A NOTE ON YIELD CURVES
IN CYCLIC WORK SOFTENING

W. R. POWELL

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W. R. Powell

Division of Engineering
Brown University
Providence, Rhode Island

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A few exploratory tests were conducted to determine the effect of cyclic work softening on the mechanical properties of a hard commercially pure aluminum. Cyclic work softening, or fatigue softening, is the loss of mechanical strength and the accompanying reduction in surface hardness which some materials show under cyclic loading [1].[2] Like hardening under cyclic loads, softening under cyclic loads is limited and usually takes place only during the first 10-25% of the fatigue life. Cyclic hardening or softening occurs only if there is plastic strain produced by each loading i.e. only if there is a mechanical hysteresis loop for the cyclic loading. Note that the material exhibits softening only in a cycle or on the reversal of load. Materials which soften cyclically do show normal stable strain hardening under monotonic loading [2].

The material was 1100-H14 aluminum (Young's modulus $= 10^7$ psi, shear modulus $= 6.4 \times 10^6$ psi, 0.2% offset yield $= 23,750$ psi in tension). The specimens were thin walled tubes with 1" O.D., 0.065" wall thickness, and 5" gage length. The tests were conducted in a combined tension and torsion testing machine [3] so that the yield curve of a specimen in σ-τ

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1.   Graduate student at Brown University, Providence, R. I.

2.   Numbers in brackets designate references at end of Note.

space could be found without having to remount the specimen. In all tests the yield criterion was taken to be 50μ in/in of offset strain. This offset is large enough to be determined accurately and still is small compared to the strain range used in the cyclic tests. The strain range, $\Delta\epsilon$, is the total width of the hysteresis as shown in Fig. 1 , and is the controlled variable in the cyclic tests. The strains were measured by a resistance strain gage bridge whose output was fed into the X-axis of an XY recorder. The load cell output was fed into the Y-axis to give the mechanical hysteresis directly. The loading rate was approximately 1/10 cycles per minute.

First the initial yield curve, shown in Fig. 2 , was found. This curve is described closely by the von Mises yield condition [4] ($\sigma^2+3\tau^2=3k^2$) with k = 8130 psi. Next one specimen was loaded cyclically in torsion. The first ten cycles were at $\Delta\epsilon$ = 0.01 and produced only a 1% decrease in the stress range, $\Delta\tau$. An additional ten cycles at $\Delta\epsilon$ = 0.02 showed a 5% decrease in $\Delta\tau$. The twenty-first cycle, also at $\Delta\epsilon$ = 0.02 , was interrupted twice, at the positions shown in Fig. 1 , to obtain the softened yield curves of the aluminum. Yield points were determined for four loading paths: increasing positive torsion, increasing negative torsion, increasing tension, and increasing tension with a fixed torsion load. The fixed torsion load used in the fourth yield point was the algebraic average of the positive and negative torsion yields - this is the center of the yield curve as drawn. Finally, the softened specimen was loaded in torsion to the limit of the testing machine to determine the softened monotonic

behavior.

The design of the testing machine introduced a complication into the tests. To maintain a firm alignment between the specimen and the load cell a constant tension load of 3500 psi was applied during the torsion tests. This tension load caused tensile deformation during cyclic torsion tests. The effect of the tensile deformation on the yield curves in Fig. 3 and Fig. 4 appears in the shift of the center line of the yield surface from the point $\sigma = 0$ to $\sigma = 3500$ psi.

The softened yield curves show the cyclic softening or reduction of the elastic range in torsion. The elastic range in the tension direction, Fig. 3 and Fig. 4 , changed very much less under the cyclic torsion. Both the position and size of the elastic domain in $\sigma - \tau$ space are determined by the strain history. Also the elastic domain appears to be smaller at the extreme of the cycle than at the midpoint. There is some reversible softening within each cycle as well as the cyclic softening from one cycle to the next. The two softened yield curves show a great deal of kinematic hardening or translation of the yield surface [4]. Simple kinematic hardening is a reasonable description of the steady cyclic behavior which exists after the transient cyclic softening has ended.

Fig. 5 shows the initial monotonic torsion stress-strain curve, the softened monotonic torsion stress-strain curve, and the 'cyclic stress-strain curve' [5]. The cyclic stress-strain curve is the locus of the tips

of the hysteresis loops for cycles of increasing or decreasing range. This cyclic curve can be approximated from any one hysteresis loop. The cyclic stress-strain curve shown in Fig. 5 was found from the hysteresis of the twentieth cycle. As expected, the cyclic curve, for this softening material, lies below the monotonic stress-strain curve.

Note that the initial and softened monotonic curves do not appear to converge within the strain range permitted by the testing machine. Thus, cyclic softening can be regarded as a permanent structural change in 1100-H14 aluminum. Also the initial monotonic and cyclic stress-strain curves do not converge; therefore, this material softens for any range of cyclic loading.

The results of these tests indicate that softening effects are mainly in the direction of the cyclic torsion loading. While the tests are too limited in scope to generalize the results with confidence, the behavior observed in irradiated copper [3] does lend support to the picture presented here.

Acknowledgement

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Figure Captions

Fig. 1.   The hysteresis loop of the twenty-first cycle of 1100-H14
aluminum. The yield curves were found at positions No. 1 and 2.

Fig. 2.   Initial yield curve of 1100-H14 aluminum. The curve is the von
Mises yield ellipse ($\sigma^2 + 3\tau^2 = 3k^2$) with k = 8130 psi.

Fig. 3.   Softened yield curve for 1100-H14 aluminum in position #1. The
dotted line is the von Mises yield ellipse for the initial
yield curve.

Fig. 4.   Softened yield curve for 1100-H14 aluminum in position #2. The
dotted line is the von Mises yield ellipse for the initial
yield curve.

Fig. 5.   Torsion stress-strain curves for 1100-H14 aluminum: (a)-initial
(hard) Monotonic curve, (b)-cyclic curve, (c)-softened monotonic curve.

FIG. I THE HYSTERESIS LOOP OF THE TWENTY-
FIRST CYCLE OF 1100-H14 ALUMINUM. THE
YIELD CURVES WERE FOUND AT POSITIONS
NO. I AND 2.

FIG. 2 INITIAL YIELD CURVE OF 1100-H14 ALUMINUM.
THE CURVE IS THE VON MISES YIELD ELLIPSE
$(\sigma^2 + 3\tau^2 = 3k^2)$ WITH $k = 8130$ psi.

FIG. 3 SOFTENED YIELD CURVE FOR 1100-H14
ALUMINUM IN POSITION #1. THE DOTTED LINE
IS THE VON MISES YIELD ELLIPSE FOR THE
INITIAL YIELD CURVE.

FIG. 4 SOFTENED YIELD CURVE FOR 1100-H14
ALUMINUM IN POSITION # 2. THE DOTTED LINE
IS THE VON MISES YIELD ELLIPSE FOR THE
INITIAL YIELD CURVE.

