instance, the method of numerical integration.

Let us assume that the center of mass moves rectilinearly with a small change of altitude so that the mass air density can be assumed constant and the angle of inclination of the trajectory to the horizon (θ) equal to zero.

Let us use the formula (5.13) assuming in it $M_p = 0$, since we consider the flight of the unguided rocket, and let us replace M_a by $M_{CT} + M_Д$. Then the equation of the rotation of the rocket with respect to the center of masses can be thusly written:

$$J_z \ddot{\vartheta} = M_B - M_{CT} - M_Д, \tag{6.18}$$

where besides the known stabilizing moment (M_{CT}) and damping moment ($M_Д$), the moment of perturbing forces (M_B) is still introduced. The action of this moment will be assumed during a small interval of time. With the motion of the rocket a similar moment always can be the effect of some briefly effective cause, for instance, a gust of wind or the change in eccentricity of the tractive force with the transition from the booster to the sustainer and others. Using the formula (3.16) for $M_z = M_{CT}$ and expression (3.11) we will write

$$M_{CT} \approx S \cdot \frac{\rho v^2}{2} \cdot l \, |m_z^\alpha| \, \alpha;$$

$$M_Д = S \rho v l^2 \, |m_Д| \, \dot{\alpha}.$$

We will designate:

$$a_1 = \frac{S\rho l^2 |m_z|}{J_z}\left|\frac{1}{m}\right|; \quad a_2 = \frac{S\rho l |m_z^{\alpha}|}{2J_z}\left|\frac{1}{m^2}\right|;$$

$$a_3 = \frac{M_p}{J_z}\left|\frac{1}{\sec^2}\right|;$$

$$f_1(t) = v;$$

$$f_2(t) = v^2.$$

Since in our case $\theta = 0$, then $\vartheta = \alpha$ and $\dot{\vartheta} = \dot{\alpha}$. Let us expand values of the moments entering the right side of the equation (6.18), and let us introduce the accepted designations:

$$\ddot{\vartheta} + a_1 f_1(t)\dot{\vartheta} + a_2 f_2(t)\vartheta = a_3.$$

As a result we obtain the linear second order differential equation with variable coefficients and a constant right side describing the process of oscillations. It cannot be solved in our case by elementary functions but can be solved on the analog computer.

For a solution to the equation we will use the method of lowering of the order of the derivative for which we will replace the obtained second order equation by the equivalent system of two first order equations considering that $\dot{\vartheta} = \omega_z$, where ω_z is the angular velocity of plane oscillations of the longitudinal axis of the rocket with respect to the center of masses:

$$\left.\begin{aligned} \dot{\omega}_z + a_1 f_1(t)\omega_z + a_2 f_2(t)\vartheta = a_3; \\ \dot{\vartheta} = \omega_z. \end{aligned}\right\} \qquad (6.19)$$

Proceeding from the principle of operation of the analog computer the differential equations describing the electrical process occurring in it with the analog formation should be the same as the differential equations describing the real process. In our case the system of differential equations describing the process realized by the electronic computer should have such form

$$\left.\begin{aligned} \frac{dU_{\omega_z}}{d\tau} + \kappa_1 U_{f_1(t)} U_{\omega_z} + \kappa_2 U_{f_2(t)} U_{\vartheta} = \kappa_3; \\ \frac{dU_{\vartheta}}{d\tau} = \kappa_4 U_{\omega_z}. \end{aligned}\right\} \qquad (6.20)$$

where U_{ω_z}, $U_{f_1(t)}$, $U_{f_2(t)}$, and U_{ϑ} are magnitudes of voltages characterizing in the corresponding scale the values ω_z, $f_1(t)$, $f_2(t)$, ϑ. The coefficients considering relationships of scales are designated κ_1, κ_2, κ_3, κ_4.

Let us present voltages in terms of real magnitudes and corresponding scale:

$$U_{\omega_z} = \mu_{\omega_z}\omega_z;$$
$$U_{f_1(t)} = \mu_{f_1(t)} f_1(t);$$
$$U_{f_2(t)} = \mu_{f_2(t)} f_2(t);$$
$$U_\vartheta = \mu_\vartheta \vartheta;$$
$$\tau = \mu_t t,$$

where $\mu_{\omega_z}\left|\frac{\text{v sec}}{1}\right|$; $\mu_{f_1(t)}\left|\frac{\text{v sec}}{\text{m}}\right|$; $\mu_{f_2(t)}\left|\frac{\text{v sec}^2}{\text{m}^2}\right|$; $\mu_\vartheta\left|\frac{\text{v}}{1}\right|$ are the corresponding scales from the voltages; $\mu_t\left|\frac{\text{v}}{\text{sec}}\right|$ is the scale of time during the transition from the real process to analog formation.

Let us introduce the written transition formulas into the system of equations and divide the first equation by $\frac{\omega_z}{\mu_t}$, and the second by $\frac{\mu_\vartheta}{\mu_t}$:

$$\left.\begin{aligned} &\frac{d\omega_z}{dt} + \kappa_1\mu_{f_1(t)}\mu_t f_1(t)\,\omega_z + \\ &+ \frac{\kappa_2\mu_{f_2(t)}\mu_\vartheta\mu_t f_2(t)\vartheta}{\mu_{\omega_z}} = \kappa_3\frac{\mu_t}{\mu_{\omega_z}}; \\ &\frac{d\vartheta}{dt} = \kappa_4\cdot\frac{\mu_{\omega_z}\mu_t}{\mu_\vartheta}\cdot\omega_z \end{aligned}\right\} \qquad (6.21)$$

So that equation (6.21) will be the same as the equation (6.19) it is necessary to calculate the coefficeients in terms of the selected scales. It is necessary to ensure the equalities:

$$\kappa_1 = \frac{a_1}{\mu_{f_1(t)}\mu_t}\left|\frac{1}{\text{v}^2}\right|;\quad \kappa_2 = \frac{a_2\mu_{\omega_z}}{\mu_{f_2(t)}\mu_\vartheta\mu_t}\left|\frac{1}{\text{v}^2}\right|;$$
$$\kappa_3 = \frac{a_3\mu_{\omega_z}}{\mu_t}|1|;\ \kappa_4 = \frac{\mu_\vartheta}{\mu_{\omega_z}\mu_t}\left|\frac{1}{\text{v}}\right|.$$

The numerical values of the scales are selected proceeding from the peculiarities of the computer in such a manner that the operating voltages at the output of the amplifiers will not exceed ±100 volts. Let us conduct multiply, and additionally let us designate

$$U_{f_{11}(t)} = \kappa_1 U_{f_1(t)};\quad U_{f_{21}(t)} = \kappa_2 U_{f_2(t)}$$

After replacement in the formula (6.20) of the infinitesimals by small finals we obtain the system of equations directly collected on the computer:

$$\left.\begin{aligned} &\frac{\Delta U_{\omega_z}}{\Delta\tau} + U_{f_{11}(t)}U_{\omega_z} + U_{f_{21}(t)}U_\vartheta = \kappa_3; \\ &\frac{\Delta U_\vartheta}{\Delta\tau} = \kappa_4 U_{\omega_z}. \end{aligned}\right\} \qquad (6.22)$$

The system of equations is very simple and can be solved on some small analog computer.

Let us describe an example of the solution on the common native small universal model [MN-7] (MH-7).

Model MN-7 is designed for the investigation of nonlinear systems to the sixth order. It consists of a power unit, resolving unit, and an electron oscillograph. The power unit provides power to the computer. The resolving unit with setting field serves for the set of the problem, introduction of initial conditions and constant coefficients, control of the amplifiers and the repetition and registration of the solution.

The cathode-ray oscillograph serves as a visual observation of the character of the investigated process. To record the process to the oscillograph must have a special photoadapter. Recording of the process can also be made on loop oscillograph. The maximum duration of studies process is 200 seconds. The graphic block diagram, composed for the solution of the described problem, is given in Fig. 100. The independent variable with the solution is time; therefore the amplifier 8 ensures a linear dependence between its output voltage and time. It is desirable beforehand at least tenatively to know the duration of the expected process for the selection of the duration of the time of analog formation . The full voltage of output of the amplifier 8 at the end of process should not exceed 100 v. Let us assume, for instance, the duration of the process is 1 sec and μ_t = = 100 v/sec, then 0.01 sec of the real process will correspond to the voltage of 1 v, 0.02 sec, 2 v, etc. Every second the voltage will be equal to 100 v. After amplifier 8 the voltage is inverted by

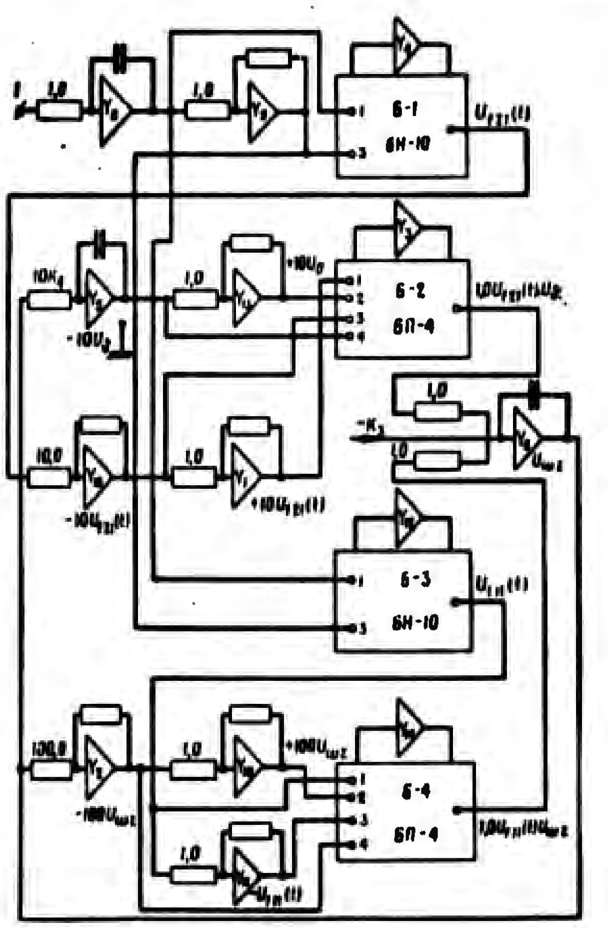


Fig. 100. Structural diagram of the MN-7 model composed for the solution of the problem of oscillations of the longitudinal axis of the rocket.

amplifier 9 and is fed to the nonlinearity blocks Б-1 and Б-3 ensuring the obtaining of functions $U_{f_{11}}(t)$ and $U_{f_{21}}(t)$. The multiplication blocks Б-2 and Б-4 issue the products $U_{f_{21}}(t)U_{\vartheta}$ and $U_{f_{11}}(t)U_{\omega_z}$. It is necessary to remember that the construction of a multiplication block is such that with the supply at its entrance of the two multipliers at the outlet we obtain a product on hundred times smaller. To obtain the full magnitude of the product it is necessary to increase the co-factors so that the product from the coefficients of the increase will equal 100. In the diagram (Fig. 100) voltages $10U_{\vartheta}$ and $10U_{f_{21}}(t)$ are sent to block Б-2 and Б-4, voltages $100U_{\omega_z}$ and $U_{f_{11}}(t)$. From the multiplication units products $U_{f_{21}}(t)U_{\vartheta}$ and $U_{f_{11}}(t)U_{\omega_z}$ are transmitted to the integrating amplifier $У_6$. Here through the timer the voltage corresponding to the perturbing moment and determined by the right side of the equation (6.22) is sent, i.e., the magnitude K_3. The time relay works in program conditions depending on the character of action of the moment.

The voltage U_{ω_z} from the output of the amplifier $У_6$ is sent in two directions: multiplication to the unit Б-4 for the production of the product $U_{f_{11}}(t)U_{\omega_z}$ and to the integrating amplifier $У_5$ whose output voltage is equal to $-10U_{\vartheta}$. Subsequently the voltage $-10U_{\vartheta}$ after inversion is sent as a multiplier to the multiplication unit Б-2.

The magnitude of $У_5$ the voltage taken from amplifier corresponding to the value ϑ expressed in scale is recorded either with the help of the cathode-ray oscillograph and photoadapter or with the help of the loop oscillograph.

Figure 101 gives an example of the recording of $U_{\vartheta} = f(t)$. In the example the voltage corresponding to the perturbing moment and determined by the magnitude K_3 was switched off at the time of the first maximum of the curve $U_{\vartheta} = f(t)$. As can be seen from Fig. 101, by the cessation of action of the perturbing moment oscillations of the longitudinal axis of the rocket quickly attenuate. Having deciphered the recording in the scale of the ordinates it is easy to obtain

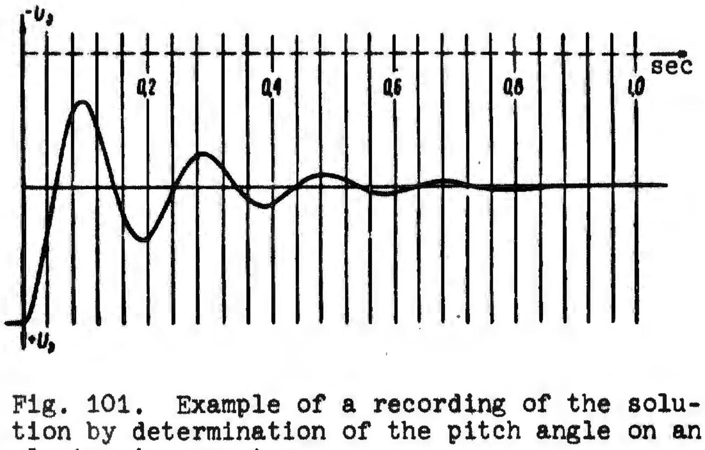


Fig. 101. Example of a recording of the solution by determination of the pitch angle on an electronic computer.

the change in the pitch angle by the time

$$\vartheta(t) = \frac{\nu U_{\vartheta(t)}}{\mu_\vartheta},$$

where ν is the scale of the oscillogram.

Amplifiers Y_3 and Y_4, Y_{12} and Y_{14} provide operation of the nonlinear units in accordance with the construction of the machine itself. The remaining amplifiers shown on the diagram (Fig. 100) and not specially stipulated are intended for the inversion of voltages.

After deciphering by the oscillograms according to the character of the change of the pitch angle ϑ and its greatest magnitude, it is possible to judge the stability of the motion of an object and the correctness of coefficients m_z^α and $m_Д$, set with the aerodynamic designing and established with the solution on the computers at a_1 and a_2. The solution to more complicated problems of the theory of flight will naturally be ensured by more complicated block diagrams. The computers applicable in this case should possess greater possibilities than the simplest computer described by us.