FIG. 5 TORSION STRESS-STRAIN CURVES FOR 1100-H14 ALUMINUM: (a)-INITIAL (HARD) MONOTONIC CURVE, (b)-CYCLIC CURVE, (c)-SOFTENED MONOTONIC CURVE.
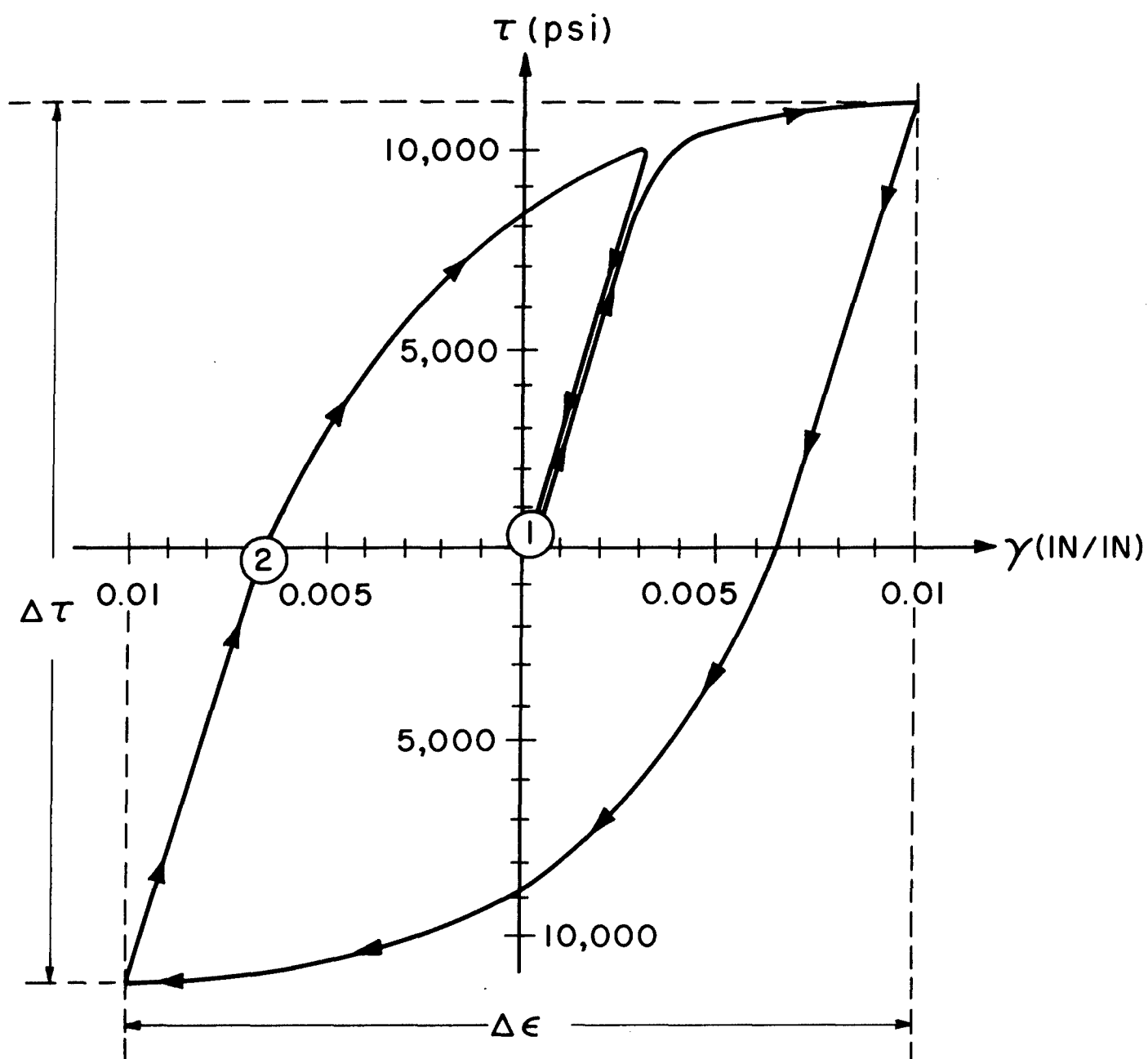