With the analog formation of complicated phenomena connected with the flight of guided rockets it is not always possible to describe reliably enough the work of separate sections of the control or the operation of some instrument by mathematical dependences accurately reflecting the physical picture of the phenomenon. In this case the property of electronic computers is used to work in combination with the actual equipment included in the general block diagram of the solution to the problem in the computer.

With use of electron computers it is necessary to remember that the solution to various kinds of problems is accompanied by certain errors which are sometimes rather considerable. Therefore in solving some problem of the theory of flight it is necessary to evaluate the accuracy of the obtained solution. The methods of evaluating errors with the realization of different dynamic systems in analog computers are described in special literature.

§ 3. Stabilization of Guided Rockets

For simplicity let us consider only the longitudinal motion of a guided rocket of the surface-to-surface class supplied by the automaton of angular stabilization. Earlier we examined the system and operation of controls which provide a stable flight of the rocket in accordance with the program of motion assigned beforehand. The fundamental pitch control circuit is shown in Fig. 102.

During operation the programming mechanism produces electrical signal corresponding to the program change of the pitch angle with time $\vartheta_{\text{пр}}(t)$. At the same time the sensing device measures the real pitch angle ϑ and transmits the corresponding signal to the converter. A comparison of signals of the program and real angles of pitch is transmitted to the steering machines and controls which in turning change direction of the movement of the rocket. The angle of mismatch by the pitch can be approximately connected with the angle of rotation of the controls with help of the so-called static coefficient of the system of control κ by the formula

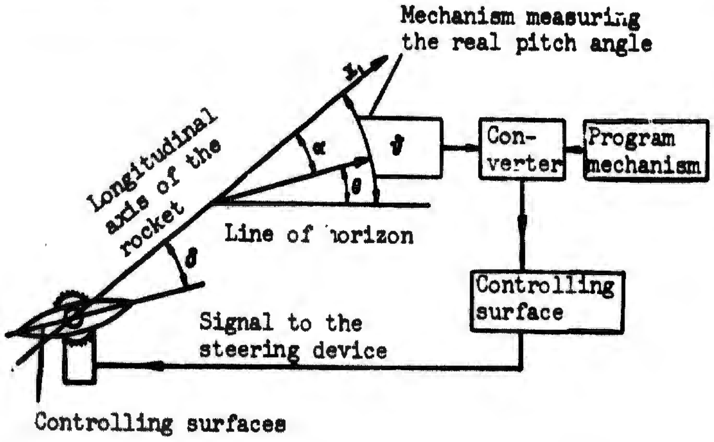


Fig. 102. Fundamental diagram of the stabilization of a guided rocket by a pitch angle.

$$\delta = \kappa(\vartheta - \vartheta_{\text{пр}}). \qquad (6.23)$$

From formula (5.13) we obtain the trim ratio when $\ddot{\vartheta} = 0$:

$$M_z + M_p = 0. \qquad (6.24)$$

Using formula (4.1) we obtain approximately $M_p = Y_{po}\delta L$, where L is taken from Fig. 78.

Having inserted the value of M_p and M_z from formula (3.16) into expression (6.24), we find the following dependence between α and δ:

$$\alpha = -\frac{2Y_{po}L\delta}{\rho v^2 Slm_z^\alpha} = -\varepsilon\delta. \qquad (6.25)$$

The minus sign shows that with a turn of the controlling surfaces to one side, for instance, clockwise, the rocket will turn to the other side (counterclockwise). Comparing formulas (6.23) and (6.25) we will have

$$\alpha = -\varepsilon\kappa(\vartheta - \vartheta_{\text{пр}}).$$

In turn for the wingless rocket the pitch angle is easily presented as the sum of angle $\vartheta = \alpha + \theta$, then

$$\alpha = \frac{\varepsilon\kappa(\vartheta_{\text{пр}} - \Theta)}{1 + \varepsilon\kappa}. \qquad (6.26)$$

To obtain the system of equations describing the controlled flight of a rocket with a stabilizing system we will return to equations (5.12) and (5.14). Let us equate in the expression (5.12) the constant angle ξ to zero and include X_p in the drag X, and Y_p in lift Y. Since the angles of incidence are small we assume $\sin\alpha = \alpha$ and $\cos\alpha = 1$. If in formula (3.15) for brevity we designate $Y_0 = S\frac{\rho v^2}{2} C_y^\alpha$ and substitute the obtained value $Y = Y_0\alpha$ in expression (5.12), then after additional simplifications and transformations from formulas (5.12) and (5.14) we have:

$$\left.\begin{array}{c} \dot{v} = \dfrac{P-X}{m} - g\cdot\sin\theta, \\ \dot{\theta} = \dfrac{(P+Y_0)\,a\kappa\,(\vartheta_{np}-\theta)}{mv\,(1+a\kappa)} - \dfrac{g\cdot\cos\theta}{v}; \\ \dot{y} = v\cdot\sin\theta; \\ \dot{x} = v\cdot\cos\theta; \\ \vartheta_{np}(t). \end{array}\right\} \qquad (6.27)$$

The system has four variables v, θ, x, y dependent on t and can be solved if the program of the pitch angle $\vartheta_{np}(t)$ and the dependence determining the change in mass m(t) are given.

After solution of the system it is possible to judge in the first approximation, without taking into account the action of the perturbing factors, the correctness of the flight of the rocket. The selection of characteristics of control instruments providing a stable flight of the rocket is based on the theory of automatic control.

§ 4. Conditions of Stable Flight of Turbojet Missiles

It is known that a fast-rotating body, for instance, a gyroscope, resists the influence of external forces. If one were to push a gyroscope lightly it will not overturn, but its axis will try to occupy its former (before the push) position and will begin oscillating.

With the motion of a fast-rotating missile the aerodynamic overturning moment tends to overturn it. But the missile, as a gyroscope, in resisting moves stably and does not overturn. Of course, the angular velocity of the rotation of such a missile should be calculated in the appropriate way.

Rocket missiles revolve due to the escape of gas from the slant-positioned nozzles (Fig. 103). The tractive force P forming with the operation of each nozzle can be expanded into two components: P_0 acts along the longitudinal axis of the missile; P_K acts perpendicular to it and directed along the tangent to the circumference passing through the centers of outlets of the nozzles. The sum of the component forces effective along the axis of the missile will transmit a forward motion to it and the sum of all P_K forces will revolve the missile around its longitudinal axis.

GRAPHIC NOT REPRODUCIBLE

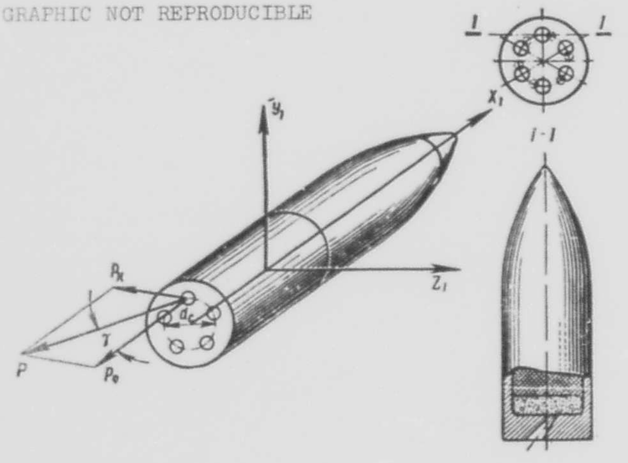


Fig. 103. Diagram of the expansion into vector components of the tractive force of a slant-positioned nozzle of a turbojet missile.

To formulate the equation describing the oscillation of the longitudinal axis of the missile in accordance with the accepted basic assumption we will consider the forward motion of the center of mass of the missile does not affect the character of oscillation near the center of mass. The trajectory of motion of the center of mass will be considered as rectilinear with a constant angle of inclination tangent to the horizon, i.e., θ = const. We will not consider the change in position of the center of mass in flight and moments of inertia.

Let us consider the influence on the missile of the basic perturbing factor of the longitudinal destabilizing aerodynamic moment. We will combine the center of mass of the missile with the beginning of rectangular coordinates the axes of which are parallel to the launch axis (Fig. 104). Let us assume that the velocity vector of the center of mass lies in the vertical plane YOX. The plane passing through the velocity vector $\overline{v}$ and body axis of the missile X_1 will be called the plane of drag. The destabilizing aerodynamic moment M acting in the plane of drag will deflect the axis of the rocket from the velocity vector on the angle of incidence. By analogy with the revolving gyroscope the angle of incidence will be at the same time the nutation angle and is usually designated by δ. The angle between the vertical plane YOX and the plane of drag is called the angle of precession and is designated by ν.

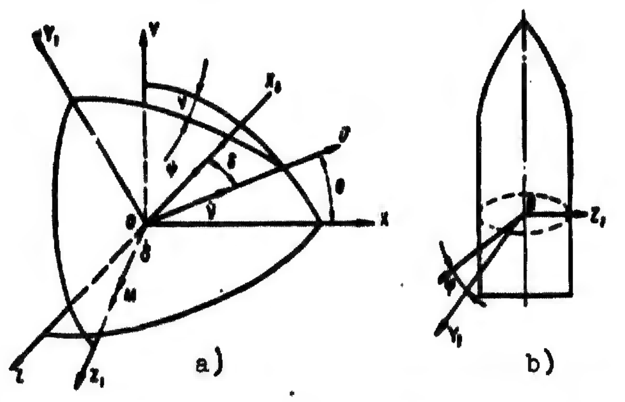


Fig. 104. Diagram of angles determining the position of the revolving missile on the trajectory: a) in a terrestrial system of coordinates; b) in a coupled system of coordinates.

Thus with the accepted assumptions the position of the missile will be determined by the three angles: δ, ν, and the angle φ determining the rotation of the missile around its longitudinal axis. Figure 104 gives the vector of the moment M and vectors of angular velocities $\dot{\delta}$, $\dot{\nu}$, and $\dot{\varphi}$. Equation of the rotation of the missile, formulated with the accepted assumptions, are integrated when v = const and with the following initial conditions: when $t = 0$; $\delta_0 = 0$; $\dot{\delta} = \dot{\delta}_0$ and $\nu = \nu_0$. As a result of integration these formulas can be obtained:

$$\nu = \nu_0 + at; \tag{6.28}$$

$$\delta = \frac{\dot{\delta}_0}{a\sqrt{\sigma}} \cdot \sin(a\sqrt{\sigma}\, t). \tag{6.29}$$

In the first formula a is the almost constant speed of precession:

$$a = \dot{\nu} = \frac{Cr_0}{A(1+\cos\delta)} \approx \frac{Cr_0}{2A}, \tag{6.30}$$

where C is the axial moment of inertia; A is equatorial moment of inertia; r_0 is the intial angular velocity of rotation of the missile about the longitudinal axis.

Since for stable missiles δ does not exceed $\sim 10^\circ$, then without great error it is possible to equate $1 + \cos\delta \approx 2$.

The nutation angle δ, besides α and $\dot{\delta}_0$, depends on the important characteristic $\sigma = 1 - \frac{\beta}{\alpha^2}$. When $\sigma < 0$ angle δ will increase continuously in the process of motion, and, consequently, the missile will be unstable. Formula (6.29) is obtained under the condition that $\sigma > 0$. In this case the nutational oscillations are presented harmonic with the period

$$T = \frac{2\pi}{\alpha\sqrt{\sigma}}.$$

With the action of air medium the oscillations gradually attenuate. Consequently so that the missile is stable on the initial stage of the flight it is necessary that $\sigma > 0$.

What constructive measures will ensure this? The condition of stability, proceeding from $\sigma > 0$, can be written thusly:

$$\frac{\beta}{\alpha^2} < 1. \tag{6.31}$$

The magnitudes on which β depends are determined in many respects by the dimensions of the missile and we designate it:

$$\beta = f[d,\ v^2,\ A,\ C_m,\ H(y)].$$

As was already mentioned, the stability of the missile is ensured owing to its rotation. So that with β assigned beforehand to have the inequality (6.31) it is necessary to increase α or, in conformity with formula (6.30), to increase the angular velocity of rotation r_0. The equation of the rotation of the missile about the longitudinal axis will be written using designations of Fig. 103:

$$C\frac{dr_0}{dt} = P \cdot n \cdot \frac{d_c}{2} \cdot \sin\gamma, \tag{6.32}$$

where n is the number of nozzle holes.

Using the formula (1.13) we replace the full tractive force P_n in terms of $\frac{G_{ceK}}{g}$ and integrate. After integration we get

$$C(r_0 - r_{0H}) = w_e \cdot \frac{d_c}{2g}(\omega - \omega_H)\sin\gamma,$$

where r_0 and r_{0H} are angular velocities of rotation of the missile about the longitudinal axis at the end and beginning of the considered section of the trajectory respectively; ω and ω_H are the fuel consumptions by jet engine toward the end and beginning of the considered section of the trajectory. After converting we obtain

$$r_0 = r_{0H} + w_e \frac{d_c}{2gC}(\omega - \omega_H)\sin\gamma. \tag{6.33}$$

Using the last formula and condition (6.31) it is possible to solve whether the turbojet missile will be stable on the initial section of the flight during the time of operation of the engine. Since the investigation was conducted with the assumptions correct for short rectilinear trajectories, it is necessary to check the stability having divided the whole section of the trajectory into a number of separate short sections. The length of the sections should be such that in order for the beginning of each of them to have $\delta_{0i} = 0$, since the formula (6.29) is obtained proceeding from this condition. For each of the sections it is necessary to take the mean vales of the speed v of the center of mass of the missile, angles θ of the slope of the trajectory, moments of inertia C and A, and other magnitudes. Of course, the characteristics of the motion of the center of mass of the missile on the powered flight trajectory should be calculated beforehand. If the condition of stability (6.31) is not observed the designer should increase the cant angle of the nozzle blocks γ to the forming diameter d_c (Fig. 104).

On the unpowered phase of the flight the curvature of the trajectory turns out to be considerable and it is impossible to assume it to be rectilinear. In this case it is necessary to consider the speed of the lowering of the tangent to the trajectory $\dot{\theta}$. On the initial rectilinear phase of the flight the nutational oscillations and precessional motion are carried out about the vector of the speed of the center of mass of the missile $\bar{v}$. On the curvilinear phase of the flight these motions are accomplished about the so-called dynamic axis of equilibrium not coinciding with the velocity vector $\bar{v}$.

If the missile in the direction of flight revolves from the left upwards to the right, the dynamic axis of equilibrium will also be deflected to the right of the velocity vector (Fig. 105). The angle o determining the position of the dynamic axis of equilibrium can be calculated by the formula

$$\delta_p = \frac{2\alpha}{\beta} |\dot{\theta}|.$$

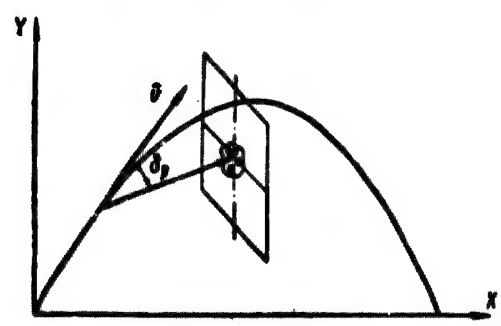


Fig. 105. Position of the dynamic axis of equilibrium of the revolving missile on the curvilinear phase of the trajectory.