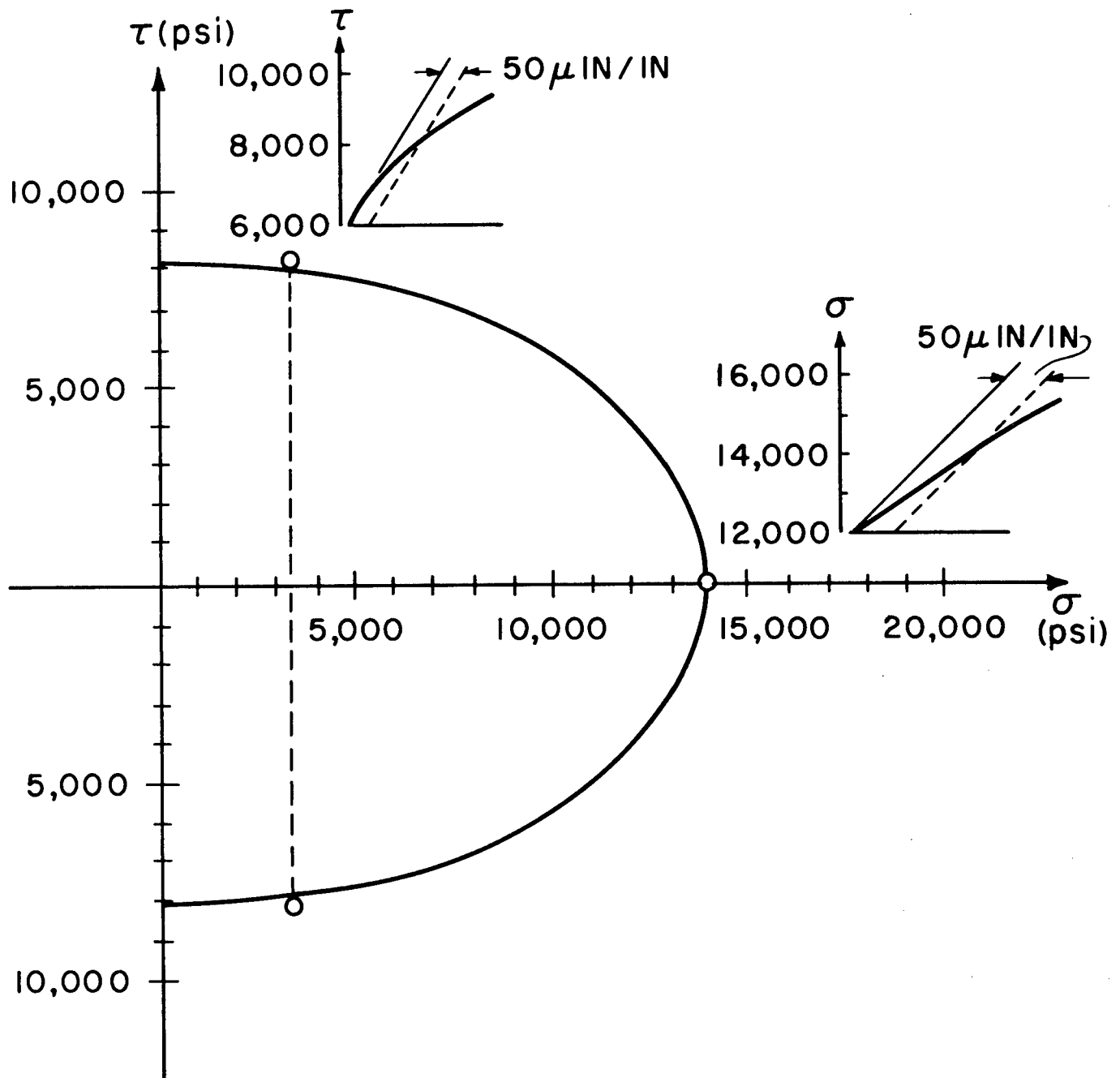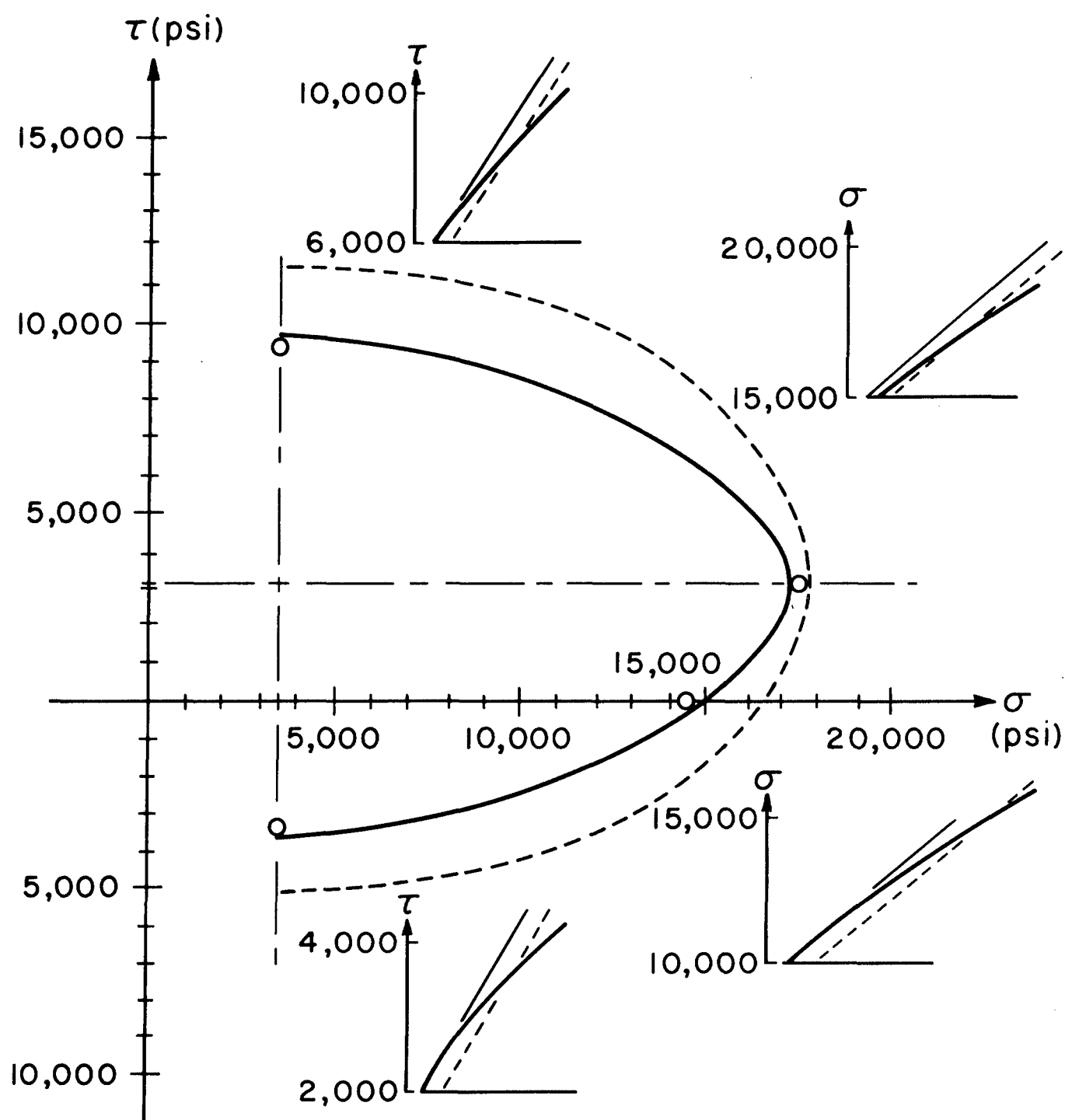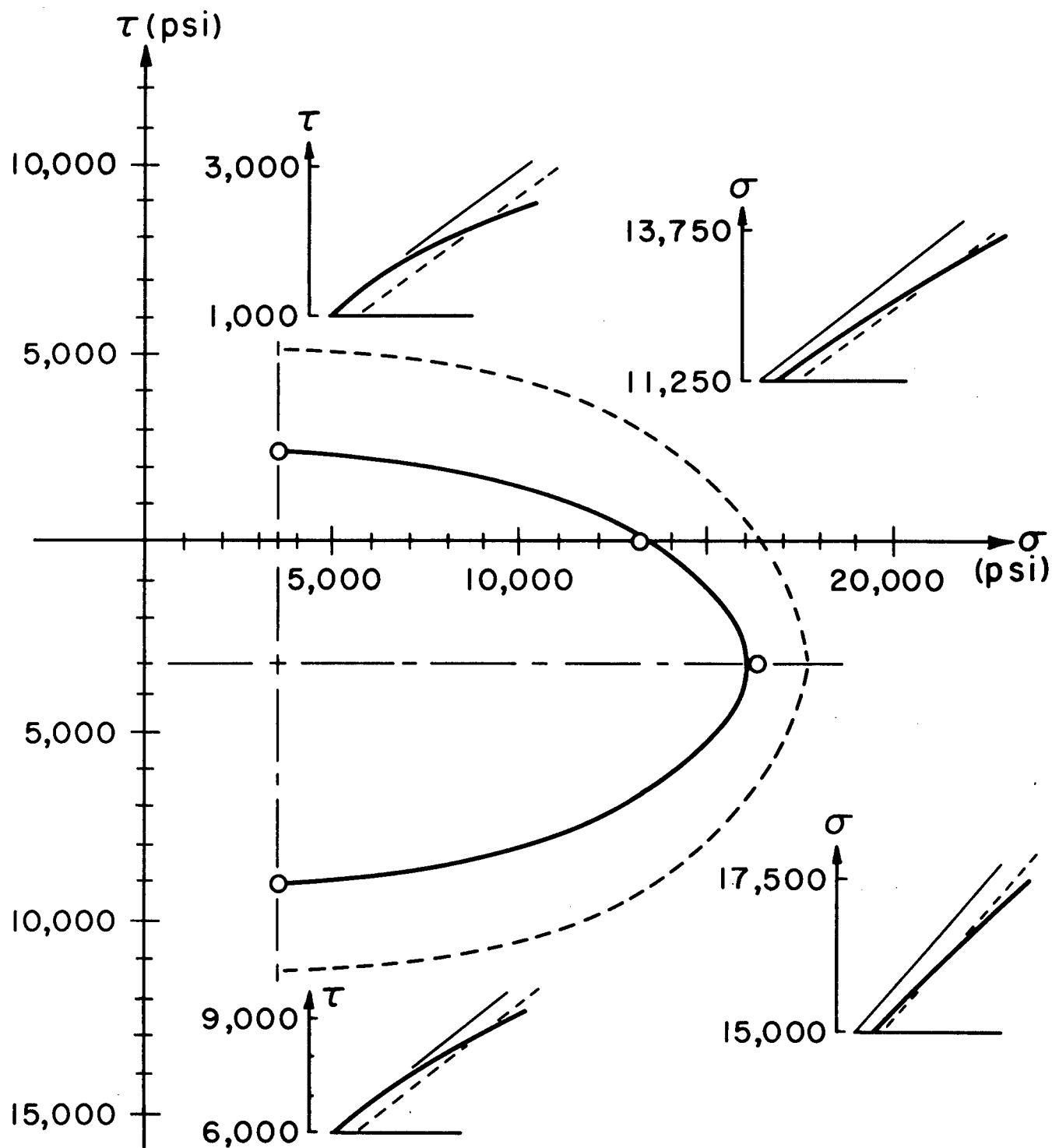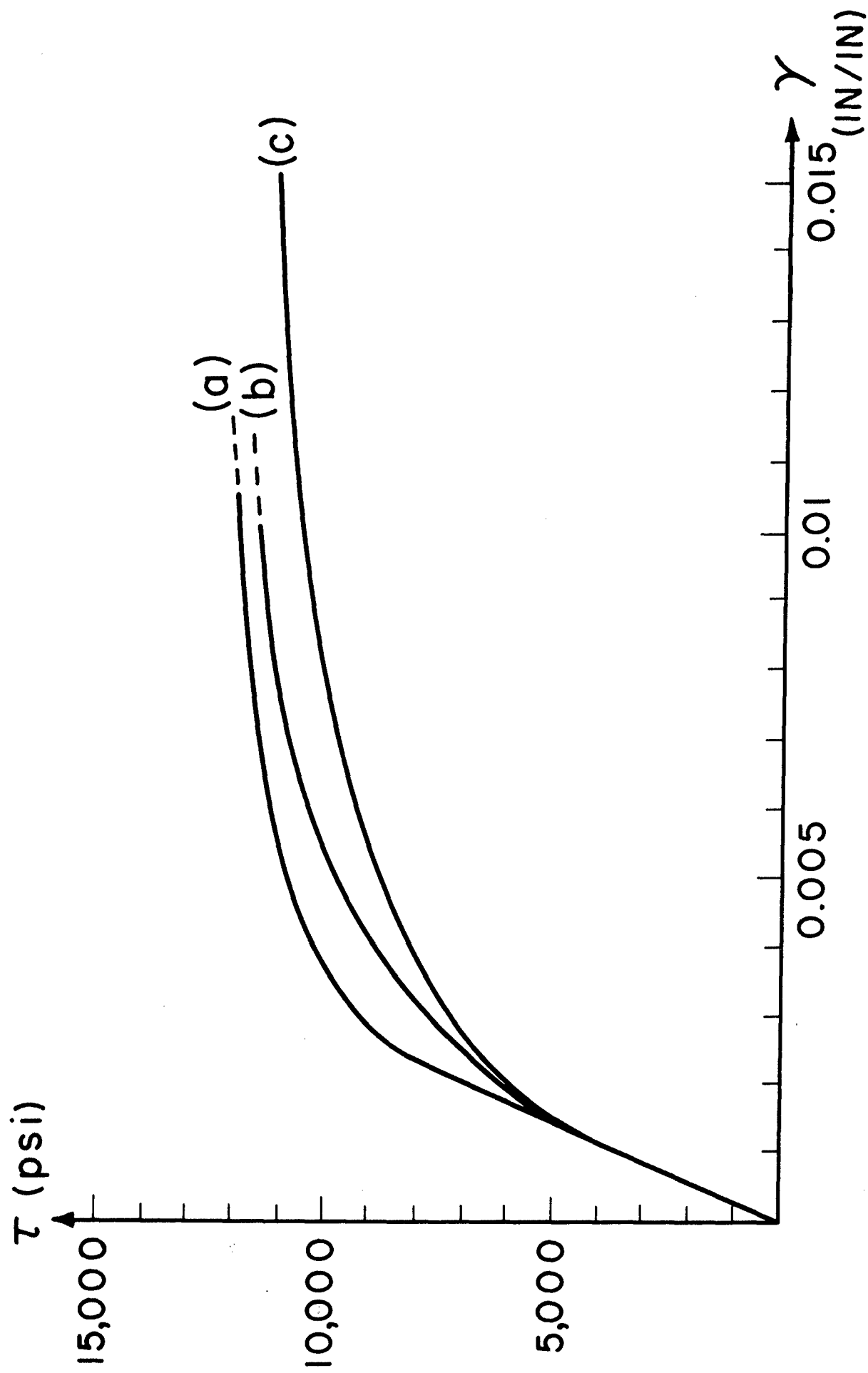
CYCLIC WORK SOFTENING