The absolute velocity value of the lowering of the tangent is determined from the second equation of the system (5.48). Then

$$\delta_p \doteq \frac{2\alpha}{\beta} \cdot \frac{g \cdot \cos\theta}{v}. \qquad (6.34)$$

With a certain value of the angle δ_p the longitudinal axis of the missile can be disposed at a considerable angle to the velocity vector sufficient for a loss in stability. Therefore the angle δ_p is selected for the criterion of stability with the motion of the revolving missile on the curvilinear phase of the flight. Let us see at what place on the trajectory the angle δ_p will attain the largest value.

The equality (6.34) can be thus transformed:

$$\delta_p = \kappa \cdot \frac{\cos\theta}{H(y)v^3}, \tag{6.35}$$

where κ is the proportionality factor.

From equality (6.35) one can see that the angle δ_p attains the largest value in the region close to the peak of the trajectory, since there the cos θ will be the largest and the product $H(y)v^3$ the least.

Since the point of trajectory at which the angle δ_p will attain the maximum is unknown beforehand, and since it can be found only by means of many repeated calculations, then we usually determine δ_p at the peak of the trajectory. The numerical value of the permissilbe magnitude δ_p depends on concrete conditions. Let us draw attention tothe contradictory nature of the conditions of stability on the initial section of the trajectory and at the peak. To guarantee the inequality (6.31) it is necessary to increase α, i.e., in accordance with formula (6.30) to increase the angular velocity of rotation r_0, and for the decrease of angle δ_p in accordance with formula (6.34) to decrease α and consequently r_0. These two contradictory requirements can be combined for angles of departure up to $\theta < 65^\circ$. Therefore ground artillery does not shoot revolving missiles with angles of elevation of more thant $\sim 65^\circ$. Proceeding from the condition of correct stabilization the full length of the revolving missiles should not be more than 5-6 caliber.

Let us note one more peculiarity of trajectories of missiles stabilized by rotation. Because of the angle δ_p the longitududinal axis of the missile for the greater part of the time of flight is along one of the sides of the vector $\bar{v}$. In accordance with Fig. 105 with a right rotation the axis of the missile in flight will deviate to the right, and the trajectory of the flight of the center of mass will be also curved to the right.

This phenomenon is called derivation. The derivation of a missile at the point of impact can be determined both experimentally and by calculation.

Turbojet missiles are designed for firing at comparatively short distances.

As compared to turbojet missiles fin-stabilized rockets, whose construction is depicted in Fig. 3, posses great possibilities with respect to the increase in distance of firing. For instance, in arming the army of the NATO countries the "Honest John" rocket with a solid-propellant rocket engine is used. With a total weight of 2.2 t weight, a warhead of 680 kilograms, and a propellant weight of 900 kilograms the rocket has a firing range of 32 km.

The essential drawback to uncontrolled fin-stabilized rockets is the great dispersion at firing. There is an especially great dispersion in a lateral direction. The main cause of dispersion is the noncoincidence of lines of force of the thrust with the center of mass and longitudinal axis of ther rocket. Moreover, due to a certain inaccuracy of the manufacture of the rocket in form it is not ideally symmetric, and the total vector of drag also does not pass through the axis of the rocket. The symmetry, of a rocket and the coincidence of its axis with the axis of the nozzle in manufacture are strictly controlled, but nevertheless there are errors. The asymmetry of the tractive force effect and aerodynamic forces is provided by additional moments effective for various rockets in various planes, which leads to the dispersion of rockets at firing. To decrease the influence of asymmetric action of the tractive force we give to the fin-stabilized rockets a rotation with a small angular velocity, the so-called "turning."

The asymmetric action of the tractive force on the rocket during flight changes its direction and deflection from the direction of firing and for each rocket turns out to be less. The dispersion also becomes less.

The turning of fin-stabilized rockets as a means of decreasing the dispersion was first indicated by the well-known Russian artilleryman General K. I. Konstantinov. In his work published in 1856 he wrote: "From the very beginning of the introduction of rockets studies were made to increase the accuracy of the flight of rockets by transmitting to it a rotation about their longitudinal axis." Subsequently the methods of rotation were indicated. Let us give one of them here. Figure 106 shows the cross section of bypasses drilled in walls of the rocket chamber. Solid propellant gases discharging through the channels owing to the influence of the reactive force turn the rocket about the longitudinal axis.

In spite of the turning the dispersion of finned unguided rockets nevertheless appear great, and the firing of such rockets, as a rule, is conducted not at single targets but at a group of targets located in a considerable area.

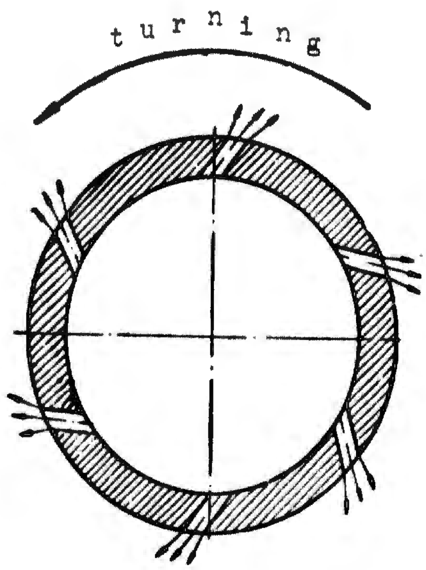


Fig. 106. Diagram of the position of bypasses of a rocket chamber of a turning rocket.

§ 5. Errors in Firing, Correction Formulas and the Dispersion of Rockets

An appraisal of the firing accuracy of rockets and missiles of barrel artillery is based on the theory of errors.

Errors of firing are divided into constant (systematic and accidental). The source of systematic errors acts equally on all shots. For instance, a longitudinal favorable wind increases the firing range opposite the calculation, and a head wind decreases it. Other meteorological factors affect the parameters of the trajectory. If one were to consider in the calculations the influence of the corresponding factor, the systematic errors can be, if not excluded completely, then to a considerable degree decreased. It is necessary to reveal the causes of systematic errors and to consider them in the preparation of firing.

Random errors are the consequence of the combination of accidental changes of separate magnitudes. The entire rocket, its separate constructive units, and components are manufactured with corresponding tolerances for dimensions, weight, and other parameters. Changes in different magnitudes within the tolerance and the diversity in the influence of meteorological factors lead to the dispersion of points of impact of rockets.

Random errors appear without definite order, and causes of their appearance are known sometimes only qualitatively and sometimes are quite unknown. Mathematical statistics and probability theories give regularities allowing the estimation of random errors and the consideration of their influence during calculation of the effectiveness action of rockets at the target.

Special complexities are encountered during calculations of errors of the firing of guided rockets, for which the number of factors affecting the deflection of the trajectory of the flight from that calculated is especially great.

Let us consider the influence on parameters of the trajectory of constantly effective factors. These factors are considered by means of transformation of differential equations describing the flight. For example, in the fifth chapter there are equations describing the motion of a rocket. These equations are different for cases of calculation of the curvature of the Earth and its rotation and without calculation of these factors. If a calculation of parameters of the trajectory is conducted without taking into account the curvature of the Earth and its rotation, then the error dependent on the method of calculation of distance will enter into the common error firing. A comparison of results of the calculation by a certain system of differential equations will give a magnitude of error dependent on the method of calculation. The calculation of the changeability of certain meteorological factors, for instance, variable in altitude and wind direction and others, requires the formulation of new differential.

In many cases new differential equations need not be formulated. An example of calculation of the curvature of the Earth without formulation of new differential equations is examined in the second chapter. In those cases when the change of some magnitude has little effect on the range, its influence can be considered with the help of corrections.

In formula (5.94) the dependence of a small change in the firing range on the small changes in magnitudes determining the distance is given. In the theory of corrections the partial derivatives

$$\frac{\partial x}{\partial v_{\kappa}} \approx \frac{\Delta x_{v}}{\Delta v_{\kappa}};\ \frac{\partial x}{\partial \Theta_{\kappa}} \approx \frac{\Delta x_{\Theta}}{\Delta \Theta_{\kappa}};\ \frac{\partial x}{\partial x_{\kappa}} \approx \frac{\Delta x_{\kappa}}{\Delta x_{\kappa}};\ \frac{\partial x}{\partial v_{\kappa}} \approx \frac{\Delta x_{\kappa}}{\Delta y_{\kappa}}$$

etc., are called correction factors. With the calculation of corrections for barrel artillery the partial derivatives $\frac{\partial x}{\partial v_0}$, $\frac{\partial x}{\partial \theta_0}$, and $\frac{\partial x}{\partial c}$ are called basic correction factors. Certain correction factors for other magnitudes can be calculated by basic correction factors. Especially widespread is the calculation of corrections for a change in meteorological factors. Basic ballistic calculations are made for normal meteorological conditions. Rockets are launched in real meteorological conditions which according to certain parameters differ from data the standard atmosphere. The influence of these deflections on the distance is considered with the help of the theory of corrections. If there are precomputed corrected coefficients, then it is not complicated to calculate corrections in distance.

Let us assume that for assigned conditions of launch the correction factor for the air temperature is $\frac{\partial x}{\partial t} = 34.3$. This means that with a change in air temperature of 1° the firing range changes 34.3 m. Let us assume that at the moment of launch the temperature decreases opposite that of the calculated by $\delta\tau_0 = 15^{\circ}$. Then the correction in distance for the change in temperature will be

$$\delta x_{\tau} = \frac{\partial x}{\partial \tau_0} \cdot \delta\tau_0 = -34{,}3 \cdot 15 = -514 \text{ м.}$$

Similarly corrections are computed separately on each of the changeable elements. The full correction in distance is calculated as the sum of separate corrections, each of which is undertaken with its sign. The calculation of corrections is conducted during the preparation of initial data for firing and with the processing of results of tests when the obtained experimental distances are reduced to normal initial conditions. Data on corrections are placed in the firing tables applied to the rocket complex or artillery system. So that it is possible to determine the deviation of meteorological factors from the normal it is necessary to conduct sounding of the atmosphere before firing with the determination of real values of meteorological elements. The method mentioned above of the calculation of corrections permits calculating them if the change in meteorological elements if constant along the whole trajectory. Therefore, the changes variable in altitude of meteorological elements are replaced by certain average values. These values obtained the name of ballistic averages. Thus, the average ballistic change in temperature is in a special form is the calculated mean deviation of temperature from the normal law, which is mentioned in the second chapter.

The concept "average ballistic wind" is also known. The calculation of correction factors and ballistic averages is a complicated problem. Special complexity are encountered during the calculation of trajectories consisting of separate sections calculated differently. In the second chapter the influence of a cross wind on a rocket missile on the powered and unpowered sections of the trajectory is described. The calculation of corrections for the wind is also a complicated and laborious problem. The actual corrections are calculated with certain errors which should enter into the general error of the firing.

It would be incorrect to judge the accuracy of firing according to one shot, since this very result is accidental. To estimate the accuracy of firing is used a mean or probable error is used. In probability theory it is proved that the probability in obtaining an error within $\pm\Delta$ is expressed by formula

$$p = \frac{h}{\sqrt{\pi}} \int_{-\Delta}^{+\Delta} e^{-h^2\Delta^2} d\Delta, \qquad (6.36)$$

determining the area under the Gaussian curve (Fig. 107):

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2\Delta^2}, \qquad (6.37)$$

where e is the base of natural logarithms; h is the constant called the degree of accuracy.

The dependence $y(\Delta)$ is called the probability density or law of distribution of continuous random variable. The Gaussian curve is called the normal law of distribution. The probability of some event is called the ration of the possible number of cases favoring the given event to the number of all possible results of the tests. Obviously the probability can change from 0 to 1. If the probability of the fact that with a given measurement the error will not exceed the bounds of $\pm$E, equal to 1/2 then this limit is called the average or probable error. Mathematically this can be written:

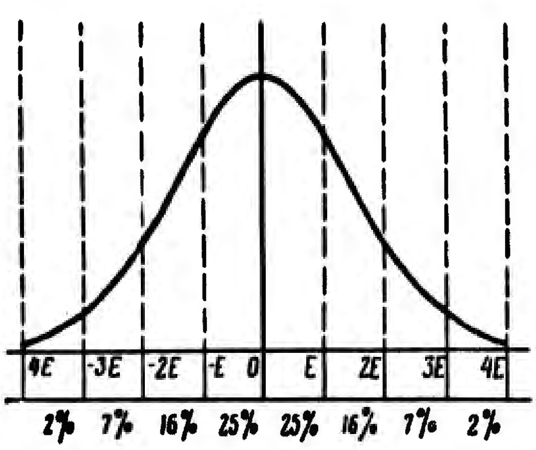


Fig. 107. Normal law of distribution of errors.

$$\frac{h}{\sqrt{\pi}} \int_{-E}^{+E} e^{-h^2\Delta^2} d\Delta = \frac{1}{2}, \qquad (6.38)$$

where E is the mean error.

According to the Gaussian curve the probability of obtaining errors in certain limits expressed in probable errors independently of the measured magnitude depends only on the number of mean errors in a selected interval.

Figure 107 gives the approximate distribution of values of probabilities of obtaining errors within the limits of $\pm$4E. The percent corresponds to the part of the area under the Gaussian curve lying within limits expresred in E. By more exact calculations limits $\pm$4E, covers 99.3% of all the area under the curve if one were to integrate within limits of $\pm\infty$. This means with a large number of tests written limits of ±4E, 99.3% of all measurements will fall. The actual value of the probable error with a finite number of tests is determined by the formula

$$E = 0{,}6745 \sqrt{\frac{\sum_{1}^{n} \Delta_i^2}{n-1}}, \qquad (6.39)$$

where Δi is the deviation of the result of measurement from the arithmetic means, and n is the number of measurements.

The measure of dispersion of impact points of rockets and missiles can be assumed to be the average or probable deviations determined by the formula (6.39). The distance dispersion is determined by the range probable error $B_{\text{Д}}$. With the vertical surface firing (shield) the probable deflection in a vertical direction $B_{\text{В}}$ is determined. With firing by site and by shield the lateral dispersion is characterized by a probable deviation in the lateral direction $B_{\text{б}}$.