I. Definition

Cyclic hardening and softening can be described by looking at the ratio $\Delta\sigma/\Delta\varepsilon$, where $\Delta\sigma$ is the stress range of the hysteresis loop and $\Delta\varepsilon$ is the strain range. These variables are shown in Fig. A-1. Cyclic softening is a decrease in $\Delta\sigma/\Delta\varepsilon$ from one cycle to the next cycle; cyclic hardening is an increase in $\Delta\sigma/\Delta\varepsilon$. During the steady cyclic state, $\Delta\sigma/\Delta\varepsilon$ is constant. If the loading is confined to the elastic range, then $\Delta\sigma/\Delta\varepsilon$ is Young's modulus, the hysteresis loop degenerates to a straight line, and no changes can occur. Experimental results [A-1] [A-2], in which the hysteresis loops of softening materials are shown, indicate that there are two changes occurring during cyclic hardening and softening. The first is a small change in the rate of strain hardening within each cycle. The second and dominant change is a change in the offset yield point as defined by an offset which is small compared to $\Delta\varepsilon$.

Most specimens show an apparent softening at the end of a fatigue test. This softening is caused by loss of cross-sectional area due to fatigue crack growth or by severe geometrical changes in the specimen; for example, axial extension of beams in reversed cantilever bending [A-3]. Cyclic softening, however, is a change in the properties of the material rather than in the geometry of the specimen, and cyclic softening occurs at the beginning of the test, not at the end.

II. Conditions Needed for Softening to Occur

In general, cyclic softening can be expected in hard materials under low-cycle fatigue conditions. Low-cycle fatigue is usually defined as fatigue at loading cycles large enough to cause failure in 10^4 or fewer cycles; it is 'low number of cycles' fatigue. The smaller cycles needed for longer fatigue lives will still have some plastic strain in each cycle, but usually do not produce cyclic softening. Most experimenters listed in Table A-I report that the rate of softening (change per cycle) is proportional to the size of the cycles. The smaller the mechanical hysteresis loop, the slower the softening. The softening rate can be zero while a substantial hysteresis loop remains.

The material must be in a hardened state initially for softening to occur; a state produced either by cold working or heat treatment. If the hardening process (e.g., quenching) induces severe residual stresses on the macroscale, then softening may be considered as the partial removal of these residual stresses [A-4]. A sample of materials which have been observed to soften, their initial conditions, and the size of the cycles is shown in Table A-I.

A material can have a range of hardness from soft (fully annealed) to hard (severely quenched or cold worked). Softening is likely to occur if the material is near the upper end of the hardness range and cyclic hardening is likely to occur if the material is near the lower end. Initial hardness near the middle of the range can result in either softening, hardening, or no change. Manson [A-5] has a method of predicting whether a

material will cyclically harden or soften. He bases his prediction on two
equations for the fatigue life of a material. If the fatigue life, N_f ,
is used as a parameter, the equations describe the steady cyclic state.
This steady cyclic state stress-strain behavior can be compared with the
initial stress-strain behavior of the material to predict the type of
transient response to cyclic loadings. A description of this method is
given in Section III.

Other experiments have shown several restrictions or limits on the
conditions in which softening may occur. Coffin and Tavernelli [A-6]
have shown that softening occurs, for materials hardened by prestraining,
only if the strain range in a cyclic test is less than the amount of the
prestrain. If the cyclic strain range, $\Delta\varepsilon$, is larger than the prestrain,
the work hardening within each cycle is the dominant process, and the
material will harden. If the cyclic strain range and the prestrain are
nearly equal, the material will show no net change in $\Delta\sigma/\Delta\varepsilon$. However,
the cyclic softening between cycles and the monotonic strain hardening
within each cycle do cause some fluctuation in $\Delta\sigma/\Delta\varepsilon$ at the beginning of
the test. An example of Coffin and Tavernelli's work is given in Fig. A-2.
Other experiments [A-7] show both softening and hardening, one after the
other, for some cyclic test ranges. Also, there is evidence [A-7] that
heat treated low carbon steel will soften only if it is sufficiently hard-
ened that the yield point is above the fatigue limit. This relation
between the monotonic yield characteristics and the fatigue characteristics
should be considered as, at most, a sufficient, and not a necessary
condition for cyclic softening.

III. The Steady Cyclic State

Cyclic hardening and softening are transient behaviors and usually disappear early in a fatigue test leaving the material in a steady cyclic state. This state is characterized by a mechanical hysteresis loop which is identical from one cycle to the next. This steady cyclic state is important because the fatigue life is more dependent on the size of the steady hysteresis loop than on the transient hysteresis loop. If the cycles are very large, failure by buckling or by ductile fracture may occur before the steady state is reached [A-2]. Buckling is more likely to occur in softening materials because of the loss of strength with each cycle. Cyclic softening usually takes a slightly larger number of cycles to die out than cyclic hardening does. This difference can be taken as the result of monotonic strain hardening within each cycle. The strain hardening helps the cyclic work hardening but negates some of the effect of the cyclic softening from one cycle to the next.

The steady cyclic state is generally described by one of two curves: the steady hysteresis loop or the 'cyclic stress-strain curve'. The shape of the cyclic stress-strain curve is found to be closely similar to the shape of the steady hysteresis loop [A-2]. Therefore, any one steady hysteresis loop can be used to generate the cyclic stress-strain curve (for loops smaller than the initial one) by putting the coordinate origin at one tip of the loop and plotting the hysteresis loop with $\Delta\sigma$ and $\Delta\varepsilon$ replaced by $\Delta\sigma/2$ and $\Delta\varepsilon/2$. Figure A-3 shows this relation between the cyclic stress-strain and the hysteresis curves.

The cyclic stress-strain curve describes the material in the steady cyclic state just as the monotonic stress-strain curve describes the material in the initial state. If both the cyclic and monotonic curves are known, the transient behavior of the material can be determined by comparing the two curves. The relation between the two curves is shown in Fig. A-4. If, at any given strain, ε' , the cyclic curve lies below (at a lower stress) the monotonic curve, then the material will soften under controlled strain cyclic loading with $\Delta\varepsilon = 2\varepsilon'$. If the cyclic curve lies above the monotonic curve, the material will harden cyclically. Similarly, if at a fixed stress, σ' , the cyclic stress-strain curve shows more strain than the monotonic curve, the material will soften under controlled stress cyclic loading with $\Delta\sigma = 2\sigma'$. If, at σ' , the cyclic curve shows less strain than the monotonic curve, the material will harden for controlled stress cycles.

Manson [A-5] states that the fatigue life, N_f, of a material is given by the two equations:

$$\Delta\varepsilon' = 3.5 \ (\sigma_u/E)N_f^{-0.12} + D^{0.6} \ N_f^{-0.6} \tag{1}$$

$$\Delta\sigma' = 3.5 \ \sigma_u N_f^{-0.12} \tag{2}$$

where   d = monotonic ductility

σ_u = ultimate tensile strength

For the steady hysteresis loop, $\Delta\sigma' = \frac{1}{2} \Delta\sigma$ and $\Delta\varepsilon' = \frac{1}{2} \Delta\varepsilon$.

For any given N_f, plotting $\Delta\sigma'$ vs. $\Delta\varepsilon'$ should yield one point on

the cyclic stress-strain curve. Therefore, the transient cyclic behavior can be determined by one monotonic test to fracture. The test gives the monotonic stress-strain curve and gives D and σ_u for the material. A cyclic curve is generated by plotting $\Delta\sigma'$ vs. $\Delta\epsilon'$ for differing values of N_f. A comparison of this derived cyclic curve and the monotonic curve, used to find D and σ_u, determines what type of transient cyclic behavior the material should exhibit. Manson's work includes an experimental study of the transient behavior of 24 different materials including 6 materials at 2 different initial hardnesses. His method predicts the correct transient behavior in 90% of the tests.

Another equation for the cyclic stress-strain curve has been offered by Morrow [A-2]. This equation, $\Delta\epsilon' = (\Delta\sigma'/E) + D'(\Delta\sigma'/\sigma_f')^{1/n'}$ does not involve the fatigue life directly, but the constants D' and σ_f' are related to the ductility and strength of the material under cyclic loading. Morrow suggests using the monotonic true fracture ductility and true fracture strength as approximations to D' and σ_f'. The exponent, n', is an experimentally determined cyclic hardening coefficient which is the slope of the log-log plot of the steady stress range, $\Delta\sigma$, vs. the steady strain range, $\Delta\epsilon$. For all the materials Morrow tested, n' is approximately 0.15.

The dependence of the cyclic stress-strain curve on the initial condition of the material is still not fully understood. There is evidence that some materials have a unique cyclic stress-strain curve which is independent of the initial condition and depends only on temperature

of testing. For other materials, the cyclic stress-strain cycle, as
experimentally determined, seems to be dependent on the initial condition
of the material.

IV. Theories of Microscopic Behavior under Cyclic Loading

Several possible explanations have been proposed to explain cyclic
softening. Most of the theories depend on the type of hardening used and
are not valid for all types of hardening.

Wood [A-4] proposed that softening of cold worked metals could be
attributed to changes in the dislocation arrays near one dimensional
defects. During monotonic straining, dislocations move until they are
stopped by grain boundaries or point defects (such as impurity or alloy-
ing atoms). Severe monotonic straining produces pile up of dislocations
at these barriers and the dislocations interact to produce stress fields
which resist the movement of additional dislocations toward the pile up.
As the stress fields intensify, the applied stress needed to start dislo-
cation motion increases and the metal work hardens. Under cyclic loadings,
dislocations move back and forth and can 'jump' from one plane of atoms
to another. These jumps enable some dislocations to bypass point defects
which block dislocation motion on only one plane. As the number of
piled up dislocations decreases with each cycle, the stress field inten-
sity decreases, and movement of dislocations during the next cycle can
start at a lower stress--the material cyclically softens. The softening
never reaches the annealed state because the bypass mechanism does not

work on dislocation pile ups at grain boundaries. The observation that slip bonds, or bands in which dislocations move, widen during cyclic tests supports Wood's theory.

This mechanism fails for quenched materials which have very small grains. As the grain size decreases, the total area of grain boundaries increases; therefore, for materials with small grains, a large percentage of the dislocations can be expected to pile up at grain boundaries. The number of pile ups which can be removed by cyclic loading should be small in materials with small grains, and very little softening due to Wood's mechanism should be expected. But experiments by Klesnil et al. [A-7] on low carbon steels show just the opposite results. The quenched steel with small mean grain size (d = 0.011 mm) softened while the steel with larger grain size (d = 0.073 mm) hardened under cyclic loading.

Feltner and Laird [A-8] have studied the dislocation arrangements in different materials before and after cyclic loadings. They divide materials into two classes: 'wavy glide' materials and 'planar glide' materials. Wavy glide materials have high stacking fault energy, little resistance to dislocation movement, and a great deal of cross-slip. Planar glide materials have low stacking fault energy, high resistance to dislocation movement, and dislocation motion or slip confined to individual planes.

The dislocation patterns after cycling of wavy glide materials show no dependence on the initial patterns, but are determined only by the range of the cyclic strains. Also, a controlled change, during a cyclic

test, in the stress range or strain range shifts the hysteresis loop to
a new position on the same cyclic stress-strain curve. The cyclic stress-
strain curve is a function only of temperature and is independent of
initial conditions.

Planar glide materials show a dislocation pattern which is influenced
both by the size of the cyclic loading and by the initial condition of the
material. Also, a change in the size of the testing cycle, for planar
glide materials, causes an irreversible change in the hysteresis loop.
The high resistance to dislocation motion prevents the material from
completely removing the dislocation arrangement produced by previous
strain history. The steady cyclic stress-strain curve is a function of
the initial condition of the material as well as the size of the cyclic
loading. This difference between the behaviors of the two types of
materials, for a change in $\Delta\varepsilon$, is illustrated by Fig. A-5.

The question of dependence of the cyclic stress-strain curve on the
initial condition does not affect the validity of Manson's method of
prediction. His formula for the cyclic stress-strain curve is for tests
at one size cycle and uses constants, D and σ_u , which are determined
by the initial conditions of the material. The experimental support for
Manson's method includes correct predictions for both types of materials:
silver with high stacking fault energy and wavy glide, and 310 stainless
steel, an austenite steel with low stacking fault energy and planar glide.

In general, monotonic straining increases the number of dislocations
and causes them to pile up at impurities and grain boundaries. Cyclic

straining pushes dislocations back and forth causing some dislocations to bypass obstructions or to annihilate each other. The reduction in the pile ups or in the number of dislocations reduces the stress fields within the metal and permits plastic flow at a lower applied stress. For annealed materials with fewer initial dislocations, the monotonic straining within each cycle dominates the process, and more dislocations are produced and piled up than are destroyed or freed. Thus, annealed materials will cyclically harden. The steady cyclic state contains an equilibrium number of dislocations--a number at which the loss of dis-locations with each load reversal matches the multiplication of disloca-tions during the monotonic part of each cycle.

A very different mechanism is believed to produce cyclic softening in dispersion hardened materials like aluminum. Laird and Thomas [A-9] report observing precipitate free bands in materials which have been loaded cyclically. The reversals of the loading break up or re-dissolve the precipitates which were initially in these bands and caused the initial hardness. No explanation is available as to why cyclic stresses have this destructive effect. It is not even settled whether the cyclic loading alone causes the loss of precipitates or whether the cyclic loading helps a natural softening process. Hanstock [A-10] also has seen unstable solutes in aluminum alloys under fatigue loading.

The precipitate free bands, like Lüders bands, yield at a lower stress than the rest of the specimen and show more deformation. Soften-ing occurs as the load reversals produce more and more soft bands.

This loss of precipitates is an irreversible process under mechanical loading. Increasing the cycle size or monotonic straining will not reproduce the precipitates. Only heat treating the material will cause the precipitates to return. Therefore, cyclic softening is a permanent structural change in dispersion hardened materials. Cycling or monotonic loading of soft aluminum will produce hardening by the normal increase in the number of dislocations and dislocation pile ups. Again, the Manson method of predicting cyclic behavior holds for the two different mechanisms in aluminum. Manson's experimental evidence includes predictions of correct behavior for Aluminum 2014-T6.

V. Models of Cyclic Softening

A model of a cyclic softening material must have the two opposing properties of strain hardening within each cycle and a yield point which decreases with the number of cycles performed. Cyclic hardening models need only the strain hardening mechanism since the strain hardening during one cycle will increase the yield point of the next cycle.

Another difficulty is that the first cycle is different from all the others. The first loading starts at the origin of the stress-strain space, the center of the hysteresis loop. The first 1/2 cycle involves only one half of the strain involved in all the other half cycles. Also many materials show a Bauschinger effect during the first reversal of load. The Bauschinger effect remains throughout the rest of the cycles and cannot produce softening. Softening will occur, however, if the

magnitude of the Bauschinger effect changes with each cycle. There are models which show the Bauschinger effect, but they do not show cyclic softening because the Bauschinger effect is constant throughout the test [A-11]. A kinematic hardening material [A-12] shows a Bauschinger effect, and Polakowski [A-13] uses a spring, dashpot, and friction model, shown in Fig. A-6, which shows a Bauschinger softening effect but not cyclic softening. These models do, however, show the behavior in the steady cyclic state.

There are several mechanisms which cannot be used in a valid cyclic softening model. A model based on an unstable (strain softening under monotonic loading) material will show cyclic softening [A-14]. However, this model does not have the strain hardening characteristic within each cycle which real cyclic softening materials do have.

Also, there is no evidence that cyclic softening can be attributed to temperature effects. Coffin [A-15] has observed softening in quenched steels tested between room temperature and 225°C. While the amount of softening is influenced by the testing temperature, just as is the mono-tonic behavior, the presence of softening was observed up to the embrit-tlement temperature. Cyclic softening can be expected within the same temperature range in which cold working occurs. Nor can cyclic softening be attributed to temperature changes in the specimen due to the cyclic loadings. Most tests are conducted at low enough rates that the specimen remains at the initial temperature. Very rapid loading rates can produce small temperature rises in the specimen. But the effect of this heating on the monotonic behavior in each cycle is far too small to explain the

5-20% change in the yield point which cyclic softening produces. Also, cyclic softening decreases with the increase in the number of cycles, while temperature changes and their effects increase.

A model based on property changes caused by time or strain-rate effects will not describe cyclic softening. The rate insensitivity is shown by Table II. This gives typical strain rates for tests in which softening of steel specimens was observed. The range is from 1 cycle/min (0.024 in/in/min) to 2800 cycles/min (28 in/in/min).

A model which is elastic-perfectly plastic or elastic-strain hardening will describe softening if the yield stress is a decreasing function of N, the number of cycles of loading performed. This model is not very useful since the function of N would still have to be determined. At present there is not even a known simple empirical formula which could be used. Attempts to derive a general empirical formula from experimental results have not been successful. All that is known is the general trend that the initial rate of softening (change per cycle) increases with the initial hardness and increases with the size of the cycles.

VI. Effects of Cyclic Softening on Mechanical Properties

It is known that cyclic softening produces a lowering of the yield point in the direction of cycling, but very little else is known about the effects of softening on other mechanical properties. Fatigue properties in general are dependent upon the steady cyclic state characteristics

of the material and not on the initial conditions. Therefore, the amount
of cyclic hardening or softening does not determine the fatigue life of a
specimen. Among the unanswered questions are: does softening influence
the monotonic ultimate strength, ductility, strain hardening rate? is
the shape of the yield surface of the material changed by softening?
is the size of the yield surface changed?

Polakowski and Palchoudhuri [A-16] softened materials cyclically
and then compared the post-softening and pre-softening monotonic tensile
stress-strain curves. The monotonic curves were in compression and were
not carried to failure, so the influence, if any, of softening on fracture
properties cannot be determined directly. The pre-softening and post-
softening curves seem to converge for strains much larger than the strains
used in the cyclic tests. This convergence indicates that the softening
effects are destroyed by monotonic strain and do not represent permanent
changes in the material. The materials used in these tests are nickel,
copper, aluminum, and alloys of the three. All these materials have
high stacking fault energies. These results agree with the theory that
high stacking fault energy or wavy glide materials have dislocation
patterns which are nearly independent of the strain history.

VII. Test Results

The tests on 1100-H14 aluminum have already been described and the
results are summarized here.

1. 1100-H14 does soften under cyclic torsion loading. The rate of

softening is much higher at $\Delta\varepsilon = 0.02$ than at $\Delta\varepsilon = 0.01$.

2. The softening produces much less shrinkage of the elastic domain in the tension direction than in the torsion.

3. There is some reversible hardening which occurs during each cycle.

4. There is kinematic hardening of the elastic domain.

5. A constant tension load during the torsion cycling appears to shift the elastic domain in the direction of increasing tension.

The kinematic hardening explains the largest part of each change in the yield curve. For these tests both the steady cyclic state and the effects of the tensile deformation can be explained by the kinematic hardening model. The effects of the tensile deformation on the tension yield point are not completely understood. Wood and Reimann [A-17] have studied this problem of time-independent creep under cyclic torsion and steady tension. They report that the tensile deformation should increase the tension yield point as much as the same amount of plastic strain in pure tension. Their results are not directly transferable because they used a cyclic hardening material and much larger strains. If Wood and Riemann's results were applied directly, the yield point in the horizontal or tension direction after cycling would be expected to be about 22,000 psi instead of the 17,000 psi maximum actually found.

VIII. Summary

Cyclic work softening is a transient behavior which occurs at the beginning of fatigue tests and reduces the mechanical strength of the material. Softening is likely to occur in hard materials under low cycle fatigue conditions. A method is described by which cyclic softening or hardening can be predicted from the monotonic properties of the material. The mechanisms which produce softening are not completely understood but appear to vary depending on the mechanisms which produce the hardening of the material. Since cyclic softening is an unstable behavior, it cannot be described by any 'stable material' models. Tests on 1100-H14 aluminum indicate that the material softens under cyclic torsion and that the softening causes only a very small reduction in the tensile strength.

IX. Concluding Remarks

Cyclic work softening is usually described as an interesting side effect in fatigue tests and still remains generally un-understood and unstudied. It is known how to predict the occurrence of cyclic softening but not how to predict the amount of softening accurately. There are three other important questions which still need to be answered: 1. What are the mechanisms which produce cyclic softening and how can they be controlled? 2. What effect does cyclic softening have on the other properties of the material? 3. Are the effects of cyclic softening large enough to be included in design and analysis? For example, does the influence of

cyclic softening on the behavior of members with stress concentrations (holes, corners, notches) have to be taken into account?

The tests conducted so far show some interesting results but much more experiment and analysis is needed. Tests are needed to determine, for many types of materials, the effects of softening on the shape and size of the yield surface and on the monotonic behavior to fracture.

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A-22. P. P. Benham, "Some Observations of Cyclic Strain-Induced Creep in Mild Steel at Room Temperature," International Journal of Mechanical Science, vol. 7, 1965.

Table A-1

Materials Which Have Been Observed to Soften

Material	Test	Author
Aluminum, 1100 prestrained 9%	axial push-pull $\Delta\epsilon = 6\%$	Raymond & Coffin [A-18]
Bronze, Phosphor cold drawn 31.5%	axial push-pull $\Delta\sigma = 114,600$ psi	Polakowski & Palchoudhuri [A-16]
Copper, Extruded H.C. fully age hardened	axial push-pull $\Delta\epsilon = 3\%$	Dugdale [A-19]
Nickel cold drawn 20%	axial push-pull $\Delta\sigma = 87,400$ psi	Polakowski & Palchoudhuri [A-16]
Steel, SAE 1111 ice brine quenched from 675°C	axial push-pull $\Delta\epsilon = 2\%$	Coffin [A-15]
Steel, SAE 1016 cold worked	axial push-pull $\Delta\epsilon = 1.15\%$	Benham & Ford [A-20]

Table A-II

Strain-rates of Tests in Which Steel Cyclically Softened

Material	Strain-rate (in/in/min)	Author
mild steel, prestrained 19.9%	0.024	Yokobori, Yamanouchi, & Yamamoto [A-21]
SAE 1111, ice brine quenched from 675°C	0.15	Coffin [A-15]
SAE 1010, cold worked	1.6	Benham [A-22]
SAE 1010, quenched	28	Klesnil, Holzmann, Lukas, & Rys [A-7]

Figure Captions

Fig. A-1.   Typical mechanical hysteresis loop for 1100-H14 aluminum.

Fig. A-2.   Sample results on prestrained aluminum.
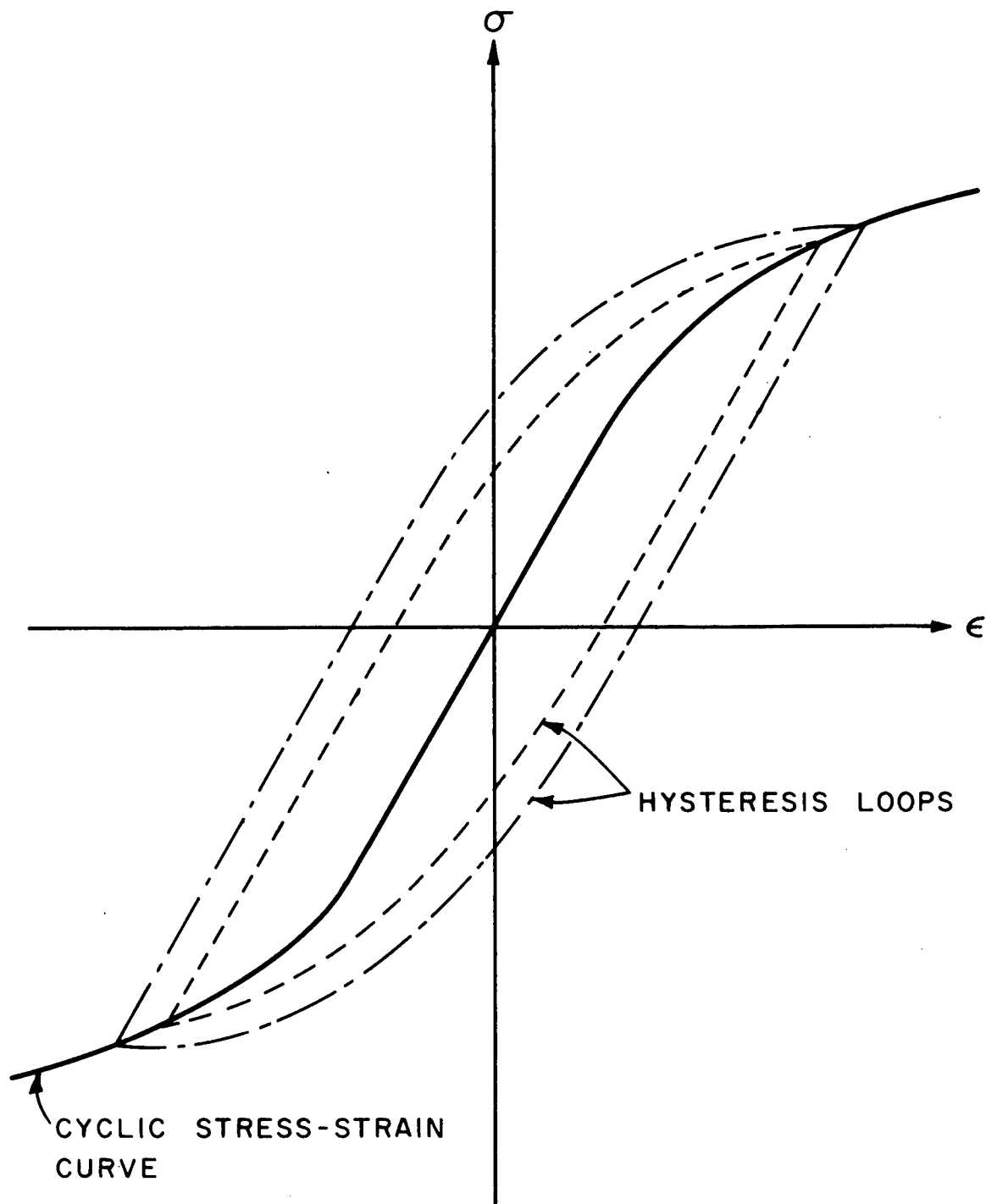
Fig. A-3.   Mechanical hysteresis loops and the cyclic stress-strain curve.
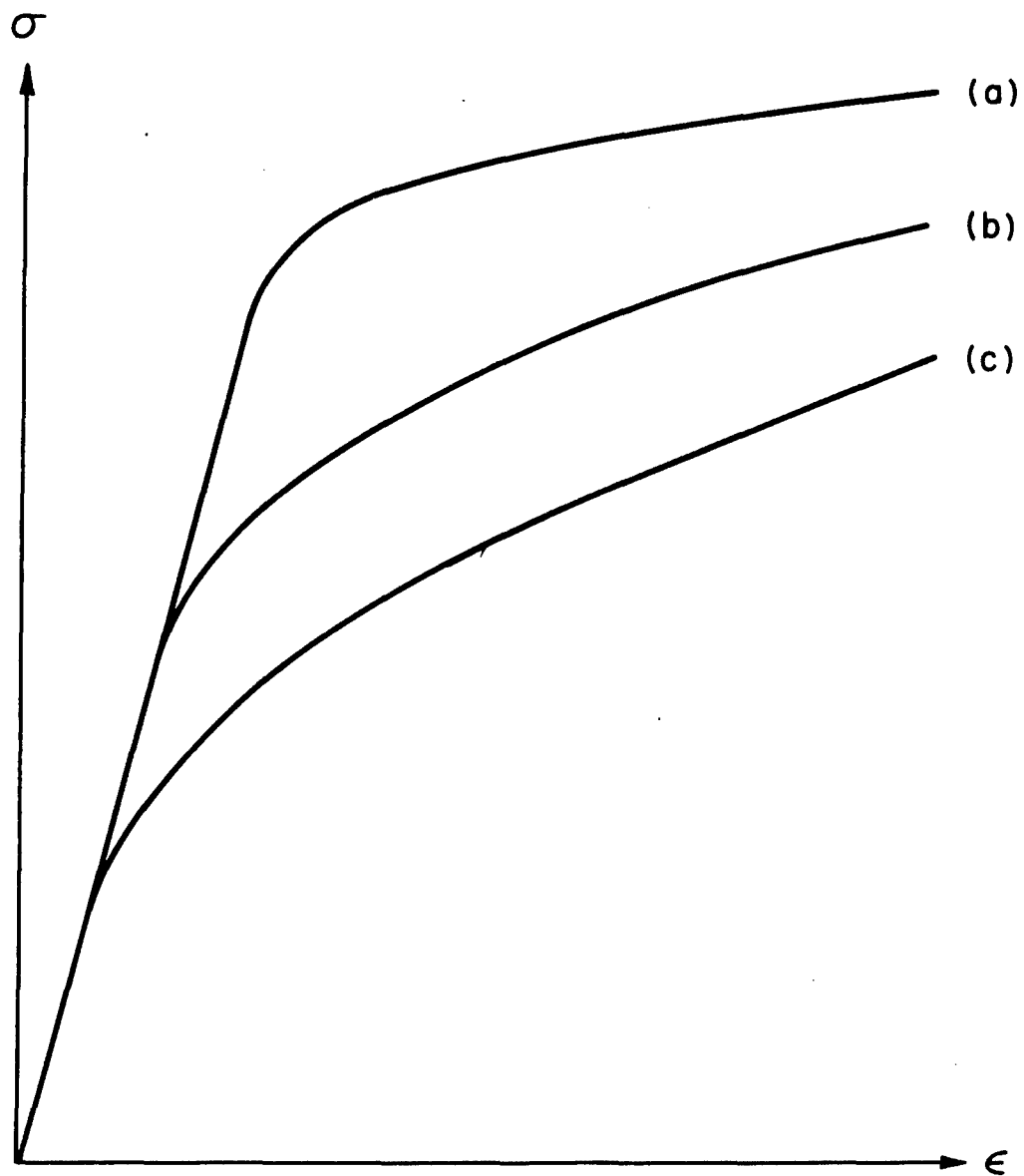
Fig. A-4.   Comparison of cyclic and monotonic stress-strain curves.

Fig. A-5.   The effects of changing the range of a cyclic test.

Fig. A-6.   Polakowski's model of a material with a Bauschinger effect and its mechanical properties.
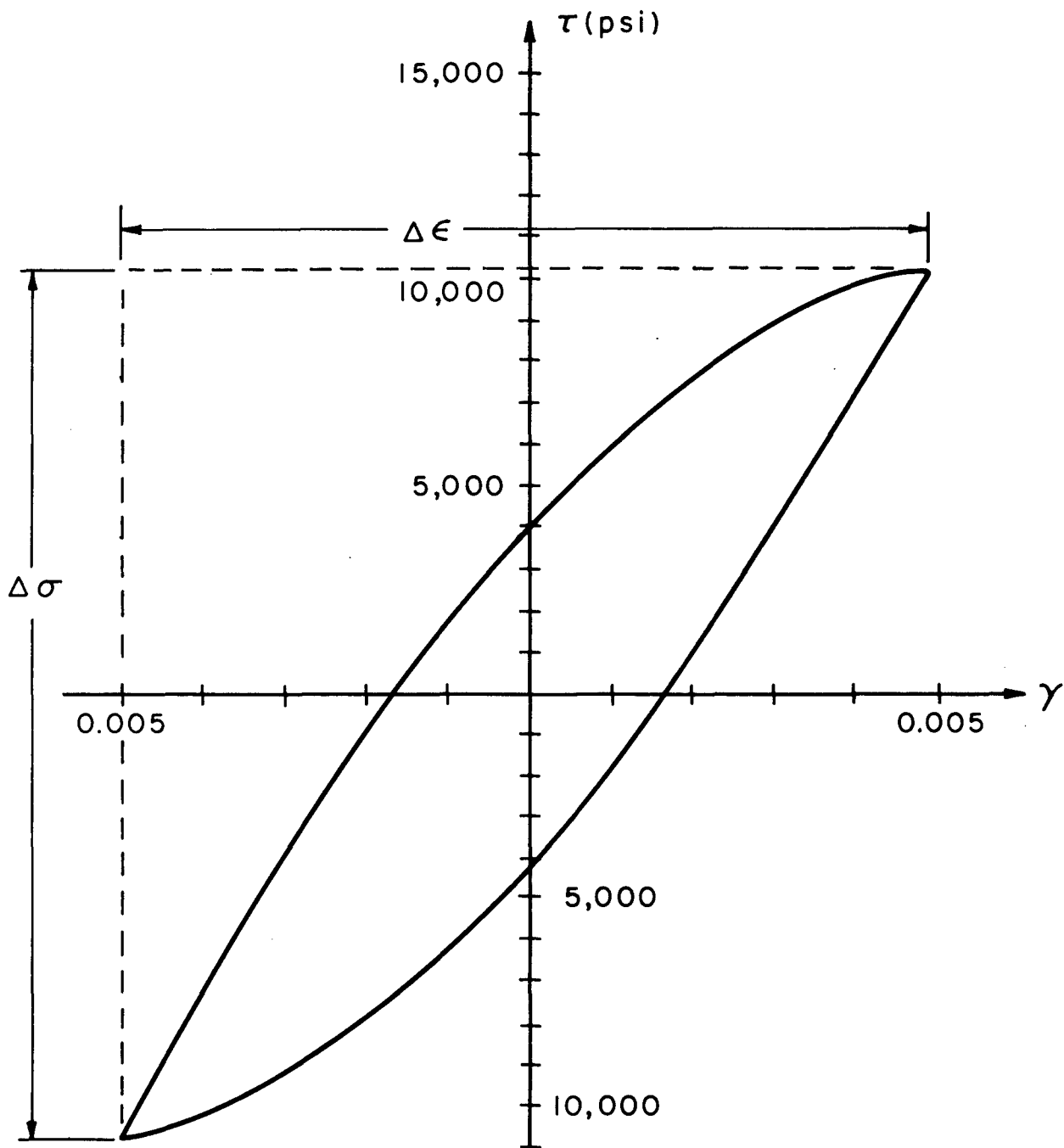
FIG. A-1 TYPICAL MECHANICAL HYSTERESIS
LOOP FOR 1100-H14 ALUMINUM
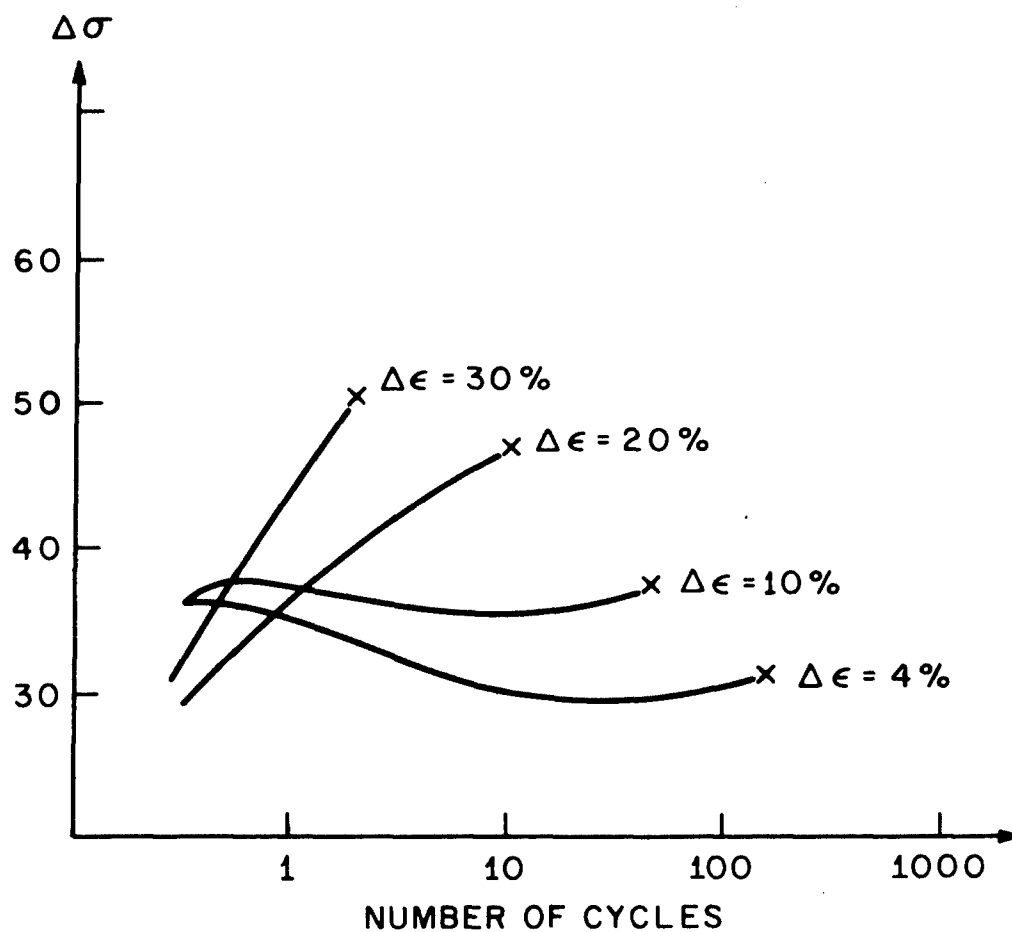
FIG. A-2 SAMPLE RESULTS ON PRESTRAINED
ALUMINUM. [A-6]

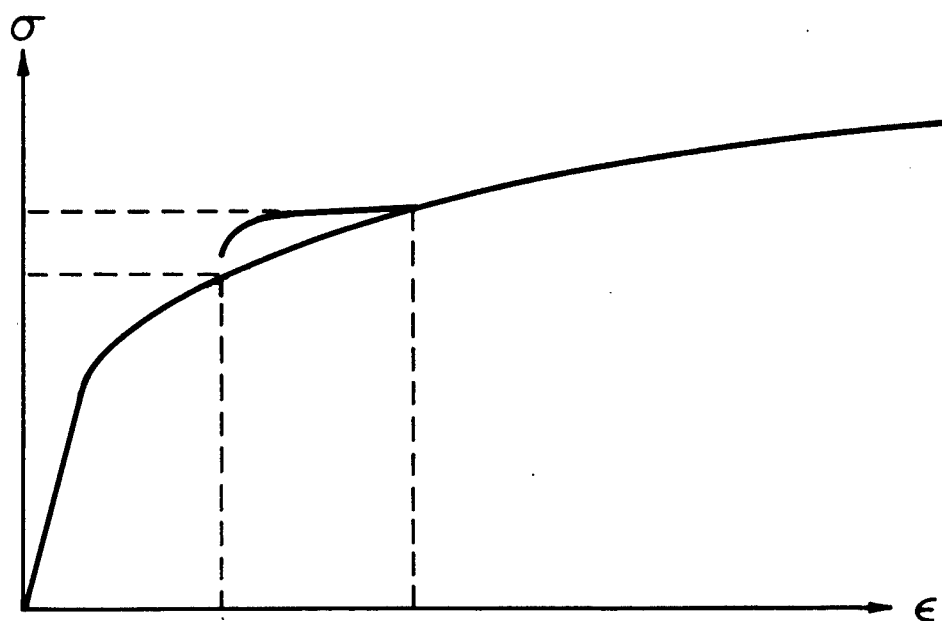FIG. A-3 MECHANICAL HYSTERESIS LOOPS AND
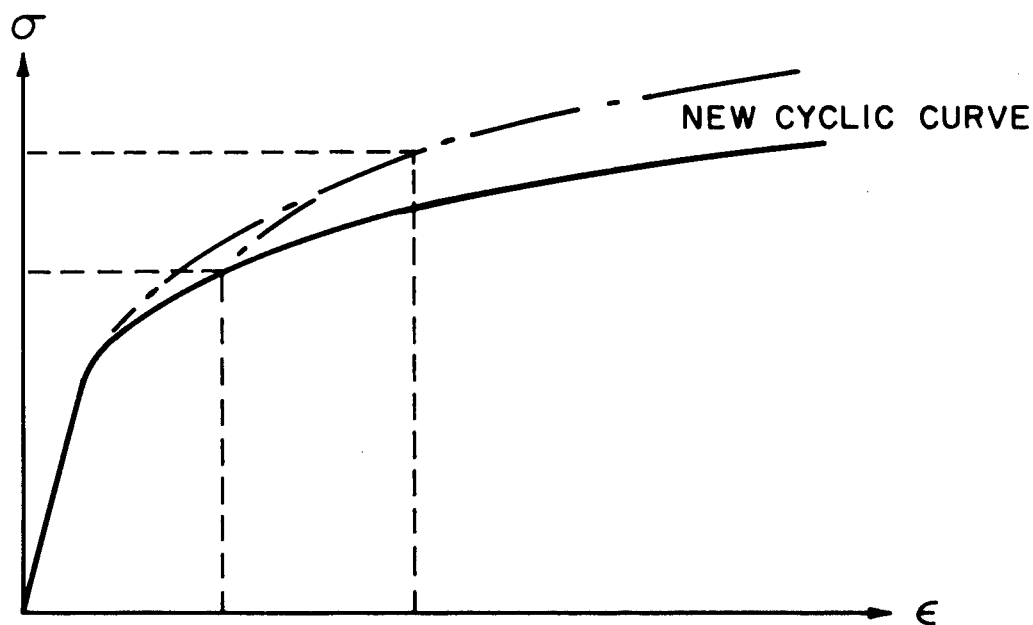THE CYCLIC STRESS-STRAIN CURVE

FIG. A-4 COMPARISON OF CYCLIC AND
MONOTONIC STRESS-STRAIN CURVES:-
(a)-Monotonic, hard (will soften),
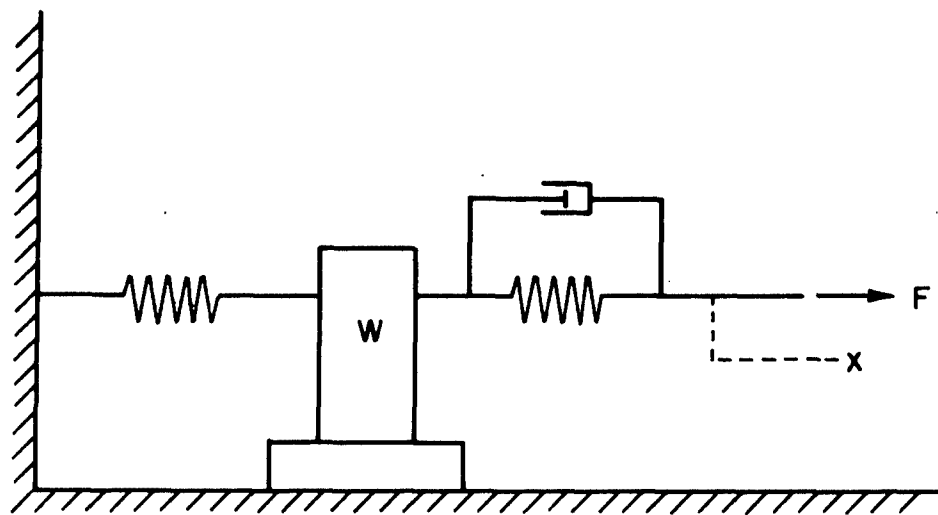(b)-Cyclic, (c) Monotonic, soft
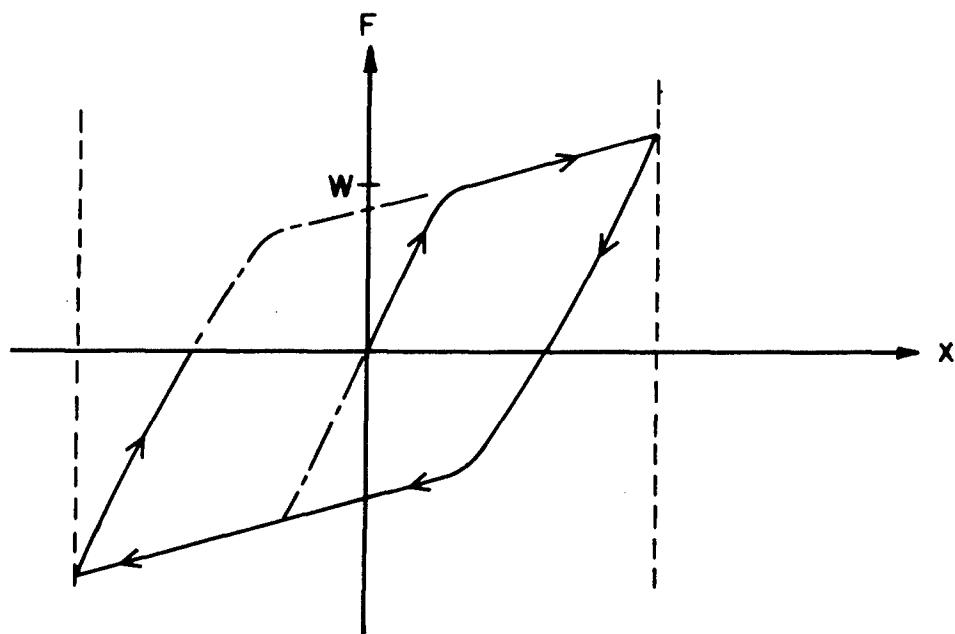(will harden).

WAVY GLIDE MATERIAL

PLANAR GLIDE MATERIAL

FIG. A-5 THE EFFECT OF CHANGING THE
RANGE OF A CYCLIC TEST.

MODEL

MECHANICAL PROPERTIES

FIG. A-6 POLAKOWSKI'S MODEL OF A
MATERIAL WITH A BAUSCHINGER
EFFECT AND ITS MECHANICAL
PROPERTIES