The named probable deviations are calculated by the formula (6.39). During firing deviations of impact points of separate rockets (missiles) are measured by some method from the selected beginning of the countdown. In the direction of firing we designate these deviations x_i and in lateral direction, z_i. Coordinates of the center of grouping of impact points are obtained as arithmetic mean values:

$$x_0 = \frac{\sum_{1}^{n} x_i}{n} \quad \text{and} \quad z_0 = \frac{\sum_{1}^{n} z_i}{n}.$$

Designating the deviations from the arithmetic mean values in terms of

$$\Delta x_i = x_i - x_0 \quad \text{and} \quad \Delta z_i = z_i - z_0,$$

by the formula (6.39) we obtain

$$B_{\text{д}} = 0{,}6745 \sqrt{\frac{\sum_{1}^{n} \Delta x_i^2}{n-1}};$$

$$B_{\text{б}} = 0{,}6745 \sqrt{\frac{\sum_{1}^{n} \Delta z_i^2}{n-1}}.$$

With a large number of tests the impact points of rockets are located in the area inside the ellipse called the ellipse of dispersion with semiaxes $4B_{\text{Д}}$ and $4B_{\text{б}}$. In the absence of abnormal shots connected with malfunctionings of the material part, within limits of the ellipse of dispersion there should be 99.3% of all the impact points. If one relates the probable deviations to the firing range we will obtain characteristics of accuracy of fire: $\frac{B_Д}{X}$ and $\frac{B_б}{X}$.

Characteristics of dispersion $B_Д$, $B_б$, and $B_в$ are total, and they depend on the dispersion of separate factors determining them.

If the probability characteristics of dispersion of separate determining factors are known, the expected characteristics $B_Д$, $B_б$, and $B_в$ can be calculated beforehand. For such calculations we use the formula of addition of mean errors known from probability theory:

$$E = \sqrt{\sum_{1}^{n} E_i^2},$$

where E_i is the mean error determined by the independent action of only one cause

Let us consider a simpler case of dispersion of missiles of barrel artillery. The dispersion of missiles occurs due to a change from shot to shot v_0, θ_0, C. The influence of dispersion of meteorological factors for one firing cannot be considered. As a rule, the causes provoking a change v_0, θ_0, and C act independently of each other. Let us designate the mean deviations: v_0 in terms of r_v, θ_0 in terms of r_θ, and C in terms of r_c. Magnitudes corresponding to a change in distance in the case of action of only one cause are:

$$\delta X_{v_0} = \frac{\partial X}{\partial v_0} \cdot r_v; \quad \delta X_\theta = \frac{\partial \theta}{\partial \theta_0} \cdot r_\theta; \quad \delta X_c = \frac{\partial X}{\partial C} \cdot r_c,$$

where $\frac{\partial x}{\partial v_0}$, $\frac{\partial x}{\partial \theta_0}$, and $\frac{\partial x}{\partial c}$ the correction factors already known us.

Since each of the causes acts independently of each other, the calculated range probable error is determined by the formula

$$B_Д = \sqrt{\left(\frac{\partial X}{\partial v_0} \cdot r_v\right)^2 + \left(\frac{\partial X}{\partial \theta_0} \cdot r_\theta\right)^2 + \left(\frac{\partial X}{\partial C} \cdot r_c\right)^2}. \qquad (6.40)$$

A reverse way of calculation of characteristics of dispersion of one of the determining magnitudes is possible according to data of full dispersion. For instance, if one were to shoot from the barrel system at an angle of elevation close to the angle of maximum range and to determine the experimental $B_Д$, then knowing that at angles close to the angle of greatest distance, $\frac{\partial X}{\partial \theta_0} \approx 0$, to find (6.40) the magnitude

$$r_c = \frac{\partial C}{\partial X} \sqrt{B_Д^2 - \left(\frac{\partial X}{\partial v_0} \cdot r_v\right)^2}.$$

The magnitude r_v is determined very simply from results of ballistic firings using the formula

$$r_v = 0{,}6745\sqrt{\frac{\sum_1^n (\Delta v_i)^2}{n-1}}, \qquad (6.41)$$

where Δv_i is the deviation of the initial speed on separate shots of v_i from the arithmetic mean speed in the group

$$v_{cp} = \frac{\sum_1^n v_i}{n};$$

$$\Delta v_i = v_{cp} - v_i.$$

It is considerably more complicated to calculate the expected characteristics of dispersion of unguided rockets for which the number of factors affecting dispersion are considerably greater. It is necessary, for instance, to know the component of dispersion dependent on the dispersion of unit pulse of fuel

$$\delta X_J = \frac{\partial X}{\partial J} \cdot r_J,$$

on the dispersion of weight of the charge

$$\delta X_\omega = \frac{\partial X}{\partial \omega} \cdot r_\omega$$

and on the inaccuracy of determination meteorological factors, especially wind.

Still more complicated is the question of the calculation of expected dispersion of guided rockets, which should be solved together with an appraisal of the accuracy of work of the control system. The matter is complicated more by the fact that each type of rocket has its errors typical for a given method of control.

As was stated in Chapter Five the control of rockets of the "surface-to-surface" in speed is realized by the linear controlling functional $\mu_v v_K$. At the same time for a more accurate control four-term controlling functional is required in accordance with the left side of the equality (5.96). Replacement of the four-term functional by monomial gives an error called the error of the control method or systematic error. In the considered case this error will be

$$\delta X_M = \mu_\theta \theta_K + \mu_x x_K + \mu_y y_K.$$

The determination of partial derivatives $\frac{\partial X}{\partial v_K}$, $\frac{\partial X}{\partial \theta_K}$ and others by linear increases $\frac{\Delta X_V}{\Delta v_K}$, $\frac{\Delta X}{\Delta \theta_K}$, and others also gives a certain error called the error of linearization. Instruments measuring the speed of the rocket also give a certain error called the instrument error. Both of these errors can also be called methodical since they depend on the control method of the rocket. It would be possible to name other sources of errors.

For rockets intended for combat with fast-moving targets the basic errors are considered to be errors dependent on maneuvering qualities of the actual rocket and instrument errors dependent on the control system. The great error in guiding to a target can be the inertness of the control complex.

Let us consider the error dependent the maneuvering qualities of a rocket. Maneuverability can be determined either by the minimum possible radius of the trajectory's curvature of the motion of the center of mass or by the maximum possible lateral acceleration $a_{пр}$, on which normal overloads depend. For example, we obtain the formula determining the possible error during the guiding of a rocket by the linear curve method described in the fifth chapter. From formula (5.39) we obtain that the distance between the rocket and target (Fig. 77) is

$$r = \frac{v_p v_ц \cdot \sin\beta}{a_{пр}}.$$

Let us determine the minimum value r for the case $p < 1$. Let us recall that p is the relation of the speed of a rocket v_p to the speed of the target $v_ц$. With a miss the function r(t) should have minimum and the derivative $\frac{dr}{dt}$ should be equal to zero. Proceeding from formula (5.37), $\frac{dr}{dt} = 0$ if the $\cos\beta = p$. Since $\sin\beta = \sqrt{1 - p^2}$ the minimum distance between the rocket and target with a miss (error of the shot) is determined by the formula

$$r_{min} = \frac{v_p v_ц \sqrt{1-p^2}}{a_{пр}}. \quad (6.42)$$

With high-speed targets in the case considered the miss will be very great. For the case of firing at relatively low-speed targets (naval or ground) the magnitude of the miss can be insignificant.

Similarly it is possible to derive formulas determining the error dependent on the maneuvering qualities of a rocket and for other methods guiding to a target.

The minimum radius of the curvature, frequently utilized in plottings of trajectories of the maneuver of a rocket and the determination of zones of striking, can be determined by the well-known formula of kinematics

$$\rho = \frac{v_p^2}{a_{пр}}.$$

It is necessary to consider that the striking of a target is determined not only by the accuracy in guiding. The calculation of the effectiveness of firing should be made taking into account all the effective factors, including factors dependent on the construction and work of the rocket warhead. This is a great problem and goes beyond the limits of this book.

CONCLUSION

The prospects of the development of rocket technology are infinite. The construction of rockets and systems of control of their flight improves at a continuously fast rate. That time is near when rockets will be supplied nuclear and nuclear-electrical engines which will incommensurably expand the possibilities of their practical application.

In close contact with aerodynamics, electronics, theory of automatic control, study of materials, and many other sciences, the theory of the flight of rockets has been developed. The requirement of the facilitating of construction and increasing the accuracy and reliability of the flight control systems forces more accurately the calculation of elements of the trajectory of the rocket and parameters of its oscillatory motion around the center of mass. This compels a rejection of a number of simplifying assumptions, requires the attraction of a more strict and accurate mathematical apparatus for the investigation of equations, and forces a seeking of new effective ways of solving the problem of the theory of flight.

For recent years the complexity of the solution of problems of the theory of flight has been characteristic. In calculations of a trajectory it is difficult to reject the calculation of the work of the control and stabilization system. With the investigation of the entry to the dense layers of the atmosphere of nose cones of long-range rockets, it is necessary to solve jointly the equations of motion, heat transfer, and ablation of protective coverings, and so forth.

The complexity of the theory of flight of rockets and its rapid developement require of persons working in this direction much preparation in the fields of mathematics, theoretical mechanics, aerodynamics, electronics, and many other sciences.

The authors hope that their labor will help to understand better the essence of phenomena accompanying the flight of rockets of different assignment and will impel a deeper and manifold study of the theory of the flight of rockets, the science without which further progress of rocket construction and strengthening of the defensive capability of our country is impossible.

LITERATURE

1. Mayyevskiy, N. V. A course in external ballistics. St. Petersburg, 1870.

2. Zabudskiy, N. A. External ballistics. St. Petersburg, 1895.

3. Tsiolkovskiy, K. E. Transactions on rocket technology. Oborongiz, 1947.

4. Meshcherskiy, I. V. Works on the mechanics of bodies of variable mass. State Technical Press, 1952.

5. Tsander, F. A. Problems of flight with the help of rockets. Oborongiz, 1947.

6. Chernyskev, M. G. Role of the Russian scientific-technical concept in the development of foundations of rocket flying. Publication of the Moscow Higher Technical School im. Baumann, 1949.

7. Bulinskiy, V. A. Dynamics of maneuvering of fighter aircraft in aerial combat. Voyenizdat, 1957.

8. Kosmodem'yanskiy, A. A. Lecture on the mechanics of bodies of variable of mass. Scientific notes of Moscow State University im. Lomonosov. Mekhanika, Issue 154, 1951.

9. Okhotsimskiy, D. Ye. The theory of motion of rockets. Applied Mathematics. Mekhanika, Vol. X, Issue 2, 1946.

10. Okhotsimskiy, D. Ye., and T. M. Eneyev. Certain variational problems connected with the launching of an artificial earth satellite. Progress of Physical Sciences. Vol. 63, Issue 1a, 1957.

11. Arzhanikov, N. S. and V. N. Mal'tsev. Aerodynamics. Oborongiz, 1956.

12. Mel'nikov, A. P. Aerodynamics of high speeds. Voyenizdat, 1961.

13. Krasnov, N. F. Aerodynamics of solids of revolution. Oborongiz, 1958.

14. Kibardin, Yu. A. and others. Atlas of gas-dynamic functions at high-speeds and high temperatures of the air flow. Gosenergoizdat, 1961.

15. Chushkin, P. I. and N. P. Shulishina. Tables of supersonic flow near blunted cones. Computer center of the Academy of Sciences of the USSR, 1961.

16. Drakin, I. I. Aerodynamic and radiant heating in flight. Oborongiz, 1961.

17. Sakharov, G. I. and others. Heating of bodies in motion with great supersonic speeds. Oborongiz, 1961.

18. Martynov, A. K. Experimental aerodynamics. Oboronigiz, 1958.

19. Vetchinkin. V. P. The dynamics of aircraft. Gosmashmetizdat, 1933.

20. Pyshnov, V. S. Dynamic stability of aircraft. Oborongiz, 1951.

21. Ostoslavskiy, I. V. Aerodynamics of aircraft. Oborongiz, 1957.

22. Lebedev, A. A., I. V. Strazheva, and G. I. Sakharov. Aeromechanics of aircraft. Oborongiz, 1955.

23. Vedrov, V. S. and M. A. Tayts. Flight tests of aircraft. Oborongiz, 1951.

24. Bodner, V. A. and M. S. Kozlov. Stabilization of aircraft and automatic pilots. Oborongiz, 1961.

25. Loytsyanskiy, L. G. and A. I. Lur'ye. A course in theoretical mechanics. State Technical Press, 1948.

26. Feodos'ev, V. I. and G. B. Sinyarev. Introduction to rocket technology. Oboronigiz, 1960.

27. Investigation of optimum conditions of the motion of rockets. Collection of translations of foreign articles edited by I. N. Sodovskiy, Oborongiz, 1959.

28. Shapiro, Ya. M. External ballistics. Oborongiz, 1946.

29. Gantmakher, F. R. and L. M. Levin. Theory of the flight of uncontrolled rockets. Fizmatgiz, 1959.

30. Ostroslavskiy, I. V. and I. V. Strazheva. The dynamics of flight. Oborongiz, 1963.

31. Pogorelov, D. A. Theory of Kepler motions of aircraft. Fizmatgiz, 1961.

32. Rosser, D., P. Newton, and G. Gross. Mathematical theory of the flight of unguided rockets. Foreign Literature Publishing House, 1950.

33. Kooy, I. and I. Yutenbogart. The dynamics of rockets. Oborongiz, 1950.

34. Lokk, A. S. Missile control. Fizmatgiz, 1958.

35. Barrer, M. and others. The motion of rockets. Foreign Literature Publishing House, 1959.

36. Devis, L. and others. External ballistics of rockets. Voyenizdat, 1961.

37. Schlichting, G. The formation of turbulence. Foreign Literature Publishing House, 1962.

38. Problems of the motion of the nose cone of long-range rockets. Collection of Articles. Foreign Literature Publishing House, 1959.

39. Vermishev, Yu. Kh. Rocket control. Voyenizdat, 1961.

40. Shternfel'd, A. A. Artificial satellites. Tekhteorizdat, 1958.

41. Aleksandrov, S. G. and R. Ye. Fedorov. Soviet satellites and spaceships. Publishing House of the Academy of Sciences of the USSR, 1961.

42. Gutkin, L. S. Principles of radio control of pilotless objects. Publication, Soviet Radio, 1959.

43. Lebedev, A. A. and L. S. Chernobrovkin. Dynamics of the flight of pilotless aircraft. Oborongiz, 1962.

44. Eterman, I. I. Mathematical machines of continuous operation. State Publishing House of Scientific and Technical and Machine Building Literature, 1957.

45. Birkgan, A. Yu. and G. P. Voskresenskiy. Programming for the digital computer "Ural-2." Publication Soviet Radio, 1962.

46. Dmitrievskiy, A. A. and V. N. Koshevoy. Physical bases of the flight of rockets. Voyenizdat, 1962.

47. Shapiro, I. I. Calculation of trajectories of ballistic missiles according to data of radar observations. Foreign Literature Publishing House, 1961.

48. Aerospace Engineering. No. 5, Vol. 21, May 1